How does suppliers' market power affect a buyer's investment?

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March 1, 2024

Abstract

Supply chain tensions have become an issue often talked about. Our paper develops a set of stylized models studying the effect of an increase in suppliers' market power on a buyer's investment decisions. First, we study the case where the buyer regularly goes to the input market to source from a known set of oligopolistic suppliers and determine the buyer's optimal investment policy. We then study the conditions under which the buyer is better off committing at the time of investment on a production schedule signing with the set of suppliers a framework agreement regulating the input price. Finally, we endogenize the size of the pool of suppliers the buyer can source from by considering a supplier's decision to exit a market if the economic circumstances are not satisfactory. The degree of competition in the input market is a key driver of a buyer's investment decision. While supply contracts favor suppliers, it leads to a delayed investment by the buyer. Cost asymmetry among suppliers may lead the buyer to delay its investment as a tradeoff between waiting for clarity about the supplier base or securing better terms with a larger pool, which is less likely to sustain.

1 Introduction

Motivation. In today's globalized and interconnected business environment, supply chains serve as the backbone of countless industries, facilitating the flow of goods and services from raw materials to end consumers. The study of supply chains has garnered attention from scholars, practitioners, and policymakers alike due to its profound implications on business performance, resilience, and sustainability.

Since the 1960s, the narrative of international trade has changed significantly from a phase of liberalized expansion (from the 1980s to the early 2010s) to a period in which supply chain disruptions and political-economic tensions affect firm decisions. Recent crises (e.g., Brexit, China–US trade war, Covid, 2021 Suez canal obstruction, draught of the Panama canal) highlight numerous challenges and complexities, often resulting in tensions that reverberate throughout the entire supply chain. Following Choi and Krause (2006), characteristics such as the length of the supply chain, the tier structure, and the connections among the parties involved in the supply chain all affect a company's expenses, its exposure to risks in the supply chain, and ultimately its capacity to invest and innovate. Such tensions pose significant hurdles to the smooth operation of client-facing downstream firms and may jeopardize entire industries when critical inputs must be sourced from a limited pool of suppliers.

A case in point is the automotive industry, which must reconsider its operational approaches to address these tensions. Governmental policies to tackle climate change and changing customer preferences favor the emergence of electric vehicles (EVs) as an alternative to and at the expense of traditional internal combustion engines (ICEs) (e.g., ban of ICEs by 2035 in Europe). Such changes have facilitated the emergence of new players (e.g., Tesla, BYD Auto), prompting incumbent Original Equipment Manufacturers (OEMs) to react, sometimes dramatically (e.g., Volkswagen) (see, e.g., Chevalier-Roignant et al., 2019). A critical input for EVs are batteries. Experts expect the global battery manufacturing market to reach a tenfold of its 2020 value by 2035 (see `https://www.oliverwyman.com/our-expertise/insights/2023/jun/where-to-find-investment-opportunities-in-the-batt` `html`). EV batteries need to be produced in very large quantities for them to be cost effective ('gigafactories'). While some OEMs (e.g., Tesla, Volkswagen/PowerCo) have decided to vertically integrate battery production, others have decided to source from more independent gigafactories (sometimes sponsored by established OEMs) such as Northvolt (e.g., BMW, Volkswagen), Verkor (e.g., Renault), or Automotive Cells Company (e.g., Stellantis and Mercedes Benz). Governments often take action to secure local battery manufacturing (e.g., US Bipartisan Infrastructure Law, Verkor sponsored by France's development bank). The supply chain for battery manufacturing is infinitely intricate and divided, with a variety of participants in each tier. To date, most manufacturers of electrodes, separators, and electrolytes, are from Asia, as do miners and processors of raw materials, component makers and battery producers. Battery cell producers are confronted with the issue of limited access to raw materials and rising energy costs. The degree of competition among battery producers (partly explained by the high fixed costs to operate gigafactories and by the high concentration of their input markets) will affect the viability of car manufacturing by OEMs and likely impact their investment decisions.

Our paper explores various characteristics of supply chain tensions, leveraging a set of simple stylized models. These models address questions pertaining to (i) the effect of industry concentration in the input market on a buyer's investment decision, (ii) the opportunity for suppliers to offer supply contracts as a way to reduce uncertainty in the output market, and (iii) how the high fixed costs of operating in the input market affects a buyer's investment decision.

Brief descriptions of the models. Our paper models a hierarchical game capturing the strategic interactions taking place among two sets of players: (a) a monopsonic buyer and (b) a pool of suppliers. The suppliers are first assumed to be symmetric (i.e., their products are substitutes to one another and suppliers face the same cost function), with their number assumed constant over time. Based on this first set of assumptions, we study two stylized models which we contrast and compare:

1. Based on the current market conditions, a monopsonic buyer sources a critical input to a pool of suppliers deciding noncooperatively on their production outputs. This perspective allows us to determine the buyer's current profit under equilibrium conditions and ultimately to determine the net present value (NPV) of this profit stream. Based on these premises, we can formulate and solve a real-options problem in which the buyer decides whether and when to launch a critical investment knowing that a share of the value created will accrue to a pool

of suppliers. Clearly, the degree of market power affects the timing of this investment.

2. Second, we consider the possibility for the buyer to sign a supply contract (Li and Kouvelis, 1999) at the time of investment specifying an input price and a production schedule/purchase orders. Here, the production levels and equilibrium prices reflect the parties' beliefs about market growth (and risk). We consider again a real-options problem and study how the change in the interactions between the buyer and the pool of suppliers compared to the first model affects the timing of the buyer's investment.

3. Third, motivated by the role played by fixed cost in battery manufacturer, we study how differential in fixed costs may lead some suppliers to forego an opportunity to service a market because doing so would not be sufficiently profitable to cover the fixed operating cost. The suppliers' decision obviously affect the buyer's investment decision.

All the stylized mathematical models, used to derive our managerial insights, are solved in closed form.

Managerial insights. We derive various insights relevant for managerial practice:

1. The **degree of competition** among the supplier base affects a buyer's investment decision. In the absence of coordination costs and quality differential, a buyer should strive to source from a larger base. This will ensure the equilibrium input price will be relatively low, with the buyer more able to wield market power and charge more for the product in the output market. The buyer's ability to source from more suppliers will hasten the buyer's investment decision. A policy implication is that governments (e.g., via the regulator) should favor a higher potential degree of competition in the input market (e.g., by developing leasing solutions to variabilize costs), so an investment by a buyer creating that input market becomes more likely.

2. To ensure more steady cash flows, the suppliers may be willing to offer a **supply contract** to the buyer. However, accepting the contract unilaterally favors the suppliers and depresses the buyer's profit. Consequently, the buyer should refrain from accepting such a supply contract as it would lead to a delayed investment. The notion of a supply contract leads to a tradeoff

for potential suppliers: such a contract ensures they are better off when they operate, but the buyer delays its investment, so the suppliers receive higher profits but later.

3. If potential suppliers were to pay a fixed cost to trade with the buyer, their willingness to trade depends on the state of demand. For low demand, the buyer would be trading with a small set of suppliers, but will be able to source from more suppliers if demand materializes further. The buyer is naturally better off trading with more suppliers because a higher degree of competition depresses the equilibrium input price. A consequence is that there are circumstantial conditions under which the buyer will decide to delay its investment, so it can benefit from more market power, while it would have invested if the set of suppliers were to sustain ("strategic ambivalence").

2 Literature review

Supply chain disruption. Our focus is not on whether diversifying one's pool of suppliers help a firm mitigate its exposure to supply chain disruption risk (see, e.g., Tomlin, 2006; Swinney and Netessine, 2009; Federgruen and Yang, 2009; Gao et al., 2019), but rather on how the degree of competition among suppliers affect equilibrium conditions in the input market and consequently a buyer's propensity to invest.

Supply contracts. In the supply chain literature (e.g., Cachon, 2003; Kouvelis and Zhao, 2015), one of the focal points is the exploration of how collaborative supply agreements can align the various components of the supply chain. This alignment aims to achieve an optimal outcome, known as the "first best," where the combined profit of the two-firm system matches that of a single-firm system.

Specifically, Li and Kouvelis (1999) consider different types of supply contracts (e.g., time-inflexible agreement) in the context of deterministic demand, but uncertain prices. One of our stylized models considers a time-inflexible agreement where the buyer and suppliers reach a consensus regarding the quantity of units to be bought. In contrast to Li and Kouvelis (1999), the input market is assumed not to be sufficiently competitive for a spot price to be readily available and, so, the market-clearing price in the input market is an outcome of strategic interactions among the suppliers and with the buyer. As in Kouvelis and Zhao (2015), we consider how the default of one of the parties in the supply chain affect the strategic interactions of the supply chain parties. However, our focus is on the implications in case of dynamic sourcing rather than how the supply contracts in case of a time-inflexible commitment would have terms adjusted for the risk of default.

In our paper, we consider a "time-flexible contract" that allows the firm to specify the purchase amount over a given period of time without specifying the exact time of purchase.

Information sharing across the supply chain. There is a large literature looking at how informational frictions is a key feature of supply chains and how they affect a firm's choice of suppliers (e.g., Simchi-Levi and Zhao, 2003; Shen et al., 2019). In contrast, we assume all key characteristics of the firms (e.g., demand and costs) are known and taken into consideration when inferring the other parties' reactions.

3 Baseline model of dynamic sourcing

There are various rationales for companies to outsource the supply of components and subassemblies such as lower cost, available capacity, quality, technology, and delivery time (see Li and Kouvelis, 1999). In many industries (e.g., paper, agriculture, electronics, textiles) the sourcing of inputs involves substantial price uncertainty, not easily hedged by trading in futures markets. The well functioning of futures markets requires a level of standardization that is often lacking for numerous raw materials. We consider an imperfect input market in which the pricing depends on the purchase decision of a buyer, itself subject to uncertainty in the output market. We first assume that the buyer sources from a given set of suppliers, with a purchase order reflecting the current level of uncertainty in the market.

Before we investigate the buyer's decision about when to launch its project, we describe the strategic interactions taking place between the buyer and the pool of suppliers. The suppliers are producing a good which can be used as input in various inputs to production processes. The buyer is a monopoly firm for one of these uses. Contrary to Li and Kouvelis (1999), we do not consider a spot price for commodities traded in the input market, under the assumption that the market is not sufficiently "liquid" for a market-clearing price to obtain from market forces. Instead, we consider an imperfect input market from which the buyer sources the input. The input is not a commodity characterized by a spot price; its price is the outcome of strategic interactions among a given set of suppliers competing à la Cournot.

Demand and cost functions. At a time $t \geq 0$ following the investment, we consider a monopsonic buyer who faces in the output market an inverse demand function $q \mapsto P(q, y) \geq 0$, which depends on the state of demand denoted by $y > 0$ observed at time $t \geq 0$. Up to a renormalisation, we assume that the buyer needs q units of input to produce q units of output.

Given an input price $c > 0$, the buyer faces a cost function given by $(c, q) \mapsto C(c, q)$, a non-negative, convex, and twice continuously differentiable function depending on the buyer's output choice $q \geq 0$. On the other hand, let $C_0(\cdot) \geq 0$ denote a supplier's cost function. In the examples studied later, we consider that the two functions C and C_0 are either linear (i.e., $C(c, q) = cq$ and $C_0(q) = c_0 q$ with $c_0 \geq 0$ an exogeneous parameter) or quadratric in the output q (i.e., $C(c, q) = \frac{1}{2}cq^2$ and $C_0(q) = \frac{1}{2}c_0 q^2$).

Equilibrium conditions. Suppose the monopsonic buyer has already invested and is sourcing at every time $t \geq 0$ a good in the input market after having observed the state of demand y. For each time $t \geq 0$, we consider a hierarchical game in which n symmetric suppliers decide on their output capacity à la Cournot. These noncooperative decisions lead to a market-clearing price, $\bar{c}_n \geq 0$, which is observed by the buyer. Given this observed input price, the buyer decides at each time $t \geq 0$ on its production output. That buyer is rational and maximizes its profit at each time $t \geq 0$.
To determine the equilibrium price in the input market, $\bar{c}_n$, we first consider the buyer's perspective given some arbitrary input price c. The buyer selects an output level $\bar{q}(y, c)$ that maximizes the time-t profit $\pi(q; y, c) := qP(q, y) - C(c, q)$. We use π_q, π_y, and π_{qq} to denote the partial derivatives. Under mild conditions, this maximum is obtained from a first-order condition, namely

$$\pi_q\left(y, \bar{q}(y, c)\right) = 0. \tag{1}$$

Now, we consider the suppliers' perspective where the function $c \mapsto \bar{q}(y, c)$ can be interpreted as the demand function of the suppliers. (By design, as the buyer is monopsonic, there is no other demand than the demand by this specific buyer.) Under standard conditions on P and C, the order quantity $\bar{q}(y,c)$ in eq. (1) increases with demand (as $\bar{q}_y = -\pi_{qy}/\pi_{qq} \geq 0$) and decreases with the input price c (as $\bar{q}_c = C_{qc}/\pi_{qq} \leq 0$).

Because the suppliers are symmetric, we focus hereafter on symmetric equilibria. To determine a symmetric Cournot-Nash equilibrium in the input market, we use the inverse demand function $Q \mapsto \bar{q}(y,\cdot)^{-1}(Q)$, which maps the total demand by the buyer Q to a price, and then compute the supplier i's *best-reply function*:

$$z \in \mathbb{R}_+ \mapsto \bar{R}(z) := \arg\max_{q_i \geq 0} \Big\{ \underbrace{q_i \; \underbrace{\bar{q}(y,\cdot)^{-1}(q_i+z)}_{\substack{\text{Inverse demand}\\ \text{function=input price}}}}_{\substack{\text{individual supplier's}\\ \text{revenues}}} - \underbrace{C_0(q_i)}_{\substack{\text{supplier's}\\ \text{cost}}} \Big\} \in \mathbb{R}_+, \tag{2}$$

where z has to be understood as $\sum_{j\neq i} q_j$.
Focusing on symmetric equilibria, we have $q_i = \bar{q}_n(y)$ for all i. Hence, the Nash-equilibrium output $\bar{q}_n(y) \geq 0$ of an individual supplier obtains from solving the following fixed-point equation,

$$\bar{q}_n(y) = \bar{R}\Big((n-1)\bar{q}_n(y)\Big). \tag{3}$$

The n-tuple $\{\bar{q}_n(y), \ldots, \bar{q}_n(y)\}$ corresponds to the symmetric Cournot-Nash equilibrium outputs among the n symmetric suppliers facing a monopsonic buyer. The total output in the input market

$$\bar{Q}_n(y) := n\bar{q}_n(y) \tag{4}$$

equals by design the order by the monopsonic buyer. The equilibrium price in the input market is then given by

$$\bar{c}_n(y) := \bar{q}(y,\cdot)^{-1}\left(\bar{Q}_n(y)\right), \tag{5}$$

while the equilibrium price in the output market is given by

$$\bar{P}_n(y) = P\left(\bar{Q}_n(y), y\right). \tag{6}$$

Clearly, the prices in the input and output markets are not decoupled, with changes in the end demand (driven by y) affecting the equilibrium price in the input market.

Unless specified otherwise, we consider henceforth an inverse demand function of the form

$$q \mapsto P(q, y) = yq^{-\delta}, \quad \delta \in (0, 1), \tag{7}$$

if the state of demand is y. Because $\frac{\partial P}{\partial q}(q, y) < 0$ and $\frac{\partial^2 P}{\partial q^2}(q, y) \geq 0$, our inverse demand function satisfies standard properties: the market-clearing price decreases with supply, in a convex manner. In this case, the price elasticity of demand is constant, given by $\frac{\mathrm{d}Q}{Q} / \frac{\mathrm{d}P}{P} = -1/\delta$. Other demand specifications are obviously possible, but are less analytically tractable. (In particular, in case of a linear inverse demand function, the optimal output decisions will be characterized by corner solutions for specific range of the demand state y. This leads to expressions for the firms' profits and net present values which are defined in a piecewise manner and are slightly less tractable.)For the state of completeness, the lemma below specifies the equilibrium input and output prices as well as the equilibrium purchase order quantity by the buyer, in case of isoelastic demand in eq. (7). We consider four cases (A, B, C, and D) to track the various combinations of cost functions:

A. $C(c, q) = cq$ and $C_0(q) = c_0 q$ with $c, c_0 \geq 0$;

B. $C(c, q) = \frac{1}{2}cq^2$ and $C_0(q) = c_0 q$ with $c, c_0 \geq 0$;

C. $C(c, q) = \frac{1}{2}cq^2$ and $C_0(q) = \frac{1}{2}c_0 q^2$ with $c, c_0 \geq 0$;

D. $C(c, q) = cq$ and $C_0(q) = \frac{1}{2}c_0 q^2$.

Lemma 1 (Equilibrium input price, purchase order quantity, and output price in case of dynamic sourcing)**.** *For the inverse demand function of eq.* (7)*, the equilibrium input price* $\bar{c}_n(\cdot)$ *in eq.* (5)*, the equilibrium output price* $\bar{P}_n(\cdot)$ *in eq.* (6)*, and the buyer's equilibrium order* $\bar{Q}_n(\cdot)$ *in eq.* (4)*, are respectively given by*

A. *$C(c, q) = cq$ and $C_0(q) = c_0 q$ with $c, c_0 \geq 0$.*

$$\bar{c}_n \equiv \frac{c_0}{1 - \frac{\delta}{n}}, \quad \bar{Q}_n(y) = \left[\frac{1-\delta}{\bar{c}_n} y\right]^{\frac{1}{\delta}}, \quad \bar{P}_n \equiv \frac{\bar{c}_n}{1-\delta}.$$

B. $\underline{C(c,q) = \frac{1}{2}cq^2 \text{ and } C_0(q) = c_0 q \text{ with } c, c_0 \geq 0}$.

$$\bar{c}_n(y) \equiv \frac{c_0}{1 - \frac{1+\delta}{n}}, \quad \bar{Q}_n(y) = \left[\frac{1-\delta}{\bar{c}_n} y\right]^{\frac{1}{1+\delta}}, \quad \bar{P}_n(y) = \left[\frac{\bar{c}_n}{1-\delta}\right]^{\frac{\delta}{1+\delta}} y^{\frac{1}{1+\delta}}.$$

C. $\underline{C(c,q) = \frac{1}{2}cq^2 \text{ and } C_0(q) = \frac{1}{2}c_0 q^2 \text{ with } c, c_0 \geq 0}$.

$$\bar{c}_n(y) = [(1-\delta)y]^{\frac{1}{\delta+2}} \left[\frac{n-1-\delta}{c_0}\right]^{-\frac{1+\delta}{\delta+2}}, \quad \bar{Q}_n(y) = \left[\frac{1-\delta}{\bar{c}_n(y)} y\right]^{\frac{1}{\delta+1}}, \quad \bar{P}_n(y) = \left[\frac{\bar{c}_n(y)}{1-\delta}\right]^{\frac{\delta}{\delta+1}} y^{\frac{1}{\delta+1}}.$$

D. $\underline{C(c,q) = cq \text{ and } C_0(q) = \frac{1}{2}c_0 q^2}$.

$$\bar{c}_n = (1-\delta)\left[(n-\delta)\frac{1-\delta}{c_0}\right]^{-\frac{\delta}{\delta+1}} y^{\frac{1}{\delta+1}}, \quad \bar{Q}_n(y) = \left[\frac{1-\delta}{\bar{c}_n(y)} y\right]^{\frac{1}{\delta}}, \quad \bar{P}_n(y) = \frac{\bar{c}_n(y)}{1-\delta}.$$

We make some observations relevant to all these cases discussed in Lemma 1. First, in line with economic intuition, the equilibrium input price $\bar{c}_n(\cdot)$ increases with the suppliers' cost parameter c_0. The input price $\bar{c}_n(y)$, however, decreases as the number of suppliers in the pool, n, increases. This is because the suppliers collectively lose market power vis-à-vis the buyer if they compete more against each other. Second, the buyer's purchase order $\bar{Q}_n(y)$ decreases with the equilibrium price in the input market, while the equilibrium price in the output market $\bar{P}_n(y)$ increases. A greater degree of competition among suppliers will thus lead the buyer to buy more units and to sell items at a lower unit price $\bar{P}_n(y)$ to the end customers. Third, the input market is subject to shocks in the output market: the buyer's purchase order $\bar{Q}_n(y)$ depends on the realization of the demand state, but so can be the equilibrium input price $\bar{c}_n(y)$ depending on the firms' cost specifications. The equilibrium input price $\bar{c}_n(\cdot)$ and purchase order $\bar{Q}_n(\cdot)$ are nondecreasing in the demand state y.

In the general case, the buyer's equilibrium profit is given by

$$\bar{\pi}_n(y) := \pi\Big(\bar{Q}_n(y); y, \bar{c}_n(y)\Big), \quad y > 0, \tag{8}$$

for $\bar{Q}_n(\cdot)$ and $\bar{c}_n(\cdot)$ given in eqs. (4) and (5) respectively, while a supplier's profit reads

$$\pi_n(y) := \bar{c}_n(y)\bar{q}_n(y) - C_0\big(\bar{q}_n(y)\big). \tag{9}$$

The firms' profit functions in eqs. (8) and (9) simplify greatly in the specific case of isoelastic demand in eq. (7).

Proposition 1 (Equilibrium profits in case of dynamic sourcing.)**.** *In the specific case with the inverse demand function in eq.* (7)*, the buyer's and suppliers' profit functions in eqs.* (8) *and* (9) *take respectively the form*

$$\bar{\pi}_n(y) = a_n y^{\epsilon} \quad \textit{and} \quad \pi_n(y) = \nu_n y^{\varepsilon}, \tag{10}$$

where $\epsilon > 0$*,* $a_n > 0$*,* $\nu_n > 0$*, and* $\varepsilon > 0$ *take different values depending on the cost combinations:*

$$\begin{cases} \epsilon = \frac{1}{\delta} & a_n = \delta\left[\frac{n-\delta}{n}\frac{1-\delta}{c_0}\right]^{\frac{1}{\delta}-1} & \nu_n = \frac{\delta}{n}\left(\frac{1-\delta}{c_0}\right)^{\frac{1-\delta}{\delta}}\left(\frac{n-\delta}{n}\right)^{\frac{1}{\delta}} & \varepsilon = \frac{1}{\delta} & \textit{in Case A,} \\ \epsilon = \frac{2}{\delta+1} & a_n = \frac{1+\delta}{2}\left[\frac{n-1-\delta}{c_0}\frac{1-\delta}{n}\right]^{\frac{1-\delta}{1+\delta}} & \nu_n = \frac{1-\delta^2}{n^2}\left(\frac{1-\delta}{c_0}\frac{n-1-\delta}{n}\right)^{-\frac{\delta}{\delta+1}} & \varepsilon = \frac{1}{\delta+1} & \textit{in Case B,} \\ \epsilon = \frac{3}{\delta+2} & a_n = \frac{1+\delta}{2}\left[\frac{n-1-\delta}{c_0}(1-\delta)\right]^{\frac{1-\delta}{2+\delta}} & \nu_n = (1-\delta)^{\frac{2}{\delta+2}}\left[\frac{n-1-\delta}{c_0}\right]^{-\frac{\delta}{\delta+2}}\frac{n+1+\delta}{2n^2} & \varepsilon = \frac{2}{\delta+2} & \textit{in Case C,} \\ \epsilon = \frac{2}{\delta+1} & a_n = (1+\delta)\left[\frac{1-\delta}{c_0}(n-\delta)\right]^{\frac{1-\delta}{\delta+1}} & \nu_n = \frac{(1-\delta)^{\frac{2-\delta}{\delta+1}}}{n^2}\left[\frac{n-\delta}{c_0}\right]^{\frac{1-\delta}{\delta+1}}\left\{n - \frac{n-\delta}{2}\left[1-\delta\right]^{\frac{\delta}{\delta+1}}\right\} & \varepsilon = \frac{2}{\delta+1} & \textit{in Case D.} \end{cases}$$

Demand dynamics. Over time, the buyer faces uncertainty due to continuous demand shocks modeled by a Brownian motion (BM) Z defined in the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. This BM generates a filtration $\mathbb{F} := (\mathcal{F}_t)_t$ that satisfies the standard conditions. Specifically, the inverse demand function $\big(P(q, Y_t)\big)_t$ evolves over time driven by the fluctuation of a geometric Brownian motion (GBM) given by

$$Y_0 = y > 0 \quad \text{and} \quad \mathrm{d}Y_t = \mu Y_t \,\mathrm{d}t + \sigma Y_t \,\mathrm{d}W_t \quad \text{with } \sigma > 0.$$

The stochastic process $(Y_t)_t$ characterizes shifts in the demand curve due to changes in consumer tastes and arrivals of substitute products over time (see, e.g., Li and Kouvelis, 1999; Bensoussan et al., 2022).

Buyer's long-term value (after investment). The buyer's discount rate, denoted $r > 0$, is assumed constant over time (Li and Kouvelis, 1999; Bensoussan et al., 2022). Let $\mathbb{E}^y$ denote the conditional expectation operator $\mathbb{E}\big[\,\cdot\,\big|Y_0 = y\big]$. The net present value (NPV) of the buyer's equilibrium profit is given by

$$\bar{u}_n(y) = \mathbb{E}^y \int_0^\infty e^{-rt} \bar{\pi}_n(Y_t) \mathrm{d}t. \tag{11}$$

Corollary 1 specifies the NPV in eq. (11) further for the case of isoelastic demand in eq. (7). To state the result, we introduce

$$\gamma \mapsto \mathcal{Q}(\gamma) := \frac{1}{2}\sigma^2 \gamma(\gamma - 1) + \mu\gamma - r. \tag{12}$$

Corollary 1 (Buyer's NPV in case of dynamic sourcing.)**.** *Consider the isoelastic demand of eq.* (7)*. If the parameter δ is such that $\epsilon > 0$ in Proposition 1 satisfies $\mathcal{Q}(\epsilon) < 0$, then the NPV in eq.* (11) *is of the form*

$$\bar{u}_n(y) = \alpha_n y^\epsilon, \quad \textit{where} \quad \alpha_n := -\frac{a_n}{\mathcal{Q}(\epsilon)} < \alpha_{n+1}, \tag{13}$$

for a_n given in Proposition 1.

If the constraint $\mathcal{Q}(\epsilon) < 0$ on the parameter is not satisfied, then the net present value explodes (i.e., $\bar{u}_n(y) = \infty$) because the profits grow in an exponential manner in a manner that is too strong compared to the discount rate. (If the suppliers face capacity limits, the buyer's profit becomes linear in the demand state at ∞ as per the discussion above and, so, a sufficient and necessary condition for finiteness is $r > \mu$.) It follows that a lower degree of market power among the pool of suppliers (i.e., a larger value for n) helps the buyer achieve a higher value from operating in this business. Independently from considerations about diversification of one's supply, sourcing from a larger pool of suppliers makes the buyer better off because the collective market power of these suppliers is reduced, which benefits the buyer.

Buyer's investment-timing problem. We are interested in studying the propensity of a buyer to invest in productive assets given a dependence on its supplier base. Given the expression for the NPV in eq. (13), we now introduce a real-options problem à la McDonald and Siegel (1986), which takes the form

$$\psi_n(y) := \sup_\tau \mathbb{E}^y \Big[e^{-r\tau} \big(\alpha_n Y_\tau^\epsilon - I \big) \Big], \tag{14}$$

where τ is selected among the $\mathbb{F}$-stopping times taking value in $[0, \infty)$ and the parameter $I \geq 0$ is a known investment cost incurred by the buyer. Let γ denote the unique positive root of $\mathcal{Q}(\cdot)$ in eq. (12). The solution of this problem is given in Proposition 2 below:

Proposition 2 (Buyer's real-options problem in case of dynamic sourcing.)**.** *Assume that* $\mathcal{Q}(\epsilon) < 0$ *as per Corollary 1. Then, the solution of eq.* (14) *is given by*

$$\psi_n(y) = \begin{cases} \left[\alpha_n \bar{y}^\epsilon - I\right] \left(\frac{y}{\bar{y}_n}\right)^\gamma, & y < \bar{y}_n := \left(\frac{\gamma}{\gamma - \epsilon} \frac{I}{\alpha_n}\right)^{\frac{1}{\epsilon}} \\ \alpha_n y^\epsilon - I, & y \geq \bar{y}_n. \end{cases}$$

The optimal investment strategy is a threshold policy, given by $\hat{\tau}_n := \inf \left\{ t \geq 0 \middle| Y_t \geq \bar{y} \right\}$.

The buyer invests if the price exceeds a level, higher than the Marshallian threshold and the NPV threshold. The buyer thus requires extra profitability from its project before undertaking investment. The investment threshold obtains by smooth fit. A classical assumption in case of linear payoff functions is that $r > \mu$ (e.g. Dixit and Pindyck, 1994), which is not sufficient here. Here, we introduce a slightly different assumption, namely $\mathcal{Q}(\epsilon) < 0$, which effectively is a constraint on the elasticity parameter δ of the inverse demand function in eq. (7). The constraint also depends on the choice of cost functions as per Corollary 1.

We also obtain the comparative results:

Corollary 2 (Comparative statics)**.** *Under the assumption of Proposition 2, the value functions and the optimal stopping times are ranked, with* $\psi_n \leq \psi_{n+1}$ *and* $\hat{\tau}_n \geq \hat{\tau}_{n+1}$.

As per Corollary 2, because of the inequalities $\alpha_{n+1} \geq \alpha_n$ in eq. (13), a larger degree of competition in the input market (and consequently a lower degree of market power) makes it more likely for the buyer to invest because it receives a larger slice of the market with less value accruing to the suppliers. The result is intuitive, as the buyer would be better off if the pool of suppliers is larger because the suppliers would then compete more against each other leading the equilibrium price in the input market to fall.

Coordination costs. The notion that the buyer is better off sourcing from a larger pool of suppliers rests on the assumption that the buyer does not incur coordination costs. If the coordination costs

$n \in \mathbb{N} \mapsto K(n)$ are nondecreasing and convex, we would be able to solve a new optimization problem determining the optimal supplier base at the time of investment. Because the optimal supplier base, denoted say $\bar{n}(y)$, is integer-valued, the gain function of the buyer's investment problem will be of the form $y \mapsto \alpha_{\bar{n}(y)} y^{\epsilon} - I - K\big(\bar{n}(y)\big)$ and be discontinuous. This would make the buyer's investment problem particularly complex (see, e.g., Bensoussan and Chevalier-Roignant, 2013), possibly with limited new insights. Obviously, in this case, the insights from Corollary 2 do not hold.

Suppliers' capacity constraints. The results in Lemma 1, Corollary 1 and proposition 2 are obtained under the assumption that the suppliers face no capacity constraints. For the sake of argument, consider symmetric capacity limits $\kappa > 0$. Because, in all cases under study, the equilibrium purchase order quantity satisfies $\bar{Q}_n'(\cdot) > 0$ with $\bar{Q}_n(0) = 0$ to $\bar{Q}_n(\infty) = \infty$, there is a unique demand state, namely $y_\star := \bar{Q}_n^{-1}(n\kappa)$, above which the supplier's constraint is binding. The supplier would then have to account for the likely capacity constraints of the suppliers when computing its net present value in eq. (11), which now is defined piecewise. For larger demand, i.e., $y \geq y_\star$, the equilibrium input price in eq. (5), now given by $\bar{q}(y,\cdot)^{-1}(n\kappa)$, is linear in y for all cases under study. Consequently, the buyer's cost $y \mapsto C(\bar{q}(y,\cdot)^{-1}(n\kappa), n\kappa)$ grows linearly with the demand state y at ∞. The equilibrium output price in eq. (7) is now $y(n\kappa)^{-\delta}$ for $y \geq y_\star$, while the buyer's purchase order is bounded by $\bar{Q}_n(y_\star) = n\kappa$ due to the suppliers' binding capacity constraints. Consequently, the buyer's revenues grow linearly in the demand state y at ∞. By optimality, the revenue contribution dominates the cost contribution for large y, so the buyer's profit is linear at ∞. The integrand thus becomes linear at ∞: the sufficient conditions $\mathcal{Q}(\epsilon) < 0$ in Corollary 1 can thus be substituted by $r > \mu$ (which is equivalent to $\mathcal{Q}(1) < 0$) in case of capacity constraints. Under suppliers' capacity constraints, the NPV will not have a simple functional form as in eq. (13), but will be defined piecewise and have convex kinks. The buyer's investment problem will not be of the form in eq. (14). The presence of convex kinks will make the study of the optimal stopping problem more involved, with the optimal strategy possibly not characterized by a threshold (e.g., Décamps et al., 2006). At any rate, we believe the additional mathematical complexity will outweigh the benefits, in terms of novel unexpected managerial insights, gained by studying a more realistic setup with capacity constraints. We also note that the capacity constraint is less of an issue if the suppliers can rely on providers of contract manufacturers if their own production capacity is limited.

4 Long-term supply contract

To reduce their exposure to uncertainty in the input market, suppliers may want to offer a supply contract to the buyer (see Li and Kouvelis, 1999). Supply contracts establish a formal agreement regarding the cost of components, the quantity of items bought, the delivery schedule, the quality of products, and other factors relevant to the procurement situation. Here, the suppliers may want to circumvent the dependence of the equilibrium input price $\bar{c}_n(y)$ in Lemma 1 on the end demand state y.

For this purpose, we consider a different hierarchical game in which, at time 0, the monopsonic buyer decides on its production capacity—assuming it produces at capacity for any future time $t \geq 0$— and liaises with a set of n suppliers to converge on an input price $\hat{c}_n \geq 0$ at which the buyer will source the good in the future (as part of a long-term framework agreement). We are examining a *time-inflexible agreement* where the buyer and suppliers reach a consensus at time 0 regarding (i) the quantity of units to be bought (Li and Kouvelis, 1999) and (ii) the product price. Given a supply contract, the buyer's decision is to determine when to purchase and how many units to purchase each time such that the expected net present value is maximized.

Buyer's long-term value (after investment). Now, the buyer is assumed to select its production capacity so as to maximize its net present value:

$$\hat{u}_n(y) := \sup_{q \geq 0} \mathbb{E}^y \left[\int_0^\infty e^{-rt} \{ qP(q, Y_t) - C(\hat{c}_n, q) \} \mathrm{d}t \right], \tag{15}$$

where $P(\cdot, y)$ is again an inverse demand function, for instance of the form in eq. (7). By assumption, the purchase order quantity is held fixed following the time-0 decision in eq. (15).

Equilibrium conditions. Again, we use backward induction to determine the equilibrium price in the input market, $\hat{c}_n$. For a given arbitrary input price $c \geq 0$, the buyer maximizes the function

$$q \mapsto u(q; y, c) := \mathbb{E}^y \left[\int_0^\infty e^{-rt} \{ qP(q, Y_t) - C(c, q) \} \mathrm{d}t \right]. \tag{16}$$

If $c \mapsto \hat{q}(y, c) := \arg\max_{g \geq 0} u(q; y, c)$ is invertible, $q \mapsto \hat{q}(y, \cdot)^{-1}(q)$ is an inverse demand function for the suppliers. To determine the Cournot-Nash equilibrium in the input market, we compute

$$Q \in \mathbb{R}_+ \mapsto \hat{R}(Q) := \arg\max_{q \geq 0} \left\{ q\hat{q}(y, \cdot)^{-1}(q + Q) - C_0(q) \right\} \in \mathbb{R}_+, \tag{17}$$

and then determine a capacity $\hat{q}_n(y) \geq 0$ such that

$$\hat{q}_n(y) = \hat{R}\left((n-1)\hat{q}_n(y)\right), \tag{18}$$

with $\{\hat{q}_n(y), \ldots, \hat{q}_n(y)\}$ corresponding to a symmetric Cournot-Nash equilibrium. The buyer's purchase order is

$$\hat{Q}_n(y) := n\hat{q}_n(y). \tag{19}$$

(The profits received by the suppliers does not depend on time as the buyer produces at capacity in all subsequent periods and the input price is agreed upon and fixed in the framework agreement. Consequently, whether the suppliers maximize their profit or the net present value of their deterministic profits does not matter.) The equilibrium input and output prices are, respectively, given by

$$\hat{c}_n := \hat{q}(y, \cdot)^{-1}\left(\hat{Q}_n(y)\right) \quad \text{and} \quad \hat{P}_n(y) := P\left(\hat{Q}_n(y), y\right), \tag{20}$$

while the buyer's NPV can now be written as

$$\hat{u}_n(y) = u\left(\hat{Q}_n(y); y, \hat{c}_n\right) \tag{15'}$$

Lemma 2 discloses the equilibrium quantities in case of isoelastic demand and stresses the difference with the earlier setting for which the results reported in Lemma 1:

Lemma 2 (Equilibrium input price, purchase order quantity, and output price in case of supply contract)**.** *We consider the inverse demand function of eq.* (7) *and recall the related quantities* $\bar{c}_n(\cdot)$*,* $\bar{Q}_n(\cdot)$*, and* $\bar{P}_n(\cdot)$ *in Lemma 1. The equilibrium input and output prices* $\hat{c}_n(\cdot)$ *and* $\hat{P}_n(\cdot)$ *in eq.* (20) *and the buyer's equilibrium order* $\hat{Q}_n(\cdot)$ *in eq.* (19) *are respectively given by*

A. $\hat{c}_n \equiv \bar{c}_n$, $\hat{Q}_n(y) = \left[\frac{r}{r-\mu}\right]^{\frac{1}{\delta}} \bar{Q}_n(y)$, *and* $\hat{P}_n \equiv \frac{r-\mu}{r}\bar{P}_n$.

B. $\hat{c}_n \equiv \bar{c}_n$, $\hat{Q}_n(y) = \left[\frac{r}{r-\mu}\right]^{\frac{1}{1+\delta}} \bar{Q}_n(y)$, *and* $\hat{P}_n(y) = \left[\frac{r-\mu}{r}\right]^{\frac{\delta}{\delta+1}} \bar{P}_n(y)$.

C. $\bar{c}_n(y) = \left[\frac{r}{r-\mu}\right]^{\frac{1}{\delta+2}} \bar{c}_n(y)$, $\hat{Q}_n(y) = \left[\frac{r}{r-\mu}\right]^{\frac{1}{\delta+2}} \bar{Q}_n(y)$, *and* $\hat{P}_n(y) = \left[\frac{r-\mu}{r}\right]^{\frac{\delta}{\delta+2}} \bar{P}_n(y)$.

D. $\hat{c}_n(y) = \left[\frac{r}{r-\mu}\right]^{\frac{1}{\delta+1}} \bar{c}_n(y)$, $\hat{Q}_n(y) = \left[\frac{r}{r-\mu}\right]^{\frac{1}{1+\delta}} \bar{Q}_n(y)$, *and* $\bar{P}_n(y) = \left[\frac{r-\mu}{r}\right]^{\frac{\delta}{1+\delta}} \bar{P}_n(y)$.

From Lemma 2, it holds in all four cases that

$$\hat{c}_n(\cdot) \geq \bar{c}_n(\cdot), \quad \hat{Q}_n(\cdot) \geq \bar{Q}_n(\cdot), \quad \text{and} \quad \hat{P}_n(\cdot) \leq \bar{P}_n(\cdot).$$

This implies that, in case of a supply contract, the suppliers are better off compared to the case of dynamic sourcing by the buyer, the suppliers selling more units $\hat{Q}_n(\cdot)$ at a higher equilibrium input price $\hat{c}_n(\cdot)$. These circumstances, however, put pressure on the buyer, who charges less to the end customers to ensure that the output market clears. The introduction of a supply contract implies a reduction of wealth to the benefits of the suppliers. Interestingly, the terms of the supply contract in Lemma 2 do not depend on the degree of uncertainty in the market. This is because the supply chain parties are considered risk-neutral and because the demand state y affects the demand in eq. (7) in a linear manner.

In this game with a time-inflexible supply contract, where the buyer decides on its production capacity (and not continually on its output), we obtain

Proposition 3 (Buyer's NPV in case of production schedule commitment.)**.** *We assume* $r > \mu$*. For the inverse demand function of eq.* (7)*, the NPV in eq.* (15) *depends on the cost specifications:*

A. $\hat{u}_n(y) = \beta_n y^{\frac{1}{\delta}}$ *where* $\beta_n := \frac{\delta}{r-\mu}\left[\left(1-\frac{\delta}{n}\right)\frac{1-\delta}{c_0}\frac{r}{r-\mu}\right]^{\frac{1}{\delta}-1}$;

B. $\hat{u}_n(y) = \beta_n y^{\frac{2}{\delta+1}}$, *where* $\beta_n := \frac{1}{r-\mu}\frac{1+\delta}{2}\left[\left(1-\frac{1+\delta}{n}\right)\frac{1-\delta}{c_0}\frac{r}{r-\mu}\right]^{\frac{1-\delta}{1+\delta}}$;

C. $\hat{u}_n(y) = \beta_n y^{\frac{3}{\delta+2}}$*, where* $\beta_n := \frac{1}{r-\mu}\frac{1+\delta}{2}\left[(n-1-\delta)\frac{1-\delta}{c_0}\frac{r}{r-\mu}\right]^{\frac{1-\delta}{2+\delta}}$.

D. $\hat{u}_n(y) = \beta_n y^{\frac{2}{\delta+1}}$*, where* $\beta_n := \frac{1+\delta}{r-\mu}\left[\frac{1-\delta}{c_0}\frac{r}{r-\mu}(n-\delta)\right]^{\frac{1-\delta}{\delta+1}}$.

Comparison with the case with dynamic sourcing. In all of the cases considered in Proposition 3, the NPV in eq. (15) can be written in the form

$$\hat{u}_n(y) = \beta_n y^{\epsilon}, \quad \text{where} \quad \beta_n \le \beta_{n+1} \text{ and } \epsilon \ge 0. \tag{21}$$

This simple form is reminiscent of the one in eq. (13). We compare the NPVs under these two distinct setups and establish:

Proposition 4 (Ranking)**.** *Under the assumptions specified in Propositions 2 and 3, it holds in all four cases that* $\beta_n \le \alpha_n$ *for all* $n \in \mathbb{N}$*.*

Following Proposition 4, the buyer is worse off by agreeing to sign a supply contract with suppliers rather than sourcing dynamically as the demand state y is realized. This outcome in Proposition 4 is not surprising because we already observed a redistribution of wealth to the benefit of suppliers in lemma 2.

Buyer's investment-timing problem. Because of the formulation in eq. (21), the solution of the real-options problem

$$\hat{\psi}_n(y) := \sup_{\tau \ge 0} \mathbb{E}^y\left[e^{-r\tau}\left(\beta_n Y_\tau^{\epsilon} - I\right)\right]$$

is of the form given in Proposition 2 but with the parameter β_n in lieu of α_n. Because of the inequality satisfied by the terms $\{\beta_n\}_n$ in eq. (21), the results in Corollary 2 also hold in this case with a commitment to the production schedule. A comparison between the optimal investment time in case of dynamic sourcing vs commitment to a production schedule can also be readily done leveraging the results of Proposition 4 and Corollary 2: the buyer is more prone to investing in a project if it sources dynamically after the investment time rather than agree to a supply contract with the suppliers.

5 Supplier dynamics

The previous stylized models rest on the assumption that the suppliers are willing to trade with the suppliers provided the input price exceeds the marginal cost. The fundamental force leading to a given set of firms in the supplier base was left unanswered. We now revise the setting to allow for dynamics in the number of firms in the supplier base, which we relate to a fixed cost of operating in the input market, due, e.g., to a calibration of the production processes to adjust the input to the buyer's requirements, fixed costs to ship the goods to the buyer, contracting cost with a contract manufacturer.

Weaker supplier's exit decision. In particular, assume two suppliers could operate at time 0. These suppliers $i \in \{1, 2\}$ are asymmetric in terms of the quasifixed cost to operate their businesses. The "strong supplier" has a competitive advantage compared to its rival as it faces a nonnegative, convex, twice continuously cost differentiable function of the form $C_0(q)$, while its rival, the "weaker supplier," faces a cost function $q \mapsto C_0(q) + K\mathbf{1}_{(0,\infty)}(q)$, with $K > 0$. Following standard economic arguments, the quasifixed cost of the weaker supplier does not affect the terms of the trade if a trade with both suppliers were to take place, but it affects the willingness of the weaker supplier to trade altogether, requesting the demand to be sufficiently large to agree on the terms.

We assume that the quasifixed cost of the weaker supplier K is sufficiently large so that there exists a unique Markov perfect equilibrium in pure strategies for the supplier's exit decision (see Georgiadis et al., 2022, Proposition 1i). Under this assumption, the weaker supplier decides to exit the business by solving the following optimal stopping problem:

$$\vartheta =: \arg\max_{\tau} \mathbb{E}^y \int_0^{\tau} e^{-rt} \big\{ \pi_2(Y_t) - K \big\} \mathrm{d}t, \tag{22}$$

where $\pi_n(\cdot)$ is given in eq. (9). This problem is akin to the one discussed in Leland (1994). In contrast, the stronger supplier, which does not incur a quasifixed cost to trade, would stay trading in the input market for any demand state $y > 0$ if the buyer decides to operate.

Buyer's long-term value (after investment). After the weaker supplier has decided to exit the business at time ϑ, the buyer can only source from one supplier, with the stronger supplier then being able to wield more market power and secure better terms of trade with the buyer. In summary, if the buyer dynamically source from the input market, its NPV is given by

$$\tilde{u}(y) := \mathbb{E}^y \Bigg[\underbrace{\int_0^{\vartheta} e^{-rt} \bar{\pi}_2(Y_t)\, \mathrm{d}t}_{\substack{\text{buyer's discounted profits} \\ \textit{before} \text{ supplier 2's exit}}} + \underbrace{\int_{\vartheta}^{\infty} e^{-rt} \bar{\pi}_1(Y_t)\, \mathrm{d}t}_{\substack{\text{buyer's discounted profits} \\ \textit{after} \text{ supplier 2's exit}}} \Bigg], \tag{23}$$

for $\bar{\pi}_n(\cdot)$ given in eq. (8) and ϑ the optimal stopping time in eq. (22). By design, if the weaker supplier is offered to trade at time 0, it will forego this opportunity forever if $\vartheta = 0$, even in the event where the demand state y increases significantly in the future. In this case, the buyer will only be able to trade with a single supplier forever. If the weaker supplier does not forego this opportunity at time 0 (i.e., $\vartheta > 0$), it will exit the market at some time in the future, namely ϑ. From that point onwards, the buyer is left with possibility to trade with a single supplier.

Proposition 5 summarizes the results for the weaker supplier's exit decision in eq. (22) and the buyer's NPV in eq. (23) in case of isoelastic demand in eq. (7) and for various combinations of the firms' cost functions.

Proposition 5 (Weaker supplier's exit and buyer's net present value under isoelastic demand)**.** *We consider the inverse demand function in eq.* (7) *and assume that δ is such that ϵ and ε in Proposition 1 satisfies $\mathcal{Q}(\epsilon) < 0$ and $\mathcal{Q}(\varepsilon) < 0$, respectively. The weaker supplier decides not to trade with the buyer at the first time*

$$\vartheta := \inf\left\{t \geq 0 \middle| Y_t \leq \tilde{y}\right\} \text{ at which the process } (Y_t)_t \text{ is below } \tilde{y} := \left(-\frac{\tilde{\gamma}}{\tilde{\gamma} - \varepsilon} \frac{\mathcal{Q}(\varepsilon)}{r} \frac{K}{\nu_2}\right)^{\frac{1}{\varepsilon}}, \tag{22'}$$

where $\tilde{\gamma}$ is the negative root of $\mathcal{Q}(\cdot)$ in eq. (12) *and ν_2 is the multiplier in eq.* (10).

Furthermore, the NPV of the buyer in eq. (23) *is given by*

$$\tilde{u}(y) = \begin{cases} \alpha_1 y^{\epsilon}, & 0 < y < \tilde{y}, \\ \alpha_2 y^{\epsilon} + \big[\alpha_1 - \alpha_2\big] \tilde{y}^{\epsilon} \left(\frac{y}{\tilde{y}}\right)^{\tilde{\gamma}}, & y \geq \tilde{y}, \end{cases} \tag{23'}$$

where α_n is specified in Corollary 1.

If the weaker supplier is offered to trade at time 0, it will forego this opportunity to if the demand state y is below $\tilde{y}$. If the demand state y is larger, the weaker supplier accepts the trade, but will exit when the demand state falls below $\tilde{y}$ in the future.

Buyer's investment-timing problem. We can now revisit the buyer's investment problem in eq. (14). This problem now reads

$$\tilde{\psi}(y) := \sup_{\tau} \mathbb{E}^y \Big[e^{-r\tau} \big\{ \tilde{u}(Y_\tau) - I \big\} \Big], \tag{24}$$

for the function $\tilde{u}(\cdot)$ in eq. (23'). Following the general theory on variational inequalities (Bensoussan and Lions, 1982), the stopping set for this problem is $\mathbb{S} := \big\{ y > 0 \big| \tilde{\psi}(y) = \tilde{u}(y) - I \big\}$.

From eq. (23'), we get $\tilde{u}'(\tilde{y}-) = \epsilon \alpha_1 \tilde{y}^{\epsilon-1}$, while $\tilde{u}'(\tilde{y}+) = \alpha_2 \epsilon \tilde{y}^{\epsilon-1} + \tilde{\gamma}\big[\alpha_1 - \alpha_2\big]\tilde{y}^{\epsilon+\tilde{\gamma}-1-\tilde{\gamma}}$. It follows that

$$\tilde{u}'(\tilde{y}+) - \tilde{u}'(\tilde{y}-) = \underbrace{\big[\alpha_2 - \alpha_1\big]}_{\substack{>0 \\ \text{from eq. (13)}}} \tilde{y}^{\epsilon-1} \underbrace{\big[\epsilon - \tilde{\gamma}\big]}_{>0} > 0.$$

So, the function $\tilde{u}(\cdot)$ has a convex kink at $\tilde{y}$.

Because of the convex kink, it is known (see, e.g., Décamps et al., 2006) that there exists a small $\eta > 0$ such that

$$\big(\tilde{y} - \eta, \tilde{y} + \eta\big) \not\subseteq \mathbb{S}.$$

In other words, close to the point $\tilde{y}$ where the weaker supplier is likely to forego trading with the buyer, the buyer will be strategically ambivalent between (i) waiting for a decrease in demand to ensure clarity with regard to the pool of suppliers, as the weaker supplier pulls out, and (ii) waiting for an increase in demand, which will at least for a short period of time, will ensure more favorable equilibrium conditions in the input market because the weaker supplier will indeed accept the trade.

6 Conclusion

Main managerial insights. Our paper derive a set of insights useful for practice. First, the level of competition among suppliers influences a buyer's investment. Accessing a broadeer supplier base would lower the input prices, which would give the buyer more market power. Second, while suppliers may offer supply contracts to stabilize their cash flows, the buyer shouild refrain from them because it gives them less market power and reduce their propensity to invest. Finally, we endogenize the size of the supplier base by introducing asymmetry with respect to the fixed operating costs. In such a case, the buyer may delay investment to gain market power.

Limitations. We identify numerous limitations. First, because in the example of demand function we consider, the demand state enters the problem linearly, uncertainty considerations do not the relative terms of the tension between dynamic sourcing and a long-term supply contract. Second, the suppliers were assumed to be able to infer the demand from the buyer and the best-reply function of their rivals. This requires a high degree of transparency, which is not realistic. We assume that purchase orders materialize into the inflow of product in the inventory. Recent events (e.g., Suez Canal Blockage, War in Ukraine, and Panama Canal drought) remind us that the buyer may want to build up inventory to achieve resilience in case of supply chain disruption. We leave these and other topics for future research.

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Appendices

A Proof of Proposition 1

Case A. Given the cost specifications, the demand from the buyer in eq. (1) becomes

$$\bar{q}(y,c) = \left(\frac{1-\delta}{c}\right)^{\frac{1}{\delta}} y^{\frac{1}{\delta}}$$

and, so, that the inverse demand faced by the suppliers is

$$q \mapsto \bar{q}(y,\cdot)^{-1}(q) = (1-\delta) y q^{-\delta}.$$

It follows by differentiation that $\bar{R}(Q)$ in eq. (2) satisfies

$$\frac{c_0}{1-\delta}\frac{1}{y} = \left(\bar{R}(Q)+Q\right)^{-\delta-1}\left[(1-\delta)\bar{R}(Q)+Q\right].$$

So, $\bar{q}_n(y)$ in eq. (18) solves

$$\begin{aligned}\frac{c_0}{1-\delta}\frac{1}{y} =& \left[n\bar{q}_n(y)\right]^{-\delta-1}\left[(1-\delta)\bar{q}_n(y)+(n-1)\bar{q}_n(y)\right] \\ =& [n\bar{q}_n(y)]^{-\delta}\left(1-\tfrac{\delta}{n}\right).\end{aligned}$$

Hence,

$$\bar{q}_n(y) = \tfrac{1}{n}\left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0}y\right]^{\frac{1}{\delta}} \tag{25}$$

It now follows from eq. (5) that

$$\bar{c}_n = (1-\delta)y\left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0}y\right]^{-1} = \frac{c_0}{1-\frac{\delta}{n}}. \tag{26}$$

We get the equilibrium output price from eqs. (6) and (7):

$$\bar{P}_n(y) \equiv \frac{c_0}{1-\frac{\delta}{n}}\frac{1}{1-\delta} = \frac{\bar{c}_n}{1-\delta}.$$

Furthermore,

$$\bar{Q}_n(y) = \left[\tfrac{1-\delta}{\bar{c}_n} y\right]^{\frac{1}{\delta}}.$$

After substitution, the buyer's profit is now:

$$\begin{aligned}
\bar{\pi}_n(y) &= \left\{ \left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0} y\right]^{\frac{1}{\delta}} y \left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0} y\right]^{-\frac{\delta}{\delta}} - \tfrac{c_0}{1-\frac{\delta}{n}} \left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0} y\right]^{\frac{1}{\delta}} \right\} \\
&= \left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0}\right]^{\frac{1}{\delta}} \left\{ \tfrac{c_0}{1-\frac{\delta}{n}}\tfrac{1}{1-\delta} - \tfrac{c_0}{1-\frac{\delta}{n}} \right\} y^{\frac{1}{\delta}} \\
&= \delta \tfrac{c_0}{1-\delta}\tfrac{1}{1-\frac{\delta}{n}} \left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0}\right]^{\frac{1}{\delta}} y^{\frac{1}{\delta}}.
\end{aligned}$$

In contrast, the supplier's profit is given by

$$\begin{aligned}
\pi_n(y) &= \left[\frac{c_0}{1-\delta} - c_0\right]\frac{1}{n}\left[(1-\tfrac{\delta}{n})\frac{1-\delta}{c_0} y\right]^{\frac{1}{\delta}} \\
&= y^{\frac{1}{\delta}} \frac{\delta}{n} \left[\tfrac{1-\delta}{c_0}\right]^{\frac{1-\delta}{\delta}} \left(\tfrac{n-\delta}{n}\right)^{\frac{1}{\delta}}.
\end{aligned}$$

Case B. When $C(c,q) = \frac{1}{2}cq^2$, we get the suppliers' inverse demand function

$$q \mapsto \bar{q}(y,\cdot)^{-1}(q) = (1-\delta) y q^{-\delta-1}.$$

In this case, $\bar{R}(Q)$ satisfies

$$0 = -c_0 + (1-\delta) y \Big\{ (\bar{R}(Q)+Q)^{-(\delta+1)} - \bar{R}(Q)(1+\delta)(\bar{R}(Q)+Q)^{-(\delta+2)} \Big\}.$$

It follows that $\bar{q}_n$ satisfies

$$0 = -\,c_0 + (1-\delta) y \Big\{ (n\bar{q}_n)^{-(\delta+1)} - \bar{q}_n(1+\delta)(n\bar{q}_n)^{-(\delta+2)} \Big\} \quad \iff \quad \bar{q}_n = \tfrac{1}{n}\left[\tfrac{1-\delta}{c_0}(1-\tfrac{1+\delta}{n}) y\right]^{\frac{1}{1+\delta}}.$$

If we substitute the total output into the inverse demand function, we get

$$\bar{c}_n(y) = (1-\delta)y\left[\left[\tfrac{1-\delta}{c_0}(1-\tfrac{1+\delta}{n})y\right]^{\frac{1}{1+\delta}}\right]^{-(\delta+1)} = (1-\delta)y\tfrac{c_0}{1-\delta}\frac{1}{1-\frac{1+\delta}{n}}\frac{1}{y} = \frac{c_0}{1-\frac{1+\delta}{n}}.$$

From the inverse demand function, the equilibrium output price is

$$\bar{P}_n(y) = \left[\frac{c_0}{1-\delta}\frac{1}{1-\frac{1+\delta}{n}}\right]^{\frac{\delta}{1+\delta}} y^{\frac{1}{1+\delta}} = \left[\frac{\bar{c}_n}{1-\delta}\right]^{\frac{\delta}{1+\delta}} y^{\frac{1}{1+\delta}}.$$

By substitution, the supplier's profit is

$$\begin{aligned}
\pi_n(y) =& \left[\frac{c_0}{1-\frac{1+\delta}{n}} - c_0\right]\frac{1}{n}\left(\frac{1-\delta}{c_0}\right)^{\frac{1}{\delta+1}}\left(1-\frac{1+\delta}{n}\right)^{\frac{1}{\delta+1}} y^{\frac{1}{\delta+1}}\\
=&\frac{c_0}{1-\frac{1+\delta}{n}}\frac{1+\delta}{n^2}\left(\frac{1-\delta}{c_0}\right)^{\frac{1}{\delta+1}}\left(1-\frac{1+\delta}{n}\right)^{\frac{1}{\delta+1}} y^{\frac{1}{\delta+1}}\\
=&c_0\frac{1+\delta}{n^2}\left(\frac{1-\delta}{c_0}\right)^{\frac{1}{\delta+1}}\left(\frac{n-1-\delta}{n}\right)^{-\frac{\delta}{\delta+1}} y^{\frac{1}{\delta+1}}\\
=&\frac{1-\delta^2}{n^2}\left(\frac{1-\delta}{c_0}\right)^{-\frac{\delta}{\delta+1}}\left(\frac{n-1-\delta}{n}\right)^{-\frac{\delta}{\delta+1}} y^{\frac{1}{\delta+1}}.
\end{aligned}$$

Case C. The inverse demand function is the one from Case B. Because $C_0(q) = \frac{1}{2}c_0q^2$, the term $\bar{R}(Q)$ in eq. (17) obtains to satisfy

$$(1-\delta)y(\bar{R}(Q)+Q)^{-(2+\delta)}\left[Q-\delta\bar{R}(Q)\right] - c_0\bar{R}(Q) = 0,$$

and, so, $\bar{q}_n$ satisfies

$$(1-\delta)y(n\bar{q}_n)^{-(2+\delta)}[n-1-\delta] - c_0 = 0 \quad\iff\quad \bar{q}_n = \tfrac{1}{n}\left[(n-1-\delta)\tfrac{1-\delta}{c_0}y\right]^{\frac{1}{2+\delta}}.$$

By substituting the total outputs of the suppliers into the inverse demand function yields the equilibrium price

$$\bar{c}_n = [(1-\delta)y]^{\frac{1}{\delta+2}}\left[\frac{n-1-\delta}{c_0}\right]^{-\frac{1+\delta}{\delta+2}}.$$

Furthermore,

$$\bar{P}_n(y) = y\left[(n-1-\delta)\tfrac{1-\delta}{c_0}y\right]^{-\frac{\delta}{2+\delta}}.$$

But we know that

$$\frac{1-\delta}{\bar{c}_n}y = \left[(1-\delta)y\tfrac{n-1-\delta}{c_0}\right]^{\frac{1+\delta}{\delta+2}} \qquad \Longrightarrow \qquad (n-1-\delta)\tfrac{1-\delta}{c_0}y = \left[\frac{1-\delta}{\bar{c}_n(y)}y\right]^{\frac{\delta+2}{\delta+1}}.$$

Hence,

$$\bar{P}_n(y) = \left[\tfrac{1-\delta}{\bar{c}_n(y)}\right]^{-\frac{\delta}{\delta+1}} y^{\frac{1}{\delta+1}} \quad \text{and} \quad \bar{Q}_n(y) = \left[\tfrac{1-\delta}{\bar{c}_n(y)}y\right]^{\frac{1}{\delta+1}}.$$

By substitution, the buyer's profit is given by

$$\begin{aligned}
\bar{\pi}_n(y) &= \left[\frac{n-1-\delta}{c_0}(1-\delta)y\right]^{\frac{1-\delta}{2+\delta}} y - \frac{1}{2}\left[(1-\delta)y\right]^{\frac{1}{\delta+2}}\left[\frac{n-1-\delta}{c_0}\right]^{-\frac{1+\delta}{\delta+2}}\left[\frac{n-1-\delta}{c_0}(1-\delta)y\right]^{\frac{2}{2+\delta}} \\
&= y^{\frac{3}{\delta+2}}\left\{\left[\frac{n-1-\delta}{c_0}\right]^{\frac{1-\delta}{\delta+2}}(1-\delta)^{\frac{1-\delta}{\delta+2}} - \frac{1}{2}(1-\delta)^{\frac{3}{\delta+2}}\left[\frac{n-1-\delta}{c_0}\right]^{\frac{1-\delta}{\delta+2}}\right\} \\
&= y^{\frac{3}{\delta+2}}\left[\frac{n-1-\delta}{c_0}\right]^{\frac{1-\delta}{\delta+2}}(1-\delta)^{\frac{1-\delta}{\delta+2}}\left\{1-\frac{1}{2}(1-\delta)^{\frac{2+\delta}{\delta+2}}\right\} \\
&= y^{\frac{3}{\delta+2}}\left[\frac{n-1-\delta}{c_0}(1-\delta)\right]^{\frac{1-\delta}{\delta+2}}\tfrac{1+\delta}{2}.
\end{aligned}$$

By substitution, the supplier's profit is

$$\begin{aligned}
\pi_n(y) &= \left[(1-\delta)y\right]^{\frac{1}{\delta+2}}\left[\tfrac{n-1-\delta}{c_0}\right]^{-\frac{1+\delta}{\delta+2}}\frac{1}{n}\left[(n-1-\delta)\frac{1-\delta}{c_0}y\right]^{\frac{1}{\delta+2}} - \frac{c_0}{2}\frac{1}{n^2}\left[(n-1-\delta)\frac{1-\delta}{c_0}\right]^{\frac{2}{\delta+2}} \\
&= \frac{y^{\frac{2}{\delta+2}}}{n^2}\left\{n(1-\delta)^{\frac{2}{\delta+2}}\left[\tfrac{n-1-\delta}{c_0}\right]^{-\frac{\delta}{\delta+2}} - \frac{c_0}{2}\left[\tfrac{n-1-\delta}{c_0}\right]^{\frac{2}{\delta+2}}\left[1-\delta\right]^{\frac{2}{\delta+2}}\right\} \\
&= \frac{y^{\frac{2}{\delta+2}}}{n^2}(1-\delta)^{\frac{2}{\delta+2}}\left[\tfrac{n-1-\delta}{c_0}\right]^{-\frac{\delta}{\delta+2}}\left\{n - \tfrac{c_0}{2}\tfrac{n-1-\delta}{c_0}\right\} \\
&= y^{\frac{2}{\delta+2}}(1-\delta)^{\frac{2}{\delta+2}}\left[\tfrac{n-1-\delta}{c_0}\right]^{-\frac{\delta}{\delta+2}}\frac{n+1+\delta}{2n^2}.
\end{aligned}$$

Case D. Here, the inverse demand function is $q \mapsto \bar{q}(y,\cdot)^{-1}(q) = (1-\delta)yq^{-\delta}$. It follows that $\bar{R}(Q)$

satisfies

$$0 = -c_0 \bar{R}(Q) + (1-\delta) y \left[\bar{R}(Q) + Q\right]^{-\delta-1} \left\{Q + (1-\delta)\bar{R}(Q)\right\}$$

and that $\hat{q}_n$ satisfies

$$0 = -c_0 + (1-\delta) y \left[n\bar{q}_n\right]^{-\delta-1} \{n-\delta\} \quad \Longleftrightarrow \quad \bar{q}_n = \frac{1}{n}\left[(n-\delta)\tfrac{1-\delta}{c_0} y\right]^{\frac{1}{1+\delta}}.$$

Hence, from the inverse demand function of the suppliers, we get

$$\bar{c}_n = (1-\delta) y \left[(n-\delta)\frac{1-\delta}{c_0} y\right]^{-\frac{\delta}{\delta+1}} = (1-\delta)\left[(n-\delta)\frac{1-\delta}{c_0}\right]^{-\frac{\delta}{\delta+1}} y^{\frac{1}{\delta+1}}.$$

It follows that

$$(n-\delta)\frac{1-\delta}{c_0} y = \left[\frac{1-\delta}{\bar{c}_n(y)} y\right]^{\frac{\delta+1}{\delta}},$$

from which

$$\bar{Q}_n(y) = \left[\frac{1-\delta}{\bar{c}_n(y)} y\right]^{\frac{1}{\delta}} \quad \text{and} \quad \bar{P}_n(y) = \frac{\bar{c}_n(y)}{1-\delta}.$$

By substitution,

$$\begin{aligned}
\pi_n(y) &= (1-\delta)^{\frac{1-\delta}{\delta+1}} \left[\frac{n-\delta}{c_0}\right]^{-\frac{\delta}{\delta+1}} y^{\frac{1}{\delta+1}} \frac{1}{n} \left[(1-\delta)y\right]^{\frac{1}{\delta+1}} \left[\frac{n-\delta}{c_0}\right]^{\frac{1}{\delta+1}} - \frac{c_0}{2}\frac{1}{n^2}\left[\frac{n-\delta}{c_0}\right]^{\frac{2}{\delta+1}} \left[(1-\delta)y\right]^{\frac{2}{\delta+1}} \\
&= \frac{y^{\frac{2}{\delta+1}}}{n^2}(1-\delta)^{\frac{2-\delta}{\delta+1}} \left[\tfrac{n-\delta}{c_0}\right]^{\frac{1-\delta}{\delta+1}} \left\{n - \frac{c_0}{2}\left[\frac{n-\delta}{c_0}\right]^{\frac{2-1+\delta}{\delta+1}} \left[(1-\delta)\right]^{\frac{2-2+\delta}{\delta+1}}\right\} \\
&= \frac{y^{\frac{2}{\delta+1}}}{n^2}(1-\delta)^{\frac{2-\delta}{\delta+1}} \left[\tfrac{n-\delta}{c_0}\right]^{\frac{1-\delta}{\delta+1}} \left\{n - \frac{n-\delta}{2}\left[1-\delta\right]^{\frac{\delta}{\delta+1}}\right\}
\end{aligned}$$

This completes the proofs of proposition 1.

B Proof of Corollary 1

The term $\bar{u}_n(y)$ in eq. (11) then becomes

$$\bar{u}_n(y) = a_n \mathbb{E}^y \int_0^\infty e^{-rt} Y_t^\epsilon \mathrm{d}t.$$

We now consider the stochastic process $(Y_t^\epsilon)_t$. By the Itô-Döblin formula,

$$\begin{aligned}\mathrm{d}Y_t^\epsilon &= \left[\tfrac{1}{2}\sigma^2 Y_t^2 \epsilon(\epsilon-1) Y_t^{\epsilon-2} + \mu Y_t \epsilon Y_t^{\epsilon-1}\right] \mathrm{d}t + \sigma Y_t \epsilon Y_t^{\epsilon-1} \mathrm{d}Z_t \\ &= \mathrm{m}(\epsilon) Y_t^\epsilon \mathrm{d}t + \sigma \epsilon Y_t^\epsilon \mathrm{d}Z_t,\end{aligned}$$

where

$$\mathrm{m}(\epsilon) := \tfrac{1}{2}\sigma^2 \epsilon(\epsilon-1) + \mu\epsilon. \tag{27}$$

So $(Y_t^\epsilon)_t$ follows a GBM. It follows by standard properties of GBMs that $\mathbb{E}^y Y_t^\epsilon = y^\epsilon e^{\mathrm{m}(\epsilon)t}$ and that

$$\mathbb{E}^y \int_0^\infty e^{-rt} Y_t^\epsilon \mathrm{d}t = y^\epsilon \int_0^\infty e^{\mathcal{Q}(\epsilon)t} \mathrm{d}t,$$

for $\mathcal{Q}(\cdot)$ given in eq. (12), converges to

$$\mathbb{E}^y \int_0^\infty e^{-rt} Y_t^\epsilon \mathrm{d}t = -\frac{y^\epsilon}{\mathcal{Q}(\epsilon)} \text{ iff } \mathcal{Q}(\epsilon) < 0.$$

C Proof of Proposition 2

Equation (14) describes the classical problem of McDonald and Siegel (1986). We drop the index n in the notation α_n and introduce the differential operator

$$\mathcal{L} := \frac{1}{2}\sigma^2 y^2 \frac{\partial^2}{\partial y^2} + \mu y \frac{\partial}{\partial y} - r\mathbf{1}. \tag{28}$$

The dynamic programming equation for the problem in eq. (14) is a variational inequality (VI):

$$\begin{cases} \max\Big\{\alpha y^\epsilon - I - \psi(y); \mathcal{L}\psi(y)\Big\} & = 0, \quad \text{a.e. } y > 0, \\ \lim\limits_{y\downarrow 0} \psi(y) & = 0, \\ \lim\limits_{y\uparrow\infty} \dfrac{\psi(y)}{y^\epsilon} & = 1. \end{cases} \tag{29}$$

We have

$$\mathcal{L}(\alpha \cdot^\epsilon - I)(y) \equiv -[r - \mathrm{m}(\epsilon)]\alpha y^\epsilon + rI \tag{30}$$

for $\mathrm{m}(\cdot)$ given in eq. (27). If $r > \mathrm{m}(\epsilon)$, $y \mapsto \mathcal{L}(\alpha \cdot^\epsilon - I)(y)$ is monotone decreasing on $(0, \infty)$ from $rI > 0$ to $-\infty$, so it has a unique root, denoted $y^\star$. We conjecture that $\{\psi > \alpha \cdot^\epsilon - I\} = (0, \bar{y}) \subset (0, y^\star)$, where $\bar{y}$ obtains by smooth fit. If this conjecture holds, then $\psi(\cdot)$ solves the free-boundary problem (FBP)

$$\begin{aligned} \psi(0+) &= 0, \\ \mathcal{L}\psi(y) &= 0, \quad \forall y \in (0, \bar{y}), \\ \psi(\bar{y}) &= \alpha \bar{y}^\epsilon - I, \\ \psi'(\bar{y}) &= \alpha\epsilon y^{\epsilon-1}. \end{aligned}$$

The function $\mathcal{Q}(\cdot)$ in eq. (12) is convex, attains its minimum at the point $\gamma_\star := -\frac{\mu - \frac{1}{2}\sigma^2}{\sigma^2}$, and satisfies $\mathcal{Q}(\pm\infty) = \infty$. Further, because $\mathcal{Q}(1) = \mathrm{m}(\epsilon) - r < 0$, the minimum is necessarily a negative minimum and $\mathcal{Q}(\cdot)$ has a positive root γ, which is unique because $\mathcal{Q}(\cdot)$ is monotone increasing on $(\max\{1; \gamma_\star\}, \infty)$. Standard computations lead us to conclude that the function $\psi(\cdot)$ given in Proposition 2 solves the FBP.

It remains to verify that this $\psi(\cdot)$ solves the variational ineq. (29). We look at the two intervals.

$(\bar{y}, \infty)$. For ψ to solve the VI in this interval, we must have $\mathcal{L}\psi = \mathcal{L}(\alpha \cdot^\epsilon - I)(y) \leq 0$. From eq. (30)

and the expression for $\bar{y}$,

$$\begin{aligned}\mathcal{L}(\alpha \cdot^{\epsilon} - I)(\bar{y}) &= = I[r - \mathrm{m}(\epsilon)]\Big[\frac{1}{1-\frac{\epsilon}{\gamma}} - \frac{1}{1-\frac{\mathrm{m}(\epsilon)}{r}}\Big] \\ &= \underbrace{-I[r-\mathrm{m}(\epsilon)]}_{<0} \int_{\frac{\mathrm{m}(\epsilon)}{r}}^{\frac{\epsilon}{\gamma}} \underbrace{\frac{1}{(1-\zeta)^2}}_{>0} \mathrm{d}\zeta.\end{aligned}$$

But, it follows from eq. (12) after simplifications that

$$\mathcal{Q}\left(\tfrac{r\epsilon}{\mathrm{m}(\epsilon)}\right) = \frac{1}{2}\frac{r-\mathrm{m}(\epsilon)}{[\mu + \frac{1}{2}\sigma^2(\epsilon-1)]^2} r\sigma^2 > 0 \text{ because } r > \mathrm{m}(\epsilon).$$

Because $\mathcal{Q}(\cdot)$ is monotone increasing on $(\max\{1;\gamma_\star\},\infty)$ and $\mathcal{Q}(\infty)=\infty$, the root γ satisfies $\gamma < \frac{r\epsilon}{\mathrm{m}(\epsilon)}$. It immediately follows $\frac{\epsilon}{\gamma} > \frac{\mathrm{m}(\epsilon)}{r}$. Hence, $\mathcal{L}(\alpha \cdot^{\epsilon} - I)(\bar{y}) < 0$. So the FBP's solution $\psi(\cdot)$ verifies the VI in the interval $(\bar{y},\infty)$.

$\underline{(0,\bar{y})}$. We want to verify that $\psi(y) \geq \alpha y^{\epsilon} - I$. We note that $\psi(\cdot)$ also reads

$$\psi(y) = \frac{\alpha\epsilon}{\gamma} y^{\gamma}\bar{y}^{\epsilon-\gamma} \text{ in the interval } (0,\bar{y}).$$

We define $\tilde{\psi}(y) := \psi(y) - \alpha y^{\epsilon} + I$. In the interval $(0,\bar{y})$,

$$\tilde{\psi}'(y) = \alpha\epsilon\left[y^{\gamma-1}\bar{y}^{\epsilon-\gamma} - y^{\epsilon-1}\right] = \alpha\epsilon y^{\epsilon-1}\left[\left(\tfrac{y}{\bar{y}}\right)^{\gamma-\epsilon} - 1\right].$$

We note that

$$\mathcal{Q}(\epsilon) = \frac{1}{2}\sigma^2\epsilon(\epsilon-1) + \mu\epsilon - r = -[r - \mathrm{m}(\epsilon],$$

for $\mathrm{m}(\cdot)$ given in eq. (27). Given the assumption $r > \mathrm{m}(\epsilon)$, it follows that $\mathcal{Q}(\epsilon) < 0$ and, so, given the behavior of $\mathcal{Q}(\cdot)$, we have $\gamma > \epsilon$. It follows that $\tilde{\psi}'(\cdot) \leq 0$ on $(0,\bar{y})$.

Further, $\tilde{\psi}(\bar{y}) = 0$ by value matching. Hence, $\tilde{\psi}(\cdot)$ necessarily decreases on $(0,\bar{y})$ from a positive value and vanishes at the right boundary. It follows that $\psi(y) \geq \alpha y^{\epsilon} - I$ and, so, that the FBP's solution $\psi(\cdot)$ solves the VI in this interval as well.

We conclude with the verification theorem. Let ψ be a supersolution of the variational ineq. (29).

For an arbitrary stopping time τ, it follows from Dynkin's formula that

$$\begin{aligned}\psi(y) &= \mathbb{E}^y\Big[e^{-r\tau}\underbrace{\psi(Y_\tau)}_{\geq \alpha Y_\tau^\epsilon - I} - \int_0^\tau e^{-rt}\underbrace{\mathcal{L}\psi(Y_t)}_{\leq 0}\,\mathrm{d}t\Big] \\ &\geq \mathbb{E}^y e^{-r\tau}\big\{\alpha Y_\tau^\epsilon - I\big\}.\end{aligned}$$

Then, a supersolution of the VI exceeds the value function. Let $\psi(\cdot)$ denote the classical solution of the VI and take $\hat{\tau} := \inf\big\{t \geq 0 \big| \psi(Y_t) \geq \alpha Y_t - I\big\}$. Proceeding similarly, we obtain that the solution of the VI is the smallest supersolution and coincides with the value function in eq. (14). This concludes the proof of Proposition 2.

D Proof of Corollary 2

Assume $N \geq n$ and that the conditions in Proposition 2 are met. It follows from eq. (13) that $\alpha_N Y_\tau^\epsilon - I \geq \alpha_N Y_\tau^\epsilon - I$ and, so, that

$$\psi_N(y) \geq \mathbb{E}^y e^{-r\hat{\tau}_n}\Big[\alpha_N Y_{\hat{\tau}_n}^\epsilon - I\Big] \geq \psi_n(y) := \mathbb{E}^y e^{-r\hat{\tau}_n}\Big[\alpha_N Y_\tau^\epsilon - I\Big].$$

Further, $\bar{y}_N \leq \bar{y}_n$ by monotonicity of the map $a \mapsto \big(\frac{\gamma}{\gamma-\epsilon}\frac{I}{a}\big)$. This completes the proof.

E Proof of Proposition 3

We assume $r > \mu$ and consider the inverse demand function in eq. (7). It follows from standard properties of GBM that the term in eq. (16) simplifies to

$$u(q; y, c) := \frac{y}{r-\mu}q^{1-\delta} - \frac{1}{r}C(c, q).$$

Case A. Because $\delta \in (0, 1)$, when $C(c, q) = cq$ where $c \geq 0$, the value-maximizing output for the supplier is

$$\hat{q}(y, c) = \left(\frac{r}{r-\mu}\frac{1-\delta}{c}\right)^{\frac{1}{\delta}} y^{\frac{1}{\delta}} \tag{31}$$

and the inverse demand faced by the suppliers is

$$q \mapsto \hat{q}(y,\cdot)^{-1}(q) = \frac{r}{r-\mu}(1-\delta) y q^{-\delta}.$$

It follows by differentiation that $\hat{R}(Q)$ in eq. (17) satisfies

$$\frac{c_0}{1-\delta}\frac{r-\mu}{r}\frac{1}{y} = \left(\hat{R}(Q)+Q\right)^{-\delta-1}\left[(1-\delta)\hat{R}(Q)+Q\right].$$

So, $\hat{q}_n(y)$ in eq. (18) solves

$$\begin{aligned}\frac{c_0}{1-\delta}\frac{r-\mu}{r}\frac{1}{y} =& \left[n\hat{q}_n(y)\right]^{-\delta-1}\left[(1-\delta)\hat{q}_n(y)+(n-1)\hat{q}_n(y)\right] \\ =& [n\hat{q}_n(y)]^{-\delta}\left(1-\tfrac{\delta}{n}\right).\end{aligned}$$

Hence,

$$\hat{q}_n(y) = \tfrac{1}{n}\left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0}\tfrac{r}{r-\mu}y\right]^{\frac{1}{\delta}} \tag{32}$$

It now follows from eqs. (20) and (31) that

$$\hat{c}_n(y) \equiv \frac{r}{r-\mu}(1-\delta)y\left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0}\tfrac{r}{r-\mu}y\right]^{-1} = \frac{c_0}{1-\frac{\delta}{n}}. \tag{33}$$

The NPV in eq. (15') now reads

$$\begin{aligned}\hat{u}_n(y) &= \frac{y}{r-\mu}\left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0}\tfrac{r}{r-\mu}y\right]^{\frac{1-\delta}{\delta}} - \frac{1}{r}\frac{c_0}{1-\frac{\delta}{n}}\left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0}\tfrac{r}{r-\mu}y\right]^{\frac{1}{\delta}} \\ &= \beta_n y^{\frac{1}{\delta}},\end{aligned}$$

where β_n is given by

$$\beta_n := \left[\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0}\tfrac{r}{r-\mu}\right]^{\frac{1}{\delta}-1}\left[\tfrac{1}{r-\mu} - \tfrac{1}{r}\tfrac{c_0}{1-\frac{\delta}{n}}\left(1-\tfrac{\delta}{n}\right)\tfrac{1-\delta}{c_0}\tfrac{r}{r-\mu}\right],$$

which simplifies as per Proposition 3.

Case B. When $C(c,q)=\frac{1}{2}cq^2$, we have

$$u(q;y,c) := \frac{y}{r-\mu}q^{1-\delta} - \frac{1}{2r}cq^2, \qquad \text{(??')}$$

from which we get the suppliers' inverse demand function

$$q \mapsto \hat{q}(y,\cdot)^{-1}(q) = \frac{r}{r-\mu}(1-\delta)yq^{-\delta-1}.$$

In this case,

$$\hat{R}(Q) := \arg\max_{q\geq 0}\Big\{q\hat{q}(y,\cdot)^{-1}(q+Q) - c_0 q\Big\} \in \mathbb{R}_+$$

satisfies

$$0 = -c_0 + \frac{r}{r-\mu}(1-\delta)y\Big\{(\hat{R}(Q)+Q)^{-(\delta+1)} - \hat{R}(Q)(1+\delta)(\hat{R}(Q)+Q)^{-(\delta+2)}\Big\}.$$

It follows that $\hat{q}_n$ satisfies

$$0 = -\,c_0 + \frac{r}{r-\mu}(1-\delta)y\Big\{(n\hat{q}_n)^{-(\delta+1)} - \hat{q}_n(1+\delta)(n\hat{q}_n)^{-(\delta+2)}\Big\},$$

which yields

$$\hat{q}_n = \tfrac{1}{n}\left[\tfrac{1-\delta}{c_0}(1-\tfrac{1+\delta}{n})\tfrac{r}{r-\mu}y\right]^{\frac{1}{1+\delta}}.$$

If we substitute the total output into the inverse demand function, we get

$$\hat{c}_n(y) = \frac{c_0}{1-\frac{1+\delta}{n}}.$$

The expression for $\hat{u}_n(\cdot)$ obtains after simplifications.

Case C. The inverse demand function is the one from Case B. Because $C_0(q)=\frac{1}{2}c_0q^2$, the term $\hat{R}(Q)$ in eq. (17) obtains to satisfy

$$\frac{r}{r-\mu}(1-\delta)y(\hat{R}(Q)+Q)^{-(2+\delta)}\Big[Q-\delta\hat{R}(Q)\Big] - c_0\hat{R}(Q) = 0,$$

and, so, $\hat{q}_n$ satisfies

$$\frac{r}{r-\mu}(1-\delta)y(n\hat{q}_n)^{-(2+\delta)}[n-1-\delta]-c_0=0 \quad\Longleftrightarrow\quad \hat{q}_n=\tfrac{1}{n}\left[(n-1-\delta)\tfrac{1-\delta}{c_0}\tfrac{r}{r-\mu}y\right]^{\frac{1}{2+\delta}}.$$

By substiting the total outputs of the suppliers into the inverse demand function yields the equilibrium price

$$\hat{c}_n=\frac{r}{r-\mu}(1-\delta)y^{\frac{1}{\delta+2}}\left[(n-1-\delta)\frac{1-\delta}{c_0}\frac{r}{r-\mu}\right]^{-1+\frac{1}{\delta+2}}.$$

The expression for $\hat{u}_n(\cdot)$ is obtained after simplifications.

Case D. Here, the inverse demand function is

$$q\mapsto\hat{q}(y,\cdot)^{-1}(q)=\frac{r}{r-\mu}(1-\delta)yq^{-\delta}.$$

It follows that $\hat{R}(Q)$ satisfies

$$0=-c_0\hat{R}(Q)+\frac{r}{r-\mu}(1-\delta)y\left[\hat{R}(Q)+Q\right]^{-\delta-1}\left\{Q+(1-\delta)\hat{R}(Q)\right\}$$

and that $\hat{q}_n$ satisfies

$$0=-c_0+\frac{r}{r-\mu}(1-\delta)y\,[n\hat{q}_n]^{-\delta-1}\{n-\delta\} \quad\Longleftrightarrow\quad \hat{q}_n=\frac{1}{n}\left[(n-\delta)\tfrac{1-\delta}{c_0}\tfrac{r}{r-\mu}y\right]^{\frac{1}{1+\delta}}.$$

Hence,

$$\hat{c}_n=\frac{r}{r-\mu}(1-\delta)y\left[(n-\delta)\frac{1-\delta}{c_0}\frac{r}{r-\mu}y\right]^{-\frac{\delta}{\delta+1}}$$

It follows the expression for $\hat{u}_n(\cdot)$ for Case D. This completes the proof.

F Proof of Proposition 4

After simplifications, it follows from Corollary 1 and proposition 3 that

$$\frac{\beta_n}{\alpha_n} = \begin{cases} \Gamma\big(\frac{1}{\delta}\big), & \text{in Case A,} \\ \Gamma\big(\frac{2}{\delta+1}\big), & \text{in Case B,} \\ \Gamma\big(\frac{3}{\delta+2}\big), & \text{in Case C,} \\ \Gamma\big(\frac{2}{\delta+1}\big), & \text{in Case D,} \end{cases}$$

where $\Gamma(\cdot)$ is defined by

$$\Gamma(\epsilon) := -\frac{\mathcal{Q}(\epsilon)}{r-\mu}\left[\frac{r}{r-\mu}\right]^{\epsilon-1}, \tag{34}$$

with $\mathcal{Q}(\cdot)$ in eq. (12). We consider the function $\Gamma(\cdot)$ in eq. (34). After simplifications,

$$\Gamma'(\epsilon) = \frac{1}{r}\left(\frac{r}{r-\mu}\right)^{\epsilon}\tilde{\Gamma}(\epsilon), \text{ where } \tilde{\Gamma}(\epsilon) := \ln\left(\frac{r}{r-\mu}\right)\big[r - \mathrm{m}(\epsilon)\big] - \mathrm{m}'(\epsilon).$$

For $\mu \geq 0$, we have

$$\tilde{\Gamma}''(\epsilon) = -\ln\left(\tfrac{r}{r-\mu}\right)\sigma^2 \leq 0.$$

so

$$\epsilon \mapsto \tilde{\Gamma}'(\epsilon) = -\ln\left(\frac{r}{r-\mu}\right)\left[\frac{1}{2}\sigma^2(2\epsilon-1)+\mu\right] - \sigma^2$$

is monotone decreasing on $(-\infty, \infty)$ from ∞ to $-\infty$ and has a unique root given by

$$\epsilon_\dagger := -\frac{1}{\sigma^2}\big[\mu - \tfrac{1}{2}\sigma^2\big] - \frac{1}{\ln(\frac{r}{r-\mu})}.$$

So the function $\tilde{\Gamma}(\cdot)$ is monotone increasing on $(-\infty, \epsilon_\dagger)$ from $-\infty$ to $\tilde{\Gamma}(\epsilon_\dagger)$ and monotone decreasing on $(\epsilon_\dagger, \infty)$ from $\tilde{\Gamma}(\epsilon_\dagger)$ to $-\infty$. We have

$$\tilde{\Gamma}(\epsilon_\dagger) = \frac{\sigma^2}{2}\frac{1}{\ln(\frac{r}{r-\mu})} + \Big[\frac{1}{2\sigma^2}\big(\mu - \frac{1}{2}\sigma^2\big)^2 + r\Big]\ln(\frac{r}{r-\mu}) > 0.$$

So $\Gamma'(\cdot)$ has a unique root in $(\epsilon_\dagger, \infty)$, which we note $\epsilon_\star$ and necessarily corresponds to a global maximum of $\Gamma(\cdot)$. The function $\Gamma(\cdot)$ is concave. Because, from eq. (27), $\Gamma(0) = 1$ and $\Gamma(1) = 1$, we necessarily have $\epsilon_\star \in (0,1)$ and $\Gamma(\epsilon) \geq 0$ for all $\epsilon \in (0,1)$. Again, $r > \mathrm{m}(\epsilon)$ is equivalent to $\epsilon < \gamma$ where γ is the root of $\mathcal{Q}(\cdot)$ in eq. (12). We have $\Gamma(\epsilon) \leq 1$ for all $\epsilon \in [1, \gamma)$.

From the above, it follows that the function $\Gamma(\cdot)$ in eq. (34) is concave on $(-\infty, \infty)$ and has a positive global maximum which is attained at a point in $(0,1)$. It satisfies $\Gamma(\cdot) \geq 1$ on $(0,1)$ and $\Gamma(\cdot) \leq 1$ on $(1, \gamma)$. We note that $\frac{1}{\delta} \geq 1$ in Case A, $\frac{2}{\delta+1} \geq$ in Case B, $\frac{3}{\delta+2} \geq$ in Case C, and $\frac{2}{\delta+1} \geq 1$ in Case D. This completes the proof.

G Proof of Proposition 5

Supplier 2's exit decision. In case of the inverse demand function in eq. (7), $\pi_n(\cdot)$ is given in Proposition 1. We have

$$\mathbb{E}^y \int_0^\tau e^{-rt}\big\{\pi_2(Y_t) - K\big\}\mathrm{d}t = \mathbb{E}^y \int_0^\tau e^{-rt}\big\{\nu_2 Y_t^\varepsilon - K\big\}\mathrm{d}t,$$

for ν_n given above. The optimal stopping time for the problem in eq. (22) is given by

$$\vartheta := \inf\big\{t \geq 0 \big| Y_t \leq \tilde{y}\big\}, \quad \text{where} \quad \tilde{y} := \Big(-\frac{\tilde{\gamma}}{\tilde{\gamma} - \varepsilon}\frac{\mathcal{Q}(\varepsilon)}{r}\frac{K}{\nu_2}\Big)^{\frac{1}{\varepsilon}}$$

where $\tilde{y}$ is given in eq. (22') and $\tilde{\gamma}$ denotes the negative root of $\mathcal{Q}(\cdot)$ in eq. (12).

Buyer's NPV. The buyer's profit $\bar{\pi}_n(\cdot)$ in eq. (8) can be expressed in simpler form following the results in Corollary 1. The buyer's NPV in eq. (23) is given by

$$\tilde{u}(y) := \mathbb{E}^y\Big[\int_0^{\vartheta} e^{-rt}\big\{-\mathcal{Q}(\epsilon)\alpha_2 Y_t^{\epsilon}\big\}\mathrm{d}t + \int_{\vartheta}^{\infty} e^{-rt}\big\{-\mathcal{Q}(\epsilon)\alpha_1 Y_t^{\epsilon}\big\}\mathrm{d}t\Big], \tag{35}$$

for α_n specified in Corollary 1. It now follows from eq. (22') that

$$\begin{aligned}\tilde{u}(y) &= -\mathcal{Q}(\epsilon)\mathbb{E}^y\Big[\alpha_2\int_0^{\infty} e^{-rt}Y_t^{\epsilon}\mathrm{d}t + \big[\alpha_1-\alpha_2\big]\int_{\vartheta}^{\infty} e^{-rt}Y_t^{\epsilon}\mathrm{d}t\Big] \\ &= \alpha_2 y^{\epsilon} + \big[\alpha_1-\alpha_2\big]\big(y\wedge\tilde{y}\big)^{\epsilon}\psi(y) \quad \text{where} \quad \psi(y) := \mathbb{E}^y e^{-r\vartheta}.\end{aligned}$$

Clearly, from eq. (22'), $\psi(y) = 1$ if $0 < y < \tilde{y}$. If $y > \tilde{y}$, the function $\psi(\cdot)$ is obtained as the solution of the second-order ODE

$$\begin{cases} \frac{1}{2}\sigma^2 y^2\psi''(y) + \mu y\psi'(y) - r\psi(y) &= 0, \text{ for } y > \tilde{y}, \\ \psi(\tilde{y}) &= 1, \\ \psi(\infty) &= 0. \end{cases}$$

This solution has a general solution of the form $y \mapsto Ay^{\gamma} + By^{\tilde{\gamma}}$, where γ and $\tilde{\gamma}$ are the roots of eq. (12). The parameter A is set to 0, so ensure the asymptotic behavior at ∞. From the boundary condition $B = (1/\tilde{y})^{\tilde{\gamma}}$. It follows that

$$\tilde{u}(y) := \alpha_2 y^{\epsilon} + \big[\alpha_1-\alpha_2\big]\big(y\wedge\tilde{y}\big)^{\epsilon}\left(\frac{y\vee\tilde{y}}{\tilde{y}}\right)^{\tilde{\gamma}},$$

which is the expression in eq. (23'). This completes the proof.