Investment timing, upper reflecting barrier, and debt-equity financing

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Abstract: This study considers how an upper reflecting barrier affects the interaction between financing and investment decisions. Here, a magnitude of upper reflecting barrier can be regarded as the degree of intense market competition. We show that fierce competition (a decrease in upper reflecting barrier) reduces the amount of debt issuance and delays investment, which decreases the credit spreads and leverage.

Keywords: Capital structure; Real options; Competition; Default; Financing constraint.

JEL classification: G31; G32; G33.

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1 Model

In this section, we begin with describing the model. As a benchmark, we provide the value functions for the cash flow not having upper reflecting barrier. Finally, we derive the value functions for the cash flow having upper reflecting barrier.

1.1 Setup

Suppose a firm with an option to install a production facility and initiate operations (e.g., sell the commodity produced). Throughout the analysis, we assume capital markets are frictionless, with a constant risk-free interest rate $r > 0$, and all agents are risk-neutral and aim to maximize their expected payoff.

Once the firm initiates operations, it incurs a one-time fixed cost, $I > 0$, to install the production facility. Thereafter, the firm receives an instantaneous cash inflow $X(t)$, which follows the geometric Brownian motion given by

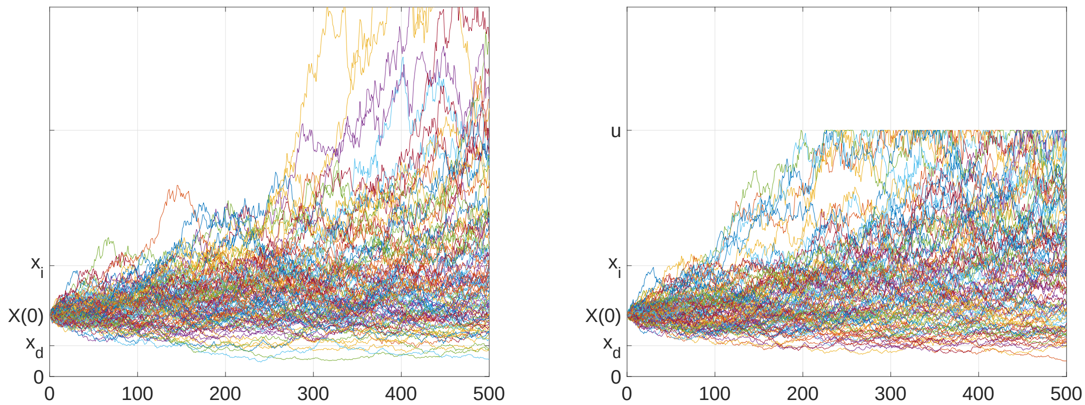
$$\frac{\mathrm{d}X(t)}{X(t)} = \mu \mathrm{d}t + \sigma \mathrm{d}z(t), \quad X(0) = x > 0, \tag{1}$$

where μ and σ are constants, and $z(t)$ denotes the Brownian motion defined by a risk-neutral probability space $(\Omega, \mathcal{F}, \mathbb{Q})$.[1] For convergence, we assume $r > \mu$.[2] In this study, the firm issues a mix of debt and equity in initiating operations. This notion of mixed financing is identical to that in Sundaresan and Wang (2007) and Shibata and Nishihara (2023) Importantly, debt benefits from the tax shield in that the firm faces a constant tax rate $\tau > 0$ on income after servicing the interest payment on the debt. For analytical convenience, this study limits the condition to perpetual debt (i.e., maturity is infinite). This assumption, as in Black and Cox (1976) and Leland (1994), simplifies the analysis without substantially altering the key economic insights. Thus, if the firm issues debt with an instantaneous coupon payment $c \geq 0$, its instantaneous cash flow is $(1-\tau)(X(t)-c)$.

In this study, the most important feature is that there exists an upper reflecting barrier. If the cash flow $X(t)$ climbs to some upper reflecting barrier u, it is immediately brought back to a slightly lower level. In technical terms, the threshold u becomes an

[1] This assumption is identical to that in Goldstein et al. (2001) and Sundaresan and Wang (2007).

[2] The assumption $r > \mu$ ensures that the value of the firm is finite. See Dixit and Pindyck (1994) and Hugonnier et al. (2015) for details.



(a) No barrier

(b) Barrier

Figure 1: Scenario with and without an upper reflecting barrier

upper reflecting barrier on the cash flow process. Following Dixit and Pindyck (1994), such an upper reflecting barrier can be thought as the threshold that a potential new entrant enters the market to initiate operations. As soon as a potential new entrant enters the market, the cash flow decreases. Two panels of Figure 1 depict the scenarios without and with upper reflecting barrier.

In the derivations of value functions, we use the following parameters:

$$\begin{aligned}
v &:= \frac{1-\tau}{r-\mu} > 0, \\
\beta &:= \frac{-(\mu-(0.5)\sigma^2)+\sqrt{(\mu-(0.5)\sigma^2)^2+2r\sigma^2}}{\sigma^2} > 1, \\
\gamma &:= \frac{-(\mu-(0.5)\sigma^2)-\sqrt{(\mu-(0.5)\sigma^2)^2+2r\sigma^2}}{\sigma^2} < 0.
\end{aligned} \tag{2}$$

1.2 Value functions without upper reflecting barrier

In this subsection, as a benchmark, we derive the value functions when there does not exist an upper reflecting barrier. The superscript "N" indicates the case without upper reflecting barrier, the subscript "0" indicates the case of all-equity financing. We assume a fixed x and c.

Suppose that a firm is financed by a mix of debt and equity. We denote the value functions of debt and equity as $E^{\mathrm{N}}(x,c)$ and $D^{\mathrm{N}}(x,c)$. These values are given as

$$E^{\mathrm{N}}(x,c) = vx - (1-\tau)\frac{c}{r} + \big(\frac{x}{x_{\mathrm{d}}^{\mathrm{N}}(c)}\big)^{\gamma}\big\{(1-\tau)\frac{c}{r} - vx_{\mathrm{d}}^{\mathrm{N}}(c)\big\}, \tag{3}$$

$$D^{\mathrm{N}}(x,c) = \frac{c}{r} + \big(\frac{x}{x_{\mathrm{d}}^{\mathrm{N}}(c)}\big)^{\gamma}\big\{(1-\alpha)vx_{\mathrm{d}}^{\mathrm{N}}(c) - \frac{c}{r}\big\}, \tag{4}$$

where $x > x_{\mathrm{d}}^{\mathrm{N}}(c)$. Here, the optimal default trigger, $x_{\mathrm{d}}^{\mathrm{N}}(c)$, is given as

$$x_{\mathrm{d}}^{\mathrm{N}}(c) = \frac{\gamma}{\gamma - 1}\frac{1-\tau}{rv}c, \tag{5}$$

where the subscript "d" represents the default strategy. These are identical to those in Black and Cox (1976). For a firm financed by an all-equity, the equity value is given as

$$E_0^{\mathrm{N}}(x) := E^{\mathrm{N}}(x,0) = vx. \tag{6}$$

In addition, we have $D^{\mathrm{N}}(x,0) = 0$ and $x_{\mathrm{d}}^{\mathrm{N}}(0) = 0$.

1.3 Value functions with upper reflecting barrier

This subsection provides the value functions when there exists an upper reflecting barrier. We assume a fixed x and c.

Suppose that a firm is financed by a mix of debt and equity. We denote the value functions of equity and debt by $E(x,c)$ and $D(x,c)$, respectively. As shown in Appendix, the value functions are obtained in the following proposition.

Proposition 1 *Suppose that there exists an upper reflecting barrier. Then, for a firm financed by a mix of debt and equity, equity value is given by*

$$E(x,c) = vx - (1-\tau)\frac{c}{r} + J(c)H_1(c)x^{\beta} + K(c)H_2(c)x^{\gamma}, \tag{7}$$

where

$$J(c) = \big((x_{\mathrm{d}}(c))^{\beta-\gamma} - \frac{\beta}{\gamma}u^{\beta-\gamma}\big)^{-1}, \tag{8}$$

$$K(c) = \big((x_{\mathrm{d}}(c))^{\gamma-\beta} - \frac{\gamma}{\beta}u^{\gamma-\beta}\big)^{-1}, \tag{9}$$

$$H_1(c) = -v\big((x_{\mathrm{d}}(c))^{1-\gamma} - \frac{1}{\gamma}u^{1-\gamma}\big) + (1-\tau)\frac{c}{r}(x_{\mathrm{d}}(c))^{-\gamma}, \tag{10}$$

$$H_2(c) = -v\big((x_{\mathrm{d}}(c))^{1-\beta} - \frac{1}{\beta}u^{1-\beta}\big) + (1-\tau)\frac{c}{r}(x_{\mathrm{d}}(c))^{-\beta}. \tag{11}$$

Here, the default trigger $x_{\mathrm{d}}(c)$ is obtained implicitly by solving the following equation with respect to x_{d}:

$$v + J(c)H_1(c)\beta x_{\mathrm{d}}^{\beta-1} + K(c)H_2(c)\gamma x_{\mathrm{d}}^{\gamma-1} = 0. \tag{12}$$

The debt value is given by

$$D(x,c) = \frac{c}{r} + J(c)H_3(c)x^{\beta} + K(c)H_4(c)x^{\gamma}, \tag{13}$$

where

$$H_3(c) = (1-\alpha)v\big((x_{\mathrm{d}}(c))^{1-\gamma} - \frac{1}{\beta}u^{1-\beta}x_d^{\beta-\gamma}\big) - \frac{c}{r}(x_{\mathrm{d}}(c))^{-\gamma}, \tag{14}$$

$$H_4(c) = (1-\alpha)v\big((x_{\mathrm{d}}(c))^{1-\beta} - \frac{1}{\beta}u^{1-\beta}\big) - \frac{c}{r}(x_{\mathrm{d}}(c))^{-\beta}. \tag{15}$$

The total firm value is defined as the sum of debt and equity values, that is, $V(x,c) := E(x,c) + D(x,c)$.

In Proposition 1, there are three important properties. First, equity value (7) has four components. The first and second components are the intrinsic value, which are identical to Black and Cox (1976). The third component corresponds to the option value of hitting the upper reflecting barrier. The fourth component is the option value of hitting the default trigger. Second, the default trigger is obtained implicitly, not explicitly. Third, debt value (13) has three components. The first component is the face value of debt, because debt does not have a maturity. The second component corresponds to the option value of hitting the upper reflecting barrier. The third component is the option valut of hitting the default trigger.

Suppose that a firm is financed by an all-equity. By substituting $c \downarrow 0$ into (7), the equity value for a firm financed by an all-equity is given as

$$E_0(x) := E(x,0) = v\big(x - \frac{1}{\beta}u^{1-\beta}x^{\beta}\big), \tag{16}$$

which is identical to that in Dixit and Pindyck (1994).[3]

[3]Substituting $c \downarrow 0$ into (12) and (13) yields $D(x,0) = 0$ and $x_{\mathrm{d}}(0) = 0$.

Consider the extreme case of $u \uparrow +\infty$. We obtain

$$
\begin{aligned}
&\lim_{u\uparrow+\infty} E(x,c) = E^{\mathrm{N}}(x,c),\\
&\lim_{u\uparrow+\infty} D(x,c) = D^{\mathrm{N}}(x,c),\\
&\lim_{u\uparrow+\infty} x_{\mathrm{d}}(c) = x_{\mathrm{d}}^{\mathrm{N}}(c),\\
&\lim_{u\uparrow+\infty} E_0(x) = E_0^{\mathrm{N}}(x),
\end{aligned}
$$

respectivly. Thus, this study includes the seminal study by Black and Cox (1976). So far, we have assumed a fixed x_{i} and c. In the next section, we derive the optimal investment and financing decisions.

2 Model solution

In this section, we derive the optimal financing and investment decisions, x_{i}^* and c^*, where "$*$" indicates the optimum. Before analyzing the decisions with upper reflecting barrier, we first briefly review the decisions without upper reflecting barrier.

2.1 Decisions without upper reflecting barrier

In this subsection, as a benchmark, we provide the financing and investment decisions without upper reflecting barrier.

Suppose that a firm is financed by all-equity. Because $E^{\mathrm{N}}(x_{\mathrm{i}}^{\mathrm{N}},0) = vx_{\mathrm{i}}^{\mathrm{N}}$, the optimal option value of the project and the investment trigger are obtained as

$$
\begin{aligned}
O_0^{\mathrm{N}}(x) &= \Big(\frac{x}{x_{\mathrm{i0}}^{\mathrm{N}*}}\Big)^{\beta}\frac{I}{\beta-1} = \max_{x_{\mathrm{i0}}^{\mathrm{N}}}\Big(\frac{x}{x_{\mathrm{i0}}^{\mathrm{N}}}\Big)^{\beta}\{vx_{\mathrm{i0}}^{\mathrm{N}} - I\},\\
x_{\mathrm{i0}}^{\mathrm{N}*} &= \frac{\beta}{\beta-1}\frac{1}{v}I = \operatorname*{argmax}_{x_{\mathrm{i0}}^{\mathrm{N}}}\Big(\frac{x}{x_{\mathrm{i0}}^{\mathrm{N}}}\Big)^{\beta}\{vx_{\mathrm{i0}}^{\mathrm{N}} - I\}.
\end{aligned}
\tag{17}
$$

Suppose that a firm is financed by a mix of debt and equity. Here, we use the following parameters:

$$
\begin{aligned}
\eta &:= \frac{\gamma-1}{\gamma}\frac{rv}{1-\tau} > 0,\\
h &:= \Big(1-\gamma\{1+\alpha\frac{1-\tau}{\tau}\}\Big)^{-1/\gamma} \geq 1,\\
\psi &:= \Big(1+\frac{\tau}{h(1-\tau)}\Big)^{-1} \leq 1.
\end{aligned}
\tag{18}
$$

r	σ	μ	I	τ	α
0.05	0.2	0.01	10	0.15	0.35

Table 1: The basic parameters

Substituting the optimal coupon payment $c^{\mathrm{N}}(x) = (\eta/h)x$ into the total firm value $V^{\mathrm{N}}(x, c^{\mathrm{N}})$ gives $V^{\mathrm{N}}(x, c^{\mathrm{N}}(x)) = (v/\psi)x$.[4] The optimal option value of the project and the investment trigger are obtained as

$$\begin{aligned} O^{\mathrm{N}}(x) &= \Big(\frac{x}{x_{\mathrm{i}}^{\mathrm{N}*}}\Big)^{\beta}\frac{I}{\beta-1} = \max_{x_{\mathrm{i}}^{\mathrm{N}}}\Big(\frac{x}{x_{\mathrm{i}}^{\mathrm{N}}}\Big)^{\beta}\Big\{\frac{v}{\psi}x_{\mathrm{i}}^{\mathrm{N}} - I\Big\}, \\ x_{\mathrm{i}}^{\mathrm{N}*} &= \frac{\beta}{\beta-1}\frac{\psi}{v}I = \operatorname*{argmax}_{x_{\mathrm{i}}^{\mathrm{N}}}\Big(\frac{x}{x_{\mathrm{i}}^{\mathrm{N}}}\Big)^{\beta}\Big\{\frac{v}{\psi}x_{\mathrm{i}}^{\mathrm{N}} - I\Big\}. \end{aligned} \tag{19}$$

In addition, we obtain the optimal financing decision as $c^{\mathrm{N}*} = c^{\mathrm{N}}(x_{\mathrm{i}}^{\mathrm{N}*})$.

From (17) and (19), we obtain the three following properties:

$$x_{\mathrm{i}}^{\mathrm{N}*} < x_{\mathrm{i0}}^{\mathrm{N}*}, \quad O^{\mathrm{N}}(x) > O_0^{\mathrm{N}}(x), \quad O^{\mathrm{N}}(x_{\mathrm{i}}^{\mathrm{N}*}) = O_0^{\mathrm{N}}(x_{\mathrm{i0}}^{\mathrm{N}*}). \tag{20}$$

The proofs for the first and second inequalities are that $x_{\mathrm{i}}^{\mathrm{N}*} = \psi x_{\mathrm{i0}}^{\mathrm{N}*}$ with $\psi \leq 1$ and $O^{\mathrm{N}}(x) = \psi^{-\beta}O_0^{\mathrm{N}}(x)$ with $\beta > 1$. The first two inequalities of (20) imply that the firm financed by a mix of debt and equity decreases the investment trigger (i.e., accelerates investment) and increases the option value of the project, compared with the firm financed by all-equity financing. The last inequality implies that the values at the time of investment are the same for two firms. We will use these three properties as a benchmark.

Figure 2 depicts O^{N} and O_0^{N} with x. The baseline values of parameters are given in Table 1. We can confirm the three properties of (20), that is, $x_{\mathrm{i}}^{\mathrm{N}*} = 0.97 < x_{\mathrm{i0}}^{\mathrm{N}*} = 1.02$, $O^{\mathrm{N}}(x) > O_0^{\mathrm{N}}(x)$ where $x < 0.97$, and $O(x_{\mathrm{i}}^{\mathrm{N}*}) = O_0(x_{\mathrm{i0}}^{\mathrm{N}*}) = 11.75$.

2.2 Decisions with upper reflecting barrier

In this subsection, we derive the optimal financing and investment decisions when there exists an upper reflecting barrier. Here, in particular, we would have a curiosity about whether the three properties in (20) are satisfied even when there exists upper reflecting barrier. We show that they are not always satisfied.

[4]Here, we obtain $c^{\mathrm{N}}(x) = (\eta/h)x = \operatorname{argmax}_{c^{\mathrm{N}}} V^{\mathrm{N}}(x, c^{\mathrm{N}}) := D^{\mathrm{N}}(x, c^{\mathrm{N}}) + E^{\mathrm{N}}(x, c^{\mathrm{N}})$ as shown in Leland (1994). In addition, see Shibata and Nishihara (2023) for the derivation of $V^{\mathrm{N}}(x, c^{\mathrm{N}}(c)) = (v/\psi)x$.

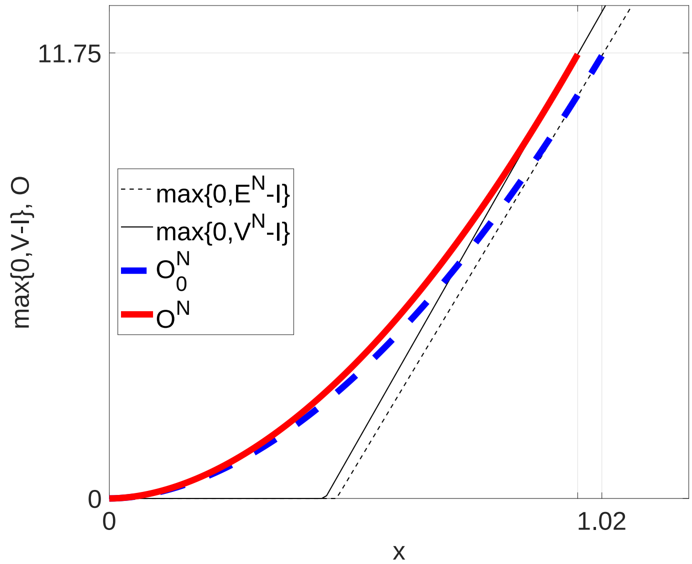


Figure 2: Option values without upper reflecting barrier

The numerical caluculation provides $x_{\mathrm{i}0}^{\mathrm{N}*} = 1.0237$, $x_{\mathrm{i}}^{\mathrm{N}*} = 0.9718$, and $O_0^{\mathrm{N}}(x_{\mathrm{i}0}^{\mathrm{N}*}) = O^{\mathrm{N}}(x_{\mathrm{i}}^{\mathrm{N}*}) = 11.7539$. We see $O^{\mathrm{N}}(x) > O_0^{\mathrm{N}}(x)$ where $x < 0.9718$.

Suppose a levered firm which is financed by a mix of debt and equity. Similar to Sundaresan and Wang (2007), the optimization problem is formulated as

$$O(x) := \max_{x_{\mathrm{i}}, c} \Big(\frac{x}{x_{\mathrm{i}}}\Big)^{\beta} \{V(x_{\mathrm{i}}, c) - I\}. \tag{21}$$

We denote the optimal investment trigger and coupon payment by x_{i}^* and c^*, where the superscript "$*$" indicates the optimum. We have the following result.

Proposition 2 *Suppose that there exists an upper reflecting barrier. Here, x_{i}^* and c^* are obtained implicitly by solving*

$$(1-\beta)x_{\mathrm{i}} - \beta\frac{\tau c}{r} + (\gamma - \beta)K(c)[H_2(c) + H_4(c)]x_{\mathrm{i}}^{\gamma} + \beta I = 0, \tag{22}$$

$$\frac{\tau}{r} + \{J(c)[H_1'(c) + H_3'(c)] + J'(c)[H_1(c) + H_3(c)]\}x_{\mathrm{i}}^{\beta} \\ + \{K(c)[H_2'(c) + H_4'(c)] + K'(c)[H_2(c) + H_4(c)]\}x_{\mathrm{i}}^{\gamma} = 0. \tag{23}$$

Suppose an unlevered firm which is financed by an all-equity, as a benchmark. Using (17), the optimization problem is formulated as

$$O_0(x) := \max_{x_{\mathrm{i}0}} \Big(\frac{x}{x_{\mathrm{i}0}}\Big)^{\beta} \{E_0(x_{\mathrm{i}0}) - I\}. \tag{24}$$

	debt-equity			all-equity		
u	$x_{\mathrm{i}}^{\mathrm{N}*}$	$O^{\mathrm{N}}(x)$	$O^{\mathrm{N}}(x_{\mathrm{i}}^{\mathrm{N}*})$	$x_{\mathrm{i}0}^{\mathrm{N}*}$	$O_0^{\mathrm{N}}(x)$	$O_0^{\mathrm{N}}(x_{\mathrm{i}0}^{\mathrm{N}*})$
$+\infty$	0.97	8.19	11.75	1.02	7.44	11.75
u	x_{i}^*	$O(x)$	$O(x_{\mathrm{i}}^*)$	$x_{\mathrm{i}0}^*$	$O_0(x)$	$O_0(x_{\mathrm{i}0}^*)$
100	1.00	8.03	12.14	1.02	7.29	11.43
10	1.71	7.31	14.77	1.02	6.37	9.99

We assume $x = 0.8$ in addition the baseline parameter values.

Table 2: Optimal investment triggers and its values

The optimal investment trigger $x_{\mathrm{i}0}^*$ is obtaind implicitly by solving

$$(1-\beta)v(x_{\mathrm{i}0} - \frac{1}{\beta}u^{1-\beta}x_{\mathrm{i}0}^{\beta}) + \beta I = 0. \tag{25}$$

These are as the same as those in Dixit and Pindyck (1994).

We now consider the properties of the option value by providing numerical calculation, because the investment trigger is obtained implicitly. Here, we assume $u \in \{10, 100, +\infty\}$, $x = 0.8$, and the baseline parameter values in Table 1. The second and fifth columns of Table 2 show $O(x)$ (option value for the levered firm) and $O_0(x)$ (option value for the unlevered firm). A decrease in u decreases $O(x)$ and $O_0(x)$. The third and sixth columns describes $O(x_{\mathrm{i}}^*)$ (value at the time of investment for the levered firm) and $O_0(x_{\mathrm{i}0}^*)$ (value at the time of investment for the unlevered firm). Interestingly, a decrease in u increases $O(x_{\mathrm{i}}^*)$, but decreases $O_0(x_{\mathrm{i}0}^*)$. To understand these results precisely, we depict $O(x)$ with x in Figure 3. Panels (a) and (b) demonstrate the two properties of $O(x)$ and $O_0(x)$, respectively. The first is that a decrease in u decreases $O(x)$. The second is that a decrease in u increases $O(x_{\mathrm{i}}^*)$, but decreases $O_0(x_{\mathrm{i}0}^*)$. We summarize the properties about the values with upper reflecting barrier in the following observation.

Observation 1 *Fierce competition (an upper reflecting barrier) always reduces the option value for the levered and unlevered firm. However, fierce competition (an upper reflecting barrier) always increases the value at the time of investment for the levered firm, but always decreases the value at the time of investment for the unlevered firm.*

We then examine the properties of the investment trigger. The first and fourth columns of Table 2 show x_{i}^* (investment trigger for the levered firm) and $x_{\mathrm{i}0}^*$ (investment trigger for the unlevered firm). A decrease in u increases x_{i}^* significantly. A decrease in u looks like invariant to x_{i0}^*, but it is not true. A decrease in u increases x_{i0}^* slightly.[5] Thus, the effects of upper reflecting barrier on investment timing for the levered firm are significantly different form those for the unlevered firm. More interestingly, although we obtain $x_{\mathrm{i}}^{\mathrm{N}*} < x_{\mathrm{i0}}^{\mathrm{N}*}$ when $u \uparrow +\infty$, we do not always obtain $x_{\mathrm{i}}^* < x_{\mathrm{i0}}^*$ when $u < +\infty$. We see, for example, $x_{\mathrm{i}}^* = 1.17 > x_{\mathrm{i0}}^* = 1.02$ when $u = 10$. We summarize the properties about the investment decisions with upper reflecting barrier in the following observation.

Observation 2 *Fierce compeititon (an upper reflecting barrier) delays the levered firm's investment (decreases the investment trigger) significantly, but delays the unlevered firm's investment slightly. The levered firm does not always accerelates investment when competition is fierce (there exists an upper reflecing barrier), whereas the levered firm always accerelates investment when competition is not fierce (there does not exist an upper reflecting barrier).*

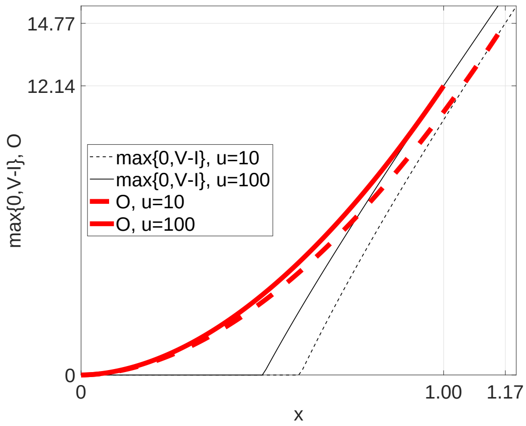
To clarify the interactions between Observations 1 and 2, we depicts $O(x_{\mathrm{i}}^*)$ with x_{i}^* in Panel (c). Suppose $u \uparrow +\infty$. We have $O^{\mathrm{N}}(x_{\mathrm{i}}^{\mathrm{N}*}) = O_0^{\mathrm{N}}(x_{\mathrm{i0}}^{\mathrm{N}*})$ and $x_{\mathrm{i}}^{\mathrm{N}*} < x_{\mathrm{i0}}^{\mathrm{N}*}$. By contrast, suppose $u < +\infty$. We then do not obtain these properties for $u < +\infty$, for example, $O^{\mathrm{N}}(x_{\mathrm{i}}^{\mathrm{N}*}) > O_0^{\mathrm{N}}(x_{\mathrm{i0}}^{\mathrm{N}*})$ and $x_{\mathrm{i}}^{\mathrm{N}*} > x_{\mathrm{i0}}^{\mathrm{N}*}$ for $u = 10$. That is, on the one hand, as u decreases, an unlevered firm increases x_{i0}^* slightly, but decreases $O_0(x_{\mathrm{i0}}^*)$. On the other hand, as u decreases, a levered firm increases x_{i}^* and $O(x_{\mathrm{i}}^*)$. To summarize, the firms' strategies and their values for $u < +\infty$ differ significantly from those for $u \uparrow +\infty$. In addition, for $u < +\infty$, the investment decisions for a levered firm differ from those for an unlevered firm.

3 Model implications

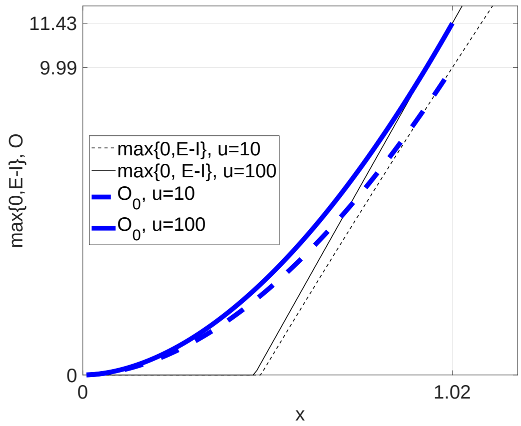
In this section, we consider some important implications of our model. Subsection 4.1 examines how fierce competition (upper reflecting barrier) affects the firm's financing and

[5] Differentiating (25) with x_{i0} and u and arranging gives

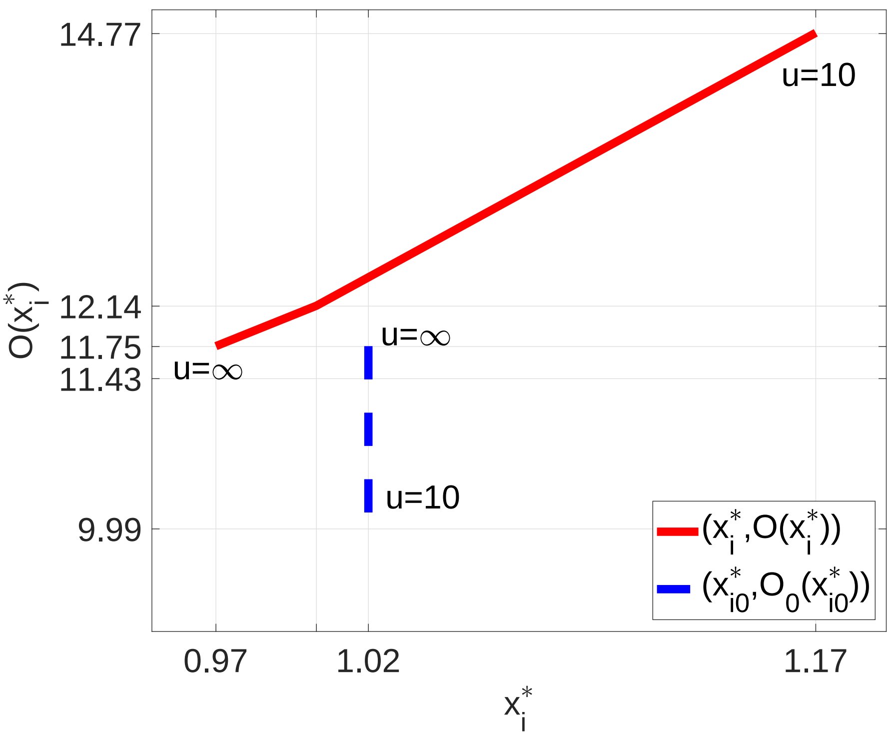
$$\frac{\mathrm{d}x_{\mathrm{i0}}}{\mathrm{d}u} = \underbrace{\Big\{1 - \Big(\frac{x_{\mathrm{i0}}}{u}\Big)^{\beta-1}\Big\}^{-1}}_{>0} \underbrace{\frac{1-\beta}{\beta}}_{<0} \underbrace{\Big(\frac{x_{\mathrm{i0}}}{u}\Big)^{\beta}}_{>0} < 0.$$



(a)



(b)



(c)

Figure 3: Option values with upper reflecting barrier

investment decisions. Subsection 4.2 investigate the effect of volatility.

3.1 Effects of fierce competition (upper reflecting barrier)

This subsection examines the effects of fierce competition (upper reflecting barrier).

Panel (a) of Figure 4 depicts $O(x)$ with x_{i} for $u = 10$. We can see that $O(x)$ is concave with x_{i} and is maximized at $x_{\mathrm{i}}^* = 1.17$. Panel (b) depicts x_{i}^* with u. An increase in u decreases x_{i}^* and converge to $x_{\mathrm{i}}^{\mathrm{N}*} = 0.97$. Panel (c) of Figure 4 depicts $V(x,c)$ with c for $u = 10$. We can see that $V(x,c)$ is concave with c and is maximized at $c^* = 0.60$.[6] Panel (d) depicts c^* with u. An increase in u increases c^* and converge to $c^{\mathrm{N}*} = 0.63$. Panel (e) of Figure 4 depicts the credit spreads cs with u, where cs is defined as $cs = c/D - r$. A decrease in u decreases cs. Panel (f) depicts the leverage D/V with u. A decrease in u decreases D/V. We summarize the results of the six panels as the following observation.

Observation 3 *Suppose that there exists an upper reflecting barrier. Fierce competition (a decrease in u) delays the levered firm's investment. Fierce competition (a decrease in u) reduces the amount of debt issuance, which leads to a decrease in credit spreads and leverage.*

The first result is consistent with those of Roques and Savva (2009) and Rodrigues (2022), where a price ceiling (cap) defers the unlevered firm's investment. The second result is new.

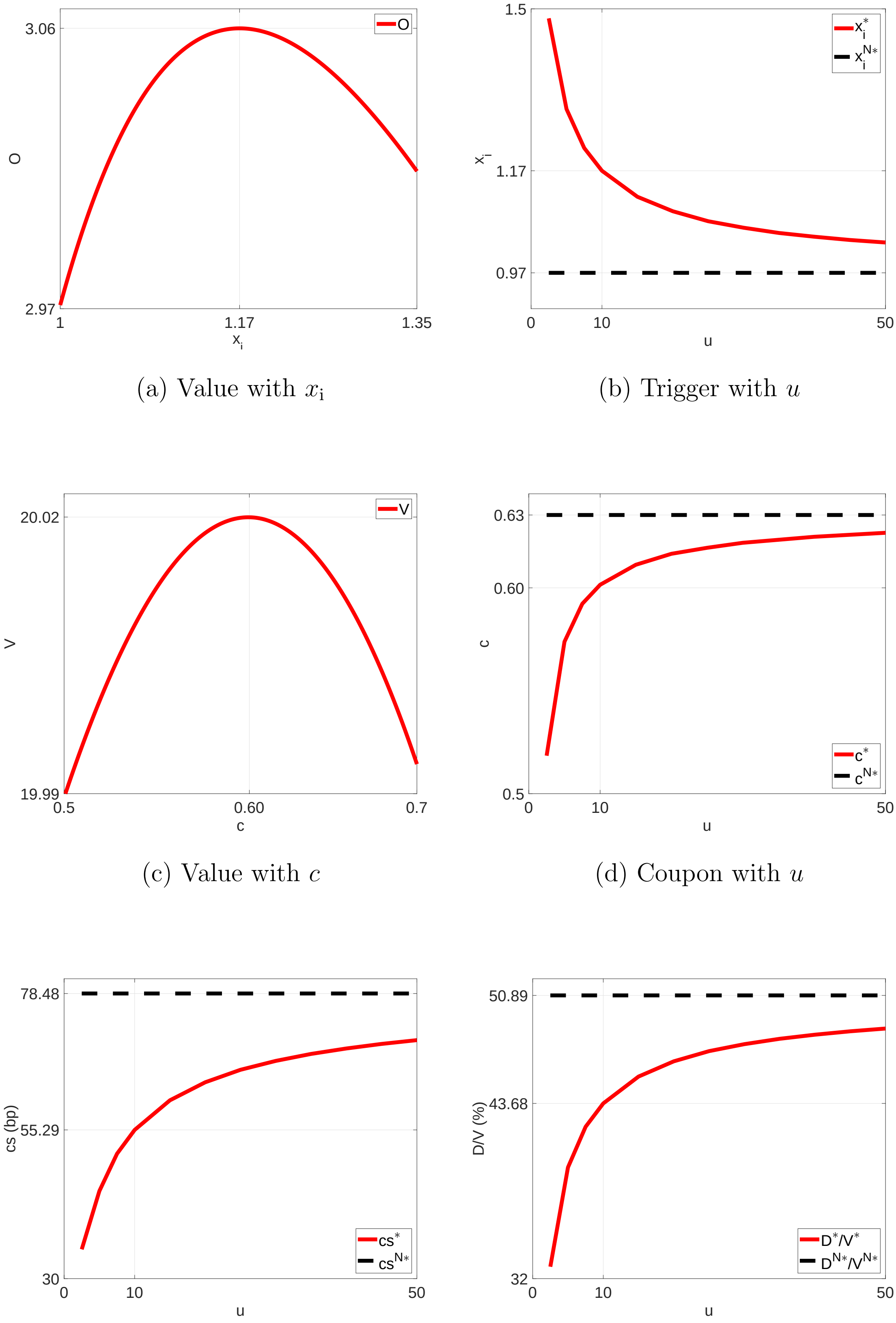
3.2 Volatility effects

This section explore the effects of volatility.

Panels (a) and (b) of Figure 5 depics x_{i}^* and c^* with u, respectively. An increase in σ increases x_{i}^* and c^*. These results are summarized in the following.

Observation 4 *Suppose that there exists an upper reflecting barrier. An increase in volatility delays the investment and increases the amount of debt value at the timei of investment.*

[6]Here, note that maximizing V with c is equivalent to maximing O with c.



(a) Value with x_{i}

(b) Trigger with u

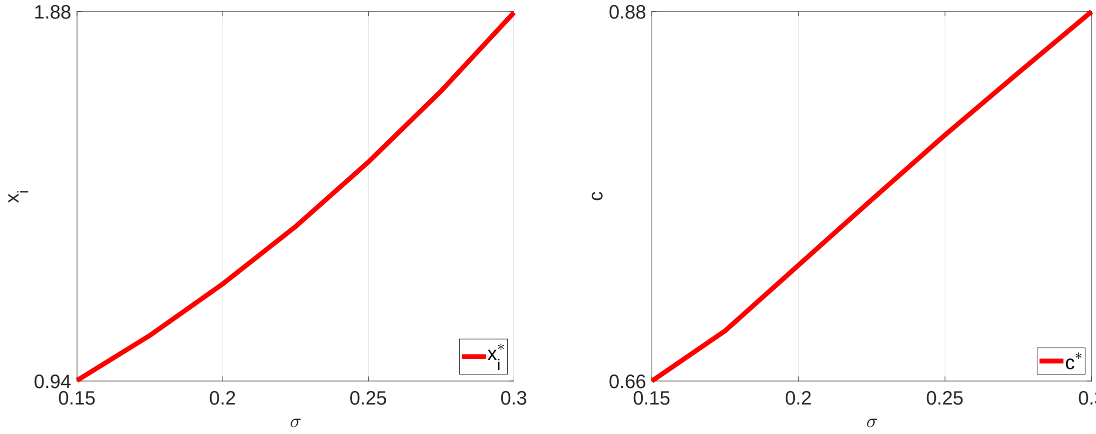
(c) Value with c

(d) Coupon with u

(e) Credit spreads with u

(f) leverage with u

Figure 4: Effects of fierce competition (upper reflecting barrier)



(a) (b)

Figure 5: Volatility effects

The first result is the same as those in the standard real options model such as Dixit and Pindyck (1994). The second result is identical to that in Sundaresan and Wang (2007).

Acknowledgments

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Appendix: Proof of Propositions

Proof of Proposition 1

Suppose that there exists an upper reflecting barrier. We derive the value functions for a firm financed by a mix of debt and equity. As $c \to 0$, we obtain the value functions for a firm financed by an all-equity

Using the standard valuation principle of Dixit and Pindyck (1994), the value function of equity $f(x)$ satisfies the following ordinary differential equation (ODE):

$$\frac{1}{2}\sigma^2 x^2 f''(x) + \mu x f'(x) - r f(x) = (1-\tau)(x-c), \tag{A.1}$$

where

$$f'(u) = 0, \quad f(x_{\mathrm{d}}) = 0. \tag{A.2}$$

Here, the first condition corresponds to the value matching condition for the upper reflecting barrier, while the second condition is the value matching condition for the default. We denote the default trigger by x_{d}, which is different from $x_{\mathrm{d}}^{\mathrm{BC}}$. The general solution of $f(x)$ is given by

$$f(x) = vx - (1-\tau)\frac{c}{r} + Ax^{\beta} + Bx^{\gamma}. \tag{A.3}$$

Using (A.3), the boundary conditions of (7) are rewritten as

$$vu + \beta A u^{\beta} + \gamma B u^{\gamma} = 0, \tag{A.4}$$

$$vx_{\mathrm{d}} - (1-\tau)\frac{c}{r} + Ax_{\mathrm{d}}^{\beta} + Bx_{\mathrm{d}}^{\gamma} = 0, \tag{A.5}$$

respectively. Thus, constants A and B are obtained as solving the simultaneous equations of (A.4) and (A.5). Similarly, we derive the value function of debt, $g(x)$, which satisfies the ODE:

$$\frac{1}{2}\sigma^2 x^2 g''(x) + \mu x g'(x) - rg(x) = c, \tag{A.6}$$

where

$$g'(u) = 0, \quad g(x_{\mathrm{d}}) = (1-\alpha)v(x_{\mathrm{d}} - \frac{1}{\beta}x_{\mathrm{d}}^{\beta}u^{1-\beta}). \tag{A.7}$$

Identical to (A.2), the first condition corresponds to the value matching condition for the upper reflecting barrier, while the second condition is the value matching condition for the default. Note that the residual value at default is $(1-\alpha)v(x_{\mathrm{d}} - \beta^{-1}x_{\mathrm{d}}^{\beta}u^{1-\beta})$ when the firm faces the upper reflectin barrier.[7] The value $v(x - \beta^{-1}x^{\beta}u^{1-\beta})$ is identical to the equity value for the alll-equity financed firm. The general solution of $g(x)$ is

$$g(x) = \frac{c}{r} + Fx^{\beta} + Gx^{\gamma}. \tag{A.8}$$

Using (A.7) and (A.8), constants F and G are obtained by solving the following simultaneous equations:

$$\beta F u^{\beta} + \gamma G u^{\gamma} = 0, \tag{A.9}$$

$$\frac{c}{r} + Fx_{\mathrm{d}}^{\beta} + Gx_{\mathrm{d}}^{\gamma} = (1-\alpha)v\big(x_{\mathrm{d}} - \frac{1}{\beta}x_{\mathrm{d}}^{\beta}u^{1-\beta}\big). \tag{A.10}$$

[7] The residual value at default is $(1-\alpha)vx_{\mathrm{d}}$ when $X(t)$ does not face an upper reflecting barrier.

We define the values of equity and debt as a function of x. However, since we assume a debt-equity financed firm in this study, the firm decides the coupon level (i.e., debt issuance) to maximize the firm value. Thus, from now on, we write the equity and debt values as $E(x,c)$ and $D(x,c)$, not $f(x)$ and $g(x)$. In addition, the firm decides default trigger to maximize its equity value, the trigger $x_{\mathrm{d}}(c)$ is obtained by satisfing $\partial E(x,c)/\partial x|_{x=x_{\mathrm{d}}}$.

Proof of Proposition 2

The firm's optimization problem is formulated as

$$\max_{x_{\mathrm{i}},c} \phi(x_{\mathrm{i}},c), \tag{A.11}$$

where

$$\phi(x_{\mathrm{i}},c) = \big(\frac{x}{x_{\mathrm{i}}}\big)^{\beta}\{V(x_{\mathrm{i}},c) - I\}. \tag{A.12}$$

Differentiating ϕ with x_{i} and c gives

$$\begin{aligned}\frac{\partial\phi}{\partial x_{\mathrm{i}}} =& \big(\frac{x}{x_{\mathrm{i}}}\big)^{\beta}\Bigg((-\beta)x_{\mathrm{i}}^{-1}\{V(x_{\mathrm{i}},c) - I\} + v \\ &+ J(c)\{H_1(c)+H_3(c)\}\beta x_{\mathrm{i}}^{\beta-1} + K(c)\{H_2(c)+H_4(c)\}\gamma x_{\mathrm{i}}^{\gamma-1}\Bigg) = 0, \qquad (\text{A.13}) \\ \frac{\partial\phi}{\partial c} =& \big(\frac{x}{x_{\mathrm{i}}}\big)^{\beta}\Bigg(\frac{\tau}{r} + \big(J(c)[H_1(c)+H_3(c)]x_{\mathrm{i}}^{\beta}\big)' + \big(K(c)[H_2(c)+H_4(c)]x_{\mathrm{i}}^{\gamma}\big)'\Bigg) = 0, \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (\text{A.14})\end{aligned}$$

respectively. Arranging (A.13) and (A.14) gives (24) and (25).

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