MTRC's Novel Infrastructure Financing Model: Rationale Based on a Stackelberg Game of Timing under Uncertainty

(Authors' names blinded for peer review)

Many municipalities or governments face challenges in financing their infrastructure. Hong Kong's transit operator designed a novel scheme whereby it receives fare revenues, but also partakes in a property-management business, exploiting the positive externalities of public transport on property prices. We develop a Stackelberg game of timing under uncertainty to explore the rationale of this scheme. The underlying mathematical problem is nontrivial because the Stackelberg's leader faces a two-dimensional optimal stopping problem (which cannot be reduced by a change of numéraire) with a gain function that is nondifferentiable due to strategic interactions. We solve this problem analytically via the intermediation of a 'penalized problem' and derive interesting, novel managerial insights. The main insight is that internalizing positive externalities provides additional revenue sources for defraying the overall costs of infrastructure investments, thereby accelerating the delivery of infrastructure.

Key words: Stackelberg game of timing, two-dimensional real options problems, dynamic programming

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1. Introduction

Well-functioning infrastructure assets facilitating mobility among places of residence, work, and leisure contribute to a city's attractiveness and competitiveness in the present globalized economy and generate follow-up opportunities for the local economy as a whole. Using such infrastructure assets should be affordable and high-quality, while the project economics should be sustainable to the extent possible. To ensure affordability for users, transit fares are often regulated and set below the total infrastructure costs made of the operating expenses, the rental cost of infrastructure goods and the environmental costs (which are often not accounted for), at the expense of project self-sustainability.

Because an infrastructure project is capital intensive, an investor may be prone to delay its launch until the present value (PV) of the revenues generated by the infrastructure project is sufficiently larger than the PV of the fixed costs (in the spirit of real options). This problem may be exacerbated when capital providers are short-sighted as the revenues from such a project are reaped over more than 30 years. A related key issue is that infrastructure investor does not internalize (as revenues) all the benefits to society from developing an infrastructure good, leading to a classical problem of underinvestment whereby a project meaningful for society is rejected for lack of profitability (see Myers 1977). Standard solutions include various forms of subsidies by municipalities or other governmental bodies. Yet, given sovereign debt concerns, policy makers often face a conflict between financing infrastructure projects and attaining other social and economic objectives (e.g., welfare programs). To accelerate the delivery of infrastructure (and ultimately bolster economic growth), it seems advised to relate more closely the financing of an infrastructure project with the externalities it generates to others.

Incorporating positive externalities into the income stream of a transit operator is an innovative approach to financing infrastructure goods. A great deal can be learned from recent experiences in Hong Kong (HK), a city in which rail transit operations are highly profitable without government subsidizing. Under a traditional transit operations model, the operator rarely benefits from the positive impact of public transport improvements on the prices of nearby properties. The financial success of HK's railway operator, the Mass Transit Railway Corporation ("MTRC"), rests on the "*Rail Plus Property*" (R+P) financing model through public-private partnerships (PPPs). Under the R+P program, the government grants MTRC exclusive property development rights of government-owned land around rail transit stations at a "before-rail" market price. (The HK government owns all land in the HK territory. Private individuals and organizations can only purchase 50-year leases that grant exclusive property development rights.) MTRC then makes transit investments and captures the land appreciation through further granting development rights to private developers at an "after-rail" market price, jointly developing the land and property, and sharing profits in agreed proportions. The land premiums (i.e., the difference between the before-rail and the after-rail market prices) and the shared profits are used to recoup the capital, operations, and maintenance costs of railway projects.

Our intent is to study the rationale of such a scheme by developing a mathematical model capturing the key tradeoffs and incentives of the various stakeholders. Uncertainty is a classical issue (also for infrastructure investments) as it affects an investor's propensity to delay a key investment (see Dixit and Pindyck 1994, Trigeorgis 1996, for an overview on real options analysis). While the property developers' decision to acquire land rights to launch a new real-estate problem is primarily driven by property prices, the R+P model suggests two businesses for MTRC, one related to the fare revenues and another real-estate business linked to the property prices. As property prices are driven by many factors besides population dynamics, it seems reasonable to assume that MTRC's income is driven by two underlying stochastic processes (modeled here as geometric Brownian motions) possibly correlated. Such strategic interactions between the two parties suggest a Stackelberg game of timing, with the infrastructure investor acting as a leader. Interestingly, the present simple framework leads to two features that make the analysis of the leader's decision nontrivial. First, optimal stopping problems involving several state variables remain underresearched (cf. literature review). Second, because of the Stackelberg structure, the leader's payoff under investment is not smooth, with a kink at the property price above which the real-estate developer invests. Because of this kink, classical techniques to solve optimal stopping problems (including the smooth-fit principle) are not guaranteed to work. To solve this problem, we proceed by deriving a weak formulation of the dynamic programming (DP) equation of optimal stopping and by constructing the solution of this DP equation via the intermediation of a "penalized problem." We prove existence and uniqueness of a solution and construct the optimal stopping rule by a Monte-Carlo scheme.

We derive various insights from this model. First and intuitively, the infrastructure investor is more prone to investing for large property prices and more substantial fare revenues. Second, we study analytically the effect of uncertainty and growth in either dimension of the investor's policy.

2. Literature review

Infrastructure investment is a key topic in management science (see, e.g., Smith and McCardle 1999, Garcia and Shen 2010, Johari et al. 2010, Mak et al. 2013, Qi et al. 2015, Yang and Lim 2018, Calvo et al. 2019, Haase et al. 2019). Given three characteristic features (irreversibility, uncertainty, and flexibility in timing), the use of real options techniques is often deemed relevant to address specific research gaps on this theme. Recently, Dahlgren and Leung (2015) for instance explored a problem of consecutive infrastructure investment decisions, modeling the original setup as an (one-dimensional) optimal stopping problem where the infrastructure owner decides on a sequence of stopping times. Thijssen (2021) considered optimal investment and abandonment decisions for projects with uncertain construction time of large-scale investment projects. In contrast, we consider a setting in which the infrastructure owner decides on a single stopping time, but introduce complexity arising from strategic interactions and from the dependence of the decision maker's policy on two state variables. Combining real-options techniques with notions from game theory (see, e.g., Chevalier-Roignant and Trigeorgis 2011) have already also proved useful to study infrastructure problems. For instance, Smit and Trigeorgis (2009) developed a discrete-time "option game" in which two competing private airports decide on the times at which to expand their capacities (by a lump sum). Silaghi and Sarkar (2021) used techniques from contract theory to study the impact of agency conflicts arising in public-private partnerships (PPPs) on the times at which an infrastructure project is launched or scrapped. These papers consider a different set of players (and payoff functions) than we do and do not focus on the positive externalities infrastructure assets have on nearby properties.

From a modeling perspective, there already are several RO studies in which two parties with conflicting interests (typically competitors) decide on their investment times. A first stream of papers (see, e.g., Grenadier 1996, Huisman and Kort 2015) discuss problems (akin to Dynkin games) in which the two parties decide simultaneous on their optimal stopping rules. Such setups do not rule out a sequence of market entries or capacity expansions as an equilibrium play. Another stream of papers (in particular Bensoussan et al. 2010, 2017) discuss problems set as Stackelberg games in which the roles of leader and follower are exogenously set (i.e., the follower cannot act before the leader has already done so). Such Stackelberg setups lead to a situation where the gain function (or "obstacle") for the leader is nondifferentiable at the state value above which the follower invests. As pointed out by the authors, the use of classical techniques (including the smooth-fit principle) is precluded in such Stackelberg games because sufficient conditions for their validity include the differentiability of the obstacle (see Strulovici and Szydlowski 2015, Theorem 4). The problem treated in this paper is closer to the second stream. Motivated by an actual infrastructure-management problem, we consider that the property developer (acting as a Stackelberg follower) decides based on the observed property prices. Because the infrastructure investor (the Stackelberg leader) has two businesses, one consisting in capturing the externalities of its investment on the property prices, its investment decision depends on two state variables. We are the first to study a two-dimensional Stackelberg game of timing.

The multiplicity of uncertainty sources was recognized early on in the real options literature (McDonald and Siegel 1986). Until recently, most two-dimensional problems were, however, effectively reducible to a one-dimensional optimal stopping problem by exploiting the gain function's homogeneity (of degree one) or equivalently by changing the numéraire (see, e.g., McDonald and Siegel 1986, Gerber and Shiu 1996, Nunes and Pimentel 2017). In our two-dimensional problem, one cannot proceed with a change of numéraire as the leader's gain function is not homogeneous (because of a fixed cost of investment). It has become a recent endeavor to construct the optimal stopping rule for two-dimensional problems in which such homogeneity cannot be exploited (see, e.g., Olsen and Stensland 1992, Hu and Øksendal 1998, Adkins and Paxson 2011, Lange et al. 2020, Compernolle et al. 2021, Dammann and Ferrari 2021). Dammann and Ferrari (2021) summarize and extend most results in the research stream, characterizing (based on an approach first developed by De Angelis et al. 2017, for a singular-control problem) the free boundary as the unique solution to a nonlinear Fredholm integral equation.

Our manuscript contributes to the literature on infrastructure investments by modeling the impact of a capture of the infrastructure owner's externalities (on property prices) on its investment decision. We provide a compelling economic rationale for the success of HK's infrastructure financing model. Technically, ours is the first study that constructs the solution of a two-dimensional Stackelberg game of timing, overcoming the difficulty related to the lack of regularity of the leader's obstacle.

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