A real options model of the supply chain with revenue-sharing and volume flexibility

Abstract

We analyze a revenue sharing contract within a decentralized supply chain, extending prior works to a multiperiod setting under buyer production flexibility. We model a buyer's capacity choice and utilization under external procurement, where quantities are obtained from a supplier firm. The supplier firm chooses a revenue sharing contract with the buyer by internalizing the impact this would have on buyer decisions relating to capacity, utilization choice and the final downstream price of the product. Our framework provides a valuation of the buyer and supplier firms under uncertainty and predictions on the optimal capacity choice of firms in downstream markets and the pricing policy of upstream firms in relation to buyer capacity constraints, the volatility and growth of downstream prices, and the elasticity of demand. We find that for a fixed revenue sharing contract a high volatility of downstream demand results in higher installed capacity by the buyer to account for future flexibility to adjust production which makes a given contract more valuable for both the buyer and supplier. We generally find however a higher revenue share claimed by the supplier when downstream demand is more volatile. In an extension of this framework we also show that suppliers could impose minimum order quantities to extract value from a buyer firm by limiting buyer's production flexibility. We also consider the decisions of a vertically integrated firm showing that the gains from vertical integration are higher when volatility is high, that is, when production flexibility is more important. We show however that the optimum vertically coordinated production can be achieved in this decentralized multiperiod setup through a combination of wholesale pricing and revenue sharing.

Keywords: vertical integration; real options; volume flexibility; supply chain;

1. Introduction

Revenue sharing contracts are quite common in practice. Their popularity has spiked in recent years with the increase in sales via online marketplaces. Revenue sharing occurs in many different types of industries such as the airline industry (Fu and Zhang, 2010), video rental (Altug and van Ryzin, 2014; Cachon and Lariviere, 2005; Giannoccaro and Pontrandolfo, 2004), newspapers (Gerchak and Khmelnitsky, 2003), electronics, e.g., Apple App Store, Google Play, or online market places such as Amazon.com (Bart et al., 2021). They are also used in franchising in sectors such as hotels, fast foods and automobile renters (Lal, 1990; Mathewson and Winter, 1985).

Thanks to their popularity in the real world, these contracts have received considerable attention in the academic literature. A recent survey by Bart et al. (2021) summarizes the operating research literature on revenue sharing contracts which has analyzed two main types of contracts: a wholesale-price contract with revenue sharing (Cachon and Lariviere, 2005), and a consignment contract with revenue sharing (Wang et al., 2004) which omits the wholesale price component. An important strand of this literature analyzes the channel performance of revenue sharing contracts in comparison with other types of contracts (Dana and Spier, 2001; Gerchak and Wang, 2004). Other studies investigate issues such as horizontal competition (Chakraborty et al., 2015; Kong et al., 2013; Krishnan and Winter, 2011; Wang and Shin, 2015; Yao et al., 2008), risk/loss-averse supply chains (Zhang et al., 2015), asymmetric information (Gerchak and Khmelnitsky, 2003; Xiao and Xu, 2018) or effort and cost sharing (Bhaskaran and Krishnan, 2009). Recent advances in this literature study revenue sharing contracts for supply chains of virtual products (Avinadav et al., 2015a and 2015b, Tan and Carrillo, 2017), behavioral laboratory experiments (Katok and Wu, 2009), sustainable supply chains (Govindan and Popiuc, 2014; Hsueh, 2014) and carbon emissions (Yang and Chen, 2018).

Most of the studies on revenue sharing propose a static framework with a one-period model and a fixed quantity to be ordered, neglecting the ability of the firms to adjust production depending on market conditions, i.e., production flexibility. Moreover, they focus on production quantities only, disregarding production capacities.[1] Our paper extends these settings to multiple periods under uncertain demand and incorporates capacity choice and volume flexibility for the buyer, such that the quantity ordered can be adjusted depending on market conditions. Production flexibility is key especially during crises since firms can temporarily stop production to reduce losses. For example, energy-intensive firms in the UK have recently warned that they might stop production due to rising energy costs.[2] Industry leaders have also warned about the risk of fallout across the entire supply chain, across manufacturing, consumer retail and other products. Moreover, several companies in Europe, and in particular in Spain, whose industry pays the highest energy price in Europe, in industries such as steel and non-ferrous metals, have already either temporarily shut down or reduced production.[3,4]

Production flexibility has been analyzed in the context of a single firm within the real options literature (Hagspiel et al., 2016; Ritchken and Wu, 2020; Sarkar, 2009, 2018). Our work is closely related to Hagspiel et al. (2016) who analyze optimal capacity investment decisions under production flexibility,to Sarkar (2018) who identifies a firm's optimal degree of operating leverage (DOL) under investment and production flexibility, and to

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[1] Two notable exceptions that analyze *supplier* capacity choice are Cachon and Lariviere (2001) and Wang and Gerchak (2003).

[2]https://www.theguardian.com/business/2021/oct/08/energy-crisis-could-halt-factory-lines-industry-leaders-warn
https://www.independent.co.uk/business/industry-leaders-warn-factories-could-stop-production-due-to-energy-costs-b1935081.html

[3] ArcelorMittal will carry out "short and selective stoppages" in several factories in Europe. Sidenor has stopped production at its largest plant for twenty days, while Fertiberia shut down one of its plants during October, while Ferroatlántica closed one of its four furnaces in Boo de Guarnizo and Asturiana de Zinc reduced its production by a few hours per day. For more details please see: https://thecorner.eu/news-spain/spain-economy/spanish-industry-starts-to-grind-to-a-halt-due-to-the-price-of-energy-the-most-expensive-in-europe/98786/.

[4] Rong and Xu (2020) study revenue sharing contracts in the manufacturing industry such as steel, automobile or non-ferrous metals, but do not consider production flexibility.

Ritchken and Wu (2020) who introduce corporate debt in this framework and analyze the impact of production flexibility on leverage and capital structure. We extend this strand of the literature by adding the supplier firm. This allows us to analyze a revenue sharing contract in the supply chain, along with its price and production level implications, as well as its channel performance. Overall, we bridge the revenue sharing literature (e.g., Cachon and Lariviere, 2004; Gianoccarro and Potrandolfo, 2004) with the real options literature on production flexibility by proposing a unified real options framework to analyze a revenue sharing contract within a decentralized supply chain under buyer production flexibility.

We model the decision making of the two parties in the supply chain as a Stackelberg (follower-leader) game, with the supplier being the leader and the buyer the follower. The buyer firm decides the optimal capacity to install and can costlessly adjust production over time with the capacity level as the upper bound. The supplier chooses the optimal revenue sharing ratio taking into account the effect that this will have on the buyer's optimal choice of installed capacity and capacity utilization rate, as well as the effect of produced quantities on prices. Our analysis quantifies the trade-offs involved in the supplier's choice of the optimal revenue sharing ratio in the presence of capacity choice and production flexibility. On the one hand, a higher revenue share has a direct positive impact on supplier value. On the other hand, a higher revenue share for the supplier has a negative impact on buyer's installed capacity and order quantities and causes a delay in buyer switching to full capacity which translate into a negative indirect effect on supplier value. However, lower order quantities also imply higher prices of the goods sold, which has a positive indirect effect on supplier value. We demonstrate that these trade-offs result in an optimal revenue sharing ratio which maximizes supplier's value.

We first analyze the optimal choice of buyer firms relating to capacity choice, utilization and end consumer prices when the supplier offers a one-for-all (fixed) revenue sharing contract. This analysis provides guidance for the conditions under which a given and fixed revenue sharing contract will be more beneficial for the supplier. Consider for example a large multinational producer (supplier) of a brand distributed to a buyer firm. We provide interesting new insights on how a fixed revenue sharing contract offered to buyers operating in different economic environments may affect their installed capacities, utilization (orders), downstream prices and eventually the value of the supplier firm. For example, while intuition would probably suggest that offering a revenue sharing contract to a buyer may be harmful for the supplier if the buyer operates under a more volatile demand environment, we show that under volume flexibility higher volatility creates a more valuable operational flexibility option for the buyer firm and thus increases its optimal installed capacity. Thus, despite the fact that under higher volatility the buyer may not utilize the full capacity immediately, the future upside potential benefits both the buyer and supplier firm. We also investigate the impact of other model parameters (e.g., highlight the effect of elasticity which has important implications for different types of products between luxury versus necessities, downstream demand level and operating costs of buyer firms, etc.), showing their impact on capacities, utilization of capacity, prices and the values of the buyer and supplier firms.

We then solve the Stackelberg (follower-leader) game in which the supplier chooses its revenue sharing terms taking into account buyer's reaction in terms of capacity choice and utilization rate and their repercussions on downstream product prices. To continue with the earlier example, for a large multinational producer (supplier), our analysis provides insights on the specific revenue sharing contracts offered to the different buyers under alternative economic settings (e.g., more volatile demand, higher operating costs, etc.). We generally find that an increase in volatility will result in a higher claim of revenue share of the supplier firm. The optimal share of revenues also increases when the operating costs faced by the supplier firm are higher or when demand becomes more inelastic and decreases when the unit operating costs of the buyer are higher or when downstream demand growth is lower. We interestingly find that there may be relatively small variation in the share of revenues for other parameters, which is driven by the counterbalancing forces that an adjustment in the level of the revenue share causes on capacities, order quantities and prices. This has important implications for the design of optimal contracts for supplier firms. For example, we find no significant variation in the share of revenues the supplier would claim when the level of downstream demand changes. This would imply for example that a large multinational producer distributing to buyer firms operating under different levels of demand could offer a one-for-all revenue contract which can achieve similar benefits for the supplier as using specific revenue sharing contracts to the different buyers (which is probably costlier).

We also extend the model to account for the common practice that suppliers impose minimum order quantities due to economies of scale in transportation and production setups (Awasthi et al., 2009; Burke et al., 2007). We quantify the negative impact on buyer's value of these constraints and show that when the constraints become binding the buyer needs to install higher capacity and engage in higher utilization of quantities in production. These effects benefit the supplier despite the dampening effect that higher quantities have on the prices at which the goods are sold.

Moreover, we investigate the channel performance of our decentralized pure revenue sharing contract by comparing the produced quantities and prices in the downstream market with those of a vertically integrated supply chain. We analyze the supply chain gain, defined as the percentage difference in total supply chain value between a centralized channel and a decentralized channel, and how it varies with model parameters such as volatility of demand, demand elasticity, retailer's share of the costs, etc. We show that a vertically integrated firm retains higher capacity and utilization rates and sells at lower prices, in line with a double marginalization problem. The percentage supply chain gain from vertically integrating production ranges between 4%-22% for the parameters considered. The gains from vertical integration are relatively higher when demand uncertainty is higher, i.e., for environments where production flexibility is more valuable. We also find that the gains from vertical integration are higher when the unit costs of production for the buyer are higher and they decrease when the supplier's unit cost decreases, when downstream demand becomes more inelastic or when demand growth is lower. As in the case of the optimal revenues sharing ratio, for certain model parameters the gains from vertical integration are almost invariant. For example, different levels of downstream demand do not appear to have a significant effect on the gains of vertical integration.

Finally, we analyze a revenue sharing contract that can overcome the double marginalization problem by achieving coordination in the supply chain. In particular, we extend our pure revenue sharing contract to a two-parameters revenue sharing contract that includes, besides the share of buyer's revenues captured by the supplier, a wholesale price paid by the buyer to the supplier. Under this contract, the optimal capacity choice and utilization rate of the buyer coincide with the optimal choice of a coordinated supply chain, so that coordination is achieved. We complement previous work (e.g., Cachon and Lariviere, 2004) by showing that under buyer production flexibility coordination with arbitrary profit division can still be achieved through a wholesale price contract with revenue sharing. In designing this contract, we ensure that a win-win condition holds, i.e., both parties obtain a higher profit under the coordinating contract than under a pure revenue sharing contract, by tuning the contract parameters. However, a certain degree of cooperation between the parties would be needed to design such a contract. Our results thus extend Gianoccarro and Potrandolfo (2004)'s results to a multiperiod setting under uncertainty with capacity choice and volume flexibility, albeit in a two-stage supply chain.

The rest of this paper is organized as follows. Section 2 describes the framework and the mathematical solution for the decentralized pure revenue sharing contract, as well as for the vertically integrated supply chain. Section 3 provides numerical sensitivity and our main results. Section 4 extends the framework to consider the case of minimum order quantities imposed by the supplier. Section 5 analyzes a two-parameters revenue sharing contract that coordinates the supply chain. Section 6 concludes.

2. The model

2.1. The model setup

The price of the good sold in the downstream market is p per unit of goods sold and given by the iso-elastic inverse demand function:

$$p = xq^{\varepsilon} \quad (1)$$

where ε is a measure of price sensitivity $-1 < \varepsilon < 0$ and x represents the demand shock. The elasticity of demand which is usually defined as the percentage sensitivity of quantity demanded to price changes is thus $\left(\frac{1}{|\varepsilon|}\right)$. Thus, a higher $|\varepsilon|$ implies a more inelastic demand.[5] The demand shock x affecting the price per unit at which the buyer can sell the goods in the downstream market follows a Geometric Brownian motion:

$$\frac{dx}{x} = \mu dt + \sigma dZ \quad (2)$$

where μ is the expected rate of change, σ is the volatility and dZ is a standard increment of a Weiner process. The demand shock x can be interpreted as the relative strength of the demand in the downstream market. We assume risk-neutrality, with r denoting the risk-free interest rate, and that $r > \mu$ such that there is a rate of return shortfall similar to a convenience yield $\delta = r - \mu$. A higher δ (while keeping r constant) captures a lower rate of growth of the good's demand in the buyer's markets. We assume that the buyer of goods selects the optimal capacity Q (Nishihara et al., 2019). Specifically, we assume at $t = 0$ the buyer needs to incur a one-time investment cost of κQ^{η}, where Q is the capacity of the goods (i.e., maximum units of goods that can be produced per unit time), Q^{η} is the amount of capital required to produce at that capacity (with $\eta > 1$), and the cost of capital is \$$\kappa$ per unit. The buyer firm faces both fixed costs of production c, as well as variable costs v. Due to variable costs, following the capacity choice the buyer selects the level of utilization of capacity q by maximizing its profits (see analysis that follows on determining the optimal q). The firm can either produce below full capacity, $q<Q$, in which case the level of production q varies with the demand shock x or at full capacity, with $q = Q$. This type of flexibility is important in many settings including, among others, car manufacturing (Hagspiel et al., 2016). The buyer continuously purchases the quantity of input goods q from the supplier which are paid in cash. For simplicity we assume that the buyer is a reseller of goods, i.e., the buyer acts as a retailer and it does not further process the goods. We do not incorporate default timing since the buyer can adjust the volume of production to limit losses when demand is not favorable. Thus, adding the optimal timing of stopping production will likely not have any major impact unless fixed costs are significant. We provide the impact of fixed costs in our sensitivity analysis.

The supplier continuously provides a quantity of input goods q to the buyer which are paid in cash and incurs the cost of production of these goods, c_S per unit sold. We model the decision making of the two firms as a Stackelberg (leader-follower) game in which the supplier acting as the Stackelberg leader first chooses how much to charge the buyer by selecting its share of the revenue α obtained from each unit sold in the downstream market. This is the only choice variable for the supplier in the problem. In turn, the choice of α affects the capacity and utilization of capacity (i.e., production) decisions of the buyer and the price of goods in the downstream markets.

2.2. The model solution

Since the supplier obtains a fraction α from the value of each unit sold this means that (1-α) remains to the buyer. The profits per dt interval for the buyer are then as follows: $\pi = ((1-\alpha)p - v)q - c = (1-\alpha)xq^{\varepsilon+1} - vq - c$. Maximizing the profits with respect to q results in the optimal level of $q = \left(\frac{(1-\alpha)x(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$. It can be seen that $\frac{dq}{dx} > 0, \frac{dq}{dv} < 0, \frac{dq}{d\alpha} < 0$ while $\frac{dq}{d\varepsilon}$ is indeterminate.[6] These effects are intuitive. For example, a higher level of demand (x) will result in a higher utilization of capacity, while a higher level of variable costs results in a lower capacity utilization.

[5] Since $|\varepsilon| < 1$ this implies that our focus is on $\frac{1}{|\varepsilon|} > 1$, i.e., an elastic demand where an increase in prices by 1% causes a more than 1% decrease in quantity. In line with previous literature, demand is assumed elastic since if demand were inelastic profits would tend to infinity as the quantities tend to zero. The same iso-elastic demand was used in Aguerrevere (2009), Dixit and Pindyck (1994), Dobbs (2004) and Silaghi and Sarkar (2020). For a review of the implications of different forms of demand functions on firms' capacity choice see Huberts et al. (2005). Also note that since our model is cast in terms of ε, where $-1 < \varepsilon < 0$, a higher ε implies a lower $|\varepsilon|$ and thus a more elastic demand ($\frac{1}{|\varepsilon|}$ becomes higher). For example, ε = -0.6> ε = -0.7, but $|\varepsilon| = 0.6 < |\varepsilon| = 0.7$.

[6] The analytic expressions for all derivatives are shown in Appendix C.

In addition, when the share of revenues of the supplier increases this creates an incentive for the buyer firm to reduce the quantities ordered. Careful inspection of the expression $\frac{dq}{d\varepsilon}$ reveals that $\frac{dq}{d\varepsilon} > 0$ when the share of revenue demanded by the supplier is relatively low and/or the relative of the price to cost ratio (x/v) is high (indicating buyers with relatively high profit margins). On the contrary, when the supplier extracts a high revenue share and/or the buyer has little profit margins then the buyer reacts by reducing order quantities when demand becomes more elastic (i.e., $\frac{dq}{d\varepsilon} < 0$). As we will show later on, these same factors appear to influence whether the buyer is more profitable in more elastic or inelastic markets.

The corresponding price is $p = xq^{\varepsilon} = \frac{v}{(1-\alpha)(\varepsilon+1)}$. Note that the price charged by the buyer firm adds a constant mark-up $\frac{1}{(1-\alpha)(\varepsilon+1)} > 1$ over the variable cost v (where $\varepsilon > -1$ is needed to ensure a positive mark-up). This mark-up increases as the share of revenue of the supplier increases thus highlighting the double marginalization effect in place. This mark-up also increases as the price sensitivity ε (in absolute terms) increases (i.e., as the demand becomes more inelastic). We find that $\frac{dp}{dv} > 0$, $\frac{dp}{d\alpha} > 0$, $\frac{dp}{d\varepsilon} < 0$. Substituting prices and quantities into buyer profit we obtain $\pi_B = A\,x^{-1/\varepsilon} - c$, where $A = -\left(\frac{v\varepsilon}{\varepsilon+1}\right)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$. In line with economic intuition, the buyer's profits depend on parameters as follows: $\frac{d\pi_B}{dx} > 0$, $\frac{d\pi_B}{dv} < 0$, $\frac{d\pi_B}{d\alpha} < 0$. However, the impact of elasticity, $\frac{d\pi_B}{d\varepsilon}$ is indeterminate and depends on the share of revenues claimed by the supplier and the relative level of x relative to v. More thorough analysis of $\frac{d\pi_B}{d\varepsilon}$ shows that when the share claimed by the supplier is high and x relative to v is low, then $\frac{d\pi_B}{d\varepsilon} < 0$, but when the share claimed by the supplier is low and when x is sufficiently higher than v, then $\frac{d\pi_B}{d\varepsilon} > 0$. Thus, when the buyer operates with relatively high profit margins then its profitability is further enhanced by a more elastic demand (and vice versa).

We observe that q increases with x ($\frac{dq}{dx} > 0$), however it cannot increase beyond Q which is the maximum capacity level. Assuming that the maximum capacity level is reached at $x = \bar{x}$ then using the optimal quantities we find that $Q = \left(\frac{\bar{x}(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$, which implies that the maximum capacity is reached at $\bar{x} = \frac{v}{(1-\alpha)(\varepsilon+1)Q^{\varepsilon}}$. Note that the threshold where full capacity is reached depends on the variable cost of production v, the installed capacity Q, the supplier's share of the price α and the elasticity of demand ε, as follows: $\frac{d\bar{x}}{dv} > 0$, $\frac{d\bar{x}}{dQ} > 0$, $\frac{d\bar{x}}{d\alpha} > 0$ and $\frac{d\bar{x}}{d\varepsilon}$ is indeterminate. Intuitively, a higher variable cost of production v, higher installed capacity Q, and higher supplier's share of the price α results in the buyer postponing production at full capacity. A more elastic demand results in an acceleration of the buyer firm entering into full scale operations when the installed capacity is small, while when the installed capacity is large a more elastic demand results in the buyer postponing switching to full capacity.

There are two operating regions depending on whether $x < \bar{x}$ or $x \geq \bar{x}$ as follows:

Region 1: $x < \bar{x}$: $p = \frac{v}{(1-\alpha)(\varepsilon+1)}$, $q = \left(\frac{(1-\alpha)x(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ and $\pi_B = A\,x^{-1/\varepsilon} - c$, with $A = -\left(\frac{v\varepsilon}{\varepsilon+1}\right)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$

Region 2: $x \geq \bar{x}$: $p = xQ^{\varepsilon}$, $q = Q$ and $\pi_B = (1-\alpha)xQ^{\varepsilon+1} - vQ - c$.

Since we assume that there is no working capital (e.g., inventory or credit) the profits are equivalent to cash flows. Following standard arguments in the real options literature (see Dixit and Pindyck, 1994) the buyer firm value $B_i(x)$ satisfies the following differential equations depending on the region of operation:

$$rB_i(x) = (r-\delta)xB_i'(x) + \frac{\sigma^2}{2}x^2B_i''(x) + \pi_{Bi}, \qquad i = 1,2. \tag{3}$$

where the last term denotes the cash flows received per dt.

The following proposition presents the buyer value in both regions.

Proposition 1 (Value of the buyer firm)

The buyer value is given by:

Region 1, $x < \bar{x}$: $$B_1(x) = \frac{A}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)} x^{-1/\varepsilon} - \frac{c}{r} + \Omega_1 x^{\beta_1} \quad (4)$$

Region 2, $x \geq \bar{x}$: $$B_2(x) = \frac{(1-\alpha)xQ^{\varepsilon+1}}{\delta} - \frac{c+vQ}{r} + \Omega_2 x^{\beta_2}, \quad (5)$$

where $$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \quad (6a)$$

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \quad (6b)$$

and Ω_1 and Ω_2 are determined from the following boundary conditions:

$$B_1(\bar{x}) = B_2(\bar{x}) \quad \text{(Value-matching)} \quad (7)$$

$$B'_1(\bar{x}) = B'_2(\bar{x}) \quad \text{(Smooth-pasting)} \quad (8)$$

Proof: The particular solutions in equations (4) and (5) are obtained by applying the differential equation in (3) the particular solution $B_i(x) = A_0 + A_1 x + A_2 x^{-\frac{1}{\varepsilon}}$. Ω_1 and Ω_2 are obtained by applying (7) and (8) respectively using equations (4) and (5) (see Appendix A for the detailed expressions). The term $\Omega_1 x^{\beta_1}$ captures the adjustment in value when the buyer moves to full capacity in region 2, while the term $\Omega_2 x^{\beta_2}$ captures the option to reduce the utilization of capacity below full capacity at level $q<Q$.

At time zero the value of the buyer firm is given by:

$$B_1^{Net}(x) = \max_Q\{B_1(x) - \kappa Q^{\eta}\} \quad \text{, if } x < \bar{x}, \quad (9)$$

else, if $x \geq \bar{x}$, the value of the buyer firm is given by:

$$B_2^{Net}(x) = \max_Q\{B_2(x) - \kappa Q^{\eta}\} \quad \text{for } x \geq \bar{x}, \quad (10)$$

Since $\bar{x}$ depends on Q¸ to find the optimal capacity we run various levels of capacity based on a dense grid of Q values where we apply (9) or (10) depending on the region being $x < \bar{x}$ or $x \geq \bar{x}$. Then the maximum value among buyer values among all grid levels defines the optimal capacity, as well as the operating region since it determines $\bar{x}$ where the firm operates.

We next move to the supplier. When the buyer is in region 1 (below full capacity), the supplier firm has the following profits per period $\pi_S = (\alpha p - c_S)q = \alpha x q^{\varepsilon+1} - c_S q$. The optimal quantity level is given by the buyer's optimization which resulted in $q = \left(\frac{(1-\alpha)x(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ (for $x < \bar{x}$) and the corresponding price is $p = xq^{\varepsilon} = \frac{v}{(1-\alpha)(\varepsilon+1)}$. Thus, the profit per dt for the supplier is: $\pi_S = Bx^{-1/\varepsilon}$ where:

$$B = \left(\frac{\alpha v}{(1-\alpha)(\varepsilon+1)} - c_S\right)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$$

The comparative statics for the supplier are as follows: $\frac{d\pi_S}{dx}0$ is indeterminate, $\frac{d\pi_S}{dc_s} < 0$, $\frac{d\pi_S}{d\alpha}$ is indeterminate, $\frac{d\pi_S}{dv}$ is indeterminate and $\frac{d\pi_S}{d\varepsilon}$ is indeterminate. Supplier's profits increase with the demand shock as long as the supplier's operating cost c_s is relatively low to allow for positive margins and they decrease with supplier production costs. The overall effect of the downstream variable costs on supplier's profit depends on which of the following two effects dominate. On the one hand, increasing variable costs reduces quantities ordered which has a negative impact on supplier profit. On the other hand, it also increases the price which positively affects supplier profit. Increasing supplier's revenue share has similar opposite indirect effects on supplier profits, in addition to a direct positive effect.[7] Finally, the effect of elasticity depends on parameter values, as in the case of the buyer.

For $x > \bar{x}$, the buyer produces at full capacity, $q = Q$ and the corresponding price is $p = xQ^{\varepsilon}$, and $\pi_S = (\alpha p - c_S)Q = \alpha x Q^{\varepsilon+1} - c_S Q$.

The supplier value satisfies the following differential equation:

$$rS_i(x) = (r-\delta)xS_i{}'(x) + \frac{\sigma^2}{2}x^2 S_i''^{(x)} + \pi_{S_i},\ i = 1,2 \tag{11}$$

The following proposition derives the value of the supplier.

Proposition 2 (Value of the supplier firm)

Region 1, $x < \bar{x}$: $\quad S_1(x) = \frac{B}{r + \left(\frac{r-\delta}{\varepsilon}\right) - 0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)} x^{-1/\varepsilon} + \Omega_1^S x^{\beta_1}$ (12)

Region 2, $x \geq \bar{x}$: $\quad S_2(x) = \frac{\alpha x Q^{\varepsilon+1}}{\delta} - \frac{c_S Q}{r} + \Omega_2^S x^{\beta_2}$, (13)

where the solutions for Ω_1^S and Ω_2^S are determined from equations

$$S_1(\bar{x}) = S_2(\bar{x}) \quad \text{(Value-matching)} \tag{14}$$

$$S'_1(\bar{x}) = S'_2(\bar{x}) \quad \text{(Smooth-pasting)} \tag{15}$$

Proof: The particular solutions in equations (12) and (13) are obtained by applying the differential equation in (11) the particular solution $S_i(x) = A_0 + A_1 x + A_2 x^{-\frac{1}{\varepsilon}}$. Ω_1^S and Ω_2^S are obtained by applying (14) and (15) respectively using equations (12) and (13) (see appendix A).

Note that the condition in equation (15) is a continuity (not an optimality) condition since the supplier value depends on the optimal choice of capacity of the buyer as described in equations (9) and (10) which also define the optimal threshold $\bar{x}$.

Finally, for comparison, we calculate the value of the firm if there is vertical integration. The vertically integrated profit per period in region 1 (unconstrained) is as follows: $\pi_V = (p - v - c_S)q - c = xq^{\varepsilon+1} - (v + c_S)q - c$. Maximizing the profits with respect to q results in the optimal level of $q_V = \left(\frac{x(\varepsilon+1)}{v+c_S}\right)^{-1/\varepsilon}$ and the corresponding price is $p_V = xq_V^{\epsilon} = \frac{v+c_S}{(\varepsilon+1)}$. Substituting this into profit we obtain $\pi = A_V\, x^{-1/\varepsilon} - c$ where $A_V = -\left(\frac{(v+c_S)\varepsilon}{\varepsilon+1}\right)\left(\frac{(\varepsilon+1)}{v+c_S}\right)^{-1/\varepsilon}$. A comparison of the vertically integrated firm with the non-coordinated profits of the buyer shows that the share of revenues of the supplier does not affect the optimally produced quantities nor the downstream prices. However, now the cost of production of these goods c_S enters the picture; the higher the cost c_S, the lower the produced quantities and the higher the price in the downstream market.

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[7] In Appendix C we show that $\frac{d\pi_S}{dv} < 0$ and $\frac{d\pi_S}{d\alpha} < 0$ when c_s is relatively small. Intuitively this implies that the effect of reduced quantities on the profitability of the supplier is more important than price increases when the supplier has high profit margins (implied by lower c_s).

Assuming that the maximum capacity level is reached at $x = \bar{x}_V$ then using the optimal quantities we find that $Q_V = \left(\frac{\bar{x}_V(\varepsilon+1)}{v+c_S}\right)^{-1/\varepsilon}$ which implies that the maximum capacity is reached at $\bar{x}_V = \frac{v+c_S}{(\varepsilon+1)Q_V^{\varepsilon}}$. Note that the threshold where full capacity is reached depends on the variable cost of production v, the cost of production of the input good c_S, the price sensitivity of demand ε, the installed capacity Q_V as follows: $\frac{d\bar{x}_V}{dv} > 0$, $\frac{d\bar{x}_V}{dc_S} > 0$, $\frac{d\bar{x}_V}{d\varepsilon}$ is indeterminate and $\frac{d\bar{x}_V}{dQ_V} > 0$.

The value of the vertically integrated firm satisfies the following differential equation:

$$rV_i(x) = (r-\delta)xV_i{}'(x) + \frac{\sigma^2}{2}x^2V_i^{\prime\prime(x)} + \pi_{V_i},\ i = 1,2 \tag{16}$$

The following proposition derives the value of the vertically integrated firm.

Proposition 3 (The value of the vertically integrated firm)

Region 1, $x < \bar{x}_V$: $$V_1(x) = \frac{A_V}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2(\frac{1}{\varepsilon})(\frac{1}{\varepsilon}+1)}x^{-1/\varepsilon} - \frac{c}{r} + \Psi_1 x^{\beta_1} \tag{17}$$

Region 2, $x \geq \bar{x}_V$: $$V_2(x) = \frac{xQ_V{}^{\varepsilon+1}}{\delta} - \frac{c+(v+c_S)Q_V}{r} + \Psi_2 x^{\beta_2}, \tag{18}$$

where β_1 and β_2 are given by (6a) and (6b).

and Ψ_1 and Ψ_2 are determined from the following boundary conditions:

$$V_1(\bar{x}_V) = V_2(\bar{x}_V) \quad \text{(Value-matching)} \tag{19}$$

$$V'_1(\bar{x}_V) = V'_2(\bar{x}_V) \quad \text{(Smooth-pasting)} \tag{20}$$

Proof: The particular solutions in equations (17) and (18) are obtained by applying the differential equation in (16) the particular solutions $V_i(x) = A_0 + A_1x + A_2x^{-\frac{1}{\varepsilon}}$. $\Psi_1$and Ψ_2 are obtained by applying (19) and (20) respectively using equations (17) and (18) (see Appendix A).

At time zero the value of the vertically integrated firm is given by:

$$V_1^{Net}(x) = \max_{Q_V}\{V_1(x) - \kappa Q_V{}^{\eta}\}, \qquad \text{if } x < \bar{x}_V, \tag{21}$$

else, if $x \geq \bar{x}$, the value of the vertically integrated firm is given by:

$$V_2^{Net}(x) = \max_{Q_V}\{V_2(x) - \kappa Q_V{}^{\eta}\}, \qquad \text{for } x \geq \bar{x}_V, \tag{22}$$

Since $\bar{x}_V$ depends on Q_V, to find the optimal capacity we run various levels of capacity based on a dense grid of Q_V values and check whether $x < \bar{x}$ in which case apply (21), else we apply (22). Then the maximum value among firm values among all grid levels defines the optimal capacity, as well as the operating region (since it determines $\bar{x}_V$) where the firm operates.

2.3. Interactions between supplier and buyer firm

In this section we provide a quantification of the trade-offs involved for the supplier firm when choosing the share of revenue taking into consideration the interactions with the buyer firm. In the region where the buyer operates unconstrained, the supplier's profits are[8]:

$$\pi_S = (\alpha p - c_S)q \tag{23}$$

By using total differentiation of the supplier profits we obtain the following:

$$\frac{d\pi_S}{da} = \underbrace{pq}_{>0} + \alpha q \underbrace{\frac{dp}{d\alpha}}_{>0} + (\alpha p - c_S)\underbrace{\frac{dq}{d\alpha}}_{<0} \tag{24}$$

Equation (24) shows that the supplier faces the following trade-offs when deciding to increase its share of revenues α. On the positive side, increasing α increases the revenue gained if quantities and prices are held fixed (first term) and increases the revenues due to higher prices (second term). On the negative side however, increasing α has an adverse effect on profits due to lower order quantities by the buyer (third term). Similar trade-offs hold when firms are in region 2 (buyer operates at full capacity) and equations (23) and (24) hold, albeit q is replaced for Q and only numerical comparative statics (not analytical) are available.

In addition to the above direct effects on profits there are also some non-linear effects that complicate the supplier's decision. These effects relate to the switching options that the buyer has between partial and full utilization of capacity and the level of its capacity choice. Although it is not possible to identify fully these non-linearities, we note some insights. First, we found earlier that $\frac{d\bar{x}}{da} > 0$ which implies that the higher the revenue share claimed by the supplier the longer the delay of the buyer switching to full capacity. In addition, $\frac{dQ}{da} < 0$ which shows that capacity is reduced when the supplier claims a larger revenue share. The combination of the above direct and indirect trade-offs determines the choice of the optimal revenue share offer that the supplier makes to the buyer firm.

3. Numerical analysis

We next provide sensitivity results and the implications of the model relating to the optimal revenue sharing offer of the supplier and the capacity and utilization of capacity of the buyer, as well as the prices of goods in the downstream markets. We assume the following base case parameters: $x = 10, \sigma = 0.2,\ v = 1, c = 0, c_S = 1, \varepsilon = -0.7,\ k = 3, \eta = 2, r = 0.05, \delta = 0.03$. Our base parameters used for r, δ and σ are in line with other real options models (e.g., Mauer and Sarkar, 2005 and Hackbarth and Mauer, 2011). η is the same as in Nishihara et al. (2019). A positive k alongside η determines an optimal capacity level for the buyer. The elasticity parameter ε is similar to Aguerrevere (2009) and Dobbs (2004).[9] We initially set $c = 0$ to avoid cases of negative profits when the volume of production is zero (we analyze $c > 0$ in our sensitivity analysis). The relative level between x and v is set to allow to retain positive values for both the supplier and buyer firms at various revenue sharing levels. Throughout the

[8] Note that taking the derivative of the supplier's profits with respect to α is equivalent to taking the derivative of the particular solution of the supplier with respect to α. Indeed, we have that $\pi_S = Bx^{-1/\varepsilon}$, while the particular solution for the supplier in region 1 is $\frac{B}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}x^{-\frac{1}{\varepsilon}}$. Formally, one can take the derivative of the supplier function in Proposition 2 which depends also on the flexibility of the buyer to switch between operating regions. These effects are incorporated in $\Omega_1^s x^{\beta_1}$ and $\Omega_2^s x^{\beta_2}$ in Proposition 2, however due to the non-linearities involved the expression of the derivative with respect to α is complicated. Instead, we try to gauge the effects by breaking down the direct revenue effects and the effect of switching between regions below.

[9] Aguerrevere (2009) uses ε = -0.625 and Dobbs (2004) an ε = -0.5. Note that the choice of ε values are restricted so that $r + \left(\frac{r-\delta}{\varepsilon}\right) - 0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right) > 0$ so that the particular solution in region 1 of Proposition 1 remains positive. Hagspiel et al. (2016) and Sarkar (2009) use a linear demand function and they also need to impose some constraints to maintain positive values on particular solutions. Specifically, they need to assume a high r and small μ and σ to maintain positive values for the particular solutions.

analysis we run a dense grid search for optimal capacity choice with increments of Q of 0.01. Similarly, for the share of revenues of the supplier we run a dense grid search with increments of α of 0.01.

3.1 Baseline results

Figure 1 shows supplier values as a function of the supplier claimed share of revenues α. The figure highlights our first important result regarding the existence of an optimal revenue sharing ratio, which is summarized as follows:[10]

[Insert Figure 1 here]

Result 1. There is an optimal sharing level α that maximizes the value of the supplier. The optimal sharing level α balances: a) the direct positive impact of a higher α on supplier revenues, b) the negative impact of a higher α on buyer's optimal capacity, c) the positive impact on prices due to lower quantities ordered, and d) a negative effect caused by a delay in the buyer firm moving to full capacity.

In order to better understand the forces involved in determining Result 1, i.e., the optimal level α, Table 1 Panel A shows how the main variables of the model change as the share of revenues claimed by the supplier (α) changes. The bold line shows the optimal pricing (revenue sharing) choice for the supplier firm. We observe that when the share of revenues of the supplier is low, the buyer selects a high capacity level and a high utilization rate. Actually, when $\alpha \leq 0.3$ the buyer starts operations at full capacity (Region 2). As α increases, the buyer reduces both the optimal capacity level and the optimal production level. The low quantities produced result in an increase in the price of the final good sold. For the supplier, an increase in α thus implies the following trade-offs which confirm the insights discussed in Altug and van Ryzin (2014). On the one hand, there is a direct positive effect, since the supplier is capturing a larger fraction of the revenues. Moreover, we have an additional indirect positive effect due to the resulting increase in the price, which increases per unit profit. On the other hand, we have an indirect negative effect since quantities sold decrease, which decreases net revenues. In addition, the buyer postpones entering into full capacity for higher α (notice that $\bar{x}$ increases with α). For small increases in α the positive effect dominates. However, at relatively large values of α the negative impact of low quantities dominates the positive effect of a higher per unit profit. Hence, we obtain an optimal level of α.

[Insert Table 1 here]

3.2 Fixed revenue sharing contract

To understand how different economic conditions affect buyer's optimal selection of capacity and its utilization rate for a given revenue sharing contract, we first run sensitivity results with respect to model parameters for a given α (we use $\alpha = 0.4$).[11] This analysis is important to understand how different types of buyer firms and their decisions affect supplier value when a one-for-all fixed contract is offered by the supplier to different buyer firms. For example, the analysis of this section can answer the question of whether a supplier value is hampered or improved when a buyer operates in different volatility of downstream demand environments.

[10] This result is general as can be seen in other sensitivity results we have run.
[11] We use a value of 0.4, instead of 0.79, the optimal α value for the benchmark parameter values, because it allows us to better illustrate the entire model including both regions, below and at full capacity.

Figure 2 shows the effect of volatility which highlights some interesting real options effects relating to operational flexibility. A higher volatility creates a more valuable operational flexibility option for the buyer firm in varying the level of production. A higher volatility thus increases the optimal capacity choice and despite the delay in moving to full capacity ($\bar{x}$ increases), the values of both the buyer and supplier improve. This is beneficial for both the buyer and the supplier since both count on the option to switch to full capacity when demand is favorable which is more valuable when volatility is high (while on the downside both firms are protected from losses due to volume flexibility, i.e., the ability to reduce production). At low enough volatility the firm starts at full capacity (see case of low volatility equal to only 8%), but operates at the lowest capacity and highest price levels. An increase in volatility from this low level of volatility increases capacity, however, the firm does not utilize all capacity and hence the firm moves to region 1 (below full capacity).[12] Further increases in volatility do not change q and p (firm continues in region 1) despite the higher installed capacity. A larger volatility thus makes it more likely for the firm to move from region 2 (full capacity) to region 1 (this is despite the increase in the threshold $\bar{x}$).

[Insert Figure 2 here]

We have investigated the effect of all parameters with extensive sensitivity analysis and their effects are summarized in the following results.

Result 2a (Buyer effect). The buyer value has the following directional effects with respect to model parameters: it increases with x, σ and r, decreases with v, k, η, c and δ, has a U-shape with respect to absolute value of ε and is invariant to c_S.

Result 2b (Supplier effect). The supplier value has the following directional effects with respect to model parameters: it increases with x, σ and r, decreases with k, η, δ, and c_S, has a U-shape with respect to absolute value of ε, an inverse U-shape with respect to v and is invariant to c.

Result 2c (Effect on capacity *Q*). The buyer's capacity Q has the following directional effects with respect to model parameters: it increases with x, σ and r, decreases with, v, absolute value of ε, k, η, δ, and is invariant to c and c_S.

Result 2d (Effect on utilization *q*). The buyer utilization of capacity q has the following directional effects with respect to model parameters: it increases with x, r and σ (flattens out for high r and σ), decreases with, v, absolute value of ε, k, δ, (remains flat for low δ and k) and is invariant to c and c_S.

While all above effects have important implications for different economic settings, we emphasize some effects that may be less intuitive. Firstly, the aforementioned effect of volatility is at first a surprising result. However, this result stems from the importance of buyer production flexibility captured in our setting. Under more volatile demand environments the buyer firm installs more capacity in order to be able to react in future favorable scenarios. Our analysis can help explain, for example, why Tesla installed a significant capacity for the production of electric cars even when demand for electric cars remained highly uncertain (see Randal, 2021).

Secondly, we highlight the effect of elasticity which has important implications for different types of products (e.g., luxury vs. necessities). A higher absolute value of ε implying a more inelastic demand, i.e., a lower $\frac{1}{|\varepsilon|}$ (e.g., implying the product becomes more of a necessity) has a significant negative impact on both capacity and utilization (see Figure 3). Thus, at higher absolute value of ε the price increases due to the lower quantities produced. The overall impact on buyer and supplier value depends on which of the two effects (lower quantities or higher price) dominates and we

[12] The downward jump in the price from the 8% volatility to 10% volatility reflects the change capacity and utilization from a region of lower capacity and production (utilization) q to a higher capacity and higher production q.

generally have a U-shape effect on values: for low absolute ε (more elastic demand) buyer and supplier values decrease since the impact on quantities is relatively more important than the impact on prices. However, this reverses for more inelastic demand levels (higher absolute ε).

[Insert Figure 3 here]

Some other parameters have very intuitive effects and are not shown for brevity. All sensitivities supporting Result 2 are shown in Appendix D. For example, higher initial demand level x or lower level of variable costs v result in an improvement in the value of the buyer and supplier firm and an increase in capacity (Q) and utilization (q). A higher cost for installing capacity k reduces the level of capacity of the buyer and has an adverse effect on both the buyer and supplier firm. A higher fixed cost of production for the buyer c does not change the buyer's capacity or utilization and thus has no effect on the threshold $\bar{x}$, the price of goods sold in the downstream market, nor on supplier value. However, higher fixed costs reduce the buyer firm value because even when reducing the volume of production to zero in unfavorable demand states, the buyer still needs to incur a fixed operational cost. A higher δ which implies a reduction in demand growth adversely affects capacity levels (Q). This has a negative effect on both the buyer and the supplier values. A higher c_s reduces only the supplier value and has no other impact on buyer's policy or values. Finally, a higher r acts in the opposite direction of δ since it effectively implies a higher drift in demand. A higher η acts in the same direction as k since it implies higher costs of installing capacity.

3.3. Optimal revenue sharing contract

We now conduct sensitivity analysis to model parameters allowing the supplier to optimize its pricing policy by optimally selecting α. That is, the supplier firm anticipates buyer's capacity and utilization decisions, and hence we solve a Stackelberg leader-follower type of game by optimizing supplier's claim of revenue share (see section 2.3). There are counterbalancing forces in place when the supplier decides to change its share α in response to a parameter change since this causes a reaction to the buyer's capacity and utilization. Due to these counterbalancing forces there is relatively small variation in optimal α. For example, when σ increases from 10% to 50% the optimal α only increases by 3% (from 79% to 82%). We find that the above effects for σ are magnified for higher v. Table 2 illustrates the effect of volatility which highlights some interesting trade-offs that need to be considered by the supplier firm when adjusting α at different volatility levels. We demonstrate the results for a higher v compared to the base case since the effects are more pronounced and thus easier to illustrate. Similar directional effects hold for our base case parameters as can be seen in reported results in the Appendix E.

[Table 2 here]

For a given revenue share, a higher σ increases the capacity of the buyer which improves supplier value even if its share of revenues remains unchanged (see earlier discussion relating to Fig.2). Thus, the supplier's decision of increasing α involves a trade-off. While increasing α has a direct positive effect on revenues, it also has a negative impact on order quantities due to lower capacity and utilization of the buyer. This negative impact is however also mitigated by the positive effect on prices of lower quantities produced. Despite the counterbalancing forces, as shown in the results of Table 2, the supplier will generally find preferable to increase its optimal claimed share for higher volatility. For example, when volatility increases from 10% to 20% the share of the supplier increases from 71% to 75%. Had the supplier not increased its share the buyer would choose an optimal capacity Q = 0.87, a utilization rate of 5.8% and the resulting price in the market would be 80.46. The supplier would then have a value of 150.36. Instead, we observe that it is optimal to increase its share from 71% to 75%, which despite the slight decrease in capacity and utilization (Q drops to 0.78 and utilization to 5.3%) results in a higher supplier value of 151.15 since there is a higher resulting equilibrium price of 93.33.

The above trade-offs characterize how the supplier chooses optimal α when there is a variation in other model parameters. For example, at a higher demand level x the buyer would increase optimal capacity and utilization if α remained fixed. Thus, the supplier would face a similar dilemma in increasing α since it would have a counterbalancing effect on optimal capacity and utilization. We have found that for some parameters these counterbalancing forces create no discernible variation in α, while for others the effects show a clearer direction. We provide the following result which summarizes the effects of model parameters on the optimal share of revenues where a clearer directional pattern could be determined. All sensitivity results are shown in the Appendix E.

Result 3 (Optimal revenue sharing contract). The optimal share of revenues α increases with σ, c_s, the absolute value of ε , r, and η and decreases with v and δ. The optimal revenue sharing contract exhibits low variation to other model parameters.

Result 3 has important implications for the design of optimal contracts for supplier firms. It suggests when there should be a significant adjustment in the claim of optimal share of revenues in response to different market conditions. For example, the results generally suggest that suppliers would require a higher optimal share for necessities (more inelastic) compared to luxury (more elastic) products, while they will tend to require a lower share of revenues when the variable costs of production of the buyer are high or when the downstream demand growth is smaller. Importantly, the result also shows that there is no significant variation in optimal α for other parameters and thus a one-for-all contract offered by the supplier to buyer firms may not be far from an optimal choice for the supplier in these situations. Thus, in cases where the supplier faces significant costs of discerning information (e.g., about the demand level in the buyer's market), offering the same contract to all buyer firms will not be far from the optimal choice.

3.4. Gains from vertical integration

We now analyze the gains from vertical integration. In Table 1, Panel B we provide the solution with vertical integration for the base case parameters. We observe that integrating production of the input results in an improvement in capacity (Q) relative to the optimal solution with non-coordinated production of α = 0.79. The gain in overall value of integrating production relative to the sum of buyer and supplier values with no coordination is 15.5%. We also note that the vertically integrated firm produces higher quantities and this results in a reduction in the price of the good offered in the market. We summarize the following main result:

Result 4a. (Coordinated vs. non-coordinated production). Relative to the non-coordinated production, integration of production results in an improvement in capacity, a gain in overall value relative to the sum of buyer and supplier values and a lower price of the good offered in the market.

Result 4a is in line with the double marginalization problem of disintegrating production where positive mark-ups are added in different stages of production resulting in higher prices (see Spengler, 1950 and Tirole, 1988, ch.4). However, our framework allows for further insights within a context of production flexibility. Due to the counterbalancing effect on buyer's capacity and utilization that the supplier needs to consider for adjusting optimal α in response to changes in parameter values (discussed in the previous section), there is a small variation in gains from vertical integration. The gains from vertical integration for alternative model parameters for which we have obtained clearer directional effects are summarized in result 4b.

Result 4b. (Gains from vertical integration). The gains for vertical integration are higher with higher σ, v and r and lower δ, c_s and lower absolute value of ε. The gains are almost invariant to other model parameters.

We note that there are generally higher gains from vertical integration when volatility is high, i.e., when operational flexibility is more valuable. However, due to counterbalancing forces involved in the choice of the supplier firm regarding α the gains are not substantially different and range from about 16% for low volatility (σ = 0.1) to about 19% for very volatile demand (σ = 0.5). Also, a higher volatility is not generally beneficial for end consumers in a decentralized setting. This is because despite the higher capacity installed when operational flexibility is present, a possible lower utilization rate may actually result in prices being higher for end consumers.

4. Minimum quantities ordered imposed by the supplier

We now consider the case in which the supplier firm imposes a minimum order restriction Q_{min}. Indeed, in practice suppliers usually impose a few restrictions such as minimum order quantities due to economies of scale in transportation and production setups (Aswathi et al., 2009; Burke et al., 2007). Just like before, the buyer also has capacity constraints with a maximum capacity Q which is reached at $\bar{x} = \frac{v}{(1-\alpha)(\varepsilon+1)Q^{\varepsilon}}$. Assuming that the minimum quantity level is reached at $x = \bar{x}_{min}$ then using the minimum quantity $Q_{min} = \left(\frac{\bar{x}_{min}(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ implies that the minimum capacity is reached at $\bar{x}_{min} = \frac{v}{(1-\alpha)(\varepsilon+1){Q_{min}}^{\varepsilon}}$.[13]

There are now three operating regions as follows:

Region 1: $x < \bar{x}_{min}$, $p = x{Q_{min}}^{\varepsilon}$, $q = Q_{min}$ and $\pi_B = (1-\alpha)x{Q_{min}}^{\varepsilon+1} - vQ_{min} - c$.

Region 2: $\bar{x}_{min} < x < \bar{x}$: $p = \frac{v}{\varepsilon+1}$, $q = \left(\frac{x(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$ and $\pi_B = A\, x^{-1/\varepsilon} - c$, with $A = -\left(\frac{v\varepsilon}{\varepsilon+1}\right)\left(\frac{(1-\alpha)(\varepsilon+1)}{v}\right)^{-1/\varepsilon}$

Region 3: $x \geq \bar{x}$: $p = xQ^{\varepsilon}$, $q = Q$ and $\pi_B = (1-\alpha)xQ^{\varepsilon+1} - vQ - c$.

The buyer firm value $B_i(x)$ satisfies the following differential equation within regions:

$$rB_i(x) = (r-\delta)xB_i{}'(x) + \frac{\sigma^2}{2}x^2B_i{}''(x) + \pi_{Bi},\ i\text{=1,2,3} \qquad (25)$$

where the last term denotes the cash flows received under that region.

Proposition 4 (Value of the buyer and optimal capacity choice with minimum order quantities)

The buyer value is given by:

Region 1, $x < \bar{x}_{min}$ $B_1(x) = \frac{(1-\alpha)x{Q_{min}}^{\varepsilon+1}}{\delta} - \frac{c+vQ_{min}}{r} + \Omega_1 x^{\beta_1}$ (26)

Region 2, $\bar{x}_{min} < x < \bar{x}$: $B_2(x) = \frac{A}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)}x^{-1/\varepsilon} - \frac{c}{r} + \Omega_2 x^{\beta_1} + \Omega_3 x^{\beta_2}$ (27)

Region 3, $x \geq \bar{x}$: $B_3(x) = \frac{(1-\alpha)xQ^{\varepsilon+1}}{\delta} - \frac{c+vQ}{r} + \Omega_4 x^{\beta_2}$, (28)

and $\Omega_1, \Omega_2, \Omega_3, \Omega_4$ are determined from the following boundary conditions:

$B_1(\bar{x}_{min}) = B_2(\bar{x}_{min})$ (Value-matching between region 1 and 2) (29)

$B'_1(\bar{x}_{min}) = B'_2(\bar{x}_{min})$ (Smooth-pasting between region 1 and 2) (30)

$B_2(\bar{x}) = B_3(\bar{x})$ (Value-matching between region 2 and 3) (31)

$B'_2(\bar{x}) = B'_3(\bar{x})$ (Smooth-pasting between region 2 and 3) (32)

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[13] In principle one could add also an abandonment threshold for the buyer which when reached it stops production altogether. This is feasible but we do not expect it will add further insights. Note also that we are assuming $Q_{min} < Q$ so that $\bar{x}_{min} < \bar{x}$. Nevertheless, in extreme cases the minimum order quantity required by the supplier could be so high that it might affect the buyer's optimal capacity choice such that $q = Q = Q_{min}$. We illustrate this case numerically.

Proof: Similar to the proof of Proposition 1.

At time zero the value of the buyer firm is given by:

$$B_1^{Net}(x) = \max_Q\{B_1(x) - \kappa Q^{\eta}\} \quad \text{, if } x < \bar{x}_{min}, \tag{33}$$

$$B_2^{Net}(x) = \max_Q\{B_2(x) - \kappa Q^{\eta}\} \quad \text{, for } \bar{x}_{min} < x < \bar{x} \tag{34}$$

$$B_3^{Net}(x) = \max_Q\{B_3(x) - \kappa Q^{\eta}\} \quad , x \geq \bar{x} \tag{35}$$

Since $\bar{x}$ and $\bar{x}_{min}$ depend on Q and Q_{min} to find the optimal capacity we run various levels of capacity based on a dense grid and check which of the three regions apply to calculate the net value of the buyer. Then the maximum value among buyer values among all grid levels defines the optimal capacity, as well as the operating region where the firm starts to operate.

The value of the supplier is derived in a similar way. The following proposition derives the value of the supplier.

Proposition 5 (Value of the supplier firm with minimum order quantities)

$$\text{Region 1, } x < \bar{x}_{min}, \quad S_1(x) = \frac{axQ_{min}{}^{\varepsilon+1}}{\delta} - \frac{c_S Q_{min}}{r} + \Omega_1^s x^{\beta_1}, \tag{36}$$

$$\text{Region 2, } \bar{x}_{min} < x < \bar{x}: \quad S_2(x) = \frac{B}{r + \left(\frac{r-\delta}{\varepsilon}\right) - 0.5\sigma^2(\frac{1}{\varepsilon})(\frac{1}{\varepsilon}+1)} x^{-1/\varepsilon} + \Omega_2^s x^{\beta_1} + \Omega_3^s x^{\beta_2} \tag{37}$$

$$\text{Region 3, } x \geq \bar{x}: \quad S_3(x) = \frac{axQ^{\varepsilon+1}}{\delta} - \frac{c_S Q}{r} + \Omega_4^s x^{\beta_2}, \tag{38}$$

where the solutions for Ω_1^s, Ω_2^s, Ω_3^s and Ω_4^s are determined from the following boundary conditions:

$$S_1(\bar{x}_{min}) = S_2(\bar{x}_{min}) \quad \text{(Value-matching between region 1 and 2)} \tag{39}$$

$$S'_1(\bar{x}_{min}) = S'_2(\bar{x}_{min}) \quad \text{(Smooth-pasting between region 1 and 2)} \tag{40}$$

$$S_2(\bar{x}) = S_3(\bar{x}) \quad \text{(Value-matching between region 2 and 3)} \tag{41}$$

$$S'_2(\bar{x}) = S'_3(\bar{x}) \quad \text{(Smooth-pasting between region 2 and 3)} \tag{42}$$

Proof: Similar to the proof of Proposition 2.

In Table 3 we investigate numerically the effect of minimum order quantities imposed by the supplier. For $Q_{min} = 0$, we obtain our base case framework in which there were no minimum requirements for the quantities ordered. Indeed, comparing the values obtained for $Q_{min} = 0$ in Table 3 with those in bold in Table 1 corresponding to an optimal α of 0.79, we can see that they coincide.

[Insert Table 3 here]

As the minimum order quantity increases buyer value decreases while the supplier value increases. For $Q_{min} = 0.5$ the constraint is not binding at the current level of *x*, the buyer chooses both a capacity and a utilization level above the minimum order quantity. The quantity produced is between the minimum order quantity and the capacity level, i.e., $Q_{min} < q < Q$ (region 2). Although not binding at t = 0, the constraint on minimum quantities $Q_{min} = 0.5$ may become binding subsequently if demand drops below $\bar{x}_{min}$. Thus, we observe a slight decrease in the value of the buyer due to the imposed constraint. As the minimum order quantity increases further to $Q_{min} = 1$ the constraint becomes binding at t = 0 and the buyer produces the minimum order quantity, which is below full capacity. Therefore, the firm is in region 1, $q = Q_{min}$<Q. For even higher values of the minimum order quantity such as 1.5 or 2, the constraint affects not only the quantities produced by the buyer, but also its capacity. In this case, the buyer firm decides to set up an optimal capacity level equal to the minimum order quantity, $q = Q = Q_{min}$.

In sum, the minimum order quantity provides a tool for the supplier to improve its value over the buyer. We summarize the following result.

Result 5. (Minimum order quantity). When the supplier has the ability to impose minimum order quantities, then it can reduce the buyer's ability to choose capacity and its production flexibility and extract more value from the buyer firm.

Result 5 highlights the importance of using minimum order quantities as a mechanism suppliers can use to extract value from a buyer. Results in Table 3 suggest that supplier value increases with minimum order quantities for a fixed revenue share ratio α, so that the supplier would set the highest minimum order quantities possible. However, in practice buyer market power or other market conditions likely balance out these effects so that in practice one observes an optimal Q_{min} exists that allows for a minimum positive buyer value.[14] As an alternative to the case above where the supplier has a fixed α and chooses minimum order quantities, the supplier may impose a fixed $Q_{min} > 0$ (justified for example by the fact that it allows him to maintain some economies of scale in production) and then optimize its share α. We find that the higher the minimum order quantity imposed, the more the supplier can increase its claim on revenues without jeopardizing a reduction in sales below a certain level. For example, when $Q_{min} = 0$ (no constraints) we found that the optimal α is 79%, while for $Q_{min} = 0.4$ the optimal α is 82%. For an even higher Q_{min}, we find that the supplier can increase α further since a minimum order quantity is guaranteed.

5. Coordination in the supply chain

Previous literature on revenue sharing has proposed revenue sharing contracts as a coordination mechanism in the supply chain (Cachon and Lariviere, 2005; Giannoccaro and Potrandolfo, 2004, among others). In particular they propose contracts that can be described by two parameters: a wholesale price w charged by the supplier to the buyer and a share of the buyer revenues α obtained by the supplier. Such contracts are shown to coordinate the supply chain, that is, to reach the maximum supply chain profit under vertical integration.

Our model can be extended to incorporate a whole price charged by the supplier to the buyer. Such a revenue sharing contract can be shown to also achieve coordination in the supply chain in our framework under buyer production flexibility. In the rest of this section we derive the contract that can coordinate the supply chain similarly to previous literature.

We modify the benchmark model by assuming that the supplier receives not only a fraction α of the buyer revenues, but also a wholesale price w per unit. Regarding notation, we will use the upper index c to denote buyer and supplier values under the coordinated case.

The profits per dt interval for the buyer are then as follows: $\pi_B^c = \big((1-\alpha)p - w - v\big)q - c = (1-\alpha)xq^{\varepsilon+1} - (w+v)q - c$. Maximizing the profits with respect to q results in the optimal level of $q = \left(\frac{(1-\alpha)x(\varepsilon+1)}{w+v}\right)^{-1/\varepsilon}$.

The supplier firm has the following profits per period $\pi_S^c = (\alpha p + w - c_S)q = \alpha x q^{\varepsilon+1} + (w - c_S)q$.

[14] We could, for example accommodate a reservation value for the buyer depending on its market power that determines an equilibrium Q_{min}.

Summing the profits of the buyer and the supplier we obtain the vertically integrated profit per period: $\pi_V = (p - v - c_S)q - c = xq^{\varepsilon+1} - (v + c_S)q - c$. Maximizing the profits with respect to q results in the optimal level of $q_V = \left(\frac{x(\varepsilon+1)}{v+c_S}\right)^{-1/\varepsilon}$.

Hence, to achieve channel coordination, we need that the optimal order quantity chosen by the retailer q corresponds to the order quantity that optimizes the SC total profit q_V. This happens if the supplier offers the retailer a wholesale price equal to:

$$w = (1 - \alpha)(v + c_S) - v = c_S - \alpha(v + c_S) < c_S \qquad (43)$$

Therefore, under the revenues sharing contract, the supplier offers the buyer a wholesale price lower than its marginal cost c_S, but in exchange receives a fraction $1 - \alpha$ of the buyer's revenue.

From equation (43) we can see that $w + v = (1 - \alpha)(v + c_S)$, that is, the total payment of the buyer consisting of wholesale price paid to supplier plus its own variable costs represents a fraction $1 - \alpha$ of the total variable costs of the whole supply chain. Thus, the buyer shares both the revenues and variable costs of the supply chain in the same proportion. Since under the coordination contract the buyer will choose the same quantity as the supply chain, $q = q_V$ and $p = p_V$, we can express the buyer's profits as a function of the total supply chain profit: $\pi_B^c = (1 - \alpha)\pi_V - \alpha c$.[15] Thus, the revenue sharing contract makes the buyer's profit function an affine transformation of the supply chain's profit function.

Assuming that the maximum capacity level is reached at $x = \bar{x}$ then using the optimal quantities we find that $Q = \left(\frac{\bar{x}(\varepsilon+1)}{v+c_S}\right)^{-1/\varepsilon}$, which implies that the maximum capacity is reached at $\bar{x} = \bar{x}_V = \frac{v+c_S}{(\varepsilon+1)Q^\varepsilon}$.

We have two operating regions for the buyer depending on whether $x < \bar{x}$ or $x \geq \bar{x}$ as follows:

Region 1: $x < \bar{x}$: $p = p_V = \frac{v+c_S}{\varepsilon+1}$, $q = q_V = \left(\frac{x(\varepsilon+1)}{v+c_S}\right)^{-1/\varepsilon}$ and $\pi_B^c = (1 - \alpha)A_V\, x^{-1/\varepsilon} - c$

Region 2: $x \geq \bar{x}$: $p = xQ^\varepsilon$, $q = Q$ and $\pi_B^c = (1 - \alpha)xQ^{\varepsilon+1} - (1 - \alpha)(v + c_S)Q - c$.

Similar to the benchmark model, the buyer firm value $B_i^c(x)$ satisfies the following differential equations depending on the region of operation:

$$rB_i^c(x)(x) = (r - \delta)xB_i^{c\prime}(x) + \frac{\sigma^2}{2}x^2 B_i^{c\prime\prime}(x) + \pi_{Bi}^c, i = 1,2. \qquad (44)$$

Proposition 6 (Value of the buyer firm under a coordinating contract)

The buyer value function is given by:

Region 1, $x < \bar{x}$:

$$B_1^c(x) = \frac{(1-\alpha)A_V}{r+\left(\frac{r-\delta}{\varepsilon}\right)-0.5\sigma^2\left(\frac{1}{\varepsilon}\right)\left(\frac{1}{\varepsilon}+1\right)} x^{-1/\varepsilon} - \frac{c}{r} + \Omega_1^c x^{\beta_1} \qquad (45)$$

Region 2, $x \geq \bar{x}$:

[15] Substituting $w + v = (1 - \alpha)(v + c_S)$ into the buyer's profit $\pi_B^c = \big((1 - \alpha)p - w - v\big)q - c$, we get $\pi_B^c = (1 - \alpha)pq - (1 - \alpha)(v + c_S)q - c = (1 - \alpha)(p - v - c_S)q - c = (1 - \alpha)\pi_V - \alpha c$.

$B_2^c(x) = \frac{(1-\alpha)xQ^{\varepsilon+1}}{\delta} - \frac{c+(v+c_S)Q}{r} + \Omega_2^c x^{\beta_2}$, (46)

and Ω_1^c and Ω_2^c are determined from the following boundary conditions:

$B_1^c(\bar{x}) = B_2^c(\bar{x})$ (Value-matching) (47)

$B_1^{c\prime}(\bar{x}) = B_2^{c\prime}(\bar{x})$ (Smooth-pasting) (48)

Proof: The particular solutions in equations (45) and (46) are obtained by applying the differential equation in (44) the particular solution $B_i^c(x) = A_0 + A_1 x + A_2 x^{-\frac{1}{\varepsilon}}$. Ω_1^c and Ω_2^c are obtained by applying (47) and (48) respectively using equations (45) and (46) (see Appendix B).

As with profit functions, we can also express the value functions of the buyer as a function of the supply chain value, and it can be shown that a similar relationship holds as in the following corollary.

Corollary 1: The buyer's value function is an affine transformation of the supply chain's value function under a coordinating contract:

$B_1^c(x) = (1-\alpha)V_1(x) - \frac{\alpha c}{r}$ (49)

$B_2^c(x) = (1-\alpha)V_2(x) - \frac{\alpha c}{r}$, (50)

Proof: It follows directly from $\Omega_1^c = (1-\alpha)\Psi_1$ and $\Omega_2^c = (1-\alpha)\Psi_2$ (see Appendix B).

Therefore, the buyer will also choose the same optimal capacity as the vertically integrated supply chain, $Q = Q_V$.

In a similar fashion it can be shown that:

$S_1^c(x) = \alpha V_1(x) + \frac{\alpha c}{r}$ and $S_2^c(x) = \alpha V_2(x) + \frac{\alpha c}{r}$.

Hence, if the revenue sharing contract satisfies the condition given in equation (43) for the wholesale price, then it will achieve channel coordination regardless of the value of α, which should however belong to the interval (0,1).

Nevertheless, this revenue sharing contract that coordinates the supply chain will only be accepted by the two parties if they both obtain higher values under this contract compared to the uncoordinated case. Hence, the value of α has to satisfy a win-win condition (Giannoccaro and Potrandolfo, 2004): $\pi_B^c \geq \pi_B$ and $\pi_S^c \geq \pi_S$.

The first inequality is equivalent to $(1-\alpha)\pi_V - \alpha c \geq \pi_B$, which implies:

$$\alpha \leq \frac{\pi_V - \pi_B}{\pi_V - c} \quad (51)$$

The second inequality is equivalent to $\alpha(\pi_V + c) \geq \pi_S$, which implies:

$$\alpha \geq \frac{\pi_S}{\pi_V + c} \quad (52)$$

Combining equations (44) and (45) we obtain:

$$\alpha \in \left[\frac{\pi_S}{\pi_V + c}, \frac{\pi_V - \pi_B}{\pi_V - c}\right] \quad (53)$$

A revenue-sharing contract (w, α) that satisfies equations (43) and (53) will thus not only coordinate the supply chain, but also be preferred by both the buyer and the supplier. We illustrate this case numerically in Figure 8. We can see that the α that coordinates the supply chain lies in the interval (0.72, 0.86), with a corresponding wholesale price ranging between (-0.71, -0.43). Note that the coordinating wholesale price is actually negative. Indeed, from equation (43) it follows that the wholesale price will be negative, i.e., $w < 0$, whenever $\alpha > \frac{c_S}{v+c_S}$. For our parameter values $\frac{c_S}{v+c_S} = 0.5$, thus it follows that the wholesale price will be negative whenever the supplier captures more than half of the revenues. This is in line with the results of Cachon and Lariviere (2004). A negative wholesale price implies that the supplier is actually subsidizing the buyer. Intuitively, if the buyer's share of the channel's cost is high, then the buyer has already a low profit margin before the supplier takes a slice of revenue. If the supplier wants to claim a large share of revenue, it must subsidize the buyer's acquisition of product. As Cachon and Lariviere (2004) argue, if we want to rule out a negative wholesale price, then a positive cost for the buyer establishes a floor on buyer profit under coordinating contracts.

[Insert Figure 4 here]

The ultimate contract design, the actual contract parameters chosen by the two parties, will depend on the relative bargaining power of the supply chain parties. However, as pointed out by Giannoccaro and Potrandolfo (2004), it is important to stress that the implementation of this contract requires a certain degree of cooperation among the supply chain parties during the contract design phase.

6. Conclusion

In this paper we bridge the revenue sharing literature with the real options literature on production flexibility. We propose a unified real options framework to analyze a revenue sharing contract within a decentralized supply chain under buyer production flexibility and demand uncertainty. We find that the double marginalization problem is exacerbated under buyer production flexibility. Indeed, a pure revenue sharing contract exhibits losses of around 4%-22% for the parameters considered compared to a vertically integrated supply chain, with lowest losses when the buyer firm operates at full capacity.

We contribute to the literature by analyzing a multiperiod setting under uncertainty and buyer production flexibility and by providing a valuation of both the buyer and supplier firm. Our findings provide guidance to managers of suppliers on the design of optimal revenue sharing contracts. In addition, we show how to incorporate minimum order quantity constraints which capture many real-world supplier requirements. We quantify the impact of minimum order quantity constraints on buyer's capacity choices, capacity utilization and prices in the downstream markets. Moreover, we extend prior work by showing that a coordinating contract which combines a wholesale pricing and revenue sharing exists in a multiperiod setting under uncertainty and production flexibility. However, such a contract would require a certain degree of cooperation between the buyer and supplier.

Our setting has overlooked several issues which could be addressed in future research. First, we focus on a single buyer and single supplier. It would be interesting to investigate how competition in either the upstream or downstream markets affects the design of the revenue sharing contract under buyer production flexibility. Second, we have focused on a two-stage supply chain with buyer production flexibility. Future work could enrich the framework to consider a three-stage supply chain. Finally, we have assumed that the buyer pays the goods in cash. However, since trade credit is also commonly used in supply chains, it would be interesting to analyze how the use of trade credit affects supplier's optimal share of revenues and buyer's production and capacity decisions.

References

Aguerrevere, F. L. (2009). Real options, product market competition, and asset returns. *The Journal of Finance*, 64 (2), 957-983.

Altug, M. S., & van Ryzin, G. (2014). Is revenue sharing right for your supply chain?. *California Management Review,* 56 (4), 53-81.

Avinadav, T., Chernonog, T. & Perlman, Y. (2015a). The effect of risk sensitivity on a supply chain of mobile applications under a consignment contract with revenue sharing and quality investment. *International Journal of Production Economics*, 168, 31-40.

Avinadav, T., Chernonog, T. & Perlman, Y. (2015b). Consignment contract for mobile apps between a single retailer and competitive developers with different risk attitudes. *European Journal of Operational Research*, 246 (3), 949-957.

Awasthi A., Chauhan, S. S., Goyal, S. K. & Proth, J-M. (2009). Supplier selection problem for a single manufacturing unit under stochastic demand. *International Journal of Production Economics*, 117, 229-233.

Bart, N., Chernonog, T. & Avinadav, T. (2021). Revenue-sharing contracts in supply chains: a comprehensive literature review. *International Journal of Production Research*, 59(21), 6633-6658.

Bhaskaran, S. R., & Krishnan, V. (2009). Effort, revenue, and cost sharing mechanisms for collaborative new product development. *Management Science*, 55(7), 1152-1169.

Burke, G., Carrillo, J., & Vakharia, A. (2007). Single versus multiple supplier sourcing strategies. *European Journal of Operational Research*, 182, 95–112.

Cachon, G. P., & Lariviere, M. A. (2001). Contracting to assure supply: How to share demand forecasts in a supply chain. *Management Science*, 47(5), 629–646.

Cachon, G. P., & Lariviere, M. A. (2005). Supply chain coordination with revenue-sharing contracts: strengths and limitations. *Management Science*, 51 (1), 30-44.

Chakraborty, T., Chauhan, S. S. & Vidyarthi, N. (2015). Coordination and competition in a Common retailer channel: Wholesale price versus revenue-sharing mechanisms. *International Journal of Production Economics*, 166, 103-118.

Dana, J. D. & Spier, K. E. (2001). Revenue sharing and vertical control in the video rental industry. *The Journal of Industrial Economics*, 49 (3), 223-245.

Dixit, A. K., & Pindyck, R. S. (1994). Investment under Uncertainty. Princeton University Press.

Dobbs, I. M. (2004). Intertemporal price cap regulation under uncertainty. *The Economic Journal*, 114 (495), 421-440.

Fu, X. & Zhang, A. (2010). Effects of Airport Concession Revenue Sharing on Airline Competition and Social Welfare. *Journal of Transport Economics and Policy*, 44 (2), 119-138.

Gerchak, Y., & Wang, Y. (2004). Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand. *Production and Operations Management*, 13 (1), 23-33.

Giannoccaro, I. & Pontrandolfo, P. (2004). Supply chain coordination by revenue sharing contracts. *International Journal of Production Economics*, 89, 131-139.

Gerchak, Y. & Khmelnitsky, E. (2003). A consignment system where suppliers cannot verify retailer's sales reports. *International Journal of Production Economics* 83, 37-43.

Govindan, N. & Popiuc, M. N. (2014). Reverse supply chain coordination by revenue sharing contract: A case for the personal computers industry. *European Journal of Operational Research*, 233, 326-336.

Hackbarth, D., & Mauer, D. C. (2011). Optimal priority structure, capital structure, and investment. *The Review of Financial Studies*, 25 (3), 747-796.

Hagspiel, V., Huisman, K. J., & Kort, P. M. (2016). Volume flexibility and capacity investment under demand uncertainty. *International Journal of Production Economics*, 178, 95-108.

Hsueh, C. F. (2014). Improving corporate social responsibility in a supply chain through a new revenue sharing contract. *International Journal of Production Economics*, 151, 214-222.

Huberts, N. F., Huisman, K. J., Kort, P. M., & Lavrutich, M. N. (2015). Capacity choice in (strategic) real options models: A survey. *Dynamic Games and Applications*, 5 (4), 424-439.

Katok, E. & Wu, D. Y. (2009). Contracting in Supply Chains: A Laboratory Investigation. *Management Science*, 55 (12), 1953-1968.

Kong, G., Rajagopalan, S. & Zhang, H. (2013). Revenue Sharing and Information Leakage in a Supply Chain. *Management Science*, 59 (3), 556-572.

Krishnan, H. & Winter, R., A. (2011). On the Role of Revenue-Sharing Contracts in Supply Chains. *Operations Research Letters*, 39 (1), 28-31.

Lal, R. (1990). Improving channel coordination through franchising. *Marketing Science*, 9 (4), 299-318.

Mathewson, F. & Winter, R. (1985). The Economics of Franchise Contracts. *The Journal of Law & Economics*, 28 (3), 503-526.

Mauer, D. C., & Sarkar, S. (2005). Real options, agency conflicts, and optimal capital structure. *Journal of Banking & Finance*, 29 (6), 1405-1428.

Nishihara, M., Sarkar, S., & Zhang, C. (2019). Agency cost of debt overhang with optimal investment timing and size. *Journal of Business, Finance & Accounting*, 46 (5-6), 784-809.

Randall, Chris, (Nov. 26, 2021). "Tesla upgrades Shanghai gigafactory production capacities". Retrieved from the web from: https://www.electrive.com/2021/11/26/tesla-upgrades-shanghai-gigafactory-production-capacities/

Ritchken, P, & Wu, Q. (2020). Capacity investment, production flexibility, and capital structure. *Production and Operations Management*, 1-21 (forthcoming).

Rong, L. & Xu, M. (2020). Impact of revenue-sharing contracts on green supply chain in manufacturing industry. *International Journal of Sustainable Engineering*, 13 (4), 316-326.

Sarkar, S. (2009). A real-option rationale for investing in excess capacity. *Managerial and Decision Economics*, 30, 119–133.

Sarkar, S. (2018). Optimal DOL (degree of operating leverage) with investment and production flexibility. *International Journal of Production Economics*, 202, 172-181.

Silaghi, F., & Sarkar, S. (2021). Agency problems in public-private partnerships investment projects. *European Journal of Operational Research*, 290 (3), 1174-1191.

Spengler, J. J. (1950). Vertical integration and antitrust policy. *Journal of Political Economy*, 58 (4), 347-352.

Tan, Y., & Carrillo, J. E. (2017). Strategic Analysis of the Agency Model for Digital Goods. *Production and Operations Management*, 26 (4), 724-741.

Tirole, J. (1988). The theory of industrial organization. MIT press.

Wang, Y. & Gerchak, Y. (2003). Capacity games in assembly systems with uncertain demand. *Manufacturing & Service Operations Management*, 5 (3), 252-267.

Wang, Y., Jiang, L. & Shen, Z. J. (2004). Channel performance under consignment contract with revenue sharing. *Management Science*, 50 (1), 34–47.

Wang, J., & Shin, H. (2015). The Impact of Contracts and Competition on Upstream Innovation in a Supply Chain. *Production and Operations Management*, 24 (1), 134-146.

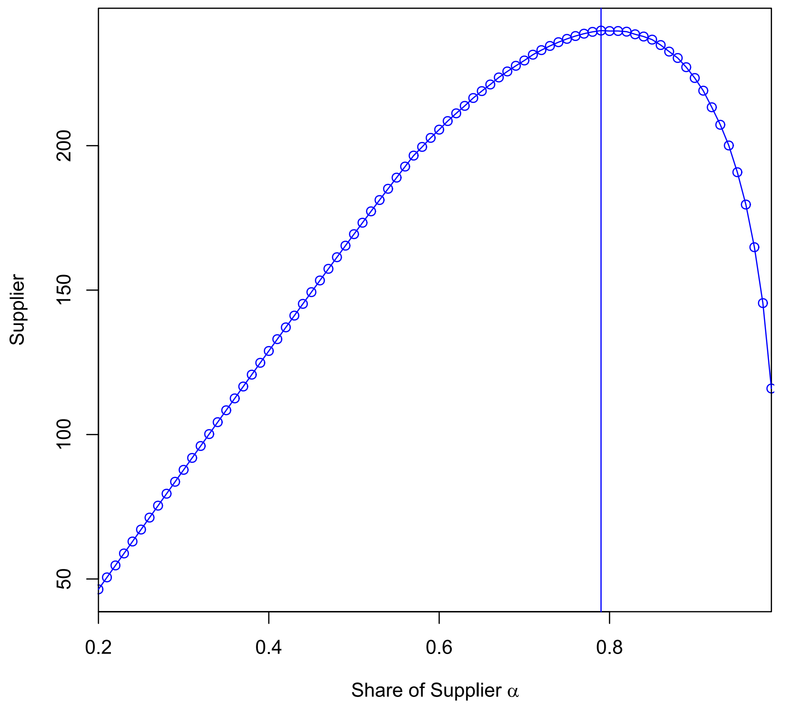
Xiao, W. & Xu, Y. (2018). Should an Online Retailer Penalize its Independent Sellers for Stockout? *Production and Operations Management*, 27 (6), 1124-1132.

Yang, H., & Chen, W. (2018). Retailer-driven carbon emission abatement with consumer environmental awareness and carbon tax: Revenue-sharing versus cost-sharing. *Omega*, 78, 179-191.

Yao, Z., Leung, S. C. & Lai, K. K. (2008). Manufacturer's revenue-sharing contract and retail competition. *European Journal of Operational Research*, 186 (2), 637-651.

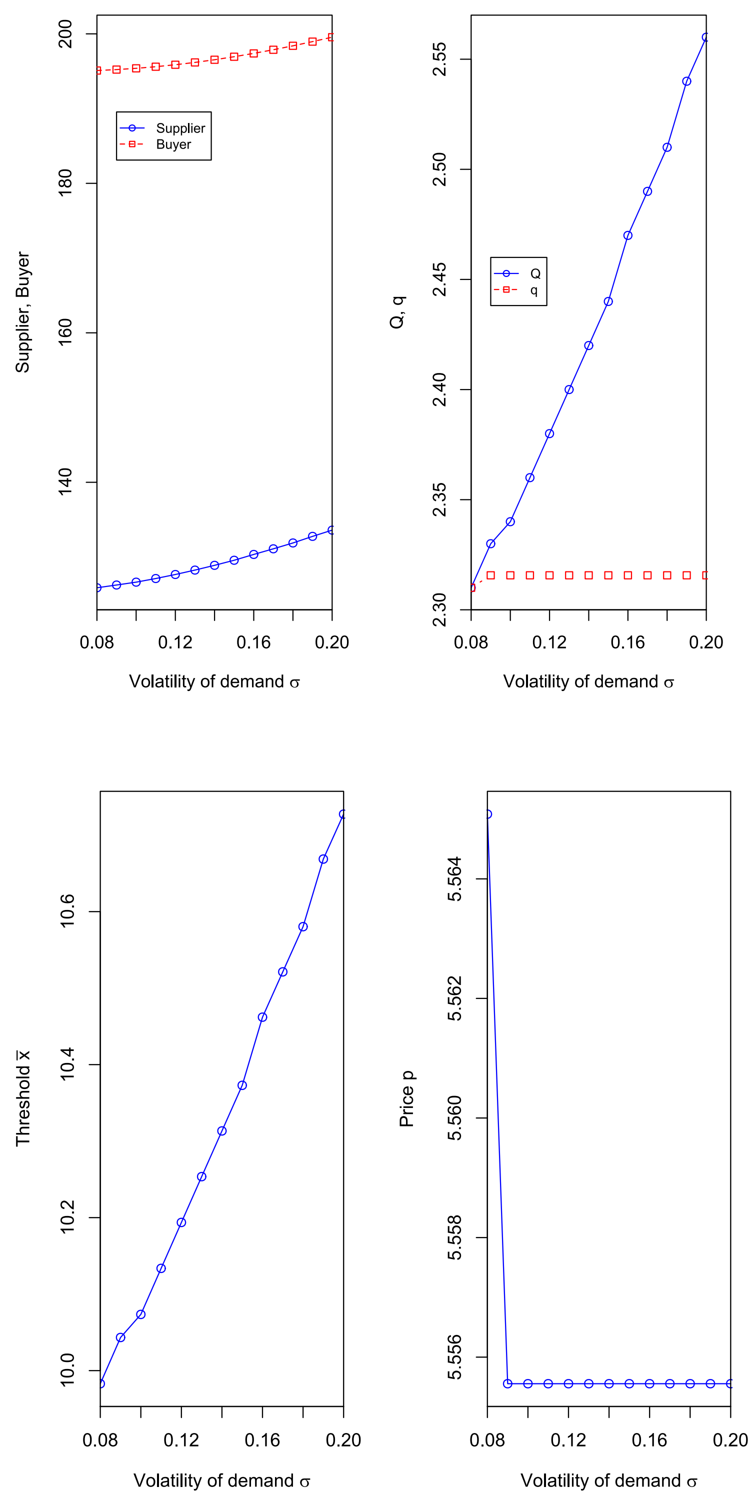
Zhang, Y., Donohue, K. & Cui, T. H. (2015). Contract preferences and performance for the loss-averse supplier: Buyback vs. revenue sharing. *Management Science*, 62 (6), 1734-1754.

Figure 1. Optimal share of supplier



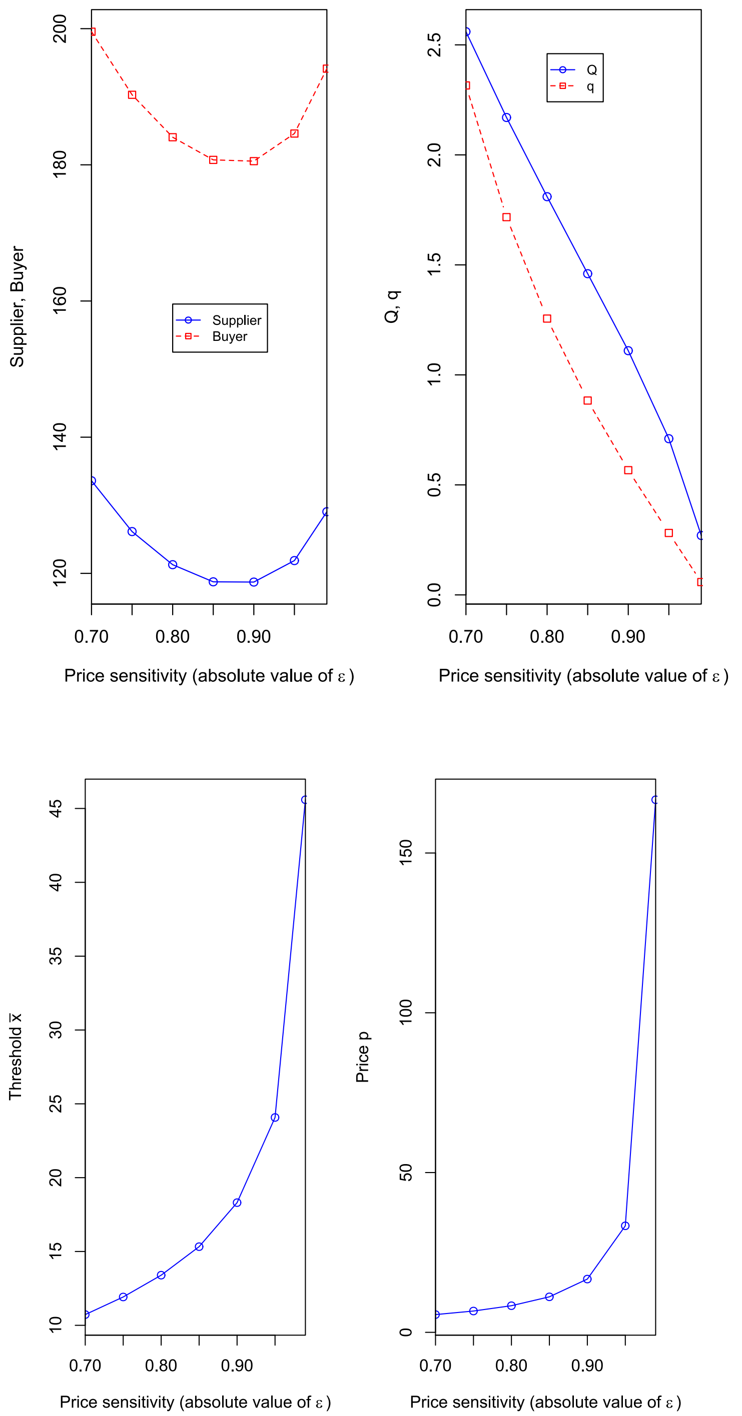
Notes: Parameters used r = 0.05, δ = 0.03, σ = 0, $v = 1$, $c = 0$, $c_s = 1$, x = 10, κ = 3, η = 2, $\varepsilon_B = -0.7$.

Figure 2. Sensitivity with respect to volatility σ



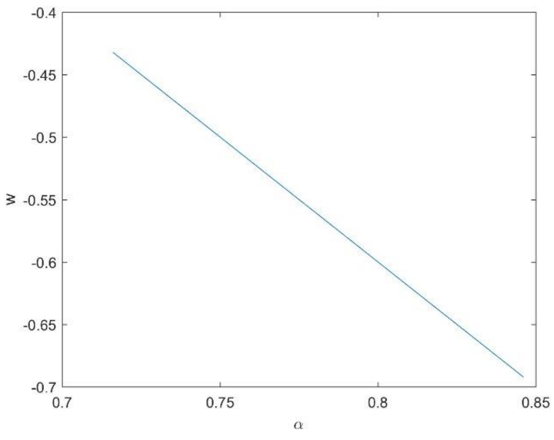
Notes: Parameters used r = 0.05, δ = 0.03, $\upsilon = 1$, $c = 0$, $c_s = 1$, x = 10, κ = 3, η = 2, $\varepsilon_B = -0.7$, $\alpha = 0.4$.

Figure 3. Sensitivity with respect to price elasticity ε



Notes: Parameters used r = 0.05, δ = 0.03, σ = 0.2. $\nu = 1$, $c = 0$, $c_s = 1$, x = 10, κ = 3, η = 2, $\alpha = 0.4$.

Figure 4. A two-parameters coordinating contract (wholesale price and revenue sharing)



Notes: Parameters used r = 0.05, δ = 0.03, σ = 0.2, $v = 1$, $c = 0$, $c_s = 1$, x = 10, κ = 3, η = 2, $\varepsilon_B = -0.7$.

Table 1. Sensitivity with respect to the share of revenues of the supplier (α)

Panel A: Non-cooperative values

α	$\bar{x}$	Region	$B^{Net}(x)$	Supplier	Q	q	(q/Q)	Price (p)
0.2	45.03	2	288.648	42.25	3.18	3.18	1.00	4.45
0.3	47.93	2	243.28	89.80	2.88	2.88	1.00	4.77
0.4	51.65	1	199.56	133.60	2.56	2.32	0.90	5.56
0.5	56.38	1	157.73	172.78	2.24	1.78	0.80	6.67
0.6	62.75	1	118.12	205.54	1.89	1.30	0.69	8.33
0.7	77.78	1	81.213	229.48	1.52	0.86	0.57	11.11
0.79	111.11	**1**	**50.89**	**239.84**	**1.17**	**0.52**	**0.44**	**15.87**
0.8	116.67	1	47.732	239.68	1.12	0.48	0.43	16.67
0.9	233.33	1	19.095	223.42	0.67	0.18	0.27	33.33

Panel B: Vertical integration

$\bar{x}_V$	Region	$V^{Net}(x)$	Q_V	q_V	(q_V/Q_V)	Price (p_V)	Gain
14.75	1	335.87	3.11	1.78	0.57	6.67	15.5%

Notes: We assume the following base case parameters: $x = 10, \sigma = 0.2,\ v = 1, c = 0, c_S = 1, \varepsilon = -0.7,\ k = 3, \eta = 2, r = 0.05, \delta = 0.03$. In Panel A, we show values varying α (share of supplier in revenues). Panel B shows the optimal values under vertical integration. Gain is calculated as the ($V^{Net}(x)$– ($B^{Net}(x)$+Supplier))/($B^{Net}(x)$+Supplier). Q increments of 0.01.

Table 2. Optimal revenue sharing contract for different levels of volatility of demand and the gains from vertical integration

σ	α	$\bar{x}$	Region	B^{Net}	Supplier	Q	(q/Q)	Price (p)
0.1	0.71	40.12	1	39.06	128.07	0.37	0.137	80.46
0.2	0.75	78.43	1	37.45	151.15	0.78	0.053	93.33
0.3	0.77	101.45	1	39.82	174.89	1	0.037	101.45
0.4	0.79	123.28	1	41.23	198.97	1.16	0.028	111.11
0.5	0.8	140.19	1	43.83	220.86	1.3	0.023	116.67

Notes: We assume the following base case parameters: $x = 10, \sigma = 0.2,\ v = 7, c = 0, c_S = 1, \varepsilon = -0.7,\ k = 3, \eta = 2, r = 0.05, \delta = 0.03$. Gain is calculated as the ($V^{Net}(x)$– ($B^{Net}(x)$+Supplier))/($B^{Net}(x)$+Supplier). Q increments of 0.01.

Table 3. Various level of minimum order quantities with fixed revenue share ratio α

Q_{min}	Region	$\overline{x}_{min}$	x	$\overline{x}$	B^{Net}	Supplier	Q	Q	Price (p)	α
0	2	0	10	17.72	50.89	239.84	1.17	0.52	15.87	0.79
0.5	2	9.77	10	17.72	50.00	242.82	1.17	0.52	15.87	0.79
1	1	15.87	10	17.72	47.18	250.26	1.17	1.00	10	0.79
1.5	1	21.08	10	21.09	42.30	267.39	1.50	1.50	7.53	0.79
2	1	25.79	10	25.79	34.18	284.20	2.00	2.00	6.16	0.79

Notes: We assume the following base case parameters: $x = 10, \sigma = 0.2,\ v = 1, c = 0, c_S = 1, \varepsilon = -0.7,\ k = 3, \eta = 2, r = 0.05, \delta = 0.03$. The share of revenues α is fixed as in the base case at 0.79. Q is optimally chosen (increments of search 0.01).