Caps, Taxes and Runs

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Abstract

In competitive industries, foreseeable policy changes lead to inevitable runs which increase the volatility of investment. We show that this phenomenon, well-known in the case of caps and quotas, also applies to taxes and subsidies, and occurs whether policy changes apply only to new entrants or to all firms equally. Looking at the case of raising taxes (or removing a subsidy) we find that runs are smaller when the policy change affects all firms. We also find that the size of the run, i.e., the size of the resulting increase in market quantity is increasing in the magnitude of the tax raise. Finally, we show that during the run firms invest faster and more massively than on the socially optimal path, and therefore welfare decreases. We show that these results have implications for a broad range of policies, such as the removal of renewable energy subsidies in some European countries.

Key words: Competitive run, Investment under uncertainty, Pigovian tax

J.E.L. codes: (C61, D41, D62)

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1. Introduction

When in the summer of 2020 the French government announced a hundred-billion euro stimulus plan, *France Relance,* to alleviate adverse consequences of the COVID-19 pandemic on economic activity, this was with the understanding that measures in the package, such as significant tax breaks for firms, would extend over a two-year period,[1] a delay presumably reflecting the expected time needed for the economy to recover. As much as policy interventions can arise unexpectedly therefore (ahead of the lockdowns measures which caused economic activity to drop, the pandemic was hardly anticipated by Western governments), there also are many circumstances where the timing of policies is to some extent known by economic actors. Such knowledge can arise because policy is explicitly constrained or closely tied to events with specific dates. It can also arise because policies are tied to events whose exact timing, though unknowable in advance, is nevertheless predictable, like economic recovery from a shock, or when environmental parameters reach a certain value. For an alternative example, consider another important policy objective, decarbonization, whose achievement rests in large part upon promoting green investment. While the road ahead is still long for many of the world's countries, in the fall of 2020, Sweden and Norway jointly announced that their green power support schemes would be phased out by 2035. Moreover, these schemes would be closed to new participants as of the beginning of 2022, because both countries deemed that their renewable energy production targets had been largely met.[2]

For many reasons, it is often preferable that economic actors anticipate policies. For businesses, aside from easing technological or organizational transitions, lower policy risk tends to accelerate investment by removing a superfluous option value of waiting. If the implementation of a policy is tied to market uncertainty however, it is possible for the anticipation of policy changes to have less desirable consequences. This is because the anticipation of a policy intervention can drive investment in competitive industries to increase too much and raise its variability significantly. The reason is that anticipated changes can lead to competitive runs, a phenomenon which has been studied generally in the context of very specific measures like quotas and caps on investments, but which we argue in this paper arises under a much broader range of policy measures.

[1] See https://www.lesechos.fr/economie-france/budget-fiscalite/plan-de-relance-les-100-milliards-deuros-seront-etales-sur-deux-ans-1224104.
[2] See https://www.reuters.com/article/us-norway-sweden-electricity-idUSKBN26922B.

The concept of a run in Economics has historically been most closely associated with movements in financial asset values. In the Diamond and Dybvig (1983) model of bank runs, rather than resulting from a gradual, dynamic process, the run arises as one of several equilibria, driven by individual beliefs. In Krugman's (1979) balance of payment crisis model, a run occurs in the face of dwindling reserves before these are depleted, as individuals protect their assets, but the framework this result is derived in is a Keynesian model without optimization. A common feature to both these classic studies is that the run springs from the knowledge that a certain stock is going to be fully exhausted, either liquid bank reserves in Diamond and Dybvig, or the central bank's foreign currency reserves in Krugman's.

Later studies have focused on runs that emerge in a market where profits are stochastic and investment is irreversible. In those studies the runs emerge in equilibrium when there is a cap on aggregate investment. As the endpoint of an investment opportunity nears, firms cease to wait for an optimal threshold to be reached and a mass of investment occurs. Applications involving this dynamic pattern of a run arise naturally in a variety of areas, ranging from foreign investment caps (Bartolini 1995), to immigration quotas (Moretto and Vergali 2010), or policies restricting land use (Di Corato, Moretto and Vergalli 2013). Some of these also incorporate an adverse externality which motivates imposing the cap.

In the present article, we study a model of competitive investment under uncertainty and describe the general conditions that can give rise to a run. These include, for example, anticipated changes in tax rates or subsidies and more broadly any measure that generates a kink in the revenue or cost functions that firms face, and thereby cause the investment threshold function to jump. Like a cap (which can be interpreted as an infinite tax) therefore, a tax on entries once a market size or industry capacity threshold is met results in a run. The threshold at which the run starts is inversely related to the size of the tax.

Our analysis shows that such a run occurs because the worsening of conditions after a policy intervention (tax or subsidy withdrawal) slows the subsequent entry process and the falls in prices that each entry would bring, thereby improving the revenue flow that existing firms enjoy. As existing firms would then enjoy above normal profit, a mass of new firms is attracted into the market generating the run. This process ultimately eliminates the possibility that the improved future revenue process might raise firm value above the normal return. More specifically, the run brings about an immediate drop in price which balances the improved future revenue process.

To verify this intuition we also study an alternative policy, a tax on flow or operating cost rather than fixed cost, which affects active and inactive firms equally. In this situation all firms have the same revenue flows after the policy intervention yet sill a run emerges, confirming that the same reasoning applies: it is not any specific future tax advantage but rather the slowing of future entries and resulting rise in revenues which dominates. This dominance springs from the endogenous nature of the revenue process – it is based on firms' entries and those occur when their net value is zero, due to free entry. Thus, the run occurs not because it rewards its winners with an advantage compared to those who were not lucky enough to enter before the policy change. On the contrary – the run is a dynamic equilibrium pattern that keeps value equal for all – at the normal return.

Finally, we turn to the welfare properties of competitive runs. In the present paper, we consider an optimal policy where the regulator chooses a tax that exactly reflects the investment externality, implying that at any moment firms face the correct economic cost. For such a policy, we find that the run reduces welfare by excessively accelerating investment, which destroys a valuable option to wait.

From a policy perspective, our analysis relates to investment in renewable energy generation for example. The effect of incentivizing policies in this area has both studied and estimated extensively (Linnerud, Andersson and Fleten 2014). In this literature, policy effects generally are estimated for small projects where individual investors are price-takers in the electricity market and both the level of policy and the electricity price fluctuate stochastically. Different policies which have been considered include lump-sum investment subsidies and feed-in tariffs which shield firms from lower electricity prices. More recently though, this literature has begun to highlight the risk associated with the withdrawal of subsidies (Nagy, Hagspiel and Kort 2021), as highlighted in our opening paragraph. We contribute to this discussion by complementing existing models through the introduction of competitive equilibrium and the run behavior this generates, both in the case of lump-sum subsidies and feed-in tariffs.

Section 2 presents the assumptions underlying our model. Section 3 studies competitive equilibrium in the absence of policy intervention. Section 4 considers a tax on investment and derives the resulting behavior, characterizing the effect of parameters on the magnitude and timing of the competitive run. In Section 5, we extend this analysis to the case of a tax on operating costs and compare the two forms of policy.

2. The model

Consider an industry consisting of an initial mass $Q_0 = Q$ of firms. Firms are identical and risk neutral. Once active, a firm is a price-taker in the market, and the price at time $t \in R_+$ is

$$P_t = X_t \cdot f(Q_t), \tag{1}$$

where Q_t is current industry capacity, $f(Q)$ is a differentiable downward-sloping inverse demand function with $\lim_{Q \to \infty} f(Q) = 0$, and X_t is an exogenous shock which evolves stochastically according to a geometric Brownian motion

$$dX_t = \mu \cdot X_t \cdot dt + \sigma \cdot X_t \cdot dZ_t \tag{2}$$

where μ is the drift of the process, $\sigma > 0$ the volatility, and dZ_t is the increment of a standard Wiener process, uncorrelated across time and satisfying $E(dZ_t) = 0$ and $E[(dZ_t)^2] = dt$ at each point in time.

Two specific examples we will later refer to are:

-constant elasticity,

$$P_t = X_t \cdot Q_t^{-\gamma} \tag{3}$$

where $\gamma \in (0,1)$ is the inverse of the elasticity of demand;

- hyperbolic,

$$P_t = \frac{X_t}{A + Q_t} \tag{4}$$

where $A > 0$.

There is free entry into the market. Once a firm has entered, it produces up to capacity and cannot exit. Initially the cost of a unit of capacity is K and the operating cost per unit of capacity is w, with at least one of these costs positive.

Once the industry reaches a predetermined capacity $\bar{Q}$ a policy intervention that permanently shifts one of the cost components takes place. The first policy we study is a shift in the cost of a capacity unit to $K' > K$. This policy can be interpreted either as a tax on investment at a constant rate τ in which case $K' = (1 + \tau) \cdot K$, or as the withdrawal of a lump-sum subsidy S in which case $K' = K + S$. The second policy we study is a shift in the operating cost to $w' > w$, once again due to an increased tax rate or to a subsidy removal. Aside from which cost component is affected, this policy shock differs from the first in that it affects all firms rather than only new entrants.

Firms have the same constant discount rate r, and we suppose $r > \mu$ to focus on the case where firm value is not infinite.

3. Industry equilibrium

We start by presenting the industry equilibrium in the absence of policy intervention, i.e., if K and w remain at their initial levels forever. Under the setup described above and in the absence of policy intervention the potential investor in this model is facing the same situation as the investors in Leahy (1993). In this section we use Leahy's analysis to present the potential investors' optimal investment policy.

The decision to enter depends on the expected profitability of this investment, and therefore takes place only when X_t is sufficiently large. Let $X^*(Q)$ denote the investment threshold given the current market quantity.[3]

Let $V(Q,X)$ denote the value of a firm that is active in the market. The standard no-arbitrage analysis, presented in Appendix A, shows that

(5) $$V(Q,X) = Y(Q) \cdot X^{\beta} + \frac{X \cdot f(Q)}{r - \mu} - \frac{w}{r}$$

[3] In the following, we will drop the time subscript for notational convenience.

where $Y(Q)$ is determined via boundary conditions and β is the positive root of the quadratic equation:

(6) $$\tfrac{1}{2}\cdot\sigma^2\cdot x^2+\left(\mu-\tfrac{1}{2}\cdot\sigma^2\right)\cdot x-r=0$$

Appendix A also shows that $\beta > 1$ and that it is decreasing in σ^2. Note that by the standard properties of the Wiener Process, the second and third terms of (5) represent the expected present value of the firm's profit flow if Q remains forever at its current level, i.e.:

(7) $$E\left\{\left[\int_{t=0}^{\infty} P(Q)-w\right]\cdot e^{-r\cdot t}\cdot dt\right\}=E\left\{\left[\int_{t=0}^{\infty} X\cdot f(Q)-w\right]\cdot e^{-r\cdot t}\cdot dt\right\}=\frac{X\cdot f(Q)}{r-\mu}-\frac{w}{r}.$$

Due to that, the first term in (5), namely $Y(Q)\cdot X^{\beta}$, accounts for how future market entries affect the value of the firm, as they increase the market quantity Q, and thus lower profitability.

Two boundary conditions are required to find $Y(Q)$ and the threshold function $X^*(Q)$. The first is the following *Value Matching* condition:

(8) $$V\left[Q, X^*(Q)\right]=K.$$

And the second is the following *Smooth Pasting* condition:

(9) $$V_X\left[Q, X^*(Q)\right] = 0.$$

The Value Matching condition holds for any threshold, not necessarily the optimal one, and states that to prevent arbitrage at the entry time the value of an idle firm (which is zero in the current free entry case) must equal that of the active firm it becomes. The Smooth Pasting condition is an optimality condition requiring that at the entry time the two value functions have the same slope with regard to *X*.

Applying (5) in (8) and (9) yields:

(10) $$X^*(Q) = \hat{\beta} \cdot (r - \mu) \cdot \frac{K + \frac{w}{r}}{f(Q)}$$

where $\hat{\beta} \equiv \frac{\beta}{\beta - 1}$. Note that $\hat{\beta} > 1$ since $\beta > 1$.

This threshold is an increasing function of Q as the larger the market quantity, the stronger the competition and, *ceteris paribus*, the higher the expected profitability required to trigger entry.

Note that according to the standard net present value rule a firm should enter as long as $X > (r - \mu) \cdot \frac{K + \frac{w}{r}}{f(Q)}$. Multiplying by the uncertainty wedge $\hat{\beta} > 1$ accounts for the presence of uncertainty and irreversibility (see Dixit and Pindyck, 1994, Ch. 5, Section 2). Also observe from (10) that $dX^*(Q)/d\beta < 0$ and this, taken together with $d\beta/d\sigma < 0$, which follows from (6), implies that $dX^*(Q)/d\sigma > 0$ which means that the higher the demand volatility, the higher the threshold triggering firm's entry.

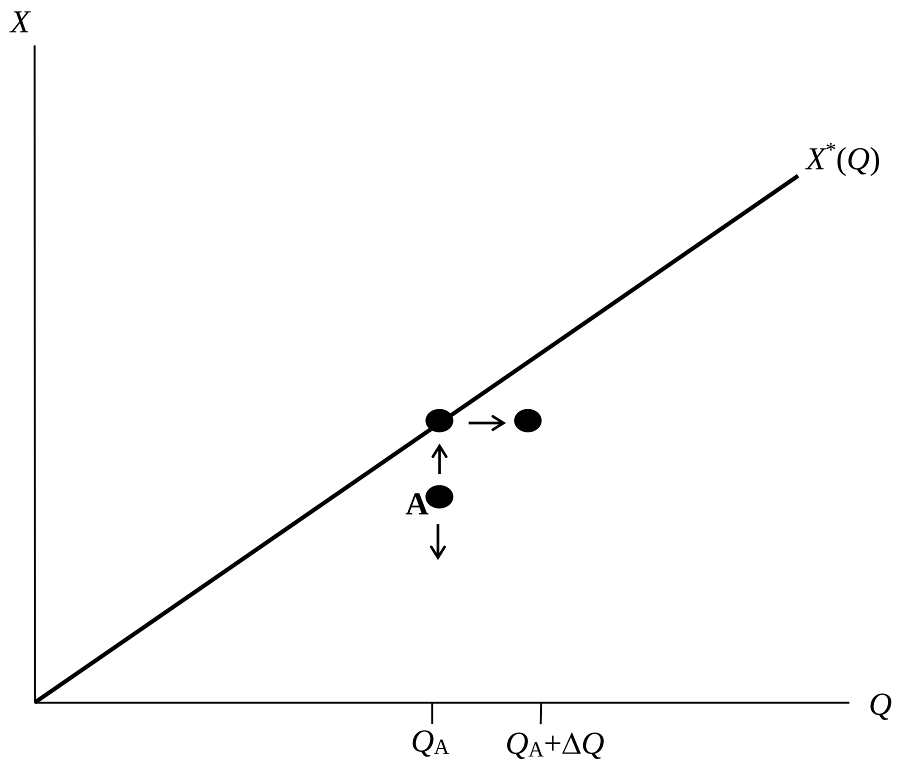

Figure 1: The entry process when there is no policy change.

Figure 1 shows the resulting gradual process of market entry. Starting at a point like A, the continuous movement of X does not change market quantity which remains Q_A until X hits $X^*(Q_A)$ and then a firm invests and enters the market. The rising Q makes $X^*(Q)$ rise too, so that X is once again below the threshold and further investment is postponed until the first time X hits the threshold function

It also follows from applying (5) in (8) and (9) that

(11) $$Y(Q) = -\frac{(\beta-1)^{\beta-1}\cdot[f(Q)]^{\beta}}{\beta^{\beta}\cdot(r-\mu)^{\beta}\cdot\left(K+\frac{w}{r}\right)^{\beta-1}} < 0,$$

which implies that the value of an active firm is negatively affected by the possible future entries of additional firms into the market. The higher is Q, the smaller this effect (in absolute terms) because with higher market output, profitability falls and future entries are expected to arrive at a slower pace as they require reaching higher thresholds. Note that *K* and *w* have a positive effect on the value of the already active firm for similar reasons.

4. Industry equilibrium with policy intervention affecting fixed cost

In this section we study the effect of a policy intervention on competitive investment. The intervention happens once the industry reaches a predetermined capacity $\bar{Q}$, and its timing is known to firms. The effect of the policy intervention in this section is to shift the cost of a capacity unit to $K' > K$.

We analyze the industry equilibrium in two steps, starting with the case where $Q > \overline{Q}$, which means that the policy change has already taken place. Then we go to the case where $Q < \overline{Q}$ and the policy change has not occurred yet.

Proposition 1. *When* $Q > \overline{Q}$ *the value of an active firm is*

$$V(Q,X) = Y^{PI}(Q) \cdot X^{\beta} + \frac{X \cdot f(Q)}{r-\mu} - \frac{w}{r}, \tag{12}$$

and the entry threshold is

$$X^{*}(Q) = \hat{\beta} \cdot (r-\mu) \cdot \frac{K' + \frac{w}{r}}{f(Q)} \tag{13}$$

where

(14) $$Y^{PI}(Q) = -\frac{(\beta-1)^{\beta-1}\cdot\left(f(Q)\right)^{\beta}}{\beta^{\beta}\cdot(r-\mu)^{\beta}\cdot\left(K'+\frac{w}{r}\right)^{\beta-1}}$$

Proof: When $Q > \overline{Q}$, no future policy changes are expected so the analysis is similar to the case analyzed in the previous section, with *K*' instead of *K*. □

We now turn to the case where $Q < \overline{Q}$ and the policy change has not yet occurred. Bartolini (1993) and Di Corato and Maoz (2019) show that the analysis of the industry equilibrium in this case starts by repeating the analysis without policy intervention presented in the previous section, and parts ways only when the condition for the firm's optimal investment is introduced. Specifically, they show that the general form of the optimality condition, which combines smooth pasting with *Complementary Slackness*, is:

(15) $$V_X[Q, X^*(Q)]\cdot\frac{dX^*(Q)}{dQ} = 0.$$

For (15) to hold, either $V_X\left[Q, X^*(Q)\right] = 0$ or $dX^*(Q)/dQ = 0$ needs to hold. In the first case, the Smooth Pasting condition captured by (9) holds and the investment threshold function is the same one captured by (10) for the case of no cap.

However, within the relevant range, $Q \le \overline{Q}$, there is an interval where the Smooth Pasting condition does not hold. To see that, recall that within the value function *V*(*Q*,*X*) as captured by (8), the term $Y(Q)\cdot X^{\beta}$ represents how future entries affect the value of the firm. Therefore, when $Q = \overline{Q}$ this value is based on firms entering under the new policy and therefore should be based on (14). Applying this to the value matching condition (8) yields that when $Q = \overline{Q}$, the investment threshold, which we denote by $\bar{X}_K$, is the root of the following equation:

(16) $$Y^{PI}(\bar{Q}) \cdot X^{\beta} + \frac{X \cdot f(\bar{Q})}{r-\mu} - \frac{w}{r} - K = 0$$

In Appendix B we prove that this equation has a unique positive root.

From (16) it follows that the Smooth Pasting condition does not hold at $\overline{Q}$. The argument is by contradiction. If the Smooth Pasting condition held, then the solution of (9) and (16) would lead to $Y(\overline{Q})$ as given by (11), which is different from $Y^{PI}(\overline{Q})$. By continuity the Smooth Pasting condition cannot hold to the left of $\overline{Q}$ either, so there exists a certain $\tilde{Q}$ such that the Smooth Pasting condition does not hold within the interval $[\tilde{Q}, \overline{Q}]$ In this interval $dX^{*}(Q)/dQ = 0$ replaces the Smooth Pasting condition as the optimality condition. This alternative form of the optimality condition implies that within that range of market sizes the threshold does not change with Q and therefore $\tilde{Q}$ is the solution of the following equation:

(17) $$X^{*}(\tilde{Q}) = \bar{X}_K.$$

Taking the value matching condition (16), evaluated at $\bar{X}_K$, and applying (14) and (17) yields the following implicit definition of $\tilde{Q}$:

(18) $$-(\hat{\beta}-1) \cdot \left(\frac{K + \frac{w}{r}}{K' + \frac{w}{r}}\right)^{\beta-1} \cdot \left[\frac{f(\bar{Q})}{f(\tilde{Q})}\right]^{\beta} + \hat{\beta} \cdot \frac{f(\bar{Q})}{f(\tilde{Q})} - 1 = 0$$

To sum up, the entry threshold is

(19) $$X^*(Q) = \begin{cases} \hat{\beta} \cdot (r-\mu) \cdot \dfrac{K + \frac{w}{r}}{f(Q)}, & Q \le \tilde{Q} \\ \bar{X}_K, & \tilde{Q} < Q \le \bar{Q} \\ \hat{\beta} \cdot (r-\mu) \cdot \dfrac{K' + \frac{w}{r}}{f(Q)}, & \bar{Q} < Q. \end{cases}$$

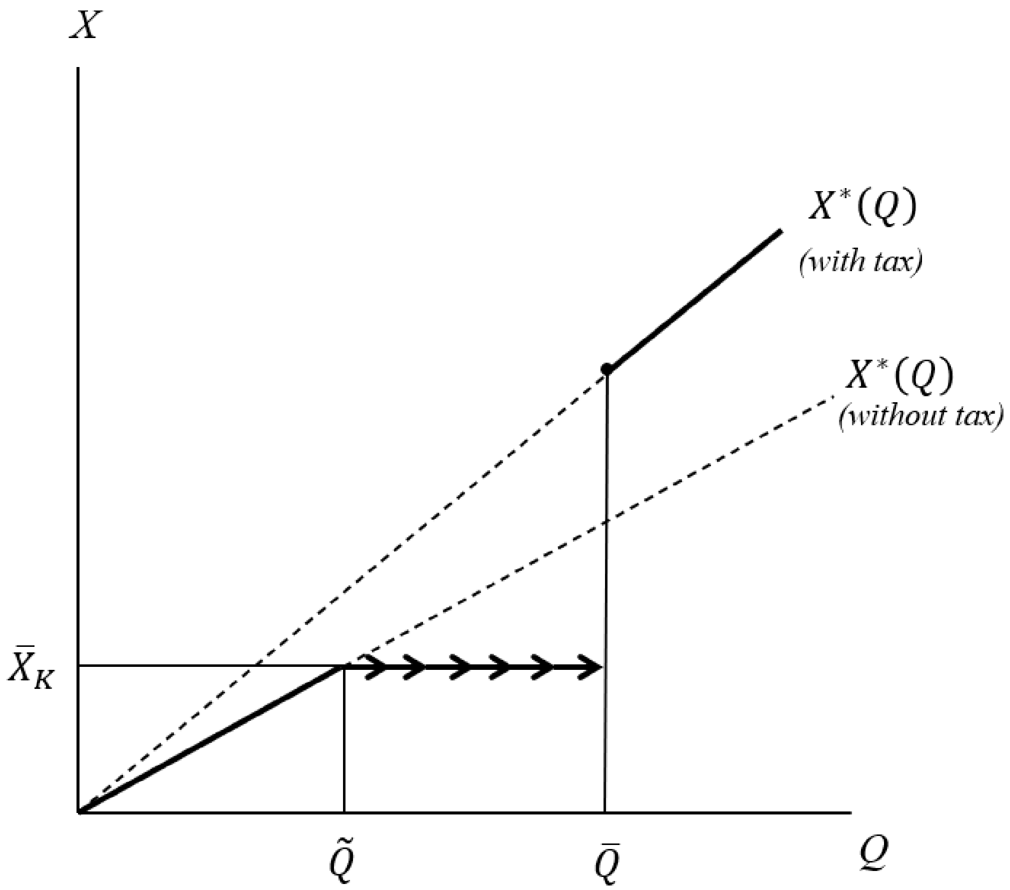

Figure 2: The entry threshold and the entry process with a policy change

Figure 2 shows the resulting entry threshold. As the figure shows, as long as $Q \le \tilde{Q}$ entry is the gradual process based on incremental increases in market quantity at points in time when the threshold $X^*(Q)$ is hit. Then, when $Q = \tilde{Q}$, a run is ignited when X hits $X^*(\tilde{Q})$ and quantity immediately rises to $\bar{Q}$. After this run, the entry process returns to be a gradual and incremental process, but now it is based on a higher threshold function, due to the new parameter value.

This *second wave* of investment is a key difference with the case of a cap on industry capacity, where investment ceases after $\bar{Q}$. Because of it, firms entering before the policy shock must anticipate a nonzero effect of further entries on their value.

Proposition 2. *The size of the run is increasing in both the magnitude of the policy shock $K' - K$ and the market volatility σ.*

Proof: see Appendix B.

5. Industry equilibrium with policy intervention affecting operating cost

In this section we study an alternative policy intervention to the one presented in the previous section. The intervention still happens once the industry reaches a predetermined capacity $\bar{Q}$, and its timing is still known to firms. Instead of altering the fixed cost of new entrants however, the effect of this policy intervention is to shift the operating cost per capacity unit to $w' > w$. We distinguish between two cases:

- the new policy applies to all firms, including those who were in the market before the change.
- the new policy applies only for the firms entering after the change

In the first case, the analysis is similar to the one conducted in previous sections, leading to the following equation for the value of the threshold at which the run occurs, which in this case we denote by $\bar{X}_0$:

$$Y^{PII}(\bar{Q}) \cdot X^{\beta} + \frac{X \cdot f(\bar{Q})}{r - \mu} - \frac{w'}{r} - K = 0 \tag{20}$$

where

$$Y^{PII}(Q) = -\frac{(\beta-1)^{\beta-1} \cdot \left(f(Q)\right)^{\beta}}{\beta^{\beta} \cdot (r-\mu)^{\beta} \cdot \left(K + \frac{w'}{r}\right)^{\beta-1}}. \tag{21}$$

In the second case, when the new policy applies only for the firms entering after the change, the analysis is somewhat different. We start this analysis with looking at a firm that were not active in the market prior to the policy change. An analysis similar to the one in previous sections yields that the value of such a firm, when active, is:

$$V(Q,X) = Y^{PII}(Q) \cdot X^{\beta} + \frac{X \cdot f(Q)}{r-\mu} - \frac{w'}{r}, \tag{22}$$

where $Y^{PII}(Q)$ is given by (21).

We now look at a firm that has entered the market before the policy change. When $Q > \bar{Q}$, i.e., after the policy change has taken place, the value of such a firm is given by:

$$V(Q,X) = Y^{PII}(Q) \cdot X^{\beta} + \frac{X \cdot f(Q)}{r-\mu} - \frac{w}{r}, \tag{23}$$

where $Y^{PII}(Q)$ is given by (21), as in the case of a firm that has entered after the policy change. The reason for that is that this term captures the change in value due to future firm's entry which is a process that affects the price dynamics, and therefore the revenue flow, but does not affect the costs flow. Thus, this term is common to both types of firms.

Turning next to the case where $Q < \overline{Q}$ and the policy change has not yet occurred, the Smooth Pasting condition (9) is replaced by the more general Complementary Slackness condition (15). In this case also there is an interval where the Smooth Pasting condition does not hold. Applying (21) and (23) in the value matching condition (8) yields that for $Q = \overline{Q}$, the investment threshold, which we denote by $\bar{X}_w$ is the root of the following equation:

(24) $$Y^{PII}(\bar{Q}) \cdot X^{\beta} + \frac{X \cdot f(\bar{Q})}{r-\mu} - \frac{w}{r} = K.$$

Proposition 4. *A policy affecting both new and existing firms results in a smaller run than a policy shock affecting new firms only.*

Proof: Consider the expression:

(25) $$Y^{PII}(\bar{Q}) \cdot X^{\beta} + \frac{X \cdot f(\bar{Q})}{r-\mu} - \frac{Z}{r} - K = 0.$$

When $Z = w'$ equation (25) becomes (20) and yields $\bar{X}_0$. When $Z = w$ equation (25) becomes (24) and its root is $\bar{X}_w$. By implicit differentiation:

(26) $$\frac{dX}{dZ} = -\frac{-\frac{1}{r}}{Y^{PII}(\bar{Q}) \cdot \beta \cdot X^{\beta-1} + \frac{f(\bar{Q})}{r-\mu}} > 0,$$

which implies that $\bar{X}_w < \bar{X}_0$ since $w' > w$. Thus, $\tilde{Q}$ is higher in the case where all firms are subject to the new policy, since, by (10), the threshold function is increasing in Q. The higher $\tilde{Q}$ in that case implies also a smaller size of the run.

6. Welfare effect of a run

To study the welfare effect of runs, we assume that the policy is a Pigovian tax, chosen to exactly reflect current economic fundamentals at all times. In the case of a subsidy withdrawal for example, this means that the subsidy level S just offsets a positive externality associated with investment as long as this externality is present, and the subsidy ceases once the industry reaches a capacity $\overline{Q}$ at which the externality is suddenly exhausted.[4] With such a policy, firms always face the true cost of investment (which is either K or K').

To assess the effect of the run, we can compare this situation to the outcome that would arise in the case of an unanticipated policy shock where the subsidy is unexpectedly removed at $\overline{Q}$. In this alternative situation, industry investment is gradual with firms investing along the threshold function $X^*(Q)$ both before and after $\overline{Q}$. The gap in industry investment due to the policy shock is reduced, from $X^*(\overline{Q}) - X^*(\tilde{Q})$ to $X^*(\overline{Q}) - \lim_{Q \to \overline{Q}^-} X^*(Q)$ (see Figure 2). Because the path of investment in the two scenarios is the same both before $\tilde{Q}$ and after $\overline{Q}$, the welfare effect of a run is entirely due to the interval $[\tilde{Q}, \overline{Q})$. Moreover, over this interval the run drives firms to accelerate investment suboptimally. To see this, note that the socially optimal investment policy coincides with the competitive equilibrium here (see Dixit and Pindyck 1994, p. 286) The effect is intuitively clear: the competitive run leads to accelerated investment as compared with the path under smooth pasting, which is the socially optimal investment timing because firms are price-takers. The competitive run therefore lowers social welfare, because it eradicates option value over this range of industry capacities. This welfare loss increases with uncertainty, although in that case the loss due to the run is offset by increased option value at higher capacities.

[4] Such a jump in the social cost is a simplification. If the subsidy was motivated by external economies of scale, we would expect the social cost of investment to decrease gradually and the socially optimal timing of the subsidy withdrawal would be more complex.

References

Bartolini, L. (1993). Competitive runs: The case of a ceiling on aggregate investment. *European Economic Review, 37*(5), 921-948.

Bartolini, L. (1995). Foreign investment quotas and rent extraction under uncertainty. *Journal of International Economics*, *38*(1-2), 25-49.

Diamond, D. W., & Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy, 91*(3), 401-419.

Di Corato, L., & Maoz, Y. D. (2019). Production externalities and investment caps: A welfare analysis under uncertainty. *Journal of Economic Dynamics and Control*, 106, 103719.

Di Corato, L., Moretto, M., & Vergalli, S. (2013). Land conversion pace under uncertainty and irreversibility: too fast or too slow?. *Journal of Economics, 110*(1), 45-82.

Dixit, A. (1989). Entry and exit decisions under uncertainty. *Journal of Political Economy, 97*(3), 620-638.

Dixit, R. K., & Pindyck, R. S. (1994). *Investment under Uncertainty*. Princeton university press.

Krugman, P. (1979). A model of balance-of-payments crises. *Journal of Money, Credit and Banking*, *11*(3), 311-325.

Leahy, J. V. (1993). Investment in competitive equilibrium: The optimality of myopic behavior. *The Quarterly Journal of Economics, 108*(4), 1105-1133.

Linnerud, K., Andersson, A. M., & Fleten, S. E. (2014). Investment timing under uncertain renewable energy policy: an empirical study of small hydropower projects. *Energy*, *78*, 154-164.

Moretto, Michele, and Sergio Vergalli. "Managing migration through conflicting policies: An option-theory perspective." *Scottish Journal of Political Economy* 57.3 (2010): 318-342.

Nagy, R. L., Hagspiel, V., & Kort, P. M. (2021). Green capacity investment under subsidy withdrawal risk. *Energy Economics, 98*, 105259.

Appendix A - The value of an active firm

In this Appendix we show that (5) represents the general form of the function $V(Q,X)$. For that, we use the standard no-arbitrage analysis of the literature on investment under uncertainty (see e.g. Dixit 1989). We start this analysis with the no-arbitrage condition:

$$r \cdot V(Q, X) \cdot dt = \left[\frac{X}{f(Q)} - w \right] \cdot dt + E[dV(Q, X)], \tag{A.1}$$

which states that the instantaneous profit, $p(Q) - w = \frac{X}{f(Q)} - w$, along with the expected instantaneous capital gain, $E[dV(Q, X)]$, which springs from a change in X, must equal the instantaneous normal return, $r \cdot V(Q, X) \cdot dt$.

By Itô's lemma:

$$\frac{E[dV(Q, X)]}{dt} = \tfrac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(Q, X) + \mu \cdot X \cdot V_X(Q, X). \tag{A.2}$$

Substituting (A.2) in (A.1) yields:

$$\tfrac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(Q, X) + \mu \cdot X \cdot V_X(Q, X) - r \cdot V(Q, X) + \frac{X}{f(Q)} - w = 0. \tag{A.3}$$

Trying a solution of the type X^b for the homogenous part of (A.3) and a linear form as a particular solution to the entire equation, yields:

(A.4) $$V(Q,X)=Z(Q)\cdot X^{\alpha}+Y(Q)\cdot X^{\beta}+\frac{X}{f(Q)\cdot(r-\mu)}-\frac{w}{r},$$

where $\alpha<0$ and $\beta>1$ solve the quadratic:

(A.5) $$\tfrac{1}{2}\cdot\sigma^{2}\cdot x\cdot(x-1)+\mu\cdot x-r=0.$$

Note that the term $\frac{X}{f(Q)\cdot(r-\mu)}-\frac{M}{r}$ represents the expected value of the flow of profits if Q remains forever at its current level. The two other elements of the RHS of (A.4) represent therefore how expected future changes in Q are affect the value of the firm.

By the properties of the Geometric Brownian Motion, when X goes to 0 the probability that it will ever hit $X^{*}(Q)$, and thus lead to an increase in Q, tends to 0. This implies:

(A.6) $$\lim_{X\to 0}\left[Z(Q)\cdot X^{\alpha}+Y(Q)\cdot X^{\beta}\right]=0.$$

Since $\alpha<0$, (A.6) implies that $Z(Q)=0$. Substituting $Z(Q)=0$ into (A.4) gives (6).

Appendix B

To show that (16) has a positive solution and that this solution is unique we define the following function:

(B.1) $$g(Q,X,K)=Y(Q)\cdot X^{\beta}+\frac{X\cdot f(Q)}{r-\mu}-\frac{w}{r}-K$$

Where $Y(Q)$ is captured by (11). Figure B.1 shows $g(Q,X,K)$ as a function of X. From (B.1) it follows that $\lim_{X\to 0} g(Q,X,K) = -\frac{w}{r} - K < 0$, and that $\lim_{X\to 0} g(Q,X,K) = -\infty$. Straight forward differentiation yields that it is an inverse u-shape function of X. By the value matching condition, (8), at $X = X^{*}(Q)$ it equals zero, and by the smooth pasting condition, (9) it also peaks at that point.

Similarly, we also define the following function:

(B.2) $$g^{PI}(Q,X,K) = Y^{PI}(Q) \cdot X^{\beta} + \frac{X \cdot f(Q)}{r-\mu} - \frac{w}{r} - K'$$

where $Y^{PI}(Q)$ is captured by (14) based on the cost parameters after the policy intervention. Figure B.1 also shows $g^{PI}(Q,X,K)$ as a function of X. A similar analysis shows that $g^{PI}(Q,X,K)$ too is an inverse function of X, maximized at the value of 0 by the threshold X^{*PI} which is based on the on the cost parameters after the policy change and captured by (13).

We now turn to the regime shift at $\bar{Q}$. For analyzing it we define:

(B.3) $$g^{Change}(Q,X,K) = Y^{PI}(Q) \cdot X + \frac{X \cdot f(Q)}{r-\mu} - \frac{w}{r} - K$$

It is a mixture of the two previous equations: it has $Y^{PI}(Q)$ and thus captures the welfare value of **future** entries which are based on the parameters after the policy change, but it also has K (and not K') and thus captures the entry cost required before the change.

Like the previous two functions, $g^{Change}(Q, X, K)$ is an inverse u-shape too, with a negative value at X =0 and approaching -∞ as X goes to infinity. Yet, it is higher from both of them for any $X > 0$ because of $K' > K$ and $Y^{PI}(Q) > Y(Q)$. Figure B.1 shows this function too.

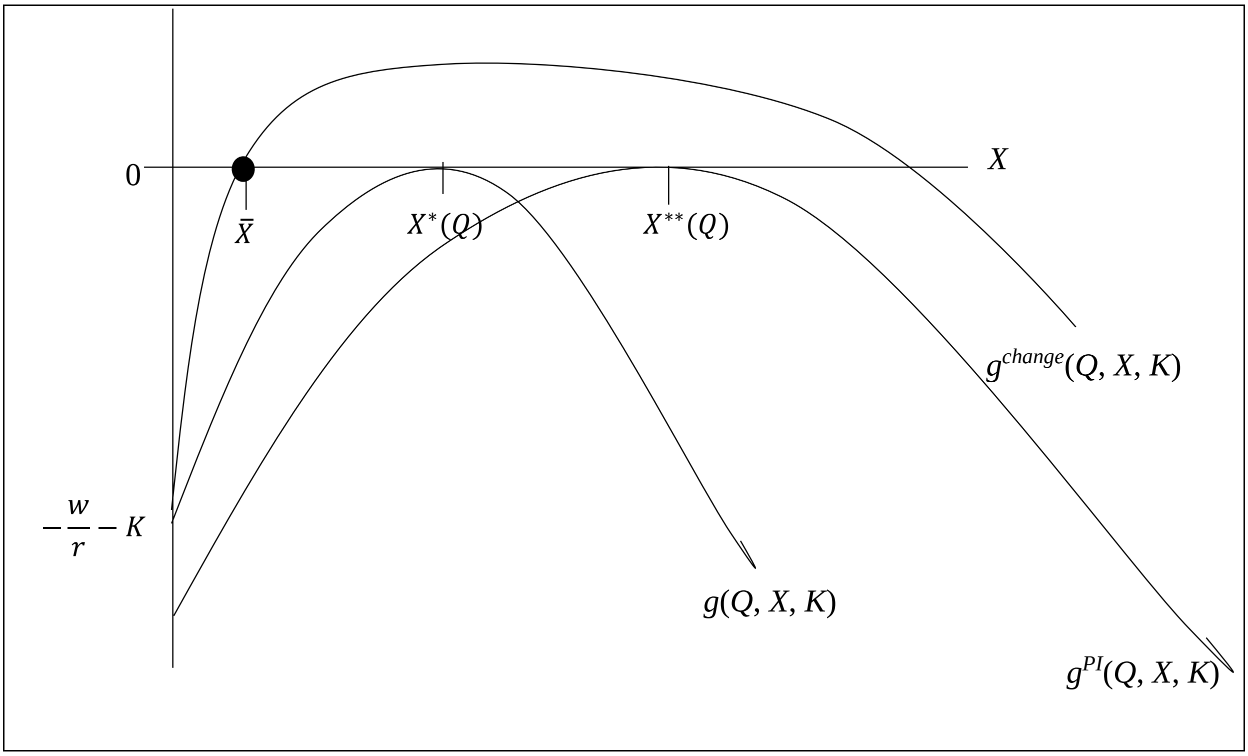

Figure B.1: The functions $g(Q, X, K)$, $g^{PI}(Q, X, K)$ and $g^{change}(Q, X, K)$.

When the value of Q these three function refer to is $\bar{Q}$, point A in *Figure B.*1 is the point where equation (16) from which the value of the threshold at which the run occurs can be found. The other value where $g^{change}(Q, X, K) = 0$ is irrelevant as the analysis in section 4 has shown that the threshold at which the run occurs is smaller then the value that (10) provides for $X^*(\bar{Q})$. Thus, this value exists, is positive and unique.

Appendix C

In this appendix we show that equation (18) that implicitly defines $\tilde{Q}$ indeed has a solution and that this solution is unique. We start by using the definition of $\hat{\beta}$ to rearrange (18) and present it as:

(C.1) $$-\left(\frac{K+\frac{w}{r}}{K'+\frac{w}{r}}\right)^{\beta-1}\cdot\left[h(\tilde{Q})\right]^{\beta}+\beta\cdot h(\tilde{Q})-\beta+1=0$$

Where $h(\tilde{Q})\equiv\frac{f(\bar{Q})}{f(\tilde{Q})}$. By the properties of the demand function (1), the function *f*(*Q*) is downward sloping, and therefore $h'(\tilde{Q})>0$, implying that $h(\tilde{Q})$ is positively related to $\tilde{Q}$. Also note that $h(\tilde{Q})$ is a positive fraction, as $\tilde{Q}<\bar{Q}$.

Viewed as a function of $h(\tilde{Q})$, the LHS of (C.1) is a concave function of $h(\tilde{Q})$. At the left-end side of the definition range of this function, i.e., when $\tilde{Q}=\bar{Q}$ and $h(\tilde{Q})=1$, the LHS of (C.1) equals $1-\left(\frac{K+\frac{w}{r}}{K'+\frac{w}{r}}\right)^{\beta-1}>0$.

We now add the reasonable assumption that as the market quantity, *Q*, goes to zero, *f*(*Q*) is sufficiently high to make the LHS of (C.1), evaluated at $\tilde{Q}\to 0$, negative. Thus, the LHS of (C.1), concavely goes from a negative to a positive value as $h(\tilde{Q})$ goes from the left-end side of its definition range to the right-end side. This asserts the existence and uniqueness of a value of $h(\tilde{Q})$, and therefore of $\tilde{Q}$, that solves (C.1).

To establish the effect that *K*' has on the size of the run we define the LHS of (C.1) as the function $F(\tilde{Q},K')$. From (C.1) it follows immediately that:

(C.2) $$F_{\tilde{Q}}(\tilde{Q},K')=-\left(\frac{K+\frac{w}{r}}{K'+\frac{w}{r}}\right)^{\beta-1}\cdot\beta\cdot\left[h(\tilde{Q})\right]^{\beta-1}\cdot h'(\tilde{Q})+\beta\cdot h'(\tilde{Q})$$

$$>-1\cdot\beta\cdot 1\cdot h'(\tilde{Q})+\beta\cdot h'(\tilde{Q})=0$$

where the inequality follows from $h(\tilde{Q})<1$ and from *K*' > *K*.