Characterizing Coordination in Investment Timing*
(PRELIMINARY AND INCOMPLETE)

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Abstract

We provide characterizations of tacit collusion (simultaneous) equilibrium in models of investment timing allowing for spillovers in both flow profits and investment costs. We validate these characterizations by applying them to common models of capacity accumulation and R&D investment, as well as to investment in an endogenously priced input. For instance, in linear demand Cournot competition, tacit collusion is likelier to arise when installed capacities and lumpy investments are both large.

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In their seminal article on preemption in games of timing, with minimal structural assumptions, Fudenberg and Tirole [8] synthetically identify conditions under which a simultaneous equilibrium may arise. There has been a recent resurgence of interest in this issue in the context of real option models of investment (see, e.g. Boyer et. al. [3], Mason and Weeds [15]). The objective of this paper is to provide a reasonably comprehensive characterization of simultaneous investment equilibria under a range of assumptions regarding firm payoffs and investment costs, and to tie the resulting results to several standard economic models of capacity competition and R&D investment.

We begin by establishing the existence of a preemption (sequential) equilibrium benchmark when allowing for indirect spillovers between firms, such as learning effects and entry barriers. We then link the existence of a simultaneous equilibrium to the presence of direct and indirect spillovers, complementarities, and the discounting term. When joint investment is not desirable, the equilibrium under which firms never invest is fully characterized. The conditions for a simultaneous investment are broadly comparable to those that arise in the repeated game framework. When joint investment is desirable, we identify conditions under which the best feasible simultaneous equilibrium, in which firms delay and invest at the same point in time, is characterized.

A specificity of this paper is to focus on two kinds of externalities, profit externalities and investment externalities. The former are well-known in real option games: one firm's investment generally impacts the flow profit of others. The latter are less well-known, and are motivated by several reasons. First, it may be as a result of the economic environment, in which there is learning, or experience effects, that one firm's investment decision has an impact on the (fixed) investment cost of the other firm. Second, in a related paper, we show that if the investment (or input) price is endogenized and there is market power, the input seller chooses to discount the first input, effectively creating an investment externality. Third, if the input is specialized, and notably in the simultnaeous investment equilibrium that is the focus of this paper, one might envisage decreasing returns to the input's production regardless of market power issues, if the two firms invest simultaneously.

The results with respect to simultaneous equilibrium are then applied to several examples: a model of capacity choice by Cournot duopolists, to investment in cost-reducing or demand-enhancing R&D, and to capacity investment in a vertical industry structure where a monopoly input supplier influences the downstream preemption race. We find, that for large enough existing capacities and investments, a simultaneous equilibrium exists regardless of discounting, and that larger installed capacities ("footholds") make a simultaneous equilibruim more likely to arise [].

1 The Model

The assumptions, most of which are standard, are described in the first part of this section. Because the paper focuses on the link between specific externalities and simultaneous equilibrium, or coordination, the relevant terminology for the remainder of the paper is then described. Finally, we outline a set of payoff functions, which are standard for this kind of model and useful to the analysis that follows.

1.1 Assumptions

Flow profits are of the form $Y_t \pi_{ij}$, with $\{i,j\} \in \{0,1\}^2$. The multiplicative shock Y_t is taken to follow a geometric Brownian motion $dY_t = \alpha Y_t dt + \sigma Y_t dZ_t$, with $Y_0 > 0$, $\alpha > 0$ (growth rate), $\sigma > 0$ (volatility), and where $(Z_t)_{t \geq 0}$ is a standard Wiener process. For example, this shock may be thought of as a measure of market size that evolves stochastically over time. The value of the multiplicative shock at the current date is hereafter denoted by y. For the equation of motion to describe a market in expansion, it is assumed that $\alpha > \frac{\sigma^2}{2}$.[1]

There are two firms. The time invariant component of flow profit, π_{ij}, depends on the previous and current investment decisions of both firms. When appropriate, $\boldsymbol{\pi}$ is used to denote the vector $(\pi_{00}, \pi_{10}, \pi_{01}, \pi_{11})$. Investment is a binary decision, and thus i takes the value 1 if the firm has invested, and j takes the value 1 if its rival has invested.[2] Investment is inherently discrete and of fixed size. For example, it may be thought of as an increase in production capacity, or as an R&D expenditure. The assumption that investment is a binary decision means that we consider a single round of investment choices by the firms. Investment is assumed to be *desirable*, that is $\pi_{1i} > \pi_{0i}$, $i \in \{0,1\}$. A second assumption regarding flow profit that is made throughout the paper is that $\pi_{10} > \pi_{01}$, that is a firm benefits more from its own investment than from its rival's.

Investment is costly, and the cost of investment may depend on previous and current investment decisions. If a single firm is the first to invest, this cost is denoted by I_L. If one firm has already invested, the cost to the second firm is denoted by I_F. Finally, if both firms invest simultaneously, this cost is denoted by I_S. Thus, a cost asymmetry may arise even though the two firms

[1] The geometric brownian motion is derived from $Y_t = Y_0 \exp\left[\left(\alpha - \frac{\sigma^2}{2}\right) t + \sigma Z_t\right]$ by using Itô's lemma.

[2] We do restrict π_{11} to be independent of the sequence of investment decisions. An alternative specification is to allow for persistent first-mover advantage by specifying $\pi_{11}^L > \pi_{11}^F$, where π_{11}^L designates the flow profit of the first firm to invest and π_{11}^F designates the flow profit of the second firm to invest (see Mason and Weeds [15] and Versaevel [18]).

are identical ex-ante. Allowing $I_L \neq I_F \neq I_S$ generalizes the analysis of some existing models, and we motivate this choice further when discussing the terminology used in the paper. When appropriate, $\mathbf{I}$ is used to denote the vector (I_L, I_F, I_S). In the analysis, the ratios of investment costs play an important role, so it is useful to define the following magnitudes: $\zeta_F \equiv \frac{I_F}{I_L}$ and $\zeta_S \equiv \frac{I_S}{I_L}$.

The interest rate, common to both firms, is r. To rule out degenerate solutions, it is assumed that $0 < \alpha < r$.

1.2 Terminology

The formal conditions to characterize the simultaneous equilibrium with respect to which coordination may arise are elaborate, but involve components that have economic significance. The following terminology is used in the rest of the paper.[3]

Investment is said to be *jointly undesirable* if $\pi_{11} \leq \pi_{00}$, and *jointly desirable* if $\pi_{11} > \pi_{00}$. This characteristic is key to determining the qualitative nature of simultaneous equilibrium, that is whether firms jointly abstain from ever investing, or jointly delay investing for a finite time. Either possibility may arise. For example, if investment consists of an advertising campaign, negative advertising may decrease market size ($\pi_{11} \leq \pi_{00}$), whereas preference-enhancing advertising may increase it ($\pi_{11} > \pi_{00}$).[4]

Investment involves a *negative profit externality* if $\pi_{i1} < \pi_{i0}$, $i \in \{0, 1\}$, and a *positive profit externality* otherwise. Note that since investment is taken to be individually desirable, if it is jointly undesirable, then there must be a negative direct externality (that is, $\pi_{00} \geq \pi_{11}$ and $\pi_{1j} > \pi_{0j}$ together imply $\pi_{i1} < \pi_{i0}$, $i, j \in \{0, 1\}$). Negative profit externalities seem natural in situations such as capacity investment, whereas positive profit externalities may be thought of as arising if investment is in R&D, when there is a large enough technological spillover.

Investment is said to involve a *negative investment externality* if $\zeta_i > 1$, $i \in \{F, S\}$, and a *positive investment externality* if $\zeta_i < 1$, $i \in \{F, S\}$. Investment externalities may arise in one of two ways. A firm's investment decision may raise or lower the cost of the next firm that invests. For example, the former ($\zeta_F > 1$) would arise if firms compete for some key resource, such as location, whereas the latter ($\zeta_F < 1$) would arise if there is a form of learning or experience effect pertaining to the investment process. A second way in which indirect externalities may arise is

[3] Comparing with the expressions for firm payoffs given in the next section (expressions (2), (3), and (4)), the externalities described here concern the levels and slopes of some of the payoff terms.

[4] See Anderson et. al. [1].

if joint investment has an impact on investment cost. This can happen either because there is a form of congestion in the provision of a key input (negative investment externality, $\zeta_S > 1$), or of synergy if an important network externality arises when the firms enter the market together (positive investment externality, $\zeta_S < 1$).

Another important characteristic of investment is supermodularity, which we refer to as *complementarity* in the text. This is captured by the ratio $\frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{00}}$, and complementarity is said to arise if this ratio is greater than one, whereas anti-complementarity is said to arise otherwise. A standard interpretation of complementarity is that one firm's investment raises the other's incentive to invest.

1.3 Payoffs

An equilibrium of the investment timing game involves a triplet of investment triggers chosen by the firms, which is denoted as (y_P, y_F^*, y_S^*). The latter two triggers result from a well-studied optimization problem (see [7]), and have the following expressions

$$y_F^* = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_{11}-\pi_{01}}I_F, \qquad y_S^* = \begin{cases} \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_{11}-\pi_{00}}I_S, & \pi_{11} > \pi_{00} \\ \infty, & \pi_{11} \leq \pi_{00} \end{cases}, \tag{1}$$

where $\beta \equiv \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$ is a standard expression in real option models.[5] The sign of $\pi_{11} - \pi_{00}$ is key to determining the nature of the simultaneous equilibrium: when this expression is strictly positive, firms jointly delay investing until a finite threshold is reached. When it is negative however, in a simultaneous equilibrium, firms jointly abstain from ever investing.

Up to the relaxation of the constraint that investment cost is invariant, the investment timing game closely follows the analyses of Boyer et. al. [3], Grenadier [9], Mason and Weeds [15]. The derivation of the preemption threshold and the characterization of equilibrium involve the following firm payoffs.

The value of a firm that invests immediately, when the current value of the multiplicative shock is y, is

$$L(y) = \begin{cases} \frac{\pi_{10}}{r-\alpha}y - I_L + \left(\frac{y}{y_F^*}\right)^{\beta}\frac{\pi_{11}-\pi_{10}}{r-\alpha}y_F^*, & y \leq y_F^* \\ \frac{\pi_{11}}{r-\alpha}y - I_S, & y > y_F^* \end{cases}, \tag{2}$$

[5] In the certainty case, that is for $\sigma = 0$, we have $\beta = \frac{r}{\alpha}$ and $\left(\frac{y}{y_i}\right)^{\beta} = e^{-r(t_i-t)}$, the continuous time discounting term.

where L is used to refer to the fact that the firm is the leader in the market with respect to investment timing. Note that after y_F^* is reached, one firm's investment immediately triggers the second's, so that investment is effectively simultaneous and the investment cost is therefore I_S.

The value of a firm that invests as a follower when the multiplicative shock reaches the threshold y_F^*, provided that the current value of the multiplicative shock is y, and provided that the rival firm invests immediately at y, is

$$F^*(y) = \begin{cases} \frac{\pi_{01}}{r-\alpha} y + \left(\frac{y}{y_F^*}\right)^{\beta} \frac{I_F}{\beta-1}, & y \leq y_F^* \\ \frac{\pi_{11}}{r-\alpha} y - I_S, & y > y_F^* \end{cases}, \tag{3}$$

where F^* is used analogously to refer to the fact that the firm invests as a follower, and also to the fact that the investment threshold y_F^* results from an optimization.

The value of a firm that invests when the multiplicative shock reaches the threshold y_S^*, provided that the current value of the multiplicative shock is y, and provided that the rival firm also invests only when the multiplicative shock reaches the threshold y_S^*, is

$$S^*(y) = \begin{cases} \frac{\pi_{00}}{r-\alpha} y + \left(\frac{y}{y_S^*}\right)^{\beta} \frac{I_S}{\beta-1}, & y \leq y_S^* \\ \frac{\pi_{11}}{r-\alpha} y - I_S, & y > y_S^* \end{cases}, \tag{4}$$

where S^* is used to denote the fact that this payoff reflects simultaneous investment by the two firms, at a threshold y_S^* that results from a straightforward optimization.

2 Sequential Investment (Preemption) Equilibrium

Simultaneous investment and the coordination problem it may generate (coordination which may or may not be achieved by means of tacit collusion) is a focus of this paper. A necessary condition for a coordination problem to arise is that there be another equilibrium solution, namely the sequential investment equilibrium. Since player roles are endogenous, the sequential investment equilibrium we consider has the nature of a preemption equilibrium. In a preemption equilibrium, firms invest sequentially, either firm may be the leader with equiprobability, and the race to be first dissipates the rents that accrue to the first investor. A preemption equilibrium is characterized by the triggers $\{y_P, y_F^*\}$, with $y_P < y_F^*$, which denote the investment thresholds for the leader and follower.[6] The preemption trigger y_P is determined by the condition $L(y_P) = F^*(y_P)$, i.e.

[6] This is a simplification. See Boyer et. al. [3], Fudenberg and Tirole [8], Huisman et. al. [11] for precise descriptions of the strategies underlying the preemption equilibrium.

firms are indifferent between investing as a leader at y_P and investing as a follower at y_F^* when the leader invests at y_P.

The existence of the preemption equilibrium when investment costs are invariant ($\zeta_F = 1$) is well-established. The same argument establishes the existence of a preemption equilibrium with asymmetric firm-specific investment costs.[7] With investment externalities, which is a source of investment cost asymmetry, the same argument applies so long as relative investment costs ζ_F are in the right range.

Proposition 1 in this section characterizes the relative investment costs for which a preemption equilibrium exists. Essentially, preemption arises whenever the investment externality is negative, or when the investment externality is not too positive and profit externalities are negative. This is to be expected: a negative investment externality means that the first firm has a lower investment cost, and negative profit externalities induce preemption by reducing the attractiveness of the joint investment phase.

Define $\underline{\zeta}(\boldsymbol{\pi}, \beta) \equiv \left[\beta \left(\frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{01}}\right)^{\beta-1} - (\beta-1)\left(\frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{01}}\right)^{\beta}\right]^{\frac{1}{\beta-1}}$. This expression appears as a lower bound on relative investment cost in Proposition 1. The following lemma partly describes the behavior of $\underline{\zeta}(\boldsymbol{\pi}, \beta)$.

Lemma 1 *For $\pi_{11} < \pi_{10}$, $\underline{\zeta}(\boldsymbol{\pi}, \beta) \in (0, 1]$ and $\frac{\partial \underline{\zeta}}{\partial \beta}(\boldsymbol{\pi}, \beta) > 0$.*

Proof Let $z \equiv \frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{01}} \in (0,1)$, so $\underline{\zeta}(z, \beta) = \left(\beta z^{\beta-1} - (\beta-1)\, z^{\beta}\right)^{\frac{1}{\beta-1}}$ is well-defined. Then $\underline{\zeta}(0, \beta) = 0$, $\underline{\zeta}(1, \beta) = 1$, and $\frac{\partial \underline{\zeta}}{\partial z}(z, \beta) = \beta z^{\beta-2}(1-z)\left(\beta z^{\beta-1} - (\beta-1)\, z^{\beta}\right)^{\frac{2-\beta}{\beta-1}} > 0$ so $\underline{\zeta}(z, \beta) \in (0, 1)$. Also, $\frac{\partial \underline{\zeta}}{\partial \beta}(z, \beta) = \frac{1}{\beta-1}\left[\frac{1-z}{\beta-(\beta-1)z} - \frac{\ln(\beta-(\beta-1)z)}{\beta-1}\right]\underline{\zeta}(z, \beta)$. Since $x \ln x \geq x - 1$ with equality if and only if $x = 1$, $(\beta - (\beta-1)\, z) \ln (\beta - (\beta-1)\, z) > (\beta-1)(1-z)$ so $\frac{\partial \underline{\zeta}}{\partial \beta}(z, \beta) < 0$. □

In what follows, we denote $\underline{\zeta}(\boldsymbol{\pi}, \beta)$ simply by $\underline{\zeta}$. The conditions on ζ_F for a preemption equilibrium to exist can now be described.

Proposition 1 *A preemption equilibrium exists whenever ζ_F is sufficiently large:*

(i) when profit externalities are non-negative ($\pi_{11} \geq \pi_{10}$), a preemption equilibrium exists if and only if investment externalities are negative ($\zeta_F > 1$);

(ii) when profit externalities are negative ($\pi_{11} < \pi_{10}$), a preemption equilibrium exists if and only if investment externalities are not too positive ($\zeta_F \geq \underline{\zeta}$).

[7] See Huisman and Kort [13].

Proof The existence of a preemption equilibrium hinges on the behavior of the difference $L(y) - F^*(y)$. Since $L(0) - F^*(0) = -I_L < 0$, preemption occurs if and only if there exists a y in $(0, y_F^*)$ such that this difference is nonnegative. Let $f(y) \equiv L(y) - F^*(y)$ so

$$f(y) = \frac{\pi_{10} - \pi_{01}}{r - \alpha} y - I_L - \left(\frac{y}{y_F^*}\right)^{\beta} \frac{I_F}{\beta - 1} \frac{\beta(\pi_{10} - \pi_{11}) + (\pi_{11} - \pi_{01})}{\pi_{11} - \pi_{01}}, \text{ all } y \leq y_F^*. \tag{5}$$

Then the preemption threshold y_P is the lower root of the equation $f(y) = 0$ in $(0, y_F^*)$, if it exists. There are two cases to consider.

(i) $\pi_{11} \geq \pi_{10}$

There are two subcases to consider. First, if $\pi_{11} - \pi_{01} > \beta(\pi_{11} - \pi_{10})$, then $f(y)$ is strictly concave in y, and $f'(\widehat{y}) = 0$ for

$$\widehat{y} = \left[\frac{\pi_{10} - \pi_{01}}{\beta\pi_{10} - (\beta - 1)\pi_{11} - \pi_{01}}\right]^{\frac{1}{\beta-1}} y_F^*.$$

The maximizer satisfies $\widehat{y} \geq y_F^*$. Otherwise, if $\pi_{11} - \pi_{01} \leq \beta(\pi_{11} - \pi_{10})$, then $f(y)$ is increasing and strictly convex in y. In both of these subcases, $f(y)$ is increasing in y over the relevant interval $(0, y_F^*)$, and therefore a preemption equilibrium exists if and only if $f(y_F^*) = I_F - I_L > 0$, i.e. if $\zeta_F > 1$.

(ii) $\pi_{11} < \pi_{10}$

In this case, $\pi_{11} - \pi_{01} \geq 0 > \beta(\pi_{11} - \pi_{10})$, so $f(y)$ is strictly concave in y, with a maximum at $\widehat{y}$. Moreover, the maximizer satisfies $\widehat{y} < y_F^*$. Therefore, a preemption equilibrium exists if and only if $f(\widehat{y}) > 0$. Evaluating and simplifying yields that $f(\widehat{y}) > 0$ if and only if $\zeta_F \geq \underline{\zeta}$. To establish this, insert the developed expressions of y_F^* and $\widehat{y}$ in (5), and the inequality follows by rearranging. □

Thus, if profit externalities are negative so it is advantageous to be the only firm to have invested, preemption occurs if there is a a negative investment externality like a location-type effect that makes investing first inherently attractive. Otherwise, if profit externalities are positive so it is disadvantageous to be the sole firm in the market to have invested, the condition is slacker and preemption occurs even witha positive investment externality, such as a learning or experience effects captured by the follower, that is not too strong. If the positive investment externality is too strong, neither firm seeks to enter first, preferring either to defer and invest as a follower, or never to invest at all, or to invest in a simultaneous equilibrium.

Having defined the preemption investment y_P, it is useful to introduce another value function (as compared with (2), (3), and (4) above) that is needed in the next section. This function describes the ex-ante expected value of a firm, at a market size $y \leq y_P$, when it anticipates that preemption occurs at y_P and that it is equally likely to enter as a leader or as a follower at that threshold.[8] Formally:

$$V_P(y) = \frac{\pi_{00}}{r-\alpha} y + \left(\frac{y}{y_P}\right)^{\beta} \left(\frac{\frac{1}{2}\pi_{10} + \frac{1}{2}\pi_{01} - \pi_{00}}{r-\alpha} y_P - \frac{1}{2} I_L\right) + \left(\frac{y}{y_F^*}\right)^{\beta} \left(\frac{\pi_{11} - \frac{1}{2}\pi_{10} - \frac{1}{2}\pi_{01}}{r-\alpha} y_F^* - \frac{1}{2} I_F\right), \text{ all } y \leq y_P. \quad (6)$$

The function $V_P(y)$ satisfies $V_P(y_P) = L(y_P) = F^*(y_P)$. It is the comparison of $S^*(y)$ with $V_P(y)$ that constitutes a valid criterion to assess whether the investment game has the features of a pure coordination game (i.e., whether the payoff from the simultaneous equilibrium is higher than the payoff under preemption, so that firms have an incentive to coordinate on the former).

Finally, the preemption threshold y_P does not have an analytic expression in general. A useful alternative is to bound its value. The bound that we use is denoted by y^* and defined by

$$y^* = \arg \max_{y_i \in [0, y_F^*]} \left(\frac{y}{y_i}\right)^{\beta} \left(\frac{\pi_{10} - \pi_{01}}{r-\alpha} y_i - I_L\right) + \left(\frac{y}{y_F^*}\right)^{\beta} \frac{\pi_{11} - \pi_{10}}{r-\alpha} y_F^*, \quad (7)$$

so that $y^* = \frac{\beta}{\beta-1} \frac{r-\alpha}{\pi_{10}-\pi_{01}} I_L$. The following lemma establishes that this threshold indeed provides a bound for the leader's trigger under preemption.

Lemma 2 *The preemption trigger satisfies $y_P < y^*$.*

Proof Since $L(0) < F^*(0)$, and since y_P is the unique solution to $L(y) = F^*(y)$, we have $y_P < y^*$ if and only if $L(y^*) > F^*(y^*)$. The latter inequality, using (2) and (3), and after a reorganization of terms, is equivalent to ζ

$${}_{F}^{\beta-1} > \beta \left(\frac{\pi_{11} - \pi_{01}}{\pi_{10} - \pi_{01}}\right)^{\beta-1} - (\beta - 1) \left(\frac{\pi_{11} - \pi_{01}}{\pi_{10} - \pi_{01}}\right)^{\beta}.$$

There are two cases to consider. (i) If profit externalities are non-negative, the right hand side of the inequality is of the form $g(z) \equiv \beta z^{\beta-1} - (\beta - 1) z^{\beta}$, which is a quasiconcave function, and attains a global maximum for $z = 1$, with $g(1) = 1$ so $g(z) \leq 1$. (ii) If profit externalities are negative, the expression on the right-hand side is positive and equal to $\underline{\zeta}^{\beta-1}$. Therefore, by Proposition 1, the inequality holds in a preemption equilibrium.□

The comparison between y_P and y^* is of interest when $y^* < y_F^*$, which holds if and only if $\zeta_F > \frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{01}}$.

[8] If initial conditions are such that $y_0 > y_P$, other issues may be raised ("mistakes" may arise, see Huisman and Kort [13]).

3 Simultaneous (Non-)Investment Equilibrium

In this section we seek to characterize the conditions for the investment timing decision to have a simultaneous equilibrium, and eventually to have the features of a pure coordination game. This requires above all to characterize the conditions for a simultaneous investment equilibrium to exist given that a preemption equilibrium exists as well. Therefore, throughout this section it is assumed that the conditions of Proposition 1 on the investment externality ζ_F hold, so a preemption equilibrium exists. When a preemption equilibrium exists, a coordination problem then arises if there is a simultaneous equilibrium solution also, which is such that no firm firms finds it profitable to invest unilaterally and thereby drive the industry to a preemption equilibrium, and which results in a higher payoff for the firms.

3.1 Joint Investment Not Desirable ($\pi_{00} \geq \pi_{11}$)

In this subsection we assume that joint investment is not desirable. In that case, in a simultaneous equilibrium both firms refrain from ever investing ($y_S^* = \infty$), so we refer to this case as the *infinite delay* case. Because investment is individually desirable, profit externalities are necessarily negative ($\pi_{i0} > \pi_{i1}$, $i \in \{0, 1\}$). By Proposition 1, for a preemption equilibrium to exist, investment externalities must therefore satisfy $\zeta_F \geq \underline{\zeta}$.

We begin by characterizing the conditions for a simultaneous equilibrium to exist. To this end, we now introduce another threshold, $\overline{\zeta}(\boldsymbol{\pi}, \beta) \equiv \frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{00}} \left(\beta \frac{\pi_{10}-\pi_{11}}{\pi_{10}-\pi_{00}}\right)^{\frac{1}{\beta-1}}$. This expression appears as an upper bound on relative investment costs in Proposition 2. The following lemma describes the behavior of $\overline{\zeta}(\boldsymbol{\pi}, \beta)$.

Lemma 3 $\underline{\zeta} < \overline{\zeta}(\boldsymbol{\pi}, \beta)$, $\overline{\zeta}(\boldsymbol{\pi}, \beta) \in \left[\frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{00}}, \infty\right)$, *and* $\frac{\partial \overline{\zeta}(\boldsymbol{\pi},\beta)}{\partial \beta} < 0$.

Proof First, let $A(\boldsymbol{\pi}, \beta) \equiv \left(\frac{\underline{\zeta}}{\overline{\zeta}(\boldsymbol{\pi},\beta)}\right)^{\beta-1} = \left(1 + \frac{1}{\beta}\frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{11}}\right)\left(\frac{\pi_{10}-\pi_{00}}{\pi_{10}-\pi_{01}}\right)^{\beta}$. Then, $A(\boldsymbol{\pi}, 1) = \frac{\pi_{10}-\pi_{00}}{\pi_{10}-\pi_{11}} \leq 1$, with an equality sign if and only if $\pi_{00} = \pi_{11}$, and

$$\frac{\partial A(\boldsymbol{\pi}, \beta)}{\partial \beta} = \left[\left(1 + \frac{1}{\beta}\frac{\pi_{11} - \pi_{01}}{\pi_{10} - \pi_{11}}\right) \ln \frac{\pi_{10} - \pi_{00}}{\pi_{10} - \pi_{01}} - \frac{1}{\beta^2}\frac{\pi_{11} - \pi_{01}}{\pi_{10} - \pi_{11}}\right]\left(\frac{\pi_{10} - \pi_{00}}{\pi_{10} - \pi_{01}}\right)^{\beta}$$

which is negative since $\ln \frac{\pi_{10}-\pi_{00}}{\pi_{10}-\pi_{01}} < 0$. Therefore, $\underline{\zeta} < \overline{\zeta}(\boldsymbol{\pi}, \beta)$.

Second, $\lim_{\beta\to 1}\left(\beta\frac{\pi_{10}-\pi_{11}}{\pi_{10}-\pi_{00}}\right)^{\frac{1}{\beta-1}} = e\lim_{\beta\to 1}\left(\frac{\pi_{10}-\pi_{11}}{\pi_{10}-\pi_{00}}\right)^{\frac{1}{\beta-1}} = \infty\,(=e)$ when $\pi_{00} > \pi_{11}\,(=\pi_{11})$, and $\lim_{\beta\to\infty}\left(\beta\frac{\pi_{10}-\pi_{11}}{\pi_{10}-\pi_{00}}\right)^{\frac{1}{\beta-1}} = 1$, so $\overline{\zeta}(\boldsymbol{\pi},\beta) \in \left[\frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{00}},\infty\right)$. Also,

$$\frac{\partial\overline{\zeta}(\boldsymbol{\pi},\beta)}{\partial\beta} = \frac{1}{(\beta-1)^2}\left[\frac{\beta-1}{\beta} - \left(\ln\beta + \ln\frac{\pi_{10}-\pi_{11}}{\pi_{10}-\pi_{00}}\right)\right]\overline{\zeta}(\boldsymbol{\pi},\beta).$$

The expression in brackets is negative since $\frac{\pi_{10}-\pi_{11}}{\pi_{10}-\pi_{00}} \geq 1$ and $\beta\ln\beta > \beta-1$ (recall that $\beta > 1$), so $\frac{\partial\overline{\zeta}(\boldsymbol{\pi},\beta)}{\partial\beta} < 0$. □

In what follows, we denote $\overline{\zeta}(\boldsymbol{\pi},\beta)$ simply by $\overline{\zeta}$. The conditions on ζ_F for a simultaneous equilibrium to exist can now be described.

Proposition 2 *Suppose that joint investment is not desirable ($\pi_{00} \geq \pi_{11}$), and that a preemption equilibrium exists ($\zeta_F \geq \underline{\zeta}$). Then, a simultaneous equilibrium exists if and only if $\zeta_F \leq \overline{\zeta}$.*

Proof A simultaneous "non-investment" equilibrium exists if and only if $y_S^*(=\infty)$ is a best-response $y_{-i} = y_S^*$, that is if $S^*(y) \geq L(y)$ over the interval $[0, y_F^*]$. Note that here, the simultaneous investment payoff has the simple form $S^*(y) = \frac{\pi_{00}}{r-\alpha}y$. Let $f(y) \equiv S^*(y) - L(y)$, so

$$f(y) = -\frac{\pi_{10}-\pi_{00}}{r-\alpha}y - \left(\frac{y}{y_F^*}\right)^{\beta}\frac{\pi_{11}-\pi_{10}}{r-\alpha}y_F^* + I_L.$$

This function is convex, and reaches a minimum at $\widehat{y} = \left(\frac{1}{\beta}\frac{\pi_{10}-\pi_{00}}{\pi_{10}-\pi_{11}}\right)^{\frac{1}{\beta-1}} y_F^*$, with $0 < \widehat{y} < y_F^*$ for all admissible parameter values. Evaluating gives

$$f(\widehat{y}) = -\frac{\pi_{10}-\pi_{00}}{\pi_{11}-\pi_{01}}\left(\beta\frac{\pi_{10}-\pi_{11}}{\pi_{10}-\pi_{00}}\right)^{-\frac{1}{\beta-1}} I_F + I_L = -\left(\overline{\zeta}\right)^{-1} I_F + I_L.$$

If $\zeta_F \leq \overline{\zeta}$, then $f(\widehat{y}) \geq 0$ and a simultaneous equilibrium exists. Conversely, if $\zeta_F > \overline{\zeta}$, we have $L(\widehat{y}) > S^*(\widehat{y})$, implying that no simultaneous non-investment equilibrium exists.□

Propositions 1 and 2 together describe the qualitative evolution of equilibrium as a function of the investment externality ζ_F. There are three equilibrium regions. First, if the relative investment cost of the second firm is relatively low ($0 \leq \zeta_F < \underline{\zeta}$), no firm wishes to enter first so there is no preemption, and only a simultaneous non-investment equilibrium. Second, in an intermediate range of relative investment cost ($\underline{\zeta} \leq \zeta_F \leq \overline{\zeta}$), both preemption and simultaneous equilibria arise, and the multiple equilibria have an intuitive pure coordination game feature (the simultaneous equilibrium dominates the preemption equilibrium, see Proposition 3 below). Third, if the relative investment cost of the second firm is relatively high ($\overline{\zeta} < \zeta_F$), only preemption (sequential investment) arises as an equilibrium. Thus as a simple guideline, comparing Propositions 1 and 2, is that greater negative investment externalities increase the likelihood of preemption but ultimately reduce the likelihood of a simultaneous infinite delay equilibrium as well.

Interestingly, the existence condition for a simultaneous equilibrium with infinite delay, given by Proposition 2, that is

$$\zeta_F \leq \frac{\pi_{11} - \pi_{01}}{\pi_{10} - \pi_{00}} \left(\beta \frac{\pi_{10} - \pi_{11}}{\pi_{10} - \pi_{00}} \right)^{\frac{1}{\beta - 1}} \equiv \overline{\zeta}, \tag{8}$$

can be given further economic interpretation. On the left-hand side of the inequality sign, ζ_F ($\equiv \frac{I_F}{I_L}$) is lower than (greater than) one if investment externalities are positive (negative). On the right-hand side of the inequality, the boundary value (in β) depends on the magnitude of the first term, $\frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{00}}$. But this term reflects (anti-)complementarities in investment. It is greater than (smaller than) one if flow profits are supermodular (submodular). These remarks lead to more intuitive sufficient conditions for (8) to hold or not.

Corollary 1 *A simultaneous equilibrium exists ($\zeta_F \leq \overline{\zeta}$) if investment externalities are positive and investments are complementary. Otherwise, a simultaneous equilibrium is more likely to arise the more positive the investment externality (the lower is ζ_F), the more complementary the investment (the greater is $\frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{00}}$), and the lower the discounting term β.*

Thus, the effect of the parameters $(\mathbf{I}, \boldsymbol{\pi}, \beta)$ can be broken down into economically identifiable components. For instance, holding the investment cost of the first firm constant, a more positive investment externality accelerates the follower's entry. As a result, the expected length of time that a firm deviating from the simultaneous equilibrium spends as an incumbent is smaller, so that the likelihood of deviation from the simultaneous equilibrium is smaller. Specifically, the impact of I_F on this incentive to deviate, when the market size is y, is

$$\frac{d\left[S^*\left(y\right) - L(y)\right]}{dI_F} = -\beta \frac{\pi_{10} - \pi_{11}}{\pi_{11} - \pi_{01}} \left(\frac{y}{y_F^*} \right)^{\beta} < 0. \tag{9}$$

With respect to the right-hand side of the condition (8), in which the term $\frac{\pi_{11}-\pi_{01}}{\pi_{10}-\pi_{00}}$ reflects the supermodularity of flow profits, one can observe that supermodularity in flow profits entails that rival investment raises one's own static incentive to invest.

Having established the condition for a simultaneous equilibrium to exist, it is natural to ask whether firms have an incentive to choose one or the other of the equilibria. In the infinite delay case, the answer is unambiguous.

Proposition 3 *The simultaneous (non-investment) equilibrium is Pareto optimal.*

Proof It is necessary to sign the difference between the payoffs in the two equilibria, which is given by

$$S^* (y) - V^P (y)$$
$$= -\left(\frac{y}{y_P}\right)^\beta \left(\frac{\frac{1}{2}\pi_{10} + \frac{1}{2}\pi_{01} - \pi_{00}}{r-\alpha} y_P - \frac{1}{2} I_L\right) - \left(\frac{y}{y_F^*}\right)^\beta \left(\frac{\pi_{11} - \frac{1}{2}\pi_{10} - \frac{1}{2}\pi_{01}}{r-\alpha} y_F^* - \frac{1}{2} I_F\right),$$

where $S^* (y)$ has the simple form $S^*(y) = \frac{\pi_{00}}{r-\alpha} y$, for all $y < y_S^* = \infty$, and where $V^P (y)$ is as defined in (6) for all $y \leq y_P$. The preemption condition $f (y_P) = 0$, where the function f is as introduced in (5), implicitly defines $\frac{y_P}{y_F^*}$. After reorganizing terms, substituting for $\left(\frac{y_P}{y_F^*}\right)^\beta$, and simplifying, $S^* (y) - V^P (y)$ can be seen to have the same sign as

$$\frac{[\beta (\pi_{10} - \pi_{11}) (\pi_{00} - \pi_{01}) - (\pi_{11} - \pi_{01}) (\pi_{10} - \pi_{00})] y_P + (\pi_{11} - \pi_{01}) (r - \alpha) I_L}{(\beta\pi_{10} - (\beta - 1) \pi_{11} - \pi_{01})}.$$

Since profit externalities are positive, the denominator is positive. Regarding the numerator, the expression in the first brackets is positive, because it is equal to $(\pi_{10} - \pi_{01}) (\pi_{00} - \pi_{11})$ when $\beta = 1$ and increasing in β. Therefore, $S^* (y) - V^P (y)$ is positive. □

Together, Propositions 2 and 3 characterize conditions under which firms face a dynamic form of a pure coordination game with respect to their choice of investment triggers, when joint investment is not desirable.

The conditions described in Corollary 1 are consistent with the ones that would emerge if the simultaneous equilibrium is sustained by tacit collusion. In that case the supermodularity term in (8) is coherent with a relatively lower cost for one firm to punish its rival for deviating from collusion. Holding the static incentive to deviate from collusion $(\pi_{10} - \pi_{00})$ constant, "more" supermodularity results in a quicker punishment insofar as y_F^* increases with the difference $\pi_{11} - \pi_{01}$.

With respect to the other term in (8), $\left(\beta\frac{\pi_{10}-\pi_{11}}{\pi_{10}-\pi_{00}}\right)^{\frac{1}{\beta-1}}$, a decrease in the interest rate r, or an increase in either the growth parameter α or the volatility parameter σ, are associated with a decrease in the discounting term β, and reduce the likelihood of existence of a simultaneous equilibrium. Finally, the lower the impact of joint investment (when $\pi_{00} \approx \pi_{11}$), the more likely is a simultaneous equilibrium to arise (Corollary 1). As for r and α specifically, the conditions on β reflect those that obtain for tacit collusion in a repeated game setting.

3.2 Joint Investment Desirable ($\pi_{00} < \pi_{11}$)

In this section we assume that joint investment is desirable ($\pi_{11} > \pi_{00}$). In this case, in a coordination equilibrium, firms invest simultaneously at a finite trigger $y_S^* = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_{11}-\pi_{00}}I_S$, so we refer to this case as the *finite delay* case. In contrast with the previous subsection (the infinite delay case), firms may now experience either positive or negative profit externalities.

Coordination may not be fully characterized as it was in the previous subsection, but a partial characterization may be determined. To do this, the central proposition in the section involves two conditions.

First, $(\mathbf{I}, \boldsymbol{\pi}, \beta)$ are taken to be such that the following holds:

Assumption A

$$\beta\frac{(\pi_{10}-\pi_{11})(\pi_{11}-\pi_{01})^{\beta-1}}{(\pi_{11}-\pi_{00})^{\beta}}\frac{I_S}{I_F}+1 \geq 0. \tag{10}$$

This condition necessarily holds if $\pi_{10} \geq \pi_{11}$ (negative profit externalities), and for β "large enough" if $\pi_{i0} < \pi_{i1}$, $i \in \{0,1\}$. It is technical in nature, and ensures t hat a key part of the difference, $S^*(y) - L(y)$, is convex. Let the threshold $\widehat{y}$ be given by:

$$\frac{\pi_{10}-\pi_{00}}{\widehat{y}^{\beta-1}} = \beta\frac{\pi_{10}-\pi_{11}}{y_F^{*\beta-1}} + \frac{\pi_{11}-\pi_{00}}{y_S^{*\beta-1}}. \tag{11}$$

It can be shown that $\widehat{y}$ is well-defined if (10) holds.

Next, let $y_L^* \equiv \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_{10}-\pi_{00}}I_L$. The trigger y_L^* is the optimal investment threshold for a leader, if the sequence of investments is predetermined. A second condition, on $(\mathbf{I}, \boldsymbol{\pi})$, is given by:

Assumption B

$$y_L^* \leq \min\{y_F^*, y_S^*\}. \tag{12}$$

Proposition 4 *Suppose that $(\mathbf{I}, \boldsymbol{\pi}, \beta)$ are such that Assumptions A and B (conditions (10) and (12)) are satisfied. Then a simultaneous equilibrium exists if and only if:*

$$\left(\frac{\overline{\zeta}}{\zeta_F}\right)^{\beta-1} + \frac{1}{\zeta_S^{\beta-1}}\left(\frac{\pi_{11}-\pi_{00}}{\pi_{10}-\pi_{00}}\right)^{\beta} \geq 1. \tag{13}$$

Proof A simultaneous equilibrium exists whenever y_S^* is a best response to $y_{-i} = y_S^*$, that is whenever $S^*(y) \geq L(y)$ for $y \in [0, y_S^*]$.

In what follows, let $\widehat{f}(y) \equiv -\frac{\pi_{10}-\pi_{00}}{r-\alpha}y + I_L + \left(\frac{y}{y_S^*}\right)^\beta \frac{I_S}{\beta-1} + \left(\frac{y}{y_F^*}\right)^\beta \frac{\pi_{10}-\pi_{11}}{r-\alpha}y_F^*$. The function $S^*(y) - L(y)$ is continuous and differentiable, with $S^*(y) - L(y) \equiv \widehat{f}(y)$ for $y \in [0, \min\{y_F^*, y_S^*\}]$. Note that $\widehat{f}'(0) = -\frac{\pi_{10}-\pi_{00}}{r-\alpha}$, and $\widehat{f}''(y) = \beta I_S \frac{y^{\beta-2}}{y_S^{*\beta}} + \beta(\beta-1)\frac{\pi_{10}-\pi_{11}}{r-\alpha}\frac{y^{\beta-2}}{y_F^{*\beta-1}}$. The function $\widehat{f}$ is convex if and only if (10) holds, in which case $\widehat{f}$ has a well-defined global minimum in $\mathbb{R}_+$, which we denote by $\widehat{y}$ (see (11) above). Moreover, after rearrangement, $\widehat{f}(\widehat{y}) = \left(1 - \frac{\widehat{y}}{y_L^*}\right) I_L$, so $\widehat{f}(\widehat{y}) \geq 0$ if and only if $\widehat{y} \leq y_L^*$, which again after some rearrangement occurs if and only if (13) holds.

In the next step of the proof, we distinguish two cases.

Case 1: $y_S^* \leq y_F^*$

If $y_S^* \leq y_F^*$, then $S^*(y) - L(y) \geq 0$ on $[0, y_S^*]$ if and only if $\min_{[0,y_S^*]} \widehat{f}(y) \geq 0$. Suppose that $y_S^* \leq \widehat{y}$, so by (11), $\beta(\pi_{10} - \pi_{11})\left(\frac{y_S^*}{y_F^*}\right)^\beta \leq (\pi_{10} - \pi_{11})\frac{y_S^*}{y_F^*}$. Then,

$$\widehat{f}(y_S^*) = \frac{-\beta\pi_{10} + \pi_{11} + (\beta-1)\pi_{00}}{\pi_{11} - \pi_{00}}\frac{I_S}{\beta-1} + I_L + \left(\frac{y_S^*}{y_F^*}\right)^\beta \frac{\pi_{10} - \pi_{11}}{r-\alpha}y_F^*$$

$$\leq -\frac{\pi_{10} - \pi_{00}}{\pi_{11} - \pi_{00}}I_S + I_L \leq 0.$$

The last inequality follows from (12). Therefore, in this case $S^*(y) - L(y) \geq 0$ on $[0, y_S^*]$ if and only if $\widehat{f}(\widehat{y}) \geq 0$.

Case 2: $y_S^* \geq y_F^*$

If $y_S^* \geq y_F^*$, then $S^*(y) - L(y) \geq 0$ on $[0, y_S^*]$ if and only if $\widehat{f}(y) \geq 0$ for $y \in [0, y_F^*]$ and $S^*(y) - L(y) = -\frac{\pi_{11}-\pi_{00}}{r-\alpha}y + I_S + \left(\frac{y}{y_S^*}\right)^\beta \frac{I_S}{\beta-1} \geq 0$ for $y \in [y_F^*, y_S^*]$. Since $S^{*\prime}(y) - L'(y) \leq 0$ for $y \in [y_F^*, y_S^*]$ with $S^*(y_S^*) - L(y_S^*) = 0$, this second inequality always holds. Therefore, $S^*(y) - L(y) \geq 0$ on $[0, y_S^*]$ if and only if $\min_{[0,y_F^*]} \widehat{f}(y) \geq 0$. Suppose that $y_F^* \leq \widehat{y}$, so by (11), $\left(\frac{y_F^*}{y_S^*}\right)^\beta \leq \frac{-(\beta-1)\pi_{10}+\beta\pi_{11}-\pi_{00}}{\pi_{11}-\pi_{00}}\frac{I_F}{I_S}$. Then,

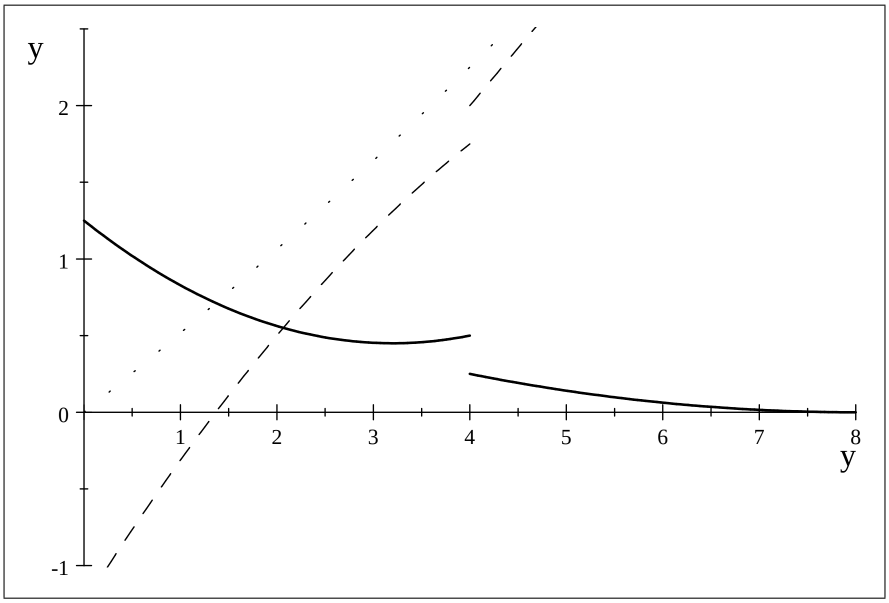
$$\widehat{f}(y_F^*) = -\frac{\pi_{11} - \pi_{00}}{\pi_{11} - \pi_{01}}\frac{\beta}{\beta-1}I_F + I_L + \left(\frac{y_F^*}{y_S^*}\right)^\beta \frac{I_S}{\beta-1}$$

$$\leq -\frac{\pi_{10} - \pi_{00}}{\pi_{11} - \pi_{01}}I_F + I_L \leq 0.$$

The last inequality follows from (12). Therefore, in this case $S^*(y) - L(y) \geq 0$ on $[0, y_S^*]$ if and only if $\widehat{f}(\widehat{y}) \geq 0$.

Combining both cases, $S^*(y) - L(y) \geq 0$ on $[0, y_S^*]$ if and only if $\widehat{f}(\widehat{y}) \geq 0$, that is if and only if (13) holds. □

The proof of Proposition 4 rests on the study of a specific function, $S^*(y) - L(y)$, over the interval $[0, y_S^*]$. This function reflects the incentive to deviate unilaterally from the candidate simultaneous investment equilibrium. Part of the difficulty of the proof is due to the behavior of this function which, depending on the model parameters $(\mathbf{I}, \boldsymbol{\pi}, \beta)$, may exhibit a discontinuity, and may or may not have an interior minimum on $[0, y_S^*]$. The proof must account for all possible cases, one of which is depicted in Figure 3.2.



3.2, Simultaneous equilibrium condition: $S^*(y)$ (dots), $L(y)$ (dash) and $S^*(y) - L(y)$ (thick curve) with $\boldsymbol{\pi} = (0.5, 1, 0.25, 0.75)$, $I_L = 1.25$, $I_F = I_S = 1$, $\beta = 2$ and $r - \alpha = 1$.

Next, as in the infinite delay case of the previous subsection, it is natural to ask when the simultaneous investment equilibrium yields higher payoffs for the firms. In contrast with the former case, the answer here is ambiguous. The simultaneous investment equilibrium Pareto dominates the sequential investment equilibrium when profit externalities are negative, but the sequential investment equilibrium may Pareto dominate when profit externalities are positive.

Lemma 4 *If joint investment is not desirable ($\pi_{00} < \pi_{11}$), then either the sequential investment or the simultaneous investment equilibrium may be Pareto optimal:*

(i) if profit externalities are negative ($\pi_{i1} > \pi_{i0}$), the simultaneous investment equilibrium is Pareto optimal;

(ii) if profit externalities are sufficiently positive($\pi_{11} > \frac{\beta}{\beta-1}\pi_{10} - \frac{1}{\beta-1}\pi_{01}$), then if investment

externalities are very negative (ζ_S large enough), the sequential investment equilibrium is Pareto optimal.

Proof As with Lemma 3, it is necessary to sign the difference between the payoffs in the two equilibria, $S^*(y) - V^P(y)$, which has the same sign as:

$$A \equiv \left(\frac{y}{y_S^*}\right)^\beta \frac{I_S}{\beta-1} + \left(\frac{y}{y_P}\right)^\beta \frac{\left[\beta\left(\pi_{10}-\pi_{11}\right)\left(\pi_{00}-\pi_{01}\right)-\left(\pi_{11}-\pi_{01}\right)\left(\pi_{10}-\pi_{00}\right)\right]\frac{y_P}{r-\alpha}+\left(\pi_{11}-\pi_{01}\right)I_L}{\left(\beta\pi_{10}-\left(\beta-1\right)\pi_{11}-\pi_{01}\right)}. \tag{14}$$

The proof of (i) is the same as for Lemma 3. To establish (ii), suppose that $\beta > \frac{\pi_{11}-\pi_{01}}{\pi_{11}-\pi_{10}}$. Then, the second term in (14) is negative. In the absence of an analytic expression of y_P, ((14)) can be partly characterized since, $y_P \in \left(\frac{r-\alpha}{\pi_{10}-\pi_{01}}I_L, \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_{10}-\pi_{01}}I_L\right)$ (the lower bound follows directly from the expression of $L(y) - F^*(y)$, the upper bound is given by Lemma 2). Substituting these bounds in for y_P and rearranging, $A < 0$ if:

$$(\beta-1)\frac{\left(\pi_{01}-\pi_{00}\right)\left(\pi_{10}-\pi_{01}\right)^{\beta-1}}{\left(\pi_{11}-\pi_{01}\right)^\beta}\zeta_S^{\beta-1} > 1.$$

□

3.2.1 Collusion with Asymmetric Investment Triggers

If certain contracting options as side payments are allowed between parties, firms might choose coordinate on investment triggers that are not symmetric. In this section, we abstract away from investment externalities and set $I_L = I_F \equiv I$. In what follows, let $\overline{\pi} \equiv \frac{\pi_{10}+\pi_{01}}{2}$. The ex-ante industry value is

$$(L+F)(y, y_L, y_F) = 2\frac{\pi_{00}}{r-\alpha}y + \left(\frac{y}{y_L}\right)^\beta\left(\frac{2\left(\overline{\pi}-\pi_{00}\right)}{r-\alpha}y_L - I\right) + \left(\frac{y}{y_F}\right)^\beta\left(\frac{2\left(\pi_{11}-\overline{\pi}\right)}{r-\alpha}y_F - I\right).$$

This function is quasiconcave, and optimizing results in the triggers $y_L^{**} = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_{10}+\pi_{01}-2\pi_{00}}I$ and $y_F^{**} = \frac{\beta}{\beta-1}\frac{r-\alpha}{2\pi_{11}-\pi_{10}-\pi_{01}}I$. The resulting firm values, which are asymmetric, are presumed to be equalized between the firms by means of side payments. Because such side payments are necessary in order for these triggers to be sustained in equilibrium, and since the resulting value is a first-best for the industry, it seems appropriate to speak of *optimal collusion* between the firms in this context.

A first point is that this asymmetric trigger optimum exists only for a restricted set of values of the model parameters.

Lemma 5 *The optimal collusion triggers* $\{y_L^{**}, y_F^{**}\}$ *are well-defined* ($0 < y_L^{**} < y_F^{**} < \infty$) *if and only if flow profits are strictly submodular* ($\pi_{00} + \pi_{11} < \pi_{10} + \pi_{01}$), *joint investment is desirable* ($\pi_{00} < \pi_{11}$), *and* $\overline{\pi} \in (\pi_{00}, \pi_{11})$.

If the conditions for optimal collusion stated in Lemma 5 do not hold, then the optimal collusion choice may either involve simultaneous investments (in which case the conditions for optimal collusion are the same as those for coordination given elsewhere in the paper), or a single monopoly investment ($y_F^{**} = \infty$) if $\overline{\pi} \geq \pi_{11}$.[9] Also, according to the lemma, optimal collusion with asymmetric triggers necessarily arises in the finite delay case, that is when coordinating firms would jointly defer investment while still performing it in finite time.

Assuming that the conditions for optimal collusion to be well-defined hold, the next proposition gives the condition for it to Pareto-dominate the coordinated equilibrium, provided that the latter exists. This involves comparing the functions $(L + F)(y, y_L^{**}, y_F^{**})$ and $2S^*(y)$ (see (4) for the latter).

Proposition 5 *Assume that the conditions of Lemma 5 hold. Optimal collusion involves a duopoly investment at the triggers* $\{y_L^{**}, y_F^{**}\}$ *if and only if:*

$$\zeta_S > 1 + 0.5\frac{\overline{\pi} - \pi_{00}}{\pi_{11} - \overline{\pi}} + 0.5\frac{\pi_{11} - \overline{\pi}}{\overline{\pi} - \pi_{00}},$$

and simultaneous investment at y_S^* *otherwise.*

Proof Evaluating,

$$(L + F)(y, y_L^{**}, y_F^{**}) - 2S^*(y)$$
$$= \frac{I}{\beta - 1}\left[\left(\frac{y}{y_L^{**}}\right)^{\beta} + \left(\frac{y}{y_F^{**}}\right)^{\beta}\right] - \frac{2I_S}{\beta - 1}\left(\frac{y}{y_S^*}\right)^{\beta}. \quad (15)$$

This expression is positive if and only if $f(\mathbf{I}, \boldsymbol{\pi}, \beta) \equiv (2\zeta_S)^{\beta-1}\left[\left(\frac{\overline{\pi}-\pi_{00}}{\pi_{11}-\pi_{00}}\right)^{\beta} + \left(\frac{\pi_{11}-\overline{\pi}}{\pi_{11}-\pi_{00}}\right)^{\beta}\right] > 1$. First, note that $f(\mathbf{I}, \boldsymbol{\pi}, 1) = 1$. Next, $\frac{\partial f}{\partial \beta}(\mathbf{I}, \boldsymbol{\pi}, \beta) = \ln\left(2\zeta_S\frac{(\overline{\pi}-\pi_{00})(\pi_{11}-\overline{\pi})}{(\pi_{11}-\pi_{00})^2}\right) f(\mathbf{I}, \boldsymbol{\pi}, \beta)$. Therefore,

[9] In which case the condition in Proposition 5 becomes,

$$(2\zeta_S)^{\beta-1}\left(\frac{\overline{\pi} - \pi_{00}}{\pi_{11} - \pi_{00}}\right) > 1.$$

$f(\mathbf{I}, \boldsymbol{\pi}, \beta)$ is positive if and only if $2\zeta_S \frac{(\overline{\pi}-\pi_{00})(\pi_{11}-\overline{\pi})}{(\pi_{11}-\pi_{00})^2} > 1$, which yields the condition in the proposition. □

Thus, a strong congestion effect (a negative simultaneous investment externality, which arises for instance if two firms order their factories or their R&D at the same date, and such that $\zeta_S > 2$ at least) makes optimal collusion more likely, provided that it is feasible.

4 Examples

The examples of this section apply the results of Section 3 to study coordination or tacit collusion in specific applications, by further specifying the economic model generating the flow profit $\boldsymbol{\pi}$, and also by endogenizing the relative investment costs ζ_F and ζ_S.

4.1 Capacity Investment with Quantity Competition

A canonical application of preemption is to capacity investment by duopolists. In a forthcoming paper [3], Boyer, Lasserre and Moreaux study industry development with Cournot duopolists that acquire lumpy capacity units over time as inverse demand grows stochastically.[10] Specifically, firms face an inverse market demand that is of the form $Y_t D(x_1 + x_2)$, where Y_t is a stochastic multiplicative shock and x_i refers to firm output, have zero marginal production cost, and engage in quantity competition. Over time, firms engage in several rounds of lumpy capacity investment over an industry development "tree".

We consider a subcase of their model in two respects. First, assume that firms have sufficient installed capacity so that a single investment round is necessary for them to reach the Cournot equilibrium output levels. Second, suppose that inverse demand is given by $D(x_1 + x_2) = 1 - x_1 - x_2$. Let k denote the existing capacity of each firm, which is assumed to be symmetric, and δ the (lumpy) increase in capacity that results from the acquisition of another unit of the specific input. The investment cost is taken to be invariant, that is $\zeta_F = 1$.

Thus, both firms are initially capacity constrained at k, and each of them may relax the constraint by investing in one additional unit of size δ. The end of the investment game is near, in that a single round of investment remains $x^c - \delta \leq k < x^c$. Firms decide non-cooperatively (contracts are ruled out) and without commitment when to invest in an additional unit. Initially,

[10] In fact, Propositions 2 and 4 effectively extend Proposition 5 of Boyer et. al. [3] to the case where duopolists have asymmetric investment costs.

with capacity k, both firms earn $Y_t \pi_{00} = Y_t D(2k) k$. When they both have capacity $k+\delta$, they may sell x^c, so that $Y_t \pi_{11} = Y_t D(2x^c) x^c$.

In this framework, we seek conditions on the parameters k and δ under which a simultaneous equilibrium exists. Moreover, we restrict attention to the finite delay case in which joint investment is not desirable ($\pi_{00} \geq \pi_{11}$) in which the simultaneous equilibrium is compeletely characterized. This allows us to partition the $\{k, \delta\}$ parameter space, so as to obtain an exhaustive representation of those cases where the investment game has the nature of a pure coordination game.

The desirability of joint investment determines the nature of any tacit collusion equilibrium that may arise. Assuming that industry revenue is quasiconcave, the desirability of joint investment may be determined by a strightforward criterion: define $x^* < x^c$ by the condition, $D(2x^*) x^* = D(2x^c) x^c$. If industry revenue is quasiconcave, then x^* is well-defined. Then, the following proposition may be established.

Proposition 6 *Investment is (strictly) jointly desirable if and only if $k < x^*$.*

Proof

If $k < x^*$, then $\pi_{00} = D(2k) k < D(2x^*) x^*$, and by definition $D(2x^*) x^* = D(2x^c) x^c$. Since a single investment round remains ($k + \delta \geq x^c$), $D(2x^c) x^c = \pi_{11}$.$\square$

Consider first the case in which joint investment is not jointly desirable, so $k \geq x^*$ ($= \frac{1}{6}$ given the linear demand specification), so the simultaneous equilibrium involves firms abstaining from ever investing, and is fully characterized by Proposition 2. Moreover, from Propositions 1 and 3, that a preemption equilibrium exists, and that the simultaneous equilibrium, when it exists, is Pareto optimal for the firms. Moreover, the simultaneous equilibrium is the best equilibrium the firms could achieve, regardless of whether or not they may collude (Lemma 5).

To characterize simultaneous equilibrium, note first that the assumption $\pi_{00} \geq \pi_{11}$ (firms abstain from investing under tacit collusion) together with the restriction that "the end of the game is near" (a single capacity investment suffices to reach the Cournot capacity), $k + \delta \geq \frac{1}{3}$, implies a first set of constraints on $\{k, \delta\}$:

$$k \in \left[\frac{1}{6}, \frac{1}{3}\right), \delta \geq \frac{1}{3} - k. \tag{16}$$

Second, a firm that invests may or may not be capacity constrained in the Cournot game. In particular, if $\delta \geq \frac{1}{2} - \frac{3}{2}k$, then $x_i^*(k) = \frac{1-k}{2} \leq k + \delta$ and firm i is not capacity cosntrained. Otherwise, the firm is at a corner in the duopoly game and $x_i^*(k) = k + \delta$. Accordingly, the flow profits π_{ij} can be computed to be $\pi_{00} = k(1-2k)$, $\pi_{11} = \frac{1}{9}$, and either $\pi_{10} = \left(\frac{1-k}{2}\right)^2$ and $\pi_{01} = \frac{k(1-k)}{2}$ (if the firms are not capacity constrained, $\delta \geq \frac{1}{2} - \frac{3}{2}k$), or $\pi_{10} = (k+\delta)(1-2k-\delta)$ and $\pi_{01} = k(1-2k-\delta)$ (if the firms are capacity constrained, $\delta \leq \frac{1}{2} - \frac{3}{2}k$). If the firms are not capacity constrained, one has $\pi_{00} + \pi_{11} \geq \pi_{01} + \pi_{10}$ if and only if $7k^2 - 4k + \frac{5}{9} \leq 0$, that is if $k \in \left(\frac{5}{21}, \frac{1}{3}\right)$. If the firms are capacity constrained, then $\pi_{00} + \pi_{11} \geq \pi_{01} + \pi_{10}$ if and only if $P(k,\delta) = 2k^2 - k + 4k\delta + \delta^2 - \delta + \frac{1}{9} \geq 0$.

By Proposition 2, in the region of $\{k, \delta\}$ for which π_{ij} is supermodular, a simultaneous equilibrium always exists. In the region for which π_{ij} is not supermodular, as $\lim_{\beta \to 1} \overline{\zeta}(\boldsymbol{\pi}, \beta) = \infty$ and $\frac{\partial \overline{\zeta}(\boldsymbol{\pi},\beta)}{\partial \beta} < 0$ (Lemma 3), a simultaneous equilibrium arises if and only if β is low enough. The condition for simultaneous equilibrium given by Proposition 2 is then:

$$1 \leq \frac{\beta\left(k^2 - 2k + \frac{5}{9}\right)\left(2k^2 - 2k + \frac{4}{9}\right)^{\beta-1}}{(3k-1)^{2\beta}} \tag{17}$$

if firms are not capacity constrained ($\delta \geq \frac{1}{2} - \frac{3}{2}k$), and:

$$1 \leq \frac{\beta\left(-2k^2 - \delta^2 - 3k\delta + k + \delta - \frac{1}{9}\right)\left(2k^2 + k\delta - k + \frac{1}{9}\right)^{\beta-1}}{\delta^{\beta}(1-3k-\delta)^{\beta}} \tag{18}$$

if firms are capacity constrained ($\delta < \frac{1}{2} - \frac{3}{2}k$).

Figure 4.1 synthesizes these results. The subset of $\{k, \delta\}$ space for which the assumptions are satisfied, that is for which a single investment remains for each firm and tacit collusion involves abstaining from investment, is comprised of four regions. For large enough "lumpy" investment (above the dashed line), firms are not capacity constrained, wheras for lower investment they are. In the (top, right) region of the parameter space bounded on the left by $k = \frac{5}{21}$ and by $P(k,\delta) = 0$, π_{ij} is supermodular and a simultaneous eqiolibrium exists for all β. Outside of this region, a simultaneous equilibrium exists (and firms may be thought of as facing a pure coordination game with respect to the strategies {invest at y_P, never invest}) for β sufficiently low, the thresholds being those described above.

Thus, the greater the existing level of capital k, all else equal, which may be interpreted as firms having a "foothold" in the market, the more likely is it that a simultaneous equilibrium exists. On the other hand, a large investment (δ) increases the likelihood of simultaneous equilibrium only if initial capacities are sufficiently large to begin with.

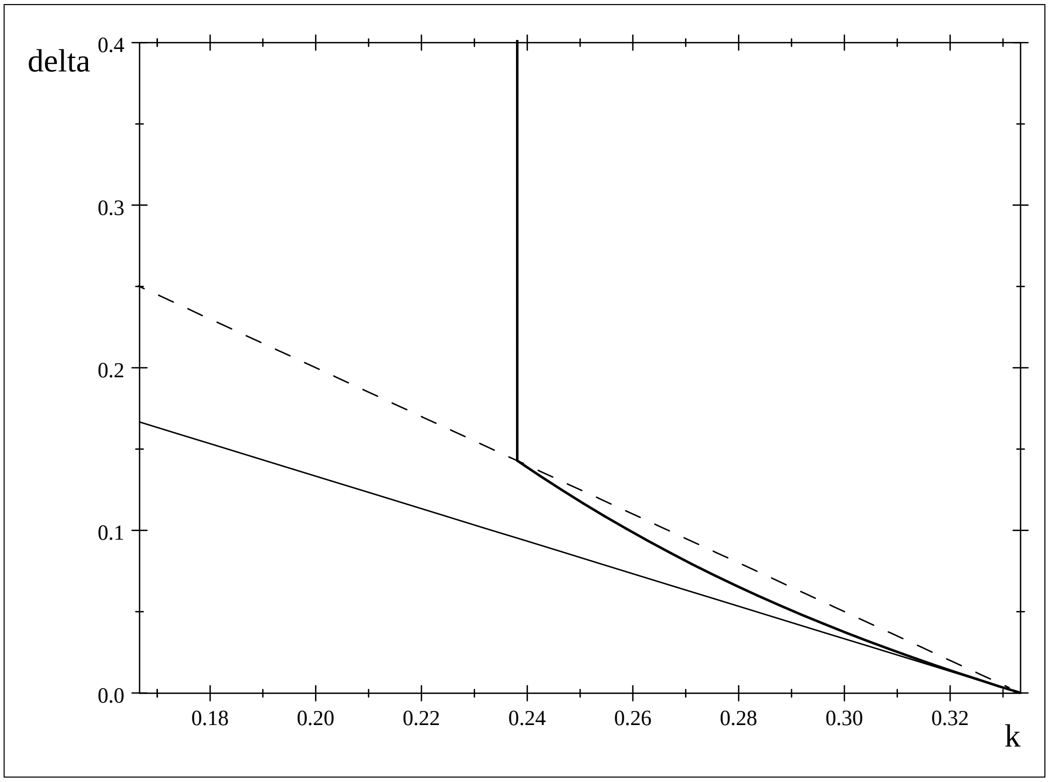


Figure 4.1: Infinite delay simultaneous equilibrium/coordination in capacity investment with linear demand and no investment externality, given initial capacity (k) and investment increment (δ).

4.2 R&D Investment with Spillovers

4.3 Endogenous Input Price and Deterrence of Collusion

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