Valuing Hydrogen-based Infrastructure Investment with Multiple Sources of Uncertainty: An application to Transportation System in Netherlands

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Abstract

The shift from a fossil-fuel to a hydrogen based transportation system requires sufficient supporting infrastructures. This paper develops a real option model to investigate the value of this investment opportunity which is able to handle the multiple uncertainties from market, political and technological aspects. The uncertain market and political uncertain factors will be transformed into a project value function which is incorporated with Geometric Brownian Motion and Jump process. Unlike the conventional jump-diffusion model, the jump in our model is designed as strictly positive to account for any favorable policy to support hydrogen fuel-cell and will only work on the drift term for a direct contribution to the underlying value. With explicit discounting of the risk of technical failure at each phase, stochastic project variation is an input of the real option framework and the sequential nature of hydrogen infrastructure investment will be interpreted as a chain of expanded call options. Moreover, we include the learning effect that will induce the cost reduction into the valuation. It appears that the early stage of infrastructure adoptions has a significant strategic value for the locked in future investment opportunities, which are dominated by the increasing power of push from technical learning, political impact and market uncertainty. However, this significance may most likely be offset due to a lower chance of investment in each stage successive of moving towards commercialization.

Key words: real options, multiple sources of uncertainty, hydrogen infrastructure investment

1. Introduction

Currently, almost all road transport is fuelled with fossil fuel sources in the Netherlands; the ever-increasing demand for transport consequently leads to more dependence on external suppliers. Additionally, various negative emissions add to global warming, which are negative to the local environment and damaging to human health (Wurster and Zittel, 2007). With the advantages of higher system efficiency and the zero tailpipe emissions, hydrogen fuel-cell stands out as a strong competitor for the future sustainable transport system (Smit et al, 2007). In spite of hydrogen being the most abundant chemical element in the universe, like electricity, hydrogen is also an energy carrier. It must be produced from a primary source and transmitted to the consumption place in order to deliver an energy service (stationary, mobile, portable). Therefore, the transition will require the establishment of a strong and reliable hydrogen fuel supply and delivery infrastructure, from production and distribution, to storage and dispensation. The series of infrastructure investments will cost billions and will take decades to complete. At the current stage, there are only few fleet projects of hydrogen fuel-cell vehicles and the related infrastructures in the world and most of them are only tested in a controlled environment. Due to the fact of technological immaturity (hydrogen on-board storage, limited range, lifetime of fuel cells) and high costs (fuel cells and sustainable hydrogen production), it is still difficult to predict whether fiscal support will be given by the Dutch government and the level of which will provided in the future. Moreover, there is significant uncertainty about the size of the market and the eventual success of the transitions even if the technology matures. All these factors still seem to act as the major barriers to enter the commercial market.

In the literature, studies of hydrogen infrastructures include identifying the optimum decisions for hydrogen transitions (Smit et al, 2006; Plotkin, 2007; Agnolucci, 2007), scenario studies (Joffe et al, 2003; Wietschel et al, 2006), simulation of refueling stations distributions (Melaina, 2003; Wurster and Zittel, 2007), economic analysis in demand, scale and network diffusion (Ogden, 1999) and cost analysis studies (Shayegan et al, 2006; Smit et al, 2006). Very few researches provide an overall valuation of the sequential investment process. The infrastructure coverage will require billions irreversible investment costs, e.g. refueling stations and hydrogen storage tanks. Given the size of the resources committed in the investments and the long investment period before realizing profits, it is necessary to employ an adequate valuation method to maximally explore the value of the investment (Abadie et al, 2008).

Traditionally, project valuation has been carried out through a simple valuation framework called Net Present Value approach, where the risk is considered undesirable. It penalizes the present value of the risky cash flows with discount factor that represents the time value of money and aversion attitude of risk. Uncertainties will thereby increase the firm's opportunity costs and raise the threshold rate of required return, which will induce investors to reject the risky projects (Trigeorgis, 1996; Cortazar, 2001). The attractiveness of option is on that it enables the investors to pay a small amount of money, in the control of profits loss, to explore the potential strategic value of the investment opportunity. As a result, investors can make more informed decisions, taking into account learning factors and managerial flexibility, which will always place the investor in a favorable position (Copeland and Keenan, 1998; Dixit and Pindyck, 1994; Trigeorgis, 2003). In summary, the superior of real option approach relies on its asymmetric structure and the way it manage the risk:

- Asymmetry between right-obligation and cost-benefit: After purchasing an option, the management has the privilege to fulfill but not necessary to do so. He may decide to exercise its right under the favorable conditions, or forgo it in that of an adverse condition. The rights and obligations related to the option are not symmetrical. Similarly, the cost of options holder pays is fixed, but through bounding the lowest possible of returns, an higher variance of returns from the underlying assets will certainly results a larger amount of future benefits, hence a greater option value. Thus real option confers large value to the investors under such valuation structure.

- Managing uncertainties to enhance the option value: More than recognizing the optimal opportunities and gaining a first mover advantage, real option could re-shape investors strategic position through managing flexibility (i.e. decision maker has the ability to defer, develop, expand, or abandon the project) to react to the changing market condition (McDonald and Siegel, 1986; Park and Herath, 2000; Copeland and Antikarov, 2001).

Dealing with uncertainty is the key to real option. It can help quantify management's ability to adapt its future plans to capitalize on favorable investment opportunities or to respond to undesirable development in a dynamic environment by cutting its losses (Andergassen and Sereno, 2009). Within the enormous real option papers, van Benthem et al (2006) are probably the only one considering its application to hydrogen infrastructure. They applied a classic model to determine the value and optimal timing of the first commercial launching through binomial lattice. The limitation of the approach is that it does not give an explicit specification of different sources of uncertainty. With the interactive among unpredictable future market, technological progress and political support, it is too weak to give just one comprehensive measurement of uncertainty. The current valuation of investments based on the option methodology assumes a continuous cash-flow generation process which is inadequate when these types of risk jointly determine the value of a new venture. Therefore, one of the main research gaps is to design a real option valuation model for the hydrogen-based transport system that capable of modeling uncertainties from market, technological and political aspects.

Other applications that share common characteristic of sequential decision making include investing in natural resources (Brennan and Schwartz, 1985; Dias, 2002; Dimitrakopoulos and Sabour, 2007), R&D (Cassimon et al, 2004; Pennings and Lint, 1996) and software developments (Chen et al, 2009). Despite investing in hydrogen infrastructure may not exactly fit into the natures, it shares the innovativeness property in a staged R&D investment and also itself as an energy infrastructure project with the payoff pattern flexible as new information may arrive at each investment stage. Referring to all the related literatures, we find several ways to deal with uncertainties those that separate the enlargement effect of market uncertainty and its opposite of private aspect (Smith & Nau, 1995; Chen et al, 2009); those consider the discontinuous arrival of information affecting the future cash flows as a jump-diffusion (Gukhal, 2004; Penning and Lint, 1997); simultaneously use multiple stochastic processes (Schwartz and Moon, 2000; Schwartz and Zozaya-Gorostiza, 2003) and papers integration with other methods, e.g. Bayesian analysis (Armstrong et al, 2004).

Our approach combines the former two approaches, but differs from the conventional jump-diffusion model in that the arrival rate of new policy information affects the value of the underlying project, instead of directly impacting the option value through the volatility. We followed the approach suggested by Cassimon et al (2010) that treats the technological uncertainties as a conditional-probability discount factor at each stage of the investment. Moreover, we incorporate the learning curve of hydrogen fuel-cell to count the value gains with the technology progress and production scale expansion as the particular importance of technical learning in measuring project value and shaping the future development opportunity. Our model is inspirited by Ansar and Sparks's work (2009) but we consider the real option context that they have focused primarily on and examine it as a multi-staged investment project where the underlying asset undergoes a stochastic process with positive jumps.

The objective of this study is to investigate the influence of market, political and technological uncertainties in hydrogen infrastructure investment through a sequential real option model. Starting from Section 2, we will introduce the investment problem and specify the choices of methodology per risk type. Then we present our model and analyze the combination of uncertainties from different sources each with its unique property in section 3. Section 4 introduces two case studies that will be evaluated from two and three sources of uncertainties respectively. Section 5 contains some general conclusions about the valuation.

2. General structure of the investment problem

We consider the commercial launching of hydrogen infrastructure investment as a multi-stage capital investment involving a sequence of real options, which starts from building the first refueling station, followed by a 10 units' expansion project, pre-commercialization and early-commercialization stages. Successfully completed these stages will trigger the commercialization. The above design of the series of market penetration phases is based on HyWays[1](2007). Generally speaking, structuring the investment in several steps enables the investor to receive more information before the final decision is made, while at the same time giving him the opportunity to stop or resume the investment at any time if it doesn't reach minimum performances, as that so large losses are avoided (Majd and Pindyck, 1987). For this project, we assume the underlying asset value (V) is equal to expected present value of the cash flows from the project. This is the value that analogs to stock price in financial option and has the variation of multiple sources of uncertainties over time. Details of its composition will be addressed in section 3. We further assume that the exercise price (I) is equivalent to the cash flows for building the infrastructure and the associated start-up costs. As indicated in Fig.1, after the first unit refueling station, introducing 10 more units is taken as the opportunity to enter a call option (C_2) with an underlying project value (V_2) and an exercise price (I_2). Moving along the transition, exercising the pre-commercial stage of investment gives the right to participate into the early-commercialization and even expand to the final stage of commercialization in our assumption. C_5 as the furthest down-stream option is a plain vanilla call option will be valued first, while C_4, C_3, C_2 and C_1 are nested in one another. This is generally suggested in the literature (Herath & Park, 2002; Shockley et al, 2002) to evaluate the sequential investment backwards. As the first real option, C_5 is only associated with the gross project value V_5 and the investment I_5. This is followed by C_4, where the option value should also include C_5, as the possible investment opportunity generated at this stage. And finally compounded to the first stage, building the 1st unit refueling station has the strategic value of all the following expansions. Comparing with the analytical solution of Compound options, such backward staged valuation allows the firm to temporatly suspends investments at a certain unfavorable time and resumes them at a later point.

[1] HyWays is an integrated project to develop the European Hydrogen Energy Roadmap, cofunded by research institutes, industry and by the European Commission.

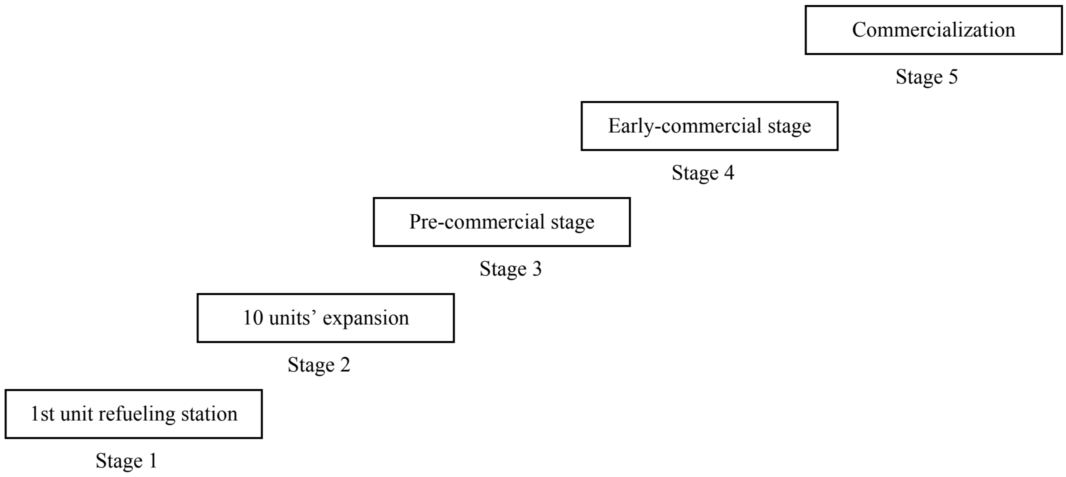

Figure 1. The multi-stage investment

2.1 Recognizing and separating uncertainties

Real option studies are usually written in a continuous time framework for the underlying dynamics. However, its application in new ventures is subject to several, qualitatively different sources of uncertainties. The uniqueness in pricing real option pricing is that they price an option independent of the risk preference of investors, in other words, it assumes them with a risk-neutral attitude. Although real option analysis is a promising tool of formulating investment decisions in uncertain environments, it could not deal with the mix of market and private risks sufficiently in the valuation procedures (Chen et al., 2009). This is due to the fact that private risk has a negative impact on real option valuation, while an increasing market risk enhances real option value. The remedy of this conflict is to adjust the project value with the private risk before entering the option model as the input. Thereby, the first step is to identify the property of the risks included in the model.

1. *Market uncertainties* are the exogenous risks associated with acceptance from the market, which relates to the compatibility of a new technology with customers' preference (Hisschemöller and van de Kerkhof, 2006). Not only is the reduction of size and weight of current fuel-cell systems required, but to attain market acceptance it is also necessary to improve the durability and the ability of functioning even further. The willingness to switch will be an evolutional change that breaks the traditional view of gasoline-based transportation. This is the uncertainty associated with the market factors outside the control of the firm that causes marginal changes in the asset value. In general, market uncertainty has a positive effect on the project value as the higher the uncertainty, the larger amount of return it could possibly provide to the investors.

2. *Political uncertainties* play an important role during the hydrogen transition by supporting R&D and series of infrastructure expansions. This is done by means of economic instruments (e.g. carbon taxes, cap and trade); command and control policies (e.g. efficiency requirements, renewable energy requirements); codes and standards for hydrogen technologies; and public

education (Bento, 2008). The occurrence of any form of simulating policy will not only increase the value of the investment monetarily, but also increases the value by building up the confidence of investors that hydrogen transition is backed by government actions and support. Without taking accounts of its competition effects, such as its main competitor electric vehicles, the political factors will have a positive impact. However, according to the current situation, due to the current immaturity, policy makers will only step in if hydrogen fuel cell starts diffusing more quickly or unless they felt a strategic necessity to develop (Melaina, 2003; von Tunzelmann et al, 2008). In addition, from the past experience with micro-CHP and Biofuel, the possibility of unexpected change of governmental policy should also be anticipated (Meijer and Hekkert, 2007). Thereby, we will subject the investment with an uncertain future climate policy, which will be treated as an external risk factor over which the investor has no control.

3. *Technological uncertainties* here refer to the chance to successful impel the transition at each development stage from technological concerns. At current stage, there are many technical problems (e.g. hydrogen storage) are still pressing for solutions. Together with the fact that fuel-cell vehicle durability under real-world conditions has not been proven yet, the chance of technical failure and abandonment should not be ignored. Clearly, this is the risk of losses for the investors and it reduces the attractiveness of the project.

The volatile demand for the product will be abstracted through the assumption of a stochastic demand evolving over time, which is modeled by a standard Geometric Brownian Motion. The component of exogenous risk associated with the proposal and adjustments of policies will be complemented by a Poisson jump process. It represents the discontinuous arrival of new information which has more than a marginal effect on the asset value. To conclude, the general effects of uncertain market and future policy decisions will significantly and rapidly alter investors' expectations about the future project returns. The technological uncertainty that is idiosyncratic to the firm will be discounted with underlying value before entering the option valuation. Moreover, the production cost will be reduced through technical learning and will be interpreted as an extra rate of return over risk-neutral pricing. A faster technology progressing will largely increase the project value and consequently the option value of this locked-in investment opportunity.

2.2 Jump in drift or in volatility

In the literature on the jump-diffusion model, the treatment of a discrete event is taken as a random jump (Merton, 1976); the jump volatility is rolling together with that of the wiener diffusion part. The main result obtained in option theory is that an option with higher volatility, in other words higher standard deviation of returns, will result in a higher option value and consequently a higher optimal threshold to trigger the investment decision. This may be construed from the asymmetrical payoff graph as the option holder does not have to exercise the option if the underlying value turns out to be at a very low value at maturity. Identically, an increasing in drift term will also raise the option value. It keeps the distribution of expected return the same and raises the underlying value to a direct percentage. In essence, the nature of the former reveals hypothetical higher returns and the option value is boosted by the risk factor.

While, the latter specifies a definite increasing in underlying value that will lead a monotone increasing to the option value.

Considering the promotions from government as a jump in the model, it will definitely act favorably to the hydrogen fuel-cell market without competition effects. However, we question the adequacy of a direct application of jump-diffusion model into the infrastructure case study; first of all, the jump-diffusion model is a continuously jumping process that initially adopted to mimic the volatile phenomenon in financial market. E.g. a stock price might be constantly jumping up and down in one day. This is contradicting to the fact that the proposition and adoptions of policy to speed up the sustainable energy transition normally takes a certain period of time, especially involves such great amount of fiscal support need to be committed. And some of political support, e.g. tax exemption, will require years of successive executions. It will never act the same as a one-off jump in the financial market; in addition, to a large extent, political intervention relies on the premise of technological maturity even for a temporary strategic push and the appearance of which is never be random. Based on the above arguments, our model relaxes such assumption and further builds on the fact that political simulation will only impact the drift term for a direct contribution to the underlying project value.

3. Model

3.1. Assumptions

For the sake of risk-neutral pricing, the assumption of complete markets and no transactions costs hold. We assume that due to the magnitude and risk level involved, the commercial phase of the project cannot be launched before the series of previous stages is completed and there is no other correlation between each investment stage beyond technical learning. The number of stages before the commercialization may be extended to more phases. In addition, we work under the assumption that the progress made in previous steps is not lost if investment is suspended temporarily. The infrastructure configuration issue of producing and distributing hydrogen will not be considered here, together with that of the competition among alternative choice of sustainable-transport systems.

3.2. Market Uncertainty

We assume the market demand level for hydrogen fuel $X = \{X_t; t \geq 0\}$ at time t as the main source of uncertainty of the project value V_t. The stochastic power of the process is dominated by the volatility of the technological maturity. Due to nature of unpredictability, it will be described by the following geometric Brownian motion process:

$$dX_t = \mu X_t dt + \sigma X_t dW_t \quad \textbf{(1)}$$

Where μ and σ represent the growth rate and the standard deviation of the demand for hydrogen as transportation fuel. The stochastic variable dW_t follows a Wiener Process in which $dW_t \sim N(0, \sqrt{dt})$.

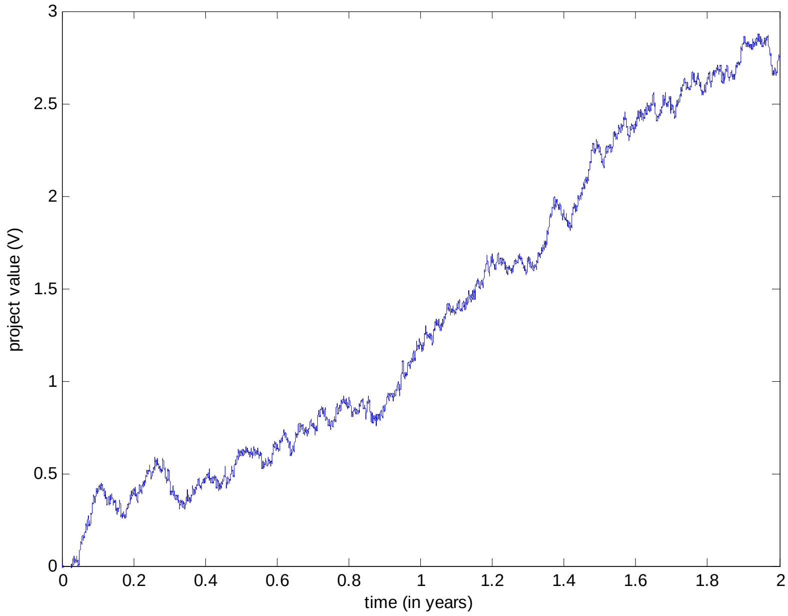

Figure 2: An example of the demand growth ($\mu = 1.4$ and $\sigma = 0.5$, 1000 steps stimulation)

Since the demand can never go negative, X_t is assumed to be lognormal distributed: e^{X_t} *(see appendix for detail).*

$$X_t = X_0 \exp(\mu t - \frac{1}{2}\sigma^2 t + \sigma W_t) \qquad (2)$$

Increasing demand will result in a scale expansion of the hydrogen production scale expansion, which will decrease the production cost. Taking account of the learning effect into account, the cumulative experience will cause unit profit $C_t = C_0 e^{b_t \delta}$ to increase with the learning speed b_t and unit cost reduction δ. Take a constant value, where $b_t = b$. The trend of project value V_t should also include the part of profit increasing $\frac{dC_t}{dt}\frac{1}{\Delta C} = b_t \delta$ and therefore will depend on both the technical learning on fuel cost and demand $V_t = f(X_t) = C_t X_t$. Here it requests a second time application of Ito'lemma:

$$dV_t = (\mu + b\delta - \frac{1}{2}\sigma^2)V_t dt + \sigma V_t dW_t \qquad (3)$$

This learning rate b_t may also be taken as the increasing rate of market penetration. Moreover, the underlying derivative (equation 3) will experience a conversion from physical measure to risk-neutral measure for the purpose of option pricing[2] in the following steps: We first define an "extra" rate of return, market price of risk ε, per unit of risk above the short term interest rate, as measured by the volatility: $\mu = r + \varepsilon\sigma - \eta$. Furthermore, the dividend rate η is considered as the opportunity loss in the expected rate of return from holding the option to complete rather than the completed project and r is the continuously compounded risk free interest rate. Then equation (4) can be written in the form

[2] Detail of Risk-neutral valuation refer to (Bingham and Kiesel, 2004)

$$\frac{dV_t}{V_t} = (r + b\delta - \eta)dt - \frac{1}{2}\sigma^2 t + \sigma(\varepsilon dt + dW_t) \tag{4}$$

Assume and define a new random process under Q measure[3]:

$$W_t^* = W_t + \int_0^t \varepsilon_s ds \quad \text{where } dW_t^* = dW_t + \varepsilon_t dt$$

W_t^* will turn out to be a Brownian motion with respect to the risk-neutral measure.

After changing of measure, the asset process (4) we can write[4]

$$\frac{dV_t}{V_t} = (r + b\delta - \eta)dt - \frac{1}{2}\sigma^2 t + \sigma dW_t^* \tag{5}$$

Hereafter, we will work with the risk neutral measure Q. A stochastic process is a sequence of probability distributions that gives the transition likelihood of future values. It adjusts the probabilities of future outcomes so that they can be incorporated in the effects of risk, which is identical to taking the expectation and further discounting risk factor in economics under risk-neutral condition (Ronn, 2002). Note that risk neutral valuation is based on financial option in a complete market; however, this is hardly the case for real investments. Therefore, the value obtained may be regarded as plausible economic valuations and not as strict non-arbitrage prices. The above dV_t formula shows that the project value therefore also follows the stochastic process with $r + b\delta$ as the expected growth rate of the project return, η is the convenience yield and σ as the volatility rate of the project value. Taking account of the logarithmic Geometric Brownian motion over market uncertainty, the project value dynamic will becomes

$$V_t = V_0 \exp[(r + b\delta - \eta)t - \frac{1}{2}\sigma^2 t + \sigma dW_t^*] \tag{6}$$

3.3. Political Uncertainty

The effect of market uncertainties on the project value is likely to involve as a continuous process, while the emergence of government policy will change the project value in a discrete contingency. According to the arguments in the previous section, the political jump will only possibly occur in the drift term; the stochastic dynamics of project value is still powered by the market demand σdW_t^*. We start with a Poisson process $\{N_t\}_{0 \le t < \infty}$, by definition, the probability to count n events ($n \ge 0$) with jump intensity parameter λ in the time interval $[s, t+s]$:

$$\mathrm{P}[N_{t+s} - N_s = n] = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \tag{7}$$

Here we define the jump q_t as an exponential compound Poisson process:

$$q_t = \exp[\alpha N_t - \lambda t(e^\alpha - 1)] \tag{8}$$

[3] This process of change of measure might be interpreted as transition of investor risk preference for the sake of a valuation in the complete market.
[4] This transformation may either through forming a martingale to apply Girsanov Theorem.

Due to the exponential setting, q_t is restricted to be non-negative. If we define $\phi = e^{\alpha} - 1$, the parameter ϕ can be interpreted as the percentage jump in the project value whenever the Poisson process happens to jump. This can be seen if we assume a process $V_t = V_0 e^{\alpha n - \lambda \phi t}$ that only drives from the political uncertainty. Substituting into ϕ, it becomes $V_t = V_0 (1+\phi)^n e^{-\lambda \phi t}$, where n counts the number of jumps with a probability $(e^{-\lambda t} (\lambda t)^n / n!)$. The occurrence of the jump represents supporting policies issued within the option life. Any governmental support of hydrogen fuel-cell technology will add extra value to the project and lead a proportional ϕV increase. Therefore, the project value V is following the dynamics of a combination of Geometric Brownian motion (equation (5)) and Jump q_t:

$$\frac{dV_t}{V_t} = (r + b\delta)dt - \frac{1}{2}\sigma^2 t + \sigma_t dW_t^* + dq_t \tag{9}$$

The level of uncertainty of the project is measured by the volatility term σ. The potential variance of the expected return is akin to the volatility.

3.4. Technological uncertainty

Investing in hydrogen infrastructure includes many stages and at every stage there is a possibility that the project may be discontinued due to catastrophic events. In option calculation, the treatment of a project failure is to compound the extra technological risk into a higher volatility. Since the option pricing is an increasing function of the uncertainty, the option value on a project with a higher probability of failure is more valuable than a project with lower technical risk (Merton, 1973). With the practical consideration, technological risks are more appropriate to be handled explicitly outside the real option model. Details of the approach may refer to Cassimon et al, (2010). Thereby, the project value will be treated with conditional probability of success at each stage first $V_i * P_i !$ and take it as the input of real option model.

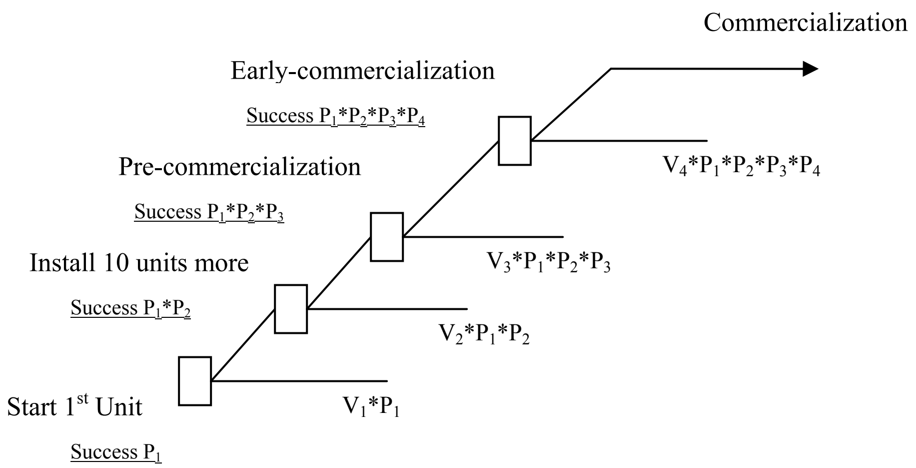

Figure 3. Project value with technical uncertainty adjustment

When there is no market uncertainty ($\sigma = 0$), no political uncertainty ($q = 0$), no learning uncertainty ($b_t\delta = b\delta$) and even no technical failure ($P_i = 100\%$), the project value in equation (10) will leave the deterministic drift term: $dV_t = (r + b\delta)V_t dt$ and the value of this investment at the end is $V_T = V_0 e^{(r+b\delta)T}$.

3.5. Option pricing

By definition, the value of the option at maturity T, worths $C = \max[V_T - I_T]$. Impact of the contingencies on the expected benefits is independent of the wiener process, that is $E[dW_t^* dq_t] = 0$.

$$V_t = V_0(1+\phi)^n \exp[(r + b\delta - \eta - \lambda\phi)t - \frac{1}{2}\sigma^2 t + \sigma dW_t] \quad (10)$$

Underlying variable V_t is log-normal distributed with expected value
$E[V_t] = e^{(r+b\delta-\eta-\lambda\phi)t} V_0(1+\phi)^n$ and variance $V[V_t] = e^{2(r+b\delta-\eta-\lambda\phi)t}[V_0(1+\phi)]^{2n}(e^{\sigma^2 t} - 1)$.

A European call option C_0 to expand worth (derivation is shown in Appendix):

$$C_0 = \sum_{n=0}^{\infty} \frac{(\lambda T)^n}{n!}[e^{(b\delta-\eta-\lambda(1+\phi))T} V_0(1+\phi)^n N(d_1) - e^{-(r+\lambda)T} I_T N(d_2)] \quad (11)$$

$$d_1 = \frac{\ln[\frac{V_0(1+\phi)^n}{I_T}] + (r + \frac{1}{2}\sigma^2 + b_t\delta - \eta - \lambda\phi)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln[\frac{V_0(1+\phi)^n}{I_T}] + (r - \frac{1}{2}\sigma^2 + b_t\delta - \eta - \lambda\phi)T}{\sigma\sqrt{T}}$$

By definition, a European option only counts the value to excise in a designated period just prior to expiration, while an American option examine the opportunity to allow the option holder to exercise at any time before it expires. In spite of the fact that the European option provides a fast computing analytical solution, valuation through the American option could extend the decision making from a point of time to a time horizon, which is closer to the practical situation. Moreover, it also increases the value of the investment as the American option comprises an early exercising value over the European option[5].

Valuing American option is to divide the option life into several discrete time intervals, for which investors can decide to exercise at any of these time prior to the final expiration. The following simulation is based on Longstaff regression algorithm (Longstaff & Schwartz, 2001; Lee et al, 2008). In mathematics term, an expected payoff of an American option is expressed as $C = \sup_{\tau \in t} E[V_\tau - I]$, where τ is the exercise moment in between $t_1 < t_2 \ldots < t_k$. The option holder may choose to exercise

[5] When there is no dividend payout, American and European call options have identical worth since it is not optimal to exercise a call option early

immediately to realize $C(V_t)$ or continue holding the option and revisit the exercise decision at the next exercise time. The value of holding is called continuation value $Q_{i,j}(V_{ij})$, which is defined as the conditional expectation of the immediate exercise:

$$Q_{i,j}(V_{ij}) = E[C_{i+1,j}(V_{i+1,j}) \big| V_{ij}] \tag{12}$$

One of the most important steps is to simulate the underlying paths *V* in equation (10). Let V_{ij} denotes the state of the Markov chain of *i*th exercise opportunity at *j*th jump and simulation for *i=1,…, k,* and *j=1,…, n*. Here, we assume that there are k exercise opportunities and therefore simulate $V_0, V_1, ..., V_k$ independent Markov Chains, This underlying dynamics includes a Wiener processes with $N(0, \sigma^2(t_i - t_{i-1})$, the Poisson part and the estimation of learning parameter $b_t\delta$. The duplication of the jump is to simulate the total number of jumps n from the Poisson distribution with positive random jump sizes ϕ_i and mean k. According to the independent uniformly distributed random variables U_i on the interval [0, t_k], if $U_i < \lambda\Delta t_i$, a jump occurs and jump size is drawn from the probability density $f(\phi_i)$. When the number of jump has been limited to one $(n = 1)$, the discrete trajectory is approximate as follows:

$$V_{t_i} = V_0 \cdot (1 + \phi_i \cdot 1_{U_i < \lambda\Delta t_i}) \cdot \exp[(b\delta + r - \eta_V - \lambda f(\phi_i))\Delta t_i + \sigma N(0,1)\sqrt{\Delta t_i}]$$

Extend the times of jump:

$$V_{t_{ij}} = V_0 \cdot (1 + f(\phi_{ij}) \cdot 1_{U_{i \le} \lambda\Delta t_i})^j \cdot \exp[(b\delta + r - \eta_V - \lambda f(\phi_{ij}))\Delta t_i + \sigma N(0,1)\sqrt{\Delta t_i}]$$

The final step is using the regression approach (Lee et al, 2008), in which the continuation value $Q_{ij}(V_{ij})$ can be represented by a linear combination on basis of the function ψ and the regression coefficient β of the current state V_{ij}:

$$Q_{i,j}(V_{ij}) = E[C_{i+1,j}(V_{i+1,j}) \big| V_{ij}] = [(\psi_1(V_{ij}), ..., \psi_k(V_{ij})](\beta_{i1}, ..., \beta_{ik})$$

The expected cash flow from the continuation of each sample path at any given time step is calculated and compared with the current payoff. The algorithm of exercising the option is when the current payoff is greater than the expected payoff from continuation. For the purpose of computational efficiency, only in-the-money paths after simulation will be selected and further considered into option pricing. Calculate the option value at the terminal node and estimate the option value by backward induction for *i=k-1,…, 1.* Determine the option value as the maximum of continuation value and immediate exercise value. The initial option value is estimated as $C_0 = \frac{1}{n}\sum_{j=1}^{n} C_{1,j}$.

4. Case study

Under the CUTE (Clean Urban Transport for Europe) project, Amsterdam has built its first hydrogen refueling station to support three fuel-cell city buses. As a two-year trial project, governors want to find out whether the whole chain of production, distribution and use of hydrogen will live up to their expectations[6]. It can be seen as the first step of the sequential investments; if the testing period results meet the requirement and there seems to be further market potential, the government or potential investors may consider expanding the investment. The first unit of refueling station in Amsterdam uses on-site Hydrogen production, which is based on IMET pressurized electrolysis technology. It has a net capacity 60 Nm^3/h with 60% efficiency and a lifetime of 20 years[7]. The initial cost is €655200 and the overall maintenance cost amounts to 3% of the overall equipment cost annually ($655200*3\%=19656$ per year). It can generate approximately 120 kg hydrogen per day with a utilization of 95% to fully support the fueling needs for three fuel-cell buses ($120*365*95\%=41610$ kg/year). The average production cost is about €8 per kg. To remain competitive with conventional vehicle[8], we assume that it can receive constant revenue of €10 if the hydrogen produced would be sold on the market without considering tax issue. The required rate of return (r_i) is assume to be 10%. Considering the probability of success at this stage is $P_1 = 90\%$ and the Net Present Value of installing the 1st unit refueling station is calculated through the formula:

$$NPV = (\sum_{t=1}^{T} \frac{\pi_t}{(1+r_i)^t}(P_i | P_{i-1}) - I_0)$$

$$= -655200 + [\frac{41610*(10-8)-19656}{1+0.1} + \frac{41610*(10-8)-19656}{(1+0.1)^2} + ... + \frac{41610*(10-8)-19656}{(1+0.1)^{20}}]*90\%$$

= €-152451

The above cost structure clearly indicates the investment will be unprofitable. However, in the following section we will demonstrate how to apply our model to more accurately assess a project's value through defining the underlying variables of a project and its potential value creation.

Suppose that the first trial has been completed successfully and that the future market of hydrogen-based transport appears sufficiently promising. Then the plan of building another 10 units of refueling stations may be initiated. The investors may consider entering a contract rather than carry out the investment right away. In that case, the investment opportunity of installing 10 additional units of refueling stations (stage 2) can be seen as an expand option. It provides the right but not the obligation to build another 10 units of refueling stations within the option life ($T = 5$ years) and will only be exercised when the project value is attractive enough. With the previous knowledge from first stage, we assume 10 units of refueling station will cost 90% of the 1st unit and the successful completion of this stage is based on the probability 95% conditional the first stage, which is the cumulative probability 0.86=0.9*0.95.

[6] www.global-hydrogen-bus-platform.com
[7] The calculation is based on data from CUTE Bus Demo[7] supported by GVB and Shell hydrogen.
[8] European objective by 2020 inscribed in the "Snapshot 2020" of the European Hydrogen and Fuel Cell Technology Platform (HFP). 1kg H2 is enough for 100 km driving. Gasoline costs approximately 10€/100km.

The overall value of the first stage investment not only includes the direct net present value, but also the inherent option value to expand to the next stage. Thereby, the contractual expand opportunity is equivalent to a call option with underlying value, that is the present value of the cumulative inflows ($V_0 = 7.9$ M), strike price ($I_T = 5.9$ M), option life ($T = 5$ years), annualized standard deviation of return ($\sigma = 30\%$). We further assume $n = 1$ that indicates that every stage there is 30% chance ($\lambda = 0.3$) that one policy will appear and bring additional $\phi V (\phi = 0.2)$ value to the investors. With respect to the learning on production cost, we take the technical learning speed $b = 0.5$ here[9]. The calculation is based on the direct application of pricing formula in equation (11) and figures in table 1&2.

This section will analyze whether after the completion of the second stage, it is worthwhile to move to the following investment phases as proposed by the model in section 3. In summary, the whole valuation includes two parts with reverse chronological valuation order. Project return V_i (Table 1: column 2) will be calculated through $V = \sum_t P_t \cdot X_t$ where $P_t \equiv$ net profit of hydrogen fuel at time t and $X_t \equiv$ yearly quantity of hydrogen demand. The first round starts from that of V need to be adjusted with technological risks at each stage. Note that here V_i equals V_0 as equation (10) at phase i, it is the initial estimated value without any uncertainty. As shown in Figure 3, a commercialization can only be realized conditional upon successful completion of the four pilot stages before. Using data from Table 1, there is a 95% likelihood that the investors will actually enter the final commercialization stage upon successful of the early-commercialization. The probability of ultimately moving to the market needs to be adjusted by the cumulative probability of success of all the optional previous stages that is 0.9*0.95*0.5*0.4*0.95 = 0.16. If one still needs a Net Present Value calculation as a benchmark together with real option, NPV one also needs to discount these by the cumulative probability of success in the previous stages $\sum_{i=1}^{n} NPV_i * P_i!$.

Stage i	Option life T (years)	Gross Project Value V_i	Investment I_T	Net Present Value NPV	Conditional Probability of Success P_i	Cumulative Probability $P_i!$
1: 1st Unit	0	0.54	0.58	-0.15	0.9	0.9
2: 10 Units	5	7.9	5.9	-1.13	0.95	0.86
3: Pre-commercialization	10	100	130	-27.3	0.5	0.43
4: Early-commercialization	10	300	360	-54.5	0.4	0.17
5: Commercialization	10	500	300	181.8	0.95	0.16

Table.1 Input data of multiple stage investments unit: M€

[9] HyWays (2009)

The second round, in contrast, involves option pricing and we need to work backwards from the furthest down-stream option to the first stage. We assume the second stage will take 5 years and the stages from pre-commercialization will take 10 years each. The option life is estimated based on the expected time to build the infrastructure and the time felt to enter the next phase. The valuation first deal with the furthest down-stream option C_4, then to the upstream real options C_3, C_2 C_1 and finally the compound hydrogen infrastructure option. The present value of the initial estimation inflows from commercialization would be V_5 = € 500*0.16=80M, which is the value of the underlying investment on which an option is purchased. The actual cash flow (Estimated Project Value at option maturity) V_5 will evolve as stochastically as V_t in equation (12) taking into consideration of both market and political uncertainty. It requires the launch cost of I_5=€ 300M as the exercise price of the option. The furthest down-stream option, C_5 is a plain option, a 5 years European option $C_5(V_5) = \max[P!_5 V_5 - P_5 ! I_5), 0]$ worth €33.9 Million with 50% volatility. Moving backwards to the Early-commercialization stage, the investors face a choice to exercise an option with 40% chances to result in the commercialization worth €33.9 M that requires € 360M investments. This option will be evaluated as $C_4(V_4) = \max[P_4 ! V_4 + C_5 - P_4 ! I_4, 0] = \max[84.9 - 61.2, 0] =$ €26.7 Million. Further rolling backwards, the first-unit refueling station has an option value $C_1(V_1) = \max[P_1 V_1 + C_2 - P_1 I_1, 0] =$ € 13.68 Million.

Model Parameter	Estimated Value
$\lambda(\phi = 0.2, n = 1)$	0.3
σ	0.5
$b(\delta = 1.02)$	0.5
η	0.018
r	0.03

Table.2 Parameter estimation

4.3. Sensitivity of parameters

First we analyze the consequences of changing the relative weight of λ and we find that there is considerable difference in its value in comparative performance. This is not particularly surprising as for a small λT almost all of the value follows the no-jump dynamics. Whilst with λT large only a small fraction of paths will have no jumps. As equation (11) states, the option value $C(V)$ will increase as ϕ raise. Figure 4(a) shows that a higher value of ϕ will increase the expected project return V which consequently raises the option value $C(V)$. Clearly, the number of jumps that represents the degree of policy support has a stronger impact on the option value. The option value is very sensitive to the parameter n .

The effect of an increasing of technological learning (or market penetration) on option value is depicted in Figure 5. Since the learning is increasing in V and an increase in total expected return increases the value of an option. The reason is that the option value is high if large leaps in productivity improvement or strong cost reductions can be expected. The uncertainty associated with the technological progress of renewable energy technologies leads to a postponement of investment in a real options model.

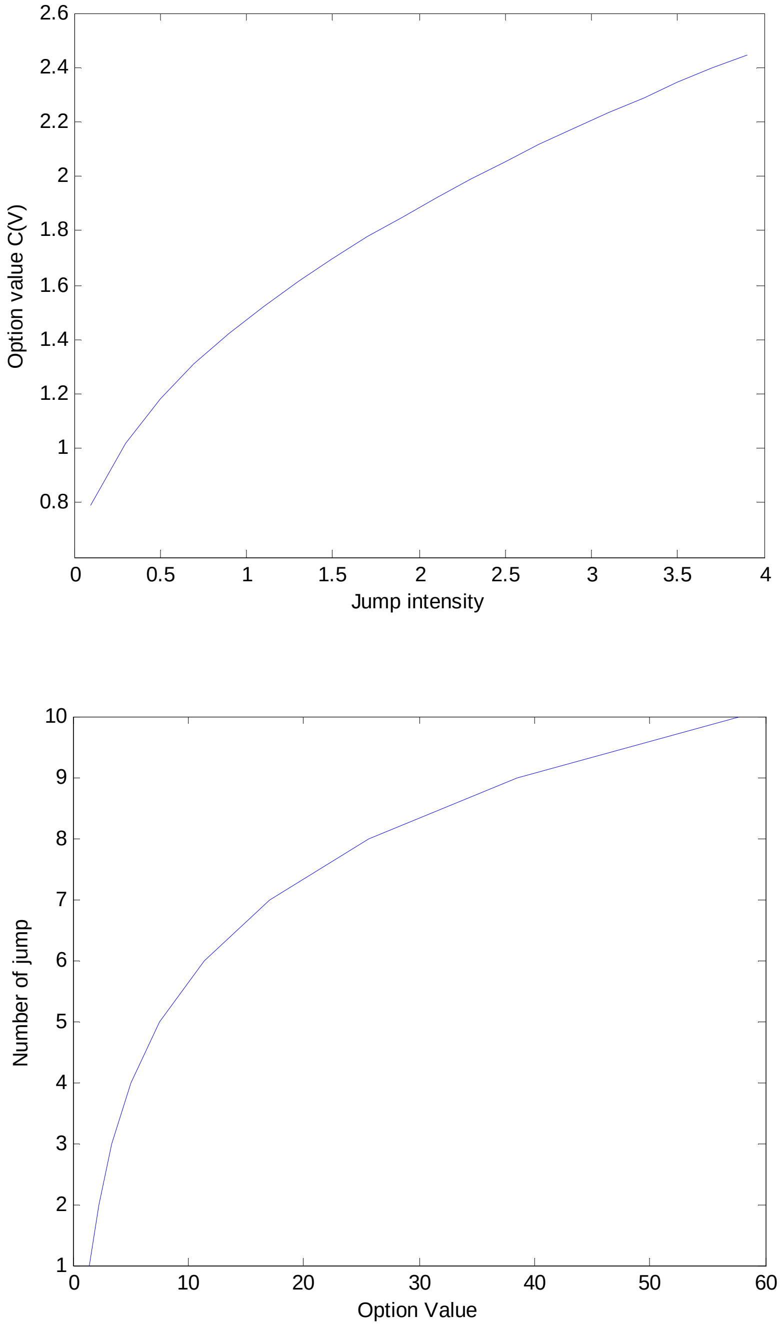

Figure 4: Jump intensity ϕ and number of jumps n with option value

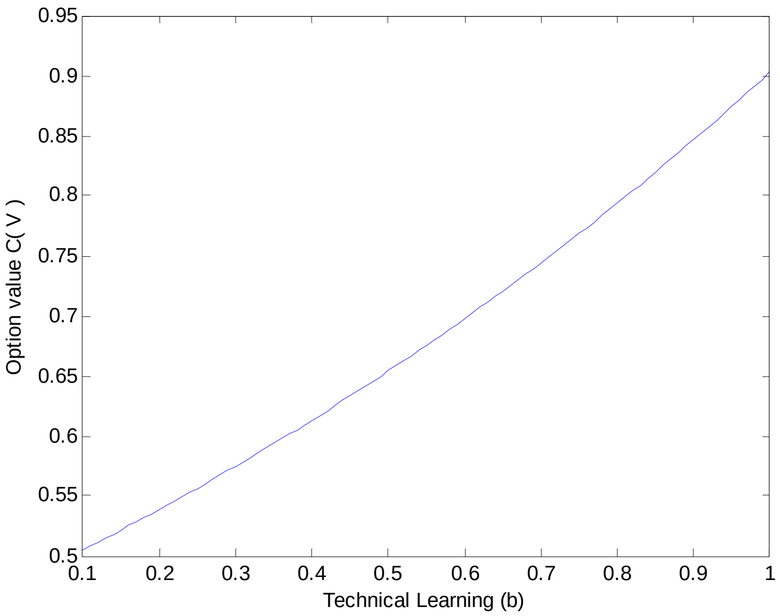

Figure 5: Learning speed b with option value

Although the value of the investment opportunity is increasing in both ϕ (fix $\lambda = 0.3$) and b, it is much more sensitive to the former. Technology diffusion in hydrogen fuel-cell is an inherently slow process, especially because such an immature technology also requires major infrastructure adaptations. Its complexity integrates with uncertainties of market acceptance that will be affected by a wide range of institutional, social and economic factors which are abstracted into a stochastic real option dynamics. What cannot be ignored is environment policy, which is certainly one of the major driving forces affecting the rate of diffusion of clean energy technologies through environment regulation aids the market pull and publicly funded R&D supports.

By rolling backwards to determine the option value, we find that the starting stages of hydrogen infrastructure investment have considerable strategic value due to the locking in of future investment opportunities in spite of its negative direct profits. While, on the other hand, this significance will be offset for a lower chance of succeeding moving into each stage towards commercialization. The investment timing is mostly determined by the speed of technological learning and level of expected market demand and most importantly the level of government support. A waiting or exercising decision is made by the tradeoff between expected technical learning, further market size and rate of cash payout to keep the investment opportunity open.

4.4. Discussion

In this paper, the stochastic underlying project is serving as an illustration of the higher uncertain phenomenon that is not related to any trading-based commodity. Since the volatility does not correlate with the jump part, our model relieves the unrealistic-looking behavior when the jumps are large. Comparing to the compound option, such staged backwards valuation allows temporarily suspended investments that are more close to the nature of the sustainable energy development. Today in the Netherlands, focus has been shifted to electric vehicles and promotions of hydrogen fuel-cell have been slowed down. While with a significant technology breakthrough, there is still a chance for hydrogen fell-cell to become a major player in the future. Hence, the flexibility of deferring to the next phase should be allowed. On the other hand, the main limitation of the model is on the assumption of a strict positive jump, which does not taking into account of any negative impact from policy makers. In addition, we do not consider the step in of policy makers with the speed of technical learning.

5. Conclusion

Investing in hydrogen infrastructure is characterized by the large up-front installation costs and multiple uncertainties interaction that result in its difficulty in predicting the value. The series investment projects have been structured into a sequential real option model; at each stage, project value has been discounted by the risk of failure and investors face a choice to determine the optimal time to exercise the option to further expand. We find that the risk of technical failure tends to undermine the expected value. On the other hand, uncertainties of market acceptance of fuel-cell vehicles commercialization and future policy regulations both have a positive effect on the option value. Direct policies of cost sharing and tax credits would accelerate the adoptions of infrastructure and forward the competitiveness of fuel-cell vehicles in the marketplace. Policy risk increases the payoffs required from the project in order to justify proceeding with the project immediately rather than waiting. But it all builds on the premise of technology breakthrough and further meets cost and performance target. It coheres with the fact that technological learning in the model has an increasing power over option value and further acts on triggering the option exercise. Waiting will increase the project payoff through technology learning while it will decrease it by opportunity loss to keep the option alive. Without supportive policies, it does not appear that the industry will actually coordinate the market and implement the infrastructure projects even though the first unit of refueling station has strategic option value of approximately €13.68 Million for the future investments.

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Appendix

Equation (2)

The main tool that we require is Ito's lemma, which says that if the random process X_t satisfies $dX_t = \mu_t dt + \sigma_t dW_t$ and if $f(X_t)$ has continuous second derivatives as a function $f(X)$ satisfies $df(X_t) = \frac{\partial f}{\partial X_t} dX_t + \frac{1}{2}\frac{\partial^2 f}{\partial X_t^2}(dX_t)^2$, where $(dX_t)^2$ is interpreted according to the rules $dt^2 = dtdW_t = dW_t dt = 0$, $(dW_t)^2 = dt$.

When $f(X_t) = \ln X_t$ $d(\ln X_t) = \frac{1}{X_t} dX_t - \frac{1}{2}\frac{1}{X_t^2}(dX_t)^2 = \frac{dX_t}{X_t} - \frac{1}{2}\frac{1}{X_t^2}(\mu_t dt + \sigma_t dW_t)^2$

$= \frac{dX_t}{X_t} - \frac{1}{2}\frac{1}{X_t^2}\sigma_t^2 X_t^2 dt$, $d(\ln X_t) = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW_t$

Thereby, $X_t = X_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t}$

The above calculation is based on (Lecture notes by Cosimano and Himonas, 2009)

Equation (11)

The European call option payoff is $C = \max(V_T - I_T, 0)$, which will be nonzero only when $V_0 \exp[\alpha n + (r + b\delta - \eta_V - \lambda\phi)T - \frac{1}{2}\sigma^2 T + \sigma\sqrt{T}\xi] - I_T > 0,\ \ \xi \sim N(0,1)$

Assume $\mu = r - \frac{1}{2}\sigma^2$

$$= V_0(1+\phi)^n \exp[(\mu + b\delta - \eta_V - \lambda\phi)T + \sigma\sqrt{T}\xi] - I_T > 0$$

Let: $z = \dfrac{\ln(\frac{I_T}{V_0(1+\phi)^n}) - (\mu + b\delta - \eta_V - \lambda\phi)T}{\sigma\sqrt{T}}$

Max ($V_0(1+\phi)^n \exp[(\mu + b\delta - \eta_V - \lambda\phi)T + \sigma\sqrt{T}\xi] - I_T$, 0) equals zero if $\xi \le z$ (the call expires out of the money) and expires in the money if $\xi > z$.

$$C_0 = e^{-rT}\int_{-\infty}^{z}(0 * \frac{1}{\sqrt{2\pi}}e^{-\frac{\xi^2}{2}})d\xi + e^{-rT}\int_{z}^{\infty}(V_0(1+\phi)^n e^{(\mu+b\delta-\eta_V-\lambda\phi)T+\sigma\sqrt{T}\xi} - I_T)\frac{1}{\sqrt{2\pi}}e^{-\frac{\xi^2}{2}}d\xi$$

$$= e^{-rT}V_0(1+\phi)^n \int_{z}^{\infty} e^{(\mu+b\delta-\eta_V-\lambda\phi)T+\sigma\sqrt{T}\xi}\frac{1}{\sqrt{2\pi}}e^{-\frac{\xi^2}{2}}d\xi - e^{-rT}I_T\int_{z}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{\xi^2}{2}}d\xi$$

$$= e^{(-r+b\delta-\eta_V-\lambda\phi)T}V_0(1+\phi)^n \int_{z}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{(\xi-\sigma\sqrt{T})^2}{2}+(\mu+\frac{1}{2}\sigma^2)T}d\xi - e^{-rT}I_T[N(-z)]$$

Substituting $x = \xi - \sigma\sqrt{T}$

$$C_0 = e^{(-r+b\delta-\eta_V-\lambda\phi)T+(\mu+\frac{1}{2}\sigma^2)T} V_0(1+\phi)^n \int_{z-\sigma\sqrt{T}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx - e^{-rT} I_T[N(-z)]$$

$$= e^{(-r+b\delta-\eta_V-\lambda\phi)T+(\mu+\frac{1}{2}\sigma^2)T} V_0(1+\phi)^n [1 - N(z-\sigma\sqrt{T})] - e^{-rT} I_T[N(-z)]$$

Because $1 - N(z) = N(-z)$ and $-\ln(\frac{I_T}{V_0(1+\phi)^n}) = \ln(\frac{V_0(1+\phi)^n}{I_T})$

$$C_0 = e^{(-r+b\delta-\eta_V-\lambda\phi)T+(\mu+\frac{1}{2}\sigma^2)T} V_0(1+\phi)^n N(-\frac{\ln(\frac{I_T}{V_0(1+\phi)^n}) - (\mu + b\delta - \eta_V - \lambda\phi)T}{\sigma\sqrt{T}} + \sigma\sqrt{T})$$

$$-e^{-rT} I_T N(-\frac{\ln(\frac{I_T}{V(1+\phi)^n}) - (\mu + b\delta - \eta_V - \lambda\phi)T}{\sigma\sqrt{T}})$$

Finally, multiplying the Possion probability, $\Pr[N_t = n] = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$ we derive:

$$C_0 = \sum_{n=0}^{\infty} \frac{\exp[-\lambda t](\lambda t)^n}{n!} e^{(b\delta-\eta_V-\lambda\phi)T} V_0(1+\phi)^n N(\frac{\ln(\frac{V_0(1+\phi)^n}{I_T} + (r - \frac{1}{2}\sigma^2 + b\delta - \eta_V - \lambda\phi)T + \sigma^2 T}{\sigma\sqrt{T}})$$

$$-e^{-rT} I_T N(\frac{\ln(\frac{V_0(1+\phi)^n}{I_T} + (r - \frac{1}{2}\sigma^2 + b\delta - \eta_V - \lambda\phi)T}{\sigma\sqrt{T}})$$

Matlab code: European option

```
function [X] = matlabcode(Callput, assetP, strike, riskFree, div, tmat, vol, Jumps, Intensity,
Learning, PoissonP)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% callput       =     Call = 1, Put = 0
% assetP        =     Underlying Asset Price
% strike        =     Strike Price of Option
% riskFree      =     Risk Free rate of interest
% div           =     Dividend Yield of Underlying
% tmat          =     Time to Maturity
% vol           =     Volatility of the Underlying
% Jumps         =     Number of Jumps per Year
% Intensity     =     Jump intensity
```

```
% Learning        =    Technical learning
% PoissonP        =    Probability of Jump
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

z = 1;
if ~callput
   z = -1;
end

dt = vol * sqrt (tmat);
df = riskFree + Learning -PoissonP*Intensity - div + 0.5 * vol ^ 2;
d1 = (log( assetP (1+Intensity)^n / strike ) + df * tmat ) / dt;
d2 = d1 - dt;
nd1 = normcdf(z * d1);
nd2 = normcdf(z * d2);
K  = poisspdf(PoissonP * tmat, Jumps);
price = z * K *(assetP(1+Intensity)^n * exp((Learning - div- PoissonP * Intensity ) * tmat) *
nd1 - strike * exp(-riskFree * tmat-PoissonP * tmat) * nd2);

X = price;
```