Evaluating pharmaceutical R&D under technical and economic uncertainty

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Abstract

This study sets up a compound option approach for evaluating pharmaceutical R&D investment projects in the presence of technical and economic uncertainties. Technical uncertainty is modeled as a Poisson jump that allows for failure and thus abandonment of the drug development. Economic uncertainty is modeled as a standard diffusion process which incorporates both up-and downward shocks. Practical application of this method is emphasized through a case analysis. We show that both uncertainties have a positive impact on the R&D option value. Moreover, from the sensitivity analysis, we find that the sensitivity of the option with respect to economic uncertainty and market introduction cost decreases when technical uncertainty increases.

Key words: Compound option; jump-diffusion process; R&D; Pharmaceutical industry.

JEL Classification: C 6; G13; G24; G30.

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1 Introduction

New drug development is a lengthy process, which is scrutinized at every stage of development by the United States Food and Drug Administration (FDA) in the USA and respective regulatory agencies in various countries. Not every compound that is tested in laboratory is eventually marketed. Only one of every 10,000 potential medicines investigated by America's research-based pharmaceutical companies makes it through the research and development pipeline and is approved for patient use by the FDA. Winning approval, on average, takes 15 years of research and development and costs over $800 Million dollars[1].

Potential new medicines pass through several crucial stages on their way from research laboratories to the pharmacy shelf. The starting point is an extensive discovery phase devoted to performing directed and highly focused research to identify and validate a therapeutic target. The development phase is focused on identifying a compound that selectively modulates the function of the target that was identified in the discovery phase. Pre-clinical studies involve tests on mammals (animal-model) and human cells. The main goals of pre-clinical studies are to understand adverse effects of the drug during clinical trials. If these tests are successful, a pharmaceutical firm applies at the public health agency (FDA or EMEA-European Medicines Agency) for the approval to starting testing in humans. The clinical testing process is known as Investigation New Drug (IND) application and includes three different phases: (a) Phase 1, (b) Phase 2, (c) Phase 3. Phase 1 involves tests on 20-100 healthy volunteers to determine safety and dosage. Phase 2 involves tests on 100–300 patient volunteers to establish the effectiveness of the drug and look for side effects. Phase 3 involves tests on 1000–5000 patient volunteers to verify effectiveness of the drug and monitor adverse reactions from long-term use. Once all three phases of the clinical trials are complete, a company analyzes all of the data. If the findings demonstrate that the potential medicine is both safe and effective, the company files a New Drug Application (NDA) with the agency for marketing approval. If the medicine is approved, or "cleared for marketing," it becomes available for patients.

The drug development process is risky in that most compounds that undergo clinical trials are abandoned without obtaining marketing approval. Reasons for research abandonment are generally grouped into 3 major categories[2]: safety, e.g. human toxicity or animal toxicity; efficacy, e.g. activity too weak or lack of efficacy; economics, e.g. commercial market too limited or insufficient return on investment. The success rate at which compounds move from phase to phase of drug development is also sometimes called *attrition rate*. Team of chemists, in general, sends an average of 10,000 new chemical entities to the pre-clinical development unit for testing. Only 250 of these will pass the criteria of activity and lack of toxic side effects which are set by the study team. Of the 250 for which IND applications are submitted to FDA, about 30 will successfully

[1] Source: Pharmaceutical Research and Manufacturers of America (PhRMA): http://www.phrma.org/innovation/.

[2] See DiMasi (2001) for further details.

complete Phase 1 trials and go on to Phase 2; about 5 of the original 250 will complete Phase 2 and go to Phase 3; and 3 to 30 of the original 250 will succeed after Phase 3. Sometimes compounds are to be dropped off during regulatory approval process[3]. A feature that is fairly unique to the pharmaceutical firm's investment project is the fact that, in general, the complete value of a project is lost upon the failure of a laboratory test. Accordingly, the failure of one of the stages results in overall project termination. Figure 1 shows a typical development path of a new medicine.

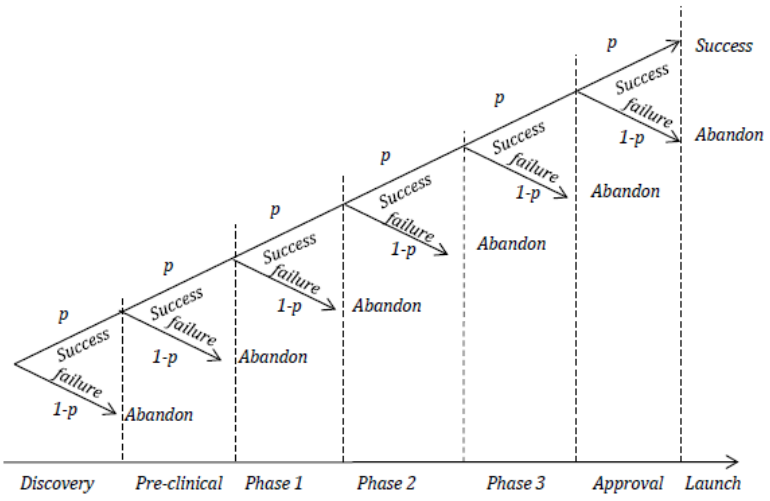

FIGURE 1: typical development path of a new medicine

In this paper, we tackle the problem of valuing pharmaceutical firm's R&D investment projects that are subject to technological uncertainty, i.e. in which the drug development activities carry a risk of failure, and where an activity's failure results in the project's overall failure. The main goal is to model the technical risk of failure as a Poisson-type jump process, in which if a drug candidate fails (e.g. is revealed not to have the desired properties or because of harmful side effects), during the drug development process, the project is abandoned, as its value falls to zero. On the other hand, if the drug candidate successfully passes laboratory and clinical tests, the company can proceed with market launch. Uncertainty related to the success/failure of R&D activities is the major concern for R&D managers in the pharmaceutical industry. If the R&D activity is unsuccessful, indeed, there is no product to commercialize. In addition to technical risks, the potential drug candidates are also exposed to a significant

[3] Source: http://www.innovation.org/index.cfm/nonav/Inside_R_&_D#link8.

amount of economic uncertainty, which is a function of factors exogenous to the project, such as general market conditions. This source of uncertainty is modeled as a standard diffusion process which incorporates both up-and downward shocks. It often appears difficult in valuing investment projects to estimate economic uncertainty of the project value, while technical risk seems easier to estimate. Furthermore, data for market introduction costs are usually not readily available. We show that, in the presence of technical risk, both parameters in our model have a lower impact on the value of the investment project as compared to the model that does not account for technical risk.

The paper is structured as follows. Section 2 provides an overview of the related economic literature. Section 3 describes the setup of the model and derives a closed-form expression for valuing compound R&D options when the underlying process follows a Poisson jump-diffusion, with the risk of complete project failure. Section 4 provides an application of this model through a case analysis. In the simulations, we analyze a pharmaceutical R&D project with the risk of failure in all R&D stages and the same absent the risk of failure to determine the value of a compound R&D option. Detailed sensitivity analyses are shown as to deepen our understanding of the determinants of the compound R&D option value. Finally, section 5 concludes the paper.

2 Literature review

Risk and flexibility involved in research and development activities make valuation of pharmaceutical firm's investment projects a non-trivial task. Traditional valuation techniques such as those using a standard discounted cash flows (DCF) analysis are of limited value in this context. Because of a high level of uncertainty, the real options valuation (ROV) is a significant alternative for valuing pharmaceutical R&D investments, but additional elements are required for a reasonable value to be obtained. In fact, the pharmaceutical firm's investment project may be interpreted as an option to expand to other (follow-on) projects. In particular, provided that the immediate "investment project" (i.e. drug discovery) succeeds, it may be opportune to start a second "project" (pre-clinical testing). Similarly, if this second "project" is successful, it may be convenient to enter a third "project" (Phase 1 clinical trials). Moreover, if this "project" is started, it may be opportune to enter a fourth "project" (Phase 2) and so on until the ultimate "project" (market launch). From a corporate finance standpoint, this is an obvious flexibility. Since management has the right to run the project actively, this flexibility has value. From a ROV standpoint, this flexibility has a form of a compound call option. Essentially, compound options are combinations of options, where an exercise of an option opens up another option. Compound options have been extensively used in the finance literature to evaluate sequential investment opportunities. Geske (1979) shows that risky securities with sequential payouts can be valued as compound options. Carr (1988) analyzes sequential compound options, of the form of options to acquire subsequent options to exchange an asset for another asset. Lee and

Paxson (2001) have applied Carr's compound exchange option formula to R&D investments valuation. In the real options theory, applications of compound options are commonly found in a number of industrial projects, but are especially relevant for pharmaceuticals where the project gives the real option to further research, or to start the implementation of the results. Consequently, compound option pricing has been proposed for valuing pharmaceutical R&D in the capital budgeting literature[4]. Shockley et al. (2003) adopt a multistage binomial option pricing model to compute the option value of an early-stage biotechnology investment. Cassimon et al., (2004) derive a closed-form expression for a N-nested compound option and have successfully applied it to assess the value of a NDA. These papers do not specify a clear distinction between technical and economic uncertainties[5]; they assume that uncertainty is one-dimensional by modeling the underlying value as a geometric Brownian process. An exception is Copeland and Antikarov (2001) who model two types of compound R&D options using binomial lattice methods. First, they model a two-phase R&D programme that depends on a single source of uncertainty. Second, they consider a rainbow type compound option in which the value of the underlying project is driven by two sources of uncertainty. Most importantly, they show how to separate technical and economic uncertainties and to model their effects on the project value using the quadrinomial approach (cf. chapters 10-11).

The contribution of this paper is in specifying a clear distinction between technical and economic uncertainties and in showing how they act together. Differently from Copeland and Antikarov (2001), we set up a compound real option model where information arrives both continuously and discontinuously over time. We stress that the standard compound option formula provides a naïve instrument for evaluating pharmaceutical R&D investment projects, since it does not allow one to take technological uncertainty (the success or failure of the project) into account. In our model there is a positive probability that the project fails due to the arrival of a technical failure. This probability of failure follows a Poisson distribution per unit of time. Combining a Poisson jump and a diffusion process, we are able to study a compound R&D option that allows for the possibility of abandoning the project at each development phase. The proposed Poisson jump-diffusion compound option model is applied in a practical business setting, through a case study application. Since it shows how to apply a compound option method to value a pharmaceutical R&D investment project it has also a straightforward practical use.

[4] R&D investments are modeled as simple European compound call options in this literature.

[5] In the real options theory, Pindyck (1993) is one of the first to make a distinction between the effect of technical and market uncertainty on real option value. Technical uncertainty in his paper relates to the cost to complete an investment project and can only be resolved by the firm by undertaking the investment project. Market uncertainty affects input costs and is external to what the firm does. He shows that both uncertainties have a positive impact on the option value. Our results for technical and economic uncertainties coincide with those obtained by Pindyck (1993), even though our measure of technical uncertainty is technical failure. Our measure seems more adequate as there is low uncertainty about the cost of completion, but a high uncertainty about the chance that the product is effective.

3 A Compound option model for evaluating pharmaceutical R&D investment projects

Our aim in this section is to set up a compound real option model for evaluating the pharmaceutical firm's investment project in the presence of technical risk of failure. For the purposes of clarity and illustration, we depart from a model that is as simple as possible, i.e. a call on a call option with two strike prices and two maturity dates. The basic intuition underlying this model is illustrated as follows. At time zero, beginning of the discovery phase, the pharmaceutical company has the option of developing and manufacturing a new drug by investing an amount I_2 (strike price of the compound option) at exercise date T_2. If this project is successful, the company has another option of proceeding with product commercialization by investing an amount I_1 (strike price of the option) at exercise date T_1, $T_2 < T_1$[6]. Figure 2 shows the compound R&D option structure.

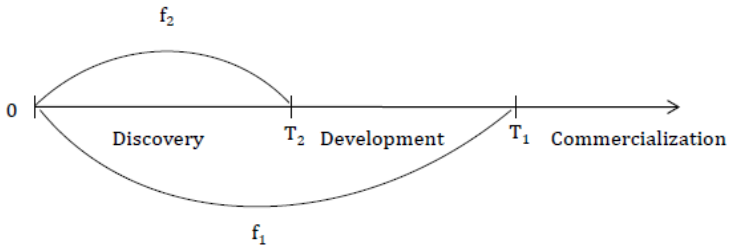

FIGURE 2: compound R&D option structure

[6] The value of a compound option, without technical risk of failure, has been derived by Geske (1979):

$$f_2(V,0) = V_0 \aleph_2(a_2, a_1; \rho) - e^{-rT_1} I_1 \aleph_2(b_2, b_1; \rho) - e^{-rT_2} I_2 \aleph_1(b_2),$$

where $f_2(V,0)$ is the value of the compound option; V_0 is the current value of the underlying asset; I_1 is the exercise price of the underlying option; I_2 is the exercise price of the compound option; T_1 is the exercise date of the option; T_2 is the exercise date of the compound option; r is the risk-free interest rate; $\aleph_2(x, y; \rho)$ is the standard bivariate normal distribution function evaluated at x and y with correlation coefficient of $\rho = \sqrt{T_2/T_1}$; $a_1 = \left(\ln\left(\frac{V_0}{I_1}\right) + \left(r + \frac{1}{2}\sigma^2\right)T_1\right) \Big/ \left(\sigma\sqrt{T_1}\right)$; $a_2 = \left(\ln\left(\frac{V_0}{v_2^*}\right) + \left(r + \frac{1}{2}\sigma^2\right)T_2\right) \Big/ \left(\sigma\sqrt{T_2}\right)$; $b_1 = a_1 - \sigma\sqrt{T_1}$; $b_2 = a_2 - \sigma\sqrt{T_2}$; σ is the volatility of the underlying asset and v_2^* is the critical value of the asset such as the underlying option is at the money at time T_2. The expression $V_0 \aleph_2(a_2, a_1; \rho)$ can be interpreted as the present value of receiving the asset at expiration of the option, contingent upon both the compound option and the underlying option finishing in the money; the expression $e^{-rT_1} I_1 \aleph_2(b_2, b_1; \rho)$ can be interpreted as the present value of paying the exercise price I_1 in that event, and finally, $e^{-rT_2} I_2 \aleph_1(b_2)$ can be interpreted as the present value of paying the exercise price I_2 contingent upon the compound option finishing in the money. See Lajeri-Chaherli (2002) for further details.

The underlying state variable in our model is the present value of all future cash flows (hereafter referred to as project value) received at time t, which is assumed to follow a mixed jump-diffusion process. Numerous previous studies have set up models of R&D investments valuation using a Poisson-type jump process[7], in which the value of the underlying project may undergo finitely many jumps in every time interval and the size of the jumps is stochastic. Examples that are well-modeled as Poisson processes include the arrivals of competitors, litigations of patent rights, innovations in technology and important breakthroughs. For the pharmaceutical firm's R&D investment project, we assume that the arrival of important information is modeled as a Poisson jump process, with only one jump in every time interval. In our model, the jump represents the possibility of a complete ruin of the project underlying a drug development process. Therefore the size of the jump is assumed to be constant and non-stochastic.

The specification of the project value dynamics in the presence of technical risk of failure is given in the following section. For the sake of clarity, we discuss the construction of the simplified jump-diffusion model starting from a more general jump-diffusion model.

3.1 Project value dynamics in the presence of technical risk of failure

The project value is uncertain during the different stages. Denoting by V_t the time $t \in [0, T_1]$ valuation of the project, we assume that V_t follows a log-normal jump-diffusion process (Merton, 1976). The underlying project value as given by the jump-diffusion model has two sources of uncertainty: the diffusion risk σdz_t (typical of ordinary businesses) which incorporates both positive and negative random fluctuations, and the term dq_t which describes the arrivals of major shocks that imply an abrupt increase/reduction in V. On average, there are λt jumps in the time interval $[0, t]$, the average relative jump size is $E[Y - 1]$ and the number of jumps is independent of the size of jumps and also independent of the remaining uncertainty in the model.

For the pharmaceutical firm's investment project, it seems more appropriate to employ a simplified version of the jump-diffusion model, in which Y is non-stochastic and there is either zero or one jump in the project value in a time interval of length t. In more specific terms, we consider the jump-diffusion model:

$$V_t = V_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)t + \sigma z_t} \phi_t, \tag{1}$$

where α is the expected rate of return on the project, σ is the standard deviation of the project, z_t is a standard Brownian process, and V_0 is the current value of the underlying project. In the above equation, ϕ_t is the variable with technological uncertainty which describes the likelihood of success of the pharmaceutical R&D project. In more specific terms, this is the exponential of the product

[7] See, for example, Pennings and Lint (1997), Martzoukos and Trigeorgis (2002) and Wu and Yen (2007).

of a Poisson random variable n_t (independent of z_t) with parameter λ (> 0), which describes the likelihood of occurrence of the jump, and a deterministic component $\ln(Y)$, which describes the jump amplitude. In particular:

$$\phi_t = e^{n_t \ln(Y)}.$$

In our study, we assume that there is only possible jump ($n_t = 0$ or $n_t = 1$)[8] and, if there is a jump, the project becomes worthless. Hence,

$$\phi_t = \begin{cases} 1 \text{ with probability } e^{-\lambda t} & \{\textit{if a technical failure does not occur}\} \\ 0 \text{ with probability } 1 - e^{-\lambda t} & \{\textit{if a technical failure occurs}\} \end{cases}$$

and its expected value is $E[\phi_t] = e^{-\lambda t}$.

We will assume that the firm is risk neutral, so that replacing the expected rate of return α in (1) by $\alpha = r + \beta\sigma$, where $r \geq 0$ denotes the risk-free interest rate and β is the market price of diffusion risk, the terminal value of the project V_t can be rewritten as:

$$V_t = V_0 e^{\left(r - \frac{1}{2}\sigma^2\right)t + \sigma z_t^*} \phi_t.$$

$z^* = z + \beta t$ is a new Brownian motion process under the risk-neutral probability measure, z^* and n_t are as above independent of each other. Note that in such a context the jump is not correlated with the general movements of the economy. It represents idiosyncratic risk that can be diversified away and have a zero market price of risk in equilibrium[9].

Risk-neutrality, meaning that the weighted average of the zero-jump and one-jump current expected value of the project equals the future value of the project, is maintained by dividing the current value of the project by the expected value of the jump. Hence,

$$V_t = \frac{V_0}{e^{-\lambda t}} e^{\left(r - \frac{1}{2}\sigma^2\right)t + \sigma z_t^*} \phi_t,$$

which implies that the deterministic drift component of the process (1) is replaced by the risk-neutral drift $\left(r + \lambda - \frac{1}{2}\sigma^2\right)$, where λ is the compensation for the technical jump risk in the time interval $[0, t]$. Because rational investors would not be willing to invest in assets yielding inadequate returns, they have to be compensate for additional jump risk[10].

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[8] From the properties of Poisson processes, we have that $\Pr(n_t = i) = e^{-\lambda t} \frac{(\lambda t)^i}{i!}$, $i = 0, 1, ...$ is the probability that there are exactly i occurrences in the time interval $[0, t]$. For a drug development process, only the first jump in the time interval $[0, t]$ is relevant, therefore $i = 0$. As a result, $\Pr(n_t = 0) = e^{-\lambda t}$, is the probability that technical failure will not have occurred at time t. Hereafter, this is referred to as the *success* probability of the pharmaceutical R&D project.

[9] This means that the jump component of V is unchanged under the risk-neutral probability measure.

[10] As shown by Merton (1976) the option price is an increasing function of λ, and therefore an option on a stock that has a positive probability of complete ruin is more valuable than an option on a stock that does not.

Hereafter, R&D investment options are valued as if the project value at any future time t is conditioned on two possible scenarios, a failure occurs and does not occur[11]. In more specific terms, let us denote by $V_t|\,(n_t = 1)$, the terminal value of the project, conditioned on knowing that a technical failure occurs during the interval $[0, t]$. This can be written as:

$$\begin{aligned} V_t|\,(n_t = 1) &= V_0 e^{\left(r+\lambda-\frac{1}{2}\sigma^2\right)t+\sigma z_t^*}\phi_t\Big|\,(n_t = 1) \\ &= 0, \end{aligned} \tag{2}$$

and its expected value is:

$$E^*\left[V_t|\,(n_t = 1)\right] = 0.$$

Moreover, let us denote by $V_t|\,(n_t = 0)$, the terminal value of the project, conditioned on knowing that a technical failure does not occur during the interval $[0, t]$. This value is:

$$\begin{aligned} V_t|\,(n_t = 0) &= V_0 e^{\left(r+\lambda-\frac{1}{2}\sigma^2\right)t+\sigma z_t^*}\phi_t\Big|\,(n_t = 0) \\ &= V_0 e^{\left(r+\lambda-\frac{1}{2}\sigma^2\right)t+\sigma z_t^*}. \end{aligned} \tag{3}$$

The expected value of $V_t|\,(n_t = 0)$, is[12]:

$$E^*\left[V_t|\,(n_t = 0)\right] = V_0 e^{(r+\lambda)t}.$$

Note that we assume for simplicity that information about the success or failure of the project is revealed at the end of each stage. Consequently, each investment option will only be exercised if all the activities scheduled to finish the R&D project have a positive outcome.

3.2 Valuing a single stage option

Consider the valuation problem of a R&D-based pharmaceutical firm who, at time zero, has an option to launch a product on the market. Let T_1 be the time of the market launch of the product, when, upon bearing the commercialization cost I_1, the firm pockets the project value V_{T_1}. The project payoff at time T_1 is $\max\{V_{T_1} - I_1, 0\}$ and let $F_1(V, t)$ denote the value at time t of this simple investment opportunity. Then, if the value of V at time T_1, is greater than I_1,

[11] The trade-off for using a more realistic jump-diffusion process is that the terminal value of the project V_t is no longer log-normal because ϕ_t is not log-normal. Within this framework the probability density function of V cannot be explicitly written. This makes valuation of compound R&D options a non-trivial task. We address this problem by conditioning on the random event occurrence, and work with the conditional variable thereafter.

[12] Since $z_t^* \sim \aleph(0, \sqrt{t})$, we have that $\psi \equiv \left(-\frac{1}{2}\sigma^2 t + \sigma z_t^*\right) \sim \aleph\left(-\frac{1}{2}\sigma^2 t, \sigma\sqrt{t}\right)$ and therefore e^{ψ} is lognormally distributed with $E^*\left[e^{\psi}\right] = e^{\left(-\frac{1}{2}\sigma^2 t+\frac{1}{2}\sigma^2 t\right)} = 1$.

the product will be marketed, i.e. the option will be exercised, while for values less than I_1 it will be abandoned.

The time zero value of this investment opportunity is the expected present value of these cash flows and is given by:

$$F_1(V,0) = e^{-rT_1} E_0^* \left[\max\left\{V_{T_1} - I_1, 0\right\}\right].$$

Valuing the investment opportunity $F_1(V,0)$ from a jump-to-ruin process is straightforward. Let us define n as the number of jumps that occur in a time interval of length T_1. The occurrence of a jump decreases the project value to zero and the random variable n takes the values of zero with probability $e^{-\lambda T_1}$ or one with probability $1 - e^{-\lambda T_1}$. By conditioning on the two scenarios, we can express $F_1(V,0)$ as a weighted sum of the call option prices given that a technical failure occurs and does not occur:

$$\begin{aligned} F_1(V,0) &= e^{-rT_1} \Pr(n=1) E_0^* \left[\max\left\{V_{T_1} - I_1, 0\right\} \middle|\ n=1\right] + \\ &\quad + e^{-rT_1} \Pr(n=0) E_0^* \left[\max\left\{V_{T_1} - I_1, 0\right\} \middle|\ n=0\right]. \end{aligned} \tag{4}$$

The value of the first expectation in (4) can be found easily. By conditioning on the occurrence of a technical failure, the terminal value of the project at time T_1 can be written as $V_{T_1}|(n=1) = 0$. This can be obtained by a straightforward application of formula (2). It follows that the expected final value, $E_0^* \left[\max\left\{V_{T_1} - I_1, 0\right\} \middle|\ n=1\right]$, of the option is worthless since its payoff becomes zero.

Now, let us concentrate on the second term in (4). The terminal value of the project, V_{T_1}, conditioned on the absence of a technical failure, is[13]:

$$V_{T_1}|(n=0) = V_0 e^{\left(r+\lambda-\frac{1}{2}\sigma^2\right)T_1 + \sigma z^*_{T_1}}.$$

The problem of computing the value of a single call option with the jump-to-zero risk reduces to the standard problem of computing the value of a single call option with an increased discount rate:

$$F_1(V,0) = e^{-rT_1} e^{-\lambda T_1} E_0^* \left[\max\left\{V_0 e^{\left(r+\lambda-\frac{1}{2}\sigma^2\right)T_1 + \sigma\sqrt{T_1}\cdot u} - I_1, 0\right\}\right].$$

Hence,

$$F_1(V,0) = e^{-\lambda T_1}\left(V_0 e^{\lambda T_1} \aleph_1(h_1) - I_1 e^{-rT_1} \aleph_1(l_1)\right), \tag{5}$$

where $\aleph_1(\cdot)$ is cumulative standard normal distribution, and the terms h_1 and l_1 are given by:

$$h_1 = \frac{\ln\left(\frac{V_0}{I_1}\right) + \left(r + \lambda + \frac{1}{2}\sigma^2\right)T_1}{\sigma\sqrt{T_1}},$$

[13] This can be obtained by a straightforward application of formula (3).

$$l_1 = h_1 - \sigma\sqrt{T_1}.$$

According to (5) the formula is the same as the Black-Scholes (1973) call option formula, given that the technical failure does not occur during the lifetime of the option, weighted by the probability of no technical failure. The expression $e^{-\lambda T_1}\left(V_0 e^{\lambda T_1}\aleph_1(h_1)\right)$ can be interpreted as the present value of receiving the future cash flows contingent on the success of the project and the exercise of the option, and the expression $I_1 e^{-(r+\lambda)T_1}\aleph_1(l_1)$ can be interpreted as the present value of paying the strike price I_1 in that event.

3.3 Valuing a compound R&D option

Consider now the valuation of a compound R&D option which, at time T_2, gives the firm the right to pay I_2 to buy another option, the underlying option, that has an exercise price I_1 and exercise date T_1. At time T_1, the underlying option gives the right to launch the product. The payoff of the compound option at time T_2 is

$$F_2(V, T_2) = \max\left\{F_1(V_{T_2}, \tau_1) - I_2, 0\right\},$$

where $F_1(V_{T_2}, \tau_1)$ stands for the value at time T_2 of a simple call option with exercise price I_1 and expire date $T_1 = T_2 + \tau_1$. Therefore, if at time T_2 the value of the option is greater than the strike price I_2 the compound option will be exercised, while for values less than I_1 it will be abandoned.

The time zero value of the compound option is the expected present value of these cash flows and is given by:

$$F_2(V, 0) = e^{-rT_2} E_0^*\left[\max\left\{F_1(V_{T_2}, \tau_1) - I_2, 0\right\}\right].$$

The evaluation of this option requires conditioning on two possible scenarios, a failure occurs and does not occur, in the intervals $[0, T_2]$ and $(T_2, T_1]$. In more specific terms, let us define n_2 and n_1 as the number of jumps that occur in the intervals $[0, T_2]$ and $(T_2, T_1]$, respectively. Recall that $\tau_1 = T_1 - T_2$. Thus the random variables n_2 and n_1 are independent Poisson variates with respective probabilities $e^{-\lambda T_2}$ and $e^{-\lambda\tau_1}$ if the jump does not occur and probabilities $1 - e^{-\lambda T_2}$ and $1 - e^{-\lambda\tau_1}$ if the jump occurs.

We know that $F_1(V_{T_2}, \tau_1)$ is given by straightforward application of formula (5). Thus:

$$F_1(V_{T_2}, \tau_1) = e^{-\lambda\tau_1}\left(V_{T_2} e^{\lambda\tau_1}\aleph_1(h_1(V_{T_2}, \tau_1)) - I_1 e^{-r\tau_1}\aleph_1(l_1(V_{T_2}, \tau_1))\right),$$

where:

$$h_1(V_{T_2}, \tau_1) = \frac{\ln\left(\frac{V_{T_2}}{I_1}\right) + \left(r + \lambda + \frac{1}{2}\sigma^2\right)\tau_1}{\sigma\sqrt{\tau_1}},$$

$$l_1(V_{T_2}, \tau_1) = h_1(V_{T_2}, \tau_1) - \sigma\sqrt{\tau_1}.$$

Consequently, the value at time zero of the compound option is:

$$\begin{aligned} F_2(V,0) &= e^{-rT_2}\Pr(n_2=1)\,E_0^*\left[\max\left\{F_1(V_{T_2},\tau_1)-I_2,0\right\}\middle|\, n_2=1\right]+ \quad (6)\\ &= e^{-rT_2}\Pr(n_2=0)\,E_0^*\left[\max\left\{F_1(V_{T_2},\tau_1)-I_2,0\right\}\middle|\, n_2=0\right], \end{aligned}$$

We know that if a technical failure occurs in the interval $[0,T_2]$, the conditional value of the project at time T_2, is:

$$V_{T_2}|\,(n_2=1)=0,$$

and consequently the expected final value, $E_0^*\left[\max\left\{F_1(V_{T_2},\tau_1)-I_2,0\right\}\middle|\, n_2=1\right]$, of the compound option is worthless since its payoff becomes zero.

On the other hand:

$$V_{T_2}|\,(n_2=0)=V_0e^{\left(r+\lambda-\frac{1}{2}\sigma^2\right)T_2+\sigma z^*_{T_2}},$$

is the terminal value of the project at time T_2, conditional on knowing that a technical failure does not occur during the interval $[0,T_2]$. Therefore, the valuation problem boils down to:

$$F_2(V,0)=e^{-rT_2}e^{-\lambda T_2}E_0^*\left[\max\left\{F_1(V_{T_2},\tau_1)-I_2,0\right\}\middle|\, n_2=0\right],$$

which can be written as:

$$F_2(V,0)=e^{-rT_2}e^{-\lambda T_2}\times$$

$$\int_{u_2}^{+\infty}\left(e^{-\lambda\tau_1}\left(v_2(u)e^{\lambda\tau_1}\aleph_1\left(\hat{h}_1\right)-I_1e^{-r\tau_1}\aleph_1\left(\hat{l}_1\right)\right)-I_2\right)n(u)\,du,$$

where $n(.)$ is the normal density function, $\hat{h}_1=h_1(v_2(u),\tau_1)$, $\hat{l}_1=l_1(v_2(u),\tau_1)$, the function $v_2:\mathbb{R}\longrightarrow\mathbb{R}$ is given by:

$$v_2(u)=V_0e^{\left(r+\lambda-\frac{1}{2}\sigma^2\right)T_2+\sigma\sqrt{T_2}\cdot u},$$

and, finally, the constant u_2 is defined implicitly by the equation:

$$u_2=\inf\left\{u\in\mathbb{R}\mid F_1(v_2(u),\tau_1)\geq I_2\right\}.$$

The value at time zero of the compound option, is:

$$\begin{aligned} F_2(V,0)=e^{-\lambda T_1}&\left(V_0e^{\lambda T_1}\aleph_2(h_2,h_1;\rho)-I_1e^{-rT_1}\aleph_2(l_2,l_1;\rho)\right)+\\ &-e^{-(r+\lambda)T_2}I_2\aleph_1(l_2), \end{aligned} \quad (7)$$

where:

$$h_1 = \frac{\ln\left(\frac{V_0}{I_1}\right) + \left(r + \lambda + \frac{1}{2}\sigma^2\right) T_1}{\sigma\sqrt{T_1}}, \qquad l_1 = h_1 - \sigma\sqrt{T_1},$$

$$h_2 = \frac{\ln\left(\frac{V_0}{V_2^*}\right) + \left(r + \lambda + \frac{1}{2}\sigma^2\right) T_2}{\sigma\sqrt{T_2}}, \quad l_2 = h_2 - \sigma\sqrt{T_2},$$

Factor $\aleph_2(x, y; \rho)$ is the standard bivariate normal distribution function evaluated at x and y with correlation coefficient of $\rho = \sqrt{T_2/T_1}$, and V_2^* is the critical value of the project such that the underlying option is at the money at time T_2, i.e.

$$v_2(u) = V_2^*,$$

where V_2^* solves the equation:

$$e^{-\lambda\tau_1}\left(V_2^* e^{\lambda\tau_1}\aleph_1\left(h_1\left(V_2^*, \tau_1\right)\right) - I_1 e^{-r\tau_1}\aleph_1\left(l_1\left(V_2^*, \tau_1\right)\right)\right) = I_2.$$

According to (7) the pricing formula is the same as the Geske's (1979) compound option formula, given that the technical failure does not occur during the intervals $[0, T_2]$ and $(T_2, T_1]$, weighted by the probabilities of no-technical failure. The expression $e^{-\lambda T_1}\left(V_0 e^{\lambda T_1}\aleph_2\left(h_2, h_1; \rho\right)\right)$ can be interpreted as the present value of receiving the future cash flows contingent on the success of the discovery and development phases and the exercise of both the compound and the underlying options. The expression $e^{-(r+\lambda)T_1} I_1 \aleph_2\left(l_2, l_1; \rho\right)$ can be interpreted as the present value of paying the strike price I_1 in that event and finally, the expression $e^{-(r+\lambda)T_2} I_2 \aleph_1\left(l_2\right)$ can be interpreted as the present value of paying the strike price I_2 contingent on the success of the discovery phase and the exercise of the compound option.

4 Case study application

To test our model in a practical business setting, it is applied to a case analysis. We provide a valuation of an R&D project using as much as possible data provided by one of the largest oncology-focused R&D companies in Europe[14]. The company is developing a pipeline of products aimed at a better treatment for cancer. R&D activity is therefore devoted to the understanding of cancer mechanisms instrumental in the definition of novel approaches to the treatment of this disease. Because of the high confidentiality of many issues, all key-dates and financial values presented in this document are modified.

According to the company's R&D programme, the drug development project moves from one stage to another according to a pre-defined stage-gate process

[14] For confidentiality reasons, we cannot disclose the name of the company, nor provide more detailed information of the invention.

as described above. Milestone projects for the drug under consideration can be easily summarized as follows. The drug R&D project started in 2003 with the discovery of a new molecule. Pre-clinical testing started in 2007 and took one year to complete. At present time (year 2009), the candidate drug has successfully completed the first phase of clinical tests on humans, and will shortly be introduced into Phase 3, during which its effect on a large number of oncological patients will be tested. The company expects to enter into Phase 3 in 2011. It often takes an average of 3 years to complete Phase 3, depending on the length of the study, and the number of volunteers. If the clinical testing is completed with a successful outcome, documentation detailing clinical results is submitted to the EMEA for approval. The company expects to file a NDA with EMEA for marketing approval in 2014[15]. As noted above, a drug product must be found to be effective and safe before it may be approved for general marketing. Provided that the drug is approved by the agency, the expected year of market launch is 2015.

4.1 Compound R&D option structure

In order to complete the drug development project, and to launch the product into the market, the company still faces two investment decisions:

(1) decision to enter into Phase 3 (year of exercise 2011);
(2) decision to launch the drug (year of exercise 2015).

The two discrete investment decisions can be considered as investment options whose values are priced by using technique of compound options. Accordingly, we perform a compound option valuation of the drug development project, as of January 2010, given what is known at the end of 2009. To transfer our theoretical model to the case study application, some specifications to the model are necessary.

The call option F_1 can be exercised at the beginning of 2015 when the company will decide to commercialize the drug or to abandon. The lifetime of the option is 5 years (exercise date) and its final payoff is equal to the difference between the 2015 value of the project and the present value of the phased investment (the capital expenditure to be made to launch the drug into the market) at year 2015. The value of the project is the present value of all future cash flows, received at the beginning of year 2015. This value is conditional on knowing that a technical failure does not occur during years 2010-2011 and 2011-2015. Provided that the project is successful, the company should exercise the call option if the value of the project is greater than the investment at year 2015. If on the other hand the project value is less that the investment, the company should abandon the option.

The compound option F_2 can be exercised at the beginning of 2011 when the company will decide to enter into Phase 3 or to abandon. The lifetime of the

[15] For simplicity's sake, we assume that all decisions regarding the completion of the R&D project will be taken at the beginning of each year.

compound option is 1 year and its final payoff is equal to the difference between the 2011 value of the underlying option and the present value of the phased investment (the capital expenditure to be made to develop and manufacture the drug) at year 2011. The 2011 value of the underlying option is conditional on knowing that a technical failure does not occur during the life of the compound option. Provided that the project is successful, the company should exercise the compound option if the value of the underlying option is greater than the investment at year 2011. If, on the other hand, the value of the underlying option is less that the investment the company should abandon the compound option.

The problem of valuing the drug development project is thus reduced to the problem of pricing the compound option F_2.

4.2 Input parameters

The variables considered when valuing the pharmaceutical R&D investment project by means of the formula (7) are illustrated in Table 1.

TABLE 1: description of the variables of the compound option model

RO variable	**Empirical equivalent**	**Value**
Current value of the underlying asset (V_0)	Project value: present value of all future cash flows (PV_{2010})	$V_0 =$ €67 Million
Strike price of the underlying option (I_1)	Present value of the phased investment at year 2015	$I_1 =$ €27 Million
Strike price of the compound option (I_2)	Present value of the phased investment at year 2011	$I_2 =$ €19 Million
Exercise date of the option (T_1)	Lifetime of the option to launch the drug	$T_1 = 5$ Years
Exercise date of the compound option (T_2)	Lifetime of the option to go into Phase 3	$T_2 = 1$ Year
Volatility of the underlying asset of the option (σ)	Volatility of the project value	σ ranges from 23% to 57%

Risk-free interest rate (r)	Annual interest rate of T-bonds with a maturity date of 5 Years	$r = 2.4\%$
Annual arrival intensity (λ)	Annual arrival intensity	$\lambda = 7.6\%$

1. Current value of the underlying asset (V_0). This is the 2010 present value of all future cash flows from the project, excluding the phased investments (i.e. capital expenditures to be made to develop, manufacture and launch the drug into the market). The reason is that they will be subtracted as exercise prices in the compound option model. Finally, technical uncertainty will be directly accounted in the compound option computation through the multiplication of the underlying value and the exercise prices by their corresponding probabilities of success. The 2010 the present value of the underlying asset can then be computed performing a standard DCF analysis[16]. We obtain that $V_0 \cong$ €67 Million.

2. Strike prices of the underlying and compound options (I_1 and I_2)**.** During the analysis of the compound option structure we identified that the present value of the phased investments of the drug development and marketing process represent the option exercise prices. Their values are presented in Table 1.

3. Exercise dates of the underlying and compound options (T_1 and T_2)**.** During the previous analysis we identified that the exercise date of the call option is equal to the lifetime of the option to launch the drug (i.e. $T_1 = 5$ Years) , and that the exercise date of the compound option is equal to the lifetime of the option to enter into Phase 3 (i.e. $T_2 = 1$ Year) .

4. Volatility of the underlying asset of the option (σ). With any option pricing model the key element to determine is volatility. When used for valuing financial options, is usually measured by the volatility of the underlying stock or a group of similar stocks. This is much more difficult with real projects[17]. One could look to the revenue or cash flow volatility (if such data are available) or use the volatility of similar projects. Some[18] have suggested using the volatility in stock prices of other firms in the same business. For our analysis, we use the stock price volatility of a NASDAQ listed biotech firm that develops a similar kind of product as a proxy for the volatility of the project value. This value can be taken from the historical stock price volatility of AMGEN Inc.[19] The range for the volatility of the project value is set at 23% to 57%.

[16] For confidentiality reasons, financial data and valuation results that can be disclosed for this project are limited.

[17] The problem with real options is that the underlying project is non-traded asset, which makes finding an estimate for the volatility difficult.

[18] See, for example, Nichols (1994).

[19] Source: http://dynamic.nasdaq.com/dynamic/nasdaqbiotech_activity.stm.

5. Risk-free interest rate (r). The value of the risk-free rate is the annual interest rate of Treasury Bonds with the same maturity date as the exercise date of the underlying option. This value is taken from Bloomberg[20] (Date: November 10, 09) and it is about 2.4% (e.g. we use a 5-Year coupon rate of U.S. Treasury Bonds).

6. Annual arrival intensity (λ). This is the annual arrival intensity of important information. The annual arrival intensity is determined based on the firm's estimations of the probabilities of success of the project in the R&D stages. These probabilities of success are determined based on the average rates in the biopharmaceutical industry and adjusted by clinical experts to better reflect specific project characteristics. Table 2 presents the milestone projects and the associated probabilities of success.

TABLE 2: milestone projects and associated probability of success

Phase	**Start date**	**Duration (Years)**	**Probability of success**
Phase 2	in progress	1	80%
Phase 3	2011	3	90%
Approval	2014	1	95%
Launch	2015		

Hence, with probability of 80% the project will show positive results in Phase 2, with probability of 90% it will show significant effectiveness in treating patients during Phase 3, and there is 95% probability that it can gain EMEA approval. This makes up 68.4% of cumulative probability that the project will be marketed. The annual arrival intensity can then be computed as:

$$e^{-\lambda T_1} = e^{-\lambda 5} = 0.684$$

Hence,

$$\lambda \cong 0.076.$$

4.3 Numerical results

In this section we provide some numerical results on compound R&D options. In order to implement the analytical solution and to study its sensitivity analysis with respect to important value drivers we use Mathematica Programming. Assuming an initial project value of €67 Million, investment costs of €27 Million and €19 Million, maturities of 5 Years and 1 Year, a volatility of 23%, a risk-free interest rate of 2.4% and an annual arrival intensity of 7.6%, we obtain:

$$F_2(67) = e^{-0.076\cdot 5}\left(67e^{0.076\cdot 5}\cdot 0.997856 - e^{-0.024\cdot 5}27\cdot 0.991902\right) +$$
$$-e^{-(0.024+0.076)\cdot 1}19\cdot 0.998265 \cong € \ 33.4 \text{ Million}.$$

[20] Source: http://www.bloomberg.com/markets/rates/index.html.

Now, instead assume no technical uncertainty (i.e. $\lambda = 0$):

$$f_2(67) = 67 \cdot 0.985953 - 27e^{-0.024 \cdot 5} \cdot 0.955848 +$$
$$-19e^{-0.024 \cdot 1} \cdot 0.995234 \cong € 24.7 \text{ Million.}$$

Numerical results show that technical uncertainty increases the value of an R&D investment opportunity, i.e. an R&D project that has a positive probability of failure is more valuable than an R&D project that does not.

The table below provides the sensitivity analyses of the compound R&D option value for different values of V_0, σ and λ.

TABLE 3: sensitivity analyses of the compound R&D option value for different values of V, σ and λ (values in Million of €)

λ		No-Technical Uncertainty			$\lambda = 0.076$			$\lambda = 0.1$		
	σ	23%	48%	57%	23%	48%	57%	23%	48%	57%
V_0										
67		24.7	28.9	31.1	33.4	35.2	36.6	35.7	37	38.1
75		32.6	36.1	38.2	41.4	42.8	44	43.7	44.7	45.7
90		47.5	50.1	52.0	56.4	57.4	58.4	58.7	59.4	60.2
100		57.5	59.6	61.4	66.4	67.2	68.1	68.7	69.2	70

The effect of these elements on the resulting compound R&D option value is as follows:

(1) an increase in the value of the underlying project will cause the compound R&D option value to increase;

(2) an increase in the volatility of the value of the underlying project will cause the compound R&D option value to increase;

(3) an increase in the annual arrival intensity will cause the compound R&D option value to increase.

Figure 3 shows the relation between the compound R&D option values and the project value. The effect of technical uncertainty is investigated through two different curves. The dashed curve illustrates the sensitivity of the compound R&D option value F_2 with the project value in the presence of technical uncertainty, while the solid curve illustrates the sensitivity of the compound R&D option value f_2 with the project value absent technical uncertainty. As before we assume: $I_1 = 27$, $I_2 = 19$, $\sigma = 23\%$, $T_1 = 5$, $T_2 = 1$, $r = 2.4\%$, $\lambda = 7.6\%$ and V_0 ranges from 20 to 100. The gap between the two curves shows the increased value by incorporating technical uncertainty into a model of compound R&D option valuation. For low values of V, the two curves are close to each other, as the option is out of the money regardless of the presence of technical failure.

Figure 4 shows the relation between the compound R&D option values and the volatility of the project value. The dashed curves illustrate the sensitivity of

the compound R&D option value F_2 with the project volatility in the presence of technical uncertainty, while the solid curve illustrates the sensitivity of the compound R&D option value f_2 with the project value absent technical uncertainty. The following parameter values are used: $V_0 = 67$, $I_1 = 27$, $I_2 = 19$, $T_1 = 5$, $T_2 = 1$, $r = 2.4\%$, $\lambda = 7.6\%$ (short dashes) and 10% (long dashes) and finally σ ranges from 20% to 100%. As figure 4 clearly shows, increasing the market volatility increases the option value. We can, moreover, make some remarks about the 'Vega' of the option, defined as the sensitivity of the option with respect to the volatility of underlying value. The Figure shows that the option 'Vega' decreases when technical uncertainty increases. As an important consequence, this means that errors in the estimate of economic uncertainty are less important when accurate estimates of significant technical risk of failure exist.

Figure 5 shows the relation between the value of the compound R&D option and the annual arrival intensity for different levels of market introduction costs ($I_1 = 14$, 27 and 54). As figure 5 clearly shows, increasing the annual arrival intensity increases the option value. We assume: $V_0 = 67$, $I_2 = 19$, $\sigma = 23\%$, $T_1 = 5$, $T_2 = 1$, $r = 2.4\%$, and λ ranges from 0 to 100%. Furthermore, when λ approaches 100%, a current value $V_0 =$€67 Million implies a very large project value when technical failure does not occur. Hence, the option will always be executed in the (rare) case of no failure. Moreover, due to high technical risk, formula (7) shows that investment costs are highly discounted, leading to a relatively high option value of €60 Million. Especially, the cost of market introduction of €27 Million hardly affects the option value as it is multiplied with a factor of $e^{-1.024 \cdot 5} = 0.006$.

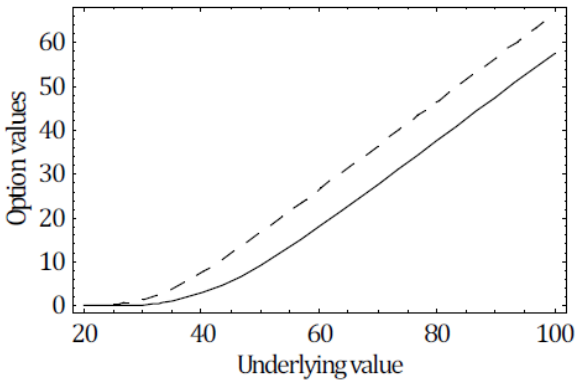

FIGURE 3: Sensitivity analysis between the value of the compound R&D options and the value of the project. We assume $I_1 = 27$, $I_2 = 19$, $\sigma = 23\%$, $T_1 = 5$, $T_2 = 1$, $r = 2.4\%$, $\lambda = 7.6\%$ and V_0 ranges from 20 to 100.

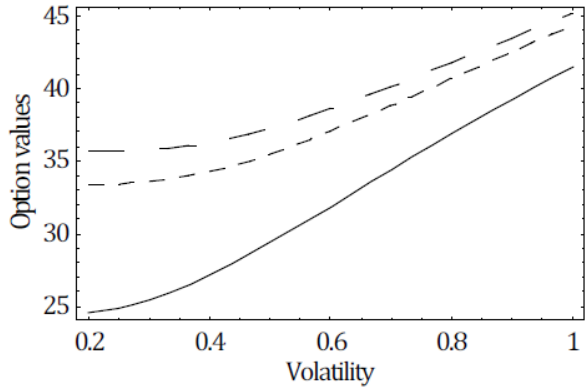

FIGURE 4: Sensitivity analysis between the value of the compound R&D options and the volatility of the project value for different levels of λ. We assume $V_0 = 67$, $I_1 = 27$, $I_2 = 19$, $T_1 = 5$, $T_2 = 1$, $r = 2.4\%$, $\lambda = 7.6\%$ (short dashes) and 10% (long dashes) and finally σ ranges from 20% to 100%.

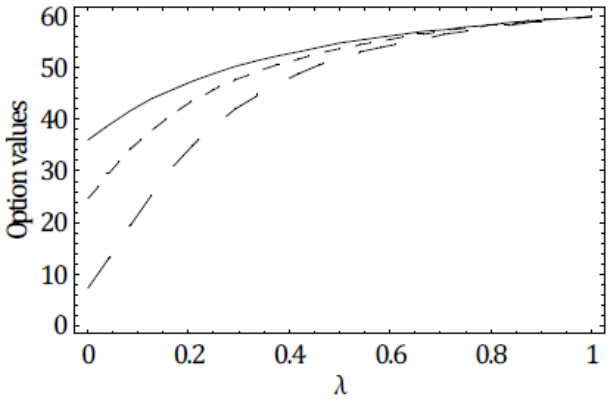

FIGURE 5: Sensitivity analysis between the value of the compound R&D option and the annual arrival intensity for different levels of market introduction costs. We assume $V_0 = 67$, $I_1 = 14$ (solid curve), 27 (short dashes) and 54 (long dashes), $I_2 = 19$, $\sigma = 23\%$, $T_1 = 5$, $T_2 = 1$, $r = 2.4\%$, and λ ranges from 0 to 100%.

5 Concluding remarks

A pharmaceutical R&D investment project can be modeled as a series of sub-projects, where investment in each step is contingent on the results obtained from the previous step. Cash flows are not obtained until the last stage, that is, until the drug is marketed. Because of this property the valuation of pharmaceutical R&D investment projects is one of the most difficult problems. Starting from the difficulty of traditional DCF methods to capture the value of these projects, the ROV literature provides advanced models, each focusing on different characteristics. In the present paper we value pharmaceutical R&D investment projects with the following characteristics: two types of uncertainty, i.e. technological and economic, and compoundness of R&D projects.

As far as we know, no compound R&D option model in the presence of technical risk of failure has been studied in the ROV literature before. Including a Poisson jump process we are able to model the nature of the drug development process and to assess the option value of pharmaceutical R&D projects in the presence of technical risk of failure in every R&D stage. Our method shows that a compound R&D option can be evaluated by conditioning on two possible scenarios, a failure occurs and does not occur, in each R&D stage. Therefore, when a technical failure occurs the investment option is abandoned, and thus the valuation problem boils down to valuing a compound R&D option only under favorable conditions.

Practical application of this method is emphasized through a case analysis. We compare an R&D project that has a positive probability of technical failure and an R&D project that does not to determine the value of a compound R&D option. Sensitivity analyses are shown as to deepen our understanding of the determinants of the compound R&D option value. We show that both uncertainties have a positive impact on the R&D option value. As a second result, from the sensitivity analysis, we find that the sensitivity of option value with respect to changes in the volatility of the underlying value and the cost of market introduction decreases in the presence of technical uncertainty.

Our method can be easily applied to valuation of sequential (R&D) investments by different industries (software development projects in the ICT industry among others) where there is a positive probability of project failure in different stages. Finally, our model can be can be extended to account for an arbitrary degree of compoundness, i.e. N-nested compound options that are useful for valuation of early-stage R&D investments and new drug applications of pharmaceutical companies[21]. This extension is included in the appendix.

APPENDIX

A generalization of the model for N-nested compound options.

[21] See, for example, Cassimon et al., (2004) where R&D projects of pharmaceutical companies are valued using 6-fold compound options.

This section shows how the model can be extended to account for an arbitrary degree of compoundness and derives an analytical expression for N-nested compound options[22].

We consider the valuation problem of a R&D-based pharmaceutical firm who, at time zero, considers to investing in a drug development project whose commercial phase cannot be launched before a R&D project consisting of N stages of investment is completed. Let T_1 be the time of the market launch of the product, when, upon bearing the commercialization cost I_1, the firm pockets the project value V_{T_1}. The project payoff at time T_1 is $\max\{V_{T_1} - I_1, 0\}$ and let $F_1(V,t)$ denote the value at time t of this simple investment opportunity. We assume that the commercialization phase is reached upon investing an amount I_k, at time period T_k, for $k = 2, ..., N$ and with $T_1 \geq T_2 \geq ... \geq T_N$. T_N is therefore the time period the project starts and I_N is the start up cost, while T_k and I_k are maturities of intermediate phases which lead up to the commercialization phase and are their respective investment costs. The $N-$staged investment problem may be viewed as a compound option and its value may be derived in a recursive way.

Let us now define a sequence of call options, with value F_k, on the call option whose value is F_{k-1}, with exercise price I_k and expiry date T_k, for $k = 2, ..., N$. The $k-$fold compound option value can be written in a recursive way and its final payoff at the option's maturity date T_k is given by:

$$F_k\left(F_{k-1}\left(V,T_k\right),T_k\right) = \max\left\{F_{k-1}\left(V,T_k\right) - I_k, 0\right\}, \tag{8}$$

for $k = 2, ..., N$ and where $F_{k-1}(V,T_k)$ stands for the price of the underlying option at T_k. According to (8), at time T_k, the firm faces the option of investing an amount I_k, gaining access to stage $k-1$ of the project whose value is $F_{k-1}(V,T_k)$, or to shut the project down. Our aim is to derive a valuation formula for the N-fold compound option, that is for $F_N(V,0)$.

Let V_k^* denote the value of V at time T_k such that $F_{k-1}(V,T_k) - I_k = 0$, for $k \geq 2$ and $V_1^* = I_1$. Then, if the value of V at time T_k, is greater than V_k^*, the compound option will be exercised, while for values less than V_k^* it will remain unexercised.

Moreover, let us define n_i the number of arrivals in the time interval $[T_{i+1}, T_i]$, $i = 1, 2, ..., N$, and let us set $T_{N+1} = 0$. Consequently, let $m_k = \sum_{i=k}^{N} n_i$ be the total number of arrivals in the interval $[0, T_k]$, for $k = 1, 2, ..., N$. The time interval $[0, T_1]$ is divided into subintervals of length $\tau_k = T_k - T_{k+1}$, for $k = 1, 2, ..., N$ with $\tau_N = T_N$. n_i takes the values of zero with probability $e^{-\lambda\tau_i}$ or one with probability $1 - e^{-\lambda\tau_i}$, $i = 1, 2, ..., N$.

Let us define:

$$h_k = \frac{\ln\left(\frac{V_0}{V_k^*}\right) + \left(r + \lambda + \frac{1}{2}\sigma^2\right)T_k}{\sigma\sqrt{T_k}}, \text{ for } k = 1, 2, ..., N,$$

[22] Agliardi and Agliardi (2005) also study N-fold compound options, in the case of variable interest rate and volatility, but without technical uncertainty.

$$l_k = h_k - \sigma\sqrt{T_k}.$$

Moreover, let x_t be the logarithmic return[23] and let the correlation between x_{T_j} and x_{T_i}, over the overlapping time interval $T_j < T_i$, conditional on observing $m_j = 0$ and $m_i = 0$, respectively, be:

$$\rho_{ij} = \sqrt{\frac{T_j}{T_i}}, \text{ for } 1 \leq i < j \leq N.$$

For any k, $1 \leq k \leq N$, let $\Xi_k^{(N)}$ denote a $k-$dimensional symmetric correlation matrix with typical element ρ_{ij}. Let $\aleph_k(l_k, ..., l_1; \Xi_k)$ denote the $k-$dimensional multinormal cumulative distribution function, with upper limits of integration $l_k, ..., l_1$ and correlation matrix Ξ_k. Finally, assuming that the number of jumps are independent of each other with a constant arrival intensity λ, the joint cumulative probability distribution function of observing $n_1 = 0$, $n_2 = 0$, ..., $n_k = 0$ in the time intervals τ_1, τ_2,..., τ_k, respectively, is $H(n_k = 0, ..., n_1 = 0; \lambda) = e^{-(\lambda\tau_1 + \lambda\tau_2 + ... + \tau_k)}$, $k = 1, 2, ..., N$.

The value at time zero of the N-fold compound option, is:

$$F_N(V, 0) = V_0 \aleph_N\left(h_N, ..., h_1; \Xi_N^{(N)}\right) +$$
$$- \sum_{j=1}^{N} I_j e^{-(r+\lambda)T_j} \aleph_{N+1-j}\left(l_N, ..., l_j; \Xi_{N+1-j}^{(N)}\right),$$

where h_j, l_j, and ρ_{ij} are as defined previously.

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