Carbon Price Uncertainty and Power Plant Greenfield Investment in Europe

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Abstract

One of the stated objective of the EU ETS policy is to incentivize investment in low-carbon or carbon-free power generation technologies. Still, so far, the uncertainty about future carbon prices and the existence of technology-dedicated incentives like subsidies for CCS and feed-in tariffs or green certificates, might indicate that the carbon price has hardly played that role.

The aim of this paper is twofold. First, the method developed should help utilities decision-makers integrate their views on carbon prices in an investment decision framework. Second, the method employed should ultimately help identify sensitivity points to guide policymakers when designing amendments to the rules governing the EU ETS.

In order to understand how corporate decisions under carbon price uncertainty are taken, we model a utility's investment decision in a multivariate real options framework. We consider a European utility that has a 10-year window to invest in a combination of various generation technologies (nuclear, IGCC, CCGT, pulverized coal and offshore wind). The model specifically account for uncertainty in carbon and power prices. The model is solved using the least-squares Monte Carlo approach (Longstaff and Schwartz, 2001 and Gamba, 2003) in order to account for various sources of uncertainty. Compared to the existing literature, we adapt the method to explicitly allow for capital rationing and choose among various technologies rather than just determining an optimal option exercise time.

Policy-wise, early results of the model indicate that attempts to limit market price volatility and / or ensure a quick reversion to long-term equilibrium are of little help when compared to giving indications regarding significant cap level at various points in time (indicative of the deterministic trend). Furthermore, the price of carbon only contributes little to shifting investment decisions towards carbon-neutral or lower carbon investments. Rather, the price of carbon is critical to short-term adjustments (fuel-switching / trading / operation planning). Finally, technology-dedicated incentives seems to better incentivize the investment in carbon-neutral or lower carbon power plants.

Keywords: EU ETS - carbon price uncertainty - real options - stochastic dynamic programming - capital rationing - least-squares Monte Carlo.

JEL: C61 - G31 - L94 - Q4 - Q54 - Q58

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Contents

List of Figures

List of Tables

1 Introduction

Carbon price uncertainty has been often invoked as one of the reasons why delay investments in power generation capacity in the EU. More specifically, the lack of long-term visibility and volatility of the European carbon price have been strongly criticized by European utilities. This paper tackles the issue of carbon price uncertainty for European utilities and tries to evaluate the claims of the power sector and find reasons why utilities corporate financiers would delay their investments in generation capacity or would favor specific investment alternatives over others.

1.1 Literature survey

1.1.1 Power plant investment and the EU ETS

Investments in power plants are unique for financial and technical reasons (Olsina et al., 2006 [1]; He, 2007 [2]). They entail large capital outflows, a large percentage of which having to be committed before the power plant is even commissioned. This often translates into long payback period and calls for reliable valuation and decision-making tools. In addition, power plant investments expose investors to several long run uncertainties: on the demand side, regarding costs (fuel and O&M), long-term electricity prices, price spikes frequency (for peakload plant valuation), technology innovation risk, regulatory risk and changes in capacity by competition. Also, these investment are characterized by a certain form of irreversibility and the option to postpone investment.

The European Union Emission Trading Scheme (EU ETS) was launched in 2005 to facilitate European member states compliance with the Kyoto protocol. The EU ETS shifts a large share of the environmental burden of EU member states to EUs stationary sources of greenhouse gases emissions (carbon dioxide most essentially). The EU ETS functions as a cap-and-trade market. Stationary sources of carbon dioxide (CO_2) emissions within the scope of the pertaining Directive (called installations) are identified and an emissions cap, corresponding to the maximum quantity of CO_2 they can emit during a given period, is imposed on them by the regulator. At the time of writing, there was three compliance periods in the EU ETS: the trial phase (phase I) between 2005 and 2007, the Kyoto phase (phase II) between 2008 and 2012 and the post-Kyoto phase (phase III) between 2013 and 2020. Stationary sources falling within the scope of the Directive are combustion installations with a capacity superior to 20 MW. Some 70% of those installations are either producing power or heat and it was estimated that 49% of them where solely producing power (Trotignon and Delbosc, 2008 [3]). The remaining installations are industrial installations from the steel, cement, refining sectors among others. Installations within the scope of the Directive have been entitled European Union Allowances (EUAs) that corresponds to the right to emit one ton of CO_2 during a specific time period. The quantity of EUAs they have been entitled corresponds to the emissions cap imposed on them. While, on average, the industrial installations have been allocated more allowances than required over the compliance periods, the power and heat portion of the EU ETS was entitled less EUAs than was expected to be needed. The prevailing allocation method during phase I and II was grandfathering: EU ETS installations emissions cap was fixed based on historical emissions. During the first two compliance periods, allowances were mostly allocated for free. In order not to disadvantage new entrants (genuine new entrants in the European power sector or extra combustion units from incumbents that would fall within the scope of the Directive), a new entrant reserve (NER) was negotiated and set aside. This NER is comprised of free allowances provided to new installations so that incumbents would not be favored as regards the EU ETS. It is expected that phase III will most probably see a move towards more auctioning for the power sector since 2013 and a gradual shift for industrial sectors with the aim of 100% auctioning by 2020. These EUAs are assets that can be traded among installations of the EU ETS. Financial intermediaries can also participate in the scheme. A cap-and-trade scheme gives the incentive to reduce emissions beyond the cap since compliance-buyers are allowed to sell emissions rights in excess of their emissions needs to those for whom it is more expensive to reduce their emissions on their own. To claim compliance, EU ETS installations must surrender as much EUAs as tons of CO_2 they have emitted over a given year. They can do so by either acquiring more EUAs (or similar assets) or by reducing their emissions. Emissions reductions in the power sector can be achieved by means of short-term operational adjustments (like fuel switching to a lower carbon content combustion fuel), investments in less carbon-emitting technologies (retrofitting power plants with carbon capture and storage or investing in a plant that emit less based on its initial characteristics) or by halting or decreasing the power plant output (and the emissions consequently).

Therefore, the theoretical impact on the power sector takes place at two level. First, the carbon price has been introduced in operational decisions. Anytime a ton of carbon is emitted in the course of the production process, the operator compares the corresponding profit margin for the production (including carbon procurement costs) with the opportunity cost of selling the allowance on the market. Some studies even talk of some emissions reduction during the first trading phase (2005-2007) in the form of fuel switching even though the cap was not that stringent (Ellerman and Buchner, 2006 [4]). Second, the carbon price can be factored in longer term decision making - namely the decision to invest in several abatement solutions. Should the carbon price be high enough, decision-makers might consider it more advantageous to invest in carbon-free or less carbon-intensive production apparel. Hoffmann (2007) [5] note that this has not been the case so far in the German power industry. He finds that while short-term operating decisions clearly have been impacted the EU ETS, this was not the case for greenfield/brownfield investment decision and R&D as well. Reasons invoked for that lack of incentives are numerous.
Most policy observers argues that the cap for phase I and II of the EU ETS has been set too low to provide an effective incentive. Others note that the effectiveness of the policy was corrupted by not following the policy tool "by the book" despite it was the condition for acceptance by the regulated: the allocation of most grandfathered allowances for free in phase I and II (instead of an auctioning process) and the new entrants and closure provisions (Ellerman, 2006 [6]). Finally, the existence of authorized flexibility mechanisms (banking, borrowing of EUAs and ability to surrender credits from Kyoto offset projects) and derogatory measures in some member states are sometimes invoked as not giving the incentive to invest in carbon-free technologies within the EU boundaries.

Contemporary to the introduction of the EU ETS, three major changes radically modified the investment decision-making for European utilities.
First, the market liberalization process has progressed in Europe (Joskow, 2008 [7]; Chevalier and Percebois, 2008 [8]). The process introduced uncertain customer demand as well as uncertain power prices which have not simplified the investment decision-making. The decision to invest is no longer a state or a monopolistic utility centralized procedure but rather a decentralized decision of generation firms aiming at maximizing profits.
Second, in parallel to the effort on curbing greenhouse gases by means of a cap-and-trade approach, the European utilities have been subject to various regulatory reforms (at the EU level and member state level) with renewable energy targets, energy efficiency measures and carbon capture and storage (CCS) objectives. These new regulations are most associated with policy instruments to provide the regulated utilities with the incentive to act in accordance with the spirit of the policies: feed-in tariffs or tradable green certificates to achieve renewable targets or tenders and subventions for the funding of demonstration CCS projects for instance. It is unclear whether their co-existence support or create distortions with the EU ETS.
Third, the second phase of the EU ETS also saw the effect of the economic and financial crisis that begun in late 2008. Between July 1st, 2008 and February 12th, 2009, the phase II carbon price was divided by 3.6 to reach EUR 8/ton given the revised expectations on future production and emissions

levels. This might to some extent have impacted the required rate of return on power plant investments upward, modified the financing decision, the prospects for valuation drivers and more fundamentally the need to undertake new investments.

1.1.2 Real option investment decision modeling

The shift towards more liberalized markets with several policy instruments triggered regained interest in electricity market modeling. Such interest revolved around three major trends (Ventosa et al., 2005 [9]): optimization models, equilibrium models and simulations models. To some extent, this paper belongs to the first trend given our focus on a single firm trying to optimize its investment plan under exogenous price developments.

In an effort to overcome the limitations of the net present value (NPV) rule under deterministic discounted cash flows (DCF)[1], the real options methodology suggests an approach that can be used to complete the traditional NPV rule. First, the real options approach (ROA) allows the decision maker to postpone the initial investment undertaken - this gives him flexibility in the investment timing (option to defer) instead of the traditional now-or-never investment decision. Second, the ROA permits the decision maker to value the operating flexibility in the underlying asset (Trigeorgis, 1996 [10]): option to alter operation scale (expand or contract), option to abandon (temporarily or definitively), option to switch (from one operating process to another) and growth options. Third, the ROA typically incorporates some way of accounting for uncertainty from simple binomial tree to stochastic price modeling for instance. Finally, initial investments are considered irreversible - a limitation of the traditional NPV rule under deterministic DCF being to assume the perfect marketability of assets being valued. This makes valuation rather irrealistic when large scale or proprietary investment are performed. Instead, the ROA takes this characteristic into account.

The ROA essentially builds on the financial options theory - and most predominantly the seminal works on option pricing by Black and Scholes and Merton, the binomial approach by Cox, Ross and Rubinstein as well on stochastic price modeling[2]. Risk-neutral valuation is also a major building block of the ROA with contingent claim analysis (replicating portfolio and use of spanning assets) and certainty-equivalent approach. Finally, most recent works especially in the face of ever more complex problem involve numerical methods to avoid solving analytically real options problems. In this respect, the landmark works on dynamic programming (Bellman, 1957 [13]) have been completed by backward-looking Monte Carlo simulations [14] and control-variate methods with numerical approximations. Reference works on the ROA include textbooks by Dixit and Pindyck (1994) [15] and Trigeorgis (1996) [10] and papers by Brennan and Schwartz (1985) [16] on multiple option framework for mine management and Pindyck (1988) [17] on the options to choose capacity under product price uncertainty.
Common applications for the ROA in the academic literature are high capital cost investments (oil fields, mines, power plants, etc.) characterized by large uncertainties in demand, supply and/or price (natural resources and R&D projects especially), long lifetime and some leeway or strategic behavior either in the initial investment decision or subsequent operating decisions[3].

[1]Conceptually, the traditional DCF approach has the following limitations: it entails accepting all the outcomes of the projects once decided upon, it is a now-or-never decision and it systematically underestimate the asset value with real options embedded. More technically, other limitations includes the difficulty to estimate future cash flows because of their stochastic nature, the risk of making errors in choosing an appropriate discount rate, etc. (He, 2007) [2].

[2]For a recent treatment on this, refer to Shreve (2004) [11] and Shreve (2006) [12].

[3]It should be reminded that the ROA is by no means a one-size-fits-all method. The method is nonetheless fraught with conceptual and implementation difficulties and has more often gained acceptance among academics rather than by decision-makers for fear of resorting to a “black box” (He, 2007 [2]).

In this respect, the very characteristics of power plant investment decisions makes it particularly relevant to use the ROA. The ROA has been applied to peak-load power plant valuation, hydro power plant valuation (taking into account the flexibility in managing the water level in its reservoir), fuel switching in IGCC plants or CHP plant optimal output scheme.
Recent applications to carbon mitigation in the power sector include focus on CCS investment in Spain (Abadie and Chamorro, 2008 [18]), in the US (Bohm et al., 2007 [19]; Sekar et al., 2007 [20] and Sekar, 2005 [21]), capacity investment decision in the EU (Laurikka, 2005 [22]; Laurikka and Koljonen, 2006 [23]; Fuss et al., 2008 [24]; Fuss et al., 2009 [25]) and investment risk quantification (Blyth et al., 2007 [26]; Yang and Blyth, 2007 [27]).

1.2 Research questions

Based on an analysis of expected generation capacity addition by European utilities, we make the following assumptions which remain to be verified by the model proposed here. First, the price of carbon might not be enough to incentivize investment in low-carbon / carbon-free generation units and could even delay such investments due to regulatory uncertainty. Second, low investments in CCS is subject to bargaining direct subventions from EC or Member States and the price of carbon might not play the incentivizing role it is supposed to. Third, investments in renewables is a direct response to renewable policies and it is unclear to what extent carbon markets are helping or distorting the incentive (and conversely, to what extent technology-dedicated incentives support or distort the EU ETS policy).
So we will discuss to what extent carbon prices direct investments towards specific low-carbon technologies. Based on the model results, we will further discuss how best to incentivize investment in low-carbon or carbon-free technologies by means of a price for price.

1.3 Assumptions

We assume a European utility operating over the French-German area. The utility has been approved to build and operate power plants on a given number of sites. Until expiration of the licenses to build for the sites (10 years from now), the utility has flexibility in (1) when to build power plants (timing option) and (2) what power plant technologies to invest in. The sites are located in France so that the utility is exposed to French power prices. This allows us to consider nuclear technology as a generating technology (while in the case of Germany that would not have been possible because of a scheduled phase-out that is still debated).
The utility investor is assumed to be either a genuine new entrant in the EU ETS or an incumbent investing in a new installation. Accordingly, he should be granted access to the new entrant reserve (NER) which puts aside EUAs for new participants in the scheme. Still, it was assumed that there were not any allowances left in the NER so that EUAs have to be purchased to initiate plant's operations in order to reflect the forthcoming situation of investors facing more generalized forms of auctioning for EUAs[4].

2 Model structure

The objective of the model is to solve an investment decision problem under uncertainty. Various methods are envisaged in the real options literature to solve such problems.
First, the analytical approximation methods attempt to solve such problems by finding a closed-form

[4]Note that since we mainly focus on the carbon price uncertainty, we are not taking into account power demand uncertainty, the impact of competition moves on market prices (by addition or removal of capacity), technical progress, transmission and network constraints (which to some extent, we acknowledge, might be critical for the valuation of intermittent sources of electricity).

solution to the partial differential equations (PDEs) at the core of the model. Two equivalent approaches are detailed in the literature. The dynamic programming approach involves breaking down the entire sequence of decisions into two components: the immediate decision and a value function that encompasses the consequences of all subsequent decisions. The contingent claims approach makes an analogy between the investment considered and a stream of costs and benefits varying through time and depending on the unfolding of uncertain events. Hence, valuation is based on underlying tradable assets. This implies some combination of traded assets that will mimic the pattern of returns from the investment project at every future date and in every future uncertain eventuality. Dixit and Pindyck (1994) [15] explain that both approaches should result in the same solutions (the only differences being the discount rate used and the way cash flows components account for uncertainty).
Given that closed-form solutions rarely exist (especially when several sources of uncertainty are considered), numerical methods have been used either to approximate solutions or to discretize continuous underlying processes. Lattice and tree methods belong to numerical methods but are plagued by the curse of dimensionality when more than one process is involved. Alternatively, Monte Carlo simulations are a numerical integration method that can be used to find a risk-neutral value of an option by sampling the range of integration. Lastly, the least-squares Monte Carlo (LSM) method (a subset of Monte Carlo methods) allows to match Monte Carlo simulations and dynamic programming which can be used to price Bermudan options (in which case the option can only be exercised at specific dates over its life) featuring several sources of uncertainty.

A typical approaches to ROA power plant valuation involves directly modeling the spark spread (the power generator profit margin per MWh) as the sole underlying process (and often as a mean-reverting process or inhomogeneous geometric Brownian motion). Given that our focus is on carbon price uncertainty, we will not model clean spark spreads or clean dark spreads[5] but rather model power and carbon price processes as distinct processes. That way, we can use the same price processes to value nuclear and wind investment alternatives and we can better observe the economic relationship between carbon and power prices.

2.1 General depiction

We use a discrete time mixed state real options decision model. In our problem, the state space is mixed (i.e. some states are continuous while others are discrete) while the action space is discrete. See figure 1 for a representation of the model.

In every period $t \in \|0; 10\|$, the investor:

- observes the state of various economic processes: (1) the remaining budget (b_t), (2) stochastic forward prices for carbon (p_t^c) and electricity (p_t^b for baseload and p_t^p for peakload) and (3) spot deterministic prices for the the feed-in tariff of the offshore wind farm (p_t^f) and for fossil fuels delivery, namely coal (p_t^k), and natural gas (p_t^g). We use S_t as the set of price state variables (excluding the budget level).
- decides to (1) invest in a combination of power plant technologies (a CCGT power plant costing I^G, a pulverized coal plant for I^K, an IGCC plant for I^I, a nuclear power plant for I^N and an offshore wind power plant for I^W) or (2) wait to invest later as long as the site license has not expired and the budget permits. The decision is indicated by the control variable x_t (the scope of actions depending on remaining budget).

[5] Cost of producing a MWh with natural gas (coal respectively) and accounting for carbon procurement costs.

Figure 1: Model structure

State variables	Choice variable	Optimal policy	Sensitivity analysis
(1) Budget **(2) Input prices** • CO_2 • Coal • Natural gas **(3) Output price** • Power **(4) Time**	**Invest in a combination of** • CCGT • Offshore wind • IGCC • Pulverized coal • Nuclear **or wait**	**Maximize the value function** (Bellman) "Immediate reward (NPV) + discounted implied value from subsequent optimal choices"	Implied optimal investments undertaken and locked-in emissions **Sensitivity study** to carbon price parameters • Price level • Price growth rate • Volatility • Mean-reversion speed

- earns a reward $f_t(b_t, x_t, S_t)$ in the form of the NPV of the investment undertaken that depends both on the states of the economic processes and the action taken at a given time t.

The investor seeks a policy of state-contingent actions $(x_0^*, x_1^*, ..., x_{10}^*)$ that will maximize the present value of current and expected future rewards, discounted at a per period factor e^{-r}:

$$\max_{x_t(.)} [\mathbb{E}_0^Q \sum_{t=0}^{10} e^{-r.t} f(b_t, x_t, S_t)]$$

Note that $\mathbb{E}_t^Q[\tilde{S}_{t+1}]$, indicating the risk-neutral expectation about the future set of stochastic state variables (S_{t+1}) conditional on knowing S_t (also known as the Equivalent Martingale Measure or EMM), is equivalent to $\mathbb{E}^Q[\tilde{S}_{t+1} \mid S_t]$. Also note that $\tilde{S}_t$ indicates that the set of stochastic state variables is actually random in time t as opposed to S_t which indicates it is known.
The use of a risk-neutral pricing framework allows us to use a the risk-free rate for discounting purpose instead of having to determine a risk-adjusted discount rate that would be bluntly applied to all cash flows whatever the risk embedded (feed-in tariffs implicitly assumed as risky as the carbon price).

2.2 Model input

2.2.1 The state variables

We now consider the various state variables.

(1) The budget constraint
The first state variable corresponds to the budget constraint. The budget is a discrete state (i.e. finite number of value taken) variable. It basically acts as a way to ensure respect of the budget constraint. Let b_t denotes the budget available to invest in period t. We begin the problem with an initial endowment of $\bar{b}$. As we progress through investment nodes, b_t can take any possible combination of investment costs between $\bar{b}$ (untapped budget) and the combination that exhaust entirely the budget granted.

The next period budget corresponds to this period's budget minus investments undertaken during this period:

$$b_{t+1} = b_t - x_t$$

Looking at recent investment programs announced by European utilities and given power plant investment costs assumptions further detailed, we set the initial endowment $\bar{b}$ at EUR 5.9 billion over the investment window. With the investment alternatives investment costs and initial budget specified, we identify 121 possible investment combinations.

(2) The price of carbon

Recent empirical papers help explain the evolution of past prices on the European carbon market. In particular, Alberola et al. (2008) [28] and Mansanet-Bataller et al. (2006) [29] have shown that carbon prices reacted to energy markets price developments (power, oil, natural gas and coal), extreme temperatures and industrial activity. Alberola and Chevallier (2009) [30] have identified that market participants would engage in intertemporal adjustments allowed by the market design of the EU ETS. Mansanet-Bataller and Pardo (2007) [31] demonstrate European carbon prices' high sensitivity to institutional announcements resulting in price shifts upon or prior announcements. Benz and Trück (2008) [32] identify stylized facts of European carbon prices: mean-reversion, jumps and spikes, and heteroskedastic volatility.

Though definitely a place to look at for guidance, the little carbon price history makes it difficult to solely rely on this literature for prospective investment decision-making. The choice of the relevant approach for modeling the carbon underlying asset must help in the long-term irreversible decision making. Still, those price drivers and stylized facts help the decision maker choose the proper carbon price modeling and parameter fitting.
Therefore, we resort to a stochastic price model to account for uncertainty in European carbon prices. We model the carbon price as a continuous state stochastic variable. This means that the investor does not know what the future prices will be (that would be a deterministic variable) but does know the price process and fitting parameters used and hence the statistical distribution associated. This approach involves using a mathematical depiction of the price dynamic for carbon, that is further calibrated and then used to simulate price paths ultimately used in generation technology valuation and investment decision-making.

The mathematical depiction typically takes the form of a general stochastic differential equation (SDE) used to model processes under uncertainty, like equity or commodity prices:

$$dX_t = \underbrace{F(X_t)dt}_{\text{drift component}} + \underbrace{G(X_t)dW_t}_{\text{diffusion component}}$$

where:

- X_t = the process variable to simulate (in our case, the price of carbon allowances, p_t^c or its natural logarithm, $ln(p_t^c)$);
- $F(X_t)$ = the drift rate function which is the trend component of the SDE. Two typical drift rate functions are commonly used in the economic and financial time series literature:
  - A "linear drift rate" taking the following shape:

$$F(X_t) = A_t + B_t X_t$$

where A_t is the intercept term of $F(X_t)$ and B_t is the first-order term of $F(X_t)$ (slope or linear growth component).

– A "mean-reverting drift rate" specification taking the following shape:

$$F(X_t) \quad = \quad \theta_t(X_t^* - X_t)$$

where θ_t is the mean reversion speed, i.e. the time it takes for the price process to go back to its long-term average level, X_t^*, to which the process eventually reverts to.

- $G(X_t)$ = the diffusion rate function expressing the behavior of the process around its trend (variability);
- W_t = a Brownian motion vector, which increments are used to model shocks to the processes;

The two main processes for carbon price found in the literature on investment decision under carbon price uncertainty so far are (1) the Geometric Brownian Motion (GBM) which is the price process basically used for stocks and (2) a typical mean-reverting (MR) process, the Ornstein-Uhlenbeck model. Those price processes are sometimes completed by adding jumps to the processes to reflect abrupt changes in climate policy[6]. Table 1 surveys price processes used for carbon prices found in the literature as well as fitting methods and data used.

Most authors have resorted to the GBM form to model the price of carbon. This is the typical form chosen for equity prices in option pricing model and implicitly makes an assumption of exponential price growth. In a policy-oriented study of investments under climate policy uncertainty, Blyth et al. (2007) [26] and Yang et al. (2008) [34] model the price of carbon as a GBM. Yang and Blyth (2007) [27] further improve their modeling of carbon price by simulating possible carbon price shocks that would represent policy-related events by adding a jump feature to the stochastic modeling (only once ten years from when the initial investment decision can be first taken). The GBM is fitted using a mix of IEA projections and judgmental input.
In an application to optimal rotation period for forest valuation, Chladná (2007) [35] resorts to a GBM fitted with the IIASA MESSAGE model. Szolgayova et al. (2008) [36] and Fuss et al. (2008) [24] assume that, while the electricity price is suggested to follow a mean-reverting process, the carbon price follows a GBM process. Again, the data used to parameterize the GBM comes from IIASA's GGI Scenario database and originally refers to the shadow price of emissions. Fuss et al. (2009) [25] use the same GBM to model the price of carbon but also add a jump process to reflect policy changes over a very long-term horizon (150 years). The size of jumps are drawn from an underlying GBM.
Abadie and Chamorro (2008) [18] resort to a stochastic model of carbon prices to evaluate the prospects of carbon capture investments in Spain. While all the other papers surveyed have been fitted using either model projections or judgmental input, they model carbon prices using a a typical GBM fitted with EU ETS Futures contracts data. Hence, they provide a risk-neutral version of GBM functional form explicitly taking into account a Futures market risk premium. They estimate the parameters using a Kalman filter procedure with EUA Futures prices between January 2006 and October 2007.

In the literature, the choice of a mean-reverting price model is an alternative to the GBM which has the drawback to allow wider price developments over time (the variance of which grows infinitely) than

[6]Other authors have suggested other functional forms for carbon price modeling taking into account more detailed price movements like price spikes or regime switching. But those stochastic modeling are not initially done for investment decision where the big picture matters the most but rather for derivatives pricing or short-term valuation purpose. See for example, Benz and Trück (2006) [32] for an application of regime-switching models and Daskalakis et al. (2007) [33] for applications of jump-diffusion models.

Table 1: Survey of carbon price stochastic modeling

	Process	Start value	Expected μ^c	Drift Risk-adjusted $\mu^c - \lambda^c$	Reversion speed θ^c	Reversion level $(P_t^c)^*$	Diffusion Instantaneous volatility σ^c	Jump	Fitting data
Abadie and Chamorro (2008)	GBM	EUR 18/t in 2007	-	3.08%	-	-	46.83%	-	1325 daily prices of the five Futures contracts maturing in phase II
Chladná (2007)	GBM	USD 5/t in 2010	3.63%	-	-	-	16.60%	-	IIASA MESSAGE model
Fuss et al. (2008) and Szolgayova et al. (2008)	GBM	EUR 5/t	5.68%	-	-	-	2.87%	-	GHG shadow prices from IIASA GGI scenario database
Fuss et al. (2009)	GBM with and without GBM jump	USD 5/t	5.00%	-	-	-	0 to 30%	frequency is from 0 to 20 years	GHG shadow prices from IIASA GGI scenario database
Yang & Blyth (2007); Blyth et al. (2007) and Yang et al. (2008)	GBM with Poisson jump	undisclosed	undisclosed	-	-	-	7.75%	+/- 100% size and once 10 years from now	In order to follow IEA scenario projections 15 years from now
Yang & Blyth (2008)	MR	USD 15/t in 2005	-	-	0.14	EUR 15/t and 15% p.a.	50.00%	-	In order to follow IEA scenario projections 15 years from now
Laurikka & Koljonen (2006)	MR	EUR 7/t in 2005	-	-	0.20	EUR 20/t or EUR 1/t in 2013	10 or 40%	-	judgemental input

mean reverting models. While models based on GBM have been used for tractability and ability to obtain closed-form expressions readily analyzable, mean reversion reflects the long-term equilibrium of production and demand. Laurikka and Koljonen (2006) [23] model the natural logarithm of the price of carbon allowances as a simple mean-reverting Ito process, namely an Ornstein-Uhlenbeck process (continuous state and discrete time). The authors assign two different values to the long-term price level (by 2013) depending on the scenario taken: EUR 20/ton in a high scenario and EUR 1/ton in a low price scenario. Similarly, the variance parameter can take the value of 10% (low volatility scenario) or 40% (high volatility scenario). For fitting the model they use a starting price of EUR 7/ton based on early forward transaction prices reported by Point Carbon in 2004. Laurikka (2005) [22] suggests a simulation model which can simultaneously deal with multiple stochastic variables (emission allowances, electricity and fuels) to estimate the value of flexibility. Again, the stochastic processes used in the simulation mimic the simplest mean-reverting process (the Ornstein-Uhlenbeck process). It is important to remark that both studies were designed prior to the entry into force of the EU ETS.

When it comes to modeling the price of carbon, we contend it is more judicious to model the price of carbon as a mean-reverting process along a linear trend for three main reasons[7].
First, we argue that carbon price long-term price drivers (the supposedly declining cap feature of the cap-and-trade policy, economic cycles oscillating around a long-term economic growth trend and technological abatement options availability) are such that a mean-reverting around a trend makes sense.
Second, even though there is no mean-reverting level as such, stakeholders actions should ensure not much price deviation (as would be implied by modeling the price of carbon as a GBM for instance) from long-term equilibrium. On the one hand, there are forces that would strive to prevent the price of carbon from reaching extremely high level. Too high a price is the sign of a cap level hardly compatible with a healthy economic activity[8]. On the other hand, there are forces eager to see the price of carbon reach a minimum threshold[9]. As such, market phases negotiations are the occasion to reset the rules in order to adjust any fundamental flaw in the market design (like the implied ban on banking between phase I and II during the trial phase). This is achieved on the regulator side by modifying the cap and other elements of policy design (flexibility, derogations, etc.). On the regulated side, lobbying, pressuring and legal challenges are the tools of the trade.
Third, commodities have often been modeled as MR processes (Pindyck, 1999 [37] and Schwartz, 1997 [38]) allowing to reflect some long-term cost of production, extraction or abatement. Even though it was argued that the GBM was as good as a MR process when applied to a ROA framework, the current carbon spot prices are so remote from what a long-term equilibrium should be that a GBM would not be appropriate. A quick look at other mandatory emissions markets (SO_2 and NOx markets in the US) long term price evolution confirms our intuition of mean-reverting prices. The strong link with other energy commodity markets known for being mean-reverting goes in the same direction. Regarding the SO_2 and NOx markets, it appears that price mean-reverted around a downward trend which can be interpreted as technological breakthroughs to allow for emissions reduction and their penetration among

[7]Of course, it is ultimately each decision maker's task to resort to the price process he deems the most appropriate. The same comment applies to the fitting of the process retained.

[8]The effect on the economy and society could be disruptive (insufficient power generation capacity, loss of international competitiveness for industries subject to carbon leakage, etc.). There exist a non-observable upper bound for the price of carbon reflecting the acceptability of compliance buyers above which their survival would be at stake (exit threshold).

[9]In this respect: the regulators (the EC and EU Member States) are urged to implement successfully the policy to justify their legitimacy to act as such. So are the politics who mandated the regulators and the international community pressuring the EU Member States to respect the engagement to reduce carbon emissions. NGOs and carbon market observers would monitor the evolution of carbon price and would publicly advocate for environmental consciousness in case things go wrong. Carbon-reducing and carbon-neutral technology developers are concerned with keeping the incentive to maintain the development of such technologies and ensuring commercial prospects thereafter or own compliance prospects. Finally, regulated entities themselves would push for meaningful carbon prices as a way to establish barriers to entry or at least increase the cost to enter the market.

compliance buyers.

We now turn to the carbon price modeling retained. Let p_t^c denote the spot price of a carbon emission allowance (in EUR/tCO_2) at time t. We assume that the p_t^c is a continuous state stochastic variable following an exogenous mean-reverting continuously-valued process with a linear trend and constant volatility (one-factor model based on the log spot price from Lucia and Schwartz, 2000 [39]):

$$\begin{cases} ln(p_t^c) & = h_t^{c*} + X_t^c \\ h_t^{c*} & = \alpha^c + \beta^c.t \quad \text{(linear deterministic trend)} \\ dX_t^c & = -\theta^c.X_t^c.dt + \sigma^c dW_t^c \end{cases}$$

In which, the log of the spot carbon price is expressed as the sum of (1) a totally predictable deterministic function of time (h_t^{c*}) and (2) a diffusion stochastic process (X_t^c) and where:

- θ^c is the constant mean reversion speed for the log of the carbon price;
- $h_t^{c*} = \alpha^c + \beta^c t$ is the linear trend for the log of the price of carbon (not a constant as in the Ornstein-Uhlenbeck model);
- σ^c represents the constant volatility of the instantaneous log-price variation;
- W_t^c is a standard Brownian motion for the log of the carbon price (providing unexpected price shocks).

The linear trend for the price of carbon can be interpreted as the long run price depending on time. In our model, this reflects future demand for abatement and future abatement options available in the marginal abatement cost curve.

Given our risk-neutral framework, we express the price of carbon according to:

$$\begin{cases} ln(\hat{p}_t^c) & = h_t^{c*} + \hat{X}_t^c \\ h_t^{c*} & = \alpha^c + \beta^c.t \quad \text{(linear deterministic trend)} \\ d\hat{X}_t^c & = \theta^c.(-\lambda^c.\frac{\sigma^c}{\theta^c} - \hat{X}_t^c).dt + \sigma^c d\hat{W}_t^c \end{cases}$$

Where the market price of risk for carbon, λ^c, is assumed to be a constant and the hat superscript used here denotes the move from the real world to the risk-neutral world.

Finally, we note that compliance with the EU ETS is most likely to be achieved by means of forward transactions thereby reflecting expectations and adjustments regarding emissions levels and availability of compliance assets. We assume that emissions allowances are purchased with annual forward or Futures contracts[10]. We decide now of the transaction terms and exchange cash versus allowances at the convened price at the maturity date.

Therefore, following Lucia and Schwartz (2000 [39]), we determine the forward price of carbon (now for a maturity T):

$$\begin{aligned} F_{0,T}^c &= \mathbb{E}_0^Q(P_T^c) \\ &= exp[\underbrace{h_T^{c*}}_{1} + \underbrace{(ln(p_0^c) - h_0^{c*}).e^{-\theta^c.T}}_{2} - \underbrace{\lambda^c.\frac{\sigma^c}{\theta^c}.(1 - e^{-\theta^c.T})}_{3} + \underbrace{\frac{(\sigma^c)^2}{4.\theta^c}.(1 - e^{-2.\theta^c.T})}_{4}] \end{aligned}$$

[10]Note that since we use a constant discount rate, there is no difference between both types of contracts [39].

In which, the log of the forward price is comprised of[11]:

1. a deterministic trend component at time T;

2. the spot price deviation from trend at t=0 multiplied by an adjustment/discount factor;

3. the mean-reverting level of the detrended process multiplied by one minus the adjustment factor;

4. the ratio of carbon price variance to its mean-reverting level multiplied by one minus twice the adjustment factor.

(3) The prices of electricity
The literature on the stochastic modeling of electricity prices (Geman, 2006 [40]; He, 2007 [2]) identifies that power prices have the following characteristics:

- High spot price volatility and volatility clustering effect (periods of high volatility tend to be followed by similar periods);

- Mean reversion to the marginal cost of production (like most commodities);

- Seasonality (intraday, weekly and annual);

- Price jumps reflecting supply shocks (power plant outage) or unexpected demand;

- Market specific prices (reflecting the existing generation mix, demand profile and incentive policies).

These characteristics pertain most to spot prices. There are two major way to model power spot prices using reduced-form models (i.e. directly modeling the time series, thereby avoiding to build an equilibrium model). First, single factor models are the simplest type of reduced-form models[12]. They basically feature the drift and diffusion components aforementioned. Second, two-factor models build on the previous category and intend to complete the analysis by giving a stochastic behavior to one of the component of the single factor models (drift or diffusion)[13].

Given that the spot market in Europe is almost exclusively an adjustment market (the real options literature involving power prices reflects largely a focus on derivatives pricing), we assume that the power plants that would be built would sell their production using exclusively forward transactions. This is quite realistic in the light of current European utilities practice. This basically means that we reduce the volatility, drop the seasonality and price jump features compared to spot price modeling and resort to a single factor model taking into account mean-reversion.

While the price of a ton of carbon is de facto EU-wide, it is not that simple for the price of a MWh generated and sold. The price of a MWh fundamentally depends on the power plant status in the generation merit order related to a given demand source (country-wide most often) and for a given time. Given the power plant investment options suggested in the next section and more exactly the capacity, availability and competing power plants, the plant should either operate as a peakload or as a baseload

[11]Note that too high a mean-reversion speed makes forward/Futures prices identical across several process generation - the trend component having an overwhelming influence over the rest.

[12]They are composed of Arithmetic Brownian Motion, Geometric Brownian Motion, the Ornstein-Uhlenbeck model, the Geometric Ornstein-Unhlenbeck model (in which the logarithm of the price follows the Ornstein-Uhlenbeck model) and the Inhomogeneous Geometric Brownian Motion (which captures both the mean reversion and the price proportional characteristics of electricity prices).

[13]Stochastic volatility, stochastic long-term equilibrium price, etc. This category also features attempts to split short-term behavior from long-term behavior, jump-diffusion models and regime-switching models.

plant. In our modeling environment, we assume that the CCGT, pulverized coal and IGCC plants would operate as peakload plants and sell their power at peakload forward prices (p_t^p). Conversely, the nuclear plant would operate as a baseload plant and sell its power at baseload forward prices (p_t^b). We suggest modeling baseload and peakload power prices as mean-reverting processes with a linear trend (just like we did for carbon). The modeling should remain the same - only the fitting of parameters should change. Additionally, sale of power generated by renewable energy sources often benefits from an incentive regime, be it tradable green certificates as in the UK or feed-in tariffs as in France. For the wind offshore investment alternative considered, we assume that the power generated can be sold at feed-in tariffs (p_t^f) over the applicable period: EUR 130/MWh for the first ten years and EUR 64/MWh for the remaining 10 years reflecting the current French feed-in tariffs[14].

Further, it should be acknowledged that the introduction of the EU ETS has hardly been neutral on the electricity prices[15]. We thus need to account for the linkages among the price processes. In the literature, two approaches have been suggested.

On the one hand, carbon and power stochastic prices can be positively correlated to account for the relationship between those prices. Szolgayva et al. (2008) [36] and Fuss et al. (2008) [24] explicitly allow for some passthrough via a positive correlation between the noises of the electricity and the carbon price processes. The increments of the Wiener processes of electricity and carbon are assumed correlated at 0.7. They assert that the positive value is implying that disturbances in the carbon price are positively reflected in those of electricity. In Laurikka and Koljonen (2006) [23], the price of carbon allowance is modeled jointly with the price of baseload electricity using a quadrinomial tree. The relationship between the two prices is summarized in a correlation factor which can take the value of either 0 or 0.5. A causality study of carbon, electricity, coal, gas and stock prices (Keppler and Mansanet-Bataller, 2009 [41]) identifies that the Granger causality relationship between carbon and electricity prices evolves from phase I to phase II. This could support the idea that simulation of power and carbon prices need to be more refined than a constant correlation factor[16].

On the other hand, some authors explicitly modeled the level of passthrough (see Laurikka and Koljonen, 2006 [23]). Consequently, the estimated price of baseload electricity is the simulated baseload price in the absence of an emissions trading scheme (a counterfactual or business-as-usual - BAU - price in other words) to which is added the price of carbon times an estimated transformation factor. That approach has the advantage to account for the potentially directional relationship from carbon prices to electricity prices while having the disadvantage to require the modeling of a forward-looking BAU electricity price. Laurikka and Koljonen (2006) [23] estimate that transformation factor between 0.22 and 0.77 depending upon the prevailing BAU electricity price.

We now turn to the modeling retained for peakload and baseload prices. The approach retained is similar to that of the price of carbon. Moving directly to the risk-neutral world:

$$\begin{cases} ln(\hat{p}_t^p) & = h_t^{p*} + \hat{X}_t^p \quad \text{(for peakload power spot price)} \\ h_t^{p*} & = \alpha^p + \beta^p.t \quad \text{(linear deterministic trend)} \\ d\hat{X}_t^p & = \theta^p.(-\lambda^p.\frac{\sigma^p}{\theta^p} - \hat{X}_t^p).dt + \sigma^p d\hat{W}_t^p \end{cases}$$

$$\begin{cases} ln(\hat{p}_t^b) & = h_t^{b*} + \hat{X}_t^b \quad \text{(for baseload power spot price)} \\ h_t^{b*}* & = \alpha^b + \beta^b.t \quad \text{(linear deterministic trend)} \\ d\hat{X}_t^b & = \theta^b.(-\lambda^b.\frac{\sigma^b}{\theta^b} - \hat{X}_t^b).dt + \sigma^b d\hat{W}_t^b \end{cases}$$

[14]For investment renewal 20 years from initial investment, we assume that the support scheme has ended and that baseload power prices apply for valuation purpose.

[15]For instance, refer to the BundesKartellamt decisions in Germany on RWE & E.ON alleged passthrough as early as 2005.

[16]We leave this point to further research.

where:

- θ^p and θ^b are the constant mean-reversion speeds for the log of peakload and baseload electricity price;
- $h_t^{p*} = \alpha^p + \beta^p.t$ is the linear trend for the log of the price of peakload power;
- $h_t^{b*} = \alpha^b + \beta^b.t$ is the linear trend for the log of the price of baseload power;
- σ^p and σ^b representing the constant volatility of the instantaneous log-price variation for peakload and baseload electricity prices;
- λ^p and λ^b are the market prices of risk for the log of the peakload and baseload power prices;
- W_t^p and W_t^b are standard Brownian motions for the log of the peakload and baseload power prices.

We now express forward prices for the stochastic processes of power price:

$$\begin{aligned} F_{0,T}^p &= \mathbb{E}_0^Q(P_T^p) \quad \text{(for peakload power forward price)} \\ &= exp[h_T^{p\,*} + (ln(p_0^p) - h_0^{p*}).e^{-\theta^p.T} - \lambda^p.\frac{\sigma^p}{\theta^p}.(1 - e^{-\theta^p.T}) \\ &\quad + \frac{(\sigma^p)^2}{4.\theta^p}.(1 - e^{-2.\theta^p.T})] \end{aligned}$$

$$\begin{aligned} F_{0,T}^b &= \mathbb{E}_0^Q(P_T^b) \quad \text{(for baseload power forward price)} \\ &= exp[h_T^{b\,*} + (ln(p_0^b) - h_0^{p*}).e^{-\theta^b.T} - \lambda^b.\frac{\sigma^b}{\theta^b}.(1 - e^{-\theta^b.T}) \\ &\quad + \frac{(\sigma^b)^2}{4.\theta^b}.(1 - e^{-2.\theta^b.T})] \end{aligned}$$

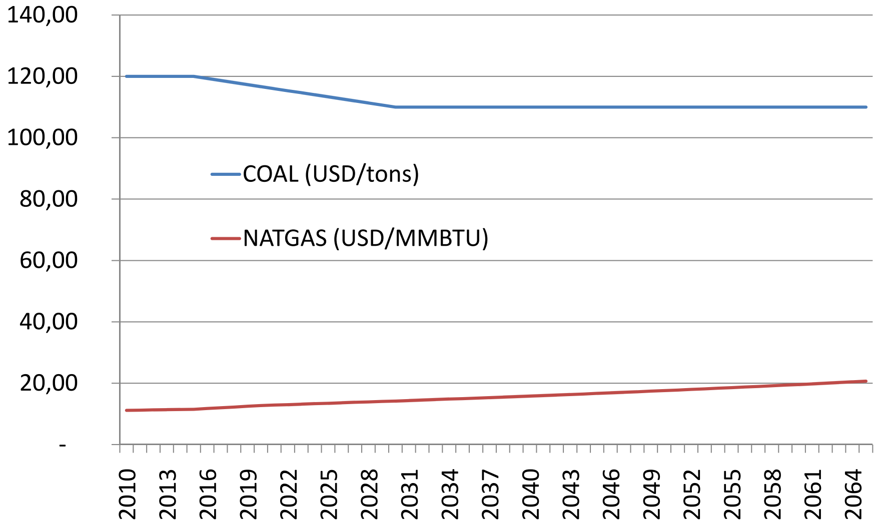
(4) Correlation among stochastic state variables
We also ensured that single price process generation would not deviate from the basic relationship among them. We therefore used constant correlation factors among the increments of the three Brownian motions involved ($\rho_{p,c}$, $\rho_{b,c}$ and $\rho_{p,b}$).

(5) The price of fossil fuels
In order to simplify the model used and strictly focus on carbon price uncertainty, we assume that fuel prices follow deterministic paths (that is, we know for sure the future prices of fuels).

Coal and natural gas are modeled as deterministic state variables consistent with the IEA 2008 price scenario assumptions (IEA, 2008 [42]). The IEA price scenario assumptions are the results of a top-down assessment of prior needs to encourage sufficient investment in supply and meet projected demand by 2030. In particular, it is assumed that the price of coal remains at USD 120/ton[17] of coal between 2010 and 2015 and linearly goes down to USD 110/ton of coal as new mining and transportation capacity becomes available. We further assume that coal prices remains at that level for the rest of our study horizon.
Similarly, the price of natural gas is expected to follow the following path in USD/MMBTU: 11.15 in 2010, 11.50 in 2015, 12.71 in 2020, 13.45 in 2025 and 14.19 in 2030. A linear interpolation between target prices and current prices is generated for the missing dates. Beyond 2030, we apply an annual

[17] We assume that 1.3705 EUR/USD consistent with the average FX rate in 2007 (WEO assumptions are expressed in 2007 USD) according to the ECB.

Figure 2: Price trajectories for fossil fuels (IEA, 2008)



growth rate of 1.077% reflecting the average growth rate between the last two target dates. Figure 2 illustrate both price trends.

Regarding uranium, we used a per MWh cost assumption instead of a dedicated price modeling given that (1) nuclear power plants either are supplemented with long-term uranium procurement contracts or the turnkey agreements incorporate such long-term contracts to begin with and (2) the volatility of nuclear ore prices and power plant valuation sensitivity to them is quite low. In particular, we assumed a nuclear fuel cost of EUR 15/MWh.

(6) Time and discount rate

We assume an investment window of 10 years starting from now (t=0). The frequency of decision points in time is annual ($t \in \|0; 10\|$). Given that power plant lifetime goes up to 40 years and building time can go up to 5 years, the horizon for simulations reaches 56 years.

The investment window retained makes our model a string of Bermudan call options with lookback features given that exercise is limited to certain dates within the life of the option and that the exercise does not necessarily kill the ability to subsequently invest in other power plants (budget permitting).

The risk-free discount rate used, r, is set at 4%.

2.2.2 The choice variable

There is a single discrete choice variable, namely the decision to invest in power plants. At any decision node in time, we may invest or wait one more period (for instance to see how the carbon price evolves). Should we wish to invest, we can invest in one power plant or a ”basket” of power plants. The investment alternatives are building a CCGT power plant (incur I^G), a supercritical pulverized coal power plant (incur I^K), an IGCC power plant (incur I^I), a nuclear power plant (incur I^N), an offshore wind power plant (incur I^W) or a combination of those. Once the initial investment cost has been incurred, we are entitled cash flows over the lifetime of the power plant.

Power plant characteristics including capital cost estimates are taken from the NEA, IEA and OECD projections (2005) [43] for European countries and from IEA (2008 [42]) when more up-to-date figures

make more sense[18].
For the purpose of the study, we considered five generation technologies:

- CCGT plants basically characterized by a moderate capital cost, high and volatile fuel procurement cost and an average carbon compliance cost;
- Supercritical pulverized coal plants characterized by a higher capital cost than CCGT plants, lower fuel procurement cost but higher carbon compliance cost than CCGT's. These power plants can be further retrofitted with CCS modules;
- IGCC power plants which gasification process makes it possible to switch burning fuel depending on relative costs, especially in the light of carbon emissions factors;
- Nuclear power plants characterized by a very high capital cost but a low fuel procurement cost and no carbon compliance cost;
- An offshore[19] wind park characterized by a high capital cost (relative to capacity) but no fuel procurement cost and no carbon compliance cost. Additionally, we assume that investment in these technologies is favored since they benefit from feed-in tariffs;

For each of those power plants, we report the value range for investment, emissions and technical data.

The CCGT plant total investment cost (initial investment cost and operation and maintenance cost over the life of the plant discounted at 8.5%) amounts to some EUR 914 million (see table 14 in annex for older cost estimates and value range in NEA, IEA and OECD, 2005 [43]). The plant takes 3 years to be built and will operate during 40 years. The thermal capacity of the power plant is set at 900 MW and its thermal efficiency is set at 59%. It is assumed that the plant will deliver power 42.5% of the year (3,723 hours). Based on this availability factor, the expected daily output for the power plant is 9,180 MWh (3.35 GWh per annum). Regarding carbon emissions, the emissions factor of the CCGT plant is assumed at 0.353 tCO_2/MWh (which amounts to 1.168 $MtCO_2$ on annual basis).
The pulverized coal plant represents a typical investment in a supercritical coal-fired plant (see table 15 in annex for value range). The pulverized coal unit total investment cost amounts to circa EUR 1,632 million. Again, the lifetime of the plant is set at 40 years and it only takes 3 years to build the plant. The thermal capacity of the plant is set at 800 MW and its thermal efficiency at 46%. With an availability factor of 42.5%, this represents 8.160 MWh on a daily basis (2.98 GWh p.a.). The emissions factor is higher than for the CCGT plant and reaches 0.728 tCO_2/MWh generated (equivalent to 2.168 $MtCO_2$ each year).
We also consider an integrated gasification combined cycle (IGCC) as an alternative to the pulverized coal plant (see table 16 in annex for value range). The IGCC plant total investment cost amounts to circa EUR 1,298 million. Lifetime of the plant is set at 40 years and 3 years are required to commission the plant. The thermal capacity of the plant is set at only 450 MW and its thermal efficiency is at 46%. With an availability factor of 42.5%, this represents 4.589 MWh on a daily basis (1.67 GWh p.a.). The emissions factor is lower than the pulverized coal plant's but still higher than the CCGT's and reaches 0.656 tCO_2/MWh generated (equivalent to 1.099 $MtCO_2$ each year).
The nuclear power plant is the first of the two carbon-free investment alternatives (see table 17 in annex for value range). The total investment cost (including discounted nuclear waste decommissioning) amounts to EUR 5,896 million. The plant takes 5 years to be built and will operate over 40 years. The

[18]Please note that recently power plant investment costs have been strongly rising (cost of materials, components, labor and lack of skilled engineers). We estimate that investment costs have more than doubled among the generation technologies considered in our study.
[19]For the sake of comparison among generation technologies in terms of generation capacity.

thermal capacity of the plant is 1,590 MW. With an availability factor of 85%, this represents 32.438 MWh on a daily basis (11.84 GWh p.a.).
The offshore wind plant is the other carbon-free investment alternative (see table 18 in annex for value range). The total investment cost reaches EUR 792 million. The wind farm takes 1 year to be built and will operate over 20 years. The average load factor of the wind farm is 42% and the capacity is 300 MW. This amounts to a potential 3.287 MWh on a daily basis (1.20 GWh p.a.).

Table 2 summarizes our assumptions for the power plant investment alternatives.

Table 2: Power plant assumption data

	CCGT Plant	PC	IGCC	Nuclear	Wind
Construction length - in years	3	3	3	5	1
Lifetime - in years	40	40	40	40	20
Thermal capacity - in MWe	900	800	450	1590	300
Thermal efficiency - in %	59	40	46	36	-
Average load factor - in %	-	-	-	-	42
Expected annual output - in GWh	3.35	2.98	1.67	11.84	1.20
CO_2 emissions factor - in tCO_2/MWh	0.353	0.728	0.656	0.000	0.000
Annual carbon emissions - in $MtCO_2$	1.168	2.168	1.099	0.000	0.000
Total investment costs - in EUR million	914	1632	1298	5896	792

Given an initial budget of EUR 5.9 billion, this implies that the budget variable can take any of the following values:

$$b_t \in \{ \underbrace{4}_{\text{After nuclear x1}} ; ...; \underbrace{5900}_{\text{Untapped}} \}, \forall t$$

and the control variable:

$$x_t \in \{ \underbrace{0}_{\text{Wait}} ; \underbrace{792}_{\text{Wind x1}} ; \underbrace{914}_{\text{CCGT x1}} ; ...; \underbrace{5896}_{\text{Nuclear x1}} \}, \forall t$$

2.2.3 The functions

The reward function
The reward function f_t identifies immediate reward from undertaking a specific choice at time t. This reward corresponds to the net present value (NPV) of given investment combination alternatives. Note that the value taken by this function depends on market prices conditions, the timing of investment, the budget level and the investment combinations decided upon. We identified 121 unique combinations of

generation technologies[20].

$$f_t(b_t, x_t, S_t) = \begin{cases} 0 & \text{for } x_t = 0, \\ NPV_t^G & \text{for } x_t = I^G, \\ NPV_t^W & \text{for } x_t = I^W, \\ NPV_t^I & \text{for } x_t = I^I, \\ NPV_t^K & \text{for } x_t = I^K, \\ 2.NPV_t^G & \text{for } x_t = 2.I^G, \\ ..., & \\ NPV_t^N & \text{for } x_t = I^N. \end{cases}$$

$$s.t.\ \ x_t \leq b_t\ , \forall t$$

Where the NPV for a given technology at time t is the sum of discounted annual cash flow minus investment cost:

$$NPV_t^{tech} \quad = \sum_{j=t+build^{tech}}^{t+build^{tech}+life^{tech}} [\Pi_j^{tech}.e^{-r.j}] - I^{tech}$$

In which:

- $\Pi_t^G = q^G.[(\frac{\sum_{j=1}^{24} F_{t-j/12,\ t}^p}{24}) - p_t^g/TE^G - (\frac{\sum_{j=1}^{24} F_{t-j/12,\ t}^c}{24}.EF^G)]$ annual cash flow for the CCGT plant;
- $\Pi_t^W = q^W.p_t^f$ annual cash flow for the wind power plant benefiting from feed-in tariffs (first 20 years);
- $\Pi_t^W = q^W.[\frac{\sum_{j=1}^{24} F_{t-j/12,\ t}^b}{24}]$ annual cash flow for the wind power plant after having benefited from feed-in tariffs (next 20 years);
- $\Pi_t^I = q^I.[(\frac{\sum_{j=1}^{24} F_{t-j/12,\ t}^p}{24}) - p_t^k/TE^I - (\frac{\sum_{j=1}^{24} F_{t-j/12,\ t}^c}{24}.EF^I)]$ annual cash flow for the IGCC plant;
- $\Pi_t^K = q^K.[(\frac{\sum_{j=1}^{24} F_{t-j/12,\ t}^p}{24}) - p_t^k/TE^K - (\frac{\sum_{j=1}^{24} F_{t-j/12,\ t}^c}{24}.EF^K)]$ annual cash flow for the pulverized coal plant;
- $\Pi_t^N = q^N.[(\frac{\sum_{j=1}^{24} F_{t-j/12,\ t}^b}{24}) - 15]$ annual cash flow for the nuclear plant;

And:

- q^G, q^W, q^I, q^K, and q^N are the annual quantities of electricity (in MWh) produced by the CCGT, wind, IGCC, pulverized coal and nuclear plant respectively;
- $F_{t-j/12,\ t}^c$, $F_{t-j/12,\ t}^p$, and $F_{t-j/12,\ t}^b$ are the forward prices (agreed upon at $t - j/12$ and settling in t) for carbon and electricity (peakload and baseload) respectively;
- p_j^g, p_j^k and p_j^f are the average annual spot prices for natural gas and coal and the feed-in tariff for offshore wind;

[20] In case the condition $x_t \leq b_t$ is not respected, we will assume, for valuation purpose, that $f_t(b_t, x_t, S_t)$ takes the value of $-\infty$.

- TE^I, TE^K and TE^G are the thermal efficiencies of the IGCC, pulverized coal and CCGT plants respectively;
- EF^I, EF^K and EF^G are the carbon emissions factors (in tCO_2/MWh) of the IGCC, pulverized coal and CCGT plant respectively;
- r corresponds to the zero-coupon rate (the risk-free rate);
- $life^G$, $life^W$, $life^I$, $life^K$ and $life^N$ are the life times of the CCGT, wind, IGCC, pulverized coal and nuclear plant respectively;
- $build^G$, $build^W$, $build^I$, $build^K$ and $build^N$ are the construction times of CCGT, wind, IGCC, pulverized coal and nuclear plant respectively;

We acknowledge that the utility, unwilling to remain exposed to price risk, would engage in forward transaction to secure future cash flows. We assume a monthly risk management meeting in which the utility hedge 1/24 of the next two years' expected production for both power and carbon prices. This seems a reasonable assumption in light of calendar contracts liquidity on market places and market practice as indicated by European utilities annual reports[21]. This implies that the price at which the power production is sold at time t is not the contemporary spot price but rather an average of the last two years calendar forward prices (on the basis of a monthly transaction for 1/24 of the annual production in year t) resulting in a smoother prices less exposed to high spot price volatility.

The value function

The principle of optimality applied to our discrete time mixed states decision models yield Bellman's recursive functional equation. Here, V_t denotes the maximum attainable sum of current and expected future rewards given that the processes are in states b_t and S_t in period t:

$$V_t(b_t, S_t) = \max_{x_t} \left\{ \underbrace{f_t(b_t, x_t, S_t)}_{\text{immediate reward component}} + \underbrace{e^{-r}.\mathbb{E}_t^Q[V_{t+1}(b_t - x_t, \tilde{S}_{t+1})]}_{\text{discounted expected reward component}} \right\} , \forall\, b_t \text{ and } \forall\, S_t \tag{1}$$

The first element of the Bellman equation corresponds to the immediate benefits (f) while the second element corresponds to the discounted expected future benefits (knowing S_t).
This latter component is also known, in the financial option terminology, as the continuation value and is estimated by OLS following the method suggested by Longstaff ans Schwartz (2001, [14]).

The post-terminal value function

Since we are in a finite horizon problem, the investor cannot invest after T periods but may earn a final reward V_{T+1} which corresponds to the remaining immediate investment opportunity of the possible investment "baskets". We assume no continuation value after T, so that at expiration:

$$V_T(b_T, S_T) = \max_{x_T} \left\{ \underbrace{f_T(b_T, x_T, S_T)}_{\text{immediate reward component}} \right\} , \forall\, b_T \text{ and } \forall\, S_T \tag{2}$$

In our backward recursion setting, this will be our starting point. With V_T, we can find recursively V_{T-1} for all states (b_T, S_T). With V_{T-1}, we can find recursively V_{T-2} for all states (b_{T-1}, S_{T-1}) and so on until $V_0(\bar{b}, S_0)$ is derived and the optimal policy established since there is no uncertainty at t=0 so that we can work our way forward into the recursion.

[21]RWE financial statements for 2008 indicates that, in fiscal year 2008, the utility actually hedged nearly 100% of its expected power production for 2009 and approximately 70% for 2010 (by selling power using forward transaction).

2.3 Calibrating the stochastic processes

It should be reminded that once the price process has been chosen, a critical step is the calibration thereof. In addition to that, it should be stressed that usually at least 30 years of historical data is required in order to properly calibrate a model (Dixit and Pindyck, 1994 [15]; Keppler et al., 2006 [44]). In our case, this is obviously impossible (in the carbon and power prices cases). Hence, the initial parameters estimated should suffice and would constitute our base case. Later, we will look at the sensitivity of the investments decided upon given the parameters.

2.3.1 Fitting the stochastic price of carbon

The literature shows that the calibration of the carbon price processes is a mix of inputs from econometric analysis of historical data, model output (like the IIASAs GGI Scenario database) and judgmental input be it a shadow price (*valeur tutélaire du carbone* in France for instance) or academic and professional expert price elicitation survey (like in Sekar, 2005 [21] and Bohm et al., 2007 [19]).

We first fit the process with historical data using econometric techniques. We will discuss the economic meaning around those parameters in the later section on parameter sensitivity study.

2.3.2 Fitting the stochastic prices of power

2.3.3 Estimating correlations among stochastic processes

We estimated correlations between the spot price of baseload electricity, peakload electricity and carbon ($\rho_{p,c}$, $\rho_{b,c}$ and $\rho_{p,b}$) using time series employed for fitting the price processes (see table 3 for the estimated correlations).

Table 3: Correlation among stochastic price processes

$\rho_{x,y}$	dW^p	dW^b	dW^c
dW^p	1.00	0.95	0.49
dW^b	-	1.0	0.55
dW^c	-	-	1.0

3 Illustrative cases

In this section, we present simpler case studies to grasp how (1) the capital rationing constraint and (2) the price uncertainty can be handled.

3.1 3-period 2-technology deterministic case

In order to illustrate how to solve the capital rationing issue, we detail calculations for a 3-period deterministic case. We consider two technologies, A and B, with investment costs of I^A and I^B irrespective of time. We are constrained by a budget of $\bar{b}$. We may invest in a combination of technologies now, next year or two years from now. To do so, we incur investment costs and benefit from resulting NPVs.
We assume the following: e^{-r} = 0.909; $\bar{b}$ = 1,000; I^A = 400; I^B = 700; NPV_t^A = 200 ,$\forall t$; NPV_t^B = 300 for $t = \{0; 1\}$ and NPV_t^B = 500 for t=2.

Finding the allowed investment combinations

The first step entails determining what are the allowed investment combinations. We are constrained by the capital rationing so that $x_t \leq b_t$, $\forall t$.
Denoting Q^A and Q^B, the quantity of technologies we invest in, we must satisfy:

$$x_t = I^A.Q^A + I^B.Q^B \leq b_t$$

Here, we easily see that the control variable can take the following values:

$$\begin{aligned} x_t &\in \{\, 0;\ I^A;\ I^B;\ 2.I^A\} \\ &\in \{\, 0;\ 400;\ 700;\ 800\} \end{aligned}$$

And the budget can therefore take the following values:

$$\begin{aligned} b_t &\in \{\, \bar{b} - 2.I^A;\ \bar{b} - I^B;\ \bar{b} - I^A;\ \bar{b}\} \\ &\in \{\, 200;\ 300;\ 600;\ 1000\} \end{aligned}$$

At t=2
We start from the last decision node at *t*=2. The value function takes the following form:

$$V_2(b_2) = \max_{x_2} \{f_2(b_2, x_2)\}$$

At the last decision node, we have no continuation value since unused budget is assumed to have no value. We consider all the possible budget levels and determine the value function accordingly[22]:

$$\begin{aligned}
V_2(\bar{b} - 2.I^A) &= \max_{x_2} \left\{ f_2(\bar{b} - 2.I^A, 0) \right\} \\
&= 0 \text{ with } x_2^*\text{=0.} \\
V_2(\bar{b} - I^B) &= \max_{x_2} \left\{ f_2(\bar{b} - I^B, 0) \right\} \\
&= 0 \text{ with } x_2^*\text{=0.} \\
V_2(\bar{b} - I^A) &= \max_{x_2} \left\{ f_2(\bar{b} - I^A, 0); f_2(\bar{b} - I^A, I^A) \right\} \\
&= \max_{x_2} \left\{ 0; NPV_2^A \right\} \\
&= \max_{x_2} \{0; 200\} \\
&= 200 \text{ with } x_2^*\text{=}I^A. \\
V_2(\bar{b}) &= \max_{x_2} \left\{ f_2(\bar{b}, 0); f_2(\bar{b}, I^A); f_2(\bar{b}, I^B); f_2(\bar{b}, 2.I^A) \right\} \\
&= \max_{x_2} \left\{ 0; NPV_2^A; NPV_2^B; 2.NPV_2^A \right\} \\
&= \max_{x_2} \{0; 200; 500; 400\} \\
&= 500 \text{ with } x_2^*\text{=}I^B.
\end{aligned}$$

At t=1
We move one step back in time to *t*=1. The value function now takes the following form since there is a continuation value component involved:

$$V_1(b_1) = \max_{x_1} \left\{ f_1(b_1, x_1) + e^{-r}.V_2(b_1 - x_1) \right\}$$

[22]For the first two budget levels, only one possibility remains, that is to do nothing/wait. The three other possible choices make us exhaust the budget limit.

We consider all the possible budget levels and determine the value function accordingly:

$$
\begin{aligned}
V_1(\bar{b}-2.I^A) &= \max_{x_1}\left\{f_1(\bar{b}-2.I^A,0)+e^{-r}.V_2(\bar{b}-2.I^A)\right\}\\
&= 0 \text{ with } x_1^*\text{=0.}\\
V_1(\bar{b}-I^B) &= \max_{x_1}\left\{f_1(\bar{b}-I^B,0)+e^{-r}.V_2(\bar{b}-I^B)\right\}\\
&= 0 \text{ with } x_1^*\text{=0.}\\
V_1(\bar{b}-I^A) &= \max_{x_1}\left\{f_1(\bar{b}-I^A,0)+e^{-r}.V_2(\bar{b}-I^A); f_1(\bar{b}-I^A,I^A)+e^{-r}.V_2(\bar{b}-2.I^A)\right\}\\
&= \max_{x_1}\left\{0+e^{-r}.NPV_2^A; NPV_1^A+0\right\}\\
&= \max_{x_1}\left\{182;200\right\}\\
&= 200 \text{ with } x_1^*\text{=}I^A.\\
V_1(\bar{b}) &= \max_{x_1}\left\{f_1(\bar{b},0)+e^{-r}.V_2(\bar{b}); f_1(\bar{b},I^A)+e^{-r}.V_2(\bar{b}-I^A);\right.\\
&\qquad \left. f_1(\bar{b},I^B)+e^{-r}.V_2(\bar{b}-I^B); f_1(\bar{b},2.I^A)+e^{-r}.V_2(\bar{b}-2.I^A)\right\}\\
&= \max_{x_1}\left\{0+e^{-r}.NPV_2^B; NPV_1^A+e^{-r}.NPV_2^A; NPV_1^B+0; 2.NPV_1^A+0\right\}\\
&= \max_{x_1}\left\{455;381;300;400\right\}\\
&= 455 \text{ with } x_1^*\text{=0.}
\end{aligned}
$$

At t=0

We move one step back in time to t=0 (now). The value function again takes the following form:

$$V_0(b_0)=\max_{x_0}\left\{f_0(b_0,x_0)+e^{-r}.V_1(b_0-x_0)\right\}$$

Compared to t=1 and t=2, we only have one possible budget level, $\bar{b}$, the initial endowment.

$$
\begin{aligned}
V_0(\bar{b}) &= \max_{x_0}\left\{f_0(\bar{b},0)+e^{-r}.V_1(\bar{b}); f_0(\bar{b},I^A)+e^{-r}.V_1(\bar{b}-I^A);\right.\\
&\qquad \left. f_0(\bar{b},I^B)+e^{-r}.V_1(\bar{b}-I^B); f_0(\bar{b},2.I^A)+e^{-r}.V_1(\bar{b}-2.I^A)\right\}\\
&= \max_{x_0}\left\{0+e^{-2r}.NPV_2^B; NPV_0^A+e^{-r}.NPV_1^A; NPV_0^B+0; 2.NPV_0^A+0\right\}\\
&= \max_{x_0}\left\{413;381;300;400\right\}\\
&= 413 \text{ with } x_0^*\text{=0.}
\end{aligned}
$$

The optimal path

$V_0(\bar{b})$ represents the maximum value that can be attained in the investment framework considered. The optimal path represents the decisions that must be taken sequentially in order to realize that maximum value. At t=0, the optimal decision is to wait (x_0^*=0), the budget remains intact. Moving forward in the tree, we look for $V_1(\bar{b})$ and again the optimal decision is to wait (x_1^*=0). Moving to the last decision node, we look for $V_2(\bar{b})$ and find that the optimal decision is to invest in one unit of technology B (x_2^*=I^B). The maximum attainable gain is realized by purchasing one unit of technology B two years from now. A now-or-never DCF framework would have yield a myopic investment in two units of technology A now, EUR 13 million less than accounting for the timing option.

3.2 3-period 2-technology stochastic case

We now add uncertainty to the NPV of one of the technologies. In particular, we generate eight price paths for one source of uncertainty (the price of baseload power). This source of uncertainty only pertains to technology B. Technology A has the same NPV whenever we decide to invest: NPV_t^A = 250

, $\forall t$. Table 4 compiles the eight price paths generated for the source of uncertainty assumed here. Note that the price of baseload power at t=0 is known for sure[23]. The set of stochastic state variables, S_t^i, denotes here solely the price of baseload power at time t on price path i.

Table 4: Illustrative case - Price paths for baseload power

S_t^i	t=0	t=1	t=2	t=3	...	t=43
1	50.52	53.29	54.80	57.51	...	150.89
2	50.52	54.05	55.92	58.43	...	159.45
3	50.52	54.24	55.87	58.72	...	154.30
4	50.52	53.20	56.18	59.57	...	167.61
5	50.52	53.81	56.04	57.90	...	151.14
6	50.52	54.69	57.89	59.35	...	154.80
7	50.52	53.04	54.84	57.08	...	153.14
8	50.52	53.88	57.08	59.80	...	152.36

Based on those price paths, we obtain eight NPV paths for technology B. We denote $NPV_t^{B,i}$, the NPV of technology B at time t on path i. Table 5 presents the hypothesized eight NPV paths for technology B.

Table 5: Illustrative case - Implied NPV paths for technology B

$NPV_t^{B,i}$	t=0	t=1	t=2
1	278	372	461
2	495	598	699
3	321	417	508
4	751	865	976
5	261	354	444
6	573	674	767
7	241	337	430
8	502	597	686

The value function now takes the following form:

$$V_t(b_t, S_t^i) = \max_{x_t} \left\{ f_t(b_t, x_t, S_t^i) + e^{-r}.\mathbb{E}_t^Q[V_{t+1}(b_t - x_t, \tilde{S}_{t+1}^i)] \right\} \text{ , } \forall i \text{ and } \forall t.$$

And at expiration

$$V_T(b_T, S_T^i) = \max_{x_T} \left\{ f_T(b_T, x_T, S_T^i) \right\} \text{ , } \forall i.$$

At t=2

We start from the last decision node at t=2. The value function takes the following form:

$$V_2(b_2, S_2^i) = \max_{x_2} \left\{ f_2(b_2, x_2, S_2^i) \right\} \text{ , } \forall i.$$

[23] A high growth rate has been retained for illustrative purpose.

At the last decision node, we have no continuation value since unused budget is assumed to have no value. We consider all the possible budget levels and determine the value function accordingly:

$$
\begin{aligned}
V_2(\bar{b}-2.I^A, S_2^i) &= \max_{x_2}\left\{f_2(\bar{b}-2.I^A, 0, S_2^i)\right\} \\
&= 0 \text{ with } x_2^*{=}0 \text{ and } \forall i. \\
V_2(\bar{b}-I^B, S_2^i) &= \max_{x_2}\left\{f_2(\bar{b}-I^B, 0, S_2^i)\right\} \\
&= 0 \text{ with } x_2^*{=}0 \text{ and } \forall i. \\
V_2(\bar{b}-I^A, S_2^i) &= \max_{x_2}\left\{f_2(\bar{b}-I^A, 0, S_2^i); f_2(\bar{b}-I^A, I^A, S_2^i)\right\} \\
&= \max_{x_2}\left\{0; NPV_2^A\right\} \\
&= \max_{x_2}\left\{0; 250\right\} \\
&= 250 \text{ with } x_2^*{=}I^A \text{ and } \forall i.
\end{aligned}
$$

The untapped budget level case (b_2=$\bar{b}$) is the only one allowing investment in technology B and hence featuring uncertainty.

$$
\begin{aligned}
V_2(\bar{b}, S_2^i) &= \max_{x_2}\left\{f_2(\bar{b}, 0, S_2^i); f_2(\bar{b}, I^A, S_2^i); f_2(\bar{b}, I^B, S_2^i); f_2(\bar{b}, 2.I^A, S_2^i)\right\} \\
&= \max_{x_2}\left\{\ 0;\ NPV_2^A;\ NPV_2^{B,i};\ 2.NPV_2^A\right\}
\end{aligned}
$$

In table 6, we detail the investment alternatives at t=2 when the budget is full and highlight in bold the maximum value and associated decision taken.

Table 6: Illustrative case - Decision nodes at t=2 and optimal decision for untapped budget

Path	0	NPV_2^A	$NPV_2^{B,i}$	$2.NPV_2^A$	x_2^*
1	0	250	461	**500**	$2.I^A$
2	0	250	**699**	500	I^B
3	0	250	**508**	500	I^B
4	0	250	**976**	500	I^B
5	0	250	444	**500**	$2.I^A$
6	0	250	**767**	500	I^B
7	0	250	430	**500**	$2.I^A$
8	0	250	**689**	500	I^B
average	0	250	**622**	500	I^B

In tables 7 and 8, we summarize the value functions and optimal decisions for each budget level and each path at t=2.

Table 7: Illustrative case - Value function vs. budget level at t=2

Path	$b_2=\bar{b}-2.I^A$	$b_2=\bar{b}-I^B$	$b_2=\bar{b}-I^A$	$b_2=\bar{b}$
1	0	0	250	500
2	0	0	250	699
3	0	0	250	508
4	0	0	250	976
5	0	0	250	500
6	0	0	250	767
7	0	0	250	500
8	0	0	250	689

Table 8: Illustrative case - Optimal decision vs. budget level at t=2

Path	$b_2 = \bar{b} - 2.I^A$	$b_2 = \bar{b} - I^B$	$b_2 = \bar{b} - I^A$	$b_2 = \bar{b}$
1	0	0	I^A	$2.I^A$
2	0	0	I^A	I^B
3	0	0	I^A	I^B
4	0	0	I^A	I^B
5	0	0	I^A	$2.I^A$
6	0	0	I^A	I^B
7	0	0	I^A	$2.I^A$
8	0	0	I^A	I^B

At t=1

The value function now takes the following form:

$$V_1(b_1, S_1^i) = \max_{x_1} \left\{ f_1(b_1, x_1, S_1^i) + e^{-r}.\mathbb{E}_1^Q[V_2(b_1 - x_1, \tilde{S}_2^i)] \right\} \ , \forall i.$$

Note that the exercise decision at t=1 cannot exploit knowledge of the future (i.e. the value taken at t=2) on a given path. We are not replacing a stochastic problem by 8 deterministic problems. Rather, we are regressing value functions discounted back at t=1 against the value of S_t^i. We are using our set of scenarios to build an approximation of the conditional expectation continuation value component. That is the key idea of the Longstaff and Schwartz method. Note that we only do so when stochasticity is involved, i.e. when we may invest in technology B[24].

We proceed like in the deterministic case by detailing the value function in t=1 for all the budget combinations.

$$\begin{aligned}
V_1(\bar{b} - 2.I^A, S_1^i) &= \max_{x_1} \left\{ f_1(\bar{b} - 2.I^A, 0, S_1^i) + e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b} - 2.I^A, \tilde{S}_2^i)] \right\} \\
&= \max_{x_1} \left\{ f_1(\bar{b} - 2.I^A, 0, S_1^i) + e^{-r}.V_2(\bar{b} - 2.I^A, S_2^i) \right\} \\
&= 0 \text{ with } x_1^*\text{=0 and } \forall i. \\
V_1(\bar{b} - I^B, S_1^i) &= \max_{x_1} \left\{ f_1(\bar{b} - I^B, 0, S_1^i) + e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b} - I^B, \tilde{S}_2^i)] \right\} \\
&= \max_{x_1} \left\{ f_1(\bar{b} - I^B, 0, S_1^i) + e^{-r}.V_2(\bar{b} - I^B, S_2^i) \right\} \\
&= 0 \text{ with } x_1^*\text{=0 and } \forall i. \\
V_1(\bar{b} - I^A, S_1^i) &= \max_{x_1} \left\{ f_1(\bar{b} - I^A, 0, S_1^i) + e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b} - I^A, \tilde{S}_2^i)]; \right. \\
&\qquad \left. f_1(\bar{b} - I^A, I^A, S_1^i) + e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b} - 2.I^A, \tilde{S}_2^i)] \right\} \\
&= \max_{x_1} \left\{ f_1(\bar{b} - I^A, 0, S_1^i) + e^{-r}.V_2(\bar{b} - I^A, S_2^i); \right. \\
&\qquad \left. f_1(\bar{b} - I^A, I^A, S_1^i) + e^{-r}.V_2(\bar{b} - 2.I^A, S_2^i) \right\} \\
&= \max_{x_1} \left\{ 0 + e^{-r}.NPV_2^A; NPV_1^A + 0 \right\} \\
&= \max_{x_1} \{227; 250\} \\
&= 250 \text{ with } x_1^*\text{=}I^A \text{ and } \forall i.
\end{aligned}$$

[24]In our general case, this will be almost always the case.

We move to the $b_1 = \bar{b}$ case.

$$\begin{aligned} V_1(\bar{b}, S_1^i) &= \max_{x_1} \Big\{ f_1(\bar{b}, 0, S_1^i) + e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b}, \tilde{S}_2^i)]; \\ &\quad f_1(\bar{b}, I^A, S_1^i) + e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b} - I^A, \tilde{S}_2^i)]; \\ &\quad f_1(\bar{b}, I^B, S_1^i) + e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b} - I^B, \tilde{S}_2^i)]; \\ &\quad f_1(\bar{b}, 2.I^A, S_1^i) + e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b} - 2.I^A, \tilde{S}_2^i)] \Big\} \\ &= \max_{x_1} \Big\{ f_1(\bar{b}, 0, S_1^i) + e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b}, \tilde{S}_2^i)]; \\ &\quad f_1(\bar{b}, I^A, S_1^i) + e^{-r}.[V_2(\bar{b} - I^A, S_2^i)]; \\ &\quad f_1(\bar{b}, I^B, S_1^i) + e^{-r}.[V_2(\bar{b} - I^B, S_2^i)]; \\ &\quad f_1(\bar{b}, 2.I^A, S_1^i) + e^{-r}.[V_2(\bar{b} - 2.I^A, S_2^i)] \} \end{aligned}$$

In our illustrative case, only one investment decision is problematic (do not invest/wait at t=1 in blue) and we will approximate the expected continuation value by performing a linear regression of $e^{-r}.V_{2,i}(\bar{b})$ against a set of basis functions for this decision. The basis functions retained in this example are the first and second powers of the power price paths.
We consider the following regression model:

$$e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b}, \tilde{S}_2^i)] \approx \phi_2(\bar{b}, S_2^i) = c_{0,1} + c_{1,1}.S_1^i + c_{2,1}.(S_1^i)^2 + e_i$$

Table 9 compiles data for the regression (dependent and independent variables). The linear regression

Table 9: Illustrative case - Sample OLS regression data

Path	$e^{-r}.V_{2,i}(\bar{b})$	S_1^i	$(S_1^i)^2$
1	455	53.29	2,840
2	635	54.05	2,922
3	462	54.24	2,942
4	887	53.20	2,831
5	455	53.81	2,896
6	697	54.69	2,991
7	455	53.04	2,813
8	624	53.88	2,903

yields the following[25]:

$$e^{-r}.\mathbb{E}_1^Q[V_2(\bar{b}, \tilde{S}_2^i)] \approx \phi_2(\bar{b}, S_2^i) = 357,959 - 13,302(S_1^i) + 123.77(S_1^i)^2$$

Coming back to the value function, we replace the conditional expectation component by its approximation (in blue):

$$\begin{aligned} V_1(\bar{b}, S_1^i) &\approx \max_{x_1} \{ f_1(\bar{b}, 0, S_1^i) + \phi_2(\bar{b}, S_2^i); f_1(\bar{b}, I^A, S_1^i) + e^{-r}.[V_2(\bar{b} - I^A, S_2^i)]; \\ &\quad f_1(\bar{b}, I^B, S_1^i) + e^{-r}.[V_2(\bar{b} - I^B, S_2^i)]; f_1(\bar{b}, 2.I^A, S_1^i) + e^{-r}.[V_2(\bar{b} - 2.I^A, S_2^i)] \} \\ &\approx \max_{x_1} \{ 0 + \phi_2(\bar{b}, S_2^i);\ NPV_1^A + e^{-r}.NPV_2^A;\ NPV_1^B + 0;\ 2.NPV_1^A + 0 \} \end{aligned}$$

In table 10, we detail the investment alternatives at t=1 and highlight in bold the maximum value.

[25]To improve the quality of the linear regression and the computation speed in more complex cases, we may exclude paths favoring investments in technology A or waiting over investment in technology B for the linear regression estimation. We would therefore build on the moneyness criteria idea used for American option pricing in the Longstaff and Schwartz paper. We do not detail that here but will consider doing so in later sections / versions of this paper.

Table 10: Illustrative case - Decision nodes at t=1 and optimal decision for untapped budget

Path	$\phi_2(\bar{b}, S_2^i)$	$NPV_1^A + e^{-r}.NPV_2^A$	$NPV_1^{B,i}$	$2.NPV_1^A$	x_1^*
1	**573**	477	372	500	0
2	561	477	**598**	500	I^B
3	**580**	477	417	500	0
4	584	477	**865**	500	I^B
5	**549**	477	354	500	0
6	662	477	**674**	500	I^B
7	**608**	477	337	500	0
8	551	477	**597**	500	I^B
average	**584**	477	527	500	0

In tables 11 and 12, we summarize the value functions and optimal decisions for each budget level and each path at t=1.

Table 11: Illustrative case - Value function vs. budget level at t=1

Path	$b_1 = \bar{b} - 2.I^A$	$b_1 = \bar{b} - I^B$	$b_1 = \bar{b} - I^A$	$b_1 = \bar{b}$
1	0	0	250	573
2	0	0	250	598
3	0	0	250	580
4	0	0	250	865
5	0	0	250	549
6	0	0	250	674
7	0	0	250	608
8	0	0	250	597

Table 12: Illustrative case - Optimal decision vs. budget level at t=1

Path	$b_1 = \bar{b} - 2.I^A$	$b_1 = \bar{b} - I^B$	$b_1 = \bar{b} - I^A$	$b_1 = \bar{b}$
1	0	0	I^A	0
2	0	0	I^A	I^B
3	0	0	I^A	0
4	0	0	I^A	I^B
5	0	0	I^A	0
6	0	0	I^A	I^B
7	0	0	I^A	0
8	0	0	I^A	I^B

At t=0

Moving step back in time to t=0, the value function again takes the following form:

$$V_0(b_0, S_0^i) = \max_{x_0} \left\{ f_0(b_0, x_0, S_0^i) + e^{-r}.\mathbb{E}_0^Q[V_1(b_0 - x_0, \tilde{S}_1^i)] \right\} \ , \forall i.$$

At t=0, we only have one possible budget level, $\bar{b}$, the initial endowment.

$$\begin{aligned}
V_0(\bar{b}, S_0^i) &= \max_{x_0} \Big\{ f_0(\bar{b}, 0, S_0^i) + e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b}, \tilde{S}_1^i)]; \\
&\quad f_0(\bar{b}, I^A, S_0^i) + e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b} - I^A, \tilde{S}_1^i)]; \\
&\quad f_0(\bar{b}, I^B, S_0^i) + e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b} - I^B, \tilde{S}_1^i)]; \\
&\quad f_0(\bar{b}, 2.I^A, S_0^i) + e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b} - 2.I^A, \tilde{S}_1^i)] \Big\} \\
&= \max_{x_0} \Big\{ f_0(\bar{b}, 0, S_0^i) + e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b}, S_1^i)]; \\
&\quad f_0(\bar{b}, I^A, S_0^i) + e^{-r}.V_1(\bar{b} - I^A, S_1^i); \\
&\quad f_0(\bar{b}, I^B, S_0^i) + e^{-r}.V_1(\bar{b} - I^B, S_1^i); \\
&\quad f_0(\bar{b}, 2.I^A, S_0^i) + e^{-r}.V_1(\bar{b} - 2.I^A, S_1^i) \} \\
&= \max_{x_0} \Big\{ \ 0 + e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b}, \tilde{S}_1^i)];\ NPV_0^A + e^{-r}.NPV_1^A; \\
&\quad NPV_0^{B,i} + 0;\ 2.NPV_0^A + 0 \Big\} \\
&= \max_{x_0} \Big\{ e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b}, \tilde{S}_1^i)];\ 477;\ NPV_0^{B,i};\ 500 \Big\}
\end{aligned}$$

Now simply discounting all cash flows back to time t=0 and averaging over the eight sample paths, we get an estimate of $e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b}, \tilde{S}_1^i)]$ [26]. We obtain in table 13 the maximum value and associated optimal decisions:

Table 13: Illustrative case - Decision nodes at t=0 and optimal decision for initial budget

Path	$e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b}, \tilde{S}_1^i)]$	$NPV_0^A + e^{-r}.NPV_1^A$	$NPV_0^{B,i}$	$2.NPV_0^A$	x_0^*
1	**521**	477	278	500	0
2	**544**	477	495	500	0
3	**527**	477	321	500	0
4	**786**	477	751	500	0
5	500	477	261	**500**	$2.I^A$
6	**613**	477	573	500	0
7	**553**	477	241	500	0
8	**543**	477	502	500	0
average	**573**	477	436	500	0

The optimal path

The optimal path represents the decisions that must be taken sequentially in order to realize that maximum average value. At t=0, we find that the optimal decision is to wait ($\hat{x}_0^*$=0) by looking at the column average in table 13. Based on this optimal decision to wait, we move forward in the tree and look for the permissible decision that maximize $V_1(\bar{b})$ on average in table 10. Again the approximated optimal decision is to wait ($\hat{x}_1^*$=0). Knowing that, we look for the permissible decision that maximize $V_2(\bar{b})$ on average in table 6 and find that the approximated optimal decision is to invest in one unit of technology B ($\hat{x}_2^*$=I^B).

The approximated optimal path ($\hat{x}_0^*$=0; $\hat{x}_1^*$=0; $\hat{x}_2^*$=I^B) is to wait two periods and then invest in one unit of technology B. It is important to note that is not the optimal decision for all the paths generated

[26] The results are identical with a linear regression in which the dependent variable is $e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b}, \tilde{S}_1^i)]$ and the independent variables are the first and second power of the known price of power at t=0. Unsurprisingly, only the intercept, equal to the average of discounted $V_1(\bar{b})$, is non null.

but an approximation of the optimal decision based on a sample of i paths. In particular, looking back in tables 13, 10 and 6, we find that the optimal decisions coincides in only one of the eight paths we generated - the others paths favor investment in technology B as early as in t=1 or investment in two units of technology A now or in t=2 [27]. But since we have no knowledge of the price paths, the approximated optimal path is the best proxy we have for decision-making.

4 General case

In this section, we jump to the general case presented in section 1 and 2. We begin by describing the general procedure employed and then present the results of the initial calibration. Finally, we perform sensitivity tests to the carbon price parameters.

4.1 Procedure

Figure 3: General procedure

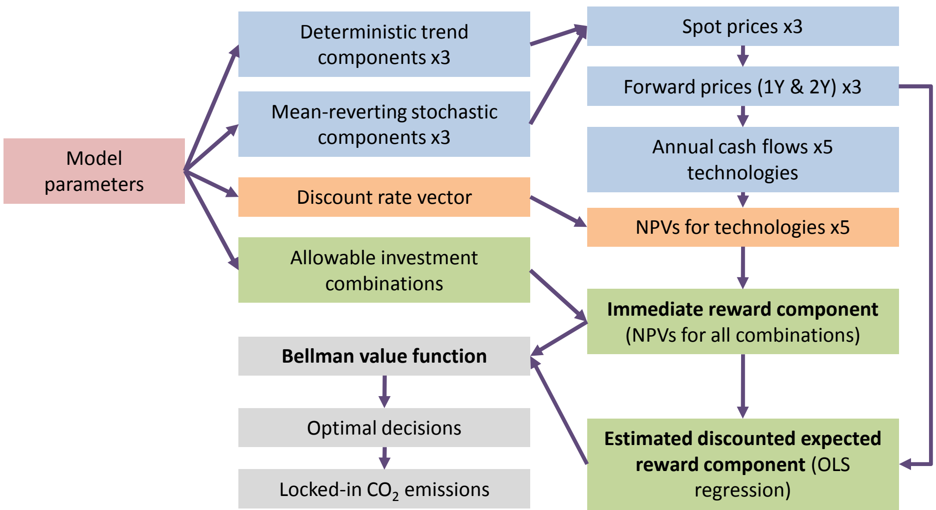


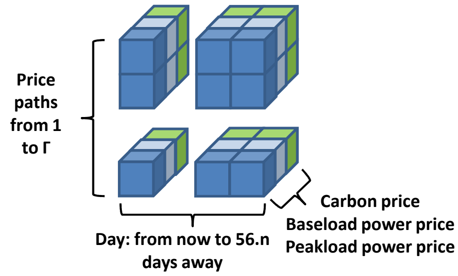
Figure 3 describe our general procedure to determine optimal decisions in our real options framework. We now look in details at each of the steps involved.

Step 1 - Simultaneously generate Γ risk-neutral paths for the stochastic state variables
We are generating jointly (since the price processes are correlated) Γ sample paths for the three price processes considered (carbon $P_t^{c,i}$, baseload power $P_t^{b,i}$ and peakload power $P_t^{p,i}$, with $i \in \|1; \Gamma\|$) according to the calibration retained over the necessary horizon (longest investment decision node + longest construction time + longest lifetime), i.e. over 672 months (56 years). We obtain an "Γ x 672 x 3" matrix with the price paths depicted in figure 4. Note that Γ is typically a large number (10,000).

As a general check, we generate plots of a sub-sample of price paths for the three stochastic state variables and report descriptive statistics on them.
Based on the Γ price paths generated and the formula for forward prices aforementioned, we compute one-year and two-year forward prices that will be subsequently used in valuation and OLS regression.

[27] Note that this is not exactly a deterministic decision framework since we resort to the OLS estimation of continuation values but this should give the general idea.

Figure 4: Sample risk-neutral spot price paths

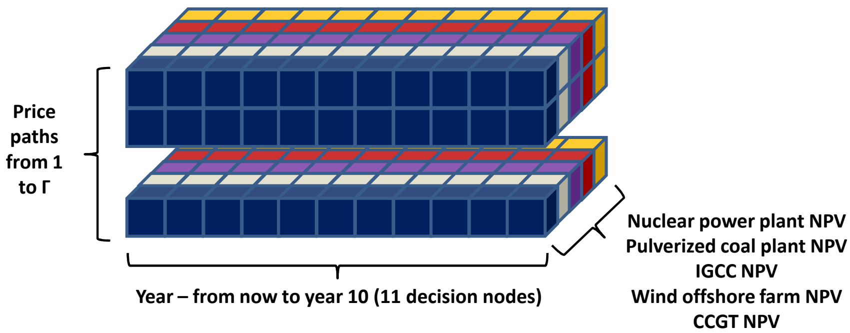


These paths are stored in the set F_t^i:

$$F_t^i = (F_{t,t+1}^{c,i}; F_{t,t+2}^{c,i}; F_{t,t+1}^{p,i}; F_{t,t+2}^{p,i}; F_{t,t+1}^{b,i}; F_{t,t+2}^{b,i}) \text{ ,} \forall i \text{ and } \forall t$$

Step 2 - Calculate Γ NPV paths for the five technologies
Based on the Γ forward sample price paths generated, we compute the NPVs for the five different technologies (nuclear $NPV_t^{N,i}$, pulverized coal $NPV_t^{K,i}$, IGCC $NPV_t^{I,i}$, wind offshore $NPV_t^{W,i}$ and CCGT, $NPV_t^{G,i}$) every year from now to ten years from now (11 investment decision nodes). With those NPVs, we are able to value any of the 104 investment combinations that can be undertaken at any time $t \in \|0; 10\|$ (budget permitting). We obtain an "Γ x 11 x 5" matrix (depicted in figure 5) with the NPVs for the five technologies considered.

Figure 5: Sample NPV paths



Again as a general check, we generate a distribution plot of the NPVs of different technology at t=0, t=5 and t=10. We report descriptive statistics on the distribution of NPVs which are important to indicate potential candidates for investment[28].

Step 3 - Determining the allowed investment combinations
Given the initial budget constraint $\bar{b}$ and investment costs I^N, I^K, I^I, I^W and I^G and denoting Q^{tech}

[28]This step might even be used as a screening procedure to eliminate consistently negative NPVs technologies to facilitate calculations.

the quantity of a given technology we invest in (Q^{tech} being an integer), we recognize that at any time, the following relation must be satisfied:

$$x_t = I^N.Q^N + I^K.Q^K + I^I.Q^I + I^W.Q^W + I^G.Q^G \leq b_t \text{ ,} \forall t.$$

We identify that the control variable can take one of the following 121 values:

$$\begin{aligned} x_t &\in \{0;\ I^W;\ I^G;\ ...;\ I^N\} \\ &\in \{0;\ 792;\ 914;\ ...;\ 5896\} \end{aligned}$$

And the budget can therefore take one of the following 121 values:

$$\begin{aligned} b_t &\in \{\bar{b} - I^N;\ ...;\ \bar{b} - I^G;\ \bar{b} - I^W;\ \bar{b}\} \\ &\in \{4;\ ...;\ 4996;\ 5108;\ 5900\} \end{aligned}$$

Step 4 - Start from the last decision node at t=10

We start from t=10, the last time we are able to invest during the investment window. At this last decision node, the continuation value is assumed to be zero. That is to say - once the investment opportunity is missed, there is no ability to generate cash flows from it. The value function takes the following form in which S_{10}^i is a set of stochastic state variables at t=10 and on path i:

$$V_{10}(b_{10}, S_{10}^i) = \max_{x_{10}} \left\{ f_{10}(b_{10}, x_{10}, S_{10}^i) \right\} \text{ ,} \forall i.$$

For all the possible budget levels at b_{10} (121) and on all the Γ paths, we compute $V_{10}(b_{10}, S_{10}^i)$. We obtain 121 "Γ paths x 121 possible decisions" tables in which we identify the immediate reward components $f_{10}(b_{10}, x_{10}, S_{10}^i)$. These are stored in the matrix $\mathcal{M}R_{10}$ (the matrix storing the reward functions). Based on those tables, we determine the maximum value among $f_{10}(b_{10}, x_{10}, S_{10}^i)$ and associated investment decision for a given remaining budget level and on a given paths. These are consigned in two "Γ paths x 121 budget levels" matrices, one for the maximum value ($\mathcal{M}V_{10}$) and one for the corresponding optimal decision ($\mathcal{M}x_{10}^*$).
Note that the condition, $x_{10} \leq b_{10}$, must be satisfied. Therefore, the calculations are eased when the remaining budget actually limits the possible investment combinations (for instance when the budget does not allow any additional investment, the only suitable course of action is to wait).
We end up this step with the matrices $\mathcal{M}R_{10}$, $\mathcal{M}V_{10}$ and $\mathcal{M}x_{10}^*$ (check matrices $\mathcal{M}R_t$, $\mathcal{M}V_t$ and $\mathcal{M}x_t^*$ in appendix for more details) in hands.

Step 5 - Moving backward in the decision-making process (from t=9 to t=1)

The value function now incorporates a continuation value and takes the following form:

$$V_9(b_9, S_9^i) = \max_{x_9} \left\{ f_9(b_9, x_9, S_9^i) + e^{-r}.\mathbb{E}_9^Q[V_{10}(b_9 - x_9, \tilde{S}_{10}^i)] \right\} \text{ ,} \forall i \text{ and } \forall ti.$$

In order to determine the value maximizing choice for all the remaining budget level (b_9) and each sample paths, we have to:

- compute $f_9(b_9, x_9, S_9^i)$ $\forall i$ and $\forall b_9$ like we did in step 4 and store the resulting 121 "Γ paths x 121 possible decisions" reward functions in matrix $\mathcal{M}R_9$;
- estimate $e^{-r}.\mathbb{E}_9^Q[V_{10}(b_9 - x_9, \tilde{S}_{10}^i)]$ $\forall i$ and $\forall b_9$ using OLS regressions like we did in the preliminary stochastic case study (else that would be clairvoyance and we would be replacing a stochastic problem by a deterministic one) and store the resulting 121 "Γ paths x 121 possible decisions" estimated continuation value functions in matrix $\mathcal{M}C_9$;

Following Cortazar et al. (2008) [45], we stress that the optimal exercise of options rather depends on expected spot prices and volatilities than on regressors being powers of all state variables (as was suggested in the reference papers by Longstaff and Schwartz, 2001 [14] and Gamba, 2003 [46]). They suggest using functions on Futures, European options or bond prices that have economic meaning. Doing so, they find that the root mean square deviation computed is lower using this reduced form function (with various number of regressors) than using Chebyshev cross products as proposed in [14]. This also partially solves the OLS regression instability and performance problems present in high-dimensional problems as is the case here.

In particular, we consider the following OLS regression model:

$$\begin{aligned} e^{-r}.\mathbb{E}_t^Q[V_{t+1}(b_t - x_t, \tilde{S}_{t+1}^i)] &\approx \phi_{t+1}(b_t - x_t, S_{t+1}^i) \\ &= c_{0,t}^{b_t-x_t} + c_{1,t}^{b_t-x_t}.F_{t,t+1}^{c,i} + c_{2,t}^{b_t-x_t}.F_{t,t+1}^{p,i} \\ &\quad + c_{3,t}^{b_t-x_t}.F_{t,t+1}^{b,i} + c_{4,t}^{b_t-x_t}.F_{t,t+2}^{c,i} \\ &\quad + c_{5,t}^{b_t-x_t}.F_{t,t+2}^{p,i} + c_{6,t}^{b_t-x_t}.F_{t,t+2}^{b,i} + e_i^{b_t-x_t} \end{aligned}$$

We regress discounted continuation value (contingent on the decision taken at t) to be found in $\mathcal{M}V_{10}$ against a set of one-year and two-year forward prices for carbon, baseload power and peakload power. It should be stressed that, contrary to the preliminary stochastic case study, we do not have to estimate a single continuation value but rather up to 121. Once estimated, we store $\phi_{10}(b_9 - x_9, S_{10}^i)$ in matrix $\mathcal{M}C_9$ (check $\mathcal{M}C_t$ in appendix for more details).

Finally, we have to:

- combine matrices $\mathcal{M}R_9$ and $\mathcal{M}C_9$ to determine $V_9(b_9, S_9^i)$, $\forall i$ and $\forall b_9$. This entails adding the $\mathcal{M}R_9$ and $\mathcal{M}C_9$ matrices and keep the maximum combined value , $\forall i$ and $\forall b_9$;
- store the resulting maximum combined value, i.e. $V_9(b_9, S_9^i)$, in $\mathcal{M}V_9$ and related optimal decisions in $\mathcal{M}x_9^*$;
- repeat the process for t = 8 until t = 1.

Step 6 - The first decision (t=0)
At t = 0, the budget variable uncertainty is resolved, we know for sure that b_0=$\bar{b}$. The value function hence takes the following form:

$$V_0(\bar{b}, S_0^i) = \max_{x_0} \left\{ f_0(\bar{b}, x_0, S_0^i) + e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b} - x_0, \tilde{S}_1^i)] \right\} \ , \forall i.$$

In order to determine the value maximizing choice for each sample paths, we do the followings:

- We compute $f_0(\bar{b}, x_0, S_0^i)$ $\forall i$ and store the resulting "Γ paths x 121 possible decisions" reward functions in matrix $\mathcal{M}R_0$. Note that this matrix is smaller to the other $\mathcal{M}R_t$ matrices since only one budget level is possible at t=0;
- At t=0, the one-year and two-year forward prices for carbon, baseload and peakload power are known as well since they are estimated based on current prices. This means that we cannot estimate $e^{-r}.\mathbb{E}_0^Q[V_1(\bar{b} - x_0, \tilde{S}_1^i)]$ using OLS regressions like we did in step 5 (as that would imply regressing NPV combinations varying upon i against static one-year forward prices independent of i). Instead, following Longstaff and Schwartz (2001 [14]) and like we did in the illustrative case study, we simply discount one year back $V_1(\bar{b} - x_0)$ $\forall i$ to be found in $\mathcal{M}V_1$. The resulting approximated continuation value is stored in $\mathcal{M}C_0$;

- We combine matrices $\mathcal{M}R_0$ and $\mathcal{M}C_0$ to determine $V_0(\bar{b}, S_0^i)$, $\forall i$. This entail adding the $\mathcal{M}R_0$ and $\mathcal{M}C_0$ matrices;
- We store the resulting value for $V_0(\bar{b}, S_0^i)$ in $\mathcal{M}V_0$ and related optimal decisions in $\mathcal{M}x_0^*$.

Step 7 - The optimal path and implied emissions

At this point, we have a set of eleven matrices $\mathcal{M}V_t$ and $\mathcal{M}x_t^*$ indicating maximum value and optimal decisions , $\forall t$ and $\forall i$.

We start from t=0 and compute averages over all paths in matrix $\mathcal{M}V_0$ (Γ x x_0 value function matrix at t=0, i.e. when budget is full). Here, we look for:

$$V_0(\bar{b}) \quad = \quad \max_{x_0} \left\{ \frac{1}{\Gamma} \sum_{i=1}^{\Gamma} [f_0(\bar{b}, x_0, S_0^i) + \phi_1(\bar{b} - x_0, S_1^i)] \right\}$$

The result suggests a peculiar optimal decision ($\hat{x}_0^*$) that is expected to maximize V_0 (which needs not be identical to what is to be found in $\mathcal{M}x_0^*$).

We move forward in time, and solve recursively the following equation $\forall t \in \|1; 9\|$:

$$V_t(\bar{b} - \sum_{k=0}^{t-1} \hat{x}_k^*) \quad = \quad \max_{x_t} \left\{ \frac{1}{\Gamma} \sum_{i=1}^{\Gamma} [f_t(\bar{b} - \sum_{k=0}^{t-1} \hat{x}_k^*, x_t, S_t^i) + \phi_{t+1}(\bar{b} - \sum_{k=0}^{t-1} \hat{x}_k^* - x_t, S_{t+1}^i)] \right\} \ , \forall t$$

When at t=10, we solve the following equation (no estimated discounted continuation value):

$$V_{10}(\bar{b} - \sum_{k=0}^{9} \hat{x}_k^*) \quad = \quad \max_{x_{10}} \left\{ \frac{1}{\Gamma} \sum_{i=1}^{\Gamma} [f_{10}(\bar{b} - \sum_{k=0}^{9} \hat{x}_k^*, x_{10}, S_{10}^i)] \right\}$$

We find a set comprised of optimal decisions ($\hat{x}_0^*$, $\hat{x}_1^*$, ..., $\hat{x}_{10}^*$).

Given the optimal path, we expect a given amount of locked-in CO_2 emissions. That amount can be estimated based on (1) the carbon emission factor of the technology we invest in, (2) the expected annual production and (3) the life length of the plants.

4.2 Model results

Price path generation

Recalling that the logarithm of the stochastic price processes are expressed as the sum of (1) a deterministic linear trend and (2) a mean-reverting stochastic component, we generate deterministic log-linear trends for the three stochastic processes based on:

$$\begin{cases} h_t^{c*} & = \alpha^c + \beta^c.t = 2.5992 + 0.0250.t \quad \text{(Carbon)} \\ h_t^{p*} & = \alpha^p + \beta^p.t = 3.9821 + 0.0075.t \quad \text{(Peakload)} \\ h_t^{b*} & = \alpha^b + \beta^b.t = 3.8061 + 0.0075.t \quad \text{(Baseload)} \end{cases}$$

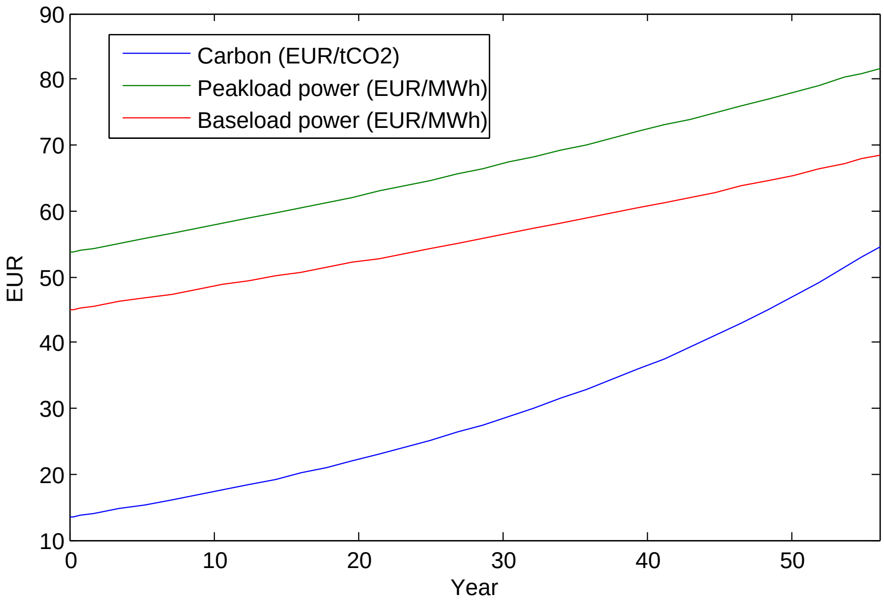
Figure 6 depicts the long-term trends suggested for the three processes:

We now move to the stochastic component of the sources of uncertainty:

$$\begin{cases} d\hat{X}_t^c & = \theta^c.(-\lambda^c.\frac{\sigma^c}{\theta^c} - \hat{X}_t^c).dt + \sigma^c d\hat{W}_t^c \quad \text{(Carbon)} \\ d\hat{X}_t^p & = \theta^p.(-\lambda^p.\frac{\sigma^p}{\theta^p} - \hat{X}_t^p).dt + \sigma^p d\hat{W}_t^p \quad \text{(Peakload)} \\ d\hat{X}_t^b & = \theta^b.(-\lambda^b.\frac{\sigma^b}{\theta^b} - \hat{X}_t^b).dt + \sigma^b d\hat{W}_t^b \quad \text{(Baseload)} \end{cases}$$

The following parameters have been estimated jointly based on a common historical sample:

Figure 6: Assumed price trends over the investment lifetime



- Mean-reverting speed parameter matrix: $\begin{pmatrix} \theta^c & 0 & 0 \\ 0 & \theta^p & 0 \\ 0 & 0 & \theta^b \end{pmatrix} = \begin{pmatrix} 2.4519 & 0 & 0 \\ 0 & 72.8955 & 0 \\ 0 & 0 & 53.6616 \end{pmatrix}$;

- Mean-reverting level parameter matrix: $\begin{pmatrix} -\lambda^c.\frac{\sigma^c}{\theta^c} \\ -\lambda^p.\frac{\sigma^p}{\theta^p} \\ -\lambda^b.\frac{\sigma^b}{\theta^b} \end{pmatrix} = \begin{pmatrix} 0.0814 \\ 0.0022 \\ 0.0030 \end{pmatrix}$ Note that the mean-reverting levels are quite low since we are dealing with a detrended variable;

- Volatility parameter matrix: $\begin{pmatrix} \sigma^c & 0 & 0 \\ 0 & \sigma^p & 0 \\ 0 & 0 & \sigma^b \end{pmatrix} = \begin{pmatrix} 0.4396 & 0 & 0 \\ 0 & 3.4488 & 0 \\ 0 & 0 & 2.7786 \end{pmatrix}$;

- Correlation matrix: $\begin{pmatrix} \rho_{c,c} & \rho_{c,p} & \rho_{c,b} \\ \rho_{p,c} & \rho_{p,p} & \rho_{p,b} \\ \rho_{b,c} & \rho_{b,p} & \rho_{b,b} \end{pmatrix} = \begin{pmatrix} 1.0000 & 0.3599 & 0.2933 \\ 0.3599 & 1.0000 & 0.9781 \\ 0.2933 & 0.9781 & 1.0000 \end{pmatrix}$.

We generate 10,000 paths based on the stochastic processes retained and the calibration parameters. One such set of sample paths is depicted in figure 7[29]. We compute spot, one-year and two-year forward contracts (see figure 8). We then compute NPV distribution (see figure 9) We run the model which indicates that the optimal decision is to invest in six CCGT plants now which locks in 284 million tons of carbon over their lifetime.

[29] We simultaneously generated paths for carbon, peakload power and baseload power mean-reverting stochastic processes. We then added the generated paths to the linear trend and took the sum to the exponential to derive three sample price paths.

Figure 7: Sample spot price paths over the investment lifetime

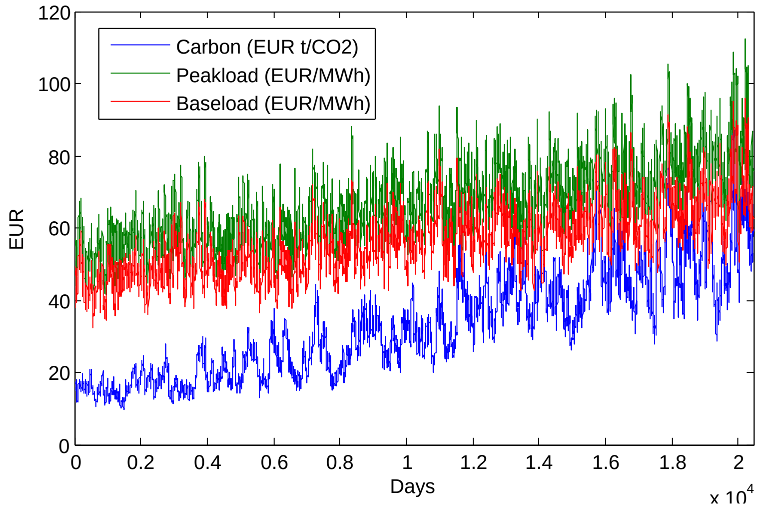
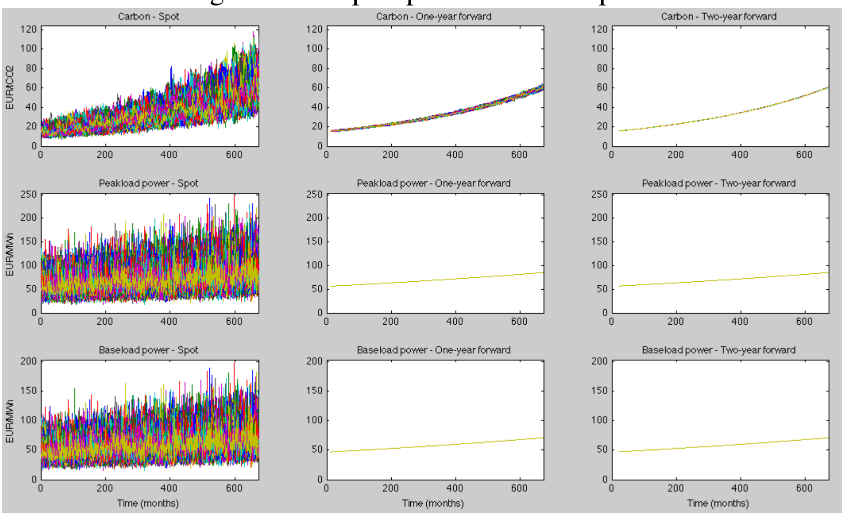


Figure 8: Sample spot and forward prices



4.3 Sensitivity study to carbon price parameters

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4.3.1 Ceteris paribus

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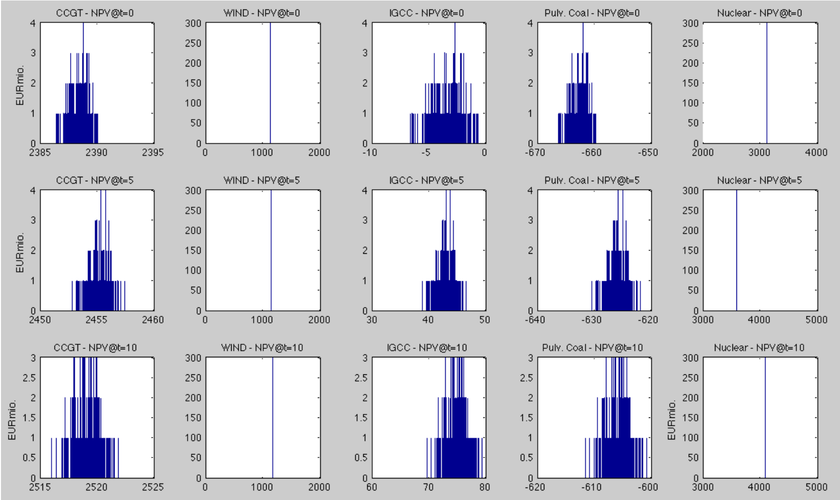
- (1) α - the intercept of the linear time trend for the carbon price (suggested range for sensitivity study: 2; 3; 4 and even higher). That would correspond to cap level or carbon reduction engagements (from G8 or from the EU for instance).
- (2) β - the slope of the linear time trend for the carbon price (suggested range for sensitivity study: -0.10;-0.05; 0.00; 0.05; 0.10). That would correspond to the annual incremental effort required by the policy.

Figure 9: NPVs for technolgies at t=0, t=5 and t=10



- (3) θ - the mean-reversion speed to the linear time trend for the carbon price (suggested range for sensitivity study: 0; 2; 4; 6)
- (4) σ^2 - the instantaneous volatility for the price of carbon (suggested range for sensitivity study : 0.00 for the deterministic case; 0.10; 0.20; 0.30; 0.40)

Versus to following results from the dynamic programming part & emissions calculation:

- Parameter (x-axis) vs. overall profit (left y-axis) and implied emissions (right y-axis).
- Composition of the investment portfolio
- Option value for waiting
- Range (+/- 1 standard deviation around mean value) of expected prices for the expected NPV of power plant alternatives for the 11 decision periods.

Expectations:

- The higher α, the higher the cost of carbon, the less likely the investor will choose to invest in CF & CCGT, the lower the profit, the lower the emissions
- The higher β, the higher the cost of carbon, the less likely the investor will choose to invest in CF & CCGT, the lower the profit, the lower the emissions
- The higher θ, the less sensitive to price shocks and the further we move to the deterministic case (?) and the higher the profit, the higher the emissions
- The higher σ^2, the higher the NPV range for the CCGT and the coal-fired plant, (the less likely we will invest in them) and thus the lower the profit and the lower the emissions

4.3.2 Stylized scenarios

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Parameterize at best to match the trading regime

- phase I

- phase II
- phase III
- termination of the ETS policy
- replacement by a tax

5 Discussion

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Of course, in no way would price be "administrated" still, prices can be supported by means of support policies to make sure that the climate policy objective remains intact. In particular, we consider:

- Cap-setting (Declining-cap within & between trading phases)
- NER & allowances fate in case of installation closure
- More flexibility mechanisms (Offset Projects & intertemporal adjustments)
- Horizon
- free-allocation vs. the impact of auctioning
- Price cap (with mean reversion speed)
- Price floor (with mean reversion speed)
- Other polices like renewables and energy efficiency (with volatility parameter)
- Help correct capital market inefficiencies (better market liquidity better informational efficiency)
- Evolution of coverage or linking to a "superior" cap-and-trade market (more constrained) (see paper by Ellis et al.).

6 Conclusion

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Appendices

Table 14: Appendix - CCGT plant data - 85% availability assumed

	Min	Max	Estimates	Comments
Construction length - in years	2	5	3	French CCGT
Lifetime - in years	40	40	40	Common assumption
Thermal capacity - in MWe	250	1200	900	French CCGT
Thermal efficiency - in %	52	60	59	French CCGT
Availability factor - in %	85	85	85	Common assumption
Expected annual output - in GWh	1.86	8.94	6.70	
ON construction costs - in EUR/kWe	318	900	524	French CCGT
O&M costs - per kWe	4	41	32	Average of French & German
CO_2 emissions factor - in tCO_2/MWh	0.333	0.412	0.353	French CCGT
Annual carbon emissions - in $MtCO_2$	0.670	3.246	2.366	
Total investment costs - in EUR million	173	630	500	

Table 15: Appendix - Pulverized coal plant data - 85% availability assumed

	Min	Max	Estimates	Comments
Construction length - in years	3	4	3	German PC
Lifetime - in years	40	40	40	Common assumption
Thermal capacity - in MWe	296	1050	800	German PC
Thermal efficiency - in %	29	47	46	German PC
Availability factor - in %	85	85	85	Common assumption
Expected annual output - in GWh	2.20	7.82	5.96	
ON construction costs - in EUR/kWe	820	1300	820	German PC
O&M costs - per kWe	9	57	50	Average of French & German
CO_2 emissions factor - in tCO_2/MWh	0.728	1.133	0.728	German PC
Annual carbon emissions - in $MtCO_2$	2.211	6.223	4.367	
Total investment costs - in EUR million	277	1253	696	

Table 16: Appendix - IGCC plant data - 85% availability assumed

	Min	Max	Estimates	Comments
Construction length - in years	3	4	3	German IGCC
Lifetime - in years	40	40	40	Common assumption
Thermal capacity - in MWe	300	450	450	German IGCC
Thermal efficiency - in %	43	51	46	German IGCC
Availability factor - in %	85	85	85	Common assumption
Expected annual output - in GWh	2.23	3.35	3.35	
ON construction costs - in EUR/kWe	1200	1692	1200	German IGCC
O&M costs - per kWe	28	81	81	German costs
$C0_2$ emissions factor - in tCO_2/MWh	0.656	0.780	0.656	German IGCC
Annual carbon emissions - in $MtCO_2$	1.742	2.198	2.198	
Total investment costs - in EUR million	516	576	576	

Table 17: Appendix - Nuclear plant data - 85% availability assumed

	Min	Max	Estimates	Comments
Construction length - in years	5	9	5	$\geq$ 5 years include studies
Lifetime - in years	40	40	40	Common assumption
Thermal capacity - in MWe	665	1600	1590	French PWR
Thermal efficiency - in %	30	37	36	French PWR
Availability factor - in %	85	85	85	Common assumption
Expected annual output - in GWh	4.95	11.91	11.84	
ON construction costs - in EUR/kWe	952	1875	1455	Average of German and French
O&M costs - per kWe	40	72	49	Average of German and French
Total investment costs - in EUR million	1006	3094	2391	

Table 18: Appendix - Offshore wind plant data

	Min	Max	Estimates	Comments
Construction length - in years	1	2	1	
Lifetime - in years	20	25	20	Most common case
Thermal capacity - in MWe	120	300	300	German offshore
Equipment availability - in %	95	95	95	Common assumption
Average load factor - in %	35	43	42	Most common case
Expected annual output - in GWh	0.36	1.13	1.20	
ON construction costs - in EUR/kWe	1431	2292	1650	German offshore
O&M costs - per kWe	38	115	58	German offshore
Total investment costs - in EUR million	238	512	512	

Table 19: Appendix - $\mathcal{M}R_t$ - Immediate reward component of the value function in t

$$\begin{pmatrix} f_t(\bar{b}-I^N,0,S_t^1) & ... & f_t(\bar{b}-I^N,I^N,S_t^1) & ... & f_t(\bar{b}-I^W,I^G,S_t^1) & ... & f_t(\bar{b},0,S_t^1) & ... & f_t(\bar{b},I^N,S_t^1) \\ f_t(\bar{b}-I^N,0,S_t^2) & ... & f_t(\bar{b}-I^N,I^N,S_t^2) & ... & f_t(\bar{b}-I^W,I^G,S_t^2) & ... & f_t(\bar{b},0,S_t^2) & ... & f_t(\bar{b},I^N,S_t^2) \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ f_t(\bar{b}-I^N,0,S_t^i) & ... & f_t(\bar{b}-I^N,I^N,S_t^i) & ... & f_t(\bar{b}-I^W,I^G,S_t^i) & ... & f_t(\bar{b},0,S_t^i) & ... & f_t(\bar{b},I^N,S_t^i) \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ f_t(\bar{b}-I^N,0,S_t^\Gamma) & ... & f_t(\bar{b}-I^N,I^N,S_t^\Gamma) & ... & f_t(\bar{b}-I^W,I^G,S_t^\Gamma) & ... & f_t(\bar{b},0,S_t^\Gamma) & ... & f_t(\bar{b},I^N,S_t^\Gamma) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & ... & -\infty & ... & NPV_t^{G,1} & ... & 0 & ... & NPV_t^{N,1} \\ 0 & ... & -\infty & ... & NPV_t^{G,2} & ... & 0 & ... & NPV_t^{N,2} \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & ... & -\infty & ... & NPV_t^{G,i} & ... & 0 & ... & NPV_t^{N,i} \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & ... & -\infty & ... & NPV_t^{G,\Gamma} & ... & 0 & ... & NPV_t^{N,\Gamma} \end{pmatrix}$$

Table 20: Appendix - $\mathcal{M}C_t$ - Continuation value component of the value function in t

$$\begin{pmatrix}
e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N,\tilde{S}^1_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N-I^N,\tilde{S}^1_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^W-I^G,\tilde{S}^1_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b},\tilde{S}^1_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N,\tilde{S}^1_{t+1})] \\
e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N,\tilde{S}^2_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N-I^N,\tilde{S}^2_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^W-I^G,\tilde{S}^2_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b},\tilde{S}^2_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N,\tilde{S}^2_{t+1})] \\
... & ... & ... & ... & ... & ... & ... & ... & ... \\
e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N,\tilde{S}^i_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N-I^N,\tilde{S}^i_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^W-I^G,\tilde{S}^i_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b},\tilde{S}^i_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N,\tilde{S}^i_{t+1})] \\
... & ... & ... & ... & ... & ... & ... & ... & ... \\
e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N,\tilde{S}^\Gamma_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N-I^N,\tilde{S}^\Gamma_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^W-I^G,\tilde{S}^\Gamma_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b},\tilde{S}^\Gamma_{t+1})] & ... & e^{-r}.\mathbb{E}_t^Q[V_{t+1}(\bar{b}-I^N,\tilde{S}^\Gamma_{t+1})]
\end{pmatrix}$$

$$= \begin{pmatrix}
\phi_{t+1}(\bar{b}-I^N,S^1_{t+1}) & ... & -\infty & ... & \phi_{t+1}(\bar{b}-I^W-I^G,S^1_{t+1}) & ... & \phi_{t+1}(\bar{b},S^1_{t+1}) & ... & 0 \\
\phi_{t+1}(\bar{b}-I^N,S^2_{t+1}) & ... & -\infty & ... & \phi_{t+1}(\bar{b}-I^W-I^G,S^2_{t+1}) & ... & \phi_{t+1}(\bar{b},S^2_{t+1}) & ... & 0 \\
... & ... & ... & ... & ... & ... & ... & ... & ... \\
\phi_{t+1}(\bar{b}-I^N,S^i_{t+1}) & ... & -\infty & ... & \phi_{t+1}(\bar{b}-I^W-I^G,S^i_{t+1}) & ... & \phi_{t+1}(\bar{b},S^i_{t+1}) & ... & 0 \\
... & ... & ... & ... & ... & ... & ... & ... & ... \\
\phi_{t+1}(\bar{b}-I^N,S^\Gamma_{t+1}) & ... & -\infty & ... & \phi_{t+1}(\bar{b}-I^W-I^G,S^\Gamma_{t+1}) & ... & \phi_{t+1}(\bar{b},S^\Gamma_{t+1}) & ... & 0
\end{pmatrix}$$

Table 21: Appendix - $\mathcal{M}V_t$ and $\mathcal{M}x_t^*$ - Value function and associated optimal decision in t

$$\begin{pmatrix} \max_{x_t}[\phi_{t+1}(\bar{b}-I^N,S^1_{t+1});...;-\infty] & ... & \max_{x_t}[\phi_{t+1}(\bar{b}-I^W,S^1_{t+1});...;-\infty] & \max_{x_t}[\phi_{t+1}(\bar{b},S^1_{t+1});...;NPV_t^{N,1}] \\ \max_{x_t}[\phi_{t+1}(\bar{b}-I^N,S^2_{t+1});...;-\infty] & ... & \max_{x_t}[\phi_{t+1}(\bar{b}-I^W,S^2_{t+1});...;-\infty] & \max_{x_t}[\phi_{t+1}(\bar{b},S^2_{t+1});...;NPV_t^{N,2}] \\ ... & ... & ... & ... \\ \max_{x_t}[\phi_{t+1}(\bar{b}-I^N,S^i_{t+1});...;-\infty] & ... & \max_{x_t}[\phi_{t+1}(\bar{b}-I^W,S^i_{t+1});...;-\infty] & \max_{x_t}[\phi_{t+1}(\bar{b},S^i_{t+1});...;NPV_t^{N,i}] \\ ... & ... & ... & ... \\ \max_{x_t}[\phi_{t+1}(\bar{b}-I^N,S^\Gamma_{t+1});...;-\infty] & ... & \max_{x_t}[\phi_{t+1}(\bar{b}-I^W,S^\Gamma_{t+1});...;-\infty] & \max_{x_t}[\phi_{t+1}(\bar{b},S^\Gamma_{t+1});...;NPV_t^{N,\Gamma}] \end{pmatrix}$$

$$\begin{pmatrix} \text{argmax}_{x_t}[\phi_{t+1}(\bar{b}-I^N,S^1_{t+1});...;-\infty] & ... & \text{argmax}_{x_t}[\phi_{t+1}(\bar{b}-I^W,S^1_{t+1});...;-\infty] & \text{argmax}_{x_t}[\phi_{t+1}(\bar{b},S^1_{t+1});...;NPV_t^{N,1}] \\ \text{argmax}_{x_t}[\phi_{t+1}(\bar{b}-I^N,S^2_{t+1});...;-\infty] & ... & \text{argmax}_{x_t}[\phi_{t+1}(\bar{b}-I^W,S^2_{t+1});...;-\infty] & \text{argmax}_{x_t}[\phi_{t+1}(\bar{b},S^2_{t+1});...;NPV_t^{N,2}] \\ ... & ... & ... & ... \\ \text{argmax}_{x_t}[\phi_{t+1}(\bar{b}-I^N,S^i_{t+1});...;-\infty] & ... & \text{argmax}_{x_t}[\phi_{t+1}(\bar{b}-I^W,S^i_{t+1});...;-\infty] & \text{argmax}_{x_t}[\phi_{t+1}(\bar{b},S^i_{t+1});...;NPV_t^{N,i}] \\ ... & ... & ... & ... \\ \text{argmax}_{x_t}[\phi_{t+1}(\bar{b}-I^N,S^\Gamma_{t+1});...;-\infty] & ... & \text{argmax}_{x_t}[\phi_{t+1}(\bar{b}-I^W,S^\Gamma_{t+1});...;-\infty] & \text{argmax}_{x_t}[\phi_{t+1}(\bar{b},S^\Gamma_{t+1});...;NPV_t^{N,\Gamma}] \end{pmatrix}$$