Valuing Investments to Enhance Energy Efficiency

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Abstract

This paper deals with the optimal time to invest in an energy efficiency enhancement. There is a broad consensus that such investments quickly pay for themselves in lower energy bills and spared emission allowances. However, investments that at first glance seem worthwhile usually are not undertaken. Our aim is to shed some light on this issue. In particular, we try to assess these projects from a financial point of view so as to attract sufficient interest from the investment community.

We consider the specific case of a firm or utility already in place that consumes huge amounts of coal and operates under restrictions on carbon dioxide emissions. In order to reduce both coal and carbon costs the firm may undertake an investment to enhance energy efficiency. We consider three sources of uncertainty: the fuel commodity price, the emission allowance price and the overall investment cost. The first one is assumed to follow a mean-reverting process while the last two are governed by geometric Brownian motions. The parameters of the coal price process and the carbon price process are estimated from actual futures prices.

The numerical parameter values are then used in a three-dimensional binomial lattice to assess the value of the option to invest. As usual, maximising this value involves determining the optimal exercise time. Thus we compute the trigger investment cost, i.e., the threshold level below which immediate investment would be optimal. A sensitivity analysis is also undertaken. Our results go some way into explaining the so-called energy efficiency paradox.

Keywords: energy efficiency, carbon constraints, efficiency gap, real options, multidimensional lattices.

JEL Codes: C6; E2; D8, G3.

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1 INTRODUCTION

This paper develops a real options evaluation of investments that enhance energy efficiency at the firm level. The model comprises three sources of risk, namely the prices of input fuel and emissions output along with the investment cost. We concentrate on the assessment from the viewpoint of self-interested firms. However, due to the presence of negative externalities and other market failures, policy makers also play an important role in this area and have a big say on the risk-reward trade-off in these investments.

Improvements in energy efficiency (EE) have put a limit to fuel consumption growth in the past (Geller et al. [12], UNF [31]). Besides they have another basic impact, namely the avoiding of greenhouse gas (GHG) emissions that go hand in hand with fossil fuel combustion (IPCC [20], IEA [14]). The key to both results is that we do not consume energy as such but energy services; therefore, it is possible to provide the same amount (level) of energy services with a lower level of energy consumption. Thus, to support governments with their implementation of EE, the IEA recommended the adoption of specific EE policy measures to the G8 summits in 2006, 2007, 2008 and 2009. They cover 25 fields of action across seven priority areas. The IEA estimates that if implemented globally without delay, the proposed actions could save around 8.2 Gt CO2/year by 2030 -equivalent to twice the European Union's current yearly emissions (IEA [18])-. Similarly, McKinsey [27] suggests that the right policies and investments in existing technologies could contribute to a reduction in global energy demand growth by at least half to 2020. To the extent that there is a price for carbon dioxide emissions, avoiding them has economic value for firms or utilities that operate in an emissionsconstrained environment (or will do so in the future). Of course, this adds to their savings in terms of reduced fuel consumption. And governments must keep in mind that the benefits of implementing EE extend beyond energy security and climate change mitigation. Experience shows that EE investments can deliver significant co-benefits -including job creation (UNEP [30]) and health improvements (Markandya and Chaibai [26])-.

There is a broad consensus that such investments quickly pay for themselves in lower energy bills. As Steven Chu, now the U.S. Secretary of Energy, puts it: "Energy efficiency isn't just low hanging fruit; it's fruit lying on the ground". He has made EE the heart of the Obama Administration's energy strategy. Tighter appliance standards are on a fast track through the Department of Energy bureaucracy. Billions of dollars from the stimulus package are pouring into programs to weatherize and retrofit homes with energy-saving technology. Also, in May 2009 the International Partnership for Energy Efficiency Cooperation (IPEEC) was officially launched. IPEEC signatories included members of the G8 - Canada, France, Germany, Italy, Japan, the Russian Federation, the United Kingdom, and the United States -, and key emerging economies such as Brazil, China, India, Mexico and the Republic of Korea. To facilitate cooperation in this key area, the IPEEC will serve as a high-level forum for facilitating a broad range of actions that yield EE gains and encourage market implementation of key EE technologies. Thus policy experiences will be exchanged among those who can subsequently initiate implementation of the best policy practices.

At the EU level, we can mention early EE policies in the form of legislation covering different activity sectors.¹ More recently the EU has adopted an ambitious policy framework regarding EE in final consumption and other energy services (Directive 2006/32/EC). This piece obliges Member States to set quantitative objectives in terms of energy savings, and measures to promote EE in the provision of energy services. A saving of 9% by the year 2016 was proposed as a reference goal; then each country had to determine the steps required to reach it. The 2008 Climate action and renewable energy package pushes these goals further into the future up to 2020 and beyond;² energy consumption must be 20% below the level forecast for that year thanks to enhanced EE in home consumption and also in manufacturing and tertiary sectors.

Despite these policies, investments that at first glance seem worthwhile usually are not undertaken. For example, around 40% of the potential energy savings from the IEA recommendations, or measures that achieve similar outcomes, remains to be captured. Why? EE continues to face pervasive barriers including insufficient information, principal-agent problems (IEA [15]), externality costs that are not reflected in energy prices, and lack of access to capital for EE investments.³

¹Directive on energy efficiency in buildings (2002/91/EC), Directive on the promotion of cogeneration (2004/8/EC), Directive on Eco-design (2005/32/EC).

²http://ec.europa.eu/environment/climat/climate_action.htm

³A comprehensive list of reasons for a lower than expected investment in EE can be found

Following Charles [3], several approaches are being used to address these issues. A first one looks for ways to influence people's energy-using behavior. In this regard some lessons can be learnt from behavioral science. Technology that brings consumers face-to-face with their energy consumptions can also play a role in promoting behavioral change. Another approach aims at fixing "market failures" or overcoming institutional roadblocks. For instance, concerning residential energy use, builders' interests and dwellers' interests typically fall apart when it comes to reducing consumption. In this case, tougher efficiency standards can change that. The Green Paper on energy efficiency (EC [8]) identifies other options to overcome the bottlenecks currently preventing cost-effective efficiencies from being captured; see also EC [9].⁴

We focus instead on the issue of financing mechanisms for EE. In particular, we feel that the situation traces in part to the challenge of attracting sufficient interest from the investment community. Mills et al. [28] point out that energy-efficiency experts (as scientists and engineers) and investment decision-makers simply do not speak the same language. Along this line, the Efficiency Valuation Organization (EVO)⁵ has launched a set of guidelines to help financial institutions evaluate the risks and quantify the benefits of end-use EE investments. These guidelines are known as the International Energy Efficiency Financing Protocol (IEEFP). They are intended to help EE projects access funding capacity at local financial institutions on commercially attractive terms.⁶

A first barrier to overcome is the traditional 'asset-based' corporate lending approach. Typically it limits lending to 70-80% of the value of the assets financed (or collateral provided). In the particular case of EE projects, there is often little or no collateral value in the equipment once installed; rather, the value is the cash flow generated by the equipment.

A second problem is that many companies that could benefit from EE projects place a low priority on investing capital or using their credit capacity to finance EE. This may be particularly acute in times when corporations are cash strapped (as the current scenario). Securing a loan to improve EE, though, would allow to reduce operating costs, thus improving the company's competitiveness and creditworthiness.

Our aim is to further contribute to bridging the gap between the two communities. We note that EE investments lend themselves to financial analysis. In particular, we focus on the valuation of the cash flows that result from investments in EE. We accomplish this by thinking of them as stochastic annuities. We also focus on the timing of the investment, i.e., on the optimal time to invest. Since EE investments are not compulsory, firms can invest immediately but also have the option to wait; and the value of this option can be significant.

Specifically, we analyze investments in EE from the viewpoint of a firm or individual that behaves rationally, i.e., in her best economic interest. The in-

in Linares [23] and Linares and Labandeira [24].

 $^{^4}$ Sá
enz de Miera and Muñoz [29] provide an overview of policy measures aimed to promote EE.

⁵This is a Washington, DC-based non-profit organization.

 $^{^{6}} http://www.evo-world.org/index.php?option=com_content&task=view&id=373&Itemid=374$

vestment or project is valued like a (real) option that is only exercised at the optimal time and is irreversible (the firm cannot disinvest should market conditions turn). The return on this investment is highly uncertain. Uncertainty emanates from energy prices and emission allowance prices, but regulatory uncertainty may come on top of them. We aim to determine the optimal time to invest or, in other words, to learn the conditions under which the investment should be undertaken.

We consider the specific case of a firm or utility already in place that consumes huge amounts of coal and operates under restrictions on carbon dioxide emissions. Obviously the price of both commodities is uncertain. Fortunately, though, both of them are regularly traded on futures markets. This allows to estimate some economic parameters that are relevant for valuation purposes. In order to reduce fuel consumption and carbon emissions the firm or utility may undertake an investment to enhance EE. The cost to such investment, however, is assumed uncertain (either the explicit cost or the intangible cost or both: Dennis [6]). Thus we consider three sources of risk. The parameters of the coal price process and the carbon price process are estimated from actual futures prices; instead, those of the investment cost are adopted ad hoc. The numerical estimates are then used in a three-dimensional binomial lattice to assess the value of the option to invest. The methodology is similar to that in Boyle et al. [2] and Gamba and Trigeorgis [11]. Our procedure precludes the possibility of negative probabilities, allows for mean-reverting stochastic processes (as opposed to standard geometric Brownian motions), and is later used to value American-type options (as opposed to European-type options). A sensitivity analysis is also undertaken.

Our (base case) results show that the firm will find it optimal to invest in an EE-raising project when the facility has reached about its half useful life. With shorter times to expiration, it is preferable not to invest even if the NPV is positive. This can help to understand the "efficiency gap" and the different perspectives sometimes adopted by engineers and economists when valuing projects. Moreover, as the investment cost increases, longer periods until expiration are required if the option to invest is to be exercised. Some policy issues can be addressed within this framework. Specifically, we briefly consider the potential effect of a public subsidy to investments that improve EE. We also highlight the impact that uncertainty (or the efforts to reduce it) can have on the optimal time to invest.

The paper is organized as follows. Section 2 briefly introduces the topic and provides some market background. Section 3 sets the theoretical framework. The particular stochastic processes for the three uncertain variables are presented. Since the physical facility where the potential enhancement in EE would take place (and hence the enhancement itself) is finite-lived, we also derive the formula for the value of a stochastic annuity. Section 4 sketches the estimation procedure and shows the numerical values for the underlying parameters. A more thorough exposition can be found in the Appendix. Section 5 explains how the three-dimensional binomial lattice is built. Section 6 comprises the general results and the sensitivity analysis, Section 7 concludes.

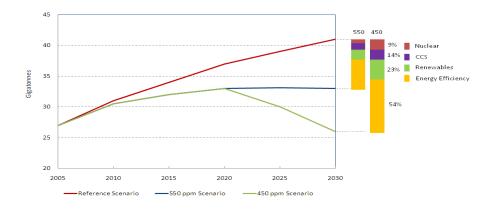


Figure 1: Reductions in energy-related CO_2 emissions in the climate-policy scenarios and desaggregation by technologies. Source: IEA [17].

2 BACKGROUND PROJECTIONS AND MAR-KET DATA

According to IEA [16], long-term stabilization of greenhouse-gas concentration at 550 or 450 ppm (parts per million of CO2 equivalent) will require substantial reductions of emissions; see Figure 1.⁷ In order to reach either of these outcomes, the energy mix should be markedly different from that of the Reference Scenario, with fossil fuels losing market share to other alternatives as renewable, nuclear power, carbon capture and storage (CCS), and improvements in EE, which is expected to account for 54% of the emissions reduction in the 450 ppm scenario.

On the other hand, reducing CO2 emissions has a cost which is not easy to estimate since many of the technologies involved are still under development. According to IEA [17] the costs per tonne of CO2 saved can fluctuate significantly depending on the sector and technology selected; see Figure 2. Some investments are very cost-effective, particularly in EE, whereas others are only economic under a high CO2 reduction incentive. In fact, EE investments could reduce CO2 emissions at no cost up to 10 Gt CO2 per year. EE is therefore the most cost-effective near-term strategy.

As mentioned above, we consider the specific case of an operating firm or utility that needs both input coal and emission allowances. Today 23% of world primary energy comes from coal. About 36% of world's electricity is produced using coal. Coal is the main fuel for electricity in USA, Germany, China, India, South Africa, Australia, and much of central Europe. Moreover 70% of world's steel is produced by coal (Franco and Diaz [10]).

But the industrial sector continues to waste energy at a staggering rate.

 $^{^{7}}$ Despite some uncertainties, those thresholds seem to set the limits if serious, or even catastrophic, effects on life and property are to be avoided.

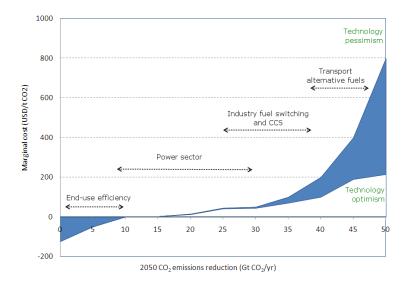


Figure 2: Marginal emission reduction costs for the global energy system, 2050. Source: IEA ([16]).

Globally, one of the most glaring inefficiencies comes from the burning of fossil fuels to generate electricity (Lindley [25]). The conversion efficiency in typical coal- or oil-fired plants is about 40 %. Still, no method for generating electricity by burning a fuel can avoid substantial inefficiency for instance through the loss of heat. The technology itself for energy-recycling is straightforward; in many cases, waste heat can be turned into electricity in exactly the same way that most power plants do it. The bad news is cost: the price tag for retrofitting waste-heat recovery technology into a plant that was not designed for it tends to send potential customers into shock. Without the appropriate knowledge, they may have a bias towards overestimating the risks while underestimating the rewards. In this regard, futures markets can play a significant role; specifically, they help companies hedge financial risks and assess potential savings that result from enhancing EE.^8

2.1 Futures contracts on fuel coal

Our sample consists of daily futures prices of coal on the New York Mercantile Exchange (NYMEX) from 01/04/2007 to 06/03/2009, or 607 trading days. Each day there is a variable number of futures prices, depending on the contracts

⁸Note that the market economics and the engineering reality are only part of the total picture. The regulatory situation is also crucial. Concerning the issue at hand, the economic stimulus package signed by President Obama on 17 February 2009 includes a modest federal subsidy for combined heat and power and recycled-energy projects.

Table 1. Summary statistics for Appalachian coal futures (NYMEX).						
	Daily data from $01/04/07$ to $06/03/09$					
	Observations Avg. Price (\$/ton) Std. Dev.					
All contracts	22,536	68.22	21.74			
1 Month	607	68.22	26.99			
5 Months	607	65.50	24.68			
10 Months	607	66.50	22.96			
15 Months	607	67.46	21.22			
20 Months	607	68.15	20.02			
25 Months	607	68.33	19.66			
30 Months	607	68.39	19.08			
35 Months	446	71.78	19.72			
40 Months	224	78.13	22.89			

maturity. The minimum number of contracts on a day is 30, and the maximum is 41 (which takes place at the final part of the sample). Table 1 shows basic statistics of the price series. Figure 3 displays futures prices of coal over the first few months of 2009 for different maturities. The usual pattern on a given day shows a futures price which increases with the time to expiration; see for instance Figure 4. The price increases flatten out. In this regard, market forces appear to put a cap on coal prices in the future.

2.2 Futures contracts on emission allowances

Concerning the futures market for EU emission allowances, prices are taken from the European Climate Exchange (ECX); the specific contract is referred to as EUA Futures. Table 2 shows summary statistics of the price series. The last part of the series includes contracts with maturity December-2013 and December-2014. These contracts thus fall beyond the Kyoto Protocol's expiration. Figure 5 displays futures prices of carbon over the most part of 2009 for different maturities. Unlike futures coal prices, now the usual pattern on a given day shows an allowance price which consistently increases with the time to expiration. This profile suggests a non-stationary path; the price grows without any apparent bound.

3 STOCHASTIC MODELS

3.1 Fuel commodity price

According to the empirical evidence in Figure 3, we assume that the spot price of a fuel commodity (say, coal) follows a long-run dynamics L_t (note that we are dealing with a long-lived facility; hence we are naturally interested in long-term

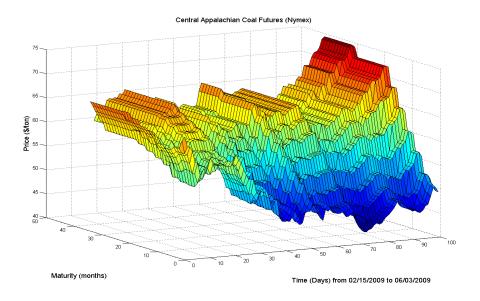


Figure 3: Central Appalachian Coal Futures prices (NYMEX).

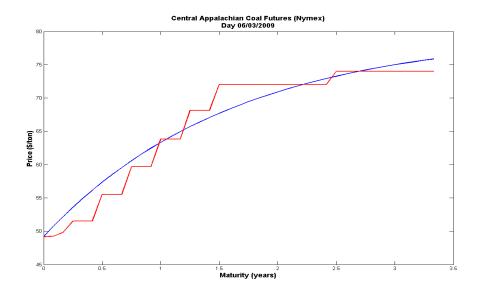


Figure 4: Coal futures prices on 06/03/2009 (NYMEX).

Table 2. Sum	Table 2. Summary statistics for EU allowance futures (ECX).				
D	aily data from	01/02/09 to $09/23/09$			
	Observations Avg. Price (€/tonne) Std. Dev.				
All contracts	1,116	18.33	3.48		
Dec 2009	186	16.43	2.67		
Dec 2010	186	17.05	2.72		
Dec 2011	186	17.86	2.80		
Dec 2012	186	19.01	2.97		
Dec 2013	186	20.59	2.96		
Dec 2014	186	22.02	3.10		

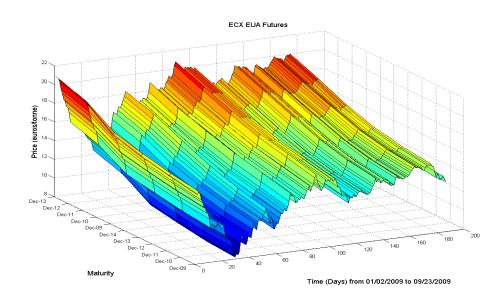


Figure 5: Futures contracts on EU allowances (ECX) over nine months.

valuations). We specify this long-run behavior as a mean-reverting stochastic process:

$$dL_t = k_L (L_m - L_t) dt + \sigma_L L_t dW_t^L, \qquad (1)$$

where:

 L_t : the current level of the spot price at time t.

 L_m : the level to which fuel price tends in the long run.

 k_L : the speed of reversion of the spot price towards its "normal" level. It can be computed as $k_L = \ln 2/t_{1/2}^L$, where $t_{1/2}^L$ is the expected half-life, i.e. the time required for the gap between L_t and L_m to halve.

 σ_L : the instantaneous volatility of the spot price, which determines the variance of L_t at t.

 dW_t^L : the increment to a standard Wiener process. It is normally distributed with mean zero and variance dt.

This specification boils down to $dL_t = \alpha_L L_t dt + \sigma_L L_t dW_t^L$ when $L_m = 0$ and $\alpha_L = -k_L$. Therefore it includes the geometric Brownian motion (GBM) as a particular case.

The time-t expectation in the physical world of the spot price at time T is:

$$E(L_T) = L_m + (L_t - L_m)e^{-k_L(T-t)}.$$

Hence we get:

$$\lim_{k_L \to \infty} E(L_T) = L_m \; ; \; \lim_{T \to \infty} E(L_T) = L_m \; .$$

Thus, for high values of k_L the model provides expected values which are close to L_m ; this amounts to the existence of little risk. In this case $(k_L >> 0)$, the expected cash flows can be discounted at the risk-free interest rate r.

In order to derive the risk-neutral behavior of the commodity price we subtract the risk premium $\lambda_L \hat{L}_t$, which we assume to be proportional to \hat{L}_t .⁹ This yields:

$$d\widehat{L}_t = \left[k_L L_m - (k_L + \lambda_L)\widehat{L}_t\right]dt + \sigma_L \widehat{L}_t dW_t^L.$$

Therefore, the time-t expected value of the spot price L_T under risk neutrality (or the time-t futures price of the commodity for delivery at T) is:

$$F(L_t, t, T) = E_t(\widehat{L}_T) = \frac{k_L L_m}{k_L + \lambda_L} + \left[L_t - \frac{k_L L_m}{k_L + \lambda_L}\right] e^{-(k_L + \lambda_L)(T-t)}.$$
 (2)

It comprises two items. The first one is the long-term equilibrium value which the estimated futures curve approaches asymptotically for longer maturities.

⁹Note that the value of λ_L can be negative in some instances. On the other hand, if the risk premium were specified as a fixed amount, independent of \hat{S}_t , then it would merely be λ_L . The ensuing formulas would be slightly different.

The second one shows the influence of the gap between the current price L_t and its equilibrium value (this influence also weakens with the passage of time).

Consider the surface that results from futures prices for different maturities over several days. On a given day i we have a number n_i of futures prices; we leave open the possibility of a growing number of prices as time goes on. Now we show that it is not necessary to know the spot price to compute the underlying parameters on a particular day. Instead, we only need to know futures prices. For maturities τ_1 and τ_2 (as seen from time 0, with $0 < \tau_1 < \tau_2$) we have:

$$F(L_{\tau_1}, \tau_1, t) = E_t(\hat{L}_{\tau_1}) = \frac{k_L L_m}{k_L + \lambda_L} + \left[L_t - \frac{k_L L_m}{k_L + \lambda_L} \right] e^{-(k_L + \lambda_L)(\tau_1 - t)},$$

$$F(L_{\tau_2}, \tau_2, t) = E_t(\hat{L}_{\tau_2}) = \frac{k_L L_m}{k_L + \lambda_L} + \left[L_t - \frac{k_L L_m}{k_L + \lambda_L} \right] e^{-(k_L + \lambda_L)(\tau_2 - t)}.$$

By taking τ_1 as the maturity of the nearest contract we get:

$$F(L_{\tau_2}, \tau_2, t) = \frac{k_L L_m}{k_L + \lambda_L} + \left[F(L_{\tau_1}, \tau_1, t) - \frac{k_L L_m}{k_L + \lambda_L} \right] e^{-(k_L + \lambda_L)(\tau_2 - \tau_1)}.$$
 (3)

This expression allows the usage of maturity gaps $\tau_2 - \tau_1$ which can be constant between futures contracts. For example, $\tau_2 - \tau_1 = 1/12$ for futures contracts with monthly maturities that are uniformly separated between them. With this formula we can estimate $k_L + \lambda_L$ and $\frac{k_L L_m}{k_L + \lambda_L}$ from actual futures prices on a given day or a set of days.¹⁰

3.2 Emission allowance price

According to the empirical evidence in Figure 5, we adopt a standard GBM process for carbon price:

$$dC_t = \alpha_C C_t dt + \sigma_C C_t dW_t^C, \tag{4}$$

where C_t denotes the price of the emission allowance at time t. The instantaneous drift rate is denoted by α_C , while σ_C stands for the instantaneous volatility of carbon price changes. In a risk-neutral setting the stochastic differential equation is:

$$d\widehat{C}_t = (\alpha_C - \lambda_C)\widehat{C}_t dt + \sigma_C \widehat{C}_t dW_t^C.$$
(5)

 λ_C stands for the premium related to carbon price risk. The expression for the futures price is a particular case of that used for the fuel commodity, specifically:

$$F(C_t, T, t) = E_t(\widehat{C}_T) = C_t e^{(\alpha_C - \lambda_C)(T - t)}.$$
(6)

 $^{^{10}}$ From these estimates we cannot derive the value of isolated parameters such as k_L, λ_L and L_m . Anyway we do not need them for valuation purposes.

Market data from futures contracts on CO_2 emission allowances along with equation (6) allow to compute the risk-adjusted drift rate $\alpha_C - \lambda_C$ of carbon price.

3.3 Overall investment cost

Let I_t denote the investment outlay at time t. The term I_t refers to the time-t present value of all the investment costs faced by the firm (whether they are disbursed all at once or sequentially over time, be they tangible or intangible, and net of whatever public subsidies received). Initially we can think of I_t as the purchase cost of new equipment that enhances EE in power plants. As such, this cost would not differ between electric utilities. However, in electric power generation, type, age, maintenance, and condition of plant differ markedly. Upgrading in this case means replacing some part of the plant and linking this newly converted part with remaining facilities. This means that installation costs can still differ if, for example, installation of the new equipment is more expensive in a plant in poorer condition (Levi and Nault [22]). Remember that we analyze the potential investment from the viewpoint of a firm or individual.

We assume I_t evolves stochastically according to a GBM:

$$dI_t = \alpha_I I_t dt + \sigma_I I_t dW_t^I. \tag{7}$$

The risk-neutral version is:

$$d\widehat{I}_t = (\alpha_I - \lambda_I)\widehat{I}_t dt + \sigma_I \widehat{I}_t dW_t^I.$$
(8)

 λ_I stands for the premium related to the risk concerning the amount to disburse. The expression for the futures price is similar to that for the emission allowance:

$$F(I_t, T, t) = E_t(\widehat{I}_T) = I_t e^{(\alpha_I - \lambda_I)(T - t)}.$$
(9)

For assessing an investment to increase EE we must determine three correlation coefficients:

$$dW_{t}^{L}dW_{t}^{C} = \rho_{LC}dt \; ; \; dW_{t}^{L}dW_{t}^{I} = \rho_{LI}dt \; ; \; dW_{t}^{C}dW_{t}^{I} = \rho_{CI}dt.$$
(10)

3.4 Value of a stochastic annuity between times τ_1 and τ_2

Assume we are deciding whether to invest or not at a given time. Therefore we need to know the present value of the prospective profits accruing to the investment, V. We deal with a stochastic income from each unit of fuel saved and each emission allowance spared. The value of this income can be computed as follows:

$$V = \int_{\tau_1}^{\tau_2} e^{-rt} F(L_t, T, t) + Q \int_{\tau_1}^{\tau_2} e^{-rt} F(C_t, T, t),$$
(11)

where Q stands for the tons of carbon dioxide avoided (or the number of allowances spared) per unit of fuel saved.¹¹ The dearest, dirtiest fuels would be the natural candidates for investments that enhance EE,¹² provided they are technically feasible. τ_1 and τ_2 enclose the operating life of the enhancement.

The value of the annuity emanates from two sources (see equations (3) and (6)):

$$V(L_0, C_0) = V_1(L_0) + V_2(C_0).$$
(12)

a) The effect of (a unit of) fuel saved:

$$V_1(L_0) = \frac{k_L L_m}{r(k_L + \lambda_L)} \left[e^{-r\tau_1} - e^{-r\tau_2} \right] + \left[L_0 - \frac{k_L L_m}{k_L + \lambda_L} \right] \frac{\left[e^{-(k_L + \lambda_L + r)\tau_1} - e^{-(k_L + \lambda_L + r)\tau_2} \right]}{k_L + \lambda_L + r}$$
(13)

c) The effect of the emission allowances spared (per unit of fuel saved):

$$V_2(C_0) = Q \frac{C_0}{\lambda_C + r - \alpha_C} [e^{-(\lambda_C + r - \alpha_C)\tau_1} - e^{-(\lambda_C + r - \alpha_C)\tau_2}].$$
 (14)

Note that this valuation only requires knowledge of the parameters derived from futures prices, i.e., those expected to prevail in a risk-neutral world. At a time when the stochastic variables take on the values (L_0, C_0) , from the estimates of $k_L + \lambda_L$, $\frac{k_L L_m}{k_L + \lambda_L}$, and $\alpha_C - \lambda_C$ we could immediately compute the value of an annuity between dates τ_1 and τ_2 .

The Net Present Value at the initial time is computed as:

$$NPV_0 = V(L_0, C_0) - I_0.$$

Similarly, at a given time t when we observe (L_t, C_t) and I_t we compute:

$$NPV_t = V(L_t, C_t) - I_t.$$

4 DATA AND ESTIMATION

Below we briefly sketch the estimation procedure and the main results. A thorough description of the whole issue can be found in the Appendix.

4.1 Parameters in the coal price process

We estimate the parameters of the coal price process regarding the long-term dynamics. We use the futures prices over the last 50 days in our sample, from 03/24/09 to 06/03/09.

¹¹Previously, if coal prices and carbon prices are quoted in different monetary units, the appropriate exchange rate will be used to convert them.

¹²Typically the cheapest fuels turn out to be the dirtiest ones.

If we undertake the valuation of future physical flows of commodities at a given time t, the delivery of which is absolutely certain, valuation should rest on the time-t futures curve. Therefore, our model must leave room for $k_L + \lambda_L$ and $\frac{k_L L_m}{k_L + \lambda_L}$ to change in value on a daily basis. Thus we recognize that the risk premium λ_L or other items change over time (as is the case in financial markets) and commodity markets). Despite the variability of $k_L + \lambda_L$ and $\frac{k_L L_m}{k_L + \lambda_L}$ with time, we are going to estimate an average value that best fits the series of daily values.¹³ We further use these average values as an estimate of future behavior.

Upon the estimation on each of the 50 days using non-linear least squares, we compute the corresponding average values. They turn out to be:

$$avg\left(k_L + \lambda_L\right) = 0.62; \quad avg\left(\frac{k_L L_m}{k_L + \lambda_L}\right) = 70.13.$$

The most relevant parameter for long-term valuations is $\frac{k_L L_m}{k_L + \lambda_L}$ because of the high value of $(k_L + \lambda_L)$ (which pushes toward quickly approaching the long-run equilibrium value). Figures 6 and 7 display their values on each day and the 95-percent confidence intervals. The average values that we adopt for our valuations below look like a reasonable compromise over the sample period. Regarding the price change volatility, we get an estimate of $\hat{\sigma}_L = 0.2850$.

4.2 Parameters in the allowance price process

The model is estimated with daily futures prices from 01/02/2009 to 09/23/2009. Previously, prices from the ECX, which are measured in \notin /tonne, are converted to \$/ton. Estimation proceeds along the same steps as before (see Appendix). The result is:

$$avg\left(\alpha_C - \lambda_C\right) = 0.056.$$

In view of Figure 8, our numerical estimate can be considered a reasonable value. We also get an estimate of volatility $\hat{\sigma}_C = 0.5622$ and a correlation coefficient $\hat{\rho}_{LC} = 0.0525$.

Risk-neutral drift rate $(\alpha_C - \lambda_C)$ over time and confidence interval

4.3 The carbon content of fuel coal

The expected growth rate of carbon prices under risk neutrality $(\alpha_C - \lambda_C)$ along with their volatility (σ_C) are fundamental components to the valuation process. Another key ingredient to valuation is the amount of carbon dioxide that is avoided for each ton of coal that ceases to be consumed. Obviously this depends on coal quality. Using data from EIA [7], we can compute the emission factors as shown in Table 3.¹⁴ In this paper we assume bituminous coal as the input fuel, hence Q = 2.4657.

¹³Something similar happens with volatility. Though it changes from one day to the next, usually it suffices to estimate one single value when trying to value long-term cash flows.

 $^{^{14}}$ One ton of Anthracite pollutes more than one tone of Bituminous coal. But a given amount of power will take less Anthracite than Bituminous coal. Sometimes, though, there is

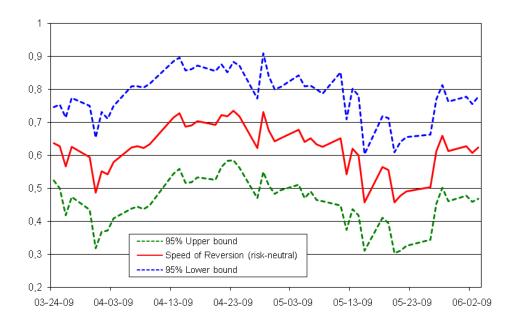


Figure 6: Risk-neutral speed of reversion $(k_L + \lambda_L)$ over time and daily confidence intervals.

Table 3. Emission factors from different coal types.				
Coal type	tons CO_2 / ton coal			
Lignite	1.3958			
Subbituminous	1.8580			
Bituminous	2.4657			
Anthracite	2.8425			

5 THE THREE-DIMENSIONAL LATTICE

Regarding multi-dimensional options, i.e., contingent claims dependent on the prices of multiple assets, closed-form solutions for their value are rarely available. Therefore one must resort to numerical methods. They fall within three main categories, namely: lattice methods, finite difference methods, and Monte Carlo simulation. Lattice methods are generally considered to be simpler, more flexible and, if dimensionality is not too large, more efficient than other methods.

A number of variations of the lattice approach have been proposed in the literature to assess the value of multivariate contingent claims. Boyle et al. [2] develop a valuation model for contingent claims involving several underlying assets. The price of each asset is assumed to be lognormally distributed (i.e., the

no choice because the type of coal that is fired is determined by the physical vicinity of coal mines.

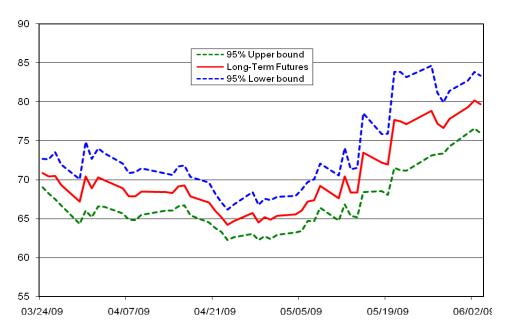


Figure 7: Long-term component $k_L L_m/(k_L + \lambda_L)$ over time and daily confidence intervals.

price process is governed by a GBM). Their numerical approximation method involves choosing jump sizes and jump probabilities so that the characteristic function of the discrete distribution converges to that of the continuous distribution. They obtain closed-form solutions for both the jump probabilities and the jump amplitudes. As they reckon, their solution technique does not guarantee that these probabilities will be positive, so this must be checked in each application. They further illustrate the accuracy of their method in the case of European options with three underlying assets.

Gamba and Trigeorgis [11] propose a binomial lattice approach (called GLT) to evaluate contingent claims whose payoff depends on multiple state variables that follow joint (correlated) GBMs. A variation of this method (called ALGT) simplifies the numerical scheme (all probabilities are equal and positive). These approaches prove to be consistent, stable and efficient. They further test the performance of the proposed approaches vis-a-vis other lattice approaches proposed for multi-dimensional option problems, among them Boyle et al. [2]. While all the lattice methods they analyze have the same order of convergence, their method dominates in terms of efficiency.

As indicated above, we assess an option to invest whose value depends on three correlated price processes. Two of them follow standard GBMs whereas the other one displays mean reversion. We derive closed-form solutions for the jump probabilities and the jump amplitudes in our case. The solution tech-

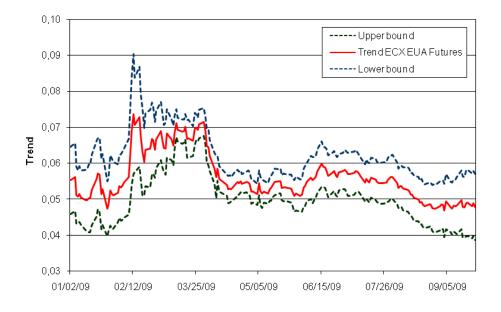


Figure 8: Risk-neutral drift rate $(\alpha_C - \lambda_C)$ over time and daily confidence intervals.

nique guarantees that these probabilities will be positive. Both jump sizes and jump probabilities will be used in our valuation of an American-type investment option.

5.1 Building the lattice

First we take natural logarithms of the (risk neutral) prices:

$$x_I \equiv \ln \widehat{I}_t; x_L \equiv \ln \widehat{L}_t; x_C \equiv \ln \widehat{C}_t$$

Applying Ito's Lemma, for the dynamics of the investment cost we have:

$$dx_I = (\alpha_I - \lambda_I - \frac{1}{2}\sigma_I^2)dt + \sigma_I dW_t^I = \nu_I dt + \sigma_I dW_t^I,$$

where $\nu_I \equiv (\alpha_I - \lambda_I - \frac{1}{2}\sigma_I^2).$

For the long-run dynamics of coal price we have:

$$dx_{L} = \left[\frac{k_{L}(L_{m} - \hat{L}_{t})}{\hat{L}_{t}} - \lambda_{L} - \frac{1}{2}\sigma_{L}^{2}\right]dt + \sigma_{L}dW_{t}^{L} = \nu_{L}dt + \sigma_{L}dW_{t}^{L}$$
where $\nu_{L} \equiv \left[\frac{k_{L}(L_{m} - \hat{L}_{t})}{\hat{L}_{t}} - \lambda_{L} - \frac{1}{2}\sigma_{L}^{2}\right],$
(16)

which can be rewritten as:

$$dx_L = \left[\frac{1}{\hat{L}_t}\frac{k_L L_m}{k_L + \lambda_L}(k_L + \lambda_L) - (k_L + \lambda_L) - \frac{1}{2}\sigma_L^2\right]dt + \sigma_L dW_t^L = \nu_L dt + \sigma_L dW_t^L$$

For the dynamics of the allowance price we have:

$$dx_C = (\alpha_C - \lambda_C - \frac{1}{2}\sigma_C^2)dt + \sigma_C dW_t^C = \nu_C dt + \sigma_C dW_t^C, \quad (17)$$

where
$$\nu_C \equiv (\alpha_C - \lambda_C - \frac{1}{2}\sigma_C^2).$$
 (18)

Note that, except for volatilities, all the parameters required for using the above formulas can be estimated in the risk-neutral world from futures prices.

With three dimensions in each node of the lattice, it is possible to move to $2^3 = 8$ different states of nature. Thus there are eight probabilities to be computed, in addition to three incremental values (Δx_I ; Δx_L ; Δx_C). For this purpose we have ten equations.

The first equation establishes that the probabilities must sum to one:

$$p_{uuu} + p_{uud} + p_{udu} + p_{udd} + p_{duu} + p_{dud} + p_{ddu} + p_{ddd} = 1.$$

The next three impose the conditions for consistency regarding the second moment:

$$\begin{split} E(\Delta x_I^2) &= (p_{uuu} + p_{uud} + p_{udu} + p_{udd})\Delta x_I^2 + (p_{duu} + p_{dud} + p_{ddu} + p_{ddd})\Delta x_I^2 = \\ &= \sigma_I^2 \Delta t + \nu_I^2 (\Delta t)^2 \simeq \sigma_I^2 \Delta t, \end{split}$$

$$E(\Delta x_L^2) = \Delta x_L^2 = \sigma_L^2 \Delta t + \nu_L^2 (\Delta t)^2 \simeq \sigma_L^2 \Delta t,$$
$$E(\Delta x_C^2) = \Delta x_C^2 = \sigma_C^2 \Delta t + \nu_C^2 (\Delta t)^2 \simeq \sigma_C^2 \Delta t.$$

When the increments Δt in the lattice are small, the term $(\Delta t)^2 \simeq 0$. These equations allow to directly compute the increments:

$$\Delta x_I = \sigma_I \sqrt{\Delta t}; \ \Delta x_L = \sigma_L \sqrt{\Delta t}; \ \Delta x_C = \sigma_C \sqrt{\Delta t}.$$

The next three equations require the probabilities to be consistent with observed correlations:

$$E(\Delta x_I \Delta x_L) = (p_{uuu} + p_{uud} - p_{udu} - p_{udd} - p_{duu} - p_{dud} + p_{ddu} + p_{ddd}) \Delta x_I \Delta x_L =$$

= $\rho_{IL} \sigma_I \sigma_L \Delta t + \nu_I \nu_L (\Delta t)^2,$

$$E(\Delta x_I \Delta x_C) = (p_{uuu} - p_{uud} + p_{udu} - p_{udd} - p_{duu} + p_{dud} - p_{ddu} + p_{ddd}) \Delta x_I \Delta x_C =$$

= $\rho_{IC} \sigma_I \sigma_C \Delta t + \nu_I \nu_C (\Delta t)^2,$

$$E(\Delta x_L \Delta x_C) = (p_{uuu} - p_{uud} - p_{udu} + p_{udd} + p_{duu} - p_{dud} - p_{ddu} + p_{ddd}) \Delta x_L \Delta x_C =$$

= $\rho_{LC} \sigma_L \sigma_C \Delta t + \nu_L \nu_C (\Delta t)^2.$

Remembering that $(\Delta t)^2 \simeq 0$ and the values for Δx_I , Δx_L , and Δx_C , we get:

$$p_{uuu} + p_{uud} - p_{udu} - p_{udd} - p_{duu} - p_{dud} + p_{ddu} + p_{ddd} = \rho_{IL}$$

$$p_{uuu} - p_{uud} + p_{udu} - p_{udd} - p_{duu} + p_{dud} - p_{ddu} + p_{ddd} = \rho_{IC}$$

 $p_{uuu} - p_{uud} - p_{udu} + p_{udd} + p_{duu} - p_{dud} - p_{ddu} + p_{ddd} = \rho_{LC}$

The last three equations establish the conditions for consistency with the first moment:

 $E(\Delta x_I) = (p_{uuu} + p_{uud} + p_{udu} + p_{udd} - p_{duu} - p_{dud} - p_{ddu} - p_{ddd})\Delta x_I = \nu_I \Delta t,$

$$E(\Delta x_L) = (p_{uuu} + p_{uud} - p_{udu} - p_{udd} + p_{duu} + p_{dud} - p_{ddu} - p_{ddd})\Delta x_L = \nu_L \Delta t_{duu}$$

 $E(\Delta x_C) = (p_{uuu} - p_{uud} + p_{udu} - p_{udd} + p_{duu} - p_{dud} + p_{ddu} - p_{ddd})\Delta x_C = \nu_C \Delta t.$ From them we derive:

$$p_{uuu} + p_{uud} + p_{udu} + p_{udd} - p_{duu} - p_{dud} - p_{ddu} - p_{ddd} = \frac{\nu_I \sqrt{\Delta t}}{\sigma_I},$$

$$p_{uuu} + p_{uud} - p_{udu} - p_{udd} + p_{duu} + p_{dud} - p_{ddu} - p_{ddd} = \frac{\nu_L \sqrt{\Delta t}}{\sigma_L},$$

$$p_{uuu} - p_{uud} + p_{udu} - p_{udd} + p_{duu} - p_{dud} + p_{ddu} - p_{ddd} = \frac{\nu_C \sqrt{\Delta t}}{\sigma_C}$$

We thus have seven equations and eight unknowns. In principle, several solutions are possible. However, we adopt the method suggested by Boyle et al. [2]. This way we get the following probabilities, which satisfy the above equations:

$$\begin{split} p_{uuu} &= \frac{1}{8} \left[1 + \rho_{IL} + \rho_{IC} + \rho_{LC} + \sqrt{\Delta t} \left(\frac{\nu_I}{\sigma_I} + \frac{\nu_L}{\sigma_L} + \frac{\nu_C}{\sigma_C} \right) \right], \\ p_{uud} &= \frac{1}{8} \left[1 + \rho_{IL} - \rho_{IC} - \rho_{LC} + \sqrt{\Delta t} \left(\frac{\nu_I}{\sigma_I} + \frac{\nu_L}{\sigma_L} - \frac{\nu_C}{\sigma_C} \right) \right], \\ p_{udu} &= \frac{1}{8} \left[1 - \rho_{IL} + \rho_{IC} - \rho_{LC} + \sqrt{\Delta t} \left(\frac{\nu_I}{\sigma_I} - \frac{\nu_L}{\sigma_L} + \frac{\nu_C}{\sigma_C} \right) \right], \\ p_{udd} &= \frac{1}{8} \left[1 - \rho_{IL} - \rho_{IC} + \rho_{LC} + \sqrt{\Delta t} \left(\frac{\nu_I}{\sigma_I} - \frac{\nu_L}{\sigma_L} - \frac{\nu_C}{\sigma_C} \right) \right], \\ p_{duu} &= \frac{1}{8} \left[1 - \rho_{IL} - \rho_{IC} + \rho_{LC} + \sqrt{\Delta t} \left(- \frac{\nu_I}{\sigma_I} + \frac{\nu_L}{\sigma_L} - \frac{\nu_C}{\sigma_C} \right) \right], \\ p_{dud} &= \frac{1}{8} \left[1 - \rho_{IL} + \rho_{IC} - \rho_{LC} + \sqrt{\Delta t} \left(- \frac{\nu_I}{\sigma_I} + \frac{\nu_L}{\sigma_L} - \frac{\nu_C}{\sigma_C} \right) \right], \end{split}$$

$$p_{ddu} = \frac{1}{8} \left[1 + \rho_{IL} - \rho_{IC} - \rho_{LC} + \sqrt{\Delta t} \left(-\frac{\nu_I}{\sigma_I} - \frac{\nu_L}{\sigma_L} + \frac{\nu_C}{\sigma_C} \right) \right]$$
$$p_{ddd} = \frac{1}{8} \left[1 + \rho_{IL} + \rho_{IC} + \rho_{LC} + \sqrt{\Delta t} \left(-\frac{\nu_I}{\sigma_I} - \frac{\nu_L}{\sigma_L} - \frac{\nu_C}{\sigma_C} \right) \right]$$

These probabilities have the same structure as those derived by Boyle et al. [2]; the terms ν_I , ν_L , ν_C , though, are different. Our development allows for mean-reverting stochastic processes, and is later used to value American-type options (unlike Boyle et al. [2] who value European-type options).

Negative probabilities cannot be accepted. To avoid this possibility we apply Bayes's Rule which decomposes the former probabilities into a product of conditional and marginal probabilities. We adopt a procedure which is similar to that in Bastian-Pinto et al. [1]. However, we consider three sources of risk (instead of two).

The conditional probabilities for x_I are:

$$p_{u} = p_{uuu} + p_{uud} + p_{udu} + p_{udd} = \frac{1}{2} + \frac{1}{2}\sqrt{\Delta t}\frac{\nu_{I}}{\sigma_{I}},$$
$$p_{d} = p_{duu} + p_{dud} + p_{ddu} + p_{ddd} = \frac{1}{2} - \frac{1}{2}\sqrt{\Delta t}\frac{\nu_{I}}{\sigma_{I}}.$$

It must be $p_u + p_d = 1$, with neither of them greater than one and less than zero. Therefore some nodes are censored as follows:

$$p_u^* = \max(0, \min(1, p_u)); p_d^* = 1 - p_u^*.$$

Now we derive the conditional probabilities of x_L in the following way:

$$p_{u/u} = \frac{p_{uuu} + p_{uud}}{p_u},$$
$$p_{d/u} = \frac{p_{udu} + p_{udd}}{p_u}.$$

These probabilities only make sense if $p_u^* > 0$, in which case it must be $p_{u/u} + p_{d/u} = 1$. Besides, they must be both between zero and one. If this does not hold at some node, we censor them as follows:

$$\begin{array}{ll} if \ p_u^* & > & 0 \ \text{then} \ p_{u/u}^* = \max(0,\min(1,p_{u/u})) \ ; \ p_{d/u}^* = 1 - p_{u/u}^*, \\ if \ p_u^* & = & 0 \ \text{then} \ p_{u/u}^* = 0 \ ; \ p_{d/u}^* = 0. \end{array}$$

Similarly:

$$\begin{array}{rcl} if \ p_d^* & > & 0 \ \text{then} \ \ p_{u/d}^* = \max(0,\min(1,p_{u/d})) \ ; \ p_{d/d}^* = 1 - p_{u/d}^*, \\ if \ p_d^* & = & 0 \ \text{then} \ p_{u/d}^* = 0 \ ; \ p_{d/d}^* = 0. \end{array}$$

In case $p_{u/u}^* > 0$ the conditional probabilities of x_E are derived as:

$$p_{u/u/u} = \frac{p_{uuu}}{p_{uuu} + p_{uud}},$$
$$p_{d/u/u} = \frac{p_{duu}}{p_{uuu} + p_{uud}}.$$

Thus we get:

$$p_{u/u/u}^* = \max(0, \min(1, p_{u/u/u})); p_{d/u/u}^* = 1 - p_{u/u/u}^*.$$

Analogously:

if $p_{d/u}^* > 0$:

$$p_{u/d/u}^* = \max(0, \min(1, p_{u/d/u})) ; p_{d/d/u}^* = 1 - p_{u/d/u}^*;$$

if $p_{u/d}^* > 0$:
$$p_{u/u/d}^* = \max(0, \min(1, p_{u/u/d})) ; p_{d/u/d}^* = 1 - p_{u/u/d}^*;$$

if
$$p_{d/d}^* > 0$$
:

 $p_{u/d/d}^*=\max(0,\min(1,p_{u/d/d}))\ ;\ p_{d/d/d}^*=1-p_{u/d/d}^*.$ In the end, the new probabilities are simply:

$$\begin{split} p_{uuu}^* &= p_u^* \cdot p_{u/u}^* \cdot p_{u/u/u}^*, \\ p_{uud}^* &= p_u^* \cdot p_{u/u}^* \cdot p_{d/u/u}^*, \\ p_{udu}^* &= p_u^* \cdot p_{d/u}^* \cdot p_{u/d/u}^*, \\ p_{udd}^* &= p_u^* \cdot p_{d/u}^* \cdot p_{d/d/u}^*, \\ p_{duu}^* &= p_d^* \cdot p_{u/d}^* \cdot p_{u/u/d}^*, \\ p_{dud}^* &= p_d^* \cdot p_{u/d}^* \cdot p_{d/u/d}^*, \\ p_{ddu}^* &= p_d^* \cdot p_{d/d}^* \cdot p_{u/d/d}^*, \\ p_{ddd}^* &= p_d^* \cdot p_{d/d}^* \cdot p_{u/d/d}^*, \\ \end{split}$$

Next we are going to value an option to invest in enhancing EE which depends on three different stochastic processes by means of a three-dimensional binomial lattice.

5.2 Deploying the lattice

The time T until maturity is subdivided into n steps each of size $\Delta t = T/n$. In our case, after the first step the initial value I_0 moves to one of two possible values, $I_0 u_I$ or $I_0 d_I$, where $u_I = e^{\sigma_I \sqrt{\Delta t}}$ and $d_I = 1/u_I = e^{-\sigma_I \sqrt{\Delta t}}$. Starting from initial values (I_0, L_0, C_0) after the first step we can compute the values $(I_0 e^{\sigma_I \sqrt{\Delta t}}, L_0 e^{\sigma_L \sqrt{\Delta t}}, C_0 e^{\sigma_C \sqrt{\Delta t}})$ with probability p_{uuu}^* . Similarly we derive the remaining nodes that arise in the first step, for example $(I_0 e^{-\sigma_I \sqrt{\Delta t}}, L_0 e^{-\sigma_L \sqrt{\Delta t}}, C_0 e^{-\sigma_C \sqrt{\Delta t}})$ with probability p_{ddd}^* .

After *i* steps, with j_I , j_L and j_C upside moves, the values $(I_0 e^{\sigma_I \sqrt{\Delta t}(2j_I - i)}, L_0 e^{\sigma_L \sqrt{\Delta t}(2j_L - i)}, C_0 e^{\sigma_C \sqrt{\Delta t}(2j_E - i)})$ will be reached. It can easily be seen that the tree branches recombine; thus, the same value results from a rise followed by a drop or the other way round. At the final time *T* the possible combinations of values can be represented by means of a cube. At the earlier moment $T - \Delta t$ another less-sized cube describes the set of feasible values. There will be some probabilities of moving from each node to eight possible states of the cube at time *T*.

This lattice is used to assess the possibility to invest in enhancing the EE level (thus saving input fuel and emission allowances) of a physical facility already in place (such as an operating coal-fired plant). Therefore, the saving opportunity is linked to the remaining life of the facility to be upgraded. We also consider that, once the decision to invest is made, it takes time for this enhancement to start working. The example below assumes that implementation takes a whole year.¹⁵ The investment opportunity is assumed to be available from initial time until T when the plant is closed down.¹⁶ However, given the time to build required, exercising the option to invest after time T - 1 will never pay off.¹⁷. So at T - 1 the value of the option at all the nodes is zero and investing makes no sense:

$$W = 0.$$

In a lattice with n time steps at the final time we will have $(n + 1)^3$ nodes; in the moment immediately before, the number will be n^3 nodes.

At earlier times, i.e., for t < T - 1, the option value at each node in the lattice is:

$$W = \max(V(I_t, L_t, C_t), e^{-r\Delta t}(p_{uuu}^*W^{+++} + p_{uud}^*W^{++-} + p_{udu}^*W^{+-+} + p_{udd}^*W^{+--} + p_{duu}^*W^{-++} + p_{dud}^*W^{-+-} + p_{ddu}^*W^{--+} + p_{ddd}^*W^{---} (19)$$

The NPV of investing immediately (i.e., exercising the option) is computed each time and it is compared with the second term, namely the value of the

¹⁵There is a lapse since the decision to invest is made until the physical units that improve EE are received, trial tests are time-consuming, and also final adjustments.

¹⁶Therefore we deal with an American-type option with three sources of risk.

¹⁷We are assuming a fixed (deterministic) useful life of the physical asset.

investment option when kept alive. The maximum between them is finally chosen. W^{+++} denotes the value reached when moving from the current node to another one where the three variables have moved upward. The latter value has been already derived since the lattice is solved backwards. Note that the value of an investment at time t < T - 1 must be computed between dates t + 1and T, i.e., one year after the investment decision until the facility's expiration at T. So in this case $\tau_1 = t + 1$ and $\tau_2 = T$.

Proceeding backwards through the lattice we get an amount which shows the value of the option to invest, which cannot be negative.¹⁸

By changing the initial values (I_0, L_0, C_0) we can derive those combinations for which the option to invest switches from worthy to worthless. These values provide the trigger prices for investing initially to be optimal. They also allow to draw the border between the "invest" region and the "wait" region.

6 RESULTS AND SENSITIVITY ANALYSIS

6.1 Results in the base case

For convenience we show again in Table 4 the parameter values adopted in the base case.

Table 4. Parameter values: base case.				
L_0	L_0 Current long-term price of coal 46.90			
C_0	Spot price of emission allowance	17.8231		
σ_I	Volatility of investment cost	0.10		
σ_L Volatility of coal price 0.2				
σ_C Volatility of allowance price		0.5254		
$k_L L_m/(k_L + \lambda_L)$ Long-term price of coal		70.13		
ρ_{LC} Correlation between coal and carbo		0.0525		
$\alpha_I - \lambda_I$ Drift rate of investment cost 0				
$\alpha_C - \lambda_C$ Drift rate of allowance price		0.056		
$k_L + \lambda_L$	Reversion coefficient of coal price	0.62		

In our computations we take 12 steps per year. The remaining life of the facility goes from 2 to 15 years. The number of steps is given by 12 * (T - 1). With 15 years this means 168 time steps. Therefore, the number of possible option values at time T - 1 when we start proceeding backwards is 4,826,809; we assign them a value of zero. Of course, this will not necessarily be so at the 4,741,632 nodes immediately before (at time T - 13/12).

We are going to make a first assessment with the initial values and assuming three possible investment costs I_0 : \$500, 750, and 1,000. The value of the option to invest W consists of the value of investing immediately (NPV) and that of

¹⁸ The three-dimensional lattice can require a lot of computer memory. It may be convenient to keep in the memory at a time only the two cubes we are working with at that time, namely those at times t and $t + \Delta t$.

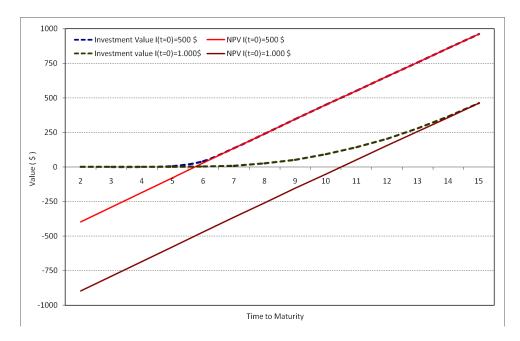


Figure 9: Value of the immediate investment and value of the option to invest.

the option to wait; it is shown in Table 5.	The option value clearly depends on
the remaining useful life of the facility.	

	Table 5. Value of the option to invest.						
	$I_0 =$	= 500	$I_0 =$	= 750	$I_0 = 1,000$		
T	W	NPV	W	NPV	W	NPV	
15	961.5	961.5	711.5	711.5	461.5	461.5	
14	859.7	859.7	609.7	609.7	365.4	359.7	
13	757.6	757.6	507.6	507.6	279.6	257.6	
12	655.2	655.2	405.2	405.2	205.1	155.2	
11	552.4	552.4	302.4	302.4	142.2	52.4	
10	449.0	449.0	199.0	199.0	91.2	-51.0	
9	345.1	345.1	117.2	95.1	52.5	-154.9	
8	240.6	240.6	60.1	-9.4	25.6	-259.4	
7	135.4	135.4	24.8	-114.6	9.6	-364.6	
6	42.7	29.5	6.9	-220.5	2.3	-470.5	
5	7.3	-77.0	0.9	-327.0	0.2	-577.0	
4	0.3	-184.0	0.0	-434.0	0.0	-684.0	
3	0.0	-291.0	0.0	-541.0	0.0	-791.0	
2	0.0	-397.0	0.0	-647.0	0.0	-897.0	

As shown in the first column ($I_0 = 500$), between year 6 and year 7 we switch from a situation in which W > NPV to another in which W = NPV; at some time in between the option to wait has become worthless. Therefore, it will be optimal to invest immediately when the remaining life at least equals that time (with $I_0 = 500$). Also, with just that time to maturity it will be optimal if the investment cost falls below $I_0 = 500$. Otherwise, for terms lower than or equal to 6 years and $I_0 = 500$, it is preferable to wait.

The blue and red lines in Figure 9 show this result. The red line describes the NPV, i.e., the value of investing immediately. As such, it is negative when there are few years left to profit from the improvement in EE, while it becomes positive for longer operation periods. The blue line describes the value of the option to invest. Since it represents a right, not an obligation, its value cannot be negative. As can be seen, with few years left, the best decision is to wait, i.e., to keep the option alive (by not investing). For longer maturities, though, the investment can pay off, and waiting no longer makes sense (the two lines overlap each other). The green and brown lines in Figure 9 above show a similar pattern for $I_0 = 1,000$. They contact each other somewhere between years 14 and 15.

6.2 Sensitivity analysis

Next we derive the threshold investment cost I^* (or optimal exercise price) in the base case and its sensitivity to changes in parameter values.

6.2.1 Sensitivity to changes in investment cost

Levi and Nault [22] consider incentive programs designed to induce firms to make a major discrete observable conversion in their production technology to mitigate damage to the environment. Their model implicitly treats the price of output as deterministic. In our case, to induce firms to enhance EE, policy makers can choose among alternative programs. Environmentalists may object to subsidies on the grounds that it is inappropriate to "bribe" firms to reduce their bills. On the other hand, a subsidy can be justified when there are positive network externalities from the investment. Specifically, an increase in the number of firms that enhance EE may bring additional benefits to such investments. For instance, economies of scale in production or installation of the EE enhancing technology could result in cost declines that increase with the number of adopting firms; there could also be learning in the EE-improving process itself. All else equal, the benefits from EE investment cost reductions are a gain in welfare.¹⁹

Cortazar et al. [4] assume both output and input prices are stochastic and use real options to evaluate investments in environmental technologies. Similarly, we use real options to assess investments that enhance EE while treating

¹⁹ For those interested in the effectiveness of subsidies to investment cost as a measure to entice firms into EE, see Jaffe and Stavins [21], and Hassett and Metcalf [13].

the prices of input coal and carbon output as stochastic along with the investment cost. Firms enjoy the flexibility in when to invest (i.e., when to enhance EE). Therefore we deal with the strategic timing of the investment. The optimal time to invest can be affected if there is a fixed time period during which the subsidy program is offered. Now, a 50% subsidy of the initial investment costs which were only available at the outset would lead us to compare the option to invest later (green curve, $I_0 = 1,000$) with the NPV of an investment with a cost $I_0 = 500$ (red curve). The lines cross between years 5 and 6, thus making it easier to undertake the investment earlier in time, some six years before closure 20

Firm decisions about when to invest may also be affected if the cost of the EE investment changes over time. For example, investment costs can decrease because of learning from related technological developments. We consider both time and uncertainty regarding investment costs. Thus, we are going to analyze the impact of changes in the volatility σ_I and the drift rate $\alpha_I - \lambda_I$. The results appear in Table 6.

	Table 6. Sensitivity to investment cost.					
	Change in σ_I		Change in $\alpha_I - \lambda_I$			
T	$\sigma_I {=} 0.10$	$\sigma_I = 0.20$	$\alpha_I - \lambda_I = 0.025$	$\alpha_I - \lambda_I = -0.025$		
15	1,019.2	902.0	1,069.1	958.7		
14	972.4	866.7	1,015.8	920.1		
13	922.5	928.5	959.8	878.0		
12	869.4	787.0	900.8	832.3		
11	812.0	741.8	838.8	782.5		
10	752.5	692.6	773.4	728.4		
9	688.3	639.0	704.5	669.7		
8	619.7	580.5	631.8	606.0		
7	546.5	516.7	555.1	537.0		
6	468.4	447.1	474.0	462.3		
5	385.0	371.0	388.4	381.6		
4	296.2	288.1	297.9	294.7		
3	202.0	198.3	202.6	201.5		
2	102.5	101.6	102.6	102.6		

As expected, a higher cost volatility σ_I raises the strain in the form of a lower level I^* for the investment cost. With 15 years to maturity, an 11.4% fall in cost is required with respect to the base case ($\sigma_I = 0.10$, $I^* = 1,019.2$). Figure 10 shows that the threshold cost is lower for higher volatilities. A more uncertain environment regarding costs leads managers to delay investments unless their

 $^{^{20}}$ We do not address the free-riding problem that can be triggered by this subsidy. Also, we leave aside the issues of fairness to firms that invested to enhance their EE levels prior to the subsidy program.

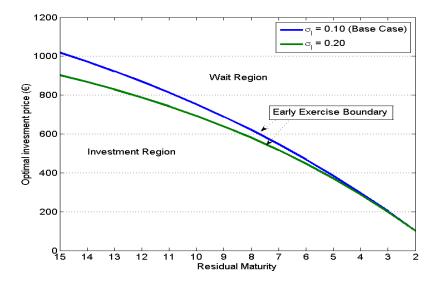


Figure 10: Threshold investment cost for different cost volatilities as a function of the facility's remaining life.

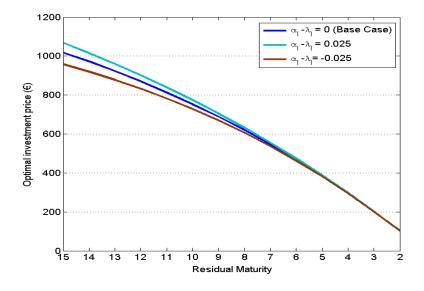


Figure 11: Threshold investment cost for different cost drift rates as a function of the facility's remaining life.

costs fall relatively to the former situation. Needless to say, as the facility gets closer to its end, the threshold cost falls consistently.

A growth rate $\alpha_I - \lambda_I = 0.025$ in the risk-neutral world makes investment easier by pushing the level I^* upward. Conversely, a rate $\alpha_I - \lambda_I = -0.025$ compounds investment at the outset, since its cost is expected to decrease in the future. Figure 11 shows this effect: if the project costs are expected to increase significantly in the future, it is relatively better to invest sooner (rather than later), so the managers are less demanding in terms of I^* . Therefore, the curve shifts upwards.

6.2.2 Sensitivity to changes in the emission allowance price

Let us consider the case of a change in the initial allowance price and allowance volatility. See Table 7.

	Table 7. Sensitivity to emission allowance.					
		Change in σ_C		Change in $\alpha_C - \lambda_C$	Change in C_0	
T	$\sigma_C = 0.25$	${m \sigma}_C{=}0.5254$	$\sigma_C = 0.75$	$\alpha_C - \lambda_C = 0.10$	$C_0 = 30.00$	
15	1,213.8	1,019.2	915.2	1,093.5	1,200.7	
14	$1,\!149.5$	972.4	873.9	1,042.0	$1,\!149.0$	
13	1,081.8	922.5	830.2	987.2	1,093.8	
12	1,010.5	869.4	783.9	928.9	1,035.0	
11	935.7	812.0	734.7	866.9	971.5	
10	857.2	752.5	682.4	800.9	903.6	
9	775.1	688.3	626.5	730.7	830.6	
8	689.4	619.7	566.8	656.1	752.2	
7	599.9	546.5	503.0	576.8	667.7	
6	506.8	468.4	434.4	492.5	576.5	
5	410.1	385.0	360.6	403.2	478.0	
4	310.1	296.2	280.9	308.8	371.5	
3	207.4	202.0	194.6	209.4	256.6	
2	103.1	102.5	101.2	105.6	132.5	

A low allowance price volatility raises significantly the threshold cost below which we would be eager to invest. Conversely, as shown in Figure 12, a higher allowance volatility feeds cautiousness in that managers require lower investment costs in order to undertake the project.

An increase in the slope, i.e., a higher allowance price expected in the future, eases investments to enhance EE. Anticipation of higher allowance prices in the future means higher savings to be reaped from improving EE. Therefore, as Figure 13 suggests, investment can be justified for higher costs.

Figure 14 shows that allowance prices have an acute impact on decisions to invest in EE. According to the last column in Table 9, if initial carbon prices are higher (30.00 instead of 17.82), managers will be persuaded to pay higher investment costs.

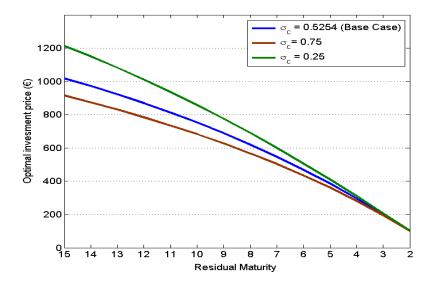


Figure 12: Threshold investment cost for different allowance volatilities as a function of the facility's remaining life.

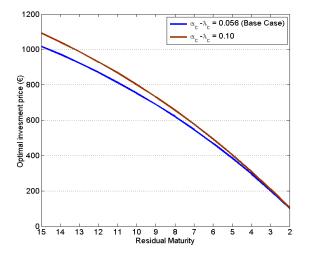


Figure 13: Threshold investment cost for different allowance drift rates as a function of the facility's remaining life.

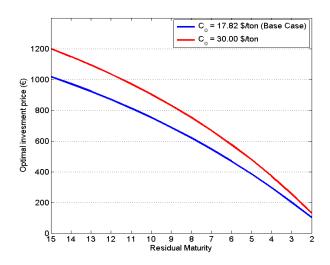


Figure 14: Threshold investment cost for different initial allowance prices as a function of the facility's remaining life.

6.2.3 Sensitivity to changes in coal price

We analyze changes in volatility and the long-term price. Volatility has a rather limited impact. This is due to the strong effect of the mean-reversion coefficient. Table 8 shows these results.

Table 8. Sensitivity to changes in σ_L .				
T	$\sigma_L = 0.25$	${oldsymbol \sigma}_L{=}~0.2850$	$\sigma_L = 0.45$	
15	1,020.6	1,019.2	1017.0	
10	753.4	752.5	751.0	
5	385.5	385.0	384.4	

Instead, the long-term price of coal has a large impact. See Table 9. Again, if coal prices are higher, investments to enhance EE will be more easily justified from a financial point of view. As shown in Figure 15, higher savings in energy bills allow the trigger cost I^* to move upwards.

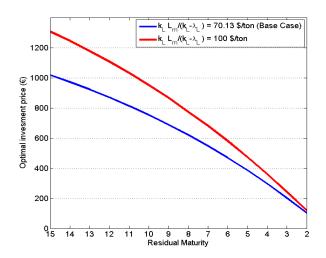


Figure 15: Threshold investment cost for different long-term coal futures prices as a function of the facility's remaining life.

	E 11 0 0 11 11 1				
	Table 9. Sensitivity to changes in $k_L L_m/(k_L + \lambda_L)$.				
T	$k_L L_m/(k_L+\lambda_L)=$ 70.13	$k_L L_m / (k_L + \lambda_L) = 100.00$			
15	1,019.2	1,306.7			
14	972.4	1,244.2			
13	922.5	1,177,7			
12	869.4	1,107.0			
11	812.0	1,031.9			
10	752.5	952.1			
9	688.3	867.3			
8	619.7	773.3			
7	546.5	681.3			
6	468.4	580.1			
5	385.0	472.7			
4	296.2	359.7			
3	202.2	241.2			
2	102.5	119.7			

7 CONCLUDING REMARKS

Investments that enhance energy efficiency (EE) can help reduce both energy and GHG emission bills. According to the IEA [19], end-use and power plants efficiency (including new appliances, more efficient gas and coal plants, switching from coal to gas and early retirements) can deliver globally 8 Gt CO_2 of abatement by 2030, the same amount than nuclear, renewable and CCS technologies together. For those savings to become a reality the IEA estimates that additional investment of 7,500 billion (\$2008) will be needed until 2030.

Although the potential of EE seems huge, it is far from being fully undertaken. Some will argue that this so-called energy efficiency paradox is not such. Given rational consumers and efficient markets, investments observed are economically optimal; any deviation from optimality would be explained by hidden costs. Others, however, would indicate that energy markets are subject to failures and barriers that explain this gap.

In any case, there is one element that can arguably explain a part of the story behind the "efficiency gap"; namely the (lack of) consideration of uncertainty when valuing potential returns on these projects. If uncertainties are not addressed conveniently decision makers can choose for inaction despite investment being profitable, or choose for action despite being unprofitable. In fact, the returns on EE investments draw heavily on variables that by their very nature are not deterministic, e.g., regulatory framework, energy prices, or emission permit restrictions.

In this paper we consider uncertain costs and revenues from projects that enhance EE; our aim is to determine the optimal time to invest. Investment is valued like a (real) option that is only exercised at the optimal time, and is irreversible. There are three sources of uncertainty: the long-term dynamics of the commodity (coal) price, the emission allowance price, and the overall investment cost. Based on a cursory look at market data, we assume that the commodity price follows a mean-reverting stochastic process. Regarding the allowance price and the investment cost we adopt a geometric Brownian motion. Parameter values for these price processes have been estimated from samples of futures prices of coal (NYMEX) and EU emission allowances (ECX). Then we can compute the value of a stochastic annuity from fuel saved and allowances spared. By subtracting the investment cost we derive the Net Present Value (NPV) of the project.

In particular, we have considered an operating physical facility already in place with a remaining useful life that ranges from 2 to 15 years. The investment to improve EE takes a whole year to be operative. The numerical estimates of the parameters are then used in a three-dimensional binomial lattice to assess the value of the option to invest. We note that our procedure precludes the possibility of negative probabilities. Maximizing the option value involves determining the optimal exercise time. Thus we compute the trigger investment cost, i.e., the threshold level below which immediate investment would be optimal. To our knowledge, a three-dimensional lattice allowing for mean-reverting processes has not been previously solved and used in any application.

Our results show the NPV of an immediate investment along with the value of the option to invest for different investment costs (\$500, 750, 1000). When the value of waiting is zero we would invest immediately. In the base case ($I_0 = 500$) investment will be optimal for remaining lives beyond some six years. For terms lower than or equal to six years it is preferable not to invest even if the NPV is positive. This finding can help understand the "efficiency gap" and the different perspectives sometimes adopted by engineers and economists when valuing projects. Moreover, as investment cost increases, exercising the option requires longer periods of useful life. Thus, doubling the cost ($I_0 = 1000$) makes investment optimal only when the remaining life is 15 years, even though the NPV of the investment would be positive for 10 years.

We have assessed several policy measures in terms of their influence on the optimal time to invest in EE improvements. Indeed, regulators can play an important role in bringing forward these investments: (a) given the external positive effects resulting from EE investments (climate change, health benefits, security of supply), a public subsidy can be justified; (b) uncertainties must be reduced where possible (e.g. regarding the EU ETS, the post-Kyoto scenario, etc.); (c) policy makers can raise carbon prices by reducing the supply of allowances. If these measures are taken in a transparent manner, within a long-term framework, so much the better.

8 Appendix: Estimation of price processes

Below we derive the numerical estimates of the underlying parameters. Our ultimate objective is to show how to value options to invest. In this respect, reasonable parameter values that can be used as a base case scenario are enough for our purposes.

8.1 Parameters in the coal price process

We estimate the parameters of the coal price process considering the long-term dynamics. We use the futures prices over 50 days ranging from 03/24/09 to 06/03/09. These days are the last days in our sample. If we took earlier dates, we would get into the price-bubble period on the commodities markets.

The estimation process consists of two steps. It has some similarities with the process followed by Cortazar and Schwartz [5]. In the first step, using the prices on each day and non-linear least-squares, we derive the curve that best fits the prices on that day, which provides an estimate of the parameters in expression (3). This estimation of the parameters refers to price behavior under risk neutrality. Our process has several advantages:

a) It allows direct usage of futures prices (we do not need spot prices which sometimes do not exist).

b) The time lapses between prices are constant. There is no initial term to maturity of varying length, which is usually given by the time between the spot price and the nearest futures price.

c) It is possible, without complicating estimation and contributing to it, to use all the futures prices available on a given day. This is not typically the case in Kalman filter-based estimations, where a limited number of futures prices is chosen.

d) It allows to use a variable number of futures prices over time. This is convenient since new contracts with longer maturities are introduced periodically, the prices of which can be of interest for long-term valuations.

e) It is possible to compute confidence intervals for the estimates of each day. The same holds for the estimates of $k_L + \lambda_L$, and $\frac{k_L L_m}{k_L + \lambda_L}$ computed as the average of the daily estimates. These daily average values are derived in the second step.

Upon the calibration on each of the 50 days, we compute the corresponding average values. They are shown in Table A1.

Table A1. Average value of the coal parameters.				
Parameter	Estimate	Std. error	t-ratio	
$\frac{\frac{k_L L_m}{k_L + \lambda_L}}{k_L + \lambda_L}$	70.13	0.6503	107.8	
$k_L + \lambda_L$	0.62	0.0103	60.05	

Regarding estimation of the volatilities, first we estimate the series of the current long-term component $E_t(L_t)$ from the nearest futures contract $F(L_t, \tau_1, t)$ using the parameters estimated for each day:

$$E_t(\widehat{L}_t) = \left[F(L_t, \tau_1, t) - \frac{k_L L_m}{k_L + \lambda_L}\right] e^{-(k_L + \lambda_L)(\tau_1 - t)} + \frac{k_L L_m}{k_L + \lambda_L}.$$

The values of $E_t(\hat{L}_t)$ that result from a regression based on equation (1) (i.e., the behavior in the physical world) allow to compute a volatility of $\hat{\sigma}_L = 0.2850$.

8.2 Parameters in the allowance price process

The model is calibrated with daily futures prices from 01/02/2009 to 09/23/2009. Previously, prices from the ECX, which are measured in \notin /tonne, are converted to $\$/\text{ton.}^{21}$ Calibration proceeds along the same steps as before. We estimate $(\alpha_C - \lambda_C)$ for each day by non-linear least squares. The result appears in Table A2. Volatility is derived by similar procedures. We get $\hat{\sigma}_C = 0.5622$. Residuals from the regression allow to compute the correlation coefficient $\hat{\rho}_{LC} = 0.0525$.

Table A2. Average value of the carbon parameters					
Parameter	Parameter Estimate Std. error t-ratio				
$\alpha_C - \lambda_C$	0.056	0.0004	115.8		

²¹1 tonne= 1.10231136 tons. The exchange rate is taken from the Bank of Spain's fixing rate. This conversion does not affect the estimate of the slope $\alpha_C - \lambda_C$. And the effect on the estimate of the volatility is very small: from $\hat{\sigma}_C = 0.5254$ (in \notin /tonne) to $\hat{\sigma}_C = 0.5622$.

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