R&D Investment and Technology Adoption*

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Abstract

We study a model of technology adoption. The firm's profit is depending on the technology that it uses. The firm holds the option to upgrade its current technology with a new one. Technology progress is modeled with a Poisson process. The firm can either rely on the external R&D market or it can setup its own R&D center. The arrival rate of new technologies in the external R&D market is constant and given. Whenever the firm chooses to setup its own R&D center it can optimaly set the arrival rate of the new technologies. However, the setup costs are increasing in the selected arrival rate.
We identify the parameter values for which its optimal for the firm to setup its own R&D center and for those parameter values we determine the optimal arrival rate. Moreover, we determine the optimal technology adoption trigger in all situations. We end the paper with comparative static results.

1 Introduction

In most technology adoption models the technological progress is exogeneously given. In this paper we extend the literature on technology adoption by assuming that the firm that adopts the new technology can influence the speed of technology adoption by setting up an R&D center. We investigate the situations in which its beneficial for a firm to use this opportunity. Furthermore, we investigate the impact on the speed of technology adoption and the technology level that is choosen by the firm.

*Please note that this is work in progress. The latest version can be downloaded from *http://center.uvt.nl/staff/huisman/researchandtechnologyadoption.pdf*

2 Model

We consider a risk neutral value maximizing firm. The firm discounts with rate $r > 0$. The profit flow at time t is denoted by $\pi_t(\theta_t)$, where θ_t denotes the technology efficiency level that the firm uses at time t. The firm initially produces with technology θ_0. We assume that the profit flow of a firm is increasing in the technology efficieny parameter, i.e $\frac{\partial \pi}{\partial \theta} > 0$. The firm can upgrade its technology once, the upgrade costs are equal to I. New technologies arrive over time according to a Poisson process with rate λ. We assume that the efficiency level of each new technology is u higher than its predecessor. So we have that

$$d\theta = \begin{cases} u & \text{with probability } \lambda dt, \\ 0 & \text{with probability } 1 - \lambda dt. \end{cases} \tag{1}$$

Furthermore, we assume that the firm can influence the speed of arrival of new technologies by setting up an R&D center. Let the setup costs for such a center be equal to $a \exp(\lambda)$ and the running cost per period c.

The model is solved in two steps. In the first step the optimal technology adoption strategy is determined given the choosen level λ. The second step uses the result of the first step to determine the optimal level of λ.

2.1 Optimal Technology Adoption

Given that we know the choosen level λ we can use the results from Huisman (2001, Chapter 2). The value of the firm after upgrading to technology θ is denoted by $V(\theta)$. We assume that the firm closes the R&D center after it has upgraded its technology. So we have that

$$V(\theta) = \int_{t=0}^{\infty} \pi_t(\theta) e^{-rt} dt = \frac{\pi(\theta)}{r}. \tag{2}$$

From Huisman (2001, Chapter 2) we know that there exists a threshold level θ^* for which it holds that the firm should upgrade its technology the first moment that θ is larger or equal than θ^*. In our model we will have that the threshold level is a function of λ, i.e. $\theta^*(\lambda)$. In order to derive this threshold we have to investigate the region for which we know that the firm will invest after the next technology arrival, i.e. the region $\{\theta \,|\theta^*(\lambda) - u < \theta \leq \theta^*(\lambda)\}$. Let $F(\theta, \lambda)$ denote the value of the firm before the firm has upgraded its technology, then the following Bellman equation should hold for F

$$F(\theta, \lambda) = \pi(\theta_0) - c + \lim_{dt \downarrow 0} \frac{1}{dt} E[dF(\theta, \lambda)]. \tag{3}$$

It holds that

$$E[dF(\theta, \lambda)] = \lambda dt (V(\theta + u) - I - F(\theta, \lambda)) + o(dt). \tag{4}$$

Substitution of (4) into (3) and rewriting gives

$$F(\theta, \lambda) = \frac{\pi(\theta_0) - c}{r + \lambda} + \frac{\lambda}{r + \lambda}(V(\theta + u) - I). \tag{5}$$

At the threshold value $\theta^* (\lambda)$ the firm is indifferent between investing right away and waiting for the next technology. In other words at the threshold the following value matching condition must hold

$$F(\theta^*(\lambda), \lambda) = V(\theta) - I. \tag{6}$$

Substitution of the derived equations and rewriting gives the following equation that implicitly defines the threshold $\theta^* (\lambda)$

$$\frac{\pi(\theta_0) - c}{r+\lambda} + \frac{\lambda}{r+\lambda}\left(\frac{\pi(\theta^*(\lambda) + u)}{r} - I\right) = \frac{\pi(\theta^*(\lambda))}{r} - I. \tag{7}$$

From Huisman (2001, Chapter 2) we know that the number of needed technology jumps n^*, the expected technology adoption time and its variance are equal to

$$n^* = \left\lceil \frac{\theta^*(\lambda) - \theta_0}{u} \right\rceil, \tag{8}$$

$$E[T^*] = \frac{n^*}{\lambda}, \tag{9}$$

$$Var[T^*] = \frac{n^*}{\lambda^2}. \tag{10}$$

In the region $\{\theta \,|\theta \leq \theta^*(\lambda) - u\}$ the same Bellman equation as (3) must hold. The difference is that in this region we have that we know that after the next technology arrival the firm will not invest for sure. So that

$$E[dF(\theta, \lambda)] = \lambda dt\,(F(\theta + u, \lambda) - F(\theta, \lambda)) + o(dt). \tag{11}$$

Substitution of (11) in (3) and rewriting gives

$$F(\theta, \lambda) = \frac{\pi(\theta_0) - c}{r+\lambda} + \frac{\lambda}{r+\lambda}(F(\theta + u, \lambda) - I). \tag{12}$$

Following the solution methods in Huisman (2001, Chapter 2) the solution of the difference equation (12) can be derived. We have that

$$F(\theta, \lambda) = \left(\frac{\lambda}{r+\lambda}\right)^{\frac{\theta^*(\lambda)-\theta}{u}} \left(\frac{\pi(\theta^*(\lambda))}{r} - \frac{\pi(\theta_0) - c}{r} - I\right) + \frac{\pi(\theta_0) - c}{r}. \tag{13}$$

2.2 Optimal Speed of Technology Arrival

Knowing the results of the previous subsection the firm will that level for the speed of arrival that maximizes its value. In mathematical words the firm solves

$$\max_{\lambda \geq 0} \left(F(\theta, \lambda) - a \exp(\lambda)^2\right). \tag{14}$$

Substitution of equation (13) into equation (14) gives rise to the following first order condition, which implicitly defines λ^*

$$\begin{aligned} &\left(\frac{\lambda}{r+\lambda}\right)^{\frac{\theta^*(\lambda)-\theta}{u}} \left(\frac{\pi(\theta^*(\lambda))}{r} - \frac{\pi(\theta_0) - c}{r} - I\right)\left(\frac{r}{\lambda(r+\lambda)}\frac{\theta^*(\lambda) - \theta}{u} + \frac{1}{u}\log\left(\frac{\lambda}{r+\lambda}\right)\frac{\partial \theta^*(\lambda)}{\partial \lambda}\right) \\ + &\left(\frac{\lambda}{r+\lambda}\right)^{\frac{\theta^*(\lambda)-\theta}{u}} \frac{\frac{\partial \pi(\theta^*(\lambda))}{\partial \theta}}{r}\frac{\partial \theta^*(\lambda)}{\partial \lambda} - a\exp(\lambda) = 0. \end{aligned} \tag{15}$$

We can solve (15) numerically if we have an expression for $\frac{\partial\theta^*(\lambda)}{\partial\lambda}$. Define $G(\lambda,\theta)$ as

$$G(\lambda,\theta) = \frac{\pi(\theta_0)-c}{r+\lambda}+\frac{\lambda}{r+\lambda}\left(\frac{\pi(\theta+u)}{r}-I\right)-\frac{\pi(\theta)}{r}+I \tag{16}$$

So that $G(\lambda,\theta^*(\lambda))=0$ and we have that

$$\frac{\partial\theta^*(\lambda)}{\partial\lambda} = -\left.\frac{\frac{\partial G(\lambda,\theta)}{\partial\lambda}}{\frac{\partial G(\lambda,\theta)}{\partial\theta}}\right|_{\theta=\theta^*(\lambda)} = \frac{\frac{\pi(\theta_0)-c}{(r+\lambda)^2}-\frac{r}{(r+\lambda)^2}\left(\frac{\pi(\theta^*(\lambda)+u)}{r}-I\right)}{\frac{\lambda}{r(r+\lambda)}\frac{\partial\pi(\theta^*(\lambda)+u)}{\partial\theta}-\frac{1}{r}\frac{\partial\pi(\theta^*(\lambda))}{\partial\theta}} \tag{17}$$

We can derive λ^* after substituting (17) into (15) and solving the resulting equation numerically.

3 Results

3.1 Example

We use the same parameters as in Huisman (2001, Chapter 2), so we take $\pi = 200\theta^2$, $r = 0.1$, $u = 0.1$, $I = 1600$. Futhermore, we take $a = 600$ and $c = 20$. In case the firm does not setup an R&D center we assume that $\lambda = 1$. In the latter case we can derive with equation (7) that $\theta^* = 2.703$, so that 18 jumps are needed and the expected adopion time is 18 years with a standard deviation of 4.243 years. The value of the firm is equal to 4172.5.

In case the firm does setup an R&D center we find that the firm should set the optimal speed of technology arrival equal to $\lambda^* = 2.356$. The value of the firm then equals 4314.3. The resulting investment trigger is equal to $\theta^*(\lambda^*) = 5.093$, so that 71 jumps are needed and the expected adoption time is 17.40 years with a standard deviation of 2.717 years. The setup costs for the R&D center are 6332.17.

We conclude that for these parameter values it is optimal to setup the R&D center as the value of the firm with the R&D center is higher than the value of the firm without the R&D center.

In Figure 1 the region in which it is optimal for the firm to setup an R&D center in the (a,c) space is drawn. We conclude that the higher are the setup costs a for the R&D center, the lower the running costs c must be and vice versa.

Figures 2-5 show the optimal size λ^*, the optimal technology adoption trigger $\theta^*(\lambda^*)$, the expected technology adoption time $E[T^*]$, and the number of needed jumps n^*, respectively, as function of the setup costs a. The running costs are taken such that the firm is exactly on the indifference boundary between setting up an R&D center and not setting up an R&D center.

The higher are the setup costs a, the lower will be the optimal size λ^* as a result the optimal technology adoption trigger $\theta^*(\lambda^*)$ will be lower. The plot for the expected technology adoption time shows two effects. In general a higher setup costs a implies a lower speed of technology arrival and a lower technology adoption trigger, which are two opposing effects on the expected technology adoption time. A lower speed of technology arrival results in a higher expected technology adoption time. However, a lower technology adoption trigger decreases the expected technology adoption time. It turns out that in the presented case the former effect dominates so that in general the trend is that the expected technology adoption time is increasing. Furthermore, we see that curve that represents the expected technology adoption time can decrease in the short run. This is caused by the fact that the number of needed jumps is a step function and in between the steps we have that λ^* decreases so that the expected technology adoption time increases until the number of needed jumps is reduced. At that moment the expected adoption time jumps down a little and starts to rise again.

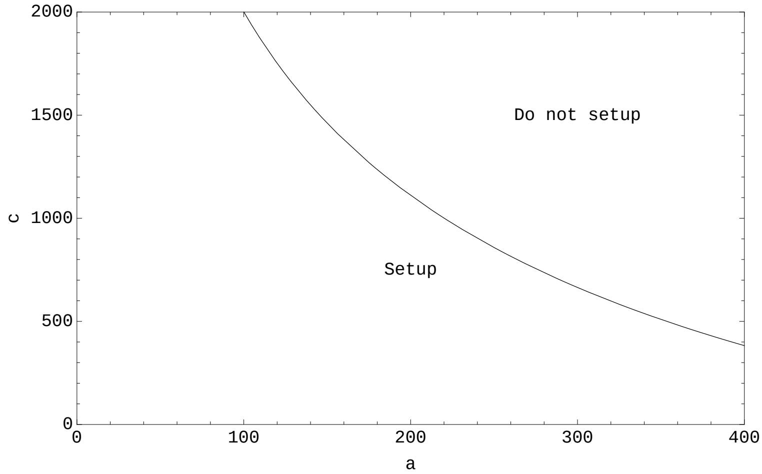

Figure 1: Region in the (a, c) space in which it is optimal for the firm to setup an R&D center.

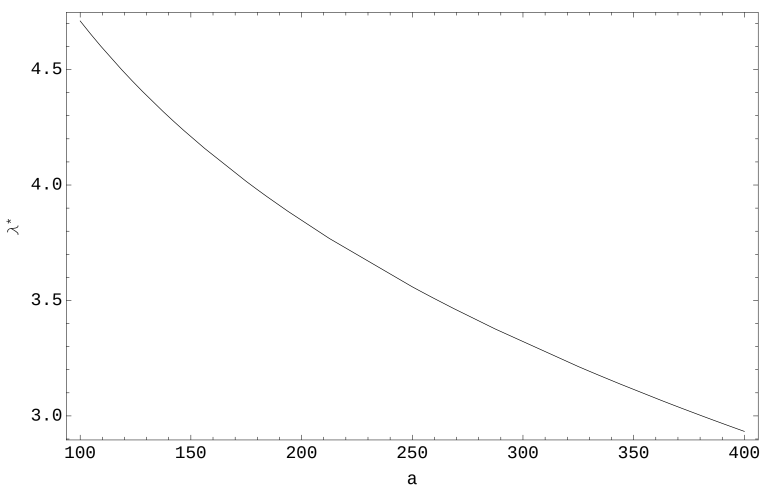

Figure 2: Optimal size λ^* on the indifference boundary as function of a.

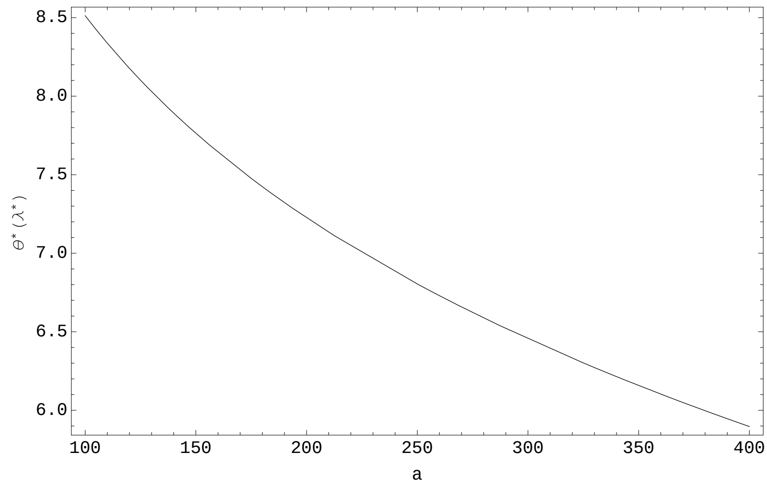

Figure 3: Optimal technology adoption trigger $\theta^*(\lambda^*)$ on the indifference boundary as function of a.

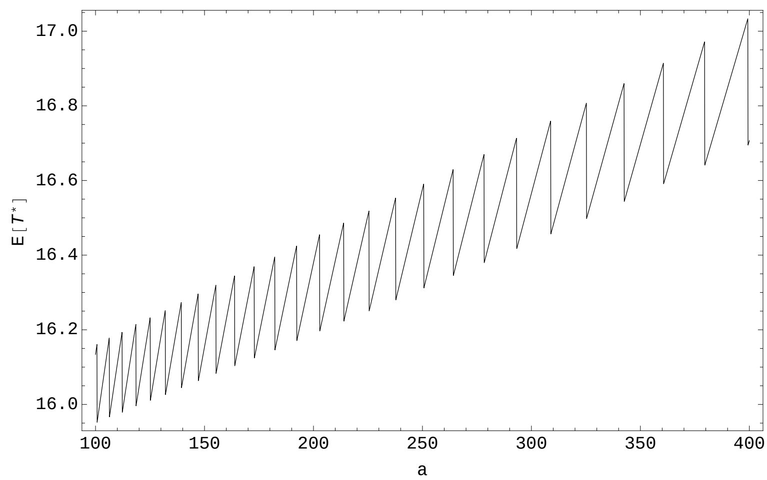

Figure 4: Expected adoption time $E[T^*]$ on the indifference boundary as function of a.

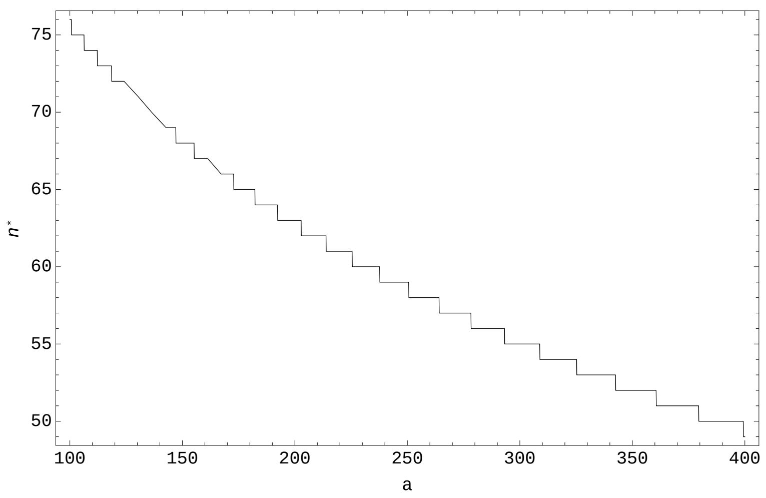

Figure 5: Number of needed jumps n^* on the indifference boundary as function of a.

4 Conclusion

In this paper we studied the combined R&D investment and technology adoption problem of the firm.

References

Huisman, K. J. M. (2001). *Technology Investment: A Game Theoretic Real Options Approach*. Kluwer Academic Publishers, Dordrecht, The Netherlands.