Continuous Rainbow Options in Co-integrated Markets

Jörg Dockendorf

Manchester Business School, Manchester, UK

Turmweg 19, D-20148 Hamburg, Germany
E-mail: dockendj@gmx.de
Tel: +49 (0) 179 217 8889

Dean A. Paxson

Manchester Business School, Manchester, UK

Booth Street West, Manchester M15 6PB, UK
E-mail: dpaxson@man.mbs.ac.uk
Tel: +44 (0) 161 275 6333

This paper develops a continuous real option model on the best of two commodities when their price spread follows an arithmetic mean-reverting process. The co-integration of the two variables effectively allows the complexity to be reduced from two sources of uncertainty to only one by focusing on the spread, which is why the model can also be applied to continuous entry/exit problems on a single mean-reverting variable. We provide a quasi-analytical solution for valuing this real rainbow option and the trigger levels when to switch between the two operating modes by incurring a switching cost. All parameters of our solution are estimated from empirical data and consistent risk-adjusted discounting is applied. We apply the theoretical model to value a polyethylene plant based on the spread between polyethylene and ethylene. The spread is shown to be stationary and the parameters of the stochastic process are estimated by OLS regression and tested for validity. The sensitivity analysis reveals important implications for the investment timing and operation decisions of investors in a flexible plant, both depending on the extent of mean-reversion in the value-driver.

JEL Classifications: D81, G31, L72

Keywords: Uncertainty, Flexibility, Real rainbow option, Continuous switching, Mean-reversion, Co-integration, Petrochemicals

1 Introduction

Industrial and agricultural applications frequently exhibit inherent options to choose between the best of two commodities. If these options are well established, the probability is high that the markets of these commodities are co-integrated to some degree. If prices drift apart, suppliers would exercise the option and switch from the less favourable to the more favourable product, typically by incurring a switching cost, until the equilibrium is re-established which is reflected in a mean-reverting price spread. In these cases, a two-factor valuation problem could then be reduced to a problem with a single stochastic factor. These co-integrated markets are found particularly when commodities have similar applications and can be substituted rather easily for one another or when the production cost of one commodity is heavily influenced by another commodity. Examples include dry (bulk) and wet (oil) markets in the shipping industry (see Sodal et al., 2007), commercial and residential uses of real estate, industrial plants with flexibility on the product mix, refining margins and other conversion processes in the chemical industry, such as the production of polyethylene which is created by polymerisation of ethylene. Both ethylene and polyethylene are traded products, so that the conversion can be considered a real rainbow option. The valuation of this rainbow option based on the conversion spread will be the subject of the empirical application.

Kulatilaka and Trigeorgis (2004) discuss the general approach of valuing switching options, including options additivity and asymmetric switching costs. Stulz (1982) and Johnson (1987) develop closed-form solutions for a European option on the maximum or minimum of two or more assets. A quasi-analytical solution to a two-factor problem, where the option is not homogenous of degree one in the stochastic variables, is provided by Adkins and Paxson (2010a) and elaborated into a general switching model for two alternative energy inputs (2010b). Dockendorf and Paxson (2009) develop real option models on the best of two commodity outputs with both single and continuous switching, including the option of temporary suspension, and apply the models to value a flexible fertilizer plant. All of the above mentioned models are based on uncertainty represented by geometric Brownian motion. The Schwartz (1997) analysis on the behaviour of commodity prices reveals that commercial commodity prices exhibit strong mean reversion. Also, Geman (2007) tests energy commodity prices for mean reversion and finds that oil and natural gas prices are mean-reverting during one period and random walk during another. Tvedt (2000) values a vessel with lay-up option in a shipping market with freight rate equilibrium and acknowledges in his conclusion that mean-reversion should be considered in the freight rate dynamics to improve the model for practical valuation. The option pricing theory on co-integrated assets has been explored by Duan and Pliska (2004), who value finite spread options on stock indices subject to time-varying volatility by means of Monte Carlo simulations. Dixit and Pindyck (1994) provide a solution to the investment problem on an asset which follows a geometric mean-reverting stochastic process, i.e. where the variable has an absorbing barrier at zero. Option valuation on mean-reverting assets is applied by Pinto et al. (2007) to the Brazilian sugar industry by approximating the prices of sugar and ethanol as discrete binomial mean-reverting processes and determining the value of switching between the two commodities within a bivariate lattice option framework. Näsäkkälä and Fleten (2005) value a flexible gas fired power plant on the basis of a spark spread with mean-reverting variations in the short term and a gBm equilibrium price in the long-term, but ignoring switching costs.

Sodal et al. (2007) value the switching option for combination carriers between the co-integrated dry and wet bulk markets by modelling the price spread as mean-reverting. The approach is based on the Bellman equation which uses for the solution of the maximisation problem a rate ρ to discount the future option values. However, such a discount rate cannot be reasonably estimated because of the options-specific risk characteristics. Sodal's empirical application confirms that the option value is highly sensitive to this discount rate ρ. The option value almost triples if ρ is reduced from 0.15 to 0.05. Furthermore, the cash flows of the static project with no switching option, which includes non-stochastic cash flows, have been discounted at the same rate ρ. We develop an option model based on the contingent-claims and the risk-neutral valuation approach and show how Sodal's solution can be transformed to be independent of ρ.

The remaining part of this paper is organised as follows: Section 2 introduces the characteristics of the mean-reverting spread, provides the present value of perpetual cash flows without switching option and then develops a model for the continuous rainbow option. Section 3 applies the continuous rainbow option to value a polyethylene plant based on an econometric model of the polyethylene-ethylene conversion spread. Specific and general implications are discussed in Section 4. Section 5 concludes and raises issues for further research.

2 Valuing the Switching Opportunity

2.1 Modelling uncertainty as a mean-reverting spread

We assume that the asset can be operated in two different modes where each operating mode is associated with a different commodity produced. The flexibility to switch between two operating modes – the base mode (denoted by '0') and the alternative mode (denoted by '1') – means that we are faced with two underlying uncertainties, which are the prices of the two commodities. In integrated markets, however, the prices of the two commodities are bound to one another by economic reasons, so that the complexity can be reduced to only one underlying uncertainty by modelling the difference between the two commodity prices as mean-reverting. Let (p) be the weighted spread of the commodity prices,

$$p = p_1 - \frac{k_0}{k_1} p_0 \ , \qquad (1)$$

where p_0 and p_1 are the commodity prices in the base and alternative mode, respectively, and k_0 and k_1 the capacities. The capacities enter into the equation in order to account for the fact that product units and output capacities of the asset may be different in the two operating modes. Hence, we unitise the spread with regard to the product sold in the alternative mode. The spread of two co-integrated commodities can be both positive or negative so that the mean-reverting process needs to be modelled as an arithmetic Ornstein-Uhlenbeck process:

$$dp = \eta(m - p)dt + \sigma dz, \qquad (2)$$

where η is the speed of reversion, m the long-run mean of the spread, σ the volatility and dz a standardised Wiener process. The expected value of p at time t is given by:

$$E[p_t] = m + (p - m)e^{-\eta t} \qquad (3)$$

and the variance of p_t:

$$Var[p_t] = \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta t}\right) \qquad (4)$$

Dixit and Pindyck (1994) determine the convenience yield (δ) of a mean-reverting process where both the drift rate and the volatility are proportional to the current level of the underlying variable (geometric mean-reversion). Applying the same logic, the parameters of the arithmetic mean-reverting process can be derived. The expected return (μ) of p is determined by the systematic risk in the stochastic fluctuations in p and is equal to the sum of convenience yield (δ) and expected increase in the level of p (α). The expected percentage change of p is the relative drift rate:

$$\alpha = \frac{\eta(m-p)}{p} = \mu - \delta \ , \tag{5}$$

While the required return μ is constant, the yield δ varies with p. Solving for the convenience yield provides

$$\delta = \mu - \frac{\eta(m-p)}{p} \ . \tag{6}$$

The dividend yield is the same in the risk-adjusted and risk-neutral world (denoted by *). With the risk-neutral drift rate $\alpha^* = r - \delta$ and the expected absolute drift being $\alpha^* p^*$, the risk-neutral process of the mean-reverting process can be specified. An alternative way of deriving the risk-neutral process is to adjust the Wiener process for the market price of risk (λ_m), $dz^* = dz - \lambda_m dt$, as outlined by Bjerksund and Ekern (1995). We deviate from the latter reference insofar as λ_m cannot be kept constant for the arithmetic mean-reversion because the relative volatility depends on p. Applying the definition provided by Hull (2006), the market price of risk for the arithmetic mean-reversion is $\lambda_m = \frac{\mu - r}{\sigma / p}$, which provides:

$$dp^* = [\eta m - (\mu - r + \eta)\ p^*]dt + \sigma dz^* . \tag{7}$$

2.2 *Discounted cash flow with no flexibility*

Assuming no operating flexibility, the present value of the asset can be calculated as the discounted cash flow. The cash flow is given by the spread less the variable operating cost, $k_1(p-c)$, where (c) is the weighted difference in variable operating cost between the two operating modes:

$$c = c_1 - \frac{k_0}{k_1} c_0 \ . \tag{8}$$

The discount factor for the volatile spread consists of a risk component and a time component. For the mean-reverting process, the risk dissipates over time so that the applicable risk discount factor would be different for each time period. This is in contrast to a geometric Brownian motion where the risk increases with time and the risk discount factor is compounded in the same way as the time discount factor. Instead of calculating the time-dependent risk discount factor for the mean-reverting process, the risk can be incorporated in the growth rate of p, as demonstrated by Bhattacharya (1978), which results in the risk-neutral process of p. Let $M = \frac{m\eta}{\eta + \mu - r}$, then from (2):

$$dp = (\eta + \mu - r)(M - p)dt + \sigma dz \tag{9}$$

In analogy to equation (3), the expected value of p in the risk-neutral scenario is then given by:

$$E[p_t] = M + (p - M)e^{-(\eta+\mu-r)t} \tag{10}$$

The risk-neutral cash-flow could either be discounted at the risk-free rate of return for an asset lifetime of T years, or be discounted in perpetuity at a higher rate taking into account the asset depreciation in the form of exponential decay. We take the latter approach since we also need to consider technological, political and environmental risk. Let the arrival rate λ of a Poisson event incorporate both, depreciation and technological risk, so that the risk-neutral cash-flow is discounted at the rate $(r+\lambda)$:

$$PV_1(p) = k_1 \int_0^{\infty} \left(E[p_t] - c\right) e^{-(r+\lambda)t} dt = k_1 \left(\frac{m}{(r+\lambda)\left(1+\frac{\mu+\lambda}{\eta}\right)} + \frac{p}{\mu+\eta+\lambda} - \frac{c}{r+\lambda} \right). \quad (11)$$

The discounted cash flow consists of three parts. Firstly, the long-term average (m) is discounted at a rate of $r(1+(\mu+\lambda)/\eta)$. This discount rate increases with the systematic risk in the stochastic fluctuations of p, represented by μ, and decreases with the speed of mean-reversion (η), because the faster p returns to its long-run average the faster the risk is dissipated. With $\eta >> (\mu+\lambda)$, the discount rate will be only slightly above the risk-free rate. Secondly, the current value of p is discounted at $(\mu+\eta+\lambda)$ which corresponds to discounting the η-decaying exponential function of p at the discount rate μ and accounting for depreciation and political/technical risk. Thirdly, the operating cost is discounted at the risk-free rate augmented by the Poisson probability.

2.3 Continuous rainbow option

We now allow for flexibility between the two operating modes. In the base mode, the commodity spread is foregone (zero cash flow). In the alternative mode, the spread is earned and variable operating costs are incurred (positive or negative cash flow). $V_0(p)$ and $V_1(p)$ represent the values of being in the respective state, each with the option to switch to the other mode. The Ornstein-Uhlenbeck process is a special case of the general Itô process of the form $dp = \alpha(p)\,dt + \sigma(p)\,dz$. Starting from this general approach, the value of an option on p, V(p), is described by the partial differential equation below:

$$\tfrac{1}{2}\sigma(p)^2 \frac{\partial^2 V}{\partial p^2} + (r-\delta)p\frac{\partial V}{\partial p} - (r+\lambda)V = 0 \quad (12)$$

The convenience yield from equation (6) is substituted for V_0 and V_1 in the above equation. In the base mode, no cash flow is earned and V_0 is solely determined by the option value.

$$\tfrac{1}{2}\sigma^2 \frac{\partial^2 V_0}{\partial p^2} + [\eta m - (\mu - r + \eta) p] \frac{\partial V_0}{\partial p} - (r + \lambda) V_0 = 0 \tag{13}$$

When operating in the alternative mode, a cash flow is earned equal to the commodity spread net of variable operating cost.

$$\tfrac{1}{2}\sigma^2 \frac{\partial^2 V_1}{\partial p^2} + [\eta m - (\mu - r + \eta) p] \frac{\partial V_1}{\partial p} - (r + \lambda) V_1 + k_1 (p - c) = 0 \tag{14}$$

A more general form of equation (13) is obtained by substituting $a = 2(r - \mu - \eta)/\sigma^2$, $b = 2\eta m/\sigma^2$ and $d = -2(r + \lambda)/\sigma^2$:

$$\frac{\partial^2 V_0}{\partial p^2} + (a\, p + b) \frac{\partial V_0}{\partial p} + d \cdot V_0 = 0 \tag{15}$$

With $\mu > r$ and $\eta > 0$, parameter (a) will always be negative. For $a < 0$, Kampke (1956, p. 416) suggests substituting $V_0 = F(x)$ and $x = \sqrt{|a|}\left(p + \frac{b}{a}\right)$ to obtain:

$$\frac{\partial^2 F}{\partial x^2} - x \frac{\partial F}{\partial x} - \frac{d}{a} F = 0 \tag{16}$$

Appendix A demonstrates how the above equation can be further transformed into the Weber equation by substituting $F = G(x) e^{\frac{1}{4}x^2}$ (see also Kampke, 1956, p. 414):

$$\frac{\partial^2 G}{\partial x^2} = \left(\frac{x^2}{4} + \frac{d}{a} - \frac{1}{2} \right) G \tag{17}$$

Spanier and Oldham (1987, p. 447) establish that the above Weber differential equation is satisfied by the parabolic cylinder function of order (-d/a) and argument (x) and (-x), represented by $D_{-d/a}(x)$ and $D_{-d/a}(-x)$, so that G(x) is determined by:

$$G(x)=A\cdot D_{-d/a}(x)+B\cdot D_{-d/a}(-x)\,, \tag{18}$$

where A and B are constant parameters and the parabolic cylinder function is defined by:

$$D_v(x)=\frac{1}{2}\cdot\frac{\sqrt{2^{v+2}\pi}}{\Gamma\left(\frac{1-v}{2}\right)}e^{-\frac{x^2}{4}}M\left(-\frac{v}{2},\frac{1}{2},\frac{x^2}{2}\right)+\frac{1}{2}\cdot\frac{-\sqrt{2^{v+3}\pi}}{\Gamma\left(\frac{-v}{2}\right)}x\,e^{-\frac{x^2}{4}}M\left(\frac{1-v}{2},\frac{3}{2},\frac{x^2}{2}\right), \tag{19}$$

with M the Kummer function:

$$M(a,b,z)=1+\frac{a}{b}z+\frac{a(a+1)}{b(b+1)}\frac{z^2}{2!}+\ldots=\sum_{k=0}^{\infty}\frac{\Gamma(a+k)}{\Gamma(b+k)}\frac{\Gamma(b)}{\Gamma(a)}\frac{z^k}{k!} \tag{20}$$

The asset in the base model is therefore valued as:

$$V_0(p)=\left(A_0\cdot D_{-d/a}\left(\sqrt{|a|}\left(p+\frac{b}{a}\right)\right)+B_0\cdot D_{-d/a}\left(-\sqrt{|a|}\left(p+\frac{b}{a}\right)\right)\right)e^{\frac{1}{4}\left(\sqrt{|a|}\left(p+\frac{b}{a}\right)\right)^2} \tag{21}$$

Concerning the asset in the alternative operating mode, the value is determined by the non-homogenous partial differential equation (14). The solution consists of the sum of the general solution to the homogeneous PDE and a particular solution to the non-homogeneous PDE. A particular solution to the non-homogeneous equation is the present value of the perpetual cash flow $k_1(p-c)$ which is the value of the operating asset without flexibility and given by equation (11), repeated below.

$$V_{1P}(p)=k_1\left(\frac{p}{\mu+\eta+\lambda}+\frac{m\eta}{(r+\lambda)(\eta+\mu+\lambda)}-\frac{c}{r+\lambda}\right) \tag{22}$$

With the substitutions $u=\frac{k_1}{\mu+\eta+\lambda}$ and $w=k_1\left(\frac{m\eta}{(r+\lambda)(\mu+\eta+\lambda)}-\frac{c}{r+\lambda}\right)$, the value of the asset operating in the alternative mode is determined by the function below:

$$V_1(p)=\left(A_1\cdot D_{-d/a}\left(\sqrt{|a|}\left(p+\frac{b}{a}\right)\right)+B_1\cdot D_{-d/a}\left(-\sqrt{|a|}\left(p+\frac{b}{a}\right)\right)\right)e^{\frac{1}{4}\left(\sqrt{|a|}\left(p+\frac{b}{a}\right)\right)^2}+u\cdot p+w \tag{23}$$

The reader can verify, that the solution to the homogenous partial differential equation based on a Bellman equation with the unspecified discount rate ρ, as provided by Sodal et al. (2007), can be transformed into the above equation by substituting $\eta \rightarrow \eta + \mu - r$, $m \rightarrow \frac{\eta m}{\eta + \mu - r}$ and $\rho \rightarrow r$ in the former, where notations apply as used in this paper. Their solution to the non-homogenous differential equation cannot be transformed in a similar way since all stochastic and non-stochastic components of the perpetual cash flows have been uniformly discounted at the rate ρ.

As Kulatilaka and Trigeorgis (2004, p. 195) state, the "valuation of the flexible project must be determined simultaneously with the optimal operating policy". So we can expect the coefficients A and B to depend on the switching boundaries given by the spread levels of p_H and p_L, where p_H triggers a switch from the base operating mode to the alternative operating mode and p_L vice versa. In order to determine the coefficients, the general form of the value functions needs to be investigated. The option value of switching from the base mode to the alternative mode needs to increase with the spread, since the spread can only be earned in the alternative mode, and to tend towards zero for large negative spreads. When operating in the alternative mode and earning the cash flow p-c, the option to switch and forego the cash flow needs to increase in value with lower (more negative) p-values and should be almost worthless for very high values of p. Figure 1 below depicts the general form of the value functions.

FIGURE 1

The parabolic cylinder function $D_v(x)$ tends towards infinity for large negative values of x and towards zero for large positive x for all v<0. It is a monotonically decreasing function in x for ($v < -0.20494$) and has one local maximum for ($-0.20494 < v < 0$). The exponential multiplier term in the option value in V_0 and V_1 makes the option values monotonically increasing and decreasing respectively for all $v<0$. For V_0, the option value of switching increases with p and becomes negligible for large negative values of p. Hence A_0 must be zero and B_0 positive. For V_1, it is the other way round, so that A_1 must be positive and B_1 zero.

Switching between operating modes occurs when the value in the new operating mode exceeds the value in the incumbent mode by the switching cost. These rules are formalised by two boundary conditions,

$$V_0(p_H) = V_1(p_H) - S_{01} \tag{24}$$

$$V_1(p_L) = V_0(p_L) - S_{10} \tag{25}$$

where S_{01} and S_{10} are the respective switching costs. V_0 and V_1 must also comply with the smooth pasting conditions at the trigger levels, p_H and p_L.

$$\frac{\partial V_0(p_H)}{\partial p} = \frac{\partial V_1(p_H)}{\partial p} \tag{26}$$

$$\frac{\partial V_0(p_L)}{\partial p} = \frac{\partial V_1(p_L)}{\partial p} \tag{27}$$

The four equations, (24), (25), (26), (27), enable us to determine the four unknown parameters B_0, A_1, p_H and p_L. The procedure to solve the system of equations is as follows:

1. Solve equation (24) for B_0 as a function of A_1, p_H and p_L
2. Solve equation (25) for A_1 as a function of p_H and p_L
3. Guess p_H and p_L (based on the general shape of the value functions)
4. Change p_H and p_L until both equations (26) and (27) are satisfied simultaneously

5. Check that both B_0 and A_1 are positive

Step 4 is a minimization problem which is solved numerically. The solution is therefore not available in closed-form. Appendix B provides the detailed equations.

3 Empirical Application: Valuing a polyethylene plant

In this empirical section, the continuous rainbow option is applied to determine the market value of a polyethylene plant which converts ethylene into polyethylene. The latter product is a plastic which is widely used in film, pipes, blow and injection moulding applications and fibres, while ethylene is the main product from the petrochemical naphtha cracking process. At first glance, this seems to be an input/output option rather than an option on the best of two outputs (rainbow option). However, both commodities are traded and ethylene could be sold to the market instead of converting it to polyethylene. In that sense, the polyethylene plant can be considered a rainbow option on ethylene and polyethylene. The flexibility is given by the option to choose between not operating the plant (base mode) and operating the plant (alternative mode). Figure 2 below depicts a simplified scheme of the transformation.

FIGURE 2

While various patented polyethylene processes are used in industry, we will focus on the slurry process for the production of high-density polyethylene (HDPE). The asset under consideration is assumed to be in Europe with an annual production capacity of 250,000 tons of HDPE, with an initial investment of an estimated €200 million.

Meyers (2004) provides specific consumption data for the slurry process which requires about 1,017 kg of ethylene for the production of 1,000 kg of polyethylene. The conversion spread is therefore defined as:

$$p = p_{polyethylene} - 1.017 \cdot p_{ethylene} \qquad (28)$$

Although other materials are required for the chemical transformation, prices of polyethylene are largely determined by ethylene as the dominant feedstock. This suggests that both prices are co-integrated, i.e. they are bound in the longer term and the difference between the two tends to revert to a long-term average which should cover operating costs of converting ethylene to polyethylene, capital costs and profit. To further explain this mechanism, consider the following scenarios. An increase in ethylene prices means higher production costs of polyethylene which will eventually lead to an increase in the market price of the latter. The extent of this price increase depends on whether the market price is more cost-driven or demand driven at that time. A cost-driven market price is much more responsive to a change of production costs than a demand-driven market price (see Figure 3). This relationship is inverse for a change in demand of polyethylene. A change of demand will lead to significant adjustments in polyethylene prices in a demand-driven market but less so in a cost-driven market. Furthermore, a polyethylene demand change will also impact on the prices of ethylene since about 60% of the global ethylene production output is used to produce polyethylene, according to estimates of Deutsche Bank (2009). While most of the remaining share is used to produce other chemical products, ethylene also has some direct applications (e.g. fuel gas for special applications or ripening of fruit).

FIGURE 3

3.1 Econometric model for the stochastic spread

As Dixit and Pindyck (1994) acknowledge, both theoretical considerations and statistical tests are important to determine whether a variable follows a mean-reverting stochastic process. Following the discussion on equilibrium mechanisms above, this section intends to econometrically test the spread for mean-reversion and then to estimate the parameters of this stochastic process. According to Brooks (2009) and Duan and Pliska (2004), a linear combination of non-stationary variables of integration order one will be stationary if the variables are co-integrated. In other words, the spread of polyethylene and ethylene prices is stationary and can be modelled as an autoregressive mean-reverting process if the two commodity prices are co-integrated. Hence, we first test the commodity prices for co-integration and the spread for stationarity. If these tests confirm the mean-reverting nature of the spread, the parameters of the Ornstein-Uhlenbeck process are determined by means of an Ordinary Least Squares regression and statistical tests are performed on the validity of the regression.

Time series with monthly data for ethylene and polyethylene prices from Jan 1991 to Dec 2009 are the basis for the empirical analysis. These prices are for delivery within Europe, i.e. gross transaction prices. Figure 4 gives a graphical representation of the historical commodity prices as well as the conversion spread. It can be seen from the figure that the two commodity prices tend to move together and the spread is more stationary although volatile.

FIGURE 4

3.1.1 Test for mean-reversion

The purpose of this chapter is to test whether the spread follows a mean-reverting/ stationary process. This can be done either directly by demonstrating that the spread is stationary or indirectly by showing that ethylene and polyethylene prices are co-integrated, because according to the Granger representation theorem, this implies that a linear combination of the two (such as the conversion spread) is stationary.

Two variables are co-integrated if their levels are non-stationary and the 1st difference in levels is stationary. An Augmented Dickey-Fuller (ADF) unit root test assumes that the series is non-stationary under the null hypothesis. Hence, the two variables are co-integrated if the ADF test statistic for each variable is not rejected on the levels but rejected on the 1st difference in levels. Co-integration is confirmed for ethylene and polyethylene prices by considering the probabilities of making an error when rejecting the null hypothesis of unit roots, as shown in Table 1: When the p-value is below 5%, the null hypothesis can be rejected with a confidence level of more than 95%. For ethylene and polyethylene prices, the null hypothesis of unit roots cannot be rejected at the 1% level but possibly at the 5% level. The hypothesis of unit roots in the 1st difference of the two commodity prices can be rejected with certainty. This means that the commodity prices tend to be non-stationary, but the change in prices is stationary.

TABLE 1

The same table also provides the ADF statistic for the spread (which is a linear combination of ethylene and polyethylene prices), for which the null hypothesis of non-stationarity is strongly rejected. Because there is the possibility that the null hypothesis might be rejected due to insufficient information, we also perform a stationarity test to confirm the above analysis. A KPSS test assumes the series is stationary under the null hypothesis. The KPSS test statistic for the spread series is 0.72, which means that the null hypothesis of stationarity is rejected at the 5% level (0.46) but not rejected at the 1% level (critical value: 0.74).

3.1.2 Regression model

The Ornstein-Uhlenbeck process for the spread (p) is specified in continuous time. In order to estimate the parameters (η, m, σ), the model needs to be converted to its discrete time equivalent. The corresponding discrete-time process of the spread is a first-order autoregressive model and can be derived from (2) and (3), see also Dixit and Pindyck (1994):

$$p_t = m\left(1-e^{-\eta}\right)+e^{-\eta}\,p_{t-1}+\varepsilon_t \tag{29}$$

where ε_t is normally distributed with mean zero and standard deviation σ_ε

$$\sigma_\varepsilon^2 = \frac{\sigma^2}{2\eta}\left(1-e^{-2\eta}\right) \tag{30}$$

It should be noted, that the parameters η and σ depend on the chosen time interval Δt which is one month. The regression is then run on equation

$$p_t = \alpha + \beta\, p_{t-1} + \varepsilon_t\,, \tag{31}$$

with

$$\hat{\eta} = -\log\hat{\beta}\,, \tag{32}$$

$$\hat{m} = \frac{\hat{\alpha}}{1-\hat{\beta}}\,, \tag{33}$$

$$\hat{\sigma} = \hat{\sigma}_{\varepsilon}\sqrt{2\frac{\log\hat{\beta}}{\hat{\beta}^2 - 1}}. \tag{34}$$

To transform the parameters η and σ from a monthly to an annual scale, multiply the mean-reversion rate by twelve and the volatility by the square root of twelve. Table 2 provides the parameter estimates of the regression model, based on the 1991-2009 monthly data of the price spread, as well as the transformed parameters for the Ornstein-Uhlenbeck process. Both parameters, α and β, are statistically significant (p-values: 0.00), thereby confirming that the model is auto-regressive. The regression estimates the mean of the spread (m) at €317/mt, the annual volatility (σ) at €198 and the mean-reversion rate (η) at 1.35.

TABLE 2

3.1.3 Statistical tests

The above regression model needs to undergo a number of diagnostic tests in order to verify its validity. The residuals of the regression should be homoscedastic, not autocorrelated and normally distributed. Further tests on the stability of the parameters and the linearity in the functional form are performed. The results of these tests are given in Table 3 and are discussed below.

TABLE 3

The distribution of the residuals ought to be of constant variance over time, i.e. homoscedastic. If this is not given, the standard error of the parameter estimates would be flawed and so would be any inference on the significance of the parameters.

However, the parameter values would be unbiased even in the presence of heteroscedasticity. The White test indicates that the probability of making an error when rejecting the null hypothesis of homoscedasticity is 0.49. We adopt the 0.05 probability level as the threshold between rejection and non-rejection. Hence, the residuals are not heteroscedastic. The autoregressive regression model already takes into account autocorrelation in the spread. We still need to test whether the model covers all of the autocorrelation. The consequences of ignoring autocorrelation in the residuals are the same as for heteroscedasticity, i.e. the parameters would be inefficient but unbiased. The Breusch-Godfrey test confirms that the residuals are not correlated. The Bera-Jarque test for normal distribution of the residuals rejects the hypothesis of normality at the 1% significance level, meaning the residuals are not normally distributed. While the residuals distribution is not skewed, it is leptokurtic (peaked relative to the normal) with a kurtosis of 4.07 (3.0 for a normal distribution). Since the kurtosis does not impact on the mean of the residuals distribution, this non-normality has no practical consequences for the validity of the regression model.

The functional form of the chosen regression model is linear. The appropriateness of this form can be tested by means of Ramsey's RESET test which adds exponential terms of the dependent variable to the regression model. With one fitted term (square of the dependent variable), the alternative hypothesis of a non-linear functional form can be rejected at the 0.05 significance level so that our chosen linear functional model is appropriate.

Parameter stability tests are intended to verify if the parameter estimates are stable over time or whether they change significantly. Performing a series of Chow tests with different breakpoints over the sampling period suggests that there might be breakpoints at the end of 1998 and 2000, as can be seen from Figure 5. Hence, parameter estimates based on data before the breakpoint would be significantly different from estimates thereafter. In the long-run the polyethylene-ethylene conversion spread depends on the conversion ratio and needs to cover operating and fixed/capital costs. With existing plants being distributed globally, any changes in these factors would happen slowly which is why there seems to be no economic justification for a sudden change in the long-term behaviour of the spread. Recursive coefficient estimates indicate that both α and β converge to stable values (see Figure 6). A CUSUM test also shows that the cumulative sum of the recursive residuals is within the 0.05 significance range at all times, suggesting that the parameters are stable.

FIGURE 5

FIGURE 6

3.2 Asset-specific parameters

The key characteristics of the polyethylene plant are given in Table 4 together with the calculation of the operating margin based on the spread as of December 2009.

TABLE 4

TABLE 5

The variable cost of production is composed of consumption material cost (see cost-breakdown provided in Table 5 above), logistics cost for the delivery of the final product within Europe, and personnel cost. About 30 people are required to operate the shifts next to a management team of about 4-6. This is under the assumption that the plant is part of a larger petrochemical complex, so that general services can be shared. Assuming annual personnel cost of €50,000 per employee, the total personnel cost amounts to €1.75 m. In case of temporary suspension of the plant operations, following a fire-and-hire strategy would endanger the know-how base. However, many European countries provide for some flexibility with regard to personnel deployment, such as flexible working-time accounts and short-time allowance. Therefore we consider 2/3 of the shift personnel cost to be variable (€1 million) so that the variable personnel cost per ton of polyethylene produced is €4. Annual maintenance cost for this kind of chemical plant is estimated at 1.5% of the investment cost (€3 million). Together with the fixed personnel cost, the total fixed operating cost amounts to €3.75 million.

As was said earlier in this paper, limited lifetime of the asset (depreciation) and specific technological and political risks associated with the investment are accounted for by a Poisson event with the arrival rate λ. The limited lifetime is modelled in the form of exponential decay, where $\phi_T = \int_{t=0}^{T} \lambda e^{-\lambda t} dt$ is the probability that the asset has reached the end of its lifetime before T. Assuming an expected lifetime of 20 years, use $T = 20$ and $\phi_{20} = 0.5$ to get the corresponding arrival rate for depreciation as $\lambda_D = 0.035$. Investing in, owning and operating a chemical plant is associated with significant technological risks, ranging from non-compliance of the chemical processes, patent conflicts, to product obsolescence. Furthermore, political risks persist over the asset lifetime, such as terrorist attacks, environmental issues or health concerns. We choose $\lambda_T = 0.045$, and get the Poisson arrival rate for the asset as $\lambda = \lambda_D + \lambda_T$.

3.3 *Asset valuation*

The theoretical models developed in Section 2 are now applied to value a polyethylene plant with the empirical data from above. As an extension, we introduce a hypothetical tax rate γ on the cash flow, so that the cash flow in the alternative operating mode becomes $(1-\gamma)k_1(p-c)$. The total asset value in the respective operating mode is then given by AV_0 and AV_1, according to $AV_{0/1} = V_{0/1} - PV(c_{fix}) + PV(tax)$, where $PV(c_{fix}) = \frac{c_{fix}}{r+\lambda}$ is the present value of the annual fixed operating cost and PV(tax) the present value of the tax break. Assuming the investment cost (I) is linearly depreciated over the depreciation period (T) for accounting purposes, as is the case in many European countries, the annual tax break during T years is $\gamma I/T$ and its present value $PV(tax) = \sum_{t=1}^{T} \frac{I}{T} \gamma \frac{1}{(1+r)^t}$. The asset values as a function of the spread and the switching boundaries are represented in graphical form in Figure 7 below.

FIGURE 7

Considering first the alternative operating state, when the plant is operated and the spread is earned, it can be seen that the asset value (AV_1) increases linearly in p for very high levels of p while the function is convex for lower levels of p. This is explained by the option to switch to the base operating mode which is relevant for lower p-values and negligible for high p-values. The value function increases steeply beyond the switching boundary p_L because the switching option would largely exceed the discounted cash flows. However, the function AV_1 is not relevant for $p<p_L$ since the operating mode is changed at p_L. AV_0 increases gradually until the option to switch and earn the spread reaches V_1-S_{01} at the switching boundary p_H. Even for highly negative p-values, it is expected that p will eventually revert to the long-run mean (m) so that the option on the spread declines only slowly towards zero for negative spread levels.

TABLE 6

Table 6 above provides the asset value with and without operating flexibility together with the switching boundaries for the standard parameters and various scenarios in order to test the sensitivity to changing parameters. For the standard parameters, we find a value of the operated plant with no flexibility of €251 million compared to an asset value with operating flexibility of €255 million, which is a 2% premium and suggests a low probability of suspending the asset operation. These asset values compare to an investment cost of about €200 million. The switching boundaries p_L and p_H lie to both sides of the variable operating cost (c), as would be expected, however, not symmetrically. Suspending the operations would be recommended at a net cash flow (p-c) of -€23.67/mt compared to restarting at €19.72/mt. This asymmetry is explained by the long-run mean of p which is significantly above the operating cost. Suspension is delayed more than resumption. The switching boundaries are distributed symmetrically to both sides of the variable operating cost if the switching cost is zero, then $p_L=p_H=c$, or if the long-run mean of the spread was identical to the variable operating cost.

Let us first validate the behaviour of the value function with regard to the parameters of the underlying uncertainty and then with regard to asset-specific parameters. When testing for zero volatility, the spread will tend towards its long-run mean (m) in a deterministic way. With $m>c$ and all stochastic elements eliminated, the option to suspend becomes irrelevant so that the operating flexible asset is valued exactly the same as the non-flexible one. Furthermore, if the plant is suspended, it would be resumed as soon as the spread exceeds the variable operating cost because with $m>c$, the net cash flows (p-c) are positive from that time on and the present value of those net cash flows exceeds the switching cost (S_{01}). Now, let the speed of mean-reversion (η) be zero so that the Ornstein-Uhlenbeck process simplifies to a Brownian motion process with no drift, $dp = \sigma dz$. For the non-flexible plant, the present value declines when mean-reversion is relaxed because the risk increases with time (volatility proportional to the square root of time). This is reflected in a higher discount rate for the spread in equation (11). As a result, the present value of €166 million is significantly lower compared to the mean-reversion case and would even not justify the investment. In contrast, the value of the flexible asset increases significantly by about 25% to €315 million when relaxing mean-reversion, which is a 90% premium on the non-flexible asset. This is consistent with real options theory because the lower the speed of mean-reversion the higher the absolute volatility and the higher the option values.

Assuming different variable operating costs, the option premium increases with higher operating cost because the probability of exercising the option (switching) increases. However, as long as the option is far in the money, $(p-c)\gg 0$, the premium is rather small. The results confirm the intuition that in the absence of switching cost, switching is optimal as soon as the spread crosses the operating cost, so that the cash flow is given by Max[p-c;0]. Although the switching cost significantly influences the switching boundaries, its effect on the asset value is minor because the current and long-run expected spread is far above the operating cost and hence the probability of suspending the plant and incurring switching costs is low. Figure 8 below illustrates the sensitivity of the switching boundaries to the variable operating cost and to the switching cost. It can be seen that while p_H and p_L move in line with the operating cost, the switching boundaries are not symmetrically distributed around the operating cost because $m \neq c$. Finally, comparing the case of an initial spread of €500/mt vs. a spread of €150/mt, the difference in asset value would be about €40 million.

FIGURE 8

It is now interesting to simulate the asset operation on the basis of historical commodity prices. Figure 9 shows the development of the polyethylene/ethylene spread over the last decade, together with the level of variable operating cost of the conversion plant and the switching boundaries. It can be seen that the plant should have been idle most of the year 2000 and be suspended in 2004 and 2005 for about one month each time. In these cases, ethylene was better sold to the market instead of polyethylene. Most of the time, however, the spread level exceeds the variable cost by far, so that sell polyethylene was the better product, which explains the rather small option premium for the flexibility of suspending the plant of 2%.

FIGURE 9

3.4 The Greek Letters

The risk measures Delta and Gamma of the asset value are provided in Figure 10. Delta is defined as the change of the asset value with changes in the spread (p), and Gamma is the change of Delta with changes in the spread. The asset value function in the base operating mode (suspension) is a convex function in p, therefore Delta is also increasing in p. When the asset is in the alternative operating mode (operation), the asset value increases with the spread and the Delta approaches a constant value for high levels of p, because the option to suspend becomes negligible. For a level of the spread lower than the switching boundary, the asset value function finds a minimum and increases again for lower levels of p in order to reflect the switching option. Hence, the Delta function is zero when the value function (V_1) is at a minimum, and negative for lower levels of the spread.

FIGURE 10

4 Implications

4.1 Implications for participants in the polyethylene industry

Three generic strategies are available to companies involved in the production of polyethylene: investing in a polyethylene plant by building a new one or buying an existing one, optimising the operations, or divesting. The model and the results from the previous section enable us to evaluate these strategies and to point out opportunities and pitfalls.

Both investment and divestment decisions require transparency on the value of the transaction asset to determine an appropriate transaction price or to compare to the investment cost. When setting up a new plant, the investment is supposed to add value and the project should be implemented at the right time to maximise the value. The polyethylene plant is valued at €255 million which compares to the investment cost of about €200 million. Hence, the investment would be positive in the current set of circumstances. We have seen that the asset value would vary by about 15% (or €40 million in absolute terms) if we vary the initial spread level between the extreme levels of €150/mt and €500/mt. Ceteris paribus, the investment is more valuable if the current spread is high. With regard to taking the actual investment decision, this needs to be interpreted in combination with the time to build (about two years) and the correlation between spread and investment cost (the model above assumes constant investment cost).

In the design phase of the new plant, decisions are taken regarding the degree of operating flexibility to be incorporated into the asset. The asset with operating flexibility has been shown to exceed that without flexibility by about €4 million. Thus, operating flexibility should be incorporated as long as it can be implemented for a cost of less than €4 million. Furthermore, a trade-off between reduced operating cost and higher investment cost is commonly encountered. For instance, if the variable operating cost of the polyethylene plant could be reduced from €128.5/mt to €100/mt, this would justify a €36 million higher investment cost.

Transparency on the spread levels triggering temporary suspension and resumption is essential for the management team operating the plant so that these critical decisions can be prepared in good time. One needs to be aware that switching boundaries change when variable operating cost (e.g. logistics cost) or the cost of ramping up or down the plant change.

4.2 General implications

The application of the continuous rainbow option has shown that the flexible asset increases in value when relaxing the mean-reversion (η) in the underlying uncertainty. This is consistent with the Smith and McCardle (1999) conclusion that the option of flexibility is worth less when the underlying variable is mean-reverting instead of random walk. For $\eta=0$, the stochastic process simplifies to a Brownian motion with no drift. Real options theory tells that the value of real options increases with volatility, and a non-stationary Brownian motion is more volatile than a stationary mean-reversion process. On the contrary, the non-flexible asset decreases significantly in value if mean-reversion is relaxed which is due to the higher discount rate. Laughton and Jacoby (1993) call these two opposing phenomena the variance and discounting effects. From this can be concluded that flexibility in assets is recommended when the value drivers are non-stationary, whereas the extra cost for flexibility might not be justified when the value-drivers are stationary.

The results also highlight the relevance of assessing the degree of co-integration of markets. If two co-integrated variables are modelled as geometric Brownian motion with the appropriate correlation, their spread would not be bound and asset values based on these variables would tend to be overstated. Instead, the spread of two co-integrated variables should be modelled as a stationary process.

In real life, decisions to realise large scale investments are typically taken at times when the uncertain value-drivers are high or near its peak, for obvious reasons. Assuming this value-driver is a commodity and follows a mean-reverting stochastic process, the probability is high that the price has reverted back towards its long-run mean by the time the investment goes on stream. In addition, investments are typically most expensive when the economy is booming and the general price level is high. The opportunity can then be seen as an anti-cyclical investment. When the economy is weak, commodity prices tend to be weaker and investment costs tend to be lower as well. This seems to be a good time to invest, so that by the time the investment is completed, the commodity price reverts back towards its long-run mean while the savings on the investment cost have been realised. The booming years of 2007/8, when the cost for large-scale investments almost doubled compared to the normal level, demonstrated that this effect can be quite significant.

5 Conclusion

This paper presents a continuous option to choose the best of two co-integrated commodities. Since the spread of two co-integrated variables can be modelled as arithmetic mean-reversion, this real rainbow option can also be interpreted as an entry/exit valuation problem on a mean-reverting stochastic variable, hence reducing the complexity from two-factor to one-factor. We develop a quasi-analytical solution for which all parameters can be estimated from empirical data.

An application of the model to value a polyethylene plant based on the spread between polyethylene and ethylene demonstrates that the option to switch between the two commodities increases when there is no mean-reversion. For the empirical data, we found a premium of the continuous rainbow option over the operation with no switching flexibility of merely 2% which is due to the mean-reverting characteristic of the spread and the on average large positive net cash flow, resulting in a low probability of switching to the alternative operating mode. When simulating zero mean-reversion, the rainbow option becomes several times as valuable as the non-flexible asset. One main implication from this is that incorporating flexibility into assets seems more promising when the value-drivers are non-stationary while the value of flexibility in co-integrated markets is more limited. On the other hand, opportunities are found in anti-cyclical investing when the value-driver is stationary because the investment can be made when prices and initial costs are low, with prices expected to revert back to their long-run mean by the time the benefits are realised. An interesting extension to the model would therefore be to determine the optimal investment timing based on a fixed investment cost and then on a stochastic investment cost proportional to the spread.

Appendix A. Proof of the transformation of the PDE into Weber's equation

Equation (15) is to be transformed on the basis of the substitutions $V_0 = F(x)$ and $x = \sqrt{|a|}\left(p + \frac{b}{a}\right)$, as suggested by Kampke (1956, p. 414). The first and second derivative functions of V are:

$$\frac{\partial V}{\partial p} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial p} = \frac{\partial F}{\partial x}\sqrt{|a|}$$

$$\frac{\partial^2 V}{\partial p^2} = \frac{\delta\left(\frac{\partial F}{\partial x}\frac{\partial x}{\partial p}\right)}{\delta p} = \frac{\delta\left(\frac{\partial F}{\partial x}\sqrt{|a|}\right)}{\delta p} = \frac{\partial^2 F}{\partial x^2}|a|$$

These derivatives are applied to equation (15), and taking into consideration that a<0, we obtain:

$$\frac{\partial^2 V}{\partial p^2} + (a\ p + b)\frac{\partial V}{\partial p} + d \cdot V = \frac{\partial^2 F}{\partial x^2}|a| + \left(a\left(\frac{x}{\sqrt{|a|}} - \frac{b}{a}\right) + b\right)\frac{\partial F}{\partial x}\sqrt{|a|} + d \cdot F$$

$$= \frac{\partial^2 F}{\partial x^2} - x\frac{\partial F}{\partial x} - \frac{d}{a} \cdot F = 0$$

Substitute $F = G(x)e^{\frac{1}{4}x^2}$ and take the derivatives:

$$\frac{\partial F}{\partial x} = \left(\frac{\partial G}{\partial x} + \frac{1}{2}x \cdot G\right)e^{\frac{1}{4}x^2}$$

$$\frac{\partial^2 F}{\partial x^2} = \left(\frac{\partial^2 G}{\partial x^2} + x\frac{\partial G}{\partial x} + \left(\frac{1}{2} + \frac{1}{4}x^2\right)G\right)e^{\frac{1}{4}x^2}$$

With the above substitutions, we obtain the Weber equation:

$$\frac{\partial^2 G}{\partial x^2} = \left(\frac{x^2}{4} + \frac{d}{a} - \frac{1}{2}\right)G$$

Appendix B. System of equations

With A_0=0 and B_1=0, equations (21) and (23) simplify to

$$V_0(p) = B_0 \cdot D_{-d/a}\left(-\sqrt{|a|}\left(p+\frac{b}{a}\right)\right) e^{\frac{1}{4}\left(\sqrt{|a|}\left(p+\frac{b}{a}\right)\right)^2}$$

and

$$V_1(p) = A_1 \cdot D_{-d/a}\left(\sqrt{|a|}\left(p+\frac{b}{a}\right)\right) e^{\frac{1}{4}\left(\sqrt{|a|}\left(p+\frac{b}{a}\right)\right)^2} + u \cdot p + w$$

With the value functions above, the two boundary conditions, (24) and (25), can be evaluated:

$$\left(B_0 \cdot D_{-d/a}\left(-\sqrt{|a|}\left(p_H+\frac{b}{a}\right)\right) - A_1 \cdot D_{-d/a}\left(\sqrt{|a|}\left(p_H+\frac{b}{a}\right)\right)\right) e^{\frac{1}{4}\left(\sqrt{|a|}\left(p_H+\frac{b}{a}\right)\right)^2}$$

$$- u \cdot p_H - w + S_{01} = 0$$

$$\left(B_0 \cdot D_{-d/a}\left(-\sqrt{|a|}\left(p_L+\frac{b}{a}\right)\right) - A_1 \cdot D_{-d/a}\left(\sqrt{|a|}\left(p_L+\frac{b}{a}\right)\right)\right) e^{\frac{1}{4}\left(\sqrt{|a|}\left(p_L+\frac{b}{a}\right)\right)^2}$$

$$- u \cdot p_L - w - S_{10} = 0$$

For the evaluation of the smooth pasting conditions, the derivative function of the parabolic cylinder function is used:

$$\frac{d}{dx} D_v(f(x)) = \left(-\frac{1}{2} f(x) D_v(f(x)) + v D_{v-1}(f(x))\right) \frac{d}{dx} f(x)$$

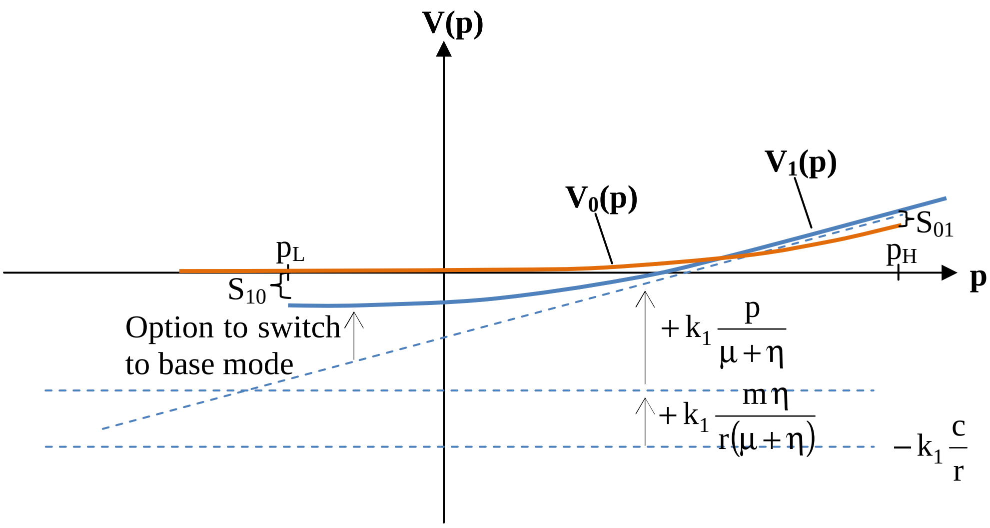
(26) can then be assessed and simplified:

$$\frac{d\sqrt{|a|}}{a} e^{\frac{1}{4}\left(\sqrt{|a|}\left(p_H+\frac{b}{a}\right)\right)^2} \left[B_0 \cdot D_{-1-d/a}\left(-\sqrt{|a|}\left(p_H+\frac{b}{a}\right)\right) + A_1 \cdot D_{-1-d/a}\left(\sqrt{|a|}\left(p_H+\frac{b}{a}\right)\right)\right] - u = 0$$

Similarly, from (27):

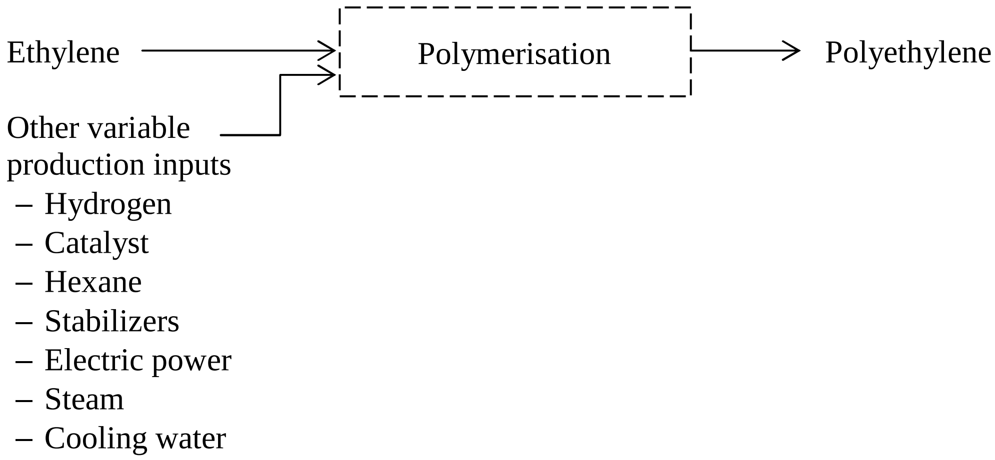
$$\frac{d\sqrt{|a|}}{a} e^{\frac{1}{4}\left(\sqrt{|a|}\left(p_L+\frac{b}{a}\right)\right)^2} \left[B_0 \cdot D_{-1-d/a}\left(-\sqrt{|a|}\left(p_L+\frac{b}{a}\right)\right) + A_1 \cdot D_{-1-d/a}\left(\sqrt{|a|}\left(p_L+\frac{b}{a}\right)\right)\right] - u = 0$$

Figure 1. General shape of the value functions V_0 and V_1



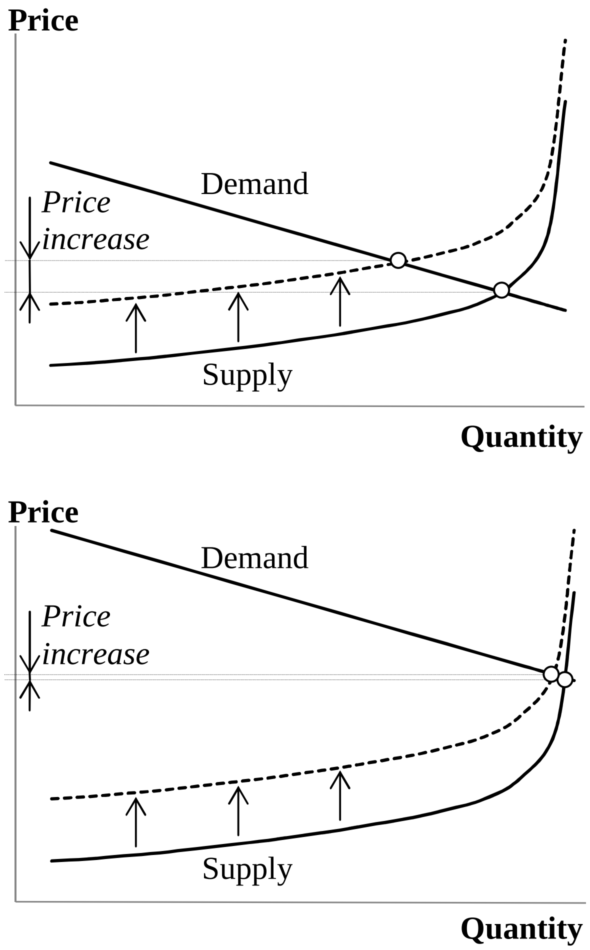
Asset values as a function of the spread (p). V_0 is the asset value in the base mode, V_1 in the alternative mode. Switching from base mode to alternative mode at p_H for a switching cost of S_{01}, reverse switching at p_L for S_{10}.

Figure 2. Simplified scheme of inputs and outputs of a polyethylene plant (HDPE)



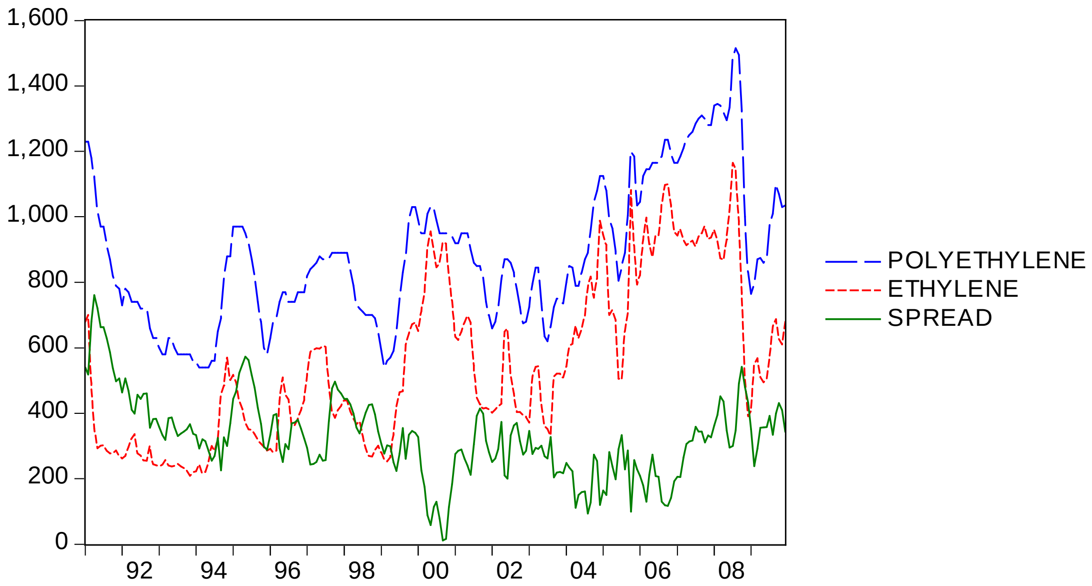
Feed components and output of the slurry polymerisation process of ethylene to high-density polyethylene (HDPE)

Figure 3. Equilibrium price changes in reaction of supply shift in cost-driven and demand-driven markets



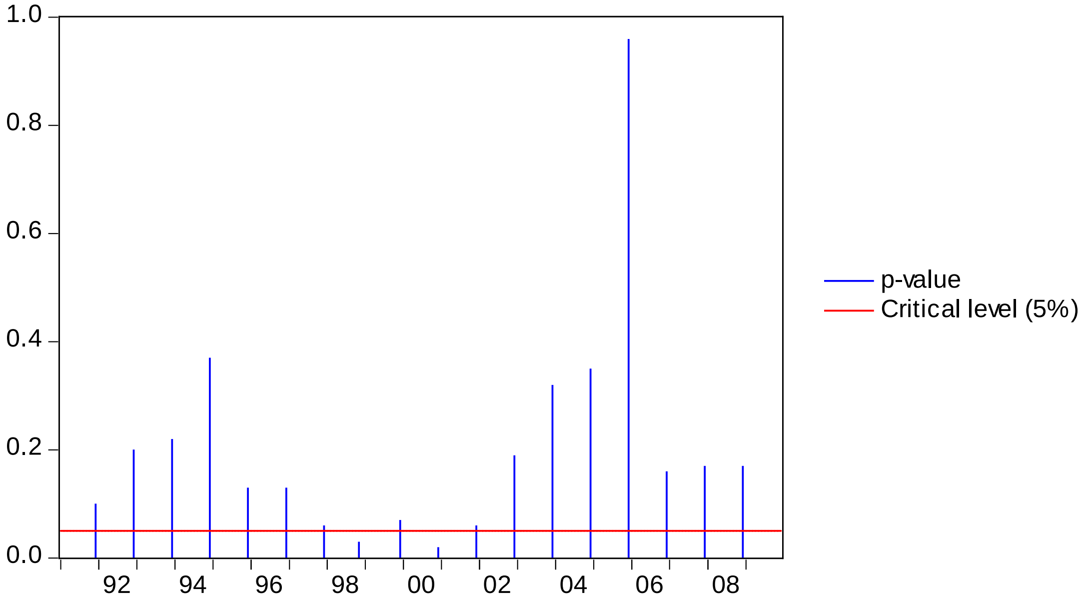
Supply/Demand equilibrium prices in cost-driven market (top) and demand-driven market (bottom). An increase in production costs leads to an upward shift in supply. Equilibrium price is more sensitive to a supply shift in a cost-driven market than in a demand-driven market.

Figure 4. Time series of commodity prices and of the spread



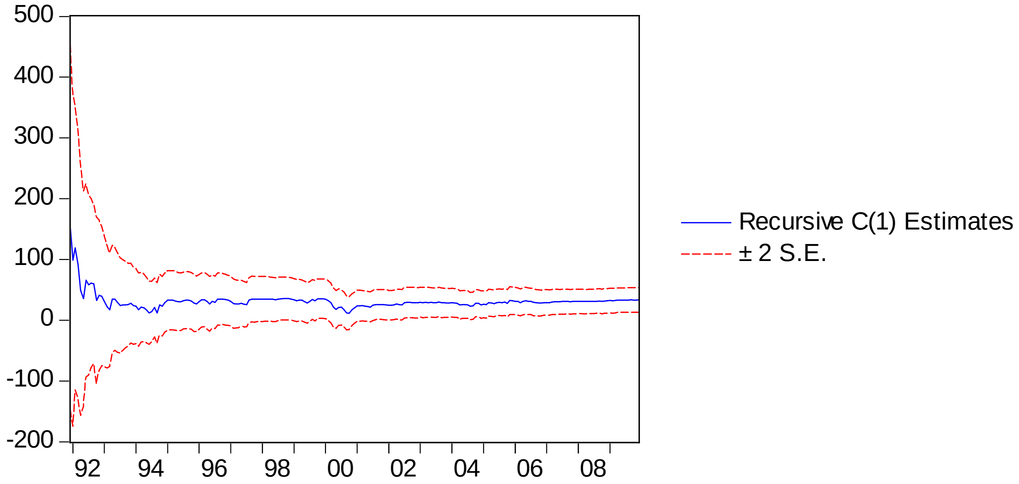
Monthly data, prices in € per metric ton and for delivery within Europe. Ethylene: spot prices. Polyethylene: HDPE quality (high-density polyethylene). Spread defined as polyethylene price less 1.017 times ethylene price.

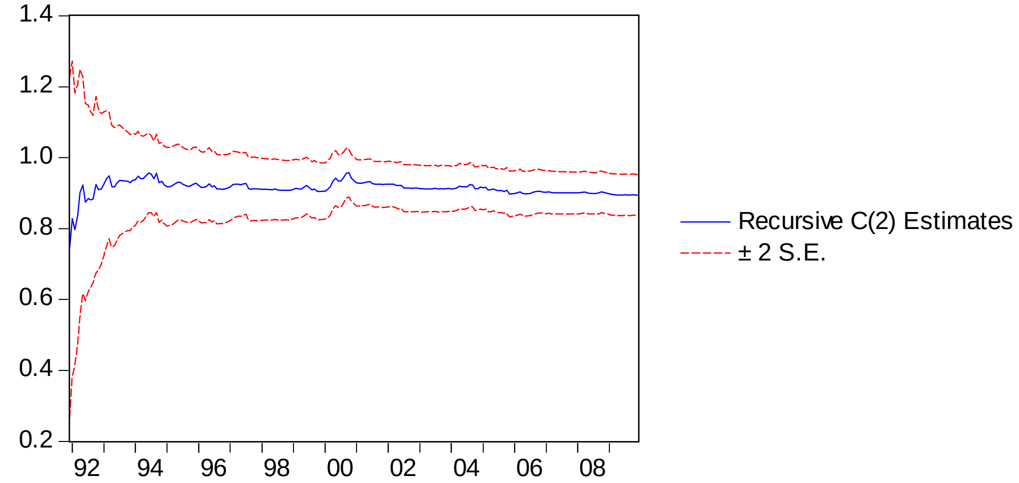
Figure 5. Chow tests on parameter stability with respect to particular breakpoints



P-value gives the probability of making an error when rejecting the null hypothesis of no breakpoint. Chow test splits the sample data into two periods divided by the breakpoint and compares the residual sums of the regressions from these sub-samples with the residual sum of the regression over the whole period. Ordinary least squares regression for the Ornstein-Uhlenbeck process: $p_t = \alpha + \beta p_{t-1} + \varepsilon_t$

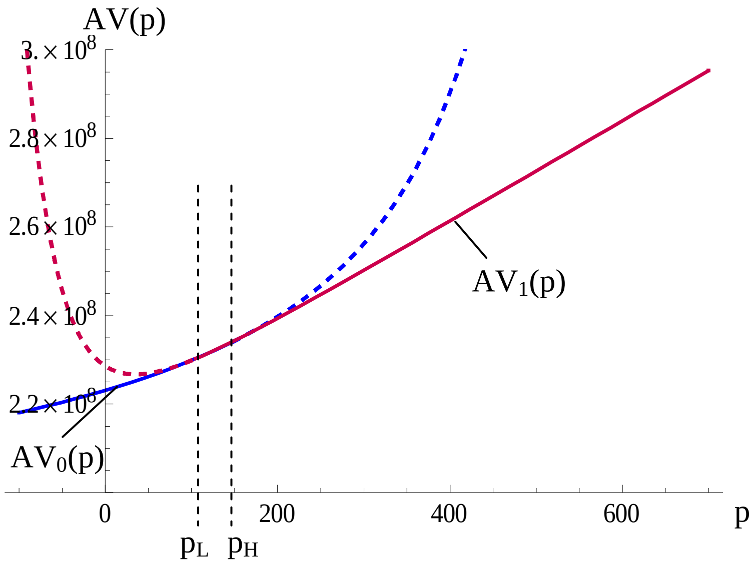
Figure 6. Recursive coefficient estimates





Regression model: $p_t = \alpha + \beta p_{t-1} + \varepsilon_t$. C(1) corresponds to α, C(2) to β. Parameter estimates start from Jan-1991 and subsequently add more data points until all data up to Dec 2009 is considered. Convergence towards a stable value indicates parameter stability.

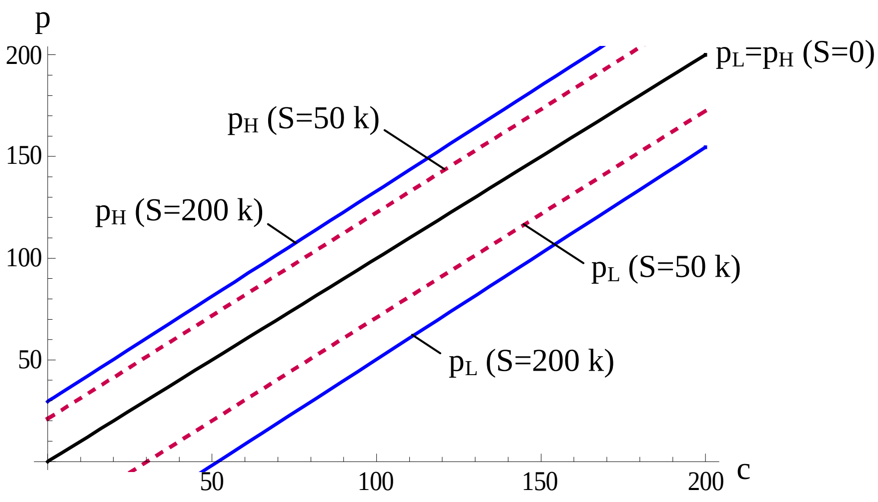
Figure 7. Asset value as a function of the spread



Spread (p) in €/mt and asset values AV_0 and AV_1 in €. Switching from not operating to operating the asset at p_H, vice versa at p_L. Asset values on dashed lines not applicable because switching of operating mode is triggered.

Long-run mean of p: m = €316.8/mt; Speed of mean-reversion of p: $\eta = 1.35$; Volatility of p: σ = €198 p.a.; Variable operating cost: c = €128.5/mt; Capacity of p: k_1 = 250,000 mt p.a.; Switching cost for resuming operation: S_{01} = €40,000; Switching cost for suspending operation: S_{10} = €20,000; Required return: $\mu = 0.10$; Risk-free rate of return: $r = 0.05$; Exponential decay and technological/political risk: $\lambda = 0.08$; Tax rate $\gamma = 0.3$; Fixed operating cost: c_{fix} = €3.75 million p.a.; Annual depreciation: €10 million over 20 years.

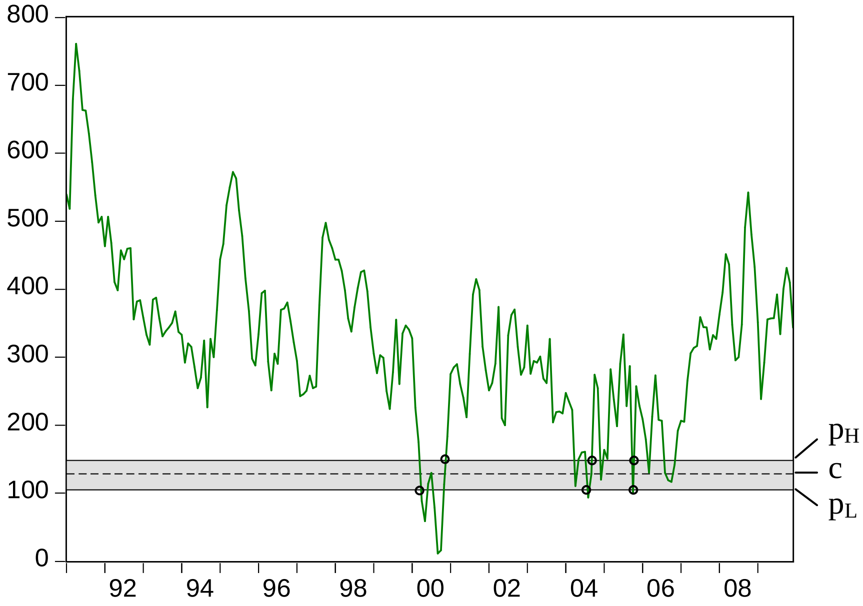
Figure 8. Switching boundaries as a function of variable operating cost



Switching boundaries p_H and p_L as a function of variable operating cost (c) and for different switching costs for resuming and suspending operation ($S_{01} = S_{10}$) of €0, €50,000 and €200,000.

Standard parameters: Current value of the spread: p = €340/mt; Long-run mean of p: m = €316.8/mt; Speed of mean-reversion of p: η=1.35; Volatility of p: σ = €198 p.a.; Capacity of p: k_1 = 250,000 mt p.a.; Required return: μ = 0.10; Risk-free rate of return: r = 0.05; Exponential decay and technological/political risk: λ = 0.08; Tax rate γ = 0.3; Fixed operating cost: c_{fix} = €3.75 million p.a.; Annual depreciation: €10 million over 20 years.

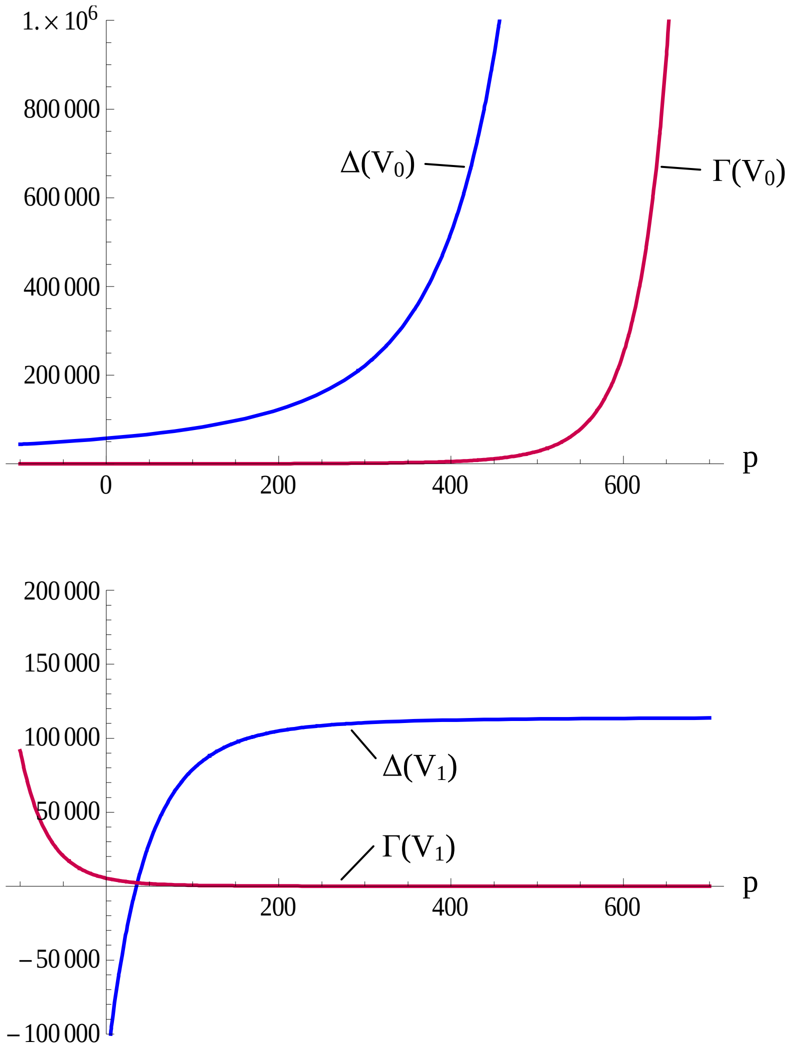
Figure 9. Time series of the spread and optimal switching points



Monthly prices for the polyethylene/ethylene spread (polyethylene price less 1.017 times ethylene price) in € per metric ton of polyethylene and for delivery within Europe. Suspend conversion of ethylene to polyethylene if the spread falls below p_L = 104.8 €/mt and resume at p_L = 148.2 €/mt.

Parameters: Variable operating cost: c = €128.5/mt; Capacity of p: k_1 = 250,000 mt p.a.; Switching cost for resuming operation: S_{01} = €40,000; Switching cost for suspending operation: S_{10} = €20,000; Required return: μ = 0.10; Risk-free rate of return: r = 0.05; Exponential decay and technological/political risk: λ = 0.08; Tax rate γ = 0.3; Fixed operating cost: c_{fix} = €3.75 million p.a.; Annual depreciation: €10 million over 20 years.

Figure 10. Delta and Gamma for asset values in base and alternative operating mode



Delta (Δ) and Gamma (Γ) for the asset value in the base operating mode (top) and in the alternative operating mode (bottom) as a function of the spread (p) in €/mt. $\Delta = \delta V / \delta p$ and $\Gamma = \delta^2 V / \delta p^2$.

Parameters: Long-run mean of p: m = €316.8/mt; Speed of mean-reversion of p: $\eta = 1.35$; Volatility of p: σ = €198 p.a.; Variable operating cost: c = €128.5/mt; Capacity of p: k_1 = 250,000 mt p.a.; Switching cost for resuming operation: S_{01} = €40,000; Switching cost for suspending operation: S_{10} = €20,000; Required return: $\mu = 0.10$; Risk-free rate of return: $r = 0.05$; Exponential decay and technological/political risk: $\lambda = 0.08$; Tax rate $\gamma = 0.3$; Fixed operating cost: c_{fix} = €3.75 million p.a.; Annual depreciation: €10 million over 20 years.

Table 1. Augmented Dickey-Fuller test for unit roots in time series

	Probability of unit roots on prices	Probability of unit roots on 1^{st} difference of prices
Ethylene	0.043	0.000
Polyethylene	0.057	0.000
Spread	0.005	0.000

MacKinnon one-sided p-values give the probability of making an error when rejecting the null hypothesis that unit roots exit. Unit roots are present if the regression $y_t = \sum_{i=1}^{12} \phi_i y_{t-i} + u_t$ yields $\Phi_i \geq 1$ for any i, where y_t is the dependent variable at time t and u_t the residual at time t. The presence of unit roots indicates that the process is non-stationary. Maximum number of lags to account for autocorrelation: 12 months.

Table 2. Regression model for the Ornstein-Uhlenbeck process of the spread

Regression parameter	Value	Std. Error	p-value
α	33.62	10.06	0.00
β	0.894	0.029	0.00
σ_{ε}	54.11		

Parameters of the Spread	Value	Unit
m	316.8	EUR/t
η_{month}	0.11	per month
η_{year}	1.35	per year
σ_{month}	57.2	EUR/t per month
σ_{year}	198.0	EUR/t per year

Ordinary least squares (OLS) regression model of the polyethylene-ethylene spread (p): $p_t = \alpha + \beta p_{t-1} + \varepsilon_t$. P-values give the probability of making an error when rejecting the null hypothesis that the respective parameter is zero.

Table 3. Diagnostic tests on regression model

Test	p-value	Interpretation
Heteroskedasticity test of residuals		
White Test		
Probability F-distribution	0.49	Do not reject the null hypothesis of homoscedasticity
Autocorrelation test of residuals		
Breusch-Godfrey		
Probability F-distribution	0.21	Do not reject the null hypothesis of no autocorrelation
Normality test of residuals		
Bera-Jarque		
Probability Chi^2-distribution	0.01	Reject the null hypothesis of normality
Test for misspecification of functional form		
Ramsey's RESET test		
Probability F-distribution	0.09	Do not reject the null hypothesis of the functional form being linear

Regression model: $p_t = \alpha + \beta p_{t-1} + \varepsilon_t$.

The White test yields the probability according to an F-distribution for the joint null hypothesis that ρ_1=0, ρ_2=0 and ρ_3=0 in the auxiliary regression of the residuals $\hat{\varepsilon}_t{}^2 = \rho_1 + \rho_2 p_{t-1} + \rho_3 p_{t-1}{}^2 + u_t$ where u_t is a normally distributed disturbance term. Squared terms are included.

The Breusch-Godfrey test yields the probability according to an F-distribution for the joint null hypothesis that ρ_i=0 for i=1..12 in the auxiliary regression of the residuals $\hat{\varepsilon}_t = \gamma_1 + \gamma_2 p_{t-1} + \sum_{i=1}^{12} \rho_i \hat{\varepsilon}_{t-i} + u_t$ where u_t is a normally distributed disturbance term. To account for autocorrelation covering 12 months, 12 lagged terms are included.

The Bera-Jarque test statistic is given by $\frac{N}{6}\left(S^2 + \frac{(K-3)^3}{4}\right)$, where $S = \frac{E\left[\varepsilon^3\right]}{\sigma^3}$ is the skewness and $K = \frac{E\left[\varepsilon^4\right]}{\sigma^4}$ the kurtosis of the residuals distribution. The Bera-Jarque statistic is distributed as a Chi-square with 2 degrees of freedom.

Ramsey's RESET test yields the probability according to an F-distribution for the null hypothesis that ρ_1=0 in the auxiliary regression of the residuals $p_t = \gamma_1 + \gamma_2 p_{t-1} + \rho_1 p_t{}^2 + u_t$ where u_t is a normally distributed disturbance term.

Table 4. Overview of asset parameters

Capacity polyethylene	k_1	250,000	mt per year
Feedstock ethylene	k_0	254,250	mt per year
Ramp-up cost	S_{01}	40	'000 €
Ramp-down cost	S_{10}	20	'000 €
Current spread	*p*	*340.0*	*€/mt polyethylene*
Logistics cost		50.0	€/mt polyethylene
Consumption materials		74.5	€/mt polyethylene
Personnel cost		4.0	€/mt polyethylene
Variable operating cost	c	128.5	€/mt polyethylene
Current margin	*p-c*	*214.9*	*€/mt polyethylene*

During the ramp-up phase the process stability is not given at all times so that the polyethylene produced is of lower quality. The ramp-up cost is then the lost income based on an estimated price reduction of €20/mt for the lower grade and a ramp-up time of 24h up to 3 days. When suspending the operations temporarily, the variable personnel costs cannot be eliminated immediately, assuming that one week's salaries will be incurred for non-productive time following a ramp-down.
As quoted commodity prices refer to delivered products, logistics cost refer to delivery of polyethylene within Europe. Current spread as of December 2009.

Table 5. Cost of consumption materials for the HDPE slurry process

Production inputs	Consumption for 1,000 kg of HDPE		Unit prices		Cost for 1,000 kg of HDPE
Catalyst					€4
Hydrogen	0.7	kg	2.4	€/kg	€1.7
Hexan	7	kg	650	€/t	€4.5
Stabilisers					€20
Steam	500	kg	25	€/t	€12.5
Electric power	600	kWh	45	€/MWh	€27.0
Cooling water	200	m^3	2.4	€ ct/m^3	€4.8
					€74.5

Main production inputs to the HDPE slurry process other than ethylene. Consumption data based on Meyers (2004). Electric power and cooling water consumption data adjusted to account for the extruder. Estimate for cost of hydrogen on natural gas basis from FVS (2004). Prices of hexan, steam and cooling water based on industry experts interview. Electric power based on average spot electricity prices at European Energy Exchange.

Table 6. Sensitivity analysis of switching boundaries and asset values

Sensitivities	**p_L** [€/mt]	**p_H** [€/mt]	**Flexible asset** **$AV_1 = V_1 - PV(c_{fix}) + PV(tax)$** [million €]	**Non-flexible asset** **$PV_1 - PV(c_{fix}) + PV(tax)$** [million €]
Standard parameters	104.83	148.22	255	251
Sensitivity to volatility				
$\sigma = 0$	115.16	128.53	251	251
Sensitivity to mean-reversion (η)				
$\eta = 0$	106.95	150.17	315	166
Operating cost sensitivity				
c = €100/mt	76.00	119.41	291	289
c = €150/mt	126.58	169.95	228	222
Switching cost sensitivity				
$S_{01} = S_{10}$ = €0	128.50	128.50	255	251
$S_{01} = S_{10}$ = €200,000	80.01	162.62	254	251
Sensitivity to current spread (p_0)				
$p_0 = 500$	104.83	148.22	273	269
$p_0 = 150$	104.83	148.22	234	229

Standard parameters: Current value of the spread: p = €340/mt; Long-run mean of p: m = €316.8/mt; Speed of mean-reversion of p: η=1.35; Volatility of p: σ = €198 p.a.; Variable operating cost: c = €128.5/mt; Fixed cost: c_{fix} = €3.75 million p.a.; Capacity of p: k_1 = 250,000 mt p.a.; Switching cost for resuming operation: S_{01} = €40,000; Switching cost for suspending operation: S_{10} = €20,000; Required return: μ = 0.10; Risk-free rate of return: r = 0.05; Exponential decay and technological/political risk: λ = 0.08; Tax rate γ = 0.3; Fixed operating cost: c_{fix} = €3.75 million p.a.; Annual depreciation: €10 million over 20 years.

References

Abramowitz, M. and I. Stegun. 1964. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. New York: Dover Publications Inc.

Adkins, R. and D. Paxson. 2010a. Renewing Assets with Uncertain Revenues and Operating Costs. *Journal of Financial and Quantitative Analysis*, forthcoming.

Adkins, R. and D. Paxson. 2010b. Reciprocal Energy Switching Options. Paper presented at the Trondheim Winter Energy Workshop, 6 January 2010, in Trondheim, Norway.

Bhattacharya, S. 1978. Project Valuation with Mean-Reverting Cash Flow Streams. *Journal of Finance* 33, 1317-1331.

Bjerksund, P., and S. Ekern. 1995. Contingent Claims Evaluation of Mean-Reverting Cash Flows in Shipping. In *Real Options in Capital Investment: Models, Strategies, and Applications*, ed. Lenos Trigeorgis. London: Praeger.

Brooks, Chris. 2008. *Introductory Econometrics for Finance*. 2nd edition. Cambridge: Cambridge University Press.

Deutsche Bank. 2009. *Petrochemicals Yearbook.*

Dickey, D. A., and W. A. Fuller. 1979. Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association* 74, 427−431.

Dixit, Avinash K., and Robert S. Pindyck. 1994. *Investment under Uncertainty*. Princeton, NJ: Princeton University Press.

Dockendorf, J. and D. Paxson. 2009. Continuous Rainbow Options on Commodity Outputs: What is the Value of Switching Facilities? Paper presented at the 13th International Conference on Real Options, 17-20 June 2009, in Braga, Portugal and Santiago, Spain.

Duan, J.-C., and S. R. Pliska. 2004. Option Valuation with Co-integrated Asset Prices. *Journal of Economic Dynamics and Control* 28, 727−754.

FVS (Forschungsverbund Sonnenenergie). 2004. Wasserstoff und Brennstoffzellen – Energieforschung im Verbund. Annual symposium, 25-26 November 2004, in Berlin

Geman, H. 2007. Mean Reversion Versus Random Walk in Oil and Natural Gas Prices. In *Advances in Mathematical Finance*, eds. Michael C. Fu, Robert A. Jarrow, Ju-Yi J. Yen and Robert J. Elliott. Basle: Birkhäuser.

Hull, J. C., 2006, *Options, Futures and Other Derivatives*, 6^{th} ed. New Jersey: Pearson

Johnson, H. 1987. Options on the Maximum or the Minimum of Several Assets. *The Journal of Financial and Quantitative Analysis* 22, 277-283.

Kamke, E. 1956. *Differenzialgleichungen – Lösungsmethoden und Lösungen*. Leipzig: Akademische Verlagsgesellschaft Geest & Portig K.G.

Kulatilaka, N., and L. Trigeorgis. 2004. The General Flexibility to Switch: Real Options Revisited. In *Real Options and Investment under Uncertainty*, eds. E. S. Schwartz and L. Trigeorgis. London: The MIT press.

Laughton, D. G. and H. D. Jacoby. 1993. Reversion, Timing Options, and Long-Term Decision-Making. *Financial Management* 22, 225-240.

Meyers, Robert A. 2004. *Handbook of Petrochemicals Production Processes*. McGraw-Hill Professional.

Näsäkkälä, E. and S.-E. Fleten. 2005. Flexibility and Technology Choice in Gas Fired Power Plant Investments. *Review of Financial Economics* 14, 371-393.

Pinto, C. B., L. Brandao and W. J. Hahn. 2007. Modelling Switching Options using Mean Reverting Commodity Price Models. Paper presented at the 11th International Conference on Real Options, 6-9 June 2007, in Berkeley.

Schwartz, E. S. 1997. The Stochastic Behaviour of Commodity Prices: Implications for Valuation and Hedging. *Journal of Finance* 52, 923−973.

Smith, J. E. and K. F. McCardle. 1999. Options in the Real World: Lessons Learned in Evaluating Oil and Gas Investments. *Operations Research* 47, 1-15.

Sodal, S., S. Koekebakker and R. Aaland. 2007. Market Switching in Shipping – A Real Option Model Applied to the Valuation of Combination Carriers. *Review of Financial Economics* 17, 183-203.

Spanier, J., and K. B. Oldham. 1987. *An Atlas of Functions*. Washington, DC: Hemisphere Publishing Company.

Stulz, R., 1982. Options on the Minimum or Maximum of Two Risky Assets: Analysis and Applications. *Journal of Financial Economics* 10, 161–185.

Tvedt, J. 2000. The Ship Lay-Up Option and Equilibrium Freight Rates. In *Project Flexibility, Agency and Competition*, eds. M. Brennan and L. Trigeorgis. Oxford: Oxford University Press.