Real Options and Signaling in Strategic Investment Games

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Abstract

This paper investigates an investment game with an incumbent and an entrant for optimal entries into a new market. The profit flows of the market involve two uncertain factors. One factor is determined at the beginning of the game and the only incumbent can observe it as private information. The other factor is described by a stochastic process for revenue flows which is common to both firms. Each firm decides the timing of the investment for the entry of the market. The profit of the incumbent is assumed to be relatively larger than that of the entrant, hence the incumbent invests earlier than the entrant. The high demand type of the incumbent can invest earlier than the low demand type. This earlier investment, however, reveals the information, so that the entrant would accelerates the timing of the investment by observing the incumbent's timing of the entry and it reduces the monopolistic profit of the incumbent. Thus, the incumbent who knows the high demand may delay the timing of the investment to hide the information strategically. I characterize this signaling effect by a weak perfect Bayesian equilibrium and investigate the values of both firms.

1 Introduction

The timing of investment of a firm is affected by uncertainty and competition of the market. The concept of real options shaded light on the nature of the strategic delay of the irreversible investment under uncertainty in contrast to the traditional net present value (NPV) model. Brennan and Schwartz (1985) and Dixit and Pindyck (1994) assert that a firm should wait for the investment even if the net present value is positive and the optimal timing of the investment

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is delayed beyond the traditional Marshallian threshold. On the other hand, a number of papers examine that the threats of preemption under a first mover's advantage or a negative externality of the investment may reduce the options of the firms and accelerate the timing of the investment. Some results are shown in a duopolistic market with symmetric firms by incorporating real option approach into the optimal stopping games, e.g. Smets (1991), Grenadier (1996), Kulatilaka and Perotti (1998), Huisman and Kort (1999) and Smit and Trigeorgis (2002). Pawlina and Kort (2006) and Kong and Kwok (2007) develop the results to two asymmetric firms, but the information about two firms are assumed to be identical

The asymmetry of information also affects the timing of the investment. Lambrecht and Perraudin (2003) considers a timing game in incomplete information for the optimal decision of the investment for two competitive firms, in which the investment costs of each firm is different and the cost of each firm is the private information of the own firm. In this setting, two firms are assumed to be identical *ex ante* and the prior distribution of the costs is followed by the same distribution. Yao-Wen and Bart M. (2007) consider the situation where one firm has complete information about the rival's investment cost but the other firm has incomplete information about the opponent's investment cost.

Grenadier and Wang (2005) considers a conflicts between managers and owners taking into the account of the asymmetric information. The model of Grenadier and Wang (2005) combined real options with contract theory.

The purpose of this paper examines an alternative factor of asymmetric information which affects the timing of the investment known as a signaling effect. I consider two asymmetric firms, an incumbent and an entrant, for the entry into a new market of a product. The demand of the market has two uncertainty factors. One factor is the potential size of the market which is determined at the beginning of the game. The factor can be observed only by the incumbent as private information due to the experience of the incumbent, but the entrant cannot obtain the information. The other factor is the fluctuation of the demand given by a stochastic process which is common to both firms. In my framework, the incumbent invests earlier than the entrant for any demand because the incumbent's profit from the market is assumed to be relatively larger than that of the entrant and the incumbent's cost of the investment is assumed to be relatively smaller than the entrant's.

If the timing of investment by the high demand type of the incumbent is earlier than low demand type, the information is revealed. Then, the entrant would accelerates the timing of the investment if the incumbent's earlier investment is observed. Since this would reduce the monopolistic profit of the incumbent with high demand type, the incumbent that knows the high demand may strategically delay the timing of the investment to hide the information.

In this paper I characterize a signaling equilibrium and investigate the values of both firms.

Grenadier (1999) examines information revelation through option exercise in which each firm has the private information about the payoff uncertainty and updates the belief for the payoff by observing exercise strategies of other firms. Grenadier (1999) mainly focus on informational cascades and the projects of the firms are not competitive each other. Hence, the strategic revelation of information is not concerned.

2 The Model

I consider that two asymmetric firms, an incumbent and an entrant have the option to wait for their optimal entry into the market of a new product. the incumbent and the entrant are denoted by firm I and firm E, respectively. The investments for the entry of both firms are assumed to be irreversible and the sunk cost of firm i's investment is denoted by K_i for $i = I, E$. The revenue flow of each firm after the entry depends on the market structure, monopoly or duopoly, and two uncertain factors of the demand.

One uncertain factor of the demand represents a stochastic process, denoted by X_t, as a standard real option setting. X_t is interpreted as the unsystematic shocks of the demand over the time and it is common to both firms.

Suppose X_t follows a geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dz$$

where μ is the drift parameter, σ is the volatility parameter and dz_t is the increment of a standard Winner process. Both firms are assumed to be risk neutral, with the risk free rate of interest r. As usual assumption of real option approach for convergence, I assume $r > \mu$.

The other uncertain factor of the demand represents a systematic risk and it is assumed to be a constant over the time. I denote the factor by θ where $\theta = H$ and $\theta = L$ means that the demand is high and low, respectively. The prior probability of drawing $\theta = H$ and $\theta = L$ are denoted by p and $1 - p$, respectively.

When only firm i enters in the market, the profit flow of firm i becomes $\pi_{i1}^{\theta} X_t$. On the other hand, when both firms enter in the market, the profit flow of firm i becomes $\pi_{i2}^{\theta} X_t$. The profit flow of the firm which has not entered in the market is assumed to be zero. I assume that $\pi_{i1}^{\theta} > \pi_{i2}^{\theta} > 0$ for $i = 1, 2$ and $\theta = H, L$.

I assume that the incumbent has several advantages to the entrant due to his experience of similar markets: the incumbent has more information, more share of the products and less cost of the investment than the entrant. In detail, the incumbent has two advantages stated as follows. First, while X_t is observable by two firms, the uncertain factor θ can be observed only by the incumbent, i.e., it is the private information of the incumbent. Secondly, I assume that K_I/π_{I2}^{L} is smaller than K_E/π_{E1}^{H}. This assumption is hold if the profit in monopoly of the incumbent is sufficiently larger than that of the entrant and/or the cost of the investment K_I is sufficiently smaller than K_E.

3 Value Functions of a Benchmark Case

Our model is one of the option exercise games which are investigated under the joint framework of real options and game theory. A number of studies such as Smets (1991), Grenadier (1996), Kijima and Shibata (2002), Kulatilaka and Perotti (1998), Huisman and Kort (1999), Huisman (2001) and Smit and Trigeorgis (2002) consider the symmetric firms in order to examine preemptive behavior of competition. In these models, if the value of the leader's optimal entry is greater than the value of the follower's entry of the best reply, then both firms want to become a leader. In this case, the leader's optimal threshold is solved by equations of an equilibrium and the value of the leader is not determined by maximizing the expected profit of either firm. Huisman (2001), Kong and Kwok (2007) and Pawlina and Kort (2006) show that this preemptive behavior and simultaneous entry would occur under the asymmetry of costs and profits. In this case obtaining the values in the equilibrium is complicated.

However, if asymmetry is sufficiently large and the initial value of both firms is sufficiently small to wait for the investment, the lower-cost firms must be the dominant leader, (see Kong and Kwok (2007) and Pawlina and Kort (2006)). By using the result of Kong and Kwok (2007), our two assumptions, $K_I/\pi_{I2}^{L} > K_E/\pi_{E1}^{H}$ and sufficiently small $X_t = x$, imply that the incumbent must be the leader and the entrant must be the follower.

Due to this setting, the decisions and the values of both firms are analyzed under the condition where the incumbent is the leader and the entrant is the follower. In next subsections, the benchmark case is solved backward. First, I consider the value of the entrant as the follower, then the value of the incumbent as the leader is discussed.

3.1 The Value of the Entrant

The value of the entrant is a function of the entrant's belief for the demand level θ. Let $u_E^*(q)$ be the value function of the entrant under the condition where the entrant invests later than the incumbent and believes the high demand occurring with probability q.

The value function is given by

$$u_E^*(q) = \max_{t_E} E^x[\int_{t_E}^{\infty} e^{-r(s-t)}(q\pi_{E2}^H + (1-q)\pi_{E2}^L)X_s ds - e^{-r(t_E - t)}K_E]$$

where E^x denotes the conditional expectation on $X_t = x$. Let $x_E^*(q)$ be the optimal threshold for the belief q, i.e., $x_E^*(q) = \inf\{t \geq 0 | X_t \geq x_E^*(q)\}$. The usual calculation of real option analysis implies

$$x_E^*(q) = \frac{\beta}{\beta - 1} \frac{r - \mu}{q\pi_{E2}^H + (1-q)\pi_{E2}^L} K_E$$

where β is defined by

$$\beta = \frac{1}{2}\left(1 - \frac{2\mu}{\sigma^2} + \sqrt{(1 - \frac{2\mu}{\sigma^2})^2 + \frac{8r}{\sigma^2}}\right).$$

Let $x_E^H = x_E^*(1)$, $x_E^L = x_E^*(0)$ and $x_E^M = x_E^*(p)$. x_E^H and x_E^L are the thresholds when the entrant believes that the demand are high and low, respectively. x_E^M is the threshold when the entrant predicts the high demand with prior probability p.

We easily find that

$$x_E^H \leq x_E^M \leq x_E^L. \tag{1}$$

3.2 The Value of the Incumbent

Let $u_I(x_I, x_E, \theta)$ be the expected profit of the incumbent with his private information of the demand θ when the incumbent invests at the threshold x_I and the entrant invests at x_E under the condition $x_I < x_E$.

$u_I(x_I, x_E, \theta)$ is given by

$$u_I(x_I, x_E, \theta) = E^x[\int_{t_I}^{t_E} e^{-r(s-t_I)}\pi_{I1}^{\theta} X_s ds - e^{-r(s-t_I)}K_E + \int_{t_E}^{\infty} e^{-r(s-t_E)}\pi_{I2}^{\theta} X_s ds],$$

where t_i is the first passage time at threshold x_i for $i = I, E$, i.e., $t_i = \inf\{t \geq 0 | X_t \geq x_i\}$. This equation can be written as

$$u_I(x_I, x_E, \theta) = E^x[\int_{t_I}^{\infty} e^{-r(s-t)} \pi_{I1}^{\theta} X_s ds - e^{-r(s-t_I)} K_E - \int_{t_E}^{\infty} e^{-r(s-t)} (\pi_{I1}^{\theta} - \pi_{I2}^{\theta}) X_s ds.]$$

Let the first term and the second term be denoted by $v_I(x_I, \theta)$ and $\Delta v_I(x_E, \theta)$, i.e.,

$$v_I(x_I, \theta) = E^x[\int_{t_I}^{\infty} e^{-r(s-t)} \pi_{I1}^{\theta} X_s ds - e^{-r(s-t_I)} K_E]$$

and

$$\Delta v_I(x_E, \theta) = E^x[\int_{t_E}^{\infty} e^{-r(s-t)} (\pi_{I1}^{\theta} - \pi_{I2}^{\theta}) X_s ds].$$

Then, $u_I(x_I, x_E, \theta)$ is given by

$$u_I(x_I, x_E, \theta) = v_I(x_I, \theta) - \Delta v_I(x_E, \theta). \tag{2}$$

where $v_I(x_I, \theta)$ is explicitly described as

$$v_I(x_I, \theta) = \left(\frac{\pi_{I1}^{\theta}}{r - \mu} x_I - K_I \right) \left(\frac{x}{x_I} \right)^{\beta}.$$

Note that

$$\Delta v_I(x_E^H, \theta) \geq \Delta v_I(x_E^M, \theta) \geq \Delta v_I(x_E^H, \theta) \tag{3}$$

because $\Delta v_I(x_E, \theta)$ is decrease in threshold x_E and (1).

If x_E is independent of the incumbent decision x_I, the second term $\Delta v_I(x_E, \theta)$ is independent of the incumbent decision x_I. In this case, hence, the incumbent maximizes the expected profit by maximizing $v_I(x_I, \theta)$. Taking into the account of the signaling effect, however, the optimal threshold of the entrant x_E depends on the threshold of the incumbent x_I.

This signaling equilibrium is examined in the next section. In this section, I consider the case in which x_E is independent of x_I. Let $x_I^*(\theta)$ be the optimal threshold of the incumbent with the private information θ under the condition that x_E is independent of x_I. Then, $v_I(x_I^*(\theta), \theta)$ is given by

$$v_I(x_I^*(\theta), \theta) = \max_{x_I} v_I(x_I, \theta) \max_{t_I} E^x[\int_{t_I}^{\infty} e^{-r(s-t)} \pi_{I1}^{\theta} X_s ds - e^{-r(s-t_I)} K_E].$$

The usual calculation of real option analysis implies

$$x_I^*(\theta) = \frac{\beta}{\beta - 1} \frac{r - \mu}{\pi_{I1}^{\theta}} K_I,$$

and

$$v_I(x_I^*(\theta),\theta) = \begin{cases} \frac{K_I}{\beta-1}\left(\frac{x}{x_I^*(\theta)}\right)^\beta & x \le x_I^*(\theta) \\ \frac{\pi_{I1}^\theta}{r-\mu}x - K_I & x > x_I^*(\theta) \end{cases}$$

$\Delta v_I(x_E,\theta)$ is given by

$$\Delta v_I(x_E,\theta) = \frac{\pi_{I1}^\theta - \pi_{I2}^\theta}{r-\mu}x_E\left(\frac{x}{x_E}\right)^\beta$$

Let $x_I^H = x_I^*(H)$ and $x_I^L = x_I^*(L)$.

$x_I^H = x_I^*(H)$ and $x_I^L = x_I^*(L)$ express the optimal threshold when the incumbent knows that the demand is high and low, respectively, if the incumbent's decision is independent of the entrant's decision.

4 Equilibrium Analysis

4.1 Definitions of the Solution

For the analysis of the signaling effect, a (weak) Perfect Bayesian Equilibrium (PBE) is applied as the solution concept. In this model, a solution concept is specified not only by a threshold for each of the players but also an entrant's belief of the demand.

Three components $\{(a_I(H), a_I(L)), a_E(\cdot), q(\cdot)\}$ is called an *assessment* where:

- $a_I(H)$ and $a_I(L)$ are incumbent's threshold for private information H and L, respectively,
- $a_E(x_I)$ is the entrant's threshold for observed incumbent's threshold x_I, and
- $q(x_I)$ is the entrant's belief for observed incumbent's threshold x_I.

A PBE is an assessment $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$ satisfying the following three conditions.

First, $a_I^*(\theta)$ is the optimal threshold of the incumbent for $\theta = H, L$ such that

$$u_I(a_I^*(\theta), a_E^*(a_I^*(\theta)), \theta) = \max_{x_I} u_I(x_I, a_E^*(x_I), \theta). \tag{4}$$

Secondly, $a_E^*(\cdot)$ is the threshold of the entrant observing the entry of the incumbent at x_I with the belief $q^*(\cdot)$ such that

$$a_E^*(x_I) = x_E^*(q^*(x_I)). \tag{5}$$

Finally, $q^*(\cdot)$ is the belief of the entrant for the high demand which is consistent to the thresholds of the incumbent observed by the entrant in the sense of Bayes rule by regarding the optimal threshold as a random variable X_I.

Let X_I be a random variable that the timing of the incumbent investment. Then $q^*(x_I) = Prob[\theta = H|X_I = x_I]$. By Bayes rule,

$$Prob[\theta = H|X_I = x_I] = \frac{Prob[X_I = x_I|\theta = H]Prob[\theta = H]}{Prob[\theta = H]Prob[X_I = x_I|\theta = H] + prob[\theta = L]Prob[X_I = x_I|\theta = L]}.$$

By $Prob[\theta = H] = p$ and $Prob[\theta = L] = 1 - p$, the condition of the consistent belief is expressed by

$$q^*(x_I) = \frac{pProb[X_I = x_I|\theta = H]}{Prob[pProb[X_I = x_I|\theta = H] + (1-p)Prob[X_I = x_I|\theta = L]}. \tag{6}$$

$Prob[X_I = x_I|\theta = H]$ and $Prob[X_I = x_I|\theta = L]$ would follow the probability distributions according to a mixed strategy of the incumbent. In Section 5, I investigate the mixed strategies of the incumbent. However, in this section, I restrict the analysis to the pure strategies, so $Prob[X_I = x_I|\theta = H]$ and $Prob[X_I = x_I|\theta = L]$ can be explicitly written as

$$Prob[X_I = x_I|\theta = H] = \begin{cases} 1 & x_I = a_I^*(H) \\ 0 & x_I \neq a_I^*(H), \end{cases} \quad Prob[X_I = x_I|\theta = L] = \begin{cases} 1 & x_I = a_I^*(L) \\ 0 & x_I \neq a_I^*(L). \end{cases} \tag{7}$$

(6) and (7) imply that

$$q^*(x_I) = \begin{cases} p & x_I = a_I^*(H) \quad \text{and} \quad x_I = a_I^*(L), \\ 1 & x_I = a_I^*(H) \quad \text{and} \quad x_I \neq a_I^*(L), \\ 1-p & x_I \neq a_I^*(H) \quad \text{and} \quad x_I = a_I^*(L). \end{cases} \tag{8}$$

If $a_I^*(H) \neq x_I$ and $a_I^*(L) \neq x_I$, any belief $q^*(x_I)$ is consistent.

Thus, a PBE in pure strategies is formally defined as follows.

Definition 4.1. *An assesment is said to be a (weak) perfect Baysian equilibrium in pure strategies (PBEP) if it satisfies (4), (5) and (8).*

A PBEP is said to be a *pooling* equilibrium if $a_I^*(H) = a_I^*(L)$. (8) implies that

$$q^*(a_I^*(H)) = q^*(a_I^*(L)) = p.$$

This means that the action of the incumbent does not convey the information about the demand in a pooling equilibrium and the entrant predicts high demand with the prior probability p in the equilibrium behavior of the incumbent $a_I^*(H) = a_I^*(L)$[1]. Hence, in the pooling equilibrium, the threshold of the entrant in the equilibrium is

$$a_E^*(a_I^*(H)) = a_E^*(a_I^*(L)) = x_I^M$$

because of $x_I^M = x_I^*(p)$.

A PBEP is said to be a *separating* equilibrium if $a_I^*(H) \neq a_I^*(L)$. In the separating equilibrium, (8) implies that

$$q^*(a_I^*(H)) = 1, \quad q^*(a_I^*(L)) = 0.$$

This means that the entrant perfectly knows the level of the demand, which *was* priorly a private information of the incumbent, by observing the action of the incumbent. The threshold of the entrant in the separating equilibrium is

$$a_E^*(a_I^*(H)) = x_E^H, \quad a_E^*(a_I^*(L)) = x_E^L.$$

4.2 Candidates of the Solution

The following two assessments are considered as candidates of the solution in this section. The first assessment is called *Truthful Revelation* defined by

$$a_I^*(H) = x_I^H, \quad a_I^*(L) = x_I^L$$
$$a_E^*(x_I) = \begin{cases} x_E^H & x_I \neq x_I^L, \\ x_E^L & x_I = x_I^L, \end{cases}$$
$$q^*(x_I) = \begin{cases} 1 & x_I \neq x_I^L, \\ 0 & x_I = x_I^L. \end{cases}$$

In Truthful Revelation, the incumbent for any demand truthfully enters to the market at the optimal threshold with respect to the demand. This truthful behavior reveals the information of the demand that the incumbent has. The entrant obtains the information about the demand by observing the incumbent's behavior and enters to the market optimally with full information. If the entrant observes that the incumbent enters to the market at neither x_I^H nor x_I^L, any belief of the entrant is consistent. In other words, the entran's beleif is assigned arbirarily in the

[1]Note that in any equilibrium, for any $x_I \neq a_I^*(H)$ in the off-equilibrium path, any $0 \leq q^*(x_I) \leq 1$ satisfies consistency condition (8). However, this has no role in the equilibrium analysis.

entrant's observation in off-equilibrium path. For this unexpected deviation of the equilibrium for the incumbent, the entrant is assumed to believe high demand in this paper.

Second assessment is called *Strategic Revelation*

$$a_I^*(H) = a_I^*(L) = x_I^L$$
$$a_E^*(x_I) = \begin{cases} x_E^H & x_I \neq x_I^L, \\ x_E^M & x_I = x_I^L, \end{cases}$$
$$q^*(x_I) = \begin{cases} 1 & x_I \neq x_I^L, \\ p & x_I = x_I^L \end{cases}$$

In Strategic Revelation, the incumbent for the high demand does not enters at the optimal threshold of the high demand but invests at the threshold of the low demand. This delay of the investment hides the information about the high demand and the entrant cannot distinguish the type of the demands by observing the incumbent's behavior. Thus, the entrant expects the level of the demand according to the prior probability and enters at the threshold for the expectation of the demand. In off-equilibrium path, the entrant is assumed to believe the high demand, as well as Truthful Revelation.

4.3 The Equilibrium Strategies

In this subsection, I analyze conditions where either of candidates, Truthful Revelation or Strategic Revelation, is a PBEP. Since both candidates are constructed by satisfying the optimality of the entrant and the consistency of the entrant's belief, it remains to consider the optimality of the incumbent for given entrant's strategy $a_E^*(\cdot)$ and belief $q^*(\cdot)$. Moreover, the low demand type of the incumbent does not have the incentive to deviate the optimal timing x_I^L because pretending the high demand type only accelerates the timing of the entrant's investment and reduce the incumbent's value. Hence, only the timing of the high type of the incumbent should be focused on.

First, suppose that Truthful Revelation is a PBEP. In Truthful Revelation, the entrant believes that the later investment of the incumbent at x_I^L reveals truthfully the information of low demand. If the incumbent with the high demand does not have the incentive for hiding the information to delay the entrant's investment, the following condition holds,

$$u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H). \tag{9}$$

Secondly, suppose that Strategic Revelation is a PBEP. In Strategic Revelation, the incumbent with information of the high demand strategically delays the investment to the optimal timing for the low demand, and the entrant cannot obtain the information about the demand. Then, the entrant observing the incumbent's investment at x_L predicts the level of the demand by prior probability p, so that the expectation of the profit is π_{E2}^M. Then the entrant enters to the market at x_E^M which is optimal for π_{E2}^M. The incumbent with information of the high demand has an incentive to hide information if the expected value for this delayed entrance at x_I^L is greater than that of the optimal entrance at the threshold of the high demand x_I^H. This condition is expressed by

$$u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^M, H). \tag{10}$$

Above arguments are summarized and proved formally in the following proposition.

Proposition 4.2. **(1)** *(9) holds if and only if Truthful Revelation is a PBEP.*

(2) *(10) holds if and only if Strategic Revelation is a PBEP.*

Proof. First, I show (1). Suppose $\pi_{I1}^H - \pi_{I2}^H \leq \xi_H(K_I, K_E, \beta)$. I will show that if assessment $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$ is Truthful Revelation, then it is a PBEP. To prove this, it is sufficient to show that the assessment satisfies three conditions: the incumbent's optimality (4), the entrant's optimality (5) and the consistency of the entrant's belief(8). By the definition, Truthful Revelation always satisfies the entrant's optimality (5) and the consistency of the belief(8), it remains to show that it satisfies the incumbent's optimality (4), i.e., for $\theta = H, L$,

$$u_I(a_I^*(\theta), a_E^*(a_I^*(\theta)), \theta) \geq u_I(x_I, a_E^*(x_I), \theta). \tag{11}$$

for any $x_I \neq a_I^*(\theta)$.

First, let $\theta = L$. Since, in Truthful Revelation, $a_I^*(L) = x_I^L$, $a_E^*(x_I^L) = x_E^L$ and $a_E^*(x_I) = x_E^H$ for any $x_I \neq x_I^L$, (11) can be expressed as $u_I(x_I^L, x_E^L, L) \geq u_I(x_I, x_E^H, L)$ for any $x_I \neq x_I^L$. Note that $\Delta v_I(x_E^H, \theta) \geq \Delta v_I(x_E^H, \theta)$ by (3). $u_I(x_I, x_E^L, L) = v_I(x_I, \theta) - \Delta(x_E, \theta)$ implies that $u_I(x_I, x_E^L, L) \geq u_I(x_I, x_E^H, L)$, because the payoff of the incumbent increases in later investment of the entrant. Since x_I^L is the optimal threshold of the incumbent, i.e. $v_I(x_I^L, \theta) = \max_{\hat{x}_I} v_I(\hat{x}_I, \theta)$, $u_I(x_I^L, x_E^L, L) \geq u_I(x_I, x_E^L, L)$. Hence, (11) hold for $\theta = L$.

Secondly, let $\theta = H$. (11) can be expressed as $u_I(x_I^H, x_E^H, H) \geq u_I(x_I, x_E^H, H)$ for any $x_I \neq x_I^L$ and $u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H)$. Since x_I^H is the optimal threshold of the incumbent,

$u_I(x_I^H, x_E^H, H) \geq u_I(x_I, x_E^H, H)$ for any $x_I \neq x_I^L$. By (9) $u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H)$. Then, Truthful Revelation is a PBEP.

Conversely, suppose that $u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^L, H)$. Then, the high demand type of the incumbent strictly increases the payoff by deviating x_I^L from $a_I^*(H) = x_I^H$ in Truthful Revelation and this means that Truthful Revelation is not a PBEP. Hence, Truthful Revelation is a PBEP, only if $\pi_{I1}^H - \pi_{I2}^H \leq \xi_H(K_I, K_E, \beta)$.

The proof of (2) is similar. □

Since $u_I(x_I^L, x_E^M, H) \leq u_I(x_I^L, x_E^L, H)$ neither Truthful Revelation nor Strategic Revelation is PBEP for $u_I(x_I^L, x_E^M, H) \leq u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^M, H)$. In this interval, the mixed strategy of the incumbent should be considered to ensure the existence of the equilibrium.

The following lemma shows that (9) and (10) can be solved for difference of the incumbent's profits between monopoly and duopoly.

Lemma 4.3. (1) *(9) holds if and only if*

$$\pi_{I1}^H - \pi_{I2}^H \leq \frac{1}{\beta}\left(\frac{K_E}{K_I}\right)^{\beta-1}\left\{\frac{(\pi_{I1}^H)^\beta - \phi(\pi_{I1}^L)^\beta}{(\pi_{E2}^H)^{\beta-1} - (\pi_{E2}^L)^{\beta-1}}\right\}, \tag{12}$$

and

(2) *(10) holds if and only if*

$$\pi_{I1}^H - \pi_{I2}^H \geq \frac{1}{\beta}\left(\frac{K_E}{K_I}\right)^{\beta-1}\left\{\frac{(\pi_{I1}^H)^\beta - \phi(\pi_{I1}^L)^\beta}{(\pi_{E2}^H)^{\beta-1} - (\pi_{E2}^M)^{\beta-1}}\right\}, \tag{13}$$

where

$$\phi = \frac{\beta\pi_{I1}^H - (\beta-1)\pi_{I1}^L}{\pi_{I1}^L}.$$

Let the right hand side of (12) and (13) be $\xi_H(K_I, K_E, \beta)$ and $\xi_M(K_I, K_E, \beta)$, i.e.,

$$\xi_H(K_I, K_E, \beta) = \tfrac{1}{\beta}\left(\tfrac{K_E}{K_I}\right)^{\beta-1}\left\{\tfrac{(\pi_{I1}^H)^\beta - \phi(\pi_{I1}^L)^\beta}{(\pi_{E2}^H)^{\beta-1} - (\pi_{E2}^L)^{\beta-1}}\right\}. \quad \xi_M(K_I, K_E, \beta) = \tfrac{1}{\beta}\left(\tfrac{K_E}{K_I}\right)^{\beta-1}\left\{\tfrac{(\pi_{I1}^H)^\beta - \phi(\pi_{I1}^L)^\beta}{(\pi_{E2}^H)^{\beta-1} - (\pi_{E2}^M)^{\beta-1}}\right\}.$$

By above arguments, the equilibrium strategies are characterized by $\xi(K_I, K_E, \beta)_H$ and $\xi(K_I, K_E, \beta)_M$. Proposition 4.4 summarizes equilibrium strategies.

Proposition 4.4. (1) $\pi_{I1}^H - \pi_{I2}^H \leq \xi_H(K_I, K_E, \beta)$ *if and only if Truthful Revelation is a PBEP.*

(2) $\pi_{I1}^H - \pi_{I2}^M \geq \xi_M(K_I, K_E, \beta)$ *if and only if Strategic Revelation is a PBEP.*

5 Equilibria in Mixed Strategies

As the discussion of the previous section, it is found that an equilibrium in the pure strategies does not exists for $u_I(x_I^L, x_E^M, H) \leq u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^L, H)$. Hence, I consider mixed strategies of the incumbent. Let $x_I(\lambda)$ be a mixed strategy of the incumbent where the incumbent chooses x_I^H with probability λ and x_I^L with probability $1-\lambda$ for $0 \leq \lambda \leq 1$. Moreover, u_I is extended to the set of mixed strategies $x_I(\lambda)$ for $0 \leq \lambda \leq 1$ and entrant strategies $a_E(\cdot)$, defined by

$$u_I(x_I(\lambda), a_E(\cdot), \theta) = \lambda u_I(x_I^H, a_E(x_I^H), \theta) + (1-\lambda) u_I(x_I^L, a_E(x_I^L), \theta)$$

for any x_E and $\theta = H, L$.

The consistent belief of the entrant for $a_I^*(H) = x_I(\lambda)$ and $a_I^*(L) = x_I^L$ is solved by Bayes rule (6). $Prob[X_I = x_I|\theta = H]$ and $Prob[X_I = x_I|\theta = L]$ are given by

$$Prob[X_I = x_I|\theta = H] = \begin{cases} \lambda & x_I = x_I^H \\ 1-\lambda & x_I = x_I^L \\ 0 & x_I \neq x_I^H, x_I^L, \end{cases} \qquad Prob[X_I = x_I|\theta = L] = \begin{cases} 1 & x_I = x_I^L \\ 0 & x_I \neq x_I^L, \end{cases} \tag{14}$$

(6) and (14) imply the consistent belief $q^*(\cdot)$ as

$$q^*(x_I^H) = \frac{p\lambda}{p\lambda + (1-p) \times 0} = 1$$

and

$$q^*(x_I^L) = \frac{p(1-\lambda)}{p\lambda + (1-p) \times 1} = \frac{p(1-\lambda)}{1 - p(1-\lambda)}.$$

If $x_I \neq x_I^H, x_I^L$, any belief $q^*(x_I)$ is consistent.

This consistent belief indicates that the entrant observing the investment at x_I^H completely learns the high demand, because only the incumbent with information of the high demand invests at x_I^H. Hence, the optimal timing of investment of the entrant observing the incumbent's investment at x_I^H is x_E^H. In contrast, since both types of the incumbents have the possibility of the investment at x_I^L, the entrant predicts the high demand according to the probability $q^*(x_I^L)$ when the entrant observes the incumbent's investment at x_I^L. The optimal timing of the investment of the entrant observing the incumbent's investment at x_I^L is $x_E^*(q^*(x_I^L))$. For simplify notation, $q^*(x_I^L)$ is denoted by q^λ and let $x_E^*(q^\lambda)$ be x_E^λ.

By above arguments, the following assessment $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$, called λ-*Hybrid Revelation*, is a general candidate of the solution, which satisfies the optimality of the entrant and the consistence of the belief.

$$a_I^*(H) = x_I(\lambda), \quad a_I^*(L) = x_I^L$$
$$a_E^*(x_I) = \begin{cases} x_E^H & x_I \neq x_I^L, \\ x_E^\lambda & x_I = x_I^L, \end{cases}$$
$$q^*(x_I) = \begin{cases} 1 & x_I \neq x_I^L, \\ 0 & x_I = x_I^L, \end{cases}$$

Note that λ-Hybrid Revelation for $\lambda = 1$ is identical to Truthful Revelation while $\lambda = 0$ is to Strategic Revelation. Hence, by solving a condition on λ where λ-Hybrid Revelation is an equilibrium for $u_I(x_I^L, x_E^M, H) \leq u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^L, H)$, an equilibrium for any case can be characterized comprehensively.

Similarly to Strategic Revealation and Truthful Revealation, the incumbent with the information of the low demand also does not have incentive to deviate the optimal timing for low demand. It remains to examine the equilibrium strategies of the high type of the incumbent. Let $\{(a_I^*(H), a_I^*(L)), a_E^*(\cdot), q^*(\cdot)\}$ be λ-Hybrid Revelation. If the high type of the incumbent does not have incentive to deviate the mixed strategy $x_I(\lambda)$ for given entrant's strategy $a_E^*(x_I^H)$, $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$ is a necessary condition of the equilibrium. To show this, suppose $u_I(x_I^H, x_E^H, H) > u_I(x_I^L, x_E^\lambda, H)$. Then,

$$u_I(x_I^H, a_E^H(\cdot), H) = u_I(x_I^H, x_E^H, H) > \lambda u_I(x_I^H, x_E^H, H) + (1-\lambda) u_I(x_I^L, x_E^\lambda, H) = u_I(x_I(\lambda), a_E^H(\cdot), H),$$

so that the incumbent has incentive to deviate from mixed strategy $x_I(\lambda)$ to pure strategy x_I^H. Conversely, suppose that $u_I(x_I^H, x_E^H, H) < u_I(x_I^L, x_E^\lambda, H)$. In this case the incumbent similarly has incentive to deviate from mixed strategy $x_I(\lambda)$ to pure strategy x_I^L.

Hence, the incumbent's mixed strategy of the equilibrium $x_I(\lambda)$ satisfies $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$ and results can be summerized as the following proposition.

Proposition 5.1. **(1)** $u_I(x_I^H, x_E^H, H) \geq u_I(x_I^L, x_E^L, H)$ *if and only if λ-Hybrid Revelation for $\lambda = 1$, which is identical to Truthful Revelation, is a PBE,*

(2) $u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^M, H)$ *if and only if λ-Hybrid Revelation for $\lambda = 0$, which is identical to Strategic Revelation, is a PBE, and,*

(3) $u_I(x_I^L, x_E^M, H) \leq u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^L, H)$ *if and only if* λ*-Hybrid Revelation for* λ *satisfying* $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$ *is a PBE.*

6 Analysis of Equilibrium Strategies and Values

In this section, I show some results of comparative statics about equilibrium strategies and values of the incumbent by numerical examples. Parameters in examples are basically set as $\mu = 0.03$, $r = 0.07$, $p = 0.5$, $\sigma = 0.2$, $x = 0.05$, $\pi_{I1}^H = 12$, $\pi_{I1}^L = 7$, $\pi_{I2}^H = 4$, $\pi_{I2}^L = 4$, $\pi_{E2}^H = 4$, $\pi_{E2}^H = 1$, $K_I = 50$ and $K_E = 100$.

First, the relation between values of the incumbent with high demand $u_I(\cdot, \cdot, H)$ and the duopoly profit of the incumbent with the high demand π_{I2}^H is examined. Figure 1 illustrates the values $u_I(x_I^H, x_E^H, H)$, $u_I(x_I^L, x_E^M, H)$, $u_I(x_I^L, x_E^L, H)$. For $\pi_{I2}^H \geq 8.0$, $u_I(x_I^H, x_E^H, H)$ is greater than $u_I(x_I^L, x_E^L, H)$. The incumbent with high type does not deviate the optimal timing of the investment truthfully, because the duopoly profit of the incumbent is sufficiently large and the incumbent does not have strong incentive to make the entrant's investment delay. Hence, the incumbent with high type enters to the market at the optimal timing of the investment for the high demand and reveals his information truthfully. In contrast, for $\pi_{I2}^H \leq 2.9$, $u_I(x_I^H, x_E^H, H)$ is less than $u_I(x_I^L, x_E^M, H)$. In this range, the incumbent with high type invests at the optimal timing for the low demand to hide information for high demand because the duopoly profit of the incumbent is small and the decrement of the incumbent's profit by the investment of the entrant is critical. The incumbent enters to the market at the optimal timing of the investment for the low demand and does not have incentive to deviate to the optimal timing of the high demand in this range. For $2.9 \leq \pi_{I2}^H \leq 8.0$, $u_I(x_I^L, x_E^M, H) \leq u_I(x_I^H, x_E^H, H) \leq u_I(x_I^L, x_E^L, H)$, the incumbent uses a mixed strategy as $\lambda-$ Hybrid Revealation. In this interval, the value of the incumbent is same as $u_I(x_I^H, x_E^H, H)$ because the mixed strategy should satisfy condition $u_I(x_I^H, x_E^H, H) = u_I(x_I^L, x_E^\lambda, H)$. Therefore, the value of hight type of the incumbent in the equilibrium strategy is identical to $u_I(x_I^H, x_E^H, H)$ for $\pi_{I2}^H \leq 2.9$ while it is $u_I(x_I^L, x_E^M, H)$ for $\pi_{I2}^H \geq 2.9$.

Figure 2 illustrates the probability λ that the incumbent with high demand invests at the optimal timing for the high demand in the equilibrium strategy, i.e., the incumbent enters to the market truthfully. For $\pi_{I2}^H \leq 2.9$, Strategic Revelation is a PBE so that $\lambda = 0$, while for

$\pi_{I2}^H \geq 8.0$, Truthful Revelation is a PBE so that $\lambda = 1$. For $2.9 < \pi_{I2}^H < 8.0$, the incumbent uses a completely mixed strategy and λ is a positive value which increases in π_{I2}^H.

Secondly, the effect of volatility is examined. Figure 3 illustrates relation between values of the incumbent with high demand and the volatility. If the volatility is small, the incumbent invests truthfully while if the volatility is large, the incumbent invests strategically. In medium range, the incumbent uses the mixed strategy.

Thirdly, the relation between the cost of the incumbent and the values is investigated. Figure 4 depicts relation between values of the incumbent with high demand and cost of the incumbent. The values are decreasing non-linearly in cost while those are increasing linearly in the profit flow. If the cost is small, Truthful Revealation is occured, while if the cost is large, Strategic Revelation is occured. For medium range, the incumbent uses a complete mixed strategy, $\lambda-$Hybrid Revelation for some $0 < \lambda < 1$, is the equilibrium.

Finally, the impact of the entrant's cost on the incumbent's cost is investigated. It is interesting that the incumbent's value is affected not only by the incumben's own cost, but also by the rival's cost because smaller entrant's cost pushing forward the entrant's investment reduces the incumbent's value. Figure 5 depicts relation between between the values of the incumbent with high demand and cost of the entrant. If the entrant's cost is large, the timing of the entrant's investment is late. Since the entrant's investment is negligible effect on the incumbent's value, the incumbent with high type invests truthfully. On the other hand, the incumbent invests strategically for small entrant's cost. For the medium interval of the entrant's cost, the incumbent uses the mixed strategy.

7 Conclusion

This paper examines investment game for an incumbent and an entrant for optimal entries into a new market in which the incumbent only has information of demand, high or low, and the entrant predict the demand by observing the incumbent's timing of the investment I invetigate whether the incumbent reveals the information truthfully or not taking into account signaling effect by using the concept of a weak perfect Bayesian equilibrium. I characterize a condition for the incumbent with information of high demand invests strategically in the equilibrium, and show that it is necessary for the incumbent to use a mixed strategy in the equilibrium under some condition.

If duopoly profit for the high demand type of the incumbent is small, the incumbent invests strategically while the incumbent does truthfully if this duopoly profit is sufficiently large. The incumbent also invests strategically, if the volatility or the cost of the incumbent is large, or the entrant's cost is small.

Further research is needed to obtain the above results analytically. We aill obtain the results by differentiating the values with respect to profit flows, costs and volatility. Some extensions of the model would be interesting. First, preemptive behavior should be considered by eliminating the assumption where the incumbent is leader and the entrant is follower. Second, other stochastic processes could be considered.

References

Brennan, M. J., and E. S. Schwartz (1985): "Evaluating Natural Resource Investments," *The Journal of Business*, 58(2), 135–157.

Dixit, A. K., and R. S. Pindyck (1994): *Investment under Uncertainty.* Princeton University Press.

Grenadier, S. R. (1996): "The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets," *Journal of Finance*, 51(5), 1653–1679.

——— (1999): "Information Revelation Through Option Exercise," *The Review of Financial Studies*, 12(1), 95–130.

Grenadier, S. R., and N. Wang (2005): "Investment Timing, Agency and Information," *Journal of Financial Economics*, 75, 493–533.

Huisman, K. J. M. (2001): *Technology and Investment: A Game Theoretic Real Options Approach.* Kluwer Academic Publishers.

Huisman, K. J. M., and P. M. Kort (1999): "Effect of Strategic Interactions on the Option Value of Waiting," Tilburg University.

Kijima, M., and T. Shibata (2002): "Real Options in a Duopoly Market with General Volatility Structure," 2002.

Kong, J. J., and Y. K. Kwok (2007): "Real Options in Strategic Investment Games between two Asymmetric Firms," *European Journal of Operational Research*, 181, 967–985.

Kulatilaka, N., and E. C. Perotti (1998): "Strategic Growth Options," *Management Science*, 44(8), 1021–1031.

Lambrecht, B., and W. Perraudin (2003): "Real Options and Preemption under Incomplete Information," *Journal of Economic Dynamics and Control*, 27, 619–643.

Pawlina, G., and P. Kort (2006): "Real Options in an Asymmetric Duopoly: Who Benefits from Your Competitive Disadvantage?," *Journal of Economics and Management*, 15, 1–35.

Smets, F. (1991): "Exporting versus FDI: The effect of uncertainty, irreversibilities and strategic interactions," Working Paper, Yale University.

Smit, H. T. J., and L. Trigeorgis (2002): "Strategic Delay in a Real Options Model of R & D Competition," *The Review of Economic Studies*, 69, 729–747.

Yao-Wen, H., and L. Bart M. (2007): "Preemptive Patenting under Uncertainty and Asymmetric Information," *Annals of Operations Research*, 151, 5–28.

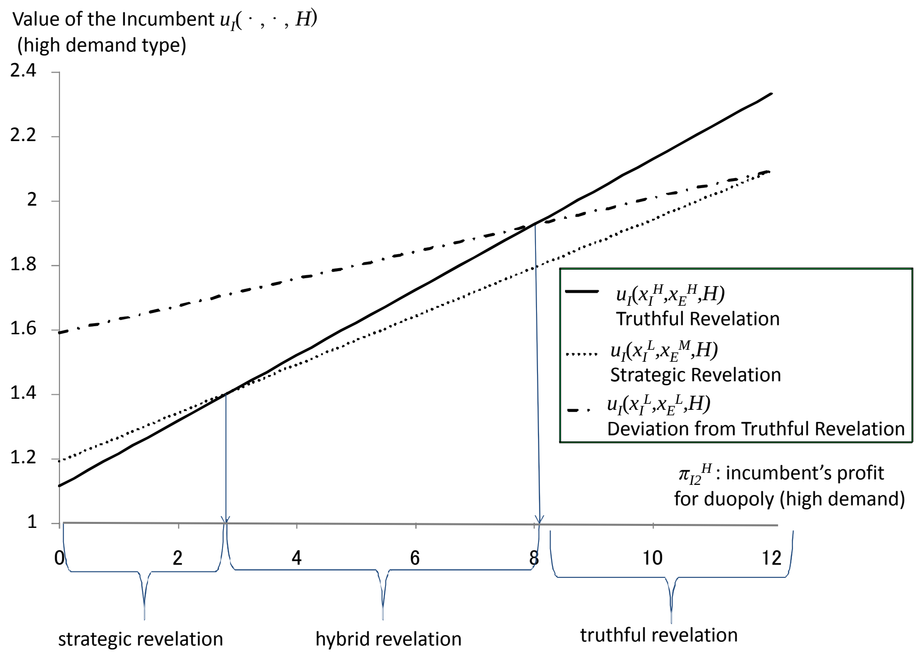

Figure 1: Values of the incumbent with high demand $u_I(\cdot,\cdot,H)$ and the duopoly profit of the incumbent with the high demand π_{I2}^H

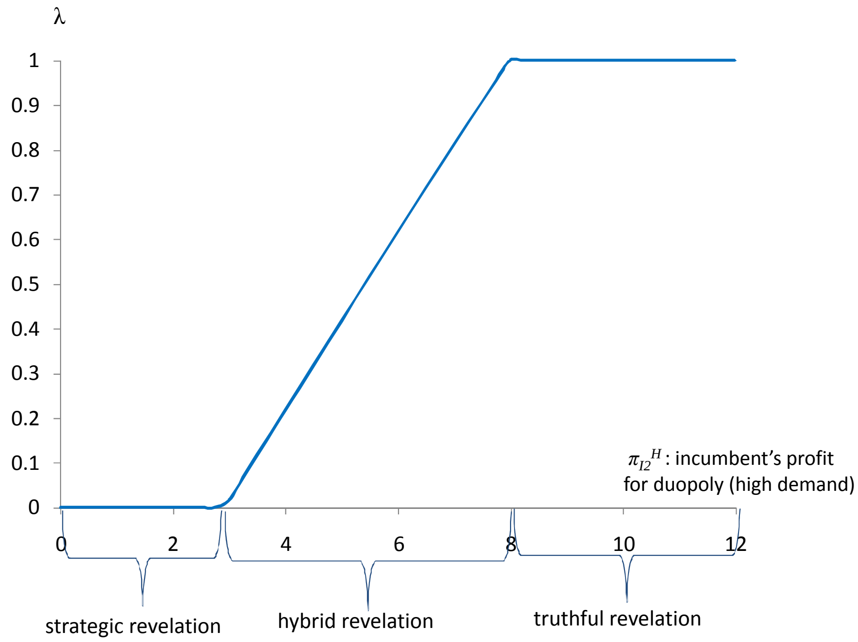

Figure 2: Probability of for investment of the high type of the incumbents at the optimal timing of the high demand and the duopoly profit of the incumbent with the high demand π^H_{I2}

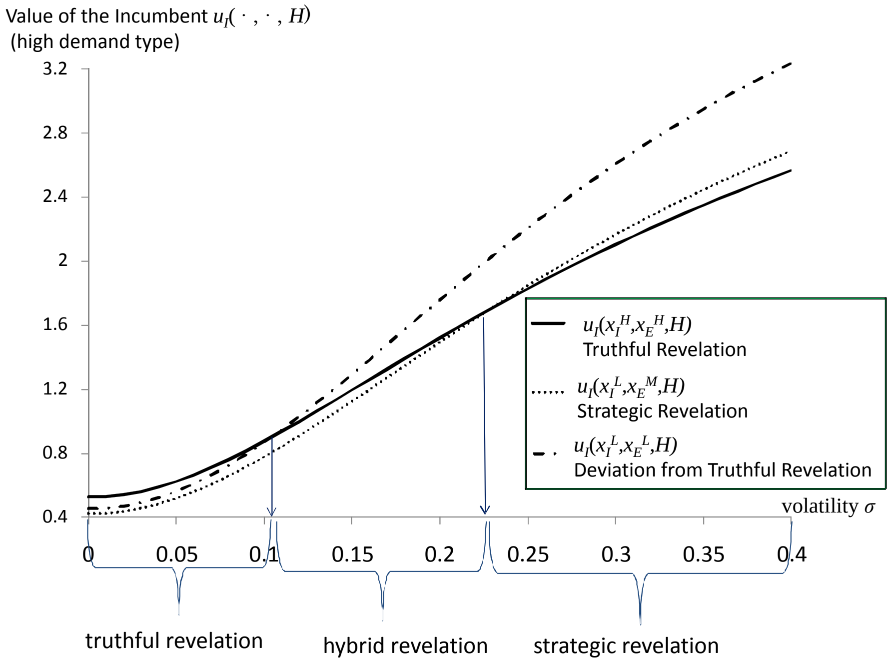

Figure 3: Values of the incumbent with high demand $u_I(\cdot,\cdot,H)$ and volatility σ

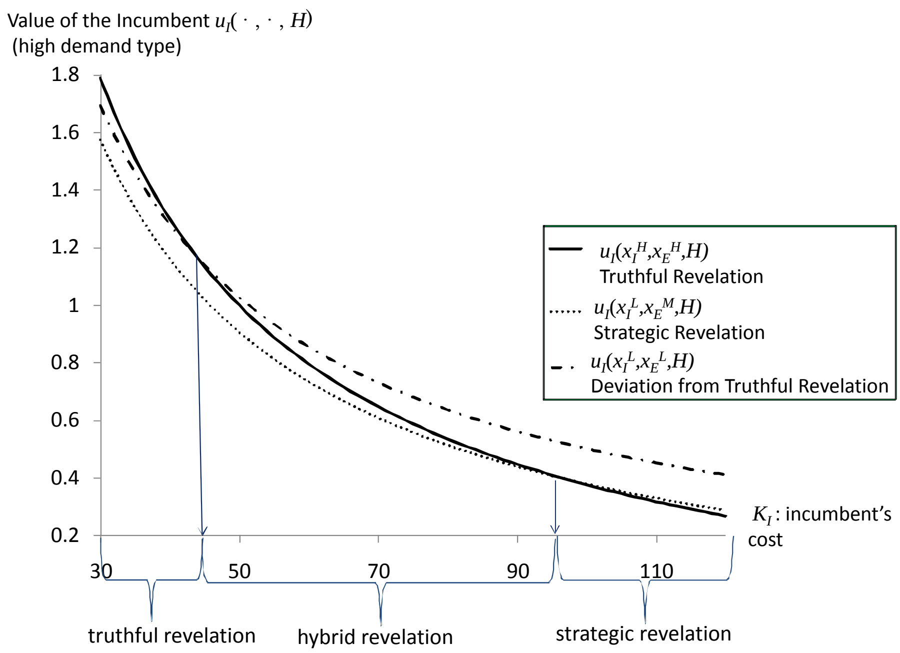

Figure 4: Values of the incumbent with high demand $u_I(\cdot,\cdot,H)$ and cost of the incumbent K_I

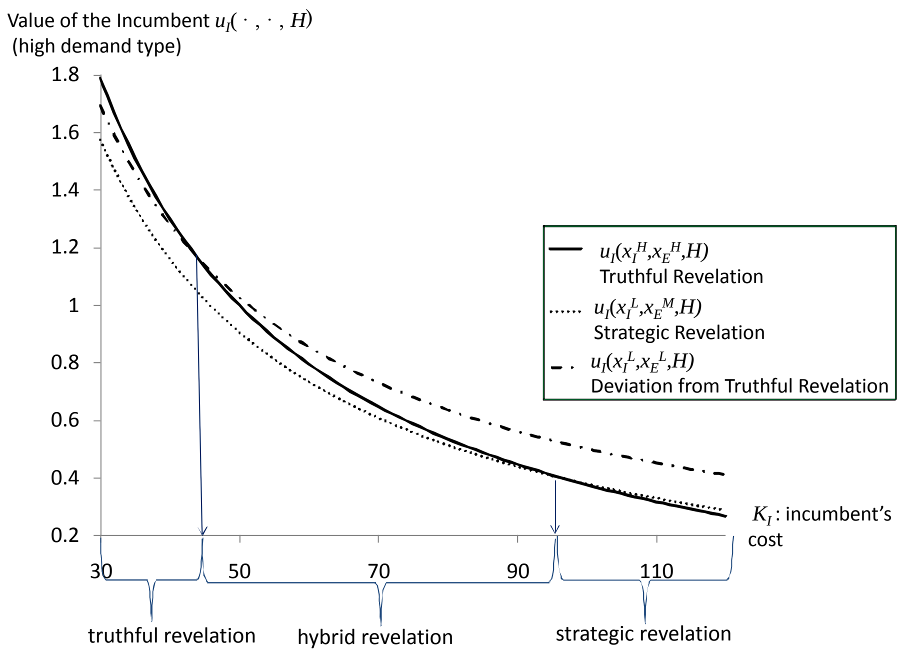

Figure 5: Values of the incumbent with high demand $u_I(\cdot,\cdot,H)$ and cost of the incumbent K_E