Gas storage valuation under limited market liquidity

Bastian Felix

University of Duisburg-Essen, Universitätsstr. 12, 45117 Essen

Bastian.Felix@uni-due.de

Oliver Woll

University of Duisburg-Essen, Universitätsstr. 12, 45117 Essen

Oliver.Woll@uni-due.de

Christoph Weber

University of Duisburg-Essen, Universitätsstr. 12, 45117 Essen

Christoph.Weber@uni-due.de

Various approaches to natural gas storage valuation applying real options theory have been developed in recent years. Postulating storage operators as price takers these methodologies ignore the important fact that most evolving gas spot markets, like the German spot market, lack of liquidity. Thus, considering storage operators as price takers does not account for interdependencies of storage operations and market prices. This paper offers a new approach to storage valuation under respect of the effect of management decisions on market prices. The within this paper proposed methodology determines the optimal production schedule and value by applying a simple finite difference scheme on the stochastic differential equation describing the storage value. The paper is organized as follows: The first section introduces the valuation problem and gives an overview above recent research on gas storage valuation. Furthermore common definitions of market liquidity are introduced. Section 2 analyses possible liquidity measures to quantify the possible impact of storage operations to market prices. Subsequent the valuation methodology is described. The fifth section applies the methodology to an exemplary German storage facility. Finally section 6 concludes.

1 INTRODUCTION

Natural gas storages offer flexibility in uncertain markets. The storage owner has the possibility to buy and inject gas, when prices are low and to sell gas, when prices are high. In uncertain markets this flexibility creates an additional value. Recent research has intensively examined this value by applying real option theory. To solve this optimal stopping problem De Jong and Walet (2005), Ludkovski and Carmona (2007) and Boogert and de Jong (2008) apply Least Squares Monte Carlo Simulations, first derived by Longstaff and Schwartz (2001) for the valuation of financial options. Whereas Holland (2007) applies a simple Monte Carlo Simulation based approach, Maragos (2002) employs a forward curve simulation to the storage valuation problem. Beside this simulation based methodologies it is possible to determine the optimal management of a storage by deriving a stochastic differential equation and solving this equation analytically (Hodges (2004)) rather numerically (Thompson et al. (2003)). Whereas these approaches consider the flexibility offered by natural gas storages, Tseng and Barz (2002) apply real options to evaluate the flexibility offered by a power plant. Hahn and Dyer (2008) determine the value to switch in between gas and oil extraction, employing a recombining tree approach. Nevertheless all of these approaches assume the owner of the asset as price taker. Thus they do not account for interdependencies between the management decision of a storage owner and the market price. As evolving (commodity) markets, e.g. the (continental) European markets for natural gas, commonly lack of market liquidity, the decisions of a storage operator may affect the market price. To cope with this limited market liquidity, we propose a advanced model for storage valuation, incorporating a market liquidity function. This approach can easily be adopted to the valuation of other flexible assets interacting with liquidity limited markets. The proposed methodology supposes that the storage operator anticipates the limited market liquidity and takes it into the account of his operating decision. As the operating decision for a natural gas storage has no fundamental long term impact on the market price, we do not account for long term interdependencies between the storage strategy and the market price. Hence, it is assumed, that a operating decision at a certain time step only affects the price at this time step.

The understanding of liquidity is quiet different in literature. In Ghysels and Pereira (2008), "an asset is liquid if large quantities can be traded in a short period of time without moving the price too much". Keynes (1930) denotes an asset as liquid if it is "realisable at short notice without loss". Amihud and Mendelson (1986) say "llliquidity can be measured by the cost of immediate execution." Geman (2007) states that "Liquidity may be measured by the size of the trade it takes to move the market." Further definitions of liquidity can be found in Brennan and Subrahmanyam (1996), Vayanos (1998) or Boyle and Guthrie (2003) In this paper, liquidity is considered as the amount of trades and the impact of the trades to the price, according to the definition of Kempf (1999)
The remainder of this paper is organized as follows: section 2 analyses possible liquidity measures to quantify the possible impact of storage operations to market prices. The third section describes the proposed valuation methodology. Subsequent, the fourth section applies the methodology to an exemplary German storage facility. Finally section 5 concludes.

2 LIQUIDITY MEASURES

In the literature, several methods for measuring liquidity have been developed. Amihud and Mendelson (1986) state "Illiquidity can be measured by the costs of immediate execution". They base their calculations on the bid-ask spread and give a model of the return-spread relation. For the impact of trading volumes to the prices, we focus on the work of Kempf (1999). He also bases his calculations on the bid-ask spread but gives a regression model of the dependency of prices and volumes. Based on this regression the liquidity is measured by the slope α of the resulting price demand function:

$$p(x) = p^0 - \alpha x \tag{1}$$

Hence a large α correspondes to an illiquid market, whereas $\alpha = 0$ represents the case of a perfect market. The liquidity measure α can depend on time t and trading volume x. Due to the lack of empirical data we assume α to be time and volume independent. To measure liquidity for a certain market commonly the order book data is necessary. In general order books of the relevant markets are not published. For our purpose it is not essential to model the order book in detail, due to the fact that the estimation of the price demand function is sufficient. This estimation is based on bid-ask spreads S and trading volume x. We define S as follows:

$$S = p^{bid} - p^{ask}. \tag{2}$$

Figure 1 illustrates the interdependence between order book, bid-ask spread, trading volume and price demand function. Applying the parameters defined above, α is calculated as $\alpha = S/x$. A fast and simple way to evaluate α is taking the average bid-ask spreads and average trading volume to calculate:

$$\hat{\alpha} = \frac{\overline{S}}{2\overline{x}}. \tag{3}$$

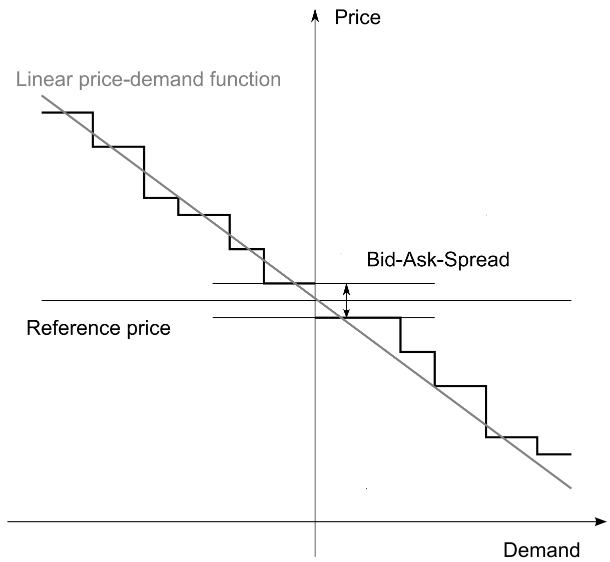

Figure 1: Exemplary order book and liquidity function

3 STORAGE VALUATION UNDER LIMITED MARKET LIQUIDITY

Subsequent to the introduction of possible liquidity measures in section 2 this section develops a new approach for the valuation of flexible assets in incomplete markets. Such assets, offering real options can be flexible e.g. power plants (Tseng and Barz (2002) or Thompson et al. (2004)) or natural gas storages (Thompson et al. (2003), Hodges (2004) or Boogert and de Jong (2008)). In evolving deregulated markets with a high degree of uncertainty, these assets offer the possibility to react flexible to changing market conditions. Nevertheless the degree of flexibility is limited by the liquidity of the considered market. Whereas recent research approaches, examining the optimal management of flexible assets in uncertain markets, postulate the asset owner as price taker in a perfect market, the proposed model copes with imperfect markets. Under the assumption, that the owner of the asset anticipates the imperfect market and takes the impact of his decision on the market (prices) into the account of his decision, the model determines the optimal value and operation of the asset simultaneously. In the following a model for the determination of the optimal operation of a natural gas storage facility is derived. This example is evaluated due to two reasons: At first, evolving (continental) European natural gas markets are still significant illiquid. Secondly, the option, offered by a natural gas storage, contains the managerial flexibility offered by other real options. In fact gas storage offers the possibility to buy or sell a good as well as to wait for changing markets conditions. Since these are the operational possibilities offered by most common operative real options the proposed example can easily be adjusted to other types of real options.

Based on Thompson et al. (2003) this section derives an optimization model for the optimal management of a gas storage facility under consideration of market liquidity constraints. As natural gas storages offer the possibility to buy and sell gas, this model can easily be adopted to assets which offer only one possible action. Beside spot market prices and market liquidity the technical constraints of a storage have a major impact on the value and operation of a storage. Therefore, in the following a brief introduction into the fundamental technical and economic storage characteristics is given.

3.1 Model Formulation

Underground storage sites can be separated into two main types: pore storages which can be aquifers, or former gas fields, and salt cavities. (IGU (2006), FERC (2004)) Whereas former gas fields offer high storage capacities, salt cavities commonly have lower capacities. Storage capacity can be divided into the amount of gas that delivers the essential technical pressure, the cushion or base gas, and the amount of gas that is utilized for the withdrawing and injection of gas, the working gas. For former gas fields the ratio of cushion gas can reach up to fifty percent of the total storage volume. Salt cavities, build by solution mining of salt domes, have lower cushion gas ratios, up to thirty-five percent of the total storage volume are used as cushion gas. (IEA (1994)) Beside the amount of gas, that can be stored, the flexibility of a storage facility is basically influenced by the maximum injection and withdrawing rates $c_{\min}$ and $c_{\max}$. Since salt cavities offer high withdrawal and injection rates, they provide higher flexibility and can be cycled on a higher frequency within a year than former gas fields. Withdrawal and injection rates are influenced by the pressure within the storage facility they depend on the storage level $I(t)$. Thus, lower inventory levels permit higher injection levels whereas higher stock levels result in larger withdrawing rates. Beside these technical constraints the storage value is also affected by the operating costs of a storage facility. As a main part of the variable costs is determined by the amount of gas that is lost during injection/ withdrawing the gas into- or out of the storage $a(I, c)$ operating costs are approximated by the lost rate of gas which is commonly taken as one percent of the withdrawn/injected gas. (Dietert and Pursell (2000)) This amount of gas is e.g. used to run the compressors that inject the gas into the storage or to fit the temperature and the pressure of the withdrawn gas to the conditions of the connected pipeline grid. These costs can be adjusted due to transaction costs for selling or buying the gas if the gas is sold at the spot market.

Beside the technical constraints of a storage facility the assumed price process of the underlying good, natural gas, has a major impact on the management and the value of the facility. Hence it is necessary to apply an appropriate price model. To cope with a broad class of underlying price processes the following jump diffusion process is proposed to describe the underlying price process:

$$dP = \mu(P, t)dt + \sigma(P, t)dX + \Phi dq. \tag{4}$$

Whereas the deterministic drift rate $\mu(P,t)$ can incorporate also a mean reversion rate. By dX the increment of a Brownian motion is denoted. Within the third term the increments of a Poisson process are defined as follows:

$$dq = \begin{cases} 1 & \text{with probability } \lambda(P,t)dt, \\ 0 & \text{with probability } (1-\lambda(P,t))dt. \end{cases} \tag{5}$$

Where λ denotes the average occurrence of a jump within a time step dt. This price process can easily be adopted to incorporate the characteristics of natural gas prices: mean reverting and jump components. Since major parts of European natural gas demand are used for domestic heating, a significant part of natural gas prices is driven by temperature and thus seasonal mean reverting. Additionally gas can be stored only at limited amount and residential consumers are almost price inelastic. Thus high demand crossing limited supply can trigger significant price jumps. On the other hand, the proposed price process can also easily be adjusted to other stochastic processes like a geometric Brownian motion.

The aim of the model is to calculate the value of the flexibility to buy or sell any amount $c(t,P,I,\alpha)$ of the underlying good within the given capacity constraints c_{min} and c_{max} under consideration of the given price P, the total gas in storage $I(t)$ and the given market impact of an action α. Taking an (non risk adjusted) interest rate of ρ into account and assuming a scrap value of zero at the end of the valuation period T, the objective function at the beginning of the valuation period can be written as follows:

$$\max_{c(t,P,I,\alpha)} E\left[\int_0^T e^{-\rho\tau}(c-a(I,c))(P-\alpha c)d\tau\right]. \tag{6}$$

Thus the value is affected by the time until the valuation period expires. In an illiquid market the operating strategy of a storage facility has an impact on the price, thus the objective function is calculated with respect to the liquidity function α. As α is assumed to be non-negative, buying an amount of the underlying good leads to increasing market prices, whereas selling the underlying results in lower prices. Therefore, the amount that is sold or bought is determined under consideration of the market impact of the selected action. To solve this scheduling problem the value at time step t is defined by:

$$V(t,P,I) = \max_c E\left[\int_t^T e^{-\rho(\tau-t)}(c-a(I,c))(P-\alpha c)d\tau\right]. \tag{7}$$

Deviding $[t,T]$ into $[t,t+dt]$ and $[t+dt,T]$, equation (7) can be rewritten as follows:

$$\begin{aligned} V(t,P,I) &= \max_c E\left[\int_t^{t+dt} ce^{-\rho(\tau-t)}(c-a(I,c))(P-\alpha c)d\tau \right.\\ &+ \left.\int_{t+dt}^T ce^{-\rho(\tau-t)}(c-a(I,c))(P-\alpha c)d\tau\right]. \end{aligned} \tag{8}$$

Using definition (7) the (basic) problem can be reformulated as dynamic programming problem:

$$\begin{aligned} V(t,P,I) &= \max_c E\left[\int_t^{t+dt} ce^{-\rho(\tau-t)}(c-a(I,c))(P-\alpha c)d\tau \right.\\ &+ \left. e^{-\rho dt}V(t+dt,P+dP,I+dI)\right]. \end{aligned} \tag{9}$$

For a sufficient small time increment dt, the first part of this equation can be understood as the immediate cash flow resulting from a decision at time step t. Whereas the second term represents the expected discounted future value after the decision at time step t. Applying a Taylor's Series expandation and $It\hat{o}s$ Lemma (Oksendal and Sulem

(2007)) to equation (9) we get:

$$\begin{aligned} V &= \max_c E\left[(c-a(I,c))(P-\alpha c)dt + (1-\rho dt)V\right. \\ &+ (1-\rho dt)\left(V_t + \frac{1}{2}\sigma^2 V_{PP} + V_P\mu - (c+a(I,c))V_I\right)dt \\ &+ \left.(1-\rho dt)\left(\sigma dX + (V^+ - V)dq\right)\right]. \end{aligned} \tag{10}$$

Whereas $V^+ = V(t, P+\Phi, I)$ represent the value of a storage if the price has jumped by a amount of ϕ. Applying *Itô* calculus, taking expectations and dividing through dt results in:

$$\begin{aligned} 0 &= \max_c \left[\frac{1}{2}\sigma^2 V_{PP} + V_t + V_P\mu - (c+a(I,c))V_I\right. \\ &+ \left.(c-a(I,c))(P-\alpha c) + \lambda E[(V^+ - V)] - \rho V\right]. \end{aligned} \tag{11}$$

For an optimization with respect to c it is sufficient to focus on the terms in (11) including c:

$$\max_{c_{\min}\leq c\leq c_{\max}} \left[-(c+a(I,c))V_I + (c-a(I,c))(P-\alpha c)\right]. \tag{12}$$

Simplifying this equation results in:

$$\max_{c_{\min}\leq c\leq c_{\max}} \left[-\alpha c^2 + (P + \alpha a(I,c) - V_I)c - a(I,c)(V_I + P)\right]. \tag{13}$$

To find the optimal control c_{opt}, which maximizes equation (13) we apply a first order condition on the first derivative of equation (13) with respect to c:

$$-2\alpha c + P + \alpha a(I,c) - V_I \stackrel{!}{=} 0. \tag{14}$$

Hence,the optimal solution $\tilde{c}_{opt}$ results as:

$$\tilde{c}_{opt} = \frac{V_I - P - \alpha a(I,c)}{-2\alpha}. \tag{15}$$

As the management of the storage is restricted by withdrawal and injection constraints we adjust $\tilde{c}_{opt}$ to obtain the optimal storage strategy with respect to this restrictions:

$$c_{opt} = \begin{cases} \max\left(\tilde{c}_{opt}, c_{\min}\right) & \text{for } \tilde{c}_{opt} < 0, \\ \min\left(\tilde{c}_{opt}, c_{\max}\right) & \text{for } \tilde{c}_{opt} > 0. \end{cases} \tag{16}$$

As for positive α the second derivative of equation (13) with respect to c is always negative, c_{opt} can be stated as a global maximum for given V_I and P. To solve the differential equation (11), border and terminal conditions are necessary. Generally this edge conditions do not differ from those in Thompson et al. (2003), anyway we state this edge conditions as they are elementary for the solution of the differential equation. The first condition can be derived regarding equation (6) where a scrap value of zero is assumed. Hence the following terminal condition can be stated:

$$V(T,P,I,c) = 0. \tag{17}$$

Afterwards the first derivatives with respect to I are considered for a full and an empty storage. For a full storage it is not possible to inject more gas, hence the incremental value considering I equals zero. Defining V_I^+ as the right hand side derivative this condition turns into:

$$V_I^+(t,P,I_{\max}) = 0. \tag{18}$$

Analogous it can be stated that $V_I^-(t,P,0) = 0$. Finally the following condition must be satisfied for the second derivative with respect to P:

$$V_{PP} \longrightarrow 0 \text{ For } P \text{ large} \tag{19}$$

$$V_{PP} \longrightarrow 0 \text{ as } P \longrightarrow 0. \tag{20}$$

Applying c_{opt} to equation (11), this differential equation is hyperbolic in I. This can cause spurious oscillations. (cf. Thompson et al. (2003)) Thus we adopt the numerical handling of this problem of Thompson et al. (2003) and apply the slope delimiter described in LeVeque (1999).

3.2 Real Options Valuation and Liquidity

The main difference of including limited liquidity into the option valuation is that it leads to a quadratic optimization problem. For a price taker, and therefore neglecting limited liquidity, the option value is a linear function of volume and the price is the slope of this function. (cf. Thompson et al. (2003)). In this case the goal is to maximize a linear objective function (OF). Figure 2 shows this case. The optimal solution depends on the slope of OF. A positive slope in addition to a positive corresponding value of OF let the right boundary being optimal (cf. OF_1, OF_2). In the case that the slope is positive but the value of the OF is negative, the optimal solution will be "doing nothing", what is equal to zero (cf. OF_3, OF_4). The same result appears when the slope of the OF is negative but the value of the OF is also negative (cf. OF_6). The case of a negative slope and the value of the OF being positive leads to the left boundary as optimal solution. Overall there are only three possible solutions. These are taking the left boundary, taking the right boundary or doing nothing. In our approach, including liquidity, the goal is to maximize a quadratic OF. Due to the fact that the price is a

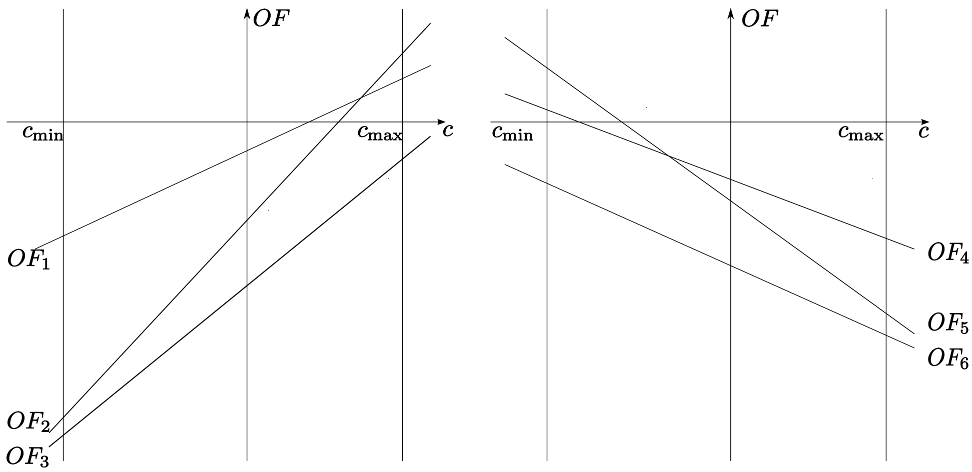

Figure 2: Linear optimization function

linear function of volume and the option value is the product of price and volume, the OF is a quadratic function. Figure 3 shows this case. The grey lines represent different OF depending on the parameter choice for the OF. It is obvious that the set of all parting points is also a quadratic function (dotted black line). The optimal solution of our problem depends on the position of the corresponding parting point. If the parting point is located in the third or fourth quadrant, the optimal solution is "doing nothing" because it is better to have zero than a negative value as objective (cf. OF_3 and OF_4). If the parting point is between the left and the right boundary, the optimal solution is the corresponding value of the parting point (cf. OF_2 and OF_5). If the parting point goes beyond the boundaries, the optimal solution is the corresponding boundary (cf. OF_1 and OF_6). Thus in our approach every value between the left and the right boundary can arise as optimal solution. Figure 4 illustrates the impact of the liquidity to the optimal control strategy. The fact that positive trading volumes have a decreasing effect on the prices, leads to an increasing function of the volume with respect to an increasing price. The slope of this function depends on the liquidity of the markets. The higher the liquidity, the higher is this slope. This is consistent to the

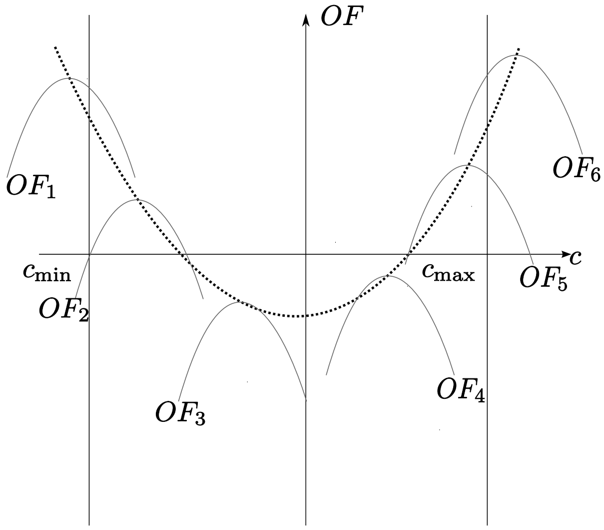

Figure 3: Quadratic optimization function

linear case, where perfect liquidity is assumed and therefore the slope of this function is infinity.

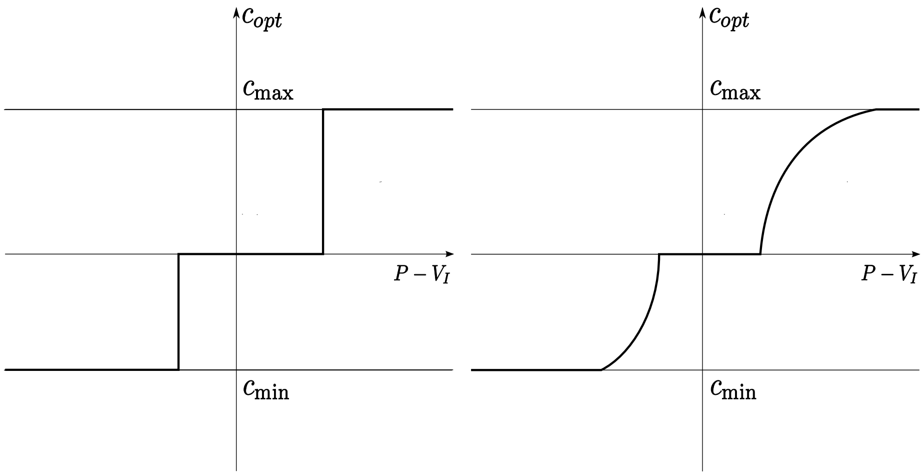

Figure 4: Comparison of the resulting storage strategies

4 APPLICATION

In this section the above derived methodology is applied to calculate the optimal schedule and the corresponding revenue of the existing gas storage Epe, owned by E.ON Ruhrgas and located in Germany. This storage offers the possibility to release and to sell gas at the spot market or to buy at the spot market and inject this gas into the storage. This flexibility is evaluated applying the above derived methodology to the exemplary storage Epe in a market with limited liquidity. Since Epe is located at the border to the Netherlands, data from the Dutch gas exchange, the Title Transfer Facility (TTF) is taken to estimate the liquidity measure and the relevant price process parameters.

To illustrate the impact of limited liquidity to the storage valuation, we compute the optimal storage schedule and storage value for different market liquidity levels. This analysis can give a hint, how storage operators have to adjust their management decisions due to a changing market liquidity measured by α.

4.1 Storage and market data

Employing a conversion factor of 113 m^3 for 1 MWh and considering the storage capacity data offered by IGU (2006), Epe offers a withdrawal rate of 451 GW per day and an injection rate of 109 GW per day. Considering the working gas capacity of 13850 GWh it is possible to refill an empty storage within 126 days and to release a full storage in 30 days. Thus the storage can be cycled more than twice in one year. Beside the working gas the storage must hold a certain amount of cushion gas to keep the essential pressure for the operation of the storage. For Epe this amount is 5585 GWh or 29 percent of the total storage capacity including cushion and working gas.

To cope with volume dependent withdrawal and injection rates, the approach of Thompson et al. (2003), applying the ideal gas law and Bernoulli´s equation, is adopted to our specific storage example. We choose this approach due to the comparability to the model of Thompson et al. (2003). Alternative less general but more detailed data, available at the storage operator (E.ON (2007)) can be applied to the methodology to cope with volume dependent injection and withdrawal rates. The operating costs of the storage are approximated by a linear cost function. This function includes transaction costs, losses for injection and withdrawing and other operating costs.

To compute the liquidity measure α for the regarded gas market, we take day ahead product data from TTF. This data is published on a daily basis and prepares the necessary information for measuring α. The available period ranges from 19th february 2007 to 5th february 2009. Applying an average Bid-Ask spread $\overline{S}$ of 1.06 $[EUR/MWh]$ and an average trading volume of $\overline{x} = 3477.32$ $[MW]$ into equation (3) results in $\alpha = 0.00015$ $\left[EUR/(MW)^2 h\right]$. In addition to the storage capacity data and the compu-

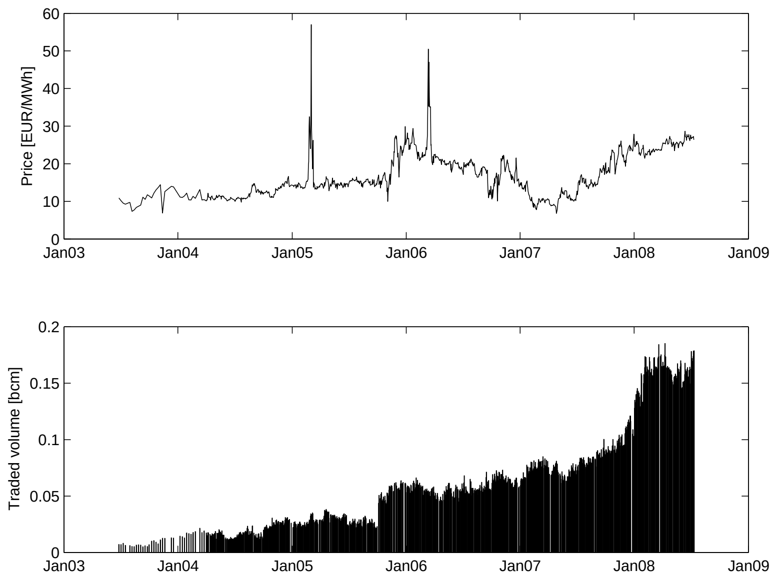

Figure 5: Prices and volumes at TTF

tation of α it is necessary to compute the relevant parameters of the underlying price process. To incorporate the main characteristics of natural gas prices: seasonality and jump components (cf. Figure 5), a mean reversion jump diffusion process is assumed as underlying price process. The parameter estimation is done applying standard procedures described in Dixit and Pindyck (1994) and Weron (2006) to the historical spot price index of the TTF in between 2004/04/01 and 2007/12/31. The parameters applied in the calculations below are summarized in Table I:

Parameter	Value	Description
ρ	0.1	interest rate
α	0.00015	liquidity measure
κ	0.005	mean reversion rate
θ	18	mean reversion level
σ	0.68	volatility
ϕ	$\mathcal{N}(0, 54.8)$	jump size distribution
λ	0.07	jump probability

Table I: Applied Parameters

4.2 Results

Implementing a simple difference scheme we solve this stochastic differential equation backwards starting at the last valuation day of 262 trading days. Computing the value of the storage it is necessary to choose the disretization sensity of dt and dP with respect to each other to derive numerically stability. (Benker (2005)) Time is measured in days. To guarantee numerical stability we applied a time step size of $dt = 1/8$ and a price step size of $dP = 0.5$. As described in Benker (2005) the following condition for numerical stability of the solution then holds:

$$dt \leq \frac{dP^2}{2}. \tag{21}$$

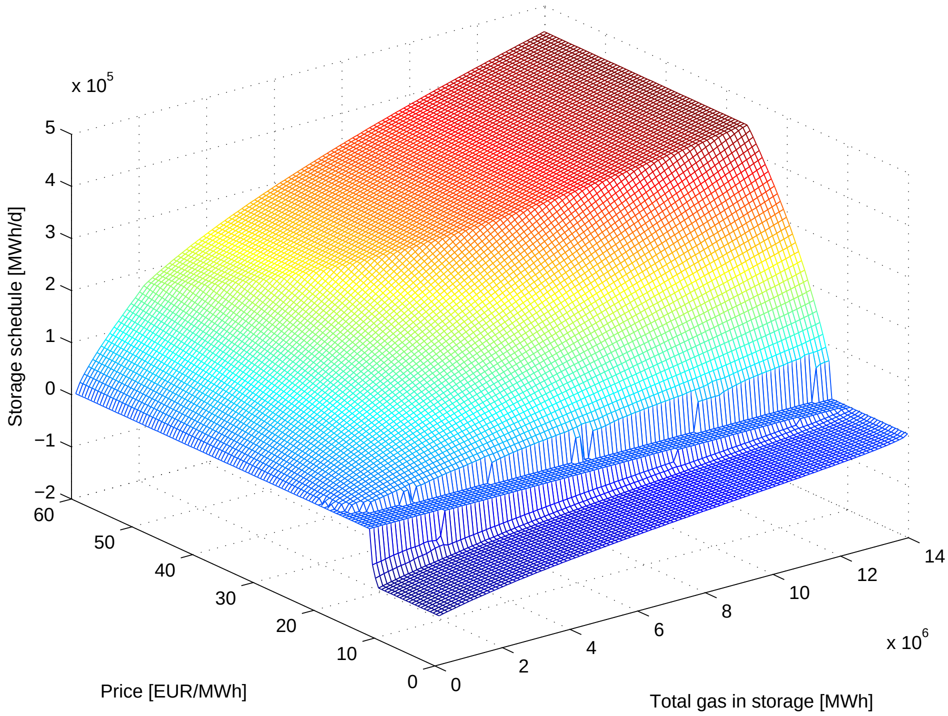

Figure 6: Storage schedule in t_0

Figure 6 depicts the resulting storage strategy at the beginning of the valuation period. There are three regions visible: injection (negative strategy), "doing nothing" and withdrawal (positive strategy). Within the injection and withdrawal regions it is apparent that the storage operator uses the total injection/ withdrawal capacity only for very low/ high prices. The operating strategies above/below these prices are adjusted due to the price impact caused by the limited market liquidity. In contrast to this operating strategy, the storage strategy in t_0 with perfect liquidity leads to decisions of the bang-bang type: For given thresholds P_{out} and P_{in} the strategy for all prices above P_{out} is to withdraw as much as possible. For all prices below P_{in} the strategy is to inject as much as possible. Figure 7 compares the value of the storage at the beginning of the valuation period with and without limited liquidity for different volume levels. As Figure 7 depicts, an empty storage at the beginning of the valuation

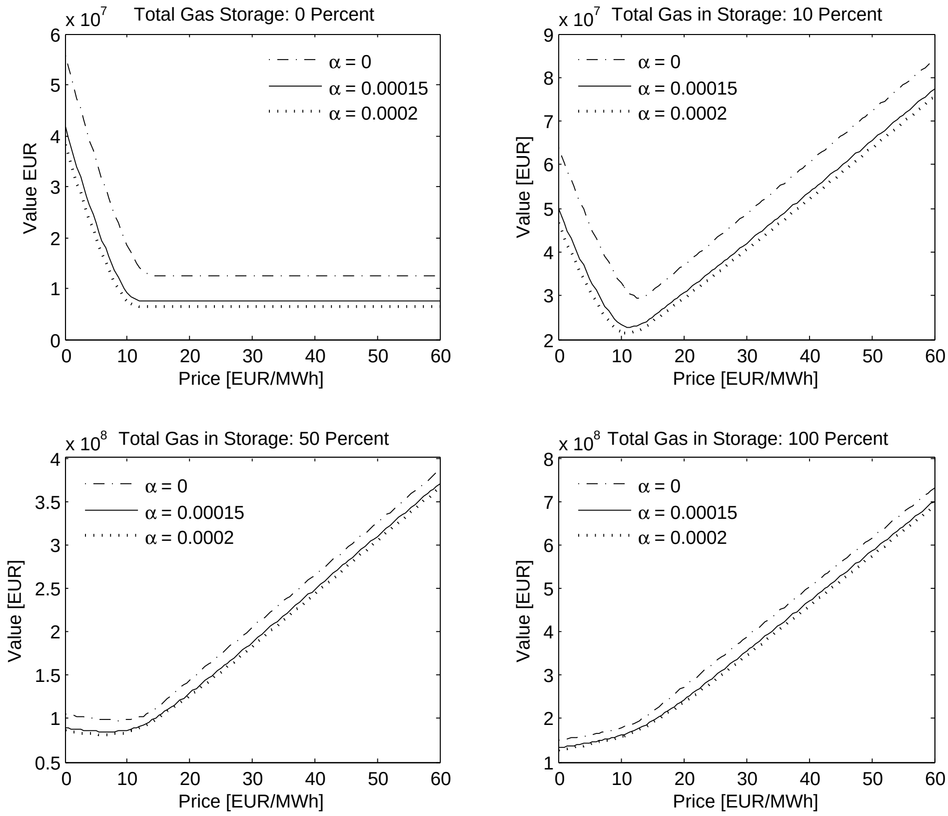

Figure 7: Value comparision for different Total Gas in Storage levels with and without liquidity.

period t_0 can be seen as a put option on the storage capacity. The storage operator has the opportunity, to buy one unit of gas and inject this unit into the storage. As the storage operator injects one unit of gas into the storage, he looses one unit of "flexibility" delivered by the storage. Hence by buying and injecting one unit of gas, the storage operator sells one unit of storage flexibility. On the other hand he obtains the option to sell this gas, when prices are high enough. Hence the value of the above mentioned put option grows with decreasing gas prices. Figure 7 illustrates further, that in an illiquid market the strike price of this put option is lower than in a market with high liquidity. In an illiquid market buying one unit of gas increases the market price. Thus, the storage operator delays his decision to inject to lower market prices.

As the storage level increases the above mentioned put option turns into a straddle. A filled storage can be construed as a call option. In an illiquid market every action of the storage operator affects the market price. This leads to lower values than in markets with perfect liquidity, for all possible volume levels. Thus, postulating a perfect market and assuming the storage operator as price taker can lead to an overestimation of the storage value.

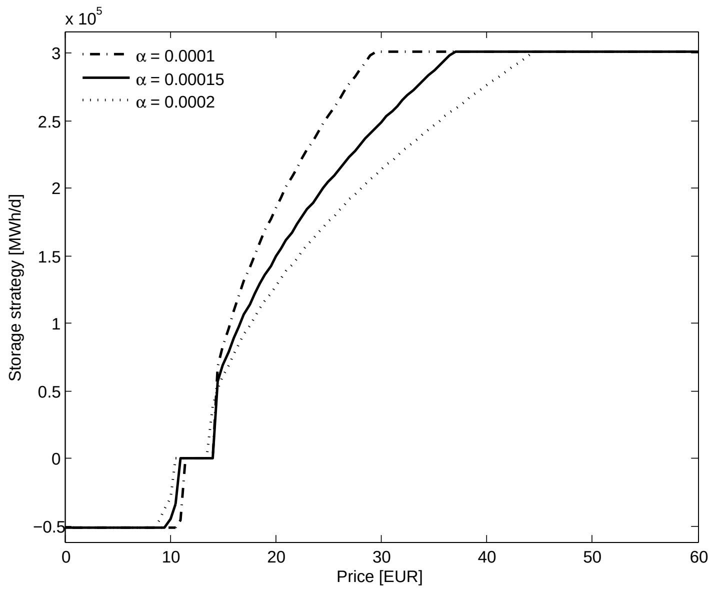

Figure 8: Strategy sensity with respect to α

Figure 8 depicts the liquidity impact on the storage strategy. It can be seen, that an increasing liquidity (measured by α) leads to significant higher withdrawal and injection volumes. In fact enhancing liquidity by fourty percent from $\alpha = 0.0002$ to $\alpha = 0.00015$ expands the withdrawal rate by more than nine percent on average. Furthermore injection rates increase by almost five percent on average. In addition, higher illiquidity enlarges the price interval where the storage operator whether withdraws nor injects gas into the storage. Thus, as decreasing liquidity reduces the return of withdrawing gas and enhances the cost of an injection the storage operator postpones injection and withdrawal decisions and waits for higher (lower) prices for an withdrawal (injection) of gas.

5 CONCLUSION

This paper has derived a new methodology to valuate the flexibility of natural gas storages in illiquid markets. This approach can be easily adopted to other assets offering flexibility. We have shown that the existence of an illiquid market decreases the storage value. Additionally the storage strategy has to be adjusted in illiquid markets. We find that an increase in illiquidity turns to a broader price interval where the storage operator whether injects not withdraws. Further all operating strategies are lowered in a market with higher illiquidity. Future research can evaluate different price processes or solve the stochastic differential equation with a different numerical method, e.g. an implicit finite difference scheme. The focus can also lie on the measurement of the liquidity risk resulting from an illiquid market. Further, under certain restrictions, the analytical solution of this problem can be of interest. Finally the problem can be evaluated considering a time and volume dependend liquidity measure.

References

Amihud, Y., Mendelson, H., 1986. Asset pricing and the bid-ask spread. Journal of Financial Economics 17 (2), 223–249.

Benker, H., 2005. Differentialgleichungen mit MATHCAD und MATLAB. Springer-Verlag Berlin Heidelberg, Berlin, Heidelberg.
URL http://dx.doi.org/10.1007/b139081

Boogert, A., de Jong, C., 2008. Gas storage valuation using a monte carlo method. The journal of derivatives 15 (3), 81–98.

Boyle, G. W., Guthrie, G. A., Oct 2003. Investment, uncertainty, and liquidity. Journal of Finance 58 (5), 2143–2166.

Brennan, M. J., Subrahmanyam, A., Jul 1996. Market microstructure and asset pricing: on the compensation for illiquidity in stock returns. Journal of financial economics 41 (3), 441–464.

De Jong, C., Walet, K., 2005. Gas storage management. In: Kaminski, V. (Ed.), Managing Energy Price Risk, 3rd Edition. Risk Books, London, pp. 631–648.

Dietert, J., Pursell, D., 2000. Underground natural gas storage.
URL http://www.worldoil.com/WO_RESEARCH/Research/062800storage.pdf

Dixit, A. K., Pindyck, R. S., 1994. Investment under uncertainty. Princeton University Press, Princeton, N.J.

E.ON, 2007. E.ON Ruhrgas AG. Wesentliche Inhalte eines Speichervertrages.
URL http://www.eon-ruhrgas.com/cps/rde/xchg/SID-3F57EEF5-C59BF9E8/er-corporate/hs.xsl/1584.htm

FERC, 2004. Federal Energy Regulatory Commission. Current state of issues concerning underground natural gas storage.
URL http://www.ferc.gov/eventcalendar/files/20041020081349-final-gs-report.pdf

Geman, H., 2007. Commodities and commodity derivatives: modeling and pricing for agriculturals, metals and energy, repr. Edition. Wiley finance series. Wiley & Sons, Chichester [u.a.].

Ghysels, E., Pereira, J. P., Sep 2008. Liquidity and conditional portfolio choice: A nonparametric investigation. Journal of Empirical Finance 15 (4), 679–699.

Hahn, W., Dyer, J., 2008. Discrete time modeling of mean-reverting stochastic processes for real option valuation. European Journal of Operational Research 184 (2), 534–548.

Hodges, S. D., 2004. The value of a storage facility, working paper. Warwick Business School.

Holland, A., 2007. Optimization of injection/withdrawal schedules for natural gas storage facilities. In: Twenty-seventh SGAI International Conference on Artificial Intelligence (AI-2007). Cambridge, England.
URL http://4c.ucc.ie/~aholland/publications/GasStorage.pdf

IEA, 1994. International Energy Agency. Natural gas transportation: organisation and regulation. Paris.

IGU, 2006. International Gas Union. UGS Data Bank.
URL http://www.igu.org/html/wgc2006/WOC2database/index.htm

Kempf, A., 1999. Wertpapierliquidität und Wertpapierpreise. Beiträge zur betriebswirtschaftlichen Forschung. Dt. Univ.-Verl. [u.a.], Wiesbaden.

Keynes, J. M., 1930. A treatise on money: Vol. 2, The applied theory of money.

LeVeque, R. J., 1999. Numerical methods for conservation laws, 2nd Edition. Lectures in mathematics. Birkhäuser, Basel [u.a.].

Longstaff, F., Schwartz, E., Sep 2001. Valuing american options by simulation: A simple least-squares approach. Review Of Financial Studies 14 (1), 113–147.

Ludkovski, M., Carmona, R., 2007. Valuation of energy storage: An optimal switching approach.
URL http://www.pstat.ucsb.edu/faculty/ludkovski/storage-CL.pdf

Maragos, S., 2002. Valuation of the operational flexibility of natural gas storage reservoirs. In: Ronn, E. I. (Ed.), Real options and energy management. Risk Books, London, pp. 431–456.

Oksendal, B. K., Sulem, A., 2007. Applied stochastic control of jump diffusions, 2nd Edition.

Thompson, M., Davison, M., Rasmussen, H., 2003. Natural gas storage valuation and optimization: A real options application.
URL http://www.apmaths.uwo.ca/~mdavison/_library/preprints/Gasstorage.pdf

Thompson, M., Davison, M., Rasmussen, H., 2004. Valuation and optimal operation of electric power plants in competitive markets. Operations Research 52 (4), 546.

Tseng, C. L., Barz, G., 2002. Short-term generation asset valuation: A real options approach. Operations Research 50 (2), 297–310.

Vayanos, D., Spr 1998. Transaction costs and asset prices: A dynamic equilibrium model. Review Of Financial Studies 11 (1), 1–58.

Weron, R., 2006. Modeling and Forecasting Electricity Loads and Prices: A Statistical Approach.