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Optimal investment timing and location under uncertainty

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Abstract We consider an investment problem such as a construction of power plants. The uncertainties, which are considered in this investment problem, are the evolution of cash flows obtained from the plant operation, and the catastrophic event, which drives the value of the project to zero due to external factors, such as earthquake. We show the model of a sequential investment as well as that of a single investment, and show the effect of the catastrophic event on the flexibility of the sequential decision by comparing the option values of the single investment and the sequential one. Additionally, in a case where both costs associated with the construction and the catastrophic event are dependent on the location, we determine simultaneously the optimal investment timing and location of the plant.

Keywords Sequential investment · Investment timing · Catastrophic event · Location of plants · Real options

1 Introduction

The option valuation theory of investments as real options has gained much attention in the past few decades. Real options theory, pioneered by Brennan and Schwartz (1985), and McDonald and Siegel (1986), and summarized in Dixit and Pindyck (1994), and Trigeorgis (1996), is a useful methodology for the economic evaluation and analysis of various investment projects under uncertainty. The timing of decision-making in various investment problems and the investment value can be shown by using real

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options theory. Especially, real options theory is useful in evaluating the investment of multi-stage sequential investment.

Many researchers have studied the analysis of sequential investments as natural resource (Brennan and Schwartz, 1985; Cortazar et al., 2001), R&D (Schwartz and Moon, 2000), information (Schwartz and Zozaya-Gorostiza, 2003), and power plants (Gollier et al., 2005; Siddiqui and Maribu, 2009). Real options theory enables us to show the value of flexibility such as multi-stage sequential investments. Particularly, in the study of the power plants investment such as Gollier et al. (2005), the sequential investment of modularity is investigated, and the value of the sequential investment is shown. Although it is found that there exists the flexibility value in the sequential decision, when a catastrophic event occurs as in Schwartz and Moon (2000) and Schwartz and Zozaya-Gorostiza (2003), the event affects the investment decision.

For example, in Japan, the Niigata-Chuetsu-Oki earthquake occurred in Niigata Prefecture on July 16th 2007. The earthquake led to automatic scram of units 2, 3, 4 and 7 of Kashiwazaki-Kariwa Nuclear Power Station, and currently, all seven units are shut down. The power plants have been constructed sequentially with the increase in the demand regarding the value of the flexibility. However, all plants were shut down because of the occurrence of the earthquake. This seems to be the loss in the value of flexibility. When there is a catastrophic risk as a earthquake, it is necessary to consider the location of the plant.

In this paper, we analyze the two-stage sequential investment taking into account the catastrophic risk. Especially, the dependence of the investment value on the catastrophic risk is shown. Additionally, we propose a model that enable us to determine the timing of the investment and the plant location simultaneously.

The remainder of this paper is organized as follows. Section 2 describes the model for the single investment and the two-stage sequential investment problems. In Section 3, we presents the model for the sequential investment with a catastrophic risk, and analyze the effect of the catastrophic risk on the investment problem. Section 4 provides the model for determining the investment timing and the location of the plant. Finally, Section 5 concludes the paper.

2 The model

In this section, we consider two investment problems such as a single investment project and a sequential investment one. The flexibility value of the sequential investment is shown by comparing each value of project.

2.1 Single investment

We begin by describing the basic investment timing model which is based on McDonald and Siegel (1986). Consider a firm that starts operating a plant of the capacity Q by incurring the investment cost I. After the investment decisions, the cash flow X_t per unit is generated. The evolution of the cash flow follows a geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \qquad X_0 = x, \tag{1}$$

where μ is the instantaneous expected growth rate of P_t, σ is the associated volatility, and W_t is a standard Brownian motion. The investment problems for the firm maximize the expected discounted value by selecting the investment time τ. Thus, the value function of the investment is represented by the following equation,

$$F_0(x) = \sup_{\tau \in \mathcal{T}} \mathbb{E}\left[\int_\tau^\infty e^{-\rho t} Q X_t dt - e^{-r\tau} I\right], \tag{2}$$

where $\mathcal{T}$ is the set of admissible stopping time, and ρ is the discount rate. Given the constant threshold of the investment x^*, the optimal investment time τ^* has the following form:

$$\tau^* = \inf\left\{t \geq 0 \mid X_t \geq x^*\right\}. \tag{3}$$

The following differential equation, which is satisfied by the investment value, is derived from Bellman equation (See, for example, Dixit and Pindyck (1994)),

$$\frac{1}{2}\sigma^2 x^2 F_0''(x) + \mu x F_0'(x) - \rho F_0(x) = 0 \tag{4}$$

The general solutions of this equation is given by the following equations,

$$F_0(x) = a_1 x^{\beta_1} + a_2 x^{\beta_2} \tag{5}$$

where a_1 and a_2 are unknown constants, and β_1 and β_2 are the positive and the negative roots of the characteristic equation $\frac{1}{2}\beta(\beta-1) + \mu\beta - \rho = 0$, respectively. The investment value must satisfy the following boundary conditions,

$$F_0(0) = 0, \tag{6}$$

$$F_0(x^*) = \frac{Qx^*}{\rho - \mu} - I, \tag{7}$$

$$F_0'(x^*) = \frac{Q}{\rho - \mu}. \tag{8}$$

Condition (6) requires that the investment option becomes zero if the cash flow is close to zero, thus, from this condition, $a_2 = 0$. Conditions (7) and (8) are the value-matching and smooth-pasting conditions, respectively. From these conditions, we can obtain the threshold value x^* and the unknown constant a_1:

$$x^* = \frac{\beta_1}{\beta_1 - 1}\frac{\rho - \mu}{Q} I, \tag{9}$$

$$a_1 = \frac{I}{\beta_1 - 1}\left[\frac{\beta_1 - 1}{\beta_1}\frac{Q}{\rho - \mu}\frac{1}{I}\right]^{\beta_1} \tag{10}$$

2.2 Sequential investment

In this section, we consider two-stage sequential investments as in Dixit and Pindyck (1994). Suppose that the total values for the investment cost of first stage, I_1 and of second stage, I_2 are equal to the investment cost of the single project, I, and likewise, for the capacity of the plant, $Q_1 + Q_2 = Q$. As in the previous section, the optimal investment rule and the investment value in the sequential investment are calculated. The firm's problem is to maximize the expected discounted value by selecting the first investment time τ_1 and the second investment time τ_2. Therefore, the value function of the two-stage sequential investment is given by the following equation.

$$F_1(x) = \sup_{\tau_1,\tau_2 \in \mathcal{T}} \mathbb{E}\left[\int_{\tau_1}^{\tau_2} e^{-\rho t} Q_1 X_t dt - e^{-\rho\tau_1} I_1 + \int_{\tau_2}^{\infty} e^{-\rho t}(Q_1+Q_2) X_t dt - e^{-\rho\tau_2} I_2,\right]. \tag{11}$$

Given the constant thresholds of first and second investments, x_1 and x_2, each optimal investment time has the following form:

$$\tau_1^* = \inf\{t \geq 0 \mid X_t \geq x_1\}, \tag{12}$$
$$\tau_2^* = \inf\{t \geq 0 \mid X_t \geq x_2\}. \tag{13}$$

We can solve the investment problem by working backwards, first finding the value of the second investment, and finally finding the value of the first investment. The differential equation, which is satisfied by the second investment value, is given by the following equation,

$$\frac{1}{2}\sigma^2 x^2 F_{12}''(x) + \mu x F_{12}'(x) - \rho F_{12}(x) + Q_1 x = 0 \tag{14}$$

The general solutions of this equation is given by the following equation,

$$F_{12}(x) = a_3 x^{\beta_1} + \frac{Q_1 x}{\rho - \mu} \tag{15}$$

where a_3 is unknown constant. For the threshold value of the second investment x_{12}, the value of the second investment must satisfy the following the value-matching and smooth-pasting conditions,

$$\begin{cases} F_{12}(x_{12}) = \frac{(Q_1+Q_2)x_{12}}{\rho-\mu} - I_2 \\ F_{12}'(x_{12}) = \frac{Q_1+Q_2}{\rho-\mu} \end{cases} \tag{16}$$

From these conditions, we can obtain the threshold value of the second investment x_{12} and the unknown constant a_3 as follows,

$$x_{12} = \frac{\beta_1}{\beta_1 - 1}\frac{\rho-\mu}{Q_2} I_2, \tag{17}$$
$$a_3 = \frac{I_2}{\beta_1 - 1}\left[\frac{\beta_1 - 1}{\beta_1}\frac{Q_2}{\rho-\mu}\frac{1}{I_2}\right]^{\beta_1}. \tag{18}$$

Likewise, the threshold value of the first investment and the value of the two-stage sequential investment are calculated. Using standard method as above, the value of the first investment is given by the following equation,

$$F_{11}(x) = a_4 x^{\beta_1} \tag{19}$$

where a_4 is unknown constant. For the threshold value of the first investment x_{11}, the value of the first investment must satisfy the following the value-matching and smooth-pasting conditions,

$$\begin{cases} F_{11}(x_{11}) = a_3 {x_{11}}^{\beta_1} + \frac{Q_1 x_{11}}{\rho-\mu} - I_1 \\ F_{11}'(x_{11}) = \beta_1 a_3 {x_{11}}^{\beta_1 - 1} + \frac{Q_1}{\rho-\mu} \end{cases} \tag{20}$$

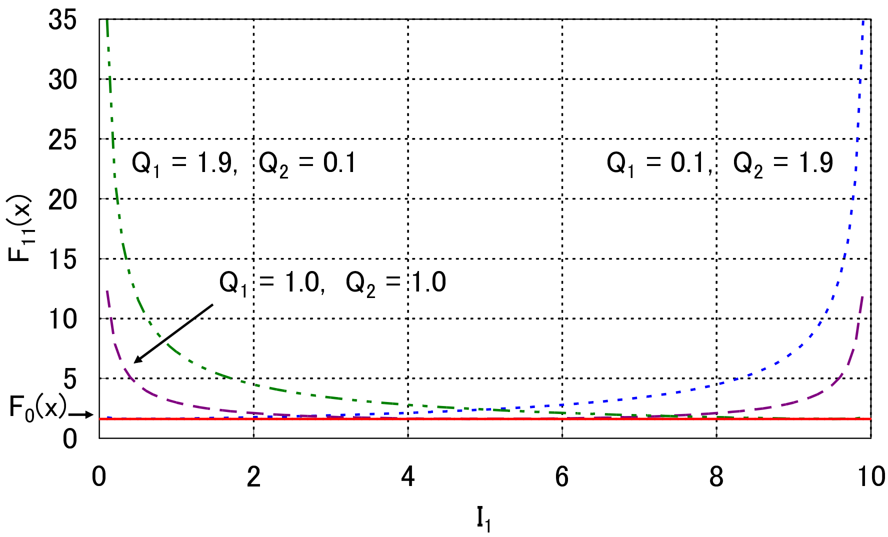

Fig. 1 Value of the flexibility for the sequential investment. Each dashed line represents the value of the sequential investment, and the solid line shows the value of the single investment.

From these conditions, the threshold value of the first investment x_{11} and the unknown constant a_4 can be obtained as follows,

$$x_{11} = \frac{\beta_1}{\beta_1 - 1} \frac{\rho - \mu}{Q_1} I_1, \tag{21}$$

$$a_4 = a_3 + \frac{I_1}{\beta_1 - 1} \left[\frac{\beta_1 - 1}{\beta_1} \frac{Q_1}{\rho - \mu} \frac{1}{I_1} \right]^{\beta_1}. \tag{22}$$

As can be seen from equations (17) and (21), the sequential exercising of these investment options must be ensured by the following condition:

$$\frac{I_1}{Q_1} < \frac{I_2}{Q_2}. \tag{23}$$

In this paper, the parameters which satisfy the above condition are used.

2.3 Numerical analysis

Using two models of the single and the sequential investments presented above, the value of the flexibility regarding the sequential investment is shown by comparing each investment value.

The threshold values of the single and the two-stage sequential investments are given by equations (9) and (21), respectively. As shown in these equations, the threshold value is dependent on the investment cost per unit capacity $\frac{I}{Q}$. Thus, the sequential project exercises the investment option earlier than the the single project when $\frac{I}{Q} > \frac{I_1}{Q_1}$, and vice versa.

The base case parameters used in this analysis are as follows: $\mu = 0.01$, $\sigma = 0.2$, $\rho = 0.04$, $I_1 = 4.0$, $I_2 = 6.0$, $Q_1 = 1.0$, $Q_2 = 1.0$ (i.e., $I = 10.0$, $Q = 2.0$), and $x = 0.1$. For these base case parameters, the threshold value of the single and the sequential projects are $x^* = 0.3686$, $x_{11} = 0.2949$, $x_{12} = 0.4423$, respectively, and the investment values of the single and the sequential projects are $F_0(x) = 1.6154$, $F_{11}(x) = 1.6540$, respectively. The investment values as a function of the first investment cost I_1 in the sequential project are shown in figure 1. Each dashed line represents the value of the sequential investment for $Q_1(Q_2) = 0.1(1.9), 1.0(1.0), 1.9(0.1)$ [1], and the solid line shows the value of the single investment. As can be seen from this figure, for each case, the value of the sequential project is larger than that of the single project. Therefore, it turns out that there exists the flexibility value of the sequential investment decision.

3 Investment with catastrophic risk

In previous section, the flexibility value of the sequential investment is demonstrated. In this section, we consider two-stage sequential investments with a catastrophic risk. The catastrophic risk implies that the occurrence of the event suddenly drives the value of the project to zero, and the life time of the project is a random variable and follows a Poisson process with intensity λ. Therefore, there is probability λdt that it will be permanently abandoned during the next short interval of time dt. By this setting, we present the dependence of the optimal investment rule on catastrophic risk, and shown the effect of the catastrophic risk on the flexibility value.

The project value with a catastrophic risk after the investment decision is given by the following equation,

$$\begin{aligned}
\mathbb{E}\left[\int_0^\infty \int_0^t e^{-\rho s} \lambda e^{-\lambda t} Q X_s \, ds \, dt\right] &= Q \int_0^\infty \lambda e^{-\lambda t} \int_0^t e^{-\rho s} x e^{\mu s} \, ds \, dt \\
&= Qx \int_0^\infty \lambda e^{-\lambda t} \frac{1 - e^{-(\rho-\mu)t}}{\rho - \mu} dt \\
&= \frac{Qx}{\rho + \lambda - \mu}.
\end{aligned} \tag{24}$$

Similarly, using a standard method in the literature as Dixit and Pindyck (1994), we can drive the following differential equations that must be satisfied by the value of the investment option with catastrophic risk for the first and second investments,

$$\frac{1}{2}\sigma^2 x^2 F_{21}''(x) + \mu x F_{21}'(x) - (\rho + \lambda) F_{21}(x) = 0, \tag{25}$$

$$\frac{1}{2}\sigma^2 x^2 F_{22}''(x) + \mu x F_{22}'(x) - (\rho + \lambda) F_{22}(x) + Q_1 x = 0. \tag{26}$$

The general solutions of these equations are given by the following equation,

$$F_{21}(x) = a_5 x^{\beta_{11}}, \tag{27}$$

$$F_{22}(x) = a_6 x^{\beta_{11}} + \frac{Q_1 x}{\rho + \lambda - \mu}, \tag{28}$$

[1] In actual case, the investment cost seems to be dependent on the capacity of the plant. In this analysis, however, various parameter values for the capacity are used in order to show the flexibility value.

where a_5 and a_6 are unknown constants, and

$$\beta_{11} = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}}.$$

Likewise, we can solve the investment problem by working backwards. For the threshold values of each stage x_{21} and x_{22}, the investment values must satisfy the following the value-matching and smooth-pasting conditions,

$$\begin{cases} F_{22}(x_{22}) = \frac{(Q_1+Q_2)x_{22}}{\rho+\lambda-\mu} - I_2, \\ F_{22}'(x_{22}) = \frac{Q_1+Q_2}{\rho+\lambda-\mu}, \end{cases} \tag{29}$$

$$\begin{cases} F_{21}(x_{21}) = a_6 {x_{21}}^{\beta_{11}} + \frac{Q_1 x_{21}}{\rho+\lambda-\mu} - I_1, \\ F_{21}'(x_{21}) = \beta_{11} a_6 {x_{21}}^{\beta_{11}-1} + \frac{Q_1}{\rho+\lambda-\mu}. \end{cases} \tag{30}$$

From these conditions, we can obtain the threshold values in each stage and unknown constants.

$$x_{22} = \frac{\beta_{11}}{\beta_{11}-1}\frac{\rho+\lambda-\mu}{Q_2} I_2, \tag{31}$$

$$a_6 = \frac{I_2}{\beta_{11}-1}\left[\frac{\beta_{11}-1}{\beta_{11}}\frac{Q_2}{\rho+\lambda-\mu}\frac{1}{I_2}\right]^{\beta_{11}}, \tag{32}$$

$$x_{21} = \frac{\beta_{11}}{\beta_{11}-1}\frac{\rho+\lambda-\mu}{Q_1} I_1, \tag{33}$$

$$a_5 = a_6 + \frac{I_1}{\beta_{11}-1}\left[\frac{\beta_{11}-1}{\beta_{11}}\frac{Q_1}{\rho+\lambda-\mu}\frac{1}{I_1}\right]^{\beta_{11}}. \tag{34}$$

Figure 2 shows the dependence of the threshold value on the Poisson intensity. The solid line represents the threshold value of first investment, and the dashed line shows the threshold value of second investment. It is shown that as the catastrophic risk increases, each threshold value increases, especially, that the gap between the two threshold grows as the catastrophic risk increases. It is found that it is difficult to enter the second stage due to the occurrence of the catastrophic event.

Figure 3 shows the dependence of the investment value on the catastrophic risk. The solid line represents the value of the sequential investment. As can be shown from this figure, as the catastrophic risk increases, the investment value decreases. Additionally, the dashed line shows the difference of the value between the single project and the two-stage sequential project. It can be seen from this figure that as the catastrophic risk increases, the difference of the value becomes smaller. It turns out that the increases in the catastrophic risk leads to a loss in the value of flexibility.

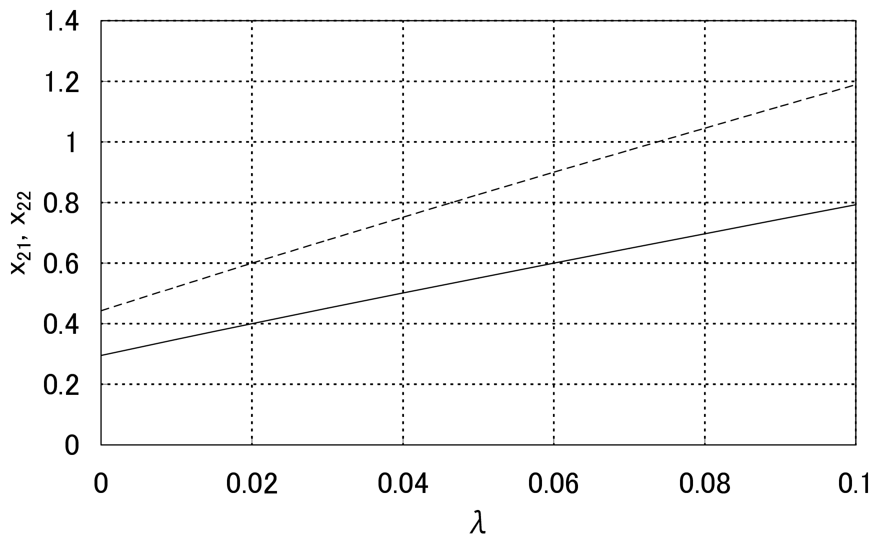

Fig. 2 Threshold values as a function of the Poisson intensity. The solid line represents the threshold value of first investment, and the dashed line shows that of second investment.

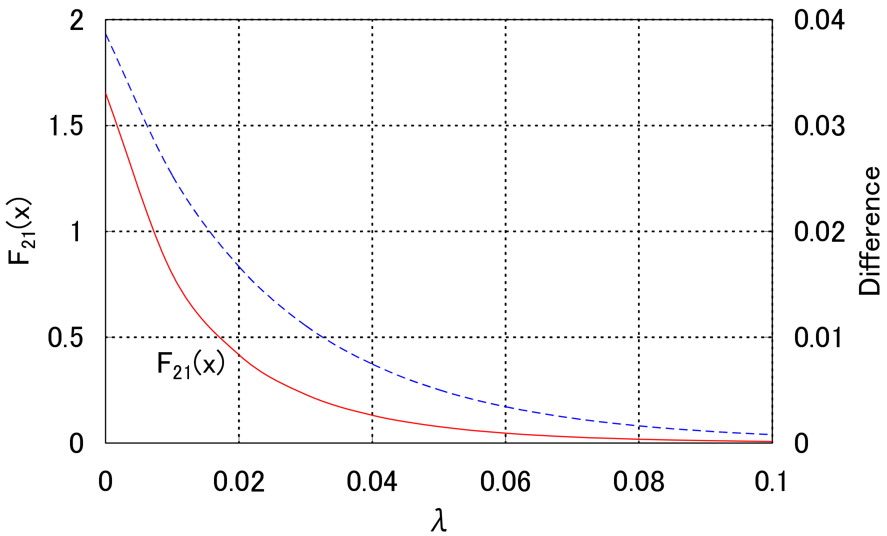

Fig. 3 Investment values as a function of the Poisson intensity. The solid line represents the value of the sequential investment. The dashed line shows the difference of the value between the single investment and the sequential one.

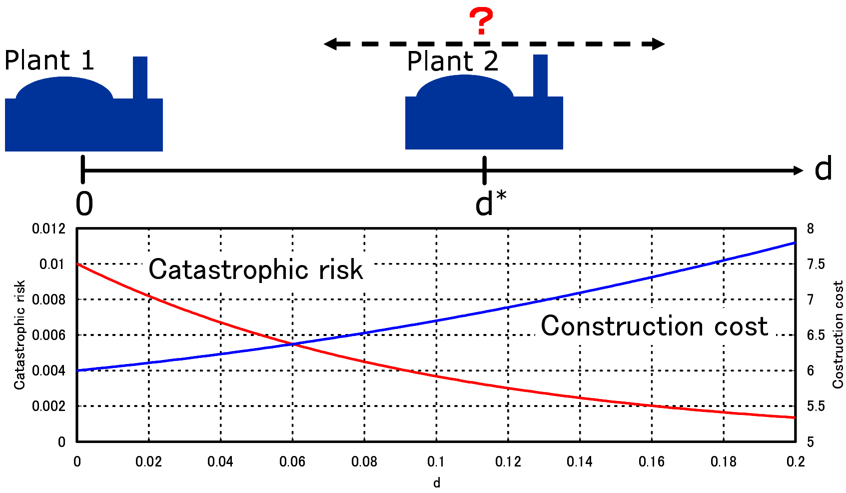

Fig. 4 Determination of the plant location, and catastrophic risk and investment cost as a function of the distance.

4 Investment timing and location

As in the previous section, we consider the two-stage sequential investment with the catastrophic risk. In this section, suppose that the catastrophic risk and the construction cost are dependent on the location. By this setting, we determine not only the investment timing but also the plant location. This means that, for the setting in previous section, the location of plants for the first and the second investments is same one.

The setting and the assumption of the model in this section are as follows. As illustrated in Figure 4, suppose that the location of plant 1 is fixed, and the location of plant 2 must be determined. Additionally, we assume that the catastrophic risk and the construction cost are dependent on the distance from the location of plant 1, d, and that the catastrophic risk (i.e., the Poisson intensity) is a decreasing function of distance, and the location cost is an increasing function. The Poisson intensity of the catastrophic event regarding plant 2 is assumed to be expressed by the following equation,

$$\lambda_2(d) = \lambda_1 e^{-\alpha d}, \tag{35}$$

where λ_1 is Poisson intensity of the catastrophic event regarding plant 1, and α is constant. The Poisson intensity of the catastrophic event regarding plant 1 is assumed to be uncorrelated with that regarding plant 2 [2]. Additionally, we assume that the investment cost regarding the construction of plant 2 is the quadratic function for the

[2] In actual catastrophic event, especially, as earthquake, the Poisson process of the catastrophic event for plant 1 appears to be correlated with that for plant 2. For simplicity, however, we assume that there is no correlation of the Poisson process between regarding plant 1 and plant 2.

distance,

$$I_2(d) = I_1 + c_1 d + c_2 d^2, \tag{36}$$

where c_1, and c_2 are constants.

The firm maximizes the value after the second investment by choosing the optimal location of the second plant. Thus, the optimal location of the second plant for any x can be obtained from this equation:

$$d^*(x) = \arg\max_{d} \frac{Q_1 x}{r + \lambda_1 - \mu} + \frac{Q_2 x}{r + \lambda_2(d) - \mu} - I_2(d) \tag{37}$$

The following nonlinear equation for the optimal location d^* is obtained from Equation (37),

$$\frac{\alpha e^{-\alpha d^*} Q_2 x_2}{(r + \lambda_2(d^*) - \mu)^2} - c_1 - 2c_2 d^* = 0. \tag{38}$$

For the threshold value of the second investment x_{32}, the value of the second investment must satisfy the following the value-matching and smooth-pasting conditions,

$$\begin{cases} a_7 {x_{32}}^{\beta_{21}(d^*)} + \frac{Q_1 x_{32}}{r+\lambda_1-\mu} = \frac{Q_1 x_{32}}{r+\lambda_1-\mu} + \frac{Q_2 x_{32}}{r+\lambda_2(d^*)-\mu} - I_2(d^*) \\ \beta_{21}(d^*) a_7 {x_{32}}^{\beta_{21}(d^*)-1} + \frac{Q_1}{r+\lambda_1-\mu} = \frac{Q_1}{r+\lambda_1-\mu} + \frac{Q_1}{r+\lambda_2(d^*)-\mu}, \end{cases} \tag{39}$$

where

$$\beta_{21}(d) = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r + \lambda_2(d))}{\sigma^2}}.$$

We can obtain the threshold value of the second investment x_{32}, the optimal location of plant 2 d^*, and the unknown constant a_7 by solving equations (38) and (39) simultaneously. Likewise, for the threshold value of the first investment x_{31}, the value of the first investment must satisfy the following the value-matching and smooth-pasting conditions,

$$\begin{cases} a_8 {x_{31}}^{\beta_{11}} = a_7 {x_{31}}^{\beta_{21}(d^*)} + \frac{Q_1 x_{31}}{r+\lambda_1-\mu} - I_1 \\ \beta_{11} a_8 {x_{31}}^{\beta_{11}-1} = \beta_{21}(d^*) a_7 {x_{31}}^{\beta_{21}(d^*)-1} + \frac{Q_1}{r+\lambda_1-\mu} \end{cases} \tag{40}$$

From these conditions, we can obtain the threshold values of the first investment x_{31} and unknown constant a_8. In this analysis, we use the base case parameters presented above, and the following parameters: $\lambda_1 = 0.01$, $\alpha = 10.0$, $c_1 = 5.0$, and $c_2 = 20.0$.

The dependence of the threshold value on the catastrophic risk is shown in figure 5. The solid line represents the threshold value in the model of this section, that is the model taking into account the location, and the dashed line shows the threshold value in the model of the previous section. It can be seen from this figure that since, for this model, the location of the plant can be determined, and the determination of the location decreases the risk, the threshold value of the investment taking into account the location becomes small compared with that without the location.

Figure 6 shows that the dependence of the investment values and the optimal location on the catastrophic risk. For the investment values, the solid line shows the value for the model in this section, $F_{31}(x)$, and the dashed line represents that of the model in previous section, $F_{32}(x)$. Although the investment values for each case decrease with the catastrophic risk, the value of the investment with the determination of the location is large compared with that without the location. This is because the investment value increases by determining the location, and avoiding the catastrophic

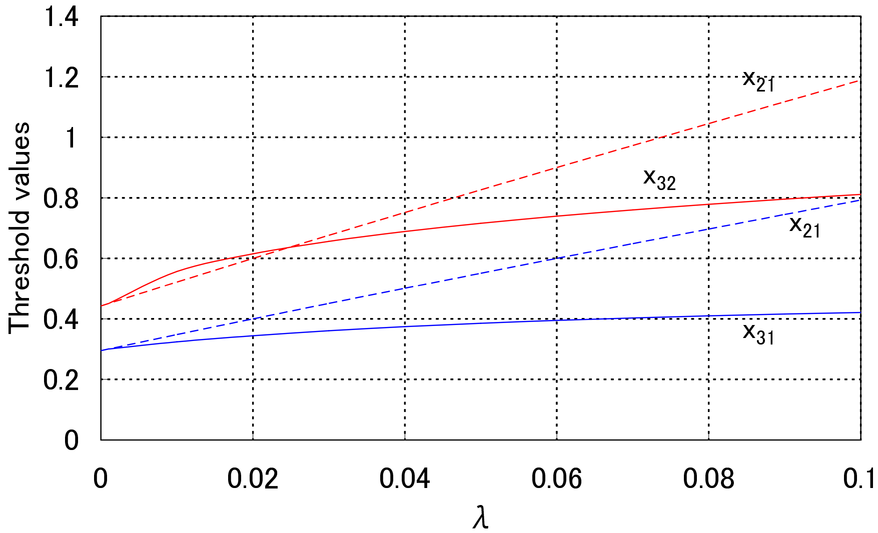

Fig. 5 Threshold values as a function of the Poisson intensity. The solid line represents the threshold value of the investment with the determination of the location. The dashed line shows the threshold value of the investment without the determination of the location.

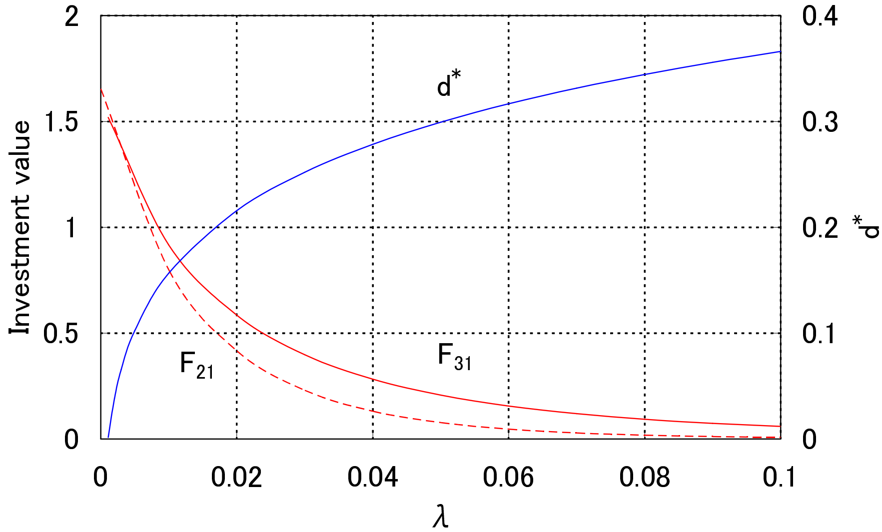

Fig. 6 Investment values and the optimal location as a function of the Poisson intensity. For the investment values, those of the investment with the determination of the location (solid line) and the investment without the determination of the location (dashed line) are shown.

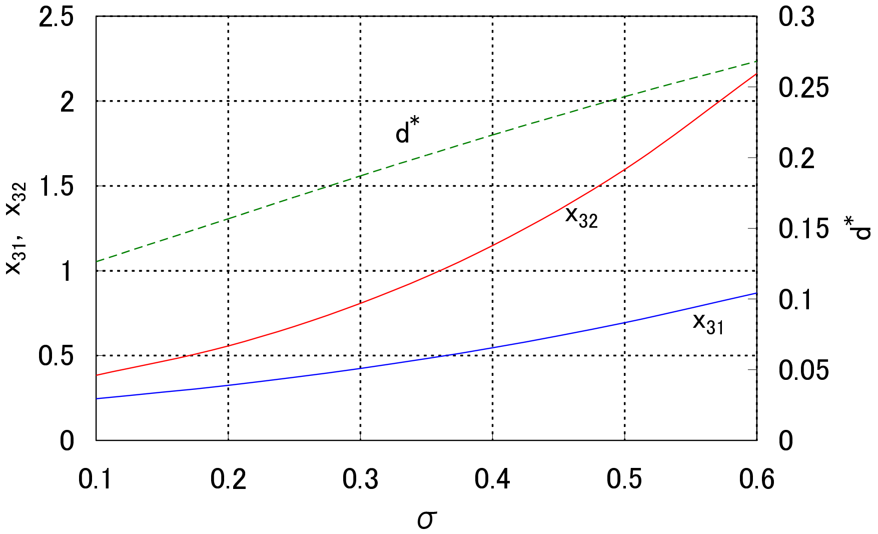

Fig. 7 Threshold values and the optimal location as a function of the volatility. The solid lines represent the threshold values of the first and second investment. The dashed line shows the optimal location.

risk. For the optimal location of the second plant, as shown in this figure, the optimal location of plant 2 becomes far from that of plant 1 as the catastrophic risk increases due to avoiding catastrophic risk.

The dependence of the threshold value and the optimal location on the volatility of cash flows are shown in Figure 7. The solid lines represent the threshold values of fist and second investments. As the volatility becomes large, the opportunity of the investment decreases. The dashed line shows the optimal location. The cash flow at the time of the investment decision becomes large as the uncertainty increases. Therefore, the decision of the optimal location tends to decrease the catastrophic risk even if the construction cost becomes large.

Figure 8 shows that the dependence of the threshold value and the optimal location on the slope coefficient of the Poisson intensity function, α, in Equation (35). The solid and dashed lines represent the threshold values of fist and second investments, and the optimal location, respectively. As can be seen from this figure, since the catastrophic risk converges on the location of plant 1 as the slope coefficient increases, the threshold values and the optimal location decreases due to decrease in the effect of avoiding the catastrophic risk. As the volatility becomes large, the opportunity of the investment decreases. On the other hand, for small slope coefficient, it is found that the threshold value for plant 2 and the optimal location decreases because of the equalization of the catastrophic risk, that is the dominant effect of the investment cost.

5 Conclusions

In this paper, we have analyzed the two-stage sequential investment when the lifetime of the project follows a Poisson process. We showed the dependence of the investment

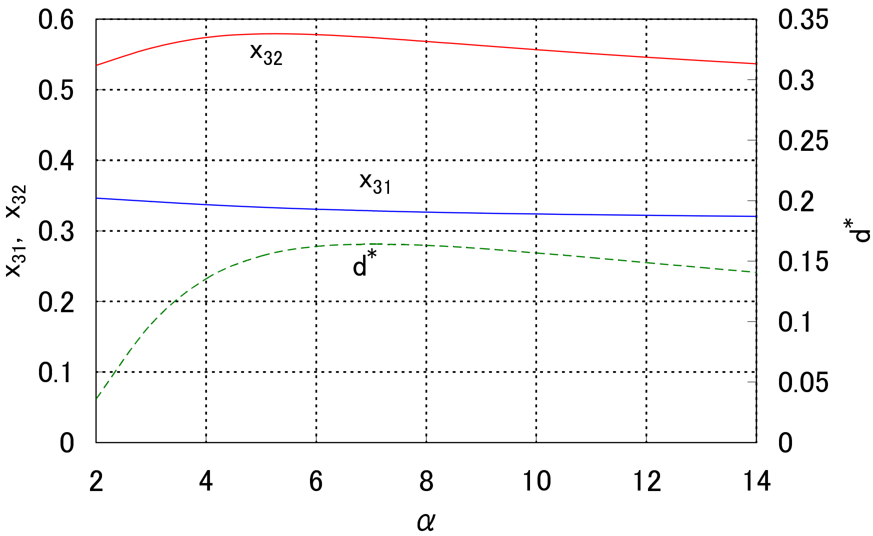

Fig. 8 Threshold values and the optimal location as a function of the slope coefficient of the Poisson intensity function. The solid lines represent the threshold values of the first and second investment. The dashed line shows the optimal location.

value on the catastrophic risk. It was founded that the catastrophic risk and decreases the flexibility value of the sequential investments. Additionally, we proposed a model for analyzing the timing and location of investments. Using this model, the effect of catastrophic risk on the optimal investment rule and location was examined.

In the future, we will analyze actual construction problems of several power plants, and problems taking into account the possibility for the restart of constructions and operations. Furthermore, we will elaborate the construction and location costs, incorporate the cost functions into the model in this paper.

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