Irreversible R&D investment with inter-firm spillovers

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Abstract

In our duopoly, an irreversible investment incorporates a significant amount of R&D, so that the improvement it introduces in production processes generates a spillover lowering the second comer's investment cost. The presence of the inter-firm spillover substantially affects the equilibrium of the dynamic game: for low – and hence realistic – spillover values, the leader delays her investment until the stochastic fundamental has reached a level such that the follower's optimal strategy is to invest as soon as he attains the spillover. This bears several interesting implications. First, because the follower invests upon benefiting from the spillover, in our equilibrium the average time delay between the two investments is short, which is realistic. Second, we show that in case of a major innovation, an optimal public policy requires a substantial intervention in favour of the investment activity; moreover, an increase in uncertainty – delaying the equilibrium – calls for higher subsidization rates. Third, we find, by means of numerical simulations, that the spillover reduces the difference in the leader's and in the follower's maximum value function. Accordingly, our model can help generating realistic market betas.

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1 Introduction

A substantial body of literature has investigated the importance of irreversibility for investment decisions in stochastic environments. In particular, in the last few years, the attention has focused on duopolistic market structures in which the optimal decision of a firm depends not only on the value of the underlying stochastic fundamental, usually profit, but also on the action undertaken by its competitor. Because large investments rarely come without a significant improvement in production methods, in some recent contributions technical progress plays a significant role.

We complement these streams of literature by analyzing a duopoly in which the lump-sum investment incorporates a relevant amount of R&D. Accordingly, it generates a spillover, which lowers the second comer's investment cost. In our duopoly game, the occurrence of the information leakage from the leader to the follower is stochastic, being governed by a Poisson variable. Because the actual attainment of the informational spillover affects the follower's investment decision, the random process of information leakage influences also the leader's efficiency advantage period.

We find that the presence of the spillover substantially affects the equilibrium of the dynamic game. In fact, in our model, for low – and hence realistic – spillover sizes, the leader delays her investment until the stochastic fundamental is so high that the follower finds optimal to invest as soon as he benefits from the spillover. This bears several interesting implications.

First, because the follower invests upon benefiting from the spillover, in our equilibrium the average time delay between the two investments is short, which – as we shall argue – is realistic. In contrast, when one calibrates the existing models with sensible parameters values, one finds long time spans separating the leader's and the follower's investments. For example, in the framework proposed by Grenadier (1996) the median time between investments varies from four to eight years when the percentage standard deviation of demand ranges between 0.05 and 0.125. While these values are adequate for the construction sector, to which the model has been originally applied by Grenadier, they seem excessive for the manufacturing one.

Second, we show that in case of a major innovation an optimal policy requires a substantial public intervention in favour of the investment activity. In the previous literature, a major innovation – inducing the fear of being preempted – triggers a socially premature investment, which calls for some disincentive. The difference in results is to be ascribed to the significant alteration of the equilibrium characteristics, involved by the presence of a modest spillover.[1] In our framework, an increase in

[1] The analysis developed in Femminis and Martini (2007) – who adopt a deterministic environ-

uncertainty – delaying the equilibrium – calls for higher subsidization rates, a result that applies independently of agents' risk aversion.

Third, the presence of a spillover weakens the dependence of the leader's and follower's maximum value functions from the fundamental. Following Cooper (2006) one can show that the differences in the value functions affect the heterogeneity in the firms' market betas. Accordingly, our model can help generating market betas that – while different among firms – do not vary excessively. This is interesting, because some recent empirical evidence suggests that the market betas show a limited dependence on the book to market ratios, and therefore on the value function (see Ang and Chen (2007) for the US stock markets).

In our framework, the behavior of the follower depends on the information he has about the new technology.

If the spillover has taken place, that is when the relevant information has already leaked out, the follower's optimal strategy is characterized by a trigger. In fact, when profits are low, the follower finds it optimal to wait, when instead the stochastic variable governing profits is sufficiently high, it is convenient for him to invest as soon as he has obtained the cost-reducing information.

When the spillover has not taken place yet, the follower finds optimal to invest paying the full cost, rather than waiting for the uncertain realization of the spillover, only when profits have reached high values. On the contrary, when profits are low, it is sensible for the follower to wait in the hope of benefiting from the cost-reducing spillover. Hence, it is natural to guess that a second threshold exists, determining the value for the fundamental that calls for the follower's investment if the spillover has not materialized.

The innovation leader takes into account such a follower's optimal behavior; as already highlighted, for realistic spillover values, the leader rationally decides to delay her investment until the stochastic fundamental reaches the threshold dictating to the follower to invest as soon as he attains the spillover. This result is best understood by considering separately an innovation granting a large cost reduction, and one involving a small cost saving.

Consider first the equilibrium prevailing in the previous literature when a major innovation is adopted. In contributions such as Smets (1991), Grenadier (1996), and Nielsen (2002) (who build on Fudenberg and Tirole (1985)), two driving forces characterize the equilibria: the length of the follower's strategic delay, and the intensity of the competitive pressure. These contributions identify a subgame perfect equilibrium, in which the second innovator delays for a long period his decision to invest.

ment – leads to similar implications.

This choice is guided by the desire to grasp the increase in profit that is driven by the drift in the stochastic fundamental. The follower's optimal choice implies a long competitive advantage period for the innovator leader, which favors the latter's payoff at the expenses of the former's one. Hence, to avoid being preempted, the first mover invests "very soon", and the investment is socially premature. The preemption possibility also implies rent equalization. The above contributions suggest that this "early" investment equilibrium is subgame perfect when the size of the innovation is large, because the per-period first innovator profits are considerable, which triggers the preemptive behavior. In our model, an increase in the spillover reduces the payoff the leader obtains investing early. In fact, it makes more convenient for the follower the policy of investing upon information disclosure, reducing the corresponding threshold. Such a decline shortens the expected cost advantage period of a leader's early investment, reducing its value. This effect proves to be strong enough to induce the leader to postpone her investment until the fundamental has gone past the trigger prescribing to the follower to invest upon the realization of the spillover. In this equilibrium the result concerning the social desirability of the investment is overturned, since it is now too delayed, which calls for some incentive.

When the investment does not significantly shrinks the unit production costs, the existing literature – disregarding the possibility of inter-firms spillovers – suggests that both firms invest simultaneously (see Pawlina and Kort (2006) for a recent and clear exposition). This happens when the per period profit has become so high that a leader cannot emerge, because the rival would immediately copy her decision. In this case, any innovator – anticipating that there will be no leadership – waits until her investment choice maximizes the joint discounted stream of net profits. The collusive flavour of this equilibrium is apparent: accordingly, it implies underinvestment with respect to the social optimum. The simultaneous investment equilibrium is subgame perfect when the size of the innovation is small, because the increase in per-period first innovator profits is not significant, which avoids preemptive behaviors, ruling out the equilibrium in which a leader invests early. When the possibility of spillovers is considered, the simultaneous equilibrium is delayed, since it implies the forsaking of the benefits stemming from the spillover. This reduces the present discounted value of the simultaneous equilibrium; it turns out that such an effect is strong enough that low sizes of the inter-firm spillover are sufficient to rule out this type of equilibrium.

Our contribution is related to several strands of literature.

Smets (1991) and Dixit and Pindyck (1994) use duopoly models to highlight the tension between the option value of waiting – that delays the firms' investment –

and the fear of being preempted – that prompts for a quick action. They identify the preemptive equilibrium with rent equalization that we have already discussed. The follower's investment is delayed by the presence of uncertainty, while the leader invests as soon as her payoff is equal to the follower's discounted one. Grenadier (1996) applies this analysis to real estate markets; he also identifies the possibility of simultaneous entry, which however depends on a high initial condition for the fundamental.

Weeds' (2002) considers irreversible investment in competing research projects, in a framework where profits evolve following a geometric brownian motion, and the discovery takes place randomly according to a Poisson distribution with constant hazard rate. She finds that, depending on the parameter's values, either an early equilibrium or a simultaneous one is subgame perfect; in the absence of externalities, she suggests that in the early (simultaneous) equilibrium firms over(under)-invest; however the simultaneous equilibrium is closer to the social optimum than the early one.

Pawlina and Kort (2006) consider an asymmetry between the two firms in the fixed investment cost. Besides identifying the early and the simultaneous equilibria, they find the possibility of a third type of subgame perfect equilibrium that they label "sequential". When the asymmetry in the investment costs is relevant, the firm bearing the highest cost has no incentive in moving first, rather it is willing to invest only when the stochastic profit has already become high enough. This gives to its opponent, that becomes the leader, the opportunity to invest at the most of its expected discounted profits. While bearing interesting positive (and normative) implications, this equilibrium still implies long expected delays between the competitors' investment dates.

Nielsen (2002) extends the standard analysis to the case of positive externalities. Under this circumstance, e.g. due to network effects, the demand, and hence the instantaneous profits for the second comer, are higher than for the first one. Hence, the second mover investment threshold is smaller than the leaders's one, so that the two firms invest simultaneously at the follower's threshold.[2] In a duopoly characterized by network effect and asymmetric information on the investment cost, Moretto (2000) highlights that for high spillovers a bandwagon strategy is adopted, so that the (joint) adoption may significantly be delayed.

Our contribution differs from Nielsen's and Moretto's ones in that for us the spillover affects the investment cost, and not the demand side. Hence, it does not

[2]Recently, Moretto (2008) finds that Nielsen's result can be extended to free-entry oligopolistic frameworks.

apply only to network externalities or to complementary goods sectors. Moreover, our approach leads to sequential entries, that are empirically more relevant than simultaneous ones, provided that the delay is not too long.

Pereira and Armada (2004) study a duopoly in which the incumbent firms may be taken over by new entrants, which can seize the two slots in the market. They find that the follower, fearing the competition of a potential new entrant, anticipates his entry, while the leader may be induced to delay her investment trigger, because the reduced length of the follower's strategic delay lowers the intensity of the competitive pressure. Despite the reduction in the follower's expected entry lag, the average time between investments is still high.

Among the duopoly games which do not take into account the uncertainty about the fundamental, it is worth mentioning the ones by Stenbacka and Tombak (1994), and by Hoppe (2000). Stenbacka and Tombak analyze the role of experience, which implies that the probability of successful implementation of an innovation for a firm is an increasing function of the time distance from its own investment date. However, the probability of success of any firm is not affected by the adoption of the rival, so that there are no spillover effects. Stenbacka and Tombak find that – in the (feedback) market equilibrium – the leader's and the follower's adoption dates are quite dispersed. In Hoppe (2000), firms are uncertain about the profitability of the innovation, which induces an asymmetry between the leader and the follower. The latter observes the leader's outcome, and hence becomes aware of the actual profitability characterizing the new technology. When it is likely to be unprofitable, the informational spillover brings about a second-mover advantage, both in the early and in the late equilibrium. A late simultaneous adoption prevails when the probability of poor performance for the new technology is particularly high, because this curtails the first mover's expected payoff. When the late equilibrium is subgame perfect, Hoppe finds that an earlier simultaneous adoption would be welfare increasing, while the result are less definite when the early equilibrium prevails. The equilibrium we describe in this paper differs from Hoppe's ones in that ours – for the empirically relevant portion of the parameters space – is characterized by a first mover advantage, that leads to rent equalization.

Murto and Keppo (2002) present a model where several firms compete for a single investment opportunity, which becomes effective only for the first firm which triggers the investment. When every firm has no information about its rival evaluation of the investment opportunity, the investment trigger is located between the monopoly benchmark, and the simple marshallian case. A similar result has been obtained by

Lambrecht and Perraudin (2003), in a model in which each firm, observing that the others have not invested, updates its beliefs about the distribution of its competitors' investment costs. Hence, each firm's inaction provides some informational spillover to its rivals. Both papers, in contrast to ours, analyze a strategic interaction that ends as soon as one firm invests.

Our modeling strategy is close to Weeds' (2002) one, since for us the randomness in profits is modeled via a geometric brownian motion, while the informational spillover takes place randomly according to a Poisson distribution with constant hazard rate. From the technical standpoint, another influential contribution is Huisman and Kort (2004). Partly building on Grenadier and Weiss (1997), they incorporate into the duopoly model the possibility that a new technology becomes available at an uncertain future date, which happens according to a constant hazard rate Poisson process. The future availability of a better technique may turn the preemption game into a second mover advantage game. The main result here is that an increase in profit uncertainty tends to delay investment, so that there is an increase in the likelihood that a new technology is introduced before the occurrence of an investment using the existing technique.

The paper proceeds in the standard way. In Section 2 we present our model, and then (Section 3) we discuss the value functions and the trigger points they imply. In Section 4 we discuss the equilibrium concept adopted in the analysis, and we compute the subgame perfect equilibrium. In Section 5 we spell out the welfare implications of our analysis, computing the optimal subsidization policy that applies in the most interesting case, namely the one of large innovations. Concluding comments in Section 6 end the paper. Three Appendixes present the analytical details, the proofs of the propositions, and the derivations of the profits and social welfare levels for the case of Cournot competition.

2 The Model

Two risk-neutral firms compete in the product market, and have the opportunity to invest in a cost-reducing process to enhance their profit flows. The cost of this irreversible investment is I for the first mover; as for the second firm introducing the innovation, the cost is I if no information has flown out of the leader firm, otherwise the follower's cost is $(1-\theta)I$, with $\theta \in (0,1)$ being the parameter capturing the spillover effect.

Several empirical studies suggest that it takes time to imitate an innovation.[3]

[3] Refer e.g. to Mansfield (1985) or to Cohen *et al.* (2002).

Accordingly, in our model, we assume that, from the time of the first investment, the informational spillover takes place randomly according to a Poisson distribution with constant hazard rate $\lambda > 0$, implying that the expected delay between the leader's investment decision and the time of information leakage is $1/\lambda$. Notice that our modeling assumption implies that $1/\lambda$ is also the minimum expected time length of the cost advantage period granted to the leader by the introduction of an improved production process.[4] Notice also that λ, θ, and I are identical for the two firms.

It would have been preferable to consider a disclosure lag characterized by a probability of information diffusion depending not only upon the time elapsed from the introduction of the innovation, but also on the follower's imitation effort:[5] nonetheless, the use of a constant hazard rate – which has been inspired by Weeds (2002), and by Huisman and Kort (2004) – seems to be the optimal compromise between analytical tractability and realism.[6]

The instantaneous profit of each firm is stochastic, but it depends also on the number of firms that have already introduced the innovation. We assume that – when no firm has invested – the profit flow for each firm can be expressed as $\Pi_0 z_t$. Π_0 is the deterministic part of the profit function: the subscript underscores the dependence of this component from the number of firms that have already invested; z_t captures the uncertainty about future profits, and it will be assumed to evolve following a geometric Brownian motion. When one firm has sunk the cost, but the other has not, the first firm's instantaneous profit is $\Pi_1^h z_t$, while the other obtains $\Pi_1^l z_t$. The superscript highlights that – in this case – profit can be *high* (for the firm which has already innovated) or *low* (for the one which has not invested yet). When both firms have innovated, they get $\Pi_2 z_t$. We introduce the following standard assumption

*A*1: $\Pi_1^h > \Pi_2 > \Pi_0 > \Pi_1^l$.

$\Pi_2 > \Pi_0$ implies that the new technology is more profitable than the older one; $\Pi_0 > \Pi_1^l$ expresses the fact that the first investment – improving the leader's compet-

[4] The first movers cost advantage period is longer than $1/\lambda$ whenever the follower does not find optimal to invest as soon as he receives the informational spillover.

[5] Both λ and θ should be influenced by the imitation effort. Jin and Troege (2006) suggest that firms can raise the spillover size, paying a convex imitation cost. We preferred not to pursue this development of the model, because our framework is already fairly complex: any further extension requires a much heavier use of numerical techniques to select the equilibrium.

[6] Modelling uncertainty by means of Brownian motions precludes what seems even simpler, i.e. the use of a fixed lenght disclosure lag, as in Femminis and Martini (2007). In fact, this would add an additional state variable to the model. Grenadier and Weiss (1997) model in a tractable way the arrival rate of a new technology, which is governed by a Brownian motion. However, their profits are deterministic, which avoids the proliferation of the state variables.

itive position – induces a deterioration of the profit for the firm who has not sunk the cost yet; when the firm that is lagging behind undertakes the project, this damages the first mover, so that $\Pi_1^h > \Pi_2$.[7]

The geometric Brownian motion z_t is described by the following expression:

$$dz_t = \alpha z_t dt + \sigma z_t d\omega,$$

where $\alpha \in (0, r)$ is the constant drift parameter measuring the expected growth rate of z_t, $\sigma > 0$ is the instantaneous standard deviation, and $d\omega$ is the increment of a standard Wiener process, where $d\omega \sim N(0, dt)$. The constant riskless interest rate is r. The restriction $\alpha < r$ is necessary to ensure that there is a strictly positive opportunity cost of holding the option to invest, so that it will not be kept forever; this restriction guarantees finite valuations for the discounted streams of expected profits.

3 Value functions and investment thresholds

As it is standard, before presenting the equilibrium, we analyze the firms' payoffs. Because we focus on the classic case of two ex-ante identical firms, it is not decided beforehand which firm will be leader or follower. Nonetheless, precisely because firms are ex-ante identical, we can study their payoffs as if their roles were pre-determined, as it is done – with no loss of generality – by Weeds (2002), Huisman and Kort (2004), and others.

For ease of exposition, we refer to the leader as if it were run by female CEO, and to the follower as if it were headed by a male CEO.

3.1 The follower's investment problem

Since the leader optimally reacts to his opponent decisions, it is easier to analyze first the follower's behavior.

Once the leader has invested, the follower behavior depends on the information he has about the new technology. We start characterizing his conduct when the relevant information has already leaked out. We proceed in this way because the follower's knowledge of the additional information is an "absorbing state": once he has obtained the information, he cannot (and he does not desire to) revert to the previous situation of ignorance. Hence, when the information has been revealed, the follower's optimal behavior cannot be influenced by his optimal choices in the "ignorance" state, while the converse is not true.

[7] The same assumptions are introduced, for example, in Pawlina and Kort (2006).

For low realizations of the state variable z_t, the follower's optimal strategy dictates to wait, while – when the state variable z_t is sufficiently high – it is convenient for him to invest as soon as he has obtained the additional information. This suggests the existence of a threshold, $\underset{\sim}{z}$, such that, for $z_t < \underset{\sim}{z}$, the follower decides to postpone the investment, sinking it at a future date, at which the expected discounted profits will be higher; while for $z_t \geq \underset{\sim}{z}$, the follower finds that the profit motive is high enough to trigger an immediate investment.

Formally, the follower solves the Bellman equation:[8]

$$\tilde{F}(z_t) = \max\left\{\Pi_1^l z_t dt + E_t\left[\tilde{F}(z_{t+dt})e^{-rdt}\right], \frac{\Pi_2}{r-\alpha}z_t - (1-\theta)I\right\}. \qquad (1)$$

In Appendix 1, we find that the follower's maximum value function, $\tilde{F}(z_t)$, is:

$$\tilde{F}(z_t) = \begin{cases} \frac{\Pi_1^l}{r-\alpha}z_t + \frac{(1-\theta)I}{\gamma-1}\left(\frac{z_t}{\underset{\sim}{z}}\right)^\gamma, & z_t \in (0, \underset{\sim}{z}) \\ \frac{\Pi_2}{r-\alpha}z_t - (1-\theta)I, & z_t \in [\underset{\sim}{z}, \infty) \end{cases}, \qquad (2)$$

The interpretation for $\tilde{F}(z_t)$ is standard, notice in particular that the second addendum in the first line represents the follower's option value of waiting until the trigger point $\underset{\sim}{z}$ is reached. This threshold is given by:

$$\underset{\sim}{z} = \frac{\gamma}{\gamma-1}\frac{r-\alpha}{\Pi_2-\Pi_1^l}(1-\theta)I, \qquad (3)$$

The comparative statics on $\underset{\sim}{z}$ gives sensible results: an increase in σ^2, and hence in γ, enlarges the investment trigger, which is reduced by an increase in the investment reward $(\Pi_2 - \Pi_1^l)$, and by an increase in the spillover parameter (which lowers the follower's investment cost). An increase in the effective discount rate $r-\alpha$ induces a larger investment trigger.

We now consider the follower's choice when the information about the new technology has not been disclosed yet.

When z_t has reached high values without the occurrence of any information leakage, it is optimal for the follower to invest paying the full cost I, instead of waiting for an uncertain spillover. Hence, it is natural to guess that a second threshold, $\bar{z}$, triggers the follower's investment, and that $\bar{z}$ is always larger than $\underset{\sim}{z}$. In fact, when there is the possibility of obtaining a "discount" on the investment cost, the profit

[8] Throughout the paper a "twiddle" above a function means that it refers to a situation in which the follower has already obtained the informational spillover.

perspectives that convince a rational firm to sink the full cost must be higher than those that triggers the investment of the reduced amount of resources.

We consider first the follower's optimal behavior for $z_t \in [\underline{z}, \infty)$. In this case the Bellman equation is

$$F(z_t) = \max\left\{ \Pi_1^l z_t dt + \lambda \left[\frac{\Pi_2}{r-\alpha} z_t - (1-\theta) I \right] dt + (1-\lambda dt) E_t \left[F(z_{t+dt}) e^{-rdt} \right], \right.$$
$$\left. \frac{\Pi_2}{r-\alpha} z_t - I \right\}. \tag{4}$$

The second addendum on the right hand side of the equation above comes from the fact that, with probability λdt, the follower benefits from the informational spillover, which triggers an immediate investment.

When $z_t \in (0, \underline{z}]$, if the information concerning the new technology are disclosed, which happens with probability λdt, the follower does not invest. Nonetheless, his maximum value function is positively affected by his better knowledge. In fact, the follower's maximum value function jumps to the level prescribed by the first line in (2). Accordingly, the maximum value function for the follower solves

$$F(z_t) = \Pi_1^l z_t dt + \lambda \left[\frac{\Pi_1^l}{r-\alpha} z_t + \frac{(1-\theta) I}{\gamma - 1} \left(\frac{z_t}{\underline{z}} \right)^{\gamma} \right] dt + (1-\lambda dt) E_t \left[F(z_{t+dt}) e^{-rdt} \right]. \tag{5}$$

In Appendix 1, we show that the value of the follower is given by:

$$F(z_t) = \begin{cases} \frac{\Pi_1^l}{r-\alpha} z_t + \frac{(1-\theta)I}{(\gamma-1)} \left(\frac{z_t}{\underline{z}} \right)^{\gamma} + E_3 z_t^{\beta_1}, & z_t \in (0, \underline{z}) \\ -\frac{\lambda}{r+\lambda}(1-\theta)I + \frac{(r-\alpha)\Pi_1^l + \lambda \Pi_2}{(r+\lambda-\alpha)(r-\alpha)} z_t + E_2 z_t^{\beta_1} + G_2 z_t^{\beta_2}, & z_t \in [\underline{z}, \bar{z}) \\ \frac{\Pi_2}{r-\alpha} z_t - I, & z_t \in [\bar{z}, \infty) \end{cases}, \tag{6}$$

where the parameters E_3, E_2, and G_2 are:

$$\begin{aligned} G_2 &= \left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1 - 1)(r+\lambda)} \right] \frac{\beta_1 - 1}{\beta_1 - \beta_2} \frac{(1-\theta)I}{\underline{z}^{\beta_2}} \\ E_2 &= \frac{\Pi_2 - \Pi_1^l}{\beta_1 (r+\lambda-\alpha)} \bar{z}^{1-\beta_1} + \\ &\quad - \left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1 - 1)(r+\lambda)} \right] \frac{\beta_2(\beta_1 - 1)}{\beta_1(\beta_1 - \beta_2)} \left(\frac{\bar{z}}{\underline{z}} \right)^{\beta_2} \frac{(1-\theta)I}{\bar{z}^{\beta_1}} \\ E_3 &= E_2 - \frac{r(1-\theta)I}{(\beta_1 - 1)(r+\lambda)\underline{z}^{\beta_1}} + \frac{\beta_2 - 1}{\beta_1 - 1} G_2 \underline{z}^{\beta_2 - \beta_1}. \end{aligned} \tag{7}$$

The threshold $\bar{z}$ is determined by the nonlinear equation:

$$\left(\frac{\bar{z}}{\underline{z}}\right)^{\beta_2}(1-\theta)I\left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)}-\frac{\beta_1 r}{(\beta_1-1)(r+\lambda)}\right]= \tag{8}$$
$$=\frac{\Pi_2-\Pi_1^l}{r+\lambda-\alpha}\bar{z}-\frac{\beta_1(r+\theta\lambda)}{(\beta_1-1)(r+\lambda)}I$$

Notice that – when $\theta=0$ – the solution for the above equation is $\bar{z}=\underline{z}$.[9] This is not surprising: when there is no spillover, the follower's decision boils down to the traditional one, since he only has to decide whether he wants to invest or to keep his option. The coincidence of the two thresholds reflects this fact.

When $\theta=1$ the solution is $\bar{z}=\frac{\beta_1(r+\lambda-\alpha)}{(\beta_1-1)(\Pi_2-\Pi_1^l)}I$, while $\underline{z}=0$ (Eq. (3)). In this particular case, because the technology adoption bears no cost to the follower, it is always optimal for him to upgrade his technique as soon as the relevant – and indeed precious – information leaks out of the leader firm. This explains why $\underline{z}=0$. When no information is revealed, the follower waits to invest until z_t has reached a value that is higher than the one characterizing the follower problem in a model with no spillover.[10]

Before commenting upon the maximum value function, we need to prove some results.

Lemma 1 $\left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)}-\frac{\beta_1 r}{(\beta_1-1)(r+\lambda)}\right]>0$ *for* $\sigma^2\in(0,\infty)$, $\lambda\in(0,\infty)$.

Proof. Refer to Appendix 2. ■

This allows to prove that:

Proposition 2 *The threshold* $\bar{z}$ *is unique; moreover* $\bar{z}>\underline{z}$ *for* $\theta\in(0,1)$, $\lambda\in(0,\infty)$.

Proof. Refer to Appendix 2. ■

Hence, our guess according to which $\bar{z}>\underline{z}$ is verified. Proposition 2 has an interesting implication

Corollary 3 $E_2, G_2>0$, *and* $E_3<0$, *for* $\sigma^2\in(0,\infty)$, $\lambda\in(0,\infty)$.

Proof. Refer to Appendix 2. ■

[9] To check this, substitute $\underline{z}$ as given by Eq. (3) in Eq. (8), and let $\theta\to 0$.

[10] The fact that – in this case – the threshold $\bar{z}$ is higher that the follower's trigger in a model with no spillovers, $\left(\text{that is } \frac{\gamma}{\gamma-1}\frac{r-\alpha}{\Pi_2-\Pi_1^l}I\right)$, is guaranteed – for $\lambda\in(0,\infty)$ – by Lemma 4, which implies $\frac{\beta_1(r+\lambda-\alpha)}{\beta_1-1}>\frac{\gamma(r-\alpha)}{\gamma-1}$.

This Corollary is useful to interpret the maximum value function (6). For $z_t \in (0,\underline{z}]$, the term $E_3 z_t^{\beta_1}$ reflects the difference between the maximum values of a follower who has not obtained the spillover, and of one who has (compare the first line in Eq. (6) with the first line in Eq. (2)). Accordingly this correction term is negative. When $z_t \in [\underline{z}, \bar{z}]$, the maximum value for a follower that has not enjoyed the spillover is characterized by two option value terms, $E_2 z_t^{\beta_1}$ and $G_2 z_t^{\beta_2}$, that are both positive. In fact, inaction grants two types of advantages to the follower. With instantaneous probability λ, he may obtain the spillover, moreover, he expects to be moved toward the investment threshold $\bar{z}$, which increases his value. The first effect is captured by $G_2 z_t^{\beta_2}$, while the second boils down into $E_2 z_t^{\beta_1}$ (notice that G_2 nullifies as $\lambda \to 0$).

3.2 The leader's investment decision

We now solve the leader's optimal decision problem, determining her payoffs.

The leader chooses her investment threshold, given that the follower will act optimally in the future. Once the leader has invested, she has no further decision to take, and her payoff is given by the present discounted value of her profits. This payoff is affected by the possibility that the follower obtains some information concerning the leader's technology. We already know that the follower's behavior is different in the two intervals $z_t \in (0, \underline{z})$, and $z_t \in [\underline{z}, \bar{z})$. This difference in the follower's behavior influences the leader's payoff, because it affects the length of her cost advantage period. Therefore, the leader's maximum value function has two different shapes in the two intervals $z_t \in (0, \underline{z})$, and $z_t \in [\underline{z}, \bar{z})$.

As a preliminary to the determination of the leader's value of investing, it is convenient to analyze her value of *having already invested*, when the follower has already obtained the spillover.

In Appendix 1, we show that the maximum value function for a leader that has sunk the cost is:[11]

$$\widetilde{\overline{L}}(z_t) = \begin{cases} \frac{\Pi_1^h}{r-\alpha} z_t + \frac{\Pi_2 - \Pi_1^h}{r-\alpha} \underline{z} \left(\frac{z_t}{\underline{z}} \right)^{\gamma}, & z_t \in (0, \underline{z}) \\ \frac{\Pi_2}{r-\alpha} z_t, & z_t \in [\underline{z}, \infty) \end{cases} . \tag{9}$$

The interpretation for the value function above is straightforward. When $z_t \geq \underline{z}$, the follower invests upon information revelation, and the leader's payoff is given by the flows of future duopoly profits, discounted at the growth-adjusted rate $r - \alpha$. If, instead, $z_t < \underline{z}$, the follower delays his investment, and the leader enjoys

[11] A bar above the maximum value function denotes that the leader has already sunk the investment cost; moreover, recall that the twiddle implies that the follower has already obtained the informational spillover.

– for a period of time of stochastic length – a cost advantage guaranteeing her the instantaneous profit $\Pi_1^h z_t$. The second addendum in the first line of Eq. (9) "corrects" the discounted profits value $\Pi_1^h z_t/(r-\alpha)$, taking account of the future reduction of instantaneous profits to $\Pi_2 z_t$, that takes place at $\underset{\sim}{z}$.

We now determine the leader's maximum value of investing in state $z_t \in (0,\ \underset{\sim}{z})$. In this interval, the leader knows that the follower – even when the informational spillover has occurred – does not invest until z_t has reached $\underset{\sim}{z}$. Hence, the leader enjoys the instantaneous profit $\Pi_1^h z_t$, which explains the first addendum on the right hand side of the equation below. The second addendum comes from the fact that, with probability λdt, the follower benefits from the informational spillover but not invests, so that the leader's maximum value function jumps to what is prescribed by the first line in Eq. (9). The third addendum is explained by the fact that, with probability $(1-\lambda dt)$ there is no information revelation, and hence the leader obtains $\bar{L}(z_{t+dt})$.

Accordingly, the leader's maximum value is the solution of

$$L(z_t) = \Pi_1^h z_t dt + \lambda\left[\tilde{\bar{L}}(z_t)\right]dt + (1-\lambda dt)E_t\left[\bar{L}(z_{t+dt})\right]e^{-rdt} - I,$$

Having determined $\tilde{\bar{L}}(z_t)$ as in the first line of Eq. (9), we exploit the fact that the value of having invested is

$$\bar{L}(z_t) = L(z_t) + I, \tag{10}$$

and – in Appendix 1 – we obtain the leader's maximum value function as the solution of

$$L(z_t) = \Pi_1^h z_t dt + \lambda\left[\frac{\Pi_1^h}{r-\alpha}z_t + \frac{\Pi_2-\Pi_1^h}{r-\alpha}\underset{\sim}{z}\left(\frac{z_t}{\underset{\sim}{z}}\right)^{\gamma}\right]dt + (1-\lambda dt)E_t\left[L(z_{t+dt})-I\right]e^{-rdt} - I. \tag{11}$$

Consider then the interval $z_t \in [\underset{\sim}{z},\ \bar{z})$. In this case, the leader knows that the market dimension is high enough to justify the immediate follower's investment upon information leakage.

Hence, we formulate the leader's maximum value of investing in state z_t as

$$L(z_t) = \Pi_1^h z_t dt + \lambda\left(\frac{\Pi_2}{r-\alpha}z_t\right)dt + (1-\lambda dt)E_t\left[L(z_{t+dt})-I\right]e^{-rdt} - I. \tag{12}$$

In the equation above, the second addendum on the right hand side comes from the fact that, with probability λdt, the follower benefits from the spillover and invests, so that the leader's instantaneous profit falls to the duopoly level (and stays there forever). The third addendum expresses the fact that, with probability $(1-\lambda dt)$ there is no information revelation, and hence the leader investing at z_t still enjoys her cost advantage.

Collecting the results explained in Appendix 1, we find that the leader's maximum value function is:

$$L(z_t)=\begin{cases}\frac{\Pi_1^h}{r-\alpha}z_t+\frac{\Pi_2-\Pi_1^h}{r-\alpha}\underline{z}\left(\frac{z_t}{\underline{z}}\right)^{\gamma}+E_6 z_t^{\beta_1}-I, & z_t\in(0,\underline{z})\\ \frac{(r-\alpha)\Pi_1^h+\lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)}z_t+E_4 z_t^{\beta_1}+G_4 z_t^{\beta_2}-I\ , & z_t\in[\underline{z},\bar{z})\\ \frac{\Pi_2}{r-\alpha}z_t-I\ , & z_t\in[\bar{z},\infty)\end{cases}\ , \tag{13}$$

where E_6, E_4, and G_4 are given by:

$$\begin{aligned}G_4&=\frac{\Pi_1^h-\Pi_2}{\Pi_2-\Pi_1^l}\frac{(1-\theta)I}{\underline{z}^{\beta_2}}\frac{\gamma}{\beta_1-\beta_2}\left[1-\frac{(\beta_1-1)(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)}\right],\\ E_4&=-\bar{z}^{(-\beta_1)}\left(G_4\bar{z}^{\beta_2}+\frac{\Pi_1^h-\Pi_2}{r+\lambda-\alpha}\bar{z}\right),\\ E_6&=E_4+\underline{z}^{(-\beta_1)}\left[G_4\underline{z}^{\beta_2}+\frac{\Pi_1^h-\Pi_2}{r+\lambda-\alpha}\underline{z}\right].\end{aligned} \tag{14}$$

For a better understanding of the economic meaning of the above parameters, it is useful to prove the following

Lemma 4 $\left[1-\frac{(\beta_1-1)(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)}\right]>0$ *for* $\sigma^2\in(0,\infty),\lambda\in(0,\infty)$.

Proof. Refer to Appendix 2. ■

Lemma 4 allows to conclude that $G_4, E_6>0$, while $E_4<0$, which implies that $L(z_t)$ needs not be monotonic neither in the interval $z_t\in(0,\underline{z})$, nor in the interval $z_t\in[\underline{z},\bar{z})$.

In fact, different forces contribute to shape the leader's maximum value function.

When $z_t\in(0,\underline{z})$, an increase in z_t enhances the instantaneous profits granted by the investment; however, it also moves the leader closer to the point in which the follower invests upon the information disclosure. The positive effect on expected profits is captured by $\frac{\Pi_1^h}{r-\alpha}z_t$, while the second addendum in the maximum value function reflects the negative effect caused by the investment carried out at $\underline{z}$ by a follower that has already benefited from the spillover (this is the same effect that can be seen in (9)). The third addendum, $E_6 z_t^{\beta_1}>0$, measures the expected loss in which a leader incurs when the follower obtains the spillover; this effect is the less relevant the lower is λ.[12] As depicted in Figure 1, the first effect tends to dominate when z_t is low, so that its effects on profits are more relevant.

[Figure 1 about here]

The interpretation of the maximum value function for $z_t \in [\underline{z}, \bar{z})$ goes as follows. The expected profit for a leader facing a constant probability of investment on behalf of her competitor, $\frac{(r-\alpha)\Pi_1^h+\lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)}z_t$, obviously grows with z_t; $E_4 z_t^{\beta_1}$ and $G_4 z_t^{\beta_2}$ are correction terms capturing the fact that an increase in z_t makes closer – on average – the attainment of the threshold $\bar{z}$ that triggers the follower's investment. Because an increase in z_t makes less likely the attainment of the spillover, the second of the two correction terms is positive (and increasing in λ).

To understand the role of the probability of information disclosure in shaping the leader maximum value function, consider that, when z_t is close to $\underline{z}$, it takes "quite a long time" to reach $\bar{z}$. Accordingly, the average length of the cost advantage period is close to $1/\lambda$, because the probability that $\bar{z}$ is reached before the relevant information are released is negligible. This suggests that, in the lower part of the interval $[\underline{z}, \bar{z})$, the leader's maximum value function is "almost linear" in z_t because the effect of z_t on profits does not change significantly with z_t itself. On the contrary, when z_t is close to $\bar{z}$, the extent of the cost advantage period is affected by the evolution of z_t. Accordingly, in this case, an increase in z_t enhances the instantaneous profits for the leader, but reduces the expected duration of her cost advantage period, which explains the contraction in (the growth of) the leader maximum value.

Figure 1 – which is drawn for a realistic value of λ – shows that the leader's maximum value function actually is "almost linear" for large parts of the interval $[\underline{z}, \bar{z})$.

Comparing Figure 1 with the pictures portraying the equilibrium for the no-spillover case (see e.g. Dixit and Pindyck (1994), Nielsen (2002), Weeds (2002)) one immediately realizes that the presence of a moderate spillover significantly reduces the difference in the leader's and in the follower's maximum value functions, and their dependence from the fundamental. Because the disparities in the value functions affects the heterogeneity in the firms' market betas (as in Cooper (2006)), our model bears the interesting implication of being able to generate betas that – while different between oligopolistic firms – do not vary excessively.

[12] When $\lambda \to 0$, we have that $\left[1 - \frac{(\beta_1-1)(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)}\right] = 0$, since $\beta_1 = \gamma$ (compare Eqs. (18) and (20)). Because in this case $G_4 = 0$, $L(z_t) = \frac{\Pi_1^h}{r-\alpha}z_t + \frac{\Pi_2-\Pi_1^h}{(r-\alpha)}\underline{z}\left(\frac{z_t}{\underline{z}}\right)^\gamma + E_6 z_t^{\beta_1}$ approaches $\frac{\Pi_1^h}{r-\alpha}z_t - \frac{(\Pi_1^h-\Pi_2)}{r-\alpha}\bar{z}^{1-\gamma}z_t^\gamma$, which is the leader's maximum value function in the traditional no-spillover model.

3.3 The simultaneous investment problem

The above analysis suggests that, if the innovation leader decides to delay her investment until z_t has reached high values (i.e. for $z_t \in [\bar{z}, \infty)$), the fixed cost is so low in comparison to the expected profits, that it is optimal for the second firm to immediately enter upon his rival's investment, without exploiting the inter-firm spillover.

In this case, the first firm is aware that – as soon as she innovates – the second firm will "immediately" follow and invest. Hence, each firm takes her decision anticipating such a follower's behavior. This leads to a candidate equilibrium where the two firms maximize their joint payoff: knowing that it will be immediately followed, each firm delays its innovation until it can get its maximum discounted sum of profits. In this context, firms remain symmetric, and the maximization of each single firm's payoff coincides with their joint maximization.[13] Notice that this solution implies that the informational spillover is never exploited.

In this case the Bellman equation for both firms is

$$S(z_t) = \max \left\{ \Pi_0 z_t dt + E_t \left[S(z_{t+dt}) e^{-rdt} \right], \frac{\Pi_2}{r-\alpha} z_t - I \right\}. \tag{15}$$

At first sight, the solution for this optimization problem seems standard: one may be inclined to think that the best strategy, for each firm, is to wait until a threshold, say z_S, is reached, and then to invest. Notice, however, that the simultaneous investment threshold must satisfy the constraint $z_S \geq \bar{z}$. In fact, if $\bar{z} > z_S$, the simultaneous equilibrium cannot be sustained: if a firm invests at z_S, her competitor best strategy is not to follow immediately. Rather his best reply – given by $F(z_t)$ – is to invest as soon as he benefits from the positive spillover, and to sink the cost at $\bar{z} > z_S$ if no information flows out of the rival.

In Appendix 1 we show that the solution is to invest at $z_S = \max\{z', \bar{z}\}$, where:[14]

$$z' = \frac{\gamma}{\gamma - 1} \frac{r - \alpha}{\Pi_2 - \Pi_0} I, \tag{16}$$

so that the cooperative maximum value function is:

[13] Weeds (2002) analyzes the case of cooperative investment decisions. When firms cooperate, if side payments are allowed, they may jointly select two different investment triggers (which of course imply different expected profits streams). If, instead, the two firms can cooperate, but they are constrained to invest at the same point, they opt for the trigger we identify in the main text. Hence our approch is equivalent to allow for cooperation, excluding the possibility of side payments.

[14] It is possible to show that, for θ strictly positive, and λ relatively high, $\bar{z} > z'$, while we have that $z' > \underline{z}$, always.

$$
S(z_t) = \begin{cases} \begin{cases} \text{if } z' \geq \bar{z} & \frac{\Pi_0}{r-\alpha} z_t + \frac{I}{\gamma-1} \left(\frac{z_t}{z_S}\right)^{\gamma} \\ \text{if } z' < \bar{z} & \frac{\Pi_0}{r-\alpha} z_t + \left(\frac{\Pi_2 - \Pi_0}{r-\alpha} z_S - I\right) \left(\frac{z_t}{z_S}\right)^{\gamma} \end{cases} & \text{for } z_t \in (0, z_S) \\ \qquad\qquad \frac{\Pi_2}{r-\alpha} z_t - I & \text{for } z_t \in [z_S, \infty) \end{cases} . \tag{17}
$$

When $z' < \bar{z}$, the value function is continuous, but not differentiable, a consequence of the constraint in the maximization process. Notice that, in this case, the function $S(z_t)$ can be interpreted as the discounted expected value of investing at z_S conditional upon being in $z_t < z_S$.

4 The competitive equilibrium

4.1 The equilibrium concept

We now focus on subgame perfect equilibria, in which it is not decided beforehand which firm is be leader or follower. Hence we build on the tradition of Fudenberg and Tirole (1985), a tradition that has been followed, among others, by Grenadier (1996), Nielsen (2002), Weeds (2002), and Huisman and Kort (2004).

Subgame perfectness requires that the equilibrium must survive all the possible off-equilibrium deviations. Hence, we need to compare the leader's payoff at any candidate equilibrium, with her payoff at any point lower than the one that is part of the proposed equilibrium. Whenever we can find a point in which the leader's payoff is higher than the discounted value of her payoff at the candidate equilibrium, the leader prefers to invest at that point rather than to wait for the proposed equilibrium, which therefore is not subgame perfect.

When the leader's payoff is higher than the follower's one, and hence there is a first mover advantage, we need to take into account the possibility of preemption by the follower. This follows from the fact that the roles of leader and follower are not pre-assigned: if the follower's payoff is lower than the leader's one, the former has an incentive to anticipate the latter's decision, becoming the leader.

When the spillover and the information leakage parameter are high, we have a second-mover advantage.[15] In this case, following Huisman and Kort (2004), we assume that each firm is assigned the task to move first with probability one half.[16]

[15] In a second mover advantage game, if the task of moving first is exogenously assigned to one of the two firms, this player – behaving optimally – obtains the lower payoff.

[16] This assumption (and therefore the equilibrium it implies) may seem arbitrary. In fact, it rules out the mixed-strategies equilibrium often referred to as a war of attrition. In a war of attrition, firms would randomize obtaining, at every point, an expected payoff equal to the leader's one. In the cases we study, our assumption does not twist the selection process in favour of a candidate equilibrium located in the interval $z_t \in [\underline{z}, z)$, simply because there is no second-mover advantage in such interval for a parameter sub-space much wider than the one for which we present our results.

As an example to clarify the equilibrium selection procedure, consider Figure 1. For the chosen parameter constellation, we have that $\bar{z}$ is the simultaneous investment trigger (i.e. $z_S = \bar{z}$, because $z' < \bar{z}$), and hence a natural candidate equilibrium. Notice that, for some $z_t \in [z_L, \bar{z})$, the leader's maximum value function $L(z_t)$ is higher than $S(z_t)$, which represents in this case also the discounted value of $L(\bar{z})$. Accordingly, the leader prefers to sink the investment cost in z_t, rather than to wait until $\bar{z}$ is reached. This is sufficient to make the simultaneous investment at $\bar{z}$ not an equilibrium. In such a case, since $L(z_t) > F(z_t)$, it is in the follower's interest to preempt the leader by investing at $z_t - dz_t$. Because the roles of innovation leader and follower are not pre-assigned, by backward induction, we conclude that the equilibrium strategy for the first innovator is to invest when the leader's payoff is equal to the follower's one (i.e. at z_L). Accordingly one firm, which becomes the leader, invests at z_L, while the other waits until $\underline{z}$, and then it invests as soon as it has benefited from the spillover; of course, if the information has not been disclosed before $\bar{z}$ is reached, at that point the follower invests anyway. Notice that we have rent equalization in the equilibrium, due to the possibility of preemption in this first-mover advantage game. Notice also that $S(z_L) > L(z_L) = F(z_L)$, but the leader cannot decide to wait, because, for some $z_t > z_L$, it is in the follower's interest to preempt the leader, which makes the outcome of investing simultaneously at $\bar{z}$ not subgame perfect.

In our set-up, the leader maximum value function may cross the follower's one more than once. This happens as the spillover parameter θ increases. In fact, an increase in θ directly benefits the follower's payoff by reducing his fixed cost; moreover, a larger spillover makes more convenient to the follower the policy of immediately investing upon information disclosure (in fact an increase in θ reduces the threshold $\underline{z}$, refer to Eq. (3)). This shortens the leader's expected cost advantage period for $z_t \in (0, \underline{z})$, reducing her value. For a sufficiently high θ, these two combined effects induce $L(z_t) < F(z_t)$ for some $z_t \in [z_L, \bar{z})$. Hence, the leader's maximum value function crosses the follower's one more than once. Notice that the fact that $L(z_t) < F(z_t)$ for some $z_t \in [z_L, \bar{z})$ does not imply that the game we are considering is of the second-mover advantage type: in the equilibrium we may still have rent equalization, as it happens for the parameter set portrayed in Figure 2. .

[Figure 2 about here]

To select the subgame perfect equilibrium, we need to proceed backward, starting from z_S. If $S(z_t) > L(z_t)$ for $z_t \in [z_L, \bar{z})$, the subgame perfect equilibrium in the interval $[z_L, \bar{z})$ prescribes simultaneous investment at z_S. If not, as it is the case with the parameters set used for Figure 2, we need to check if the first rent equalization point that we find moving backward toward z_L is the equilibrium. Suppose that $z_S = \bar{z}$ – as it is in our specific case – and call $\tilde{z}$ the first rent equalization point at the left of $\bar{z}$. To verify whether this point actually represents a subgame perfect equilibrium, one must check whether the discounted expected value of investing at $\tilde{z}$, conditional upon being in $z_t \in [z_L, \tilde{z})$, is higher than $L(z_t)$. When this is the case, the leader prefers to wait until $\tilde{z}$ is reached, rather than to sink the investment cost in z_t. Hence $\tilde{z}$ is the subgame perfect equilibrium. When this is not the case, we need to move to the left to the next candidate equilibrium. Figure 2 depicts a case in which $\tilde{z}$ actually is the subgame perfect equilibrium. In general, this procedure must be iterated until the subgame perfect equilibrium is found (which may well happen for a leader investing at z_L).

In our highly non linear model, the equilibrium can be selected only by means of numerical techniques; before resorting to the use of computations, we prove that at least one rent-equalization equilibrium exists in $z_t \in (0, \bar{z})$. This is accomplished by means of

Theorem 5 *$L(z_t)$ crosses $F(z_t)$ at least once for $z_t \in (0, \bar{z})$.*

Proof. Refer to Appendix 2. ■

4.2 Equilibrium selection

The equilibrium cannot be identified analytically, due to the high degree of non linearity of our model. Hence, we now present some numerical results.[17] In particular, we shall highlight the portion of the parameter space in which the equilibrium investment trigger for the leader is higher than $\underline{z}$. When this is the case, in fact, the follower best strategy is to invest as soon as he gets the spillover, so that the average time distance between the leader's and the follower's investment dates is close to $1/\lambda$, which implies realistic investment lags for the follower. Hence, in what follows, we will determine – for any given λ – the value $\theta(\lambda)$ such that the spillover parameter is high enough that the leader's subgame perfect equilibrium investment trigger is higher than $\underline{z}$.

To limit the range of relevant values for λ, consider that, in his classic study, Mansfield (1985) reports that in 41% of cases it takes less than twelve months to

[17] Our routine has been written in Matlab, and it is based on a discretization of the space $[\theta \text{ x } \lambda]$, for $\theta \in [0.03, 0.25]$ and $\lambda \in [0.2, 0.6]$. We have used 72.000 gridpoints, however, our results do not appreciably change for any number of evaluation points larger than 4.500. This routine is available upon request from the authors.

the innovator's rival to obtain the relevant information. More recently, Cohen *et al.* (2002) compute that the average adoption lag for unpatented process innovation is 2.03 and 3.37 years in Japan, and in the US, respectively. These contributions leads to think that – recasting the innovation lag in our terms – λ should be comprised between 0.3 and 0.5; accordingly, we simulate the model for $\lambda \in [0.2, 0.6]$.

In our analysis, we fix the discount rate r to 0.04, which is consistent with computing calendar time in years. Then we notice that the level of the irreversible investment does not play any substantial role: the effect of an higher I is to postpone all the equilibria, without changing their relative convenience. Hence, we choose $I = 100$ with no loss of generality. As for α, we fix it at 0.02 simply because we have verified that – moving it in the interval [0.01, 0.03] – does not appreciably modify our result. The role of uncertainty is much more significant. An increase in the standard deviation for dz_t has relevant effects on $\theta(\lambda)$. As we shall detail later, an higher uncertainty increases the investment triggers, and the value of waiting, and hence plays a role in the equilibrium selection process. Hence, we shall present the result for $\sigma \in \{0.03, 0.1\}$. While the second value may seem high, it has been adopted in various studies to stylize the role of sector-specific uncertainty (see e.g. Grenadier (1996), Pawlina and Kort (2006)). The first value has been chosen to portray the polar case of a relatively stable sector.[18]

Another key element is given by the post-investment profit levels. In fact, a significant profit increase for the leader – in absence of spillovers – favours the preemptive equilibrium, as originally suggested by Fudenberg and Tirole (1985), and verified in the stochastic settings by many contributions (see, in particular Nielsen (2002), and Weeds (2002)). On the contrary, with no spillover, an investment yielding only a modest profit increase to the front runner tends to induce the selection of a simultaneous equilibrium (see Pawlina and Kort (2006), or, again, Weeds (2002)). Accordingly, we analyze two different scenarios.

We first consider a major innovation, that is the introduction of a production techniques yielding a significant cost reduction. In this case, the leader – when she is the unique innovator – grasps large profits in comparison to the ones obtained by the follower. In fact, the cost advantage she enjoys induces her to significantly increase her market share. To simulate this case, we normalize Π_0 to unity, and we assume: $\Pi_1^h = 4$, $\Pi_1^l = 0.25$, and $\Pi_2 = 2.25$.[19]

[18] The choice of the value for the low variance sector has been influenced by Guiso and Parigi (1999), who – using a panel of Italian firms – find a coefficient of variation of one-year ahead expected demand as low as 0.023.

[19] In Appendix 3, we show that these values are coherent with Cournot competition in the final product market when the innovation size, denoted by x, is 0.50 of the market dimension.

Then we portray a minor innovation. In this case, the cost reduction is modest, so that it is not convenient for the leader to sizably expand her production at the follower's expenses. Hence the leader – even when she is the unique innovator – does not enjoy a profits so much larger than the follower's one. Accordingly, to depict a minor innovation, besides normalizing $\Pi_0 = 1$ as before, we assume $\Pi_1^h = 1.21$, $\Pi_1^l = 0.9025$, and $\Pi_2 = 1.1025$.[20]

The case of an important innovation is depicted in Figure 3, which shows the threshold $\theta(\lambda)$ for $\sigma \in \{0.03, 0.1\}$.

[Figure 3 about here]

For $\theta \geq \theta(\lambda)$, the leader's subgame perfect equilibrium investment trigger is higher than $\underset{\sim}{z}$, while, for $\theta < \theta(\lambda)$, the preemptive equilibrium with the leader investing at z_L prevails. To understand this behavior, notice that, for a given λ, an increase in θ reduces the leader's payoff. In fact, an increase in θ makes more convenient to the follower the policy of immediately investing upon information disclosure, reducing the threshold $\underset{\sim}{z}$. This shortens the leader's expected cost advantage period for $z_t \in [z_L, \underset{\sim}{z})$, reducing her value. This first limits, and then eliminates, the range for $z_t \in (0, \underset{\sim}{z})$ such that $L(z_t) > F(z_t)$, ruling out the possibility of an equilibrium in which the leader invests at $z_L < \underset{\sim}{z}$ (i.e. of a first mover advantage "early" equilibrium). Hence, an increase in θ, for a given λ, favours the selection, as a subgame perfect equilibrium, of a leader's investment trigger higher than $\underset{\sim}{z}$.

The effects of a larger λ are subtler, but they need to be scrutinized to understand why the threshold $\theta(\lambda)$ is decreasing in λ. Notice that an increase in this parameter does not affect $\underset{\sim}{z}$ (refer to Eq. (3)), and therefore does not significantly reduce the cost advantage period for a leader investing in the early stages of the game. Nonetheless, a larger probability of information spillover does reduce the value of a leader investing in $z_t \in [\underset{\sim}{z}, \bar{z})$, while obviously benefiting the follower's expected profits. Accordingly, the two payoff functions meet at a later $\tilde{z}$, which implies an higher current value for the equilibrium. (Refer again to Figure 2, and consider that the process we are describing shifts upward $F(z_t)$, and downward $L(z_t)$). This higher value twists the equilibrium selection process toward leader's investment triggers that are higher than $\underset{\sim}{z}$. Accordingly, a higher λ requires a lower θ for the early equilibrium to be dominated.

In words, a large probability of releasing relevant information induces the leader to delay her investment, in order to grasp large benefits from the increased market

[20] In this case, the values in the main text are consistent with Cournot competition in the final product market, when the innovation size amounts to 0.05 of the market dimension.

dimension during her limited cost advantage period. This effect proves to be strong enough to sustain – even for a relatively small spillover size – the equilibrium in which the leader invest after $\underline{z}$.

A larger σ enhances quite significantly the threshold $\theta(\lambda)$. The intuition for this result is simple: an increase in uncertainty has the usual effects on the follower's optimal choices: it delays the thresholds $\underline{z}$ and $\bar{z}$.[21] The increase in the follower's value of waiting, delaying his investment triggers, not only increases the follower's payoff, but it also benefits the leader's value. In particular, she enjoys, in the early stages of the game, a longer cost advantage period, because the follower's optimal policy dictates him to invest – upon information disclosure – at $\underline{z}$. This obviously acts in favour of the subgame perfectness of the early equilibrium.

Figure 4 shows the threshold $\theta(\lambda)$ for a minor innovation, again for $\sigma \in \{0.03, 0.1\}$. While for $\theta \geq \theta(\lambda)$, the leader's subgame perfect equilibrium investment trigger is higher than $\underline{z}$, when θ is low it is the simultaneous investment that prevails.

[Figure 4 about here]

To understand this case, consider first that the simultaneous investment strategy is optimal once the market dimension, and hence the potential increase in profits due to the innovation, have reached high values. In this case, an innovation leader cannot emerge, because the rival would immediately copy her decision. The existing literature suggests that the simultaneous investment equilibrium is subgame perfect when the size of the innovation is small, because the per-period first innovator profits are not significant, which avoids preemptive behaviors, and hence an equilibrium in which a leader invests in $z_L < \underline{z}$.

When the follower's best reply is to invest immediately upon information disclosure, (i.e. for $z_t \in [\underline{z}, \bar{z})$), an higher θ reduces – for a given λ – the difference between the leader's and the follower's payoffs. While the negative effect on the leader is limited, because the cost advantage period is essentially governed by λ (unless z_t is "very close" to $\bar{z}$), the follower significantly benefits from the lower investment cost. This increases the follower's value function, postponing the candidate equilibrium point $\tilde{z}$, which acts against the subgame perfectness of the simultaneous equilibrium.[22]

In words, in the interval $z_t \in [\underline{z}, \bar{z})$, the presence of a spillover induces the leader to delay her investment; in fact, she is aware that it is not in the follower's interest to

[21] As for $\underline{z}$, the effect can be verified analytically from Eqs (3), and (18), following the usual steps expounded in Dixit and Pindyck (1994).

[22] Even if Figure 2 has been drawn for an high cost reduction, it may be helpful to visualize the effects of an higher θ on the value functions.

preempt her, because he knows that, waiting, he will obtain a reduction in the sunk cost. This increases the firms' expected values in this equilibrium, which therefore tends to dominate the simultaneous solution.

An increase in λ benefits the follower while harming the leader's payoff. Hence, it acts in favour of the subgame perfectness of the equilibrium in $[\underline{z}, \bar{z})$, which therefore dominates for lower values of the spillover parameter.

In this case also, a larger σ enhances appreciably the threshold $\theta(\lambda)$. The intuition for this result is simple: an increase in uncertainty delays the threshold $z_S = \max\{z', \bar{z}\}$.[23] The increase in z_S benefits both firms' values, which obviously acts in favour of the subgame perfectness of the simultaneous investment equilibrium.

The evaluation of our result involves a thorny issue, namely the assessment of the actual size of the spillover parameter.

Some early literature (see Mansfield et al. (1981)) suggests that the ratio of the imitator's cost to the one of the first innovator is 0.65; more recent contributions estimate the role of technological externalities from production functions. Los and Verspagen (2000), find the role of "external R&D" to be extremely important for U.S. manufacturing firms. Actually, they find an elasticity of output to external R&D of the order of 0.5 - 0.6. Ornaghi (2006) estimates that, in Spain, the elasticity of output with respect to "technological spillovers" is of the order of 0.2 of the elasticity of output to *own* R&D.[24]

These data are suggestive of the fact that the role of inter-firm spillovers is actually relevant; they also support the view that there is quite a significant inter-sectoral variation of the importance of spillovers. Accordingly, we believe that our result apply at least to the industrial sectors in which, due to the geographical or technological proximity of the producers, the spillovers are likely to be relevant.

5 Optimal subsidization

To underscore the policy relevance of our results, we present an exercise in which a benevolent planner chooses the optimal tax/subsidization rate of investment in a duopoly characterized by the elements we have depicted so far.

In dealing with this issue, we have adopted a second best perspective: for us, neither the number of firms acting in the market nor the way they compete in the second stage quantity game lies within the regulatory power of the benevolent plan-

[23] As for z', the effect can be verified analytically from Eqs (16), and (18), following Dixit and Pindyck (1994)

[24] In both papers the technological spillover variable is a weighted sum of the R&D expenditures of the firms belonging to a specific sector.

ner. Hence, what this non-omnipotent planner chooses is the timing of innovation, which is affected via the subsidy (or the tax) on investment.[25] The planner decisions are based on welfare; in particular for our simulation we use the welfare levels – computed *à la* Marshall – that can be obtained under the Cournot decentralized solution, for the market described in Appendix 3. The instantaneous welfare levels are discounted at the same rate, r, that is used by firms.

The details concerning the computation of the welfare function are provided in Appendix 1. Here, we analyze the consequence of the changes in the investment triggers that are induced by a proportional subsidization of the fixed investment cost. In our exercises subsidy levels are decided upon at time 0, and left unchanged thereafter. In particular, we focus on the case of a major innovation introduced in a not very volatile sector ($\sigma = 0.03$). Figure 5 shows the welfare maximizing subsidization rates for $\theta > \theta(\lambda)$, the parameters configuration being the one used to generate Figure 3.

[Figure 5 about here]

Figure 6 is drawn for comparison: it shows the optimal subsidization rate called for by a "preemptive equilibrium". In other words, Figure 6 shows the optimal subsidization rate that applies for θ below the threshold $\theta(\lambda)$; it also shows the optimal subsidization that would have applied, had the strategy of investing at z_L been subgame perfect for the leader, even for $\theta > \theta(\lambda)$.

[Figure 6 about here]

The fact that the optimal policy portrayed in Figure 6 implies the taxation of the investment is not surprising: in such an equilibrium configuration the first mover invests "very soon" to avoid being preempted, and the R&D investment is socially excessive, so that it must be delayed via taxation (see again Fudenberg and Tirole (1985), but also Riordan (1992), and others). Notice that our result implies that the optimal tax rate is virtually independent from λ. This happens because – in the early equilibrium – when the leader invest, she is "virtually sure" that the follower obtains the spillover before $\underline{z}$. Accordingly, also the trigger point z_L is "almost independent" from λ, which makes this result.

[25] This approach is standard in the literature: see e.g. Hoppe (2000), and Weeds (2002). The first best equilibrium for an omnipotent planner implies the presence of only one firm: whenever there are non-decreasing returns in the innovation size or probability, it is optimal to have only one firm to innovate and cover the entire market at the marginal (post-innovation) cost.

Once the existence of the equilibrium in $[\underline{z}, \bar{z})$ has been recognized, the picture changes quite dramatically: an optimal policy requires a substantial public intervention in favour of the investment activity.

We have also found that an increase in uncertainty – delaying the equilibrium – calls for higher subsidization rates, a result that applies independently of agents' risk aversion.

When one considers the case of a minor innovation, the results are less striking: in this case, it is the simultaneous equilibrium than tends to prevail with low spillover. The collusive flavour of this equilibrium implies underinvestment, which calls for positive subsidization. In this case, our result implies that the policies aimed at stimulating R&D have to be less sizeable than suggested before, because the under-investing equilibrium in $[\underline{z}, \bar{z})$ is closer to the social optimum than the simultaneous equilibrium.

6 Concluding remarks

What drives the result in our model, is not the fact that an increasing spillover progressively postpones the leader adoption date in the "early" equilibrium. While this happens, the crucial aspect is that a different equilibrium of the dynamic game emerges. In fact, for low – and hence realistic – spillover, we find a subgame perfect rent equalization equilibrium in which the leader invests much later. Actually, she delays her investment until the stochastic fundamental is high enough that the follower's invests as soon as he obtains the spillover.

The model could be extended in various ways. First, the spillover size parameter, and the probability of benefiting from the spillover could be endogenized, while alternative stochastic processes for profits could be assumed, such as those exhibiting mean reversion. These assumptions would generate similar qualitative results.

The paper has focused on the symmetric duopoly case. If firms costs are instead allowed to differ, the identities of the leader and of the follower could be defined, with the more efficient firm receiving a greater payoff. In this case, it would be interesting to analyze how Pawlina and Kort (2006) sequential equilibrium affects the selection of the subgame perfect equilibrium. We leave this point for future research, our conjecture being that it would be possible to obtain a sequential equilibrium in which the leader invests after $\underline{z}$.

An increase in the number of firms is problematic. As explained by Fudenberg and Tirole (1985), with three or more identical firms, equilibrium is made complicated by the fact that rent equalization need not to hold.

7 References

Ang, A. and J. Chen, "CAPM over the long run: 1926-2001", *Journal of Empirical Finance*, Vol. 14 (2007), pp. 1-40.

Cohen, W.M., Goto A., Nagata A., Nelson R.R., and Walsh, J.P., "R&D spillovers, patents and the incentives to innovate in Japan and the United States", *Research Policy*, Vol. 31 (2002), pp. 1349-1367.

Cooper, I., "Asset Pricing Implications of Nonconvex Adjustment Costs and Irreversibility of Investment", *Journal of Finance*, Vol. 61 (2006), pp. 139-70.

Dixit A. K., and R. S. Pindyck, *Investment under Uncertainty*, Princeton University Press, Princeton (1994).

Femminis G. and G. Martini, "Spillovers, disclosure lags, and incentives to innovate. Do oligopolies over-invest in R&D?", ITEMQ Working paper nr. 44 (2007), Università Cattolica di Milano.

Fudenberg, D., and Tirole, J., "Preemption and Rent Equalization in the Adoption of New Technology." *Review of Economic Studies*, Vol. 52 (1985), pp. 383–401.

Grenadier S.R., "The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets", *Journal of Finance*, Vol. 51 (1996), pp. 1653-1679

Grenadier S.R. and A.M. Weiss, "Investment in Technological Innovations: An Option Pricing Approach", *Journal of Financial Economics*, Vol. 44 (1997), pp. 397-416.

Guiso L. and G. Parigi, "Investment and demand uncertainty", *Quarterly Journal of Economics*, Vol. 114 (1999), pp. 185–27.

Hoppe, H.C., "Second-mover advantages in the strategic adoption of new technology under uncertainty." *International Journal of Industrial Organization*, Vol. 18 (2000), pp. 315–338.

Huisman K.J.M. and P.M. Kort, "Strategic technology adoption taking into account future technological improvements: A real options approach", *European Journal of Operation Research*, Vol. 159 (2004), pp. 705-728.

Jin, J.Y., and Troege, M., "R&D Competition and Endogenous Spillovers", *The Manchester School*, Vol. 74 (2006), pp. 40–51.

Lambrecht B. and W. Perraudin, "Real options and preemption under incomplete information", *Journal of Economic Dynamics and Control*, Vol. 27 (2003), pp. 619-643.

Los, B. and B. Verspagen "R&D spillovers and productivity: Evidence from U.S. manufacturing data", *Empirical Economics,* Vol. 25 (2000), pp. 127-148.

Mansfield, E., "How Rapidly Does New Industrial Technology Leak Out?" *Journal of Industrial Economics*, Vol. 34 (1985), pp. 217–223.

Mansfield, E., Schwartz, M., and Wagner, S., "Imitation Costs and Patents: An Empirical Study" *Economic Journal*, Vol. 91 (1981), pp. 907–918.

Murto, P. and J. Keppo, "A Game Model of Irreversible Investment under Uncertainty", *International Game Theory Review,* Vol. 4, (2002), pp. 127-140.

Moretto, M., "Irreversible investment with uncertainty and strategic behavior", *Economic Modelling*, Vol. 17 (2000), pp. 589-617.

Moretto, M., "Competition and irreversible investments under uncertainty", *Information Economics and Policy*, Vol. 20 (2008), pp. 75-88.

Nielsen, M.J., “Competition and irreversible investment” *International Journal of Industrial Organization*, Vol. 20 (2002), pp. 731–743.

Ornaghi C. “Spillovers in product and process innovations: Evidence from manufacturing firms”, *International Journal of Industrial Organization*, Vol. 24 (2006), pp. 349-380.

Pawlina G. and P.M. Kort, “Real Options in an Asymmetric Duopoly: Who Benefits from Your Competitive Disadvantage?” *Journal of Economics and Management Studies,* Vol. 15 (2006), pp. 1-35.

Pereira, P.J. and M.R. Armada, “The Optimal Decision to Invest in a Duopoly Market for (Two) Positioned Companies when there are Hidden Competitors”, *mimeo,* University of Minho (2004).

Riordan, M.H., “Regulation and preemptive technology adoption.” *RAND Journal of Economics* Vol. 23 (1992), pp. 334-349.

Smets, F. “Exporting versus FDI: The Effect of Uncertainty, Irreversibilities and Strategic Interactions”, (1991) Working Paper, Yale University.

Stenbacka, R. and Tombak, M.H., “Strategic timing of adoption of new technologies under uncertainty.” *International Journal of Industrial Organization*, Vol. 12 (1994), pp. 387–411.

Weeds, H., “Strategic Delay in a Real Options Model of R&D Competition.” *Review of Economic Studies*, Vol. 69 (2002), pp. 729–747.

8 Appendix 1: Details on the value functions

8.1 The follower has obtained the spillover

We guess that the follower's maximum value function is

$$\tilde{F}(z_t) = C_1 z_t + D_1 z_t^{\gamma},$$

where C_1, D_1, and γ are undetermined coefficients, while the threshold $\underline{z}$ must of course be determined endogenously.

Ito's Lemma guarantees that, for $z_t < \underline{z}$:

$$E_t\left[\tilde{F}(z_{t+dt})e^{-rdt}\right] = \tilde{F}(z_t) + \frac{\partial \tilde{F}(z_t)}{\partial z_t}\alpha z_t dt + \frac{\partial^2 \tilde{F}(z_t)}{\partial z_t^2}\frac{\sigma^2}{2} z_t^2 dt - r\tilde{F}(z_t)dt.$$

Following the strategy commonly used in the literature, we now exploit the expression above into Eq. (1), and we use our guess to obtain that, for $z_t \in (0,\underline{z}]$,

$$0 = \Pi_1^l z_t + \left(C_1 + \gamma D_1 z_t^{\gamma-1}\right)\alpha z_t + \gamma(\gamma-1) D_1 z_t^{\gamma}\frac{\sigma^2}{2} - r(C_1 z_t + D_1 z_t^{\gamma}).$$

The above equation implies that $C_1 = \frac{\Pi_1^l}{r-\alpha}$, and that γ is the positive root of:[26]

$$\gamma\alpha + \gamma(\gamma-1)\frac{\sigma^2}{2} - r = 0. \tag{18}$$

The usual value-matching and smooth pasting conditions determine D_1, and $\underline{z}$:

$$\begin{cases} \frac{\Pi_1^l}{r-\alpha}\underline{z} + D_1 \underline{z}^{\gamma} = \frac{\Pi_2}{r-\alpha}\underline{z} - (1-\theta)I \\ \frac{\Pi_1^l}{r-\alpha} + \gamma D_1 \underline{z}^{\gamma-1} = \frac{\Pi_2}{r-\alpha} \end{cases}.$$

It is immediate to verify that the system above yields $\underline{z}$ as in (3), and $D_1 = \frac{(1-\theta)I}{\gamma-1}\underline{z}^{-\gamma}$.

8.2 The follower has not obtained the spillover

For $z_t \in [\underline{z}, \bar{z})$, i.e. when the follower's optimal strategy is to wait, we guess that his maximum value function is

$$F(z_t) = A_2 + C_2 z_t + E_2 z_t^{\beta_1} + G_2 z_t^{\beta_2}, \tag{19}$$

where A_2, C_2, E_2, G_2, β_1, and β_2 are undetermined coefficients; the threshold $\bar{z}$ must be determined endogenously.

We apply Ito's Lemma to $E\left[F(z_{t+dt})e^{-(r+\lambda)dt}\right]$, we use the resulting expression into Eq. (4), and we use (19) to obtain that, for $z_t \in [\underline{z}, \bar{z})$,

[26] The negative root of the quadratic equation must be discarded because its use would imply that $\lim_{z_t \to 0} \tilde{F}(z_t) \neq 0$.

$$
\begin{aligned}
0 = {} & \Pi_1^l z_t + \lambda \left[\frac{\Pi_2}{r-\alpha} z_t - (1-\theta) I \right] + \left(C_2 + \beta_1 E_2 z_t^{\beta_1 - 1} + \beta_2 G_2 z_t^{\beta_2 - 1} \right) \alpha z_t + \\
& + \left[\beta_1(\beta_1 - 1) E_2 z_t^{\beta_1 - 2} + \beta_2(\beta_2 - 1) G_2 z_t^{\beta_2 - 2} \right] z_t^2 \frac{\sigma^2}{2} + \\
& - (r+\lambda)(A_2 + C_2 z_t + E_2 z_t^{\beta_1} + G_2 z_t^{\beta_2}).
\end{aligned}
$$

The above equation implies: $A_2 = -\frac{\lambda}{r+\lambda}(1-\theta)I$, and $C_2 = \frac{(r-\alpha)\Pi_1^l + \lambda \Pi_2}{(r+\lambda-\alpha)(r-\alpha)}$; β_1, and β_2 are the roots of:[27]

$$
\beta\alpha + \beta(\beta - 1)\frac{\sigma^2}{2} - (r+\lambda) = 0. \tag{20}
$$

To pin down the undetermined E_2, G_2, and the threshold $\bar{z}$, we can exploit the value-matching and smooth pasting conditions at $\bar{z}$. This gives:

$$
\begin{cases}
-\frac{\lambda}{r+\lambda}(1-\theta)I + \frac{(r-\alpha)\Pi_1^l + \lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)} \bar{z} + E_2 \bar{z}^{\beta_1} + G_2 \bar{z}^{\beta_2} = \frac{\Pi_2}{r-\alpha}\bar{z} - I \\
\frac{(r-\alpha)\Pi_1^l + \lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)} + \beta_1 E_2 \bar{z}^{\beta_1 - 1} + \beta_2 G_2 \bar{z}^{\beta_2 - 1} = \frac{\Pi_2}{r-\alpha}
\end{cases}. \tag{21}
$$

Of course, we need to postpone the determination of E_2, G_2, and $\bar{z}$, until when we are able to identify a third equation, completing system (21).

Our tentative solution for the follower's maximum value function in the interval $z_t \in (0,\ \underline{z}]$ is

$$
F(z_t) = C_3 z_t + D_3 z_t^{\gamma} + E_3 z_t^{\beta_1}, \tag{22}
$$

where C_3, D_3, and E_3 are undetermined coefficients, while γ, and β_1 are pinned down by the quadratic equations (18) and (20), respectively.[28]

Our guess (22) readily gives:

$$
\begin{aligned}
0 = {} & \Pi_1^l z_t + \lambda \left[\frac{\Pi_1^l}{r-\alpha} z_t + \frac{(1-\theta)I}{\gamma - 1} \left(\frac{z_t}{\underline{z}} \right)^{\gamma} \right] + \left(C_3 + \gamma D_3 z_t^{\gamma - 1} + \beta_1 E_3 z_t^{\beta_1 - 1} \right) \alpha z_t + \\
& + \left[\gamma(\gamma - 1) D_3 z_t^{\gamma - 2} + \beta_1(\beta_1 - 1) E_3 z_t^{\beta_1 - 2} \right] z_t^2 \frac{\sigma^2}{2} - (r+\lambda)\left(C_3 z_t + D_3 z_t^{\gamma} + E_3 z_t^{\beta_1} \right).
\end{aligned}
$$

The above equation implies that $C_3 = \frac{\Pi_1^l}{r-\alpha}$, and that $D_3 = \frac{(1-\theta)I}{\gamma-1}\underline{z}^{-\gamma}$.

At $\underline{z}$, due to the follower optimizing behavior, the value-matching and smooth pasting conditions between the maximum value functions (19) and (22) must apply. This yields:

[27] In this case, the negative root of the quadratic equation cannot be discarded because we are considering an interval, $z_t \in [\underline{z},\ \bar{z}]$, that does not contain 0.

[28] The negative roots of equation (20) must obviously be discarded, since the limit, for $z_t \to 0$, of the maximum value function defined by the Bellman equation (5) must be 0. It is easy to verify that γ, must actually fulfill equation (18).

$$\begin{cases} \frac{\Pi_1^l}{r-\alpha}\underline{z} + \frac{(1-\theta)I}{\gamma-1} + E_3\underline{z}^{\beta_1} = -\frac{\lambda}{r+\lambda}(1-\theta)I + \frac{(r-\alpha)\Pi_1^l+\lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)}\underline{z} + E_2\underline{z}^{\beta_1} + G_2\underline{z}^{\beta_2} \\ \frac{\Pi_1^l}{r-\alpha} + \gamma\frac{(1-\theta)I}{\gamma-1}\underline{z}^{(-1)} + \beta_1 E_3\underline{z}^{\beta_1-1} = \frac{(r-\alpha)\Pi_1^l+\lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)} + \beta_1 E_2\underline{z}^{\beta_1-1} + \beta_2 G_2\underline{z}^{\beta_2-1} \end{cases}. \tag{23}$$

The four equations in (21) and (23) determine E_2, E_3, G_2, and the threshold $\bar{z}$.

8.3 Value of a leader who has invested

In the interval $z_t \in (0,\ \underline{z})$ the maximum value function $\tilde{\tilde{L}}(z_t)$, can be obtained starting from its recursive form:

$$\tilde{\tilde{L}}(z_t) = \Pi_1^h z_t dt + E_t\left[\tilde{\tilde{L}}(z_{t+dt})e^{-rdt}\right]. \tag{24}$$

We then guess that

$$\tilde{\tilde{L}}(z_t) = C_5 z_t + D_5 z_t^{\gamma}, \tag{25}$$

where C_5, and D_5 are undetermined coefficients, while γ is the positive root of Equation (18).

Using a now familiar procedure, we apply Ito's Lemma to $E\left[\tilde{\tilde{L}}(z_{t+dt})e^{-rdt}\right]$, we use the resulting expression into Eq. (24), and we exploit the tentative solution (25) to obtain

$$0 = \Pi_1^h z_t + \left(C_5 + \gamma D_5 z_t^{\gamma-1}\right)\alpha z_t + \left[\gamma(\gamma-1)D_5 z_t^{\gamma-2}\right] z_t^2\frac{\sigma^2}{2} - r(C_5 z_t + D_5 z_t^{\gamma}),$$

which gives: $C_5 = \frac{\Pi_1^h}{r-\alpha}$. The still undetermined coefficient D_5 is obtained by means of a value matching condition. In fact, at $\underline{z}$, the value of being the leader given that the informational spillover has occurred, is identical to the expected stream of profits obtained when both the firms have sunk the fixed cost. In fact, there the follower is investing.

Accordingly, at $\underline{z}$, we have $\tilde{\tilde{L}}(z_t) = F(z_t) + (1-\theta)I$, and hence:

$$\frac{\Pi_1^h}{r-\alpha}\underline{z} + D_5\underline{z}^{\gamma} = \frac{\Pi_2}{r-\alpha}\underline{z},$$

so that: $D_5 = \frac{\Pi_2 - \Pi_1^h}{r-\alpha}\underline{z}^{1-\gamma}$.

When $z_t \in [\underline{z},\ \bar{z}]$, we have that

$$\tilde{\tilde{L}}(z_t) = \frac{\Pi_2}{r-\alpha}z_t.$$

8.4 Maximum value function for the leader

Our tentative solution for the leader's value of investing in the interval $z_t \in (0,\ \underline{z})$ is

$$L(z_t) = C_6 z_t + D_6 z_t^{\gamma} + E_6 z_t^{\beta_1} - I, \tag{26}$$

As usual, C_6, D_6, and E_6 are undetermined coefficients, while we shall verify that γ, and β_1 are the positive roots of the quadratic equations (18) and (20), respectively.[29]

Applying Ito's Lemma to $E\left[\bar{L}(z_{t+dt})e^{-rdt}\right]$, using the resulting expression into Eq. (11), and exploiting equations (9), (26), and (10), we obtain

$$0 = \Pi_1^l z_t + \lambda \left[\frac{\Pi_1^h}{r-\alpha} z_t + \frac{\Pi_2 - \Pi_1^h}{r-\alpha} \underline{z} \left(\frac{z_t}{\underline{z}} \right)^{\gamma} \right] + \left(C_6 + \gamma D_6 z_t^{\gamma-1} + \beta_1 E_6 z_t^{\beta_1 - 1} \right) \alpha z_t +$$
$$+ \left[\gamma(\gamma-1) D_6 z_t^{\gamma-2} + \beta_1(\beta_1 - 1) E_6 z_t^{\beta_1 - 2} \right] z_t^2 \frac{\sigma^2}{2} +$$
$$-(r+\lambda)(C_6 z_t + D_6 z_t^{\gamma} + E_6 z_t^{\beta_1}),$$

which implies that $C_6 = \frac{\Pi_1^h}{r-\alpha}$, and that $D_6 = \frac{\Pi_2 - \Pi_1^h}{r-\alpha} \underline{z}^{1-\gamma}$; it is easy to verify that γ, and β_1 fulfill equations (18) and (20). Notice that E_6 is still to be determined.

At $\underline{z}$, due to the leader optimizing behavior, a value-matching, and a smooth pasting conditions must apply between the maximum value functions (26), and the one that shall be valid in $[\underline{z}, \bar{z}]$.

Our tentative solution for the leader's value of investing in the interval $z_t \in [\underline{z}, \bar{z}]$ is

$$L(z_t) = C_4 z_t + E_4 z_t^{\beta_1} + G_4 z_t^{\beta_2} - I, \tag{27}$$

while the value of having invested is given by Equation (10).

C_4, E_4, and G_4 are coefficients to be determined, while β_1, and β_2 are the roots of Equation (20).[30]

We apply Ito's Lemma to $E\left[\bar{L}(z_{t+dt})e^{-rdt}\right]$, we use the resulting expression into Eq. (12), and we use the tentative solutions (27)-(10) to obtain

$$0 = \Pi_1^l z_t + \lambda \left(\frac{\Pi_2}{r-\alpha} z_t \right) + \left(C_4 + \beta_1 E_4 z_t^{\beta_1 - 1} + \beta_2 G_4 z_t^{\beta_2 - 1} \right) \alpha z_t +$$
$$+ \left[\beta_1(\beta_1 - 1) E_4 z_t^{\beta_1 - 2} + \beta_2(\beta_2 - 1) G_4 z_t^{\beta_2 - 2} \right] z_t^2 \frac{\sigma^2}{2} +$$
$$-(r+\lambda)(C_4 z_t + E_4 z_t^{\beta_1} + G_4 z_t^{\beta_2}).$$

The above equation implies that $C_4 = \frac{(r-\alpha)\Pi_1^h + \lambda \Pi_2}{(r+\lambda-\alpha)(r-\alpha)}$; and that β_1, and β_2 actually are the roots of Equation (20).

Notice that, if the leader invests at $\bar{z}$, the follower immediately reacts by following suit. Hence the two firms value are the same: this provides the value matching condition $L(\bar{z}) = F(\bar{z})$, that helps pinning down the undetermined coefficients E_4, and G_4.[31] The value matching condition at $\bar{z}$ is

$$\frac{(r-\alpha)\Pi_1^h + \lambda \Pi_2}{(r+\lambda-\alpha)(r-\alpha)} \bar{z} + E_4 \bar{z}^{\beta_1} + G_4 \bar{z}^{\beta_2} - I = \frac{\Pi_2}{r-\alpha} \bar{z} - I \tag{28}$$

As already remarked, at $\underline{z}$, a value-matching, and a smooth pasting conditions between the maximum value functions (26), and (27) must apply. This yields:

[29] As before, we discard the negative roots of equation (20).

[30] Again, the negative root of the quadratic equation must not be discarded because we are concerned with the interval, $z_t \in [\underline{z}, \bar{z}]$.

[31] Because at $\bar{z}$ there is no optimal choice on the part of the leader, there is no corresponding smooth pasting condition in this case (see Weeds, (2002)).

$$\left\{ \begin{array}{c} \frac{\Pi_1^h}{r-\alpha}\underline{z} + \frac{\Pi_2-\Pi_1^h}{r-\alpha}\underline{z} + E_6\underline{z}^{\beta_1} - I = \frac{(r-\alpha)\Pi_1^h+\lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)}\underline{z} + E_4\underline{z}^{\beta_1} + G_4\underline{z}^{\beta_2} - I \\ \frac{\Pi_1^h}{r-\alpha} + \gamma\frac{\Pi_2-\Pi_1^h}{r-\alpha} + \beta_1 E_6\underline{z}^{\beta_1-1} = \frac{(r-\alpha)\Pi_1^h+\lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)} + \beta_1 E_4\underline{z}^{\beta_1-1} + \beta_2 G_4\underline{z}^{\beta_2-1} \end{array} \right. . \tag{29}$$

The three equations in (28) and (29) determine E_4, E_6, and G_4, as in (14).

8.5 Maximum value function for the simultaneous investment problem

We now assume that no firm has invested before the threshold $\bar{z}$ has been reached, and we guess that, for $z_t \in [\bar{z}, z_S)$, it is optimal for the firms to delay their investment. In this case the tentative solution for their maximum value function is

$$S(z_t) = C_7 z_t + D_7 z_t^{\gamma},$$

where C_7, D_7, and γ are undetermined coefficients, while the threshold z_S must be determined endogenously, taking account of the constraint $z_S \geq \bar{z}$.

Following our usual strategy, we exploit the Ito differential for $E_t\left[S(z_{t+dt})e^{-rdt}\right]$, and our guess above, to reformulate Eq. (15) – for $z_t \in [\bar{z}, z_S)$ – as:

$$0 = \Pi_0 z_t + \left(C_7 + \gamma D_7 z_t^{\gamma-1}\right)\alpha z_t + \gamma(\gamma-1)D_7 z_t^{\gamma}\frac{\sigma^2}{2} - r(C_7 z_t + D_7 z_t^{\gamma}).$$

The above equation implies that $C_7 = \frac{\Pi_0}{r-\alpha}$, and that γ is the positive root of Eq. (18) (as usual, the negative root of the quadratic equation must be discarded).

Assuming for the moment that $z' \geq \bar{z}$, we determine D_7, and $z_S = z'$ by means of the usual value-matching and smooth pasting conditions. These give:

$$\left\{ \begin{array}{c} \frac{\Pi_0}{r-\alpha}z' + D_7 z'^{\gamma} = \frac{\Pi_2}{r-\alpha}z' - I \\ \frac{\Pi_0}{r-\alpha} + \gamma D_7 z'^{\gamma-1} = \frac{\Pi_2}{r-\alpha} \end{array} \right. .$$

It is immediate to verify that the system above determines $z_S = z'$ as in Eq. (16), and $D_7 = \frac{I}{\gamma-1}z'^{(-\gamma)}$. Notice that the maximum value function

$$S(z_t) = \frac{\Pi_0}{r-\alpha}z_t + \frac{I}{\gamma-1}\left(\frac{z_t}{z'}\right)^{\gamma},$$

gives the expected present discounted value of investing at z', conditional on being at z_t.(Refer to Dixit and Pindyck (1994).)

When $z' \geq \bar{z}$, this qualifies the solution. When $z' < \bar{z}$, the constraint $z_S \geq \bar{z}$ is binding, and the two competitors are not free to choose when to invest. Accordingly, the smooth-pasting condition does not apply, and the solution is determined by the value matching condition:

$$\frac{\Pi_0}{r-\alpha}\bar{z} + D_7\bar{z}^{\gamma} = \frac{\Pi_2}{r-\alpha}\bar{z} - I,$$

which gives: $D_7 = \left(\frac{\Pi_2-\Pi_0}{r-\alpha}\bar{z} - I\right)\bar{z}^{(-\gamma)}$. In this case the maximum value function is relevant for $z_t < \bar{z}$, because it provides the expected present discounted value of investing at $\bar{z}$.

8.6 The Social Welfare function

The welfare levels depend on the number of firms that have already sunk the cost. Let $M_i z_t$ be the instantaneous welfare level that is obtained when $i = 0, 1, 2$ firms have already invested, and the market dimension variable takes value z_t.

First, we consider the case in which the leader has already invested, while the follower still needs to sink his cost. For $z_t \in [\underline{z}, \bar{z})$, the follower shall invest immediately after he enjoys the spillover (or he shall invest at $\bar{z}$ if the fundamental gets there before the information disclosure takes place). Hence, when one firm has already invested but the information leakage has not occurred, the welfare $W(z_t)$, for $z_t \in [\underline{z}, \bar{z})$, is given by:

$$W(z_t) = M_1 z_t dt + \lambda dt \left[\frac{M_2}{r-\alpha} z_t - (1-\theta)I \right] + (1-\lambda dt) E_t \left[W(z_{t+dt}) e^{-rdt} \right], \quad (30)$$

where the second addendum on the right hand side comes from the fact that, with probability λdt, the follower benefits from the informational spillover and invests, because $z_t \geq \underline{z}$, so that the instantaneous welfare jumps to $M_2/(r-\alpha)$. With probability $(1-\lambda dt)$ there is no information revelation, and hence no investment.

In this case, our guess for Eq. (30) is:

$$W(z_t) = F + G z_t + H z_t^{\beta_1},$$

where F, G, and H are undetermined coefficients, while β_1 is the positive root of Eq. (20).

Using our standard procedure, we apply Ito's Lemma to $E\left[W(z_{t+dt})e^{-rdt}\right]$, we use the resulting expression into Eq. (30), and we exploit the tentative solution above to obtain

$$\begin{aligned} 0 = & \frac{(r-\alpha)M_1 + \lambda M_2}{r-\alpha} z_t - \lambda(1-\theta)I + \alpha G z_t + \alpha \beta_1 H z_t^{\beta_1} + \\ & + \beta_1(\beta_1 - 1) H z_t^{\beta_1} \frac{\sigma^2}{2} - (r+\lambda)(F + G z_t + H z_t^{\beta_1}), \end{aligned}$$

which gives: $F = -\frac{\lambda}{r+\lambda}(1-\theta)I$, and $G = \frac{(r-\alpha)M_1 + \lambda M_2}{(r-\alpha)(r+\lambda-\alpha)}$ The still undetermined coefficient H is obtained by means of a value matching condition. In fact, at $\bar{z}$, the social value of the leader's investment is identical to the value of the investment performed by both firms, net of its cost. Accordingly, we have:

$$W(\bar{z}) = -\frac{\lambda}{r+\lambda}(1-\theta)I + \frac{(r-\alpha)M_1 + \lambda M_2}{(r-\alpha)(r+\lambda-\alpha)} \bar{z} + H \bar{z}^{\beta_1} = \frac{M_2}{r-\alpha} \bar{z} - I,$$

which yields: $H = \bar{z}^{(-\beta_1)} \left[\frac{M_2 - M_1}{r-\alpha+\lambda} \bar{z} - \frac{r+\theta\lambda}{r+\lambda} I \right]$.

We now analyze the welfare effects of the leader's investment.

Suppose first that the leader optimal decision is to sink the R&D cost at $\tilde{z} > \underline{z}$. In this case, for $z_t \in (0, \underline{z})$, i.e. when no firm has invested, the welfare function is:

$$W(z_t) = M_0 z_t dt + E_t \left[W(z_{t+dt}) e^{-rdt} \right], \quad (31)$$

and the solution we propose is:

$$W(z_t) = Nz_t + Pz_t^{\gamma}.$$

From the above tentative solution, where N, and P, are undetermined coefficients and γ is given by Eq. (18), we readily obtain:

$$0 = M_0 z_t + \alpha N z_t + \alpha\gamma P z_t^{\gamma} + \gamma(\gamma-1)Pz_t^{\gamma}\frac{\sigma^2}{2} - r(Nz_t + Pz_t^{\gamma}),$$

which gives: $N = M_0/(r-\alpha)$.

To pin down the coefficient P, notice that, at $\tilde{z}$, the social value of the future investments must be equal to the value of the first investment, net of its cost, which implies:

$$N\tilde{z} + P\tilde{z}^{\gamma} = F + G\tilde{z} + H\tilde{z}^{\beta_1} - I,$$

and therefore,

$$P = \tilde{z}^{(-\gamma)}\left\{\frac{\lambda\left(M_2 - M_0\right) + (r-\alpha)(M_1 - M_0)}{r+\lambda-\alpha}\tilde{z} - \frac{r+\lambda(2-\theta)}{r+\lambda}I + \right.$$
$$\left. + \left[\frac{\lambda\left(M_2 - M_1\right)}{r+\lambda-\alpha}\bar{z} - \frac{r+\theta\lambda}{r+\lambda}I\right]\left(\frac{\tilde{z}}{\bar{z}}\right)\right\}^{\beta_1}.$$

Collecting the above result, one obtains the following welfare function,

$$W(z_t) = \begin{cases} \frac{M_0}{r-\alpha}z_t + Pz_t^{\gamma} & z_t \in (0, \tilde{z}) \\ -\frac{\lambda}{r+\lambda}(1-\theta)I + \frac{(r-\alpha)M_1 + \lambda M_2}{(r+\lambda-\alpha)(r-\alpha)}z_t + \left[\frac{\lambda(M_2-M_1)}{r+\lambda-\alpha} - \frac{r+\theta\lambda}{r+\lambda}I\right]\left(\frac{z_t}{\bar{z}}\right)^{\beta_1} & z_t \in [\tilde{z}, \bar{z}) \\ \frac{M_2}{r-\alpha}z_t - I & z_t \in [\bar{z}, \infty) \end{cases},$$

which has been used to generate Figure 5.

When the leader optimal decision of investing at $z_L < \underset{\sim}{z}$ is part of the subgame perfect equilibrium, we need to determine the social value of the investment of the leading firm for $z_t \in (z_L, \underset{\sim}{z})$.

In this case, we may formulate the leader's maximum value of investing in state z_t as

$$W(z_t) = M_1 z_t dt + \lambda\tilde{W}(z_t)dt + (1-\lambda dt)E_t\left[W(z_{t+dt})e^{-rdt}\right], \tag{32}$$

where $\tilde{W}(z_t)$ is the social value of the investment performed by the leader when the follower has benefited from the spillover, but he has not invested yet. Hence, the second addendum on the right hand side comes from the very fact that, with probability λdt, the follower benefits from the informational spillover but not invests.

It would now be easy to show that

$$\tilde{W}(z_t) = \frac{M_1}{r-\alpha}z_t + \left[\frac{M_2 - M_1}{r-\alpha}\underset{\sim}{z} - (1-\theta)I\right]\left(\frac{z_t}{\underset{\sim}{z}}\right)^{\gamma}.$$

Our tentative solution for Eq. (32) is

$$W(z_t) = Qz_t + Rz_t^{\beta_1} + Sz_t^{\gamma},$$

where obviously Q, R, and S, are undetermined coefficients, and γ and β_1 are given, respectively, by Eqs. (18), and (20). From the above tentative solution we readily obtain $Q = \frac{M_1}{r-\alpha}$, and $S = \left[\frac{M_2-M_1}{r-\alpha}\underline{z} \ -(1-\theta)I\right]\underline{z}^{-\gamma}$. As for R, we notice that, at $\underline{z}$, the value matching condition

$$Q\underline{z} + R\underline{z}^{\beta_1} + S\underline{z}^{\gamma} = F + G\underline{z} + H\underline{z}^{\beta_1}$$

must hold. This readily gives

$$R = \underline{z}^{(-\beta_1)}\left\{\left[\frac{M_2-M_1}{r+\lambda-\alpha}\bar{z} - \frac{r+\theta\lambda}{r+\lambda}I\right]\left[\left(\frac{\underline{z}}{\bar{z}}\right)^{\beta_1} - \frac{\underline{z}}{\bar{z}}\right] - \theta I\right\}.$$

Finally, we shall determine the social welfare when no firm has invested, i.e. for $z_t \in (0, z_L]$. In this case the welfare function is given again by (31), and the solution we propose is:

$$W(z_t) = Tz_t + Uz_t^{\gamma}.$$

It is easy to see that $T = M_0/(r-\alpha)$; as for U, we need to resort to the value matching condition

$$Tz_L + Uz_L^{\gamma} = Qz_L + Rz_L^{\beta_1} + Sz_L^{\gamma} - I$$

which requires that – at the leader's investment trigger z_L – the social value of the future investments must be equal to the net value of the first investment.

Some calculation gives:

$$U = z_L^{-\gamma}\left\{\frac{M_1-M_0}{r-\alpha+\lambda}z_L + \left\{\left[\frac{M_2-M_1}{r-\alpha+\lambda}\bar{z} - \frac{r+\theta\lambda}{r+\lambda}I\right]\left[\left(\frac{\underline{z}}{\bar{z}}\right)^{\beta_1} - \frac{\underline{z}}{\bar{z}}\right] - \theta I\right\}\left(\frac{z_L}{\underline{z}}\right)^{\beta_1} + \right.$$
$$\left. + \left[\frac{M_2-M_1}{r-\alpha}\underline{z} \ -(1-\theta)I\right]\left(\frac{z_L}{\underline{z}}\right)^{-\gamma} - I\right\}$$

In sum, when the leader optimal decision is to invest at $z_L < \underline{z}$, the social welfare function is

$$W(z_t) = \begin{cases} \frac{M_0}{r-\alpha}z_t + Uz_t^{\gamma} & z_t \in (0, z_L) \\ \frac{M_1}{r-\alpha} + \left\{\left[\frac{M_2-M_1}{r-\alpha+\lambda}\bar{z} - \frac{r+\theta\lambda}{r+\lambda}I\right]\left[\left(\frac{\underline{z}}{\bar{z}}\right)^{\beta_1} - \frac{\underline{z}}{\bar{z}}\right] - \theta I\right\}\left(\frac{z_t}{\underline{z}}\right)^{\beta_1} + & \\ \quad + \left[\frac{M_2-M_1}{r-\alpha}\underline{z} \ -(1-\theta)I\right]\left(\frac{z_t}{\underline{z}}\right)^{\gamma} & z_t \in [z_L, \underline{z}) \\ \frac{(r-\alpha)M_1+\lambda M_2}{(r+\lambda-\alpha)(r-\alpha)}z_t + \left[\frac{\lambda(M_2-M_1)}{r+\lambda-\alpha} - \frac{r+\theta\lambda}{r+\lambda}I\right]\left(\frac{z_t}{\bar{z}}\right)^{\beta_1} - \frac{\lambda}{r+\lambda}(1-\theta)I & z_t \in [\underline{z}, \bar{z}) \\ \frac{M_2}{r-\alpha}z_t - I & z_t \in [\bar{z}, \infty) \end{cases},$$

which has been used to used to generate Figure 6.

9 Appendix 2: Proofs

Proof of Lemma 1.

$\left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1-1)(r+\lambda)}\right] > 0$ implies $\left[\frac{\gamma(r-\alpha)}{(\gamma-1)r} - \frac{\beta_1(r+\lambda-\alpha)}{(\beta_1-1)(r+\lambda)}\right] > 0$. Define $F(\lambda,\sigma^2) = \left[\frac{\gamma(r-\alpha)}{(\gamma-1)r} - \frac{\beta_1(r+\lambda-\alpha)}{(\beta_1-1)(r+\lambda)}\right]$, and notice that $F(0,\sigma^2) = 0$, since, in this case

$\beta_1 = \gamma$. Now compute

$$\frac{\partial F(\lambda,\sigma^2)}{\partial\lambda} = \frac{1}{(\beta_1-1)^2(r+\lambda)^2}\left[(r+\lambda-\alpha)(r+\lambda)\frac{\partial\beta_1}{\partial\lambda} - \alpha\beta_1(\beta_1-1)\right].$$

If $\frac{\partial F(\lambda,\sigma^2)}{\partial\lambda} > 0$, then $F(\lambda,\sigma^2) > 0$, hence we now show that $\frac{\partial F(\lambda,\sigma^2)}{\partial\lambda} > 0$. From Eq. (20), it is immediate to obtain

$$\frac{\partial\beta_1}{\partial\lambda} = \frac{2}{2\alpha + (2\beta_1-1)\sigma^2}.$$

Accordingly,

$$\frac{\partial F(\lambda,\sigma^2)}{\partial\lambda} = \frac{2(r+\lambda-\alpha)(r+\lambda) - \alpha\beta_1(\beta_1-1)\left[2\alpha+(2\beta_1-1)\sigma^2\right]}{(\beta_1-1)^2(r+\lambda)^2\left[2\alpha+(2\beta_1-1)\sigma^2\right]}.$$

Because the denominator of the above expression is positive, $\frac{\partial F(\lambda,\sigma^2)}{\partial\lambda} > 0$ if $G(\lambda,\sigma^2) = 2(r+\lambda-\alpha)(r+\lambda) - \alpha\beta_1(\beta_1-1)\left[2\alpha+(2\beta_1-1)\sigma^2\right] > 0$.

Hence, we now study $G(\lambda,\sigma^2)$. Using the fact that $\sigma^2 = \frac{2(r+\lambda-\alpha\beta_1)}{\beta_1(\beta_1-1)}$ (exploit Eq. (20)), we obtain:

$$G(\lambda,\sigma^2) = 2(r+\lambda-\alpha)(r+\lambda) - 2\alpha\left[\alpha\beta_1(\beta_1-1) + (r+\lambda-\alpha\beta_1)(2\beta_1-1)\right],$$

that is:

$$G(\lambda,\sigma^2) = 2(r+\lambda-\alpha)(r+\lambda) - 2\alpha\left[(2\beta_1-1)(r+\lambda) - \alpha\beta_1^2\right],$$

Notice, first, that $\lim_{\sigma^2\to 0} G(\lambda,\sigma^2) = 0$ because $\lim_{\sigma^2\to 0}\beta_1 = \frac{r+\lambda}{\alpha}$; hence, if $\frac{\partial G(\lambda,\sigma^2)}{\partial\sigma^2} > 0$, then $G(\lambda,\sigma^2) > 0$, for $\sigma^2 \in (0,\infty)$, $\lambda \in (0,\infty)$.

Because $\frac{\partial G(\lambda,\sigma^2)}{\partial\sigma^2} = -4\alpha\left(r+\lambda-\alpha\beta_1\right)\frac{\partial\beta_1}{\partial\sigma^2}$, since we have that $r+\lambda-\alpha\beta_1 > 0$ (refer again to Eq. (20)), and $\frac{\partial\beta_1}{\partial\sigma^2} < 0$, the proof is completed. ■

Proof of Proposition 2.

Because $\left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1-1)(r+\lambda)}\right] > 0$, we have that $\lim_{\bar{z}\to 0} l.h.s.(8) = \infty$, and that $\lim_{\bar{z}\to\infty} l.h.s.(8) = 0$. Notice, moreover that $\frac{\partial(l.h.s.(8))}{\partial\bar{z}} < 0$, and that $\frac{\partial^2(l.h.s.(8))}{(\partial\bar{z})^2} > 0$, because $\beta_2 < 0$.

The right hand side of Equation (8) is linear and increasing in $\bar{z}$. Hence, its second derivative in $\bar{z}$ is nought, and $\bar{z}$ is unique.

Notice that

$$r.h.s.(8)|_{\bar{z}=\underline{z}} = \left[\frac{r-\alpha}{(r+\lambda-\alpha)}\frac{\gamma}{\gamma-1}(1-\theta) - \frac{\beta_1}{\beta_1-1}\frac{r+\theta\lambda}{r+\lambda}\right]I,$$

while

$$l.h.s.(8)|_{\bar{z}=\underline{z}} = \left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1-1)(r+\lambda)}\right](1-\theta)I.$$

Hence,

$$r.h.s.(8)|_{\bar{z}=\underline{z}} < l.h.s.(8)|_{\bar{z}=\underline{z}},$$

which completes the proof. ■

Proof of Corollary 3.
Recall that $\beta_2 < 0$: hence, the fact that $E_2, G_2 > 0$ is obvious from Lemma 1.
As for E_3, consider that:

$$E_3 = \frac{\Pi_2 - \Pi_1^l}{\beta_1(r+\lambda-\alpha)}\bar{z}^{1-\beta_1} - \frac{r(1-\theta)I}{(\beta_1-1)(r+\lambda)\underline{z}^{\beta_1}} - \frac{\beta_2}{\beta_1}G_2\bar{z}^{\beta_2-\beta_1} + \frac{\beta_2-1}{\beta_1-1}G_2\underline{z}^{\beta_2-\beta_1}.$$

Exploiting Equation (3), the above expression may be written as:

$$\begin{aligned} E_3 = & \frac{\Pi_2 - \Pi_1^l}{\beta_1(r+\lambda-\alpha)}\left(\bar{z}^{1-\beta_1} - \underline{z}^{1-\beta_1}\right) + \\ & + \left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1-1)(r+\lambda)}\right]\frac{(1-\theta)I}{\beta_1\underline{z}^{\beta_1}} + \\ & -\frac{\beta_2}{\beta_1}G_2\left(\bar{z}^{\beta_2-\beta_1} - \underline{z}^{\beta_2-\beta_1}\right) - \frac{\beta_2}{\beta_1}G_2\underline{z}^{\beta_2-\beta_1} + \frac{\beta_2-1}{\beta_1-1}G_2\underline{z}^{\beta_2-\beta_1}, \end{aligned}$$

which, using the definition for G_2 in (7), simplifies to:

$$E_3 = \frac{\Pi_2 - \Pi_1^l}{\beta_1(r+\lambda-\alpha)}\left(\bar{z}^{1-\beta_1} - \underline{z}^{1-\beta_1}\right) - \frac{\beta_2}{\beta_1}G_2\left(\bar{z}^{\beta_2-\beta_1} - \underline{z}^{\beta_2-\beta_1}\right).$$

If $\bar{z}$ were equal to $\underline{z}$, we would have $E_3 = 0$. Notice, moreover, that:

$$\frac{\partial E_3}{\partial \bar{z}} = (1-\beta_1)\frac{\Pi_2 - \Pi_1^l}{\beta_1(r+\lambda-\alpha)}\bar{z}^{-\beta_1} - \frac{(\beta_2-\beta_1)\beta_2}{\beta_1}G_2\bar{z}^{\beta_2-\beta_1-1} < 0.$$

■

Proof of Lemma 4.
$\left[1 - \frac{(\beta_1-1)(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)}\right] > 0$ requires $(\gamma-1)(r+\lambda-\alpha) > (\beta_1-1)(r-\alpha)$. γ is the positive root of Eq. (18), so that $\gamma - 1 = -\frac{\alpha}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$, while, from Eq. (20) implies: $\beta_1 - 1 = -\frac{\alpha}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}}$. Hence, the above inequality can be written as:

$$\begin{aligned} & \left[-\frac{\alpha}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}\right](r+\lambda-\alpha) > \\ & \qquad > \left[-\frac{\alpha}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}}\right](r-\alpha). \end{aligned}$$

When $\lambda = 0$, the left and the right hand sides of the expression above are identical.
Notice, however, that the first derivative with respect to λ of the left hand side is positive, while the second derivative is nought. As for the right hand side of the expression above, notice that the first derivative is positive, while the second one is negative. Hence, to prove the lemma, it suffices to show that the derivative of the left hand side – evaluated at $\lambda = 0$ – is higher that the derivative of the right hand

side, i.e. that:

$$\left[-\frac{\alpha}{\sigma^2}-\frac{1}{2}+\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}\right] > \left[\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}\right]^{-1}\frac{(r-\alpha)}{\sigma^2}.$$

Multiplying both sides by $\left[\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}\right]$, and rearranging, we obtain:

$$\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2} > \frac{(r-\alpha)}{\sigma^2}+\left(\frac{\alpha}{\sigma^2}+\frac{1}{2}\right)\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}},$$

which readily becomes:

$$\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{r+\alpha}{\sigma^2} > \left(\frac{\alpha}{\sigma^2}+\frac{1}{2}\right)\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}.$$

Squaring both sides of the above expression gives:

$$\begin{aligned}&\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^4+\left(\frac{r+\alpha}{\sigma^2}\right)^2+2\left(\frac{r+\alpha}{\sigma^2}\right)\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2 > \\ &\qquad > \left[\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2\alpha}{\sigma^2}\right]\left[\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}\right].\end{aligned}$$

Some simplifications give:

$$\left(\frac{r+\alpha}{\sigma^2}\right)^2 > \frac{4\alpha r}{\sigma^4},$$

which is always verified, since $r > \alpha$. ■

Proof of Theorem 5.

Notice, from (6), that $\lim_{z_t \to 0} F(z_t) = 0$, while $\lim_{z_t \to 0} L(z_t) = -I$. Now consider the difference $L(z_t) - F(z_t)$ in the interval $z_t \in [\underline{z}, \bar{z}]$. Grouping terms one obtains:

$$L(z_t) - F(z_t) = \frac{\Pi_1^h - \Pi_1^l}{r+\lambda-\alpha} z_t + (E_4 - E_2) z_t^{\beta_1} + (G_4 - G_2) z_t^{\beta_2} - \frac{r+\theta\lambda}{r+\lambda} I.$$

Exploiting (14), and (7) one can write:

$$E_4 - E_2 = \bar{z}^{\beta_2-\beta_1}\left(\frac{\beta_2}{\beta_1}G_2 - G_4\right) - \frac{\bar{z}^{1-\beta_1}}{r+\lambda-\alpha}\left(\Pi_1^h - \Pi_2 + \frac{\Pi_2 - \Pi_1^l}{\beta_1}\right).$$

Taking advantage of the expression above, we obtain:

$$\begin{aligned}\frac{\partial\left[L(z_t) - F(z_t)\right]}{\partial z_t} = &\frac{\Pi_1^h - \Pi_1^l}{r+\lambda-\alpha} + \beta_2 (G_4 - G_2) z_t^{\beta_2 - 1} + \\ &+ \left[\bar{z}^{\beta_2-\beta_1}\left(\beta_2 G_2 - \beta_1 G_4\right) - \frac{\beta_1 \bar{z}^{1-\beta_1}}{r+\lambda-\alpha}\left(\Pi_1^h - \Pi_2 + \frac{\Pi_2 - \Pi_1^l}{\beta_1}\right)\right] z_t^{\beta_1 - 1}.\end{aligned}$$

Hence, we have that

$$\left.\frac{\partial\left[L(z_t)-F(z_t)\right]}{\partial z_t}\right|_{z_t=\bar{z}} = \frac{\Pi_1^h-\Pi_1^l}{r+\lambda-\alpha}+\beta_2(G_4-G_2)\bar{z}^{\beta_2-1}+$$
$$+\bar{z}^{\beta_2-1}\left(\beta_2 G_2-\beta_1 G_4\right)-\frac{\beta_1}{r+\lambda-\alpha}\left(\Pi_1^h-\Pi_2+\frac{\Pi_2-\Pi_1^l}{\beta_1}\right),$$

which boils down to:

$$\left.\frac{\partial\left[L(z_t)-F(z_t)\right]}{\partial z_t}\right|_{z_t=\bar{z}} = \frac{\left[\Pi_1^h-\Pi_1^l-\beta_1(\Pi_1^h-\Pi_2)-\left(\Pi_2-\Pi_1^l\right)\right]}{r+\lambda-\alpha}+(\beta_2-\beta_1)G_4\bar{z}^{\beta_2-1}.$$

We now substitute out G_4 using (14), and we obtain:

$$\left.\frac{\partial\left[L(z_t)-F(z_t)\right]}{\partial z_t}\right|_{z_t=\bar{z}} =$$
$$=(\Pi_1^h-\Pi_2)\left\{\frac{1-\beta_1}{r+\lambda-\alpha}-\frac{1-\gamma}{r-\alpha}\left[1-\frac{(\beta_1-1)(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)}\right]\left(\frac{\bar{z}}{\underline{z}}\right)^{\beta_2-1}\right\}.$$

where we have exploited Eq. (3). The above expression can be written as:

$$\left.\frac{\partial\left[L(z_t)-F(z_t)\right]}{\partial z_t}\right|_{z_t=\bar{z}} =$$
$$=\frac{(\Pi_1^h-\Pi_2)(1-\beta_1)}{r+\lambda-\alpha}\left\{1+\left[1-\frac{(\gamma-1)(r+\lambda-\alpha)}{(\beta_1-1)(r-\alpha)}\right]\left(\frac{\bar{z}}{\underline{z}}\right)^{\beta_2-1}\right\}.$$

Lemma 4 guarantees that both addenda inside the big curly brackets are positive, so that the derivative is negative. Because $L(z_t)$, and $F(z_t)$ are continuous, this completes the proof. ■

10 Appendix 3: A Cournot interpretation for payoffs and welfare levels

Consider an industry composed of two firms, i and j, which, in each (infinitesimally short) period, are involved in a two–stage interaction: first they decide whether to innovate or not, and then they compete *à la* Cournot. Firms' horizon is infinite. Market demand is linear and equal to: $P = a\sqrt{z_t} - bQ$, where P is the market clearing price and $Q = q_i + q_j$ is the total quantity supplied.

Each firm has a unit cost of production $c\sqrt{z_t}$. The assumption that both the market dimension parameter a, and the unit cost c are influenced by the same disturbance is widely used in the literature (Huisman and Kort (2004), Pawlina and Kort (2006), Cooper (2006), Moretto (2008)). In fact, it greatly simplifies the analysis. To avoid excessive analytical intricacies, several other contributions admit only a few possible demand levels, or ignore variable cost (see e.g. Grenadier (1996), Nielsen (2002)). We think that the approach we follow is the optimal compromise between analytical tractability and "realism".

In each period t firm i (and j) decides whether to invest in R&D or not. This investment immediately yields a cost-reducing process innovation, which shrinks the unit production cost by an amount $x\sqrt{z_t}$, with $x < c$. Hence firm i's post–innovation production cost is $C(q_i) = (c - x)q_i\sqrt{z_t}$.

Each firm's payoff depends not only on its adoption date but also on its rival's one. If both firms have not invested up to period t, their individual profits in the Cournot subgame at t are those of the pre–innovation stage, i.e.

$$\Pi_0 z_t = \frac{A^2}{9b} z_t, \tag{33}$$

where $A = a - c$. The subscript indicates the number of firms that have innovated at time t. The instantaneous welfare (computed *à la Marshall* as the sum of consumers' and producers' surpluses) is then equal to:

$$M_0 z_t = \frac{4}{9} \frac{A^2}{b} z_t. \tag{34}$$

If instead only one firm, say firm i, invests in R&D at t, it benefits of an efficiency advantage, and obtains a higher market share. The market price at t decreases in comparison with the pre-innovation level, while the individual profits become:

$$\Pi_1^h z_t = \frac{(A + 2x)^2}{9b} z_t; \Pi_1^l z_t = \frac{(A - x)^2}{9b} z_t, \tag{35}$$

where the superscript h denotes variables pertaining to the firms that has already invested, while l refers to the firms which has not innovated yet. Notice that $\Pi_1^h > \Pi_1^l$, $\Pi_1^h > \Pi_0$ and $\Pi_1^l < \Pi_0$, as required by Assumption 1. Because $q_j^l = \frac{A-x}{3b}$, to preserve the duopolistic structure characterizing our market we need to assume $A > x$. This hypothesis implies that, in a Cournot environment, the cost-reducing innovation is non$-$drastic. In case of asymmetric behavior at t, welfare is:

$$M_1 z_t = \frac{8A(A + x) + 11x^2}{18b} z_t, \tag{36}$$

with $M_1 > M_0$.

Finally, we need to compute the outcomes when both firms have innovated at t. In this case, being more efficient, they both produce more than in the *status quo*; therefore, the market price is lower. Individual profits at t are:

$$\Pi_2 z_t = \frac{(A + x)^2}{9b} z_t. \tag{37}$$

Obviously, $\Pi_1^h > \Pi_2$, as required by Assumption 1; notice, moreover, that the difference between Π_1^h and Π_2 is increasing in x: when only one firm enjoys a cost advantage, she obtains a larger market share while benefiting from an higher price to cost margin.

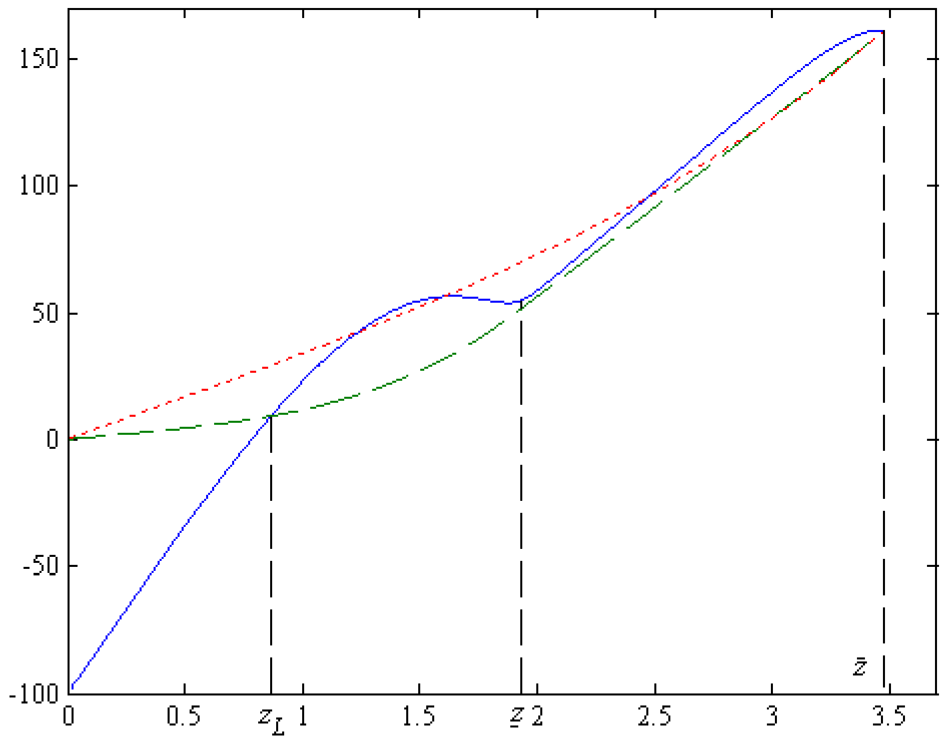
When both firms have innovated, the social welfare is:

$$M_2 z_t = \frac{4(A + x)^2}{9b} z_t, \tag{38}$$

with $M_2 > M_1$.

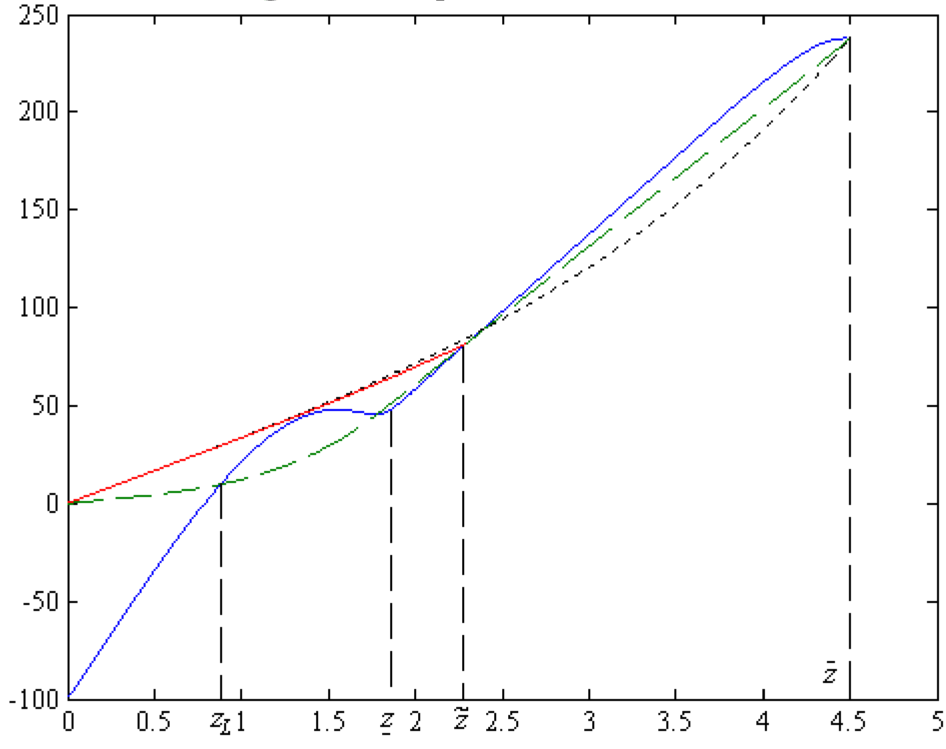
When firms simultaneously invest in R&D, individual profits rise from (33) to (37) and welfare jumps from (34) to (38). Alternatively, firms may behave asymmetrically, so that there are both an innovation leader and a follower. Under these circumstances individual profits first change from (33) to (35) (and welfare from (34) to (36)) and then from (35) to (37) (and welfare from (36) to (38)).

Figure 1: Maximum value functions



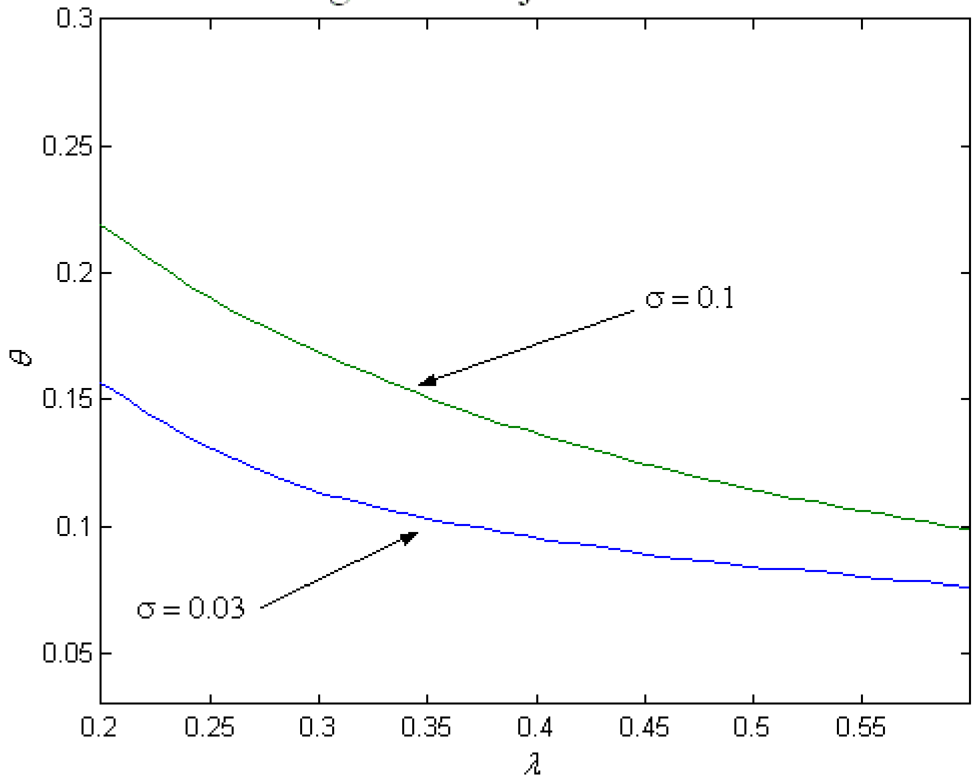
Leader's (continuous line) and Follower's (dashed line) value functions for: $r = 0.04$, $a = 0.01$, $s = 0.03$, $q = 0.07$, $l = 0.40$, $I = 100$. $P_0 = 1$, $P_1^h = 4$, $P_1^l = 0.25$, and $P_2 = 2.25$. The dotted line represents both $S(z_t)$, and the discounted value of $L(\bar{z}) = F(\bar{z})$.

Figure 2: Equilibrium selection



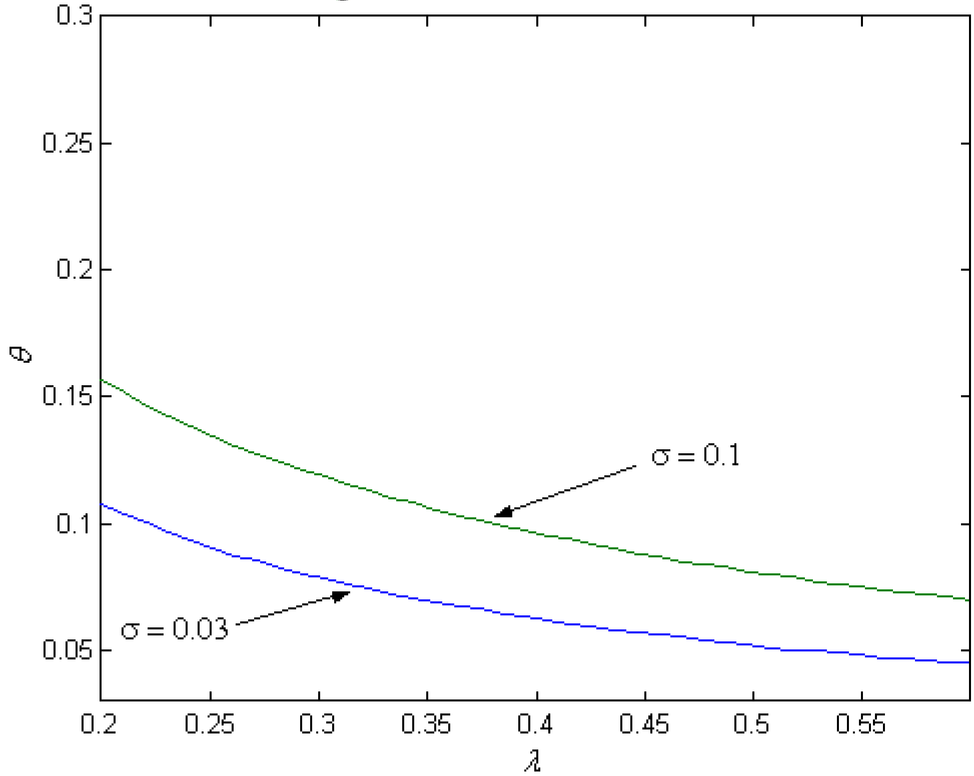
Leader's (continuous line) and Follower's (dashed line) value functions for: $r = 0.04$, $a = 0.01$, $s = 0.03$, $q = 0.12$, $l = 0.40$, $I = 100$. $P_0 = 1$, $P_1^h = 4$, $P_1^l = 0.25$, and $P_2 = 2.25$. The dotted line represents both $S(z_t)$, and the discounted value of $L(\bar{z}) = F(\bar{z})$. The continuous line ending at $\tilde{z}$ represents the discounted value of $L(\tilde{z}) = F(\tilde{z})$.

Figure 3: Major innovation



In the areas above the $q(l)$ frontiers the leader delays her investment at least up to $\underline{z}$.

Figure 4: Minor innovation



In the areas above the $q(l)$ frontiers the leader delays her investment at least up to $\underline{z}$.

Figure 5: Optimal subsidization rates

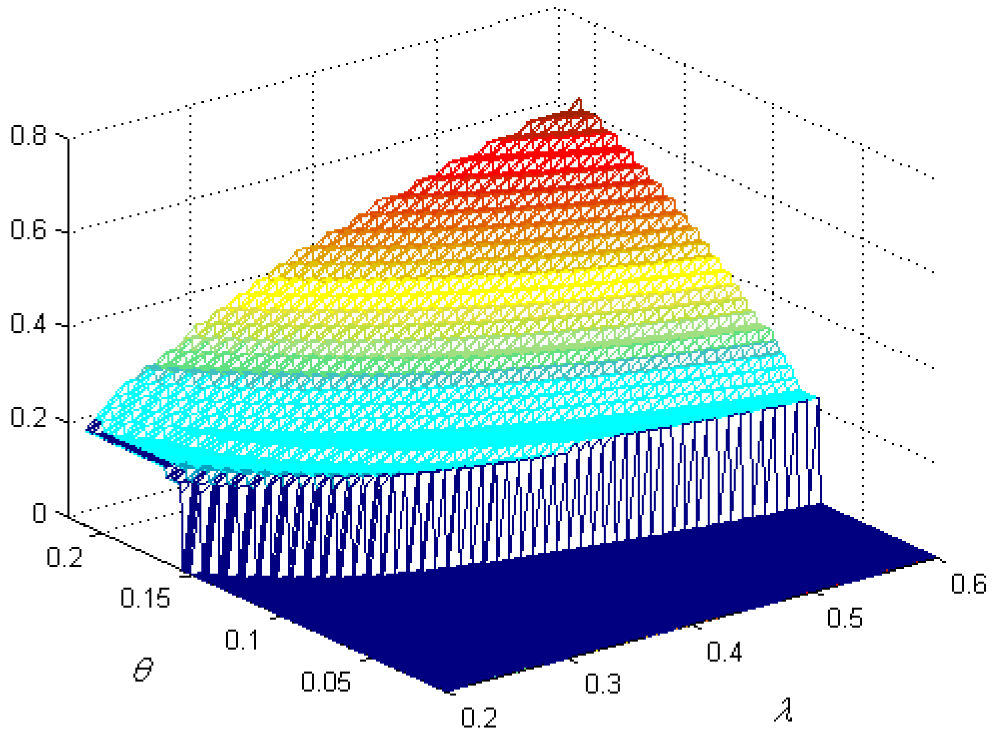


Figure 6: Optimal subsidization rates - early equilibrium