Strategic Capacity Investment Under Uncertainty

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Abstract

Contrary to most of the papers in the literature of investment under uncertainty we study models that not only capture the timing, but also the size of the investment. We consider a monopoly setting as well as a duopoly setting and compare the results with the standard models in which the firms do not have the capacity choice. Our main results are the following. First, for low uncertainty values the follower chooses a higher capacity than the leader and for high uncertainty values the leader chooses a higher capacity. Second, compared to the model without capacity choice, the monopolist and the follower invest later in a higher capacity for higher values of uncertainty. However, the leader will invest earlier in a higher capacity for higher values of uncertainty. The reverse results apply for lower values of uncertainty.

1 Introduction

When entering a new market it is not only the timing that is important, but also the scale of the investment. By investing at a large scale the firm takes a risk in case of uncertain demand. In particular, revenue may be too low to defray the investment cost if ex-post demand turns out to be disappointingly low. On the other hand, large scale investment gives a high revenue in case of a high demand realization and makes it less attractive for other firms to enter the same market and thus reduce demand for the incumbent firm.

This paper analyzes the optimal timing and size of this entry problem by considering a firm's capital investment project where undertaking the investment implies that the firm obtains a production plant. In particular, the firm can decide how much output the production plant is able to produce where the amount increases with the sunk cost investment. This is a real option problem, but, however, the bulk of real option models only determines the optimal timing of an investment project of given size (see Dixit and Pindyck (1994) and Trigeorgis (1996) for an overview). This also holds for the strategic real option models where competition between firms is taken into account. The latter area is surveyed in Grenadier (2000) and in Huisman *et al.* (2004).

We consider a monopoly setting as well as a duopoly setting and compare the results with the standard models in which the firms do not have the capacity choice. For the monopoly case where our starting point is Dixit (1993) (see also Decamps *et al.* (2006)) we find that for higher levels of uncertainty the monopolist invests later in a higher quantity. This result is comparable to the result in Dangl (1999). The difference in the setup is that Dangl (1999) assumes a slightly different inverse demand curve and furthermore assumes that the firm can adjust its output in downtimes. Bøckman *et al.* (2008) apply the model of Dangl (1999) to study the investment in small hydropower projects. Bar-Ilan and Strange (1999) compare lumpy investment with incremental investment. Our results are comparable for their lumpy investment setup, uncertainty delays investment and increases the size.

In the duoploy model, our main results are the following. First, for low uncertainty values the follower chooses a higher capacity than the leader and for high uncertainty values the leader chooses a higher capacity. Second, compared to the model without capacity choice, the monopolist and the follower invest later in a higher capacity for higher values of uncertainty. However, the leader will invest earlier in a higher capacity for higher values of uncertainty. The inverse results apply for lower values of uncertainty. Wu (2006) also studies a duopoly model in which firms can choose the timing and the size of their investment. In his setup the market is growing until some uncertain point in time and decreasing afterwards. As the first investor knows that the market will start decreasing some time in the future it will choose a smaller capacity than the second investor. In this way the first investor can make sure that it will be a monopolist in case the market has decreased enough.

2 Monopoly

We consider a framework with one firm that can undertake an investment to enter a market. The price at time t in this market is given by

$$P\left(t\right)=X\left(t\right)\left(1-\eta Q\left(t\right)\right), \tag{1}$$

where $Q\left(t\right)$ is total market output, η a constant, and $X\left(t\right)$ an exogeneous shock process. We assume that $X\left(t\right)$ follows a geometric Brownian motion:

$$dX\left(t\right)=\mu X\left(t\right)dt+\sigma X\left(t\right)d\omega\left(t\right), \tag{2}$$

in which $\mu>0$ is the growth rate, $d\omega\left(t\right)$ is the increment of a Wiener process, and $\sigma>0$ is a constant. The inverse demand function (1) is a special case of, e.g., (Dixit and Pindyck, 1994, Chapter 9), where they have $P=XD\left(Q\right)$ with $D\left(Q\right)$ unspecified. Inverse demand being linear in quantity has been adopted also in, e.g., Pindyck (1988), He and Pindyck (1992), Aguerrevere (2003), and Wu (2006). The firm is risk neutral and discounts against rate $r > \mu$.

A firm can become active on this market by investing in capacity. A unit of capacity costs δ. This implies that a firm investing in a plant with capacity Q, incurs investment costs being equal to δQ. The firm always produces up to capacity. In Subsection 2.1 we assume that the firm can invest only once. We study the case where the firm can make two investments in Subsection 2.2. We compare the model of Subsection 2.1 with the same model but without capacity choice in Subsection 2.3.

2.1 One Investment

In this subsection we study the market entry of a single firm. The corresponding investment problem is solved as an optimal stopping problem in dynamic programming. Let V denote the value of the firm then the investment problem that the firm is facing can be formalized as follows:

$$V\left(X\left(t\right),Q\right) = \max_{T \geq t, Q \geq 0} E\left[\exp\left(-rT\right)\left(\int_{\tau=0}^{\infty} QX\left(\tau\right)\left(1-\eta Q\right)d\tau - \delta Q\right)\right],$$

where T is the stopping time at which the investment is made and Q the quantity that the firm chooses. The expectation is conditional on the information that is available at time t. Let X^* be the value of X at which the firm is indifferent between investing and not investing. The corresponding quantity is denoted by $Q^*\left(X^*\right)$. For $X > X^*$ we are in the stopping region where it is optimal to invest immediately. When $X < X^*$ demand is (still) too low to undertake the investment. Then we are in the continuation region where the firm thus waits with investing. The optimal investment policy can be found in two steps. In the first step, given the current level of the geometric Brownian motion, X, the corresponding optimal value of Q is found by solving

$$\max_{Q \geq 0} E\left[\int_{\tau=0}^{\infty} QX\left(\tau\right)\left(1-\eta Q\right)d\tau - \delta Q\right].$$

The solution is given by

$$Q^*\left(X\right) = \frac{1}{2\eta}\left(1 - \frac{\delta\left(r-\mu\right)}{X}\right). \tag{3}$$

In the next step the optimal investment threshold X^* is derived. Appendix A.1 shows the derivation of the results that are presented in the next proposition.

Proposition 1 *The value of the monopolist that can invest only once is equal to*

$$V\left(X\right) = \begin{cases} AX^{\beta} & \text{if } X < X^*, \\ \frac{\left(X-\delta\left(r-\mu\right)\right)^2}{4X\eta\left(r-\mu\right)} & \text{if } X \geq X^*, \end{cases}$$

where the optimal investment trigger X^* and the corresponding optimal capacity level Q^* are given by:

$$X^* = \frac{\beta+1}{\beta-1}\delta\left(r-\mu\right), \tag{4}$$

$$Q^* = \frac{1}{\left(\beta+1\right)\eta}. \tag{5}$$

Furthermore, it holds that

$$\beta = \tfrac{1}{2} - \tfrac{\mu}{\sigma^2} + \sqrt{\left(\tfrac{1}{2} - \tfrac{\mu}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}},$$

$$A = \frac{\delta\left(\frac{\beta+1}{\beta-1}\delta\left(r-\mu\right)\right)^{-\beta}}{\left(\beta^2-1\right)\eta}.$$

Note that equation (5) is equivalent to equation (34) in Dixit (1993)[1]. From equation (3) we conclude that the optimal capacity level is increasing in X, indicating the level of demand at the moment of investment. At a higher level of X it is profitable for the firm to have a higher capacity so that the total profit flow increases.

Next we carry out some comparative statics analysis. First of all, we have (cf. Dixit and Pindyck (1994)): $\frac{\partial\beta}{\partial\sigma} < 0$, $\frac{\partial\beta}{\partial\mu} < 0$, and $\frac{\partial\beta}{\partial r} > 0$. Furthermore, differentiating (4) and (5) with respect to β gives

$$\frac{\partial X^*}{\partial\beta} = -\frac{2\delta\left(r-\mu\right)}{\left(\beta-1\right)^2} < 0, \tag{6}$$

$$\frac{\partial Q^*}{\partial\beta} = -\frac{1}{\left(\left(\beta+1\right)\eta\right)^2} < 0. \tag{7}$$

We conclude that, like the standard real options result, increased uncertainty raises X^* and thus delays investment. However, here we also find that increased uncertainty raises Q^* as well. This confirms Dixit (1993) who concludes that greater volatility systematically leads to the adoption of larger projects. Figure 1 presents the uncertainty results for a specific example.

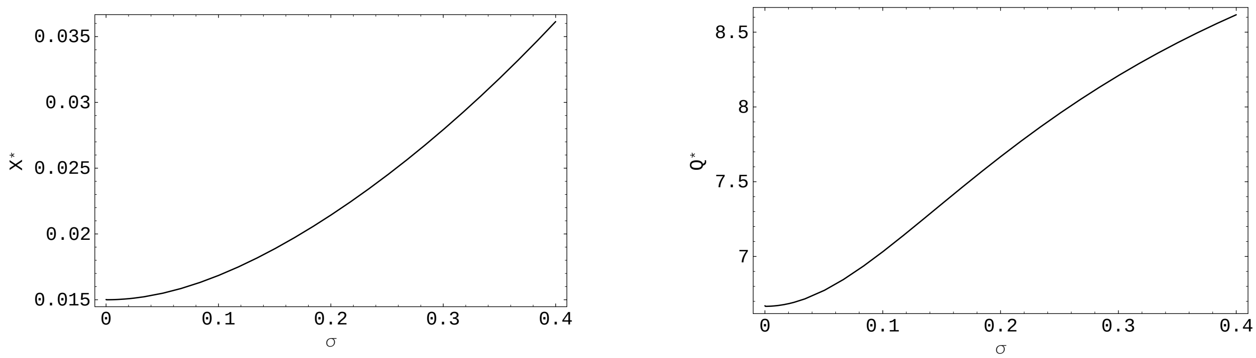
Furthermore, we can derive from equation (5) that if uncertainty goes to infinity (if σ goes to ∞, then β goes to 1) the optimal capacity approaches $\frac{1}{2\eta}$ from below, which is the optimal output level for a monopolist in the corresponding static Cournot game in which there are no investment costs associated with the capacity, i.e. the capacity is already in place.

2.2 Two Investments

In this subsection we investigate the consequences for the investment policy if the firm has two (instead of one) investment opportunities. The first investment brings the capacity of the firm from 0 to Q_1 and the second investment from Q_1 to Q_2. To rule out disinvestment we assume that $Q_2 > Q_1 > 0$. The model is solved backwards. This means that first for a given capacity level Q_1 the second investment is analyzed. After that the first investment is studied given the optimal investment behavior for the second investment. We refer the reader to Appendix A.2 for the derivation of the following proposition.

[1] To see this, note that the total investment cost in Dixit (1993) is denoted by K and the profit flow in that paper is equal to PX, where P follows a geometric Brownian motion and X is equal to $F(K)$, being the production function. In our model the total investment cost is equal to δQ and the profit flow is equal to $XQ\left(1-\eta Q\right)$, where X follows a geometric Brownian motion.

Equation (5) can be found by taking $F(K) = \left(1-\eta\frac{K}{\delta}\right)\frac{K}{\delta}$ and maximizing $\frac{(F(K))^\beta}{K^\beta}$ with respect to K as suggested in Dixit (1993).



(a) Investment trigger

(b) Quantity

Figure 1: Optimal investment trigger X^* and optimal quantity Q^* as function of σ. The plots are based on the following parameter values: $\mu = 0.05$, $r = 0.1$, $\delta = 0.1$, and $\eta = 0.05$.

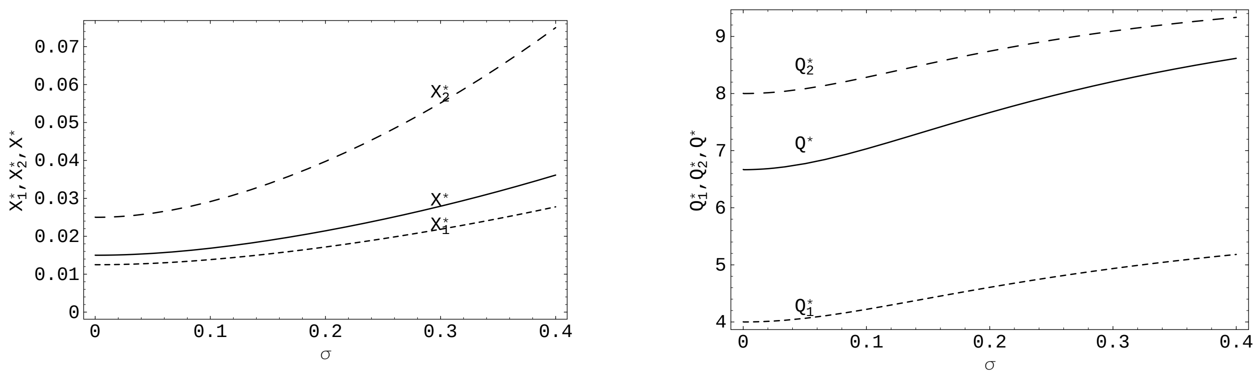
Proposition 2 *Consider a monopolist that can invest twice in time. The optimal investment triggers X_1^* and X_2^* and the corresponding optimal capacity levels Q_1^* and Q_2^* are implicitly given by the following equations:*

$$
\begin{aligned}
1 - \frac{\beta Q_1^* \eta}{(1 - \eta Q_1^*)} - 2\left(\frac{\beta(1 - 2\eta Q_1^*)}{(\beta + 1)(1 - \eta Q_1^*)}\right)^{\beta} &= 0, && (8)\\
X_1^*(Q_1^*) &= \frac{\beta\delta(r - \mu)}{(\beta - 1)(1 - \eta Q_1^*)}, && (9)\\
X_2^*(Q_1) &= \frac{(\beta + 1)\delta(r - \mu)}{(\beta - 1)(1 - 2\eta Q_1)}, && (10)\\
Q_2^*(Q_1) &= \frac{1 + (\beta - 1)\eta Q_1}{(\beta + 1)\eta}. && (11)
\end{aligned}
$$

In Figure 2 the relationship of the investment triggers and the optimal levels of capacity with the level of uncertainty are plotted. From Figure 2 we conclude that adding an additional investment opportunity does not lead to new economic insights, since the result is qualitatively similar. If we compare it to the case where the firm can invest only once,we conclude that, first the firm makes its first investment earlier. Second, the size of the first investment is smaller. Third, the second investment is later. Fourth, the final capacity of the firm is larger. Fifth, the final capacity converges to $\frac{1}{2\eta}$ if uncertainty goes to infinity (as in the one investment case).

2.3 Comparison

In this subsection we analyze the influence of capacity choice. We do so by taking the monopoly model with one investment as basis. We compare the model with capacity choice with the model without capacity choice. Let the capacity in this latter model be equal to Q_M^{fix}, then the investment thresholds is equal to



(a) Investment trigger

(b) Quantity

Figure 2: Optimal investment triggers X_1^*, X_2^*, and X^* and optimal quantities Q_1^*, Q_2^*, and Q^* as function of σ. The plots are based on the following parameter values: $\mu = 0.05$, $r = 0.1$, $\delta = 0.1$, and $\eta = 0.05$.‘

(substitute Q_M^{fix} into equation (37))

$$X_M^{fix} = \frac{\beta\delta\,(r-\mu)}{(\beta-1)\left(1-\eta Q_M^{fix}\right)}. \tag{12}$$

The difference of the investment behavior in the two models can be explained by looking at the difference of the two investment thresholds, i.e. by investigating

$$\frac{X_M^{fix}}{X_M^*} = \frac{\beta}{(\beta+1)\left(1-\eta Q_M^{fix}\right)}. \tag{13}$$

We study the effect of uncertainty in the following way. Given a particular level of uncertainty σ let us fix Q_M^{fix} to the corresponding Q_M^*. For that particular level of uncertainty the investment thresholds are equal. From (13) we conclude that X_M^{fix} will be lower than X_M^* for higher values of uncertainty and vice versa. Knowing that Q_M^* increases with uncertainty we conclude that, compared to the model without capacity choice, for a higher level of uncertainty the monopolist will invest later in a larger capacity and vice versa.

We illustrate this effect in the following example. We take $r = 0.1$, $\mu = 0.05$, $\delta = 0.1$, $\eta = 0.05$. We set the capacities equal for $\sigma = 0.1$. This gives $Q_M^{fix} = 7.03$. In the next figures the thresholds and the optimal quantities for the two models are plotted as function of the uncertainty paramater. We indeed see that lowering uncertainty leads to earlier investment with a smaller capacity and vice versa higher uncertainty leads to later investment with a higher capacity.

3 Duopoly

This section adds competition to the investment problem in which we determine the optimal timing and quantity of the firm. Therefore we extend the model of the previous section with an additional firm. We

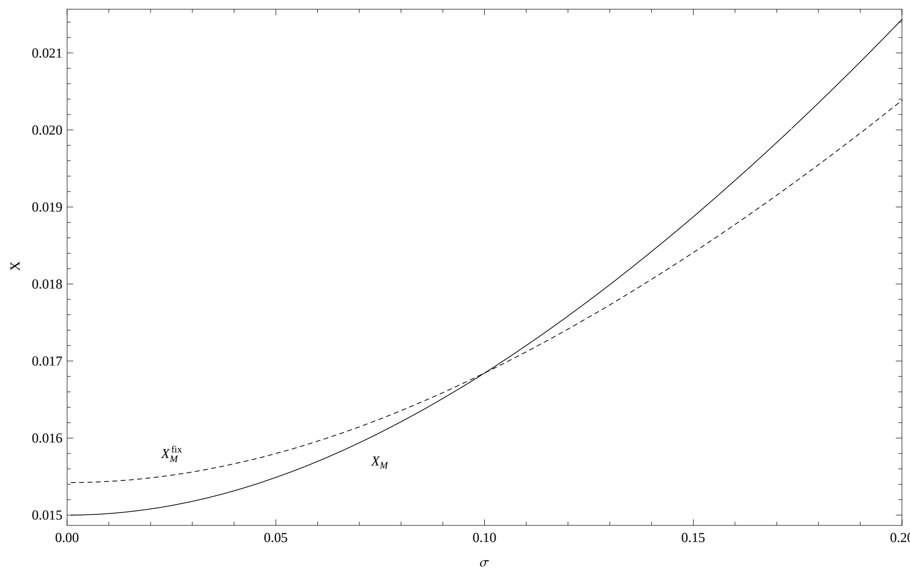


Figure 3: Comparison of triggers X_M and X_M^{fix} for the following parameter values $\mu = 0.05$, $r = 0.1$, $\delta = 0.1$, and $\eta = 0.05$.

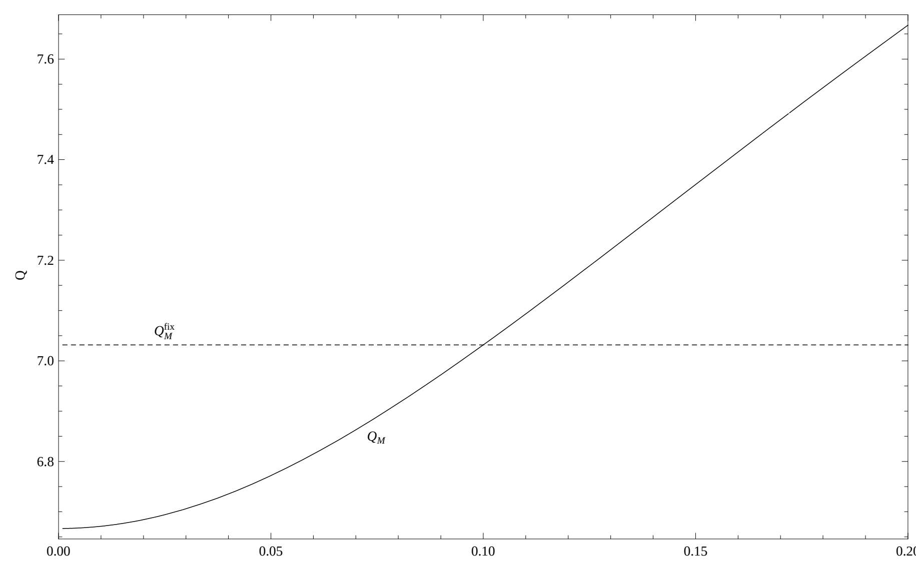


Figure 4: Comparison of quantities Q_M and Q_M^{fix} for the following parameter values $\mu = 0.05$, $r = 0.1$, $\delta = 0.1$, and $\eta = 0.05$.

denote by Q_L (Q_F) the capacity of the first (second) investor, so that $Q = Q_L + Q_F$. The derivations of the propositions are given in Appendix B. In Subsection 3.1 we study the game in which each firm can make a single capacity investment and the firm roles are assigned exogenous to the firms. The next step is to assign the firm roles endogenous, which is done in Subsection 3.2. We compare that model with the same model without capacity choice in Subsection 3.3.

3.1 Exogenous firm roles

As is standard in the literature, also this game is solved backwards in time, i.e. we start with the analysis of the follower leading to the optimal follower quantity Q_F^* and the follower's threshold level of the stochastic demand parameter denoted by X_F^*. We do this for every given level of the leader quantity, so that we obtain functions $Q_F^*(Q_L)$ and $X_F^*(Q_L)$.

Proposition 3 *Given the current level of X and the capacity level Q_L of the leader, the optimal capacity level for the follower $Q_F^*(X, Q_L)$ is equal to*

$$Q_F^*(X, Q_L) = \frac{1}{2\eta}\left(1 - \eta Q_L - \frac{\delta(r-\mu)}{X}\right). \tag{14}$$

The value function of the follower $V_F^*(X, Q_L)$ is given by

$$V_F^*(X, Q_L) = \begin{cases} A_F(Q_L) X^{\beta} & \text{if } X < X_F^*(Q_L), \\ \frac{(X(1-\eta Q_L) - \delta(r-\mu))^2}{4X\eta(r-\mu)} & \text{if } X \geq X_F^*(Q_L), \end{cases} \tag{15}$$

where

$$X_F^*(Q_L) = \frac{\beta+1}{\beta-1}\frac{\delta(r-\mu)}{1-\eta Q_L}, \tag{16}$$

$$A_F(Q_L) = \left(\frac{(\beta-1)(1-\eta Q_L)}{(\beta+1)\delta(r-\mu)}\right)^{\beta}\frac{(1-\eta Q_L)\delta}{(\beta+1)(\beta-1)\eta}, \tag{17}$$

so that

$$Q_F^*(Q_L) \equiv Q_F^*(X_F^*(Q_L), Q_L) = \frac{\beta+1}{\beta-1}\frac{\delta(r-\mu)}{1-\eta Q_L}.$$

The next step is to study the problem of the leader. The leader takes into account the strategy of the follower. The follower has two possibilities: investing at the same time as the leader or investing later. Given the current level of X the leader knows that the follower will invest later if it chooses its capacity Q_L such that $X_F^*(Q_L)$ is larger than X. We refer to this strategy as a *deterrence* strategy. Of course, normally entry by the follower is deterred only temporarily, because the follower does enter at the moment that X hits $X_F^*(Q_L)$. Note that from equation (16) it follows that the leader can (temporarily) deter the investment of the follower for any X by choosing Q_L larger than $\widehat{Q}_L(X)$, where

$$\widehat{Q}_L(X) = \frac{1}{\eta}\left(1 - \frac{(\beta+1)\delta(r-\mu)}{(\beta-1)X}\right). \tag{18}$$

This also implies that the follower immediately invests if $Q_L \leq \widehat{Q}_L$. There exists an X level, which will be denoted by X_2^{det}, such that for X larger than X_2^{det}, at which it is not optimal for the leader to play the deterrence strategy. In other words, for these X values it holds that the optimal leader capacity in case of a deterrence strategy, Q_L^{det}, is lower than $\widehat{Q}_L(X)$. Then the level of demand is that high that it is always optimal for the follower to enter at the same time as the leader. Furthermore, for small values of X the leader will not play the deterrence strategy, as then the demand level is that low that it will result in a negative value. This lower bound for the deterrence strategy is labeled X_1^{det}. The following proposition summarizes the deterrence strategy.

Proposition 4 *The leader will consider the deterrence strategy whenever the current level of X is in the interval $\left(X_1^{det}, X_2^{det}\right)$, where X_1^{det} is implicitly defined by*

$$\frac{X_1^{det}}{r-\mu} - \delta - \left(\frac{X_1^{det}(\beta-1)}{(\beta+1)\delta(r-\mu)}\right)^{\beta} \frac{\delta}{\beta-1} = 0, \tag{19}$$

and

$$X_2^{det} = \frac{2(\beta+1)}{\beta-1}\delta(r-\mu). \tag{20}$$

The optimal capacity level $Q_L^{det}(X)$ of the leader for its deterrence strategy is implicitly determined by

$$\frac{X\left(1-2\eta Q_L^{det}\right)}{r-\mu} - \delta - \left(\frac{X(\beta-1)\left(1-\eta Q_L^{det}\right)}{(\beta+1)\delta(r-\mu)}\right)^{\beta} \frac{\left(1-(\beta+1)\eta Q_L^{det}\right)\delta}{(\beta-1)\left(1-\eta Q_L^{det}\right)} = 0, \tag{21}$$

The value function for the leader's deterrence strategy, when the leader invests at X, $V_L^{det}(X)$, is given by

$$V_L^{det}(X) = \frac{XQ_L^{det}(X)\left(1-\eta Q_L^{det}(X)\right)}{r-\mu} - \delta Q_L^{det}(X) - \left(\frac{X(\beta-1)\left(1-\eta Q_L^{det}(X)\right)}{(\beta+1)\delta(r-\mu)}\right)^{\beta} \frac{\delta Q_L^{det}(X)}{\beta-1}. \tag{22}$$

Given that X is sufficiently low, for the deterrence strategy the optimal investment threshold X_L^{det} and the corresponding quantity Q_L^{det} is given by

$$\begin{aligned} X_L^{det} &= \frac{\beta+1}{\beta-1}\delta(r-\mu), \\ Q_L^{det} &= \frac{1}{(\beta+1)\eta}. \end{aligned}$$

Alternatively the leader can choose an *accommodation* strategy. If it chooses its capacity Q_L lower than or equal to $\widehat{Q}_L(X)$, the investment will trigger the follower to make its investment immediately afterwards. Since the leader is the first firm that makes the investment decision, we assume that the leader becomes the Stackelberg leader in the duopoly that is formed after the two investments are made. As with the deterrence strategy there exists an X interval in which the leader will consider this strategy. For low X values, the optimal leader quantity in the accommodation strategy is too high, i.e. $Q_L^{acc}(X) > \widehat{Q}_L(X)$, to trigger direct follower investment. In other words there exists an X level, denoted by X_1^{acc} such that the leader can only play the accommodation strategy for X values larger than X_1^{acc}. In the following proposition the accommodation strategy of the leader is described.

Proposition 5 *The leader will consider the accommodation strategy whenever the current level of X is larger than or equal to X_1^{acc}, where*

$$X_1^{acc} = \frac{\beta+3}{\beta-1}\delta\left(r-\mu\right). \tag{23}$$

The optimal capacity level Q_L^{acc} for the leader's accommodation strategy is given by

$$Q_L^{acc}\left(X\right) = \frac{1}{2\eta}\left(1-\frac{\delta\left(r-\mu\right)}{X}\right). \tag{24}$$

The value of the accommodation strategy, when the leader invests at X, is equal to

$$V_L^{acc}\left(X\right) = \frac{\left(X-\delta\left(r-\mu\right)\right)^2}{8X\eta\left(r-\mu\right)}. \tag{25}$$

The optimal investment threshold for the accommodation strategy is given by

$$X_L^{acc} = \frac{\beta+1}{\beta-1}\delta\left(r-\mu\right), \tag{26}$$

$$Q_L^{acc} = \frac{1}{\left(\beta+1\right)\eta}. \tag{27}$$

Since $X_L^{acc} < X_1^{acc}$, the optimal investment threshold X_L^{acc} has in fact no meaning since the demand parameter has to admit at least the value X_1^{acc} before the follower invests at the same time as the leader. In Figure 5 the functions Q_L^{det}, $\widehat{Q}_L$, and Q_L^{acc} are plotted as function of X. The boundary values X_1^{det}, X_2^{det}, and X_1^{acc} are also plotted in that figure. One can see that X_1^{det} is the smallest X value for which the leader chooses a positive capacity. The follower postpones its investment if the leader chooses a capacity that is larger than $\widehat{Q}_L\left(X\right)$, i.e. above the dashed line. That is why X_2^{det} is equal to the intersection point of $\widehat{Q}_L\left(X\right)$ and Q_L^{det}. For X larger than or equal to X_2^{det} the optimal capacity of the leader in the deterrence strategy, $Q_L^{det}\left(X\right)$, is smaller than the capacity that ensures deterrence, $\widehat{Q}_L\left(X\right)$. Furthermore, the leader can succesfully use its accommodation strategy for X values larger than X_1^{acc}. Below that boundary the optimal capacity of the leader is larger than the capacity that ensures direct entry of the follower, $\widehat{Q}_L\left(X\right)$. The optimal capacity of the leader depends on the strategy that the leader chooses: deterrence or accommodation. For X less than X_1^{acc} the leader can only choose deterrence and for X larger than X_2^{det} the leader can only choose accommodation. For $X \in \left(X_1^{acc}, X_2^{det}\right)$ either the deterrence or the accommodation strategy maximizes the leader's value. In case X is less than X_L^{det} the leader will wait with investment until X hits X_L^{det} for the first time. The following proposition describes the optimal leader strategy.

Proposition 6 *Given the current level of X the optimal capacity of the leader is equal to*

$$Q_L^*\left(X\right) = \begin{cases} Q_L^{det}\left(X_L^{det}\right) & \text{if } X \in \left[0, X_L^{det}\right), \\ Q_L^{det}\left(X\right) & \text{if } X \in \left[X_L^{det}, \widehat{X}\right), \\ Q_L^{acc}\left(X\right) & \text{if } X \in \left[\widehat{X}, \infty\right), \end{cases}$$

where $\widehat{X}$ is such that

$$\widehat{X} = \inf\left(\widehat{X} \in \left(X_1^{acc}, X_2^{det}\right)\middle| V_L^{acc}\left(X\right) = V_L^{det}\left(X\right)\right). \tag{28}$$

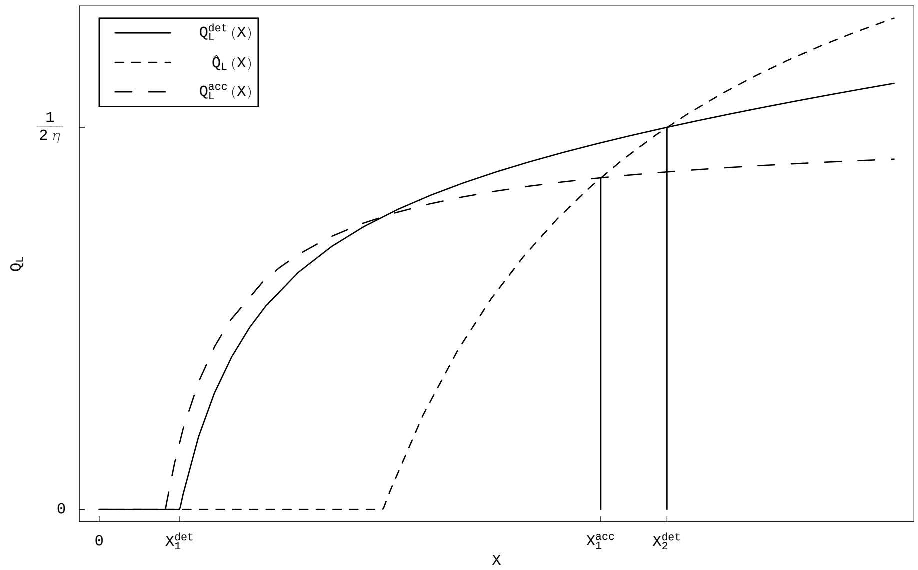


Figure 5: Q_L^{det}, $\widehat{Q}_L$, and Q_L^{acc} as function of X.

The value of the leader is given by

$$V_L^*(X) = \begin{cases} \left(\frac{X}{X_L^{det}}\right)^{\beta} V_L^{det}\left(X_L^{det}\right) & \text{if } X \in \left[0, X_L^{det}\right), \\ V_L^{det}(X) & \text{if } X \in \left[X_L^{det}, \widehat{X}\right), \\ V_L^{acc}(X) & \text{if } X \in \left[\widehat{X}, \infty\right). \end{cases} \tag{29}$$

All numerical examples show that $X_L^{det} < X_1^{acc}$, which implies that the leader will only play the accomodation strategy if the starting value of X is larger than $\widehat{X}$. The following proposition gives the investment threshold for the exogenous leader.

Proposition 7 *Given the current level of X the investment threshold for the exogenous leader is equal to*

$$X_L^* = \begin{cases} X_L^{det} & \text{if } X \in \left[0, X_L^{det}\right), \\ X & \text{if } X \in \left[X_L^{det}, \infty\right). \end{cases}$$

The optimal capacity for the leader $Q_L^*(X)$ and the optimal capacity of the follower $Q_F^*(X)$ are plotted in Figure 6. Figure 7 shows the value functions for the leader and the follower.

The following proposition is proved in Appendix B.

Proposition 8 *The strategy boundaries X_1^{det}, X_2^{det}, and X_1^{acc} are increasing with uncertainty. Furthermore, the region in which the leader can choose between the two strategies $X \in \left(X_1^{acc}, X_2^{det}\right)$ decreases with uncertainty.*

Since X_1^{acc} is increasing with uncertainty, we conclude that the X region in which the leader uses the deterrence strategy increases with uncertainty.

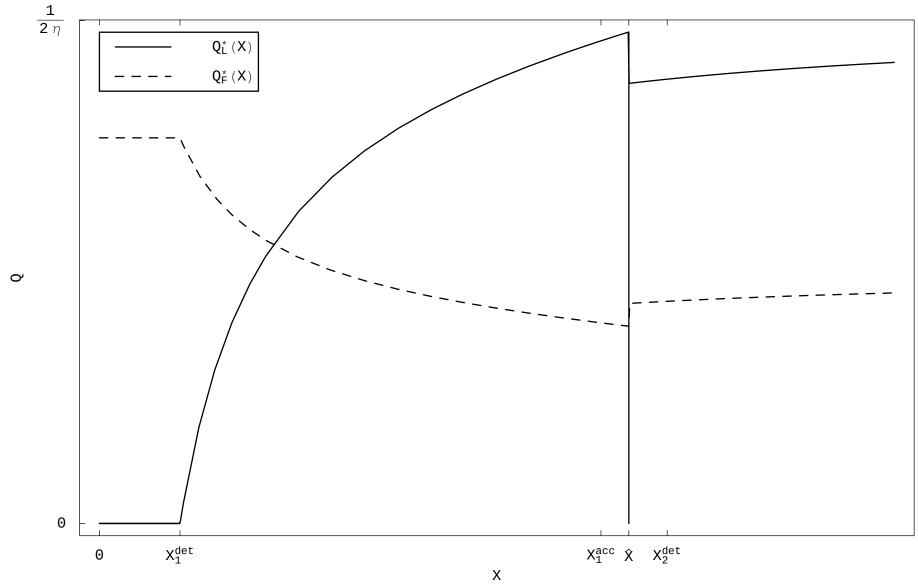


Figure 6: Optimal quantities for the leader (Q_L^*) and the follower (Q_F^*) as function of X.

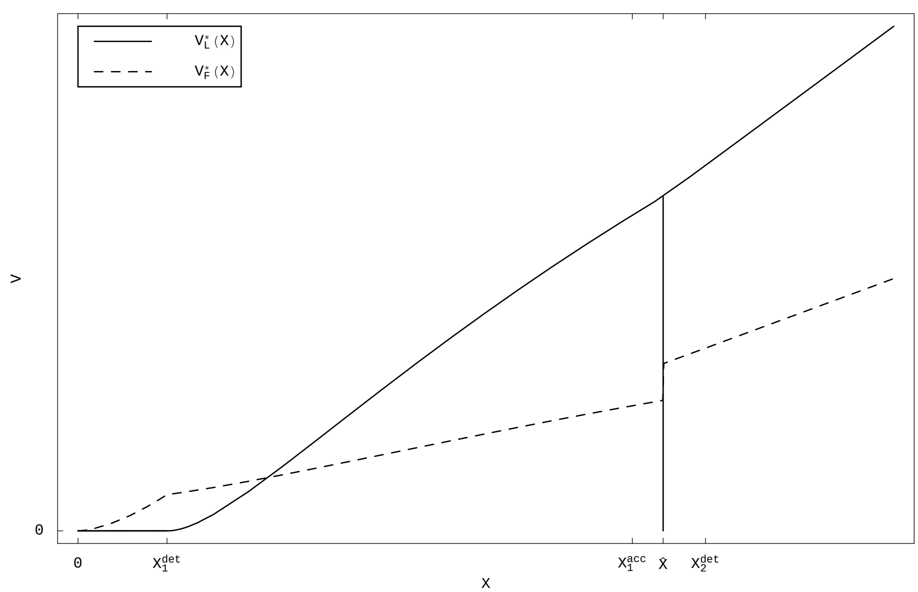


Figure 7: Optimal value functions for the leader (V_L^*) and follower (V_F^*) as function of the X at which the leader invests.

3.2 Endogeneous firm roles

In this subsection we use the knowledge of the previous subsection to analyze the model with endogenous firm roles. We use the preemption principle that is described in Fudenberg and Tirole (1985) and extended to stochastic models in e.g. Huisman (2001). The preemption trigger X_P is the solution of the following equation

$$V_L^*\left(X_P\right) = V_F^*\left(X_P, Q_L^*\left(X_P\right)\right).$$

The following theorem describes the equilibrium in the capacity choice game. The proof is given in the Appendix.

Theorem 1 *The first firm (leader) invests at X_P in capacity $Q_L^*\left(X_P\right)$ and the second firm (follower) invests at X_F^* in capacity $Q_F^*\left(Q_L^*\left(X_P\right)\right)$.*

It turns out that for low values of uncertainty the leader capacity is lower than the follower capacity and for high values of uncertainty the leader chooses a larger capacity than the follower.

3.3 Comparison

Like in Subsection 2.3 we analyze the addition of the capacity choice by comparing the model with and without capacity choice. Fixing the quantity of the leader to Q_L^{fix} and the quantity of the follower Q_F^{fix} leads to the following expression for the investment trigger of the follower:

$$X_F^{fix} = \frac{\beta\delta\left(r-\mu\right)}{\left(\beta-1\right)\left(1-\eta\left(Q_L^{fix}+Q_F^{fix}\right)\right)}.$$

The investment trigger of the leader, X_P^{fix}, is implicitly defined by the following equation

$$\frac{X_P^{fix}Q_L^{fix}\left(1-\eta Q_L^{fix}\right)}{r-\mu} - Q_L^{fix}\delta + \left(\frac{X_P^{fix}}{X_F^{fix}}\right)^{\beta}\left(Q_F^{fix}\delta - \frac{X_F^{fix}Q_F^{fix}\left(1-\eta Q_F^{fix}\right)}{r-\mu}\right) = 0.$$

The comparison for the follower is comparable to the comparison of the monopolist, because at the time the follower invests, the leader has already invested, so that the follower cannot influence the decision of the leader anymore. So, compared to the model without capacity choice, for a higher level of uncertainty the follower will invest later in a larger capacity. For the leader we were not able to derive an analytical comparison of the thresholds in the two models. Therefore we do the comparison in an example. We have set $r = 0.1$, $\mu = 0.05$, $\delta = 0.1$, $\eta = 0.05$. That leads to $Q_L^{fix} = 4.94$ and $Q_F^{fix} = 5.30$. From Figure 8 we conclude that, compared to the model with fixed capacity, the leader invests earlier in the model with flexible capacity when uncertainty increases and later when uncertainty decreases. Although the investment trigger for the first investor in the model with flexible capacity goes up, it increases slower than the investment trigger of the first investor in the model with fixed capacity. Note that this result is opposite to the result that we

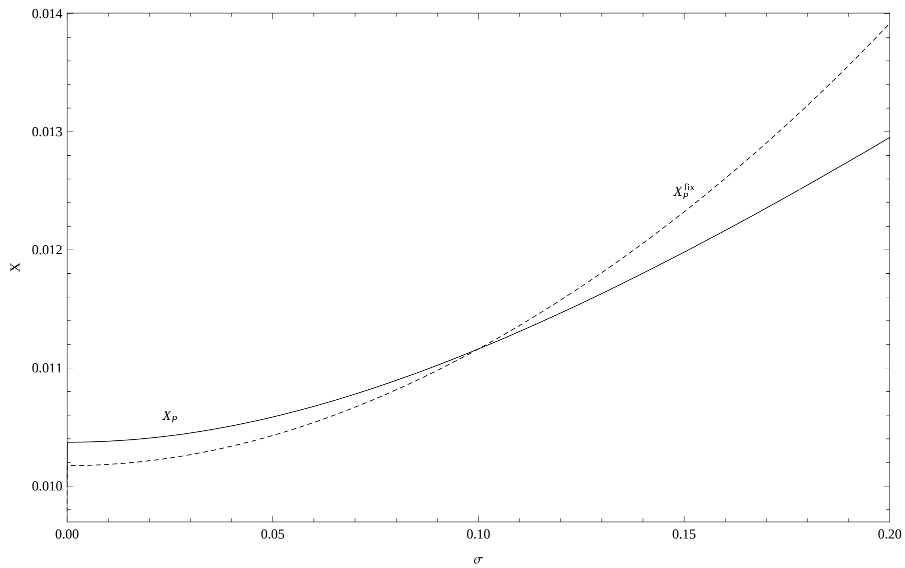


Figure 8: Comparison of triggers X_P and X_P^{fix} for the following parameter values $\mu = 0.05$, $r = 0.1$, $\delta = 0.1$, and $\eta = 0.05$.

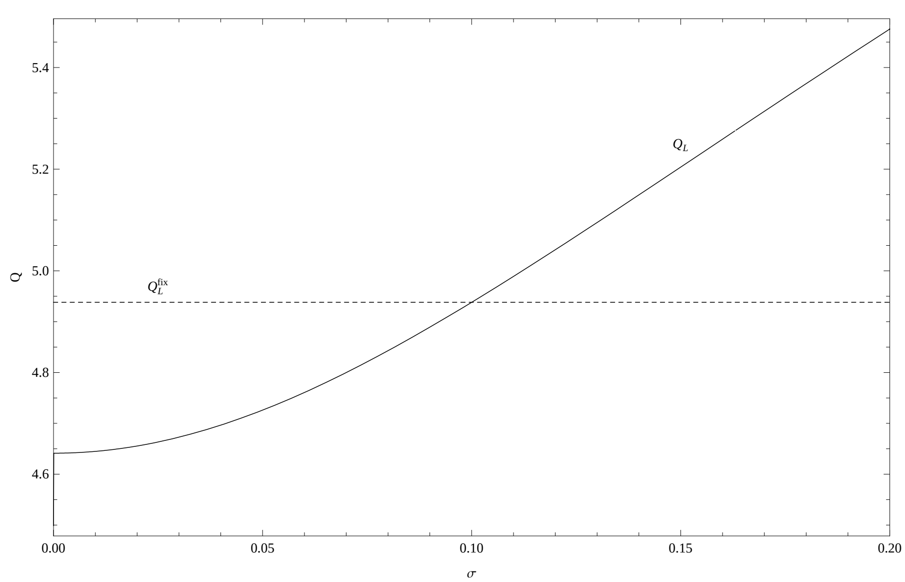


Figure 9: Comparison of quantities Q_L and Q_L^{fix} for the following parameter values $\mu = 0.05$, $r = 0.1$, $\delta = 0.1$, and $\eta = 0.05$.

found for the monopolist in Section 2.3. Furthermore, we conclude from Figure 9 that the leader will invest in a higher capacity when uncertainty increases and in a lower capacity when uncertainty decreases.

The results for the follower are indeed comparable to the results of the monopolist. Figure 10 shows that the investment trigger in the model with flexible capacity increases faster than the investment trigger in the model with fixed capacity in case the uncertainty increases. The capacity that the follower chooses in the model with flexible capacity increases with uncertainty (see Figure 11).

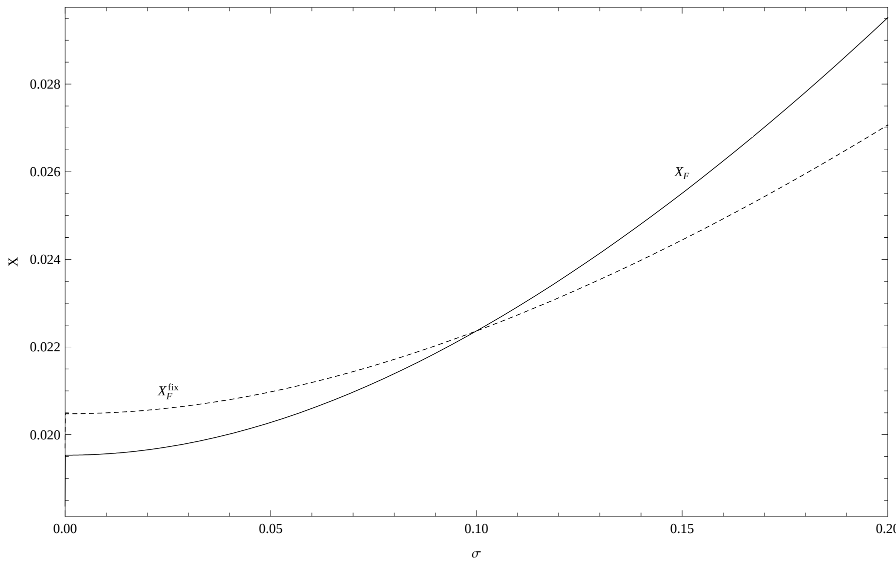


Figure 10: Comparison of triggers X_F and X_F^{fix} for the following parameter values $\mu = 0.05$, $r = 0.1$, $\delta = 0.1$, and $\eta = 0.05$.

4 Conclusions

A Monopoly

A.1 One Investment

A.1.1 Stopping Region

The profit of this firm at time t is denoted by $\pi(t)$ and is equal to

$$\pi(t) = P(t)\, Q(t) = X(t)\, Q(t)\, (1 - \eta Q(t))\,. \tag{30}$$

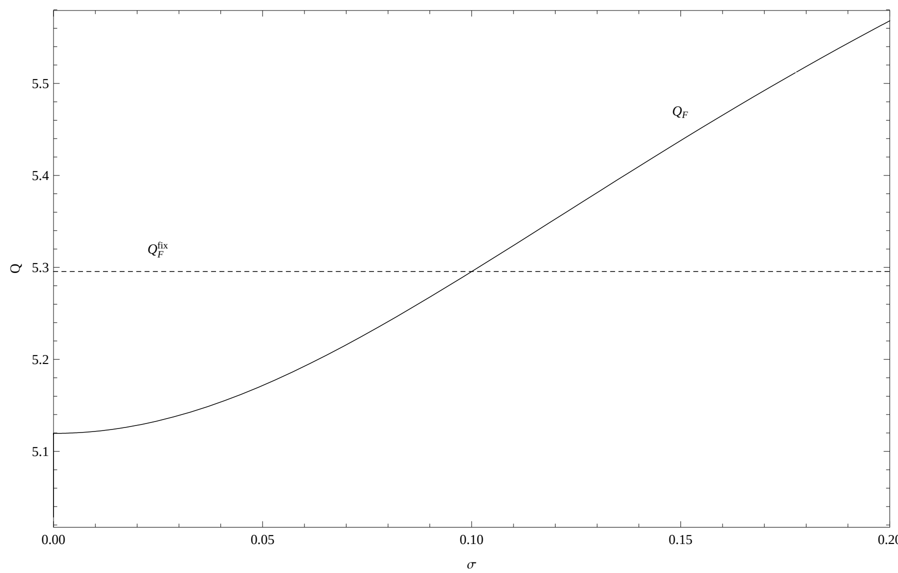


Figure 11: Comparison of quantities Q_F and Q_F^{fix} for the following parameter values $\mu = 0.05$, $r = 0.1$, $\delta = 0.1$, and $\eta = 0.05$.

We denote by $V(X, Q)$ the expected value of the firm at the moment of investment given that the current level of $X(t)$ is X and the firm invests in Q units of capital. Then it holds that

$$V(X, Q) = E\left[\int_{t=0}^{\infty} \pi(t) \exp(-rt)\, dt - \delta Q\right] = \frac{XQ(1-\eta Q)}{r-\mu} - \delta Q. \tag{31}$$

Maximizing with respect to Q gives the optimal capacity size Q^* for every given level of X :

$$Q^*(X) = \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{X}\right).$$

A.1.2 Continuation Region

Standard real options analysis (e.g, Dixit and Pindyck (1994)) shows that the value of the option to invest, denoted by F, is equal to

$$F(X) = AX^{\beta}, \tag{32}$$

where β is the positive root of the quadratic polynomial

$$\tfrac{1}{2}\sigma^2\beta^2 + \left(\mu - \tfrac{1}{2}\sigma^2\right)\beta - r = 0. \tag{33}$$

and is thus given by

$$\beta = \tfrac{1}{2} - \tfrac{\mu}{\sigma^2} + \sqrt{\left(\tfrac{1}{2} - \tfrac{\mu}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}}. \tag{34}$$

To determine the indifference level X^* we employ the value matching and smooth pasting conditions:

$$F(X^*) = V(X^*, Q), \tag{35}$$

$$\left.\frac{\partial F(X)}{\partial X}\right|_{X=X^*} = \left.\frac{\partial V(X,Q)}{\partial X}\right|_{X=X^*}. \tag{36}$$

Substitution of (31) and (32) into (35) and (36) and solving for X^* gives

$$X^*(Q) = \frac{\beta\delta(r-\mu)}{(\beta-1)(1-\eta Q)}. \tag{37}$$

From (3) and (37) we obtain the results that are presented in Proposition 1.

A.2 Two Investments

A.2.1 Second Investment

The value of the firm at the moment of the second investment when the capacity of the firm increases from Q_1 to Q_2 is equal to

$$V_2(X, Q_1, Q_2) = \frac{XQ_2(1-\eta Q_2)}{r-\mu} - \delta(Q_2 - Q_1). \tag{38}$$

Before the second investment the value is equal to

$$F_2(X, Q_1) = \frac{XQ_1(1-\eta Q_1)}{r-\mu} + A_2 X^\beta. \tag{39}$$

Let us denote the trigger of the second investment by X_2^*. The value matching and smooth pasting conditions are given by

$$\frac{XQ_1(1-\eta Q_1)}{r-\mu} + A_2 X^\beta = \frac{XQ_2(1-\eta Q_2)}{r-\mu} - \delta(Q_2 - Q_1), \tag{40}$$

$$\frac{Q_1(1-\eta Q_1)}{r-\mu} + \beta A_2 X^{\beta-1} = \frac{Q_2(1-\eta Q_2)}{r-\mu}. \tag{41}$$

Solving these equations gives

$$X_2^* = \frac{\beta}{\beta-1}\frac{(r-\mu)\delta}{(1-\eta(Q_1+Q_2))}, \tag{42}$$

$$A_2 = \frac{(X_2^*)^{1-\beta}}{\beta}\frac{(Q_2-Q_1)(1-\eta(Q_1+Q_2))}{r-\mu}. \tag{43}$$

The optimal Q_2 is determined by solving

$$\max_{Q_2>Q_1}\left[\frac{XQ_2(1-\eta Q_2)}{r-\mu} - \delta(Q_2-Q_1)\right]. \tag{44}$$

The first order condition is given by

$$\frac{X(1-\eta Q_2)}{r-\mu} - \frac{XQ_2\eta}{r-\mu} - \delta = 0, \tag{45}$$

which gives

$$Q_2^*(X) = \frac{1}{2\eta}\left(1 - \frac{(r-\mu)\delta}{X}\right). \tag{46}$$

Solving the system of equations (42) and (46) leads to the equations (8) and (9).

A.2.2 First Investment

The value of the firm at the moment of the first investment is equal to

$$V_1(X,Q_1) = \frac{XQ_1(1-\eta Q_1)}{r-\mu} - \delta Q_1 + A_2 X^{\beta}. \tag{47}$$

Before the first investment the value is given by

$$F_1(X) = A_1 X^{\beta}. \tag{48}$$

Value matching and smooth pasting results in the following equations:

$$\begin{aligned} A_1 X^{\beta} &= \frac{XQ_1(1-\eta Q_1)}{r-\mu} - \delta Q_1 + A_2 X^{\beta}, &(49)\\ \beta A_1 X^{\beta-1} &= \frac{Q_1(1-\eta Q_1)}{r-\mu} + \beta A_2 X^{\beta-1}, &(50)\end{aligned}$$

which give

$$\begin{aligned} X_1^* &= \frac{\beta}{\beta-1}\frac{(r-\mu)\delta}{(1-\eta Q_1)}, &(51)\\ A_1 &= A_2 + \frac{(X_1^*)^{1-\beta}}{\beta}\frac{Q_1(1-\eta Q_1)}{r-\mu}. &(52)\end{aligned}$$

The optimal Q_1 can be determined by maximizing the value of the firm at the moment of the first investment

$$\max_{Q_1\geq 0}\left[\frac{XQ_1(1-\eta Q_1)}{r-\mu} - \delta Q_1 + A_2(Q_1)X^{\beta}\right]. \tag{53}$$

The first order condition is given by

$$\frac{X(1-\eta Q_1)}{r-\mu} - \frac{XQ_1\eta}{r-\mu} - \delta + \frac{\partial A_2(Q_1)}{\partial Q_1}X^{\beta} = 0. \tag{54}$$

Note that

$$\begin{aligned} A_2(Q_1) &= \frac{\delta(1-2\eta Q_1)(X_2^*(Q_1))^{-\beta}}{(\beta-1)(\beta+1)\eta}\\ &= \frac{\delta(1-2\eta Q_1)}{(\beta-1)(\beta+1)\eta}\left(\frac{\beta+1}{\beta-1}\frac{(r-\mu)\delta}{(1-2\eta Q_1)}\right)^{-\beta}\\ &= \frac{\delta}{(\beta-1)(\beta+1)\eta}\left(\frac{(\beta+1)(r-\mu)\delta}{(\beta-1)}\right)^{-\beta}(1-2\eta Q_1)^{\beta+1}, &(55)\end{aligned}$$

so that

$$\begin{aligned} \frac{\partial A_2(Q_1)}{\partial Q_1} &= \frac{\delta}{(\beta-1)(\beta+1)\eta}\left(\frac{(\beta+1)(r-\mu)\delta}{(\beta-1)}\right)^{-\beta}(\beta+1)(1-2\eta Q_1)^{\beta}\cdot -2\eta\\ &= -\frac{2\delta}{(\beta-1)}\left(\frac{(\beta+1)(r-\mu)\delta}{(\beta-1)(1-2\eta Q_1)}\right)^{-\beta}\\ &= -\frac{2\delta(X_2^*(Q_1))^{-\beta}}{(\beta-1)}. &(56)\end{aligned}$$

Substitution of (56) into (54) gives

$$\frac{X(1-\eta Q_1)}{r-\mu} - \frac{XQ_1\eta}{r-\mu} - \delta - \frac{2\delta}{(\beta-1)}\left(\frac{X}{X_2^*(Q_1)}\right)^{\beta} = 0. \tag{57}$$

Substitution of (8) and (51) into equation (57) results in equation (10).

B Duopoly

Finally we establish our economic results based on the outcome of this game.

B.1 Follower

The value function of the follower at the moment of investment is denoted by V_F^*, it depends on X, Q_L, and Q_F, and is equal to

$$V_F^*(X, Q_L, Q_F) = \frac{XQ_F(1 - \eta(Q_L + Q_F))}{r - \mu} - \delta Q_F. \tag{58}$$

Maximizing with respect to Q_F gives the optimal capacity size of the follower, given the level X and the capacity size of the leader Q_L:

$$Q_F^*(X, Q_L) = \frac{1}{2\eta}\left(1 - \eta Q_L - \frac{\delta(r - \mu)}{X}\right). \tag{59}$$

Before the follower has invested, thus when $X < X_F^*(Q_L)$, the firm holds an option to invest. the option value is

$$F_F(X) = A_F X^\beta. \tag{60}$$

Solving the corresponding value matching and smooth pasting conditions gives

$$X_F^*(Q_L, Q_F) = \frac{\beta}{\beta - 1}\frac{\delta(r - \mu)}{(1 - \eta(Q_L + Q_F))}. \tag{61}$$

We conclude that (after solving the system of equations (59) and (61))

$$X_F^*(Q_L) = \frac{\beta + 1}{\beta - 1}\frac{\delta(r - \mu)}{1 - \eta Q_L}, \tag{62}$$

$$Q_F^*(Q_L) = \frac{1 - \eta Q_L}{(\beta + 1)\eta}. \tag{63}$$

Substitution (63) into (58) gives equation (15).

B.2 Leader

The value function of the leader at the moment of investment for the deterrence strategy is given by

$$V_L^{det}(X, Q_L) = \frac{XQ_L(1 - \eta Q_L)}{r - \mu} - \delta Q_L - \left(\frac{X}{X_F^*(Q_L)}\right)^\beta \left(\frac{X_F^*(Q_L)Q_L\eta Q_F^*(Q_L)}{r - \mu}\right). \tag{64}$$

Substitution of (62) and (63) into this equation results in

$$V_L^{det}(X, Q_L) = \frac{XQ_L(1 - \eta Q_L)}{r - \mu} - \delta Q_L - \left(\frac{X(\beta - 1)(1 - \eta Q_L)}{(\beta + 1)\delta(r - \mu)}\right)^\beta \frac{\delta Q_L}{\beta - 1}. \tag{65}$$

Maximizing with respect to Q_L gives the following first order condition

$$\phi(X, Q_L) \equiv \frac{X(1 - 2\eta Q_L)}{r - \mu} - \delta - \left(\frac{X(\beta - 1)(1 - \eta Q_L)}{(\beta + 1)\delta(r - \mu)}\right)^\beta \frac{(1 - (\beta + 1)\eta Q_L)\delta}{(\beta - 1)(1 - \eta Q_L)} = 0. \tag{66}$$

Solving (66) gives $Q_L^{det}(X)$.

Setting $Q_L = 0$ in equation (66) gives equation (19). Define

$$\psi(X) = \frac{X}{r-\mu} - \delta - \left(\frac{X(\beta-1)}{(\beta+1)\delta(r-\mu)}\right)^{\beta} \frac{\delta}{\beta-1},$$

then we have that

$$\begin{aligned} \psi(0) &= -\delta < 0, \\ \psi(X_F^*(0)) &= \frac{\delta}{\beta-1} > 0, \\ \frac{\partial \psi(X)}{\partial X} &= \frac{1}{r-\mu}\left(1 - \frac{\beta}{\beta+1}\left(\frac{X(\beta-1)}{(\beta+1)\delta(r-\mu)}\right)^{\beta-1}\right). \end{aligned}$$

For $X \in (0, X_F^*(0))$ it holds that

$$\frac{\partial \psi(X)}{\partial X} > 0, \tag{67}$$

so that we have shown that X_1^{det} exists.

For the accommodation strategy the value function of the leader is given by

$$V_L^{acc}(X, Q_L) = \frac{XQ_L(1-\eta(Q_L + Q_F^*(Q_L)))}{r-\mu} - \delta Q_L. \tag{68}$$

Substitution of (63) into (68) and maximizing with respect to Q_L gives

$$Q_L^{acc}(X) = \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{X}\right). \tag{69}$$

Equation (88) is the result of the substitution of equation (69) into equation (68).

The leader will only use its accommodation strategy if the optimal quantity $Q_L^{acc}(X)$ leads to immediate investment of the follower. So it should hold that

$$X_F^*(Q_L^{acc}(X)) \leq X. \tag{70}$$

We define $\widehat{X}_1$ as

$$\widehat{X}_1 = X_F^*\left(Q_L^{acc}\left(\widehat{X}_1\right)\right). \tag{71}$$

Subsitution of (62) and (69) into (71) gives

$$\widehat{X}_1 = \frac{\beta+1}{\beta-1} \frac{\delta(r-\mu)}{1 - \eta\frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{\widehat{X}_1}\right)}. \tag{72}$$

Rearranging gives

$$\widehat{X}_1 = \frac{\beta+3}{\beta-1}\delta(r-\mu). \tag{73}$$

Furthermore, the leader cannot use the deterrence strategy anymore if we have that

$$X_F^*\left(Q_L^{det}(X)\right) \leq X. \tag{74}$$

Let us define $\widehat{X}_2$ as

$$X_F^*\left(Q_L^{det}\left(\widehat{X}_2\right)\right) = \widehat{X}_2. \tag{75}$$

To determine $\widehat{X}_2$ we substitute equation (62) for X into (66):

$$\frac{\frac{\beta+1}{\beta-1}\frac{\delta(r-\mu)}{1-\eta Q_L}\left(1-2\eta Q_L\right)}{r-\mu} - \delta - \left(\frac{\frac{\beta+1}{\beta-1}\frac{\delta(r-\mu)}{1-\eta Q_L}\left(\beta-1\right)\left(1-\eta Q_L\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right)^{\beta}\frac{\left(1-\left(\beta+1\right)\eta Q_L\right)\delta}{\left(\beta-1\right)\left(1-\eta Q_L\right)} = 0. \tag{76}$$

Some rearrangement gives

$$\frac{\beta+1}{\beta-1}\frac{1-2\eta Q_L}{1-\eta Q_L} - \delta - \frac{\left(1-\left(\beta+1\right)\eta Q_L\right)\delta}{\left(\beta-1\right)\left(1-\eta Q_L\right)} = 0, \tag{77}$$

so that

$$Q_L = \frac{1}{2\eta}. \tag{78}$$

Substitution of (78) into (62) gives

$$\widehat{X}_2 = \frac{2\left(\beta+1\right)}{\beta-1}\delta\left(r-\mu\right). \tag{79}$$

Before the leader has invested, thus when $X < X_L^{det}$, the firm holds an option to invest. the option value is

$$F_L^{det}\left(X\right) = A_L^{det}X^{\beta}. \tag{80}$$

The value matching and smoothing pasting conditions to determine X_L^{det} are given by:

$$\begin{aligned} A_L^{det}X^{\beta} &= \frac{XQ_L\left(X\right)\left(1-\eta Q_L\left(X\right)\right)}{r-\mu} - \delta Q_L\left(X\right) - \left(\frac{X\left(\beta-1\right)\left(1-\eta Q_L\left(X\right)\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right)^{\beta}\frac{\delta Q_L\left(X\right)}{\beta-1} && (81) \\ \beta A_L^{det}X^{\beta-1} &= \frac{Q_L\left(X\right)\left(1-\eta Q_L\left(X\right)\right) + X\frac{\partial Q_L}{\partial X}\left(1-2\eta Q_L\left(X\right)\right)}{r-\mu} - \delta\frac{\partial Q_L}{\partial X} && (82) \\ &- \left(\frac{X\left(\beta-1\right)\left(1-\eta Q_L\left(X\right)\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right)^{\beta}\frac{\delta\left(Q_L\left(X\right)\left(\beta\left(1-\eta Q_L\left(X\right)\right)-\left(\beta+1\right)X\eta\frac{\partial Q_L}{\partial X}\right)+X\frac{\partial Q_L}{\partial X}\right)}{X\left(\beta-1\right)\left(1-\eta Q_L\left(X\right)\right)}. \end{aligned}$$

Substitution of (82) into (81) gives

$$\begin{aligned} & \frac{XQ_L\left(X\right)\left(1-\eta Q_L\left(X\right)\right)}{r-\mu} - \frac{XQ_L\left(X\right)\left(1-\eta Q_L\left(X\right)\right)+X^2\frac{\partial Q_L}{\partial X}\left(1-2\eta Q_L\left(X\right)\right)}{\beta\left(r-\mu\right)} \\ - & \ \delta Q_L\left(X\right) + \frac{\delta X}{\beta}\frac{\partial Q_L}{\partial X} \\ + & \left(\frac{X\left(\beta-1\right)\left(1-\eta Q_L\left(X\right)\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right)^{\beta}\left(\frac{\delta X\frac{\partial Q_L}{\partial X}\left(1-\left(\beta+1\right)\eta Q_L\left(X\right)\right)}{\beta\left(\beta-1\right)\left(1-\eta Q_L\left(X\right)\right)}\right) \\ = & \ 0. \end{aligned} \tag{83}$$

In order to be able to use equation (83) to calculate X_L^{det} we need an expression for $\frac{\partial Q_L}{\partial X}$. Total differentiation of equation (66) gives

$$\frac{\partial\phi f\left(X,Q_L\right)}{\partial Q_L}\frac{\partial Q_L}{\partial X} + \frac{\partial\phi\left(X,Q_L\right)}{\partial X} = 0. \tag{84}$$

Rewriting gives

$$\frac{\partial Q_L}{\partial X} = -\frac{\frac{\partial\phi(X,Q_L)}{\partial X}}{\frac{\partial\phi(X,Q_L)}{\partial Q_L}}. \tag{85}$$

We have that

$$\frac{\partial \phi\left(X,Q_L\right)}{\partial X} = \frac{1-2\eta Q_L}{r-\mu} - \left(\frac{X\left(\beta-1\right)\left(1-\eta Q_L\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right)^{\beta-1} \frac{\beta\left(1-\left(\beta+1\right)\eta Q_L\right)}{\left(\beta+1\right)\left(r-\mu\right)}, \tag{86}$$

$$\frac{\partial \phi\left(X,Q_L\right)}{\partial Q_L} = -\frac{2\eta X}{r-\mu} + \left(\frac{X\left(\beta-1\right)\left(1-\eta Q_L\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right)^{\beta} \frac{\beta\eta\delta\left(2-\left(\beta+1\right)\eta Q_L\right)}{\left(\beta-1\right)\left(1-\eta Q_L\right)^2}. \tag{87}$$

Combining (85), (86), and (87) gives

$$\frac{\partial Q_L}{\partial X} = \frac{\frac{1-2\eta Q_L}{r-\mu} - \left(\frac{X(\beta-1)(1-\eta Q_L)}{(\beta+1)\delta(r-\mu)}\right)^{\beta-1} \frac{\beta(1-(\beta+1)\eta Q_L)}{(\beta+1)(r-\mu)}}{\frac{2\eta X}{r-\mu} - \left(\frac{X(\beta-1)(1-\eta Q_L)}{(\beta+1)\delta(r-\mu)}\right)^{\beta} \frac{\beta\eta\delta(2-(\beta+1)\eta Q_L)}{(\beta-1)(1-\eta Q_L)^2}}. \tag{88}$$

The leader threshold X_L^{det} and the corresponding quantity Q_L^{det} can be calculated by first substituting equation (88) into equation (83) and then simultaneously solving the resulting equation and equation (66). Doing this gives

$$X_L^{det} = \frac{\beta+1}{\beta-1}\delta\left(r-\mu\right),$$

$$Q_L^{det} = \frac{1}{\left(\beta+1\right)\eta}.$$

For the accommodation strategy the value matching and smooth pasting condition are given by:

$$A_L^{acc}X^{\beta} = \frac{\left(X-\delta\left(r-\mu\right)\right)^2}{8X\eta\left(r-\mu\right)}, \tag{89}$$

$$\beta A_L^{acc}X^{\beta-1} = \frac{X^2-\delta^2\left(r-\mu\right)^2}{8X^2\eta\left(r-\mu\right)}. \tag{90}$$

Substitution of (90) into (89) gives

$$\frac{\left(X-\delta\left(r-\mu\right)\right)^2}{8X\eta\left(r-\mu\right)} - \frac{X^2-\delta^2\left(r-\mu\right)^2}{8\beta X\eta\left(r-\mu\right)} = 0. \tag{91}$$

Rearranging gives

$$\frac{\left(\beta\left(X-\delta\left(r-\mu\right)\right)-\left(X+\delta\left(r-\mu\right)\right)\right)\left(X-\delta\left(r-\mu\right)\right)}{8\beta X\eta\left(r-\mu\right)} = 0. \tag{92}$$

Since from (89) it follows that $X=\delta\left(r-\mu\right)$ is not a valid solution, we have that

$$X_L^{acc} = \frac{\beta+1}{\beta-1}\delta\left(r-\mu\right). \tag{93}$$

B.2.1 Economic Analysis

We know from the literature (e.g., Dixit and Pindyck (1994)) that

$$\frac{\partial\beta}{\partial\sigma} < 0.$$

Futhermore, we have that

$$\begin{aligned}
\frac{\partial X_1^{acc}}{\partial \beta} &= \frac{-4\delta\left(r-\mu\right)}{\left(\beta-1\right)^2} < 0, \\
\frac{\partial X_2^{det}}{\partial \beta} &= \frac{-4\delta\left(r-\mu\right)}{\left(\beta-1\right)^2} < 0, \\
\frac{X_2^{det}}{X_1^{acc}} &= \frac{2\left(\beta+1\right)}{\beta+3}, \\
\frac{\partial \frac{X_2^{det}}{X_1^{acc}}}{\partial \beta} &= \frac{4}{\left(\beta+3\right)^2} > 0.
\end{aligned}$$

Concerning X_1^{det} it holds that

$$\psi\left(X_1^{det},\beta\right) = \frac{X}{r-\mu} - \delta - \left(\frac{X\left(\beta-1\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right)^{\beta}\frac{\delta}{\beta-1} = 0$$

So that

$$\left.\frac{\partial\psi\left(X,\beta\right)}{\partial X}\right|_{X=X_1^{det}}\frac{\partial X_1^{det}}{\partial\beta} + \left.\frac{\partial\psi\left(X,\beta\right)}{\partial\beta}\right|_{X=X_1^{det}} = 0.$$

Rewriting gives

$$\frac{\partial X_1^{det}}{\partial\beta} = -\frac{\left.\frac{\partial\psi(X,\beta)}{\partial\beta}\right|_{X=X_1^{det}}}{\left.\frac{\partial\psi(X,\beta)}{\partial X}\right|_{X=X_1^{det}}}.$$

We know from (67) that $\left.\frac{\partial\psi(X,\beta)}{\partial X}\right|_{X=X_1^{det}} > 0$. Furthermore,

$$\frac{\partial\psi\left(X,\beta\right)}{\partial\beta} = -\frac{\delta}{\beta^2-1}\left(\frac{X\left(\beta-1\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right)^{\beta}\left(1+\left(\beta+1\right)\log\left(\frac{X\left(\beta-1\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right)\right),$$

so that

$$\left.\frac{\partial\psi\left(X,\beta\right)}{\partial\beta}\right|_{X=X_1^{det}} > 0$$

if and only if

$$1+\left(\beta+1\right)\log\left(\frac{X_1^{det}\left(\beta-1\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right) < 0. \tag{94}$$

Define $\overline{X} = \frac{\beta}{\beta-1}\delta\left(r-\mu\right)$, then $X_1^{det} < \overline{X}$, as it holds that

$$\overline{X} < \frac{\beta+1}{\beta-1}\delta\left(r-\mu\right),$$

$$\psi\left(X_1^{det},\beta\right) = 0,$$

$$\frac{\partial\psi\left(X\right)}{\partial X} > 0 \text{ for } X \in \left(0,\frac{\beta+1}{\beta-1}\delta\left(r-\mu\right)\right),$$

and

$$\psi\left(\overline{X},\beta\right) = \frac{\delta}{\beta-1}\left(1-\left(\frac{\beta}{\beta+1}\right)^{\beta}\right) > 0.$$

So that (94) holds if

$$1+\left(\beta+1\right)\log\left(\frac{\overline{X}\left(\beta-1\right)}{\left(\beta+1\right)\delta\left(r-\mu\right)}\right) < 0.$$

Substitution of the definition of $\overline{X}$ gives

$$1+(\beta+1)\log\left(\frac{\beta}{\beta+1}\right)<0.$$

Define the function $\gamma(\beta)$ as follows

$$\gamma(\beta)=1+(\beta+1)\log\left(\frac{\beta}{\beta+1}\right).$$

We have that

$$\begin{aligned}\gamma(1) &= 1+2\log\left(\frac{1}{2}\right)<0,\\ \lim_{\beta\to\infty}\gamma(\beta) &= 0,\\ \frac{\partial\gamma(\beta)}{\partial\beta} &= \frac{1}{\beta}+\log\left(\frac{\beta}{\beta+1}\right)>0.\end{aligned}$$

The last equation holds since $\beta>1$ and

$$\begin{aligned}\left.\frac{\partial\gamma(\beta)}{\partial\beta}\right|_{\beta=1} &= 1+\log\left(\frac{1}{2}\right)>0,\\ \lim_{\beta\to\infty}\frac{\partial\gamma(\beta)}{\partial\beta} &= 0,\\ \frac{\partial^2\gamma(\beta)}{\partial\beta^2} &= -\frac{1}{\beta^2+\beta^2}<0.\end{aligned}$$

We conclude that $\left.\frac{\partial\psi(X,\beta)}{\partial\beta}\right|_{X=X_1^{det}}>0$ and therefore $\frac{\partial X_1^{det}}{\partial\beta}<0$.

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