

Real Options, Product Market Competition, and Asset Returns^{*}

Felipe L. Aguerrevere[≡]

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Abstract

The effects of strategic behavior on asset returns are studied in a model of incremental investment with operating flexibility. We show how the interaction of competition and production and investment decisions influences the relation between industry structure and expected rates of return. The effect of competition on asset returns depends on the level of demand for the industry output. When demand is low firms in less concentrated industries earn higher returns. As demand increases and growth options become more valuable firms in more concentrated industries earn higher returns. We compare the predictions of our model with recent empirical evidence on industry structure and average rates of return by Hou and Robinson (2005).

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[≡] University of Alberta School of Business, Edmonton, Alberta, Canada T6G 2R6.
Tel. (780) 492-8288 Fax (780) 492-3325
E-mail: felipe.aguerrevere@ualberta.ca

Introduction

A recent empirical study on industry concentration and stock returns by Hou and Robinson (2005) finds that firms in less concentrated industries earn higher returns, even after controlling for size, book-to-market, momentum, and other known return predictors. Hou and Robinson consider two possible explanations for their results. The first is that firms in less concentrated industries are riskier because they engage in more innovation, thus commanding higher expected returns. The second explanation is that barriers to entry in highly concentrated industries insulate firms from aggregate demand shocks. Therefore firms in more concentrated industries have lower distress risk and earn lower returns. There are a number of other reasons why industry concentration affects stock returns, hence this empirical evidence suggests a need for asset-pricing models which explicitly incorporate features of product markets as determinants of asset returns.

This article studies the effects of competitive interactions among firms on asset returns in a real options framework. We analyze the asset pricing implications of product market competition by examining how the strategic behavior of market participants affects their equilibrium investment and production decisions.

Our model extends the approach of Grenadier (2002), which derives the equilibrium investment strategies of firms in a Cournot-Nash framework, by introducing an operating option that allows firms to vary their capacity utilization in response to changes in demand. Specifically, capital is the only factor in production in our model and on any given period each firm chooses its output to maximize its current profit. This choice depends on current demand and it is constrained by the firm's current production capacity. In addition, each firm must condition its output choice on the output choices of the other firms, which are also constrained by their capacity levels. All the firms can expand their production capacity by investing in additional capacity and they must determine when to exercise their investment opportunities. Each firm investment strategy is conditional on its competitors' investment strategies. Therefore investment and operating decisions arise from equilibrium in the product market which reflects strategic interactions among market participants.

To understand the link between the effect of product market competition on the firms operating and investment decisions and their expected returns we separate the total value of all firms into two components, the value of the assets in place and the value of the growth opportunities. Once we know how the firms' strategic behavior affects the values and returns of each of these components separately, then the net effect of competition on the risk and return of every firm will depend on the weighting of its assets in place and its growth opportunities on its total value.

The value of the assets in place is the present value of the future cash flows generated by the firm's current production capacity. Hence the return on the assets in place depends on the effect of competition on the riskiness of these cash flows. In our model production costs introduce leverage which increases the risk of the firms' cash flows. On the other hand the option to reduce output in response to a fall in demand diminishes the effect of leverage on risk. However, the value of this option decreases with the number of firms in the industry because with more competition the firms have less power to reduce output. Therefore the firm's assets in place in more competitive industries earn higher returns as their cash flows are riskier.

The value of the growth options derives from the firms' ability to decide when to invest in additional capacity. Because the future value of any additional unit of capacity is uncertain, there is an opportunity cost to investing today. Thus the optimal investment rule is to invest when the value of the additional capacity exceeds the investment cost by an amount that is the value of the option to invest. However, increased competition leads the firm to invest earlier to avoid losing the investment opportunity to its competitors. Therefore, the value of the option to invest decreases with more firms in the market. The presence of growth option increases the total risk of a firm because these investment opportunities have implicit leverage. This leverage arises from fixed development costs. Thus, if the value of the growth options relative to the total value of a firm decreases with more competition then the risk and expected return of the firm also decreases.

As seen above the assets in place in more competitive industries earn higher returns, whereas the value of the growth options decreases with more firms in the market. Then it follows that the net effect of competition on the risk and return of the firms depends on the weighting of the assets in place and growth options on the total value of the firm.

Regardless of the number of firms in the industry, growth options become more valuable as demand increases because they are more likely to be exercised. It follows that the value of the growth options as a proportion of the total value of the firm increases with the level of demand. Therefore when the demand level is low firms in less concentrated industries earn higher returns because their assets in place are riskier and the value of their growth options is relatively low, but as demand increases and expansion becomes more likely firms in more concentrated industries will earn higher returns because their growth options are more valuable.

This article contributes to a growing research literature pioneered by Berk, Green, and Naik (1999) that links firms' real investment decisions and asset return dynamics. This literature includes Gomes, Kogan, and Zhang (2003), Kogan (2004), Carlson, Fisher and Giammarino (2004), Cooper (2005), and Zhang (2005a, 2005b). These papers provide models that relate risk and return dynamics to firm specific-characteristics such as size and book-to-market. Specifically, if assets in place and growth options have different sensitivities to changing economic conditions, then their systematic risk is different. The relative weight of assets in place and growth options changes as the firm value changes. Hence, the firm's true conditional systematic risk or beta can vary. However, the empirical methods fail to capture this variation. By endogenizing expected returns through firm-level decisions, the papers in this emerging literature show how the true conditional beta can be proxied by firm characteristics such as size and book-to-market. Thus, by providing theoretical structure for risk and return dynamics this literature explains the economic mechanisms behind the observed empirical regularities¹. We contribute to this literature by showing how competitive interactions among firms in a given industry affect their risk dynamics.

In addition to explaining a link between industry structure and asset returns through the effect of competition on firm decisions our framework is also helpful in expanding our understanding on the role of some key assumptions in the existing literature. For example, in a monopolistic setting, Carlson, Fisher and Giammarino (2004), and Cooper (2005) rely on operating leverage to explain the book-to-market effect or value premium

¹ For example, Fama and French (1992, 1993) provide empirical evidence on the ability of size and book-to-market to explain the cross-section of stock returns.

in stock returns. Intuitively, when firms' revenue fall due to a fall in their output price, equity values fall relative to installed capital which can be proxied by book value. If the fixed operating costs are proportional to installed capital, the risk of the firm increases because of higher operating leverage. In contrast Kogan (2004) can obtain the value premium without production costs in perfect competition. In his model investment is irreversible and firms always utilize all their installed capital. Therefore, to guarantee market clearing either prices or supply must adjust in response to a shock in demand. When installed capacity considerably exceeds the optimal level, the output price and the firm value must adjust to absorb a shock in demand. Therefore, asset prices are more sensitive to changing economic conditions and returns have more systematic risk. As firm value increases, investment in new capital becomes more attractive. The new supply will absorb the effect of changes in demand on the firm value and reduce its systematic risk. Therefore the expected return decreases as the value of the firm increases relative to its installed capital, which is the value premium. Kogan obtains the value premium without production costs because in a perfectly competitive industry the value of the options to invest in additional capacity is zero. However, in a monopolistic market the risk and expected return of the firm increases as its value increases because the increased contribution of the growth options to the total risk of the firm is greater than the reduction in risk resulting from new investment. Hence, to obtain the value premium Carlson, Fisher and Giammarino (2004), and Cooper (2005) introduce operating leverage to make assets in place riskier than growth options. But when firms always utilize all their capacity increasing competition does not affect the sensitivity of their operating cash flows to changes in demand. Thus, as discussed above, operating flexibility is required to explain why firms in more competitive industries earn higher returns.

Our work is also related to Aguerrevere (2003) who shows how operating flexibility affects the behavior of equilibrium output prices in a model of strategic capacity expansion. He demonstrates that when firms have the ability to vary their capacity utilization in response to a shock in demand, the output price volatility is increasing in the number of firms in the industry.

This article is organized as follows. Section 2 presents our model of capacity choice and operating flexibility. Section 3 derives the value of the firms in the market. Section 4 examines the effect of competition on expected returns. Section 5 concludes.

2. The Model

Our model extends the model of Grenadier (2002), which derives the equilibrium investment strategies of firms in a Cournot-Nash framework, by introducing an operating option that allows firms to vary their capacity utilization in response to changes in demand.

Consider an industry composed of n firms producing a single non-storable good. At time t , each firm i produces $q_i(t)$ units of output. The output price is a function of the industry output and a stochastic demand shock. Specifically we assume the following simple form for the inverse demand curve:

$$P(t) = Y(t) \cdot Q(t)^{-1/\gamma} \quad (1)$$

Where $P(t)$ is the output price, $Y(t)$ is an exogenous shock to demand, $Q(t) = \sum_{i=1}^n q_i(t)$ is the industry output, and the constant $\gamma > 1$ is the elasticity of demand. With this assumed functional form changes in the variable Y will be reflected in parallel shifts to the demand curve. Thus Y can be thought as the relative strength of the demand side of the market. Conditions affecting the strength of demand include the level of industrial production, household income, etc. The demand shock evolves as geometric Brownian motion

$$dY(t) = \mu Y(t)dt + \sigma Y(t)dZ(t) \quad (2)$$

where μ is the instantaneous proportional change in Y per unit time, σ is the instantaneous standard deviation per unit time, and Z is a standard Wiener process. Both μ and σ are constant.

Firms operate a simple production technology. Each unit of installed capacity can produce one unit of output per unit time at a cost $C_i(q_i) = cq_i$. Where c is the marginal

cost, which is the same constant for all firms. At any time t each firm chooses its output to maximize its current profit. For each firm the optimal output choice is constrained by its installed capacity, which is denoted by $K_i(t)$. Specifically, at any time t the firms play a static Cournot game. Each firm chooses its output to maximize its profit. This choice depends on current demand and it is constrained by the firm's current production capacity. In addition, each firm must condition its output choice on the output choices of the other firms, which are also constrained by their capacity levels.

At any time t , each firm can invest in additional capital to increase its production capacity by an infinitesimal increment $dK_i(t)$. The price of a new unit of capacity is a constant I . If $I = 1$, then firm's i capital K_i can be interpreted as the book value of the firm's assets.

Each firm chooses its production capacity $K_i(t)$ to maximize its value, conditional on the capacity choices of its competitors. Thus the optimal investment decision is an endogenous Nash equilibrium solution in investment strategies. Production capacity is the strategic variable and each firm must condition its capacity choice on the strategies of its competitors. For each firm i , let $K_{-i} = (K_1, \dots, K_{i-1}, K_{i+1}, \dots, K_n)$ denote the strategies of firm i 's competitors. An n -tuple of strategies (K_1^*, \dots, K_n^*) is a Nash industry equilibrium if

$$K_i^* = K_i(Y, K_{-i}^*), \quad i = 1, \dots, n \quad (3)$$

To simplify the analysis we assume that the industry is composed of n identical firms. That is, all the firms start with the same initial capacity and, thus, they all have the same size at any time. Thus, our analysis focuses on a symmetric Nash equilibrium.

Let $K(t)$ denote the total industry installed capacity. Since all firms are identical it follows that $K_i(t) = K(t)/n$, and $K_{-i}(t) = (n-1)K(t)/n$. Thus, by focusing on a symmetric equilibrium, the state space is reduced and the firms condition their investment and production decision on the level of the demand shock Y and the total industry capacity K . For example the instantaneous profit that firm i earns at time t when the industry capacity is $K(t)$ and the demand parameter is $Y(t)$ is given by

$$\pi_i(K(t), Y(t)) = \max_{0 \leq q_i(t) \leq K(t)/n} \left[(Y(t)Q(t))^{-1/\gamma} q_i(t) - cq_i(t) \right] \quad (4)$$

From the symmetric equilibrium assumption we get

$$\pi_i(K, Y) = \begin{cases} \frac{1}{n} \left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} Y \right)^\gamma & \text{for } Y < \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \\ \frac{YK^{\frac{\gamma-1}{\gamma}}}{n} - \frac{cK}{n} & \text{for } Y > \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \end{cases} \quad (5)$$

3. Valuation

Following Carlson, Fisher and Giammarino (2004) we assume the existence of traded assets that can hedge demand uncertainty. Specifically, let M be the price of a riskless asset with dynamics $dM(t) = rM(t)dt$ where r is the (constant) risk-free rate of interest, and let X be the price of a risky asset which evolves as geometric Brownian motion

$$dX(t) = \eta X(t)dt + \sigma X(t)dZ(t) \quad (6)$$

The risky asset and the demand shock are driven by the same Brownian motion Z and have the same instantaneous standard deviation σ of relative changes. Thus, they are perfectly correlated. The difference in their drifts is $\delta = \eta - \mu$. To ensure that the value of the firm is finite we must have $\delta > 0$. Since the relative changes in X and Y are perfectly correlated we can construct a portfolio with X and M that exactly replicates the dynamics of the firm value. To find the value of the firm we use the traded assets X and M to define a new probability measure Q under which the process $Z^*(t) = Z(t) + \frac{\eta - r}{\sigma}t$ is a standard Brownian motion. Under this risk neutral measure, the demand shock follows the process

$$dY(t) = (r - \delta)Y(t)dt + \sigma Y(t)dZ^*(t) \quad (7)$$

Let $V_i(K, Y)$ be the value of firm i when the industry capacity is K and the level of the demand parameter is Y . The firm's problem is to choose the path of capacity expansion

that maximizes the present value of its future cash flows. Thus, each firm solves the following optimal control problem

$$V_i(K(t), Y(t)) = \max_{\{K_i(s): s > 0\}} E^Q \left\{ \int_0^{\infty} e^{-rt} [\pi_i(K(t), Y(t)) dt - I dK_i(t)] \right\} \quad (8)$$

As in Pindyck (1988), and He and Pyndyck (1992) we approach the solution to this problem by examining the firm's incremental investment decision.² The opportunity to invest in an additional unit of capacity is analogous to a perpetual American call option. The underlying asset is the value of an extra unit of capacity and the exercise price is the cost of investing in this unit.

Therefore, the solution to the firm's capacity choice problem involves two steps. First, the value of an extra unit of capacity must be determined. Second, the value of the option to invest in this unit must be determined together with the decision rule for exercising this option. This decision rule is the solution to the optimal capacity problem.

The value of a marginal unit of capacity is the present value of the expected flow of profits from this unit. Given the current capacity K , and demand Y , $\Delta F_i(K, Y)$ denotes the value of a marginal unit of capacity. An expression for ΔF_i is derived in the appendix.

After obtaining the value of the marginal unit of capacity, we can value the option to invest in this unit. Let $\Delta G_i(K, Y)$ denote the value of this option when the current capacity level is K . The exercise price of this option is equal to the investment cost I . Therefore, for any level of committed capacity K , $\Delta G_i(K, Y)$ is a perpetual American call option whose value depends on Y . Hence, there will be a threshold value at which it will be optimal to exercise this option. Specifically, for any K and n there will exist a threshold, $Y^n(K)$, such that the option to build an additional unit of capacity will be exercised the first time that Y equals or exceeds $Y^n(K)$.

The appendix shows that the solution to the investment threshold in an industry composed by n identical firms is of the form

$$Y^n(K) = v_n K^{1/\gamma} \quad (9)$$

² The value of the firm can be derived as the solution to an optimal instantaneous control problem. He and Pindyck (1992) show that the solution to this type of capacity choice problem can be obtained by examining the firm's incremental investment decision.

where the expression for v_n is given in the appendix.

It follows from (9) that $v_n = Y^n(K)K^{-1/\gamma}$ is the output price at which firms expand their capacity. The appendix shows that given the number of firms in the industry n , v_n is a constant which is independent of the industry capacity K . Thus the endogenous output price process has a constant upper reflecting barrier at v_n .

Once we solve for the firms optimal investment we can derive an expression for their value. Given the industry capacity K and the current value of Y , we can write the value of firm i , V_i , as the sum of two parts:

$$V_i(K, Y) = F_i(K, Y) + G_i(K, Y) \quad (10)$$

where $F_i(K, Y)$ is the value of the firm's assets in place, and $G_i(K, Y)$ is the value of the firm's growth options. Therefore, to get the value each firm we need to obtain the value its assets in place and the value its growth options.

The value of the assets in place is the value of the firm's installed capacity. When the industry capacity is K , each firm i has K/n units of capacity. This capacity provides each firm a cash flow stream of $\pi_i(K(t), Y(t))$ which is the profit given by (5). Also notice that in valuing the assets in place we have to take into consideration the effect of future investment on the value of the installed capacity. Specifically, the fact that firms can increase their capacity when the demand factor Y hits $Y^n(K)$ cuts off some of the upside potential for prices and profits. Therefore to determine the value of the assets in place we first find the present value of the profit flow $\pi_i(K(t), Y(t))$ and then we adjust this value for the impact of future increases in industry capacity on the value of the firm current capacity.

The appendix shows that the present value of the profit flow $\pi_i(K(t), Y(t))$ is $\frac{K}{n} J(K, Y)$

where $J(K, Y)$ is the value of one unit of capacity which it is given by

$$J(K, Y) = \begin{cases} \left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} YK^{-1/\gamma} \right)^\gamma + B(K)Y^\lambda & \text{for } Y < \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \\ \frac{YK^{-1/\gamma}}{\delta} - \frac{c}{r} + A(K)Y^\alpha & \text{for } Y > \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \end{cases} \quad (11)$$

The expressions for the functions $A(K)$ and $B(K)$, and the constants α and λ are given in the appendix.

To facilitate the analysis of the results on the effect of competition on asset returns presented in Section 4 below, we provide an explanation for the expression for the value of one unit of capacity in (12). Equation (5) gives the profit function for a firm with K/n units of capacity. Thus the profit per unit is

$$\pi U_i(K, Y) = \frac{\pi_i(K, Y)}{K/n} = \begin{cases} \left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} YK^{-1/\gamma} \right)^\gamma & \text{for } Y < \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \\ YK^{-1/\gamma} - c & \text{for } Y > \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \end{cases} \quad (12)$$

If the installed capacity is always used the value of one unit of capacity is

$$\frac{YK^{-1/\gamma}}{\delta} - \frac{c}{r}$$

Thus, when $Y > n\gamma c K^{1/\gamma}/(n\gamma - 1)$, the term $A(K)Y^\alpha$ in (11) is the value of the option to reduce output should Y decrease. This option is valuable because firms can reduce their output when $Y < n\gamma c K^{1/\gamma}/(n\gamma - 1)$ and earn a profit of $\left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} YK^{-1/\gamma} \right)^\gamma$ per unit of capacity. This profit is greater than $YK^{-1/\gamma} - c$ which is the profit per unit if installed capacity is always used. Therefore, $A(K)$ is positive for all n .

When the installed capacity is not fully used, i.e. when $Y < n\gamma c K^{1/\gamma}/(n\gamma - 1)$, the present value of the profits per unit of capacity is

$$\frac{\left(\frac{c}{n\gamma - 1}\right) \left(\frac{n\gamma - 1}{n\gamma c} Y K^{-1/\gamma}\right)^\gamma}{r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2 / 2}$$

The output of each firm and its capacity utilization increases as Y increases. However, when the industry capacity is K , each firm can produce up to K/n units of output. Therefore the term $B(K)Y^\lambda$ in (11) represents the impact of the capacity constraint on the value of the firms assets. This impact can be negative or positive depending on the degree of competition. In other words the sign of $B(K)$ depends on the number of firms in the industry. To understand how competition affects the sign of $B(K)$ we look at the influence of capacity constraints in the two most extreme cases of competition, namely monopoly and perfect competition. For the monopolist the capacity constraint reduces the profit that would otherwise be earned if the capacity was larger. That is for $n = 1$,

$$\left(\frac{c}{\gamma - 1}\right) \left(\frac{\gamma - 1}{\gamma c} Y K^{-1/\gamma}\right)^\gamma > Y K^{-1/\gamma} - c \text{ for all } Y. \text{ Hence, when capacity is not completely}$$

utilized, i.e. when $Y < \gamma c K^{1/\gamma}/(\gamma - 1)$, the term $B(K)Y^\lambda$ represents the reduction in the value of the firm's assets in place if Y rises over $\gamma c K^{1/\gamma}/(\gamma - 1)$ and capacity binds. Thus, for $n = 1$ $B(K)$ is negative. In the case of perfect competition, which in the model is obtained as the limit as n approaches infinity, the profit when capacity is not fully used is zero because the output price is equal to the marginal cost c . But when the capacity is completely utilized, i.e. when $Y > c K^{1/\gamma}$, the profit per unit is $Y K^{-1/\gamma} - c > 0$. Therefore, $B(K)$ is positive and $B(K)Y^\lambda$ represents the value of the option to earn a positive profit. The fact that $B(K)$ is negative for monopoly and positive for perfect competition suggest that $B(K)$ is increasing in the number of firms in the market n , and that there exist a number N such that $B(K)$ is negative if $n \leq N$ and $B(K)$ is positive if $n > N$. This number N depends on the parameters of the model.

The value of one unit of capacity for fixed K , $J(K, Y)$ in equation (11) does not take into consideration the effect of future investment on the value of the installed capacity. However the fact that firms can increase their capacity when the demand factor Y hits $Y^n(K)$ removes some of the upside potential for prices and profits, so the value of one unit

of capacity must be less than $J(K, Y)$. Denote by $H(K, Y)$ the value of one unit of capacity. The appendix shows that

$$H(K, Y) = J(K, Y) + E(K)Y^\lambda \quad (13)$$

Where $E(K)Y^\lambda$ represents the impact of future increases in industry capacity on the value of the firm current capacity. Since increased supply has a negative effect on output prices and, therefore, on future cash flows, the sign of $E(K)$ is negative.

Therefore, the value of firm i 's assets in place is

$$F_i(K, Y) = \frac{K}{n} H(K, Y) \quad (14)$$

Finally, the appendix shows that value of the growth options is

$$G_i(K, Y) = C(K)Y^\lambda. \quad (15)$$

Where the function $C(K)$ is positive and decreasing in K .

4. Expected Returns

This section analyzes the effect of product market competition on asset returns by examining how the strategic behavior of firms affects their betas. The demand factor Y is the source of risk in our model, and the beta of a firm measures the sensitivity of relative changes in the firm's value to relative changes in the demand factor. Thus, a firm's beta is the elasticity of its market value with respect to the demand factor. Formally, if the demand factor has a beta of one, then the beta of firm i is given by³

$$\beta_i(K, Y) = \frac{Y}{V_i(K, Y)} \frac{\partial V_i(K, Y)}{\partial Y} \quad (16)$$

³ For a formal derivation of equation (16) see the proof of proposition 2 in Carlson, Fisher and Giammarino (2004)

Our model assumes that investment is totally irreversible. To highlight the importance of irreversible investment in generating our results on effect of product market competition on asset returns below, we first compute the beta of the firm when investment is totally reversible. Without investment frictions the value of the firm is

$$V_i(K, Y) = \frac{\frac{1}{n} \left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} Y \right)^\gamma}{r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2 / 2}$$

and beta is $\beta_i(K, Y) = \gamma$. Therefore when investment is totally reversible the beta of the firm is a constant independent of the number of firms in the market. Hence, irreversible investment is required to generate our results on competition and asset returns.

We study the effect of competition on asset returns by analyzing the betas of the assets in place and growth options. The beta of the assets in place is

$$\beta_F(K, Y) = \frac{Y}{F_i(K, Y)} \frac{\partial F_i(K, Y)}{\partial Y} = \frac{Y}{H(K, Y)} \frac{\partial H(K, Y)}{\partial Y} \quad (17)$$

Were the last equality in (17) follows from equation (14). From (17) and (13) we can get the following expression for the beta of the assets in place

$$\beta_F(K, Y) = \beta_J(K, Y) + \frac{E(K)Y^\lambda}{H(K, Y)} (\lambda - \beta_J(K, Y)) \quad (18)$$

Where $\beta_J(K, Y)$ is the beta of the one unit of capacity for fixed K , and it is given by

$$\beta_J(K, Y) = \begin{cases} \gamma + (\lambda - \gamma) \frac{B(K)Y^\lambda}{J(K, Y)} & \text{for } Y < \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \\ 1 + \frac{c/r}{J(K, Y)} + (\alpha - 1) \frac{A(K)Y^\alpha}{J(K, Y)} & \text{for } Y > \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \end{cases} \quad (19)$$

The interpretation of (18) is as follows. In the previous section we showed that the value of one unit of capacity, $H(K, Y)$, is equal to the value of one unit of capacity for fixed K ,

$J(K, Y)$, adjusted for the impact of future increases in industry capacity on the value of the firm current capacity. New investment affects the beta of the assets in place because the additional supply from the added capacity buffers the effect of changes in demand on the firm's value. Thus, the first term in equation (18) is the beta of the value of the profit stream provided by the firm's current capacity, and the second term takes into account the effect of future investment on the beta of the assets in place.

To understand how product market competition affects the riskiness of the assets in place we first analyze how the number of firms in the market affects the beta of each firm's current capacity, $\beta_J(K, Y)$. When $Y > n\gamma cK^{1/\gamma}/(n\gamma - 1)$ the expression for the beta of the firm's capacity in Equation (19) has three terms. The first term is the firm's revenue beta. When a firm produces at full capacity its output is fixed and its revenue varies linearly with Y . Thus, the revenue beta is equal to the beta of demand, which is assumed to be one. With fixed production the total production cost is constant. This introduces leverage which increases the risk of the firm's assets. Hence, the second term captures the leverage effect of fixed production costs. This leverage effect is more pronounced in more competitive markets because the value of one unit of capacity $J(K, Y)$ decreases with the number of firms in the market. The third term derives from the option to reduce output in response to a fall in demand. This option diminishes the effect of leverage on risk, and consequently reduces beta⁴. However, the value of this option decreases with the number of firms in the industry. The reason is that with more competition the firms have less power to reduce output. Therefore, when capacity is fully used the beta of the firms' current capacity is increasing in the number of firms in the industry.

When $Y < n\gamma cK^{1/\gamma}/(n\gamma - 1)$ the expression for the beta of the firm's capacity in Equation (19) has two terms. When capacity is not fully used each firm varies its output in response to changes in demand. The first term is the beta of the firm's profits. This is also the beta of a firm that operates without capacity constraint. Therefore, the second term captures the impact of the capacity constraint on the risk of the firm's assets. For a given level of capacity K , the level of demand at which the firms will produce at full capacity, $n\gamma cK^{1/\gamma}/(n\gamma - 1)$, decreases with n . The reason is that for any specified level of demand Y the industry output increases with more firms in the market. Thus increased competition

⁴ The sign of this term is negative because $\alpha < 0$ and $A(K) > 0$.

raises the probability that firms will use their capacity sooner. It follows that the exposure to changes in demand also increases because, as seen above, production at full capacity is riskier with more competition. Therefore, the beta of the firms' current capacity is increasing in the number of firms in the industry.

For a given level of capacity Figure 1 illustrates the relationship between the beta of the firm capacity and the level of demand for a monopoly, a duopoly, a 5-firm oligopoly, and a 10-firm oligopoly. The beta is increasing in the number of firms in the industry. Observe that for the level of capacity specified in the figure $K = 100$, the beta values for an industry with n firms are obtained for levels of demand between 0 and $Y^n(K)$. For levels larger than $Y^n(K)$ the industry capacity is greater than K because all the firms will optimally add capacity as soon as demand reaches $Y^n(K)$. Notice also that $Y^1(K) > Y^2(K) > Y^5(K) > Y^{10}(K)$. The reason is that with more competition each firm has to expand its capacity sooner to avoid losing the investment opportunity to its competitors. In the appendix we prove that for any given K , the investment trigger $Y^n(K)$ is decreasing in n .

The example in Figure 1 illustrates the effect of competition on the beta of the firm capacity for a given level of industry capacity that is independent of the number of firms in the market. Equivalently, we can also analyze the effect of competition on expected returns by allowing the current industry capacity to depend on the degree of competition. Specifically, let K^n be the total capacity for an n -firm industry. The endogenous industry capacity is increasing in the number of firms. Specifically, in the appendix we show that for any number of firms n ,

$$K^n = \left[\frac{n\gamma - 1}{n(\gamma - 1)} \right]^\gamma K^1 \quad (20)$$

thus the capacity of an n -firm industry is equal to the monopoly capacity times a factor that is greater than one, increasing in n , and converges to $\gamma/(\gamma - 1)$ as n increases to infinity.

However, the investment threshold for new capacity, $Y^n(K^n)$ is independent of the number of firms in the market. To prove this result it suffices to show that $Y^n(K^n) = Y^1(K^1)$ for all n . Substituting Equation (20) into Equation (A5) in the appendix proves this result. Thus, by allowing the industry capacity to be dependent on the number of firms in

the market we can compare the betas resulting from different number of firms in the market over the same interval for the demand values.

Figure 2 provides an example of the effect of competition on the beta of the firm capacity when the relationship between industry capacity and the number of firms is given by (20). Beta is increasing in the number of firms in the industry for any level of Y between 0 and the investment threshold, $Y^n(K^n)$. Figure 2 also shows the level of Y above which firms will produce at full capacity, which is denoted by Y^c . This level is the same for all n when the relationship between industry capacity and the number of firms is given by (20) because $n\gamma c(K^n)^{1/\gamma}/(n\gamma - 1)$ is the same for all n . Notice that in the example illustrated in Figure 2 the beta of the firm's capacity is monotonically decreasing in Y for the case of 1 and 2-firm industries. But for the case of 5 and 10-firm industries beta increases in Y when capacity is not fully used, i.e. when $Y < Y^n(K^n)$, and decreases in Y when firms produce at full capacity, i.e. when $Y > Y^n(K^n)$. Regardless of the number of firms in the market the beta of the firm's capacity is decreasing in Y when capacity is fully used. The reason is that total revenue increases as Y increases while total production costs are constant. Consequently leverage drops, causing risk to decrease. The explanation why the relationship between beta and the level of demand depends on the degree of competition when capacity is not fully used is as follows. When Y increases the probability that firms will use their capacity sooner increases. As explained above, when firms produce at full capacity the leverage introduced by constant production costs is more pronounced with more firms in the market because more competition reduces the value of the option to cut output if demand falls. Thus, when this leverage is sufficiently large beta will be increasing in Y when capacity is not fully used.

Figure 3 illustrates the effect of competition on the beta of the assets in place with the same parameters used for Figure 2. Beta is increasing in the number of firms in the industry for any level of Y between 0 and the investment threshold, $Y^n(K^n)$. Figure 3 also shows the level of Y above which firms will produce at full capacity, which is denoted by Y^c . Recall that the difference between the value of the firm's capacity and value of the assets in place is that the latter takes into consideration the effect of new investment on profits. When $Y = Y^n(K^n)$, the beta of the assets in place is zero because the production from new capacity exactly offsets the effect of demand volatility on the assets value.

The beta of the growth options is

$$\beta_G(K, Y) = \frac{Y}{G_i(K, Y)} \frac{\partial G_i(K, Y)}{\partial Y} = \lambda \quad (21)$$

Where the last equality follows from Equation (15). Equation (21) shows that the beta of the growth options is a constant independent of the industry capacity and the demand factor. The reason is that the option to invest in additional capacity is a perpetual American call option. Since $\lambda > 1$, the beta of the growth options is greater than the demand beta. This follows from the leverage effect arising from the fixed investment costs.

The beta of the firm is the value-weighted average of the betas of the assets in place and growth options. Regardless of the number of firms in the market, the value of the growth options increases as Y increases, as investment in new capacity becomes more attractive. On the other hand the value of the growth options declines as the number of firms increases because increased competition reduces the incentive of waiting to invest as firms fear preemption. Thus, the ratio of the value of the growth options to the value of the firm decreases with more competition. This result combined with the result on competition and the beta of the assets in place above implies that the net effect of competition on firm return depends on the level of demand.

Figure 4 illustrates the beta of the firm for different number of firms in the market, when the relationship between industry capacity and the number of firms is given by (20). When demand is low the proportion of the assets in place in the total value of the firm is larger and firms in more competitive markets earn higher returns. As demand increases and growth options become more valuable, firms in more concentrated industries earn higher returns.

Our result that firms in more competitive industries earn higher returns when demand is low is consistent with Hou and Robinson (2005) finding that difference between the rates of return for firms in the most competitive industries and firms in the most concentrated industries grows as the economy contracts.

In order to get a better understanding on how the different components of the value of the firm affect its return it is useful to consider two special cases.

Case 1: $c = 0$. This is the base case in Grenadier (2002). Since the variable production cost is zero, the firms always produce at full capacity. In this case we can get closed form solutions for the investment threshold and the value of the firm.

$$v_n = \left(\frac{\lambda}{\lambda - 1} \right) \left(\frac{n\gamma}{n\gamma - 1} \right) \delta I \quad (22)$$

The value of one unit of capacity and is $H(K, Y) = E(K)Y^\lambda + J(K, Y)$, where

$$E(K) = -\frac{v_n^{1-\lambda}}{\lambda\delta} K^{-\lambda/\gamma} \quad (23)$$

and

$$J(K, Y) = \frac{YK^{-1/\gamma}}{\delta} \quad (24)$$

The value of the growth options is $G_i(K, Y) = C(K)Y^\lambda$, where

$$C(K) = \frac{\gamma}{\lambda - \gamma} \frac{I}{n\gamma - 1} v_n^{-\lambda} \frac{K^{\gamma-\lambda}}{n} = \frac{K}{n} \frac{\gamma}{\lambda - \gamma} c(K) \quad (25)$$

Where $c(K)Y^\lambda$ is the value of the option to invest in one unit of capacity.

The relation between the industry capacity for a n -firm industry and the monopoly capacity in (20) also obtains in this case.

The beta of the assets in place is

$$\beta_F(K, Y) = \left[1 + (\lambda - 1) \frac{E(K)Y^\lambda}{H(K, Y)} \right] \quad (26)$$

$\beta_F(K, Y)$ is decreasing in Y because its derivative with respect to Y is negative. By substituting (20) into (23), (24) and (26) we can show that for all n

$$\beta_F(K^n, Y) = \beta_F(K^1, Y) \quad (27)$$

The beta of the firm is

$$\beta_i(K, Y) = \beta_F(K, Y) + \frac{C(K)Y^\lambda}{V_i(K, Y)} (\lambda - \beta_F(K, Y)) \quad (28)$$

To get (28) notice that the beta of the firm is the value weighted average of the betas of the assets in place and growth options. The derivative of $C(K)$ with respect to n is negative and the limit of $C(K)$ as n approaches infinity is 0. Combining this result with the result of equation (27) we get that the beta, and therefore the risk premium, is always higher for firms in more concentrated industries. This is inconsistent with the evidence in Hou and Robinson (2005).

This case is also useful in understanding why Kogan (2004) obtains the value premium without production costs. For this purpose we use the following alternative expression for the beta of the firm

$$\beta_i(K, Y) = \left[1 + (\lambda - 1) \frac{\left(E(K) + \frac{\gamma}{\lambda - \gamma} c(K) \right) Y^\lambda}{H(K, Y) + \frac{\gamma}{\lambda - \gamma} c(K)} \right] \quad (29)$$

The value of the firm increases as Y increases. Therefore we can get the relation between the beta and the firm's book-to-market ratio $IK_i / V_i(K, Y)$ by looking at the sign derivative of $\beta_i(K, Y)$ with respect to Y . It is straightforward to show that this sign is the same as the sign of $E(K) + \frac{\gamma}{\lambda - \gamma} c(K)$. In a monopoly, i.e, $n = 1$, $E(K) + \frac{\gamma}{\lambda - \gamma} c(K) > 0$ and beta is decreasing in the book-to-market ratio. As n increases $c(K)$ decreases and the sign of $E(K) + \frac{\gamma}{\lambda - \gamma} c(K)$ will become negative and beta will be increasing in the book-to-market ratio, which is the value premium. Kogan (2004) analyzes the effect of irreversible investment on assets prices in perfect competition. This corresponds to $c(K) = 0$ in our case, and the beta of the firm is given by (22) when n increases to infinity. Furthermore, Kogan's explanation on how the value premium arises in his model is similar to our intuition as to why $E(K)$ is negative.

In sum it is possible to obtain the value premium in a model without production costs if the number of firms in the industry is sufficiently large, but the risk premium will decrease with more competition. Carlson, Fisher and Giammarino (2004) and Cooper (2005) introduce operating leverage to obtain the value premium in a monopolistic

industry. Next we investigate the introduction of operating leverage in the model of Case 1.

Case 2: $c > 0$ with no operating flexibility, which means that the firm always produces at full capacity and pays a total production cost cK/n per period. In this case we can also get closed form solutions for the investment threshold and the value of the firm. The expressions for v_n , $E(K)$ and $C(K)$ are similar to (18), (19) and (21) with I changed for $I + c/r$. The reason is that when a firm invests in an additional unit of capacity the total cost not only includes the purchase cost I but also the committed future payments whose present value is c/r . The value of one unit of capacity for fixed K is

$$J(K, Y) = \frac{YK^{-1/\gamma}}{\delta} - \frac{c}{r} \quad (30)$$

The beta of the assets in place is

$$\beta_F(K, Y) = \left[1 + (\lambda - 1) \frac{E(K)Y^\lambda}{H(K, Y)} + \frac{c/r}{H(K, Y)} \right] \quad (31)$$

Comparing with (26) the expression in (31) has an additional term which represents the effect of operating leverage on beta. We can show that $\beta_F(K^n, Y)$ increases as n increases. This follows from the effect of operating leverage on the firm's cash flows. Specifically, as n increases the revenue per unit $Y[K^n]^{-1/\gamma}$ declines whereas the unit cost c is constant for all n .

The beta of the firm is

$$\beta_i(K, Y) = \left[1 + (\lambda - 1) \frac{\left(E(K) + \frac{\gamma}{\lambda - \gamma} c(K) \right) Y^\lambda}{H(K, Y) + \frac{\gamma}{\lambda - \gamma} c(K)} + \frac{c/r}{H(K, Y) + \frac{\gamma}{\lambda - \gamma} c(K)} \right] \quad (32)$$

Now the effect of competition on beta depends on the combined effects of operating leverage and growth options. When Y is small the operating leverage effect dominates

and beta increases as n increases. As Y increases to the investment threshold the growth option effect dominates and beta decreases as n increases.

We showed above that without operating costs the beta of a monopolistic firm is decreasing in the book-to-market ratio. This contrasts with the empirical evidence on the value premium. By introducing operating leverage beta can be decreasing in Y if the leverage effect dominates the growth options effect. This is why the assumption of operating leverage is crucial in obtaining the value effect in Carlson, Fisher and Giammarino (2004) and Cooper (2005).

Comparing cases 1 and 2 we see that it is possible to obtain the value premium without operating costs if there is “sufficient” competition in the market, but beta declines with the number of firms in the market for all Y . By introducing fixed production cost we saw when beta will be increasing with competition and why operating leverage is required to obtain the value premium in a monopolistic industry. However, recent evidence by Xing and Zhang (2004) fails to find much support for the operating leverage hypothesis of Carlson, Fisher and Giammarino (2004) and Cooper (2005). By measuring operating leverage as the elasticity of operating profits with respect to sales Xing and Zhang (2004) find that value firms have slightly lower operating leverage than growth firms. This is consistent with our model. Since firms do not need to use all their installed capacity when demand declines, the operating flexibility in our model implies that high book-to-market firms have lower operating leverage. Furthermore, we could use this measure of operating leverage to test our result that the option to vary output in response to changes in demand can explain the concentration premium documented in Hou and Robinson (2005). By forming portfolios based on industry concentration and the elasticity of operating profits with respect to sales we could test how the concentration premium changes with changes in operating flexibility.

5. Conclusion

In this paper we study the effect of product market competition on asset returns in a real-options framework. We analyze how industry structure affects asset returns by

examining how the strategic behavior of market participants affects their equilibrium investment and production decisions. We show how the option to vary output in response to changes in demand can explain the concentration premium documented in Hou and Robinson (2005). Production costs introduce leverage which increases the risk of the firms' cash flows. On the other hand the option to reduce output in response to a fall in demand diminishes the effect of leverage on risk. However, the value of this option decreases with the number of firms in the industry because with more competition the firms have less power to reduce output. Therefore the firm's assets in place in more competitive industries earn higher returns as their cash flows are riskier. However, since firms can expand their production capacity by investing in additional capacity, the effect of competition on asset returns depends on the level of demand.

The presence of growth options increases the total risk of a firm because these investment opportunities have implicit leverage that arises from fixed development costs. Growth options are more valuable as demand increases because they are more likely to be exercised. But, the value of the option to invest decreases with more firms in the market because the increased competition leads the firm to invest earlier to avoid losing the investment opportunity to its competitors. Therefore, as demand increases and growth options become more valuable firms in more concentrated industries earn higher returns.

In sum the effect of competition on asset returns depends on the level of demand. When demand is low firms in less concentrated industries earn higher returns. As demand increases and the value of the growth options as a proportion of the total value of the firm increases firms in more concentrated industries earn higher returns.

Our assumption of identical firms could be relaxed to allow for firms of different sizes and costs structures. This extension would provide greater realism to the model, but the loss of the simplifying feature of a symmetric equilibrium would greatly diminish the tractability of the model.

Appendix

Capacity Choice

The solution to the firm's capacity choice problem involves two steps. First, the value of an extra unit of capacity must be determined. Second, the value of the option to invest in this unit must be determined together with the decision rule for exercising this option. This decision rule is the solution to the optimal capacity problem.

The Value of a Marginal Unit of Capacity

Given the current capacity K , and demand Y , $\Delta F_i(K, Y)$ denotes the value of a marginal unit of capacity. The value of a marginal unit of capacity is the present value of the expected flow of profits from this unit. It follows from (4) that the profit from the marginal unit of capacity at any time t is

$$\Delta \pi_i(K, Y(t)) = \max \left[\frac{n\gamma - 1}{n\gamma} YK^{-1/\gamma} - c, 0 \right] \quad (\text{A1})$$

Thus, after it is completed, each incremental unit of capacity will be used only when the additional profit it generates is positive, i.e. when $Y > n\gamma c K^{1/\gamma} / (n\gamma - 1)$.

Following standard arguments, the value of the marginal unit of capacity satisfies the following differential equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 \Delta F_i}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial \Delta F_i}{\partial Y}(K, Y) - r \Delta F_i(K, Y) + \Delta \pi_i(K, Y) = 0$$

where $\Delta \pi_i$ is given by (A1). In the region where $Y < n\gamma c K^{1/\gamma} / (n\gamma - 1)$, ΔF_i satisfies the equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 \Delta F_i}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial \Delta F_i}{\partial Y}(K, Y) - r \Delta F_i(K, Y) = 0$$

Subject to the boundary condition

$$\Delta F_i(K, 0) = 0$$

Therefore, the solution is of the form

$$\Delta F^1(K, Y) = \Delta B(K) Y^\lambda$$

Where λ is the positive root of the characteristic equation

$$\frac{\sigma^2}{2} \xi(\xi - 1) + (r - \delta)\xi - r = 0 \quad (\text{A2})$$

In the region where $Y > n\gamma c K^{1/\gamma}/(n\gamma - 1)$, ΔF_i satisfies the equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 \Delta F_i}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial \Delta F_i}{\partial Y}(K, Y) - r \Delta F_i(K, Y) + \frac{n\gamma - 1}{n\gamma} Y K^{-1/\gamma} - c = 0$$

Subject to the boundary condition

$$\lim_{Y \rightarrow \infty} \Delta F_i(K, Y) = \frac{n\gamma - 1}{n\gamma} \frac{Y K^{-1/\gamma}}{\delta} - \frac{c}{r}$$

Which implies a solution of the form

$$\Delta F^2(K, Y) = \Delta A(K) Y^\alpha + \frac{n\gamma - 1}{n\gamma} \frac{Y K^{-1/\gamma}}{\delta} - \frac{c}{r} \quad (\text{A3})$$

Where α is the negative root of equation (A2). To solve for $\Delta A(K)$ and $\Delta B(K)$ we consider the point $Y = n\gamma c K^{1/\gamma}/(n\gamma - 1)$, where the two regions meet. $\Delta F(K, Y)$ must be continually differentiable across $Y = n\gamma c K^{1/\gamma}/(n\gamma - 1)$, therefore $\Delta A(K)$ and $\Delta B(K)$ are the solutions to the system of equations

$$\Delta F^1\left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right) = \Delta F^2\left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right)$$

$$\frac{\partial \Delta F^1}{\partial Y}\left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right) = \frac{\partial \Delta F^2}{\partial Y}\left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right)$$

Solving this system we get

$$\Delta A(K) = \frac{c \left(\frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \right)^{-\alpha} \left(\frac{\lambda}{r} - \frac{\lambda - 1}{\delta} \right)}{(\lambda - \alpha)} \quad (\text{A4})$$

$$\Delta B(K) = \frac{c \left(\frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \right)^{-\lambda} \left(\frac{\alpha}{r} - \frac{\alpha - 1}{\delta} \right)}{(\lambda - \alpha)}$$

The Decision to Invest in the Marginal Unit

Having valued the marginal unit of capacity, we can now value the option to invest in this unit. Let $\Delta G_i(K, Y)$ denote the value of this option when the current capacity level is K . The exercise price of this option is equal to the cost of construction. Therefore, for any level of committed capacity K , $\Delta G_i(K, Y)$ is a perpetual American call option whose value depends on Y . Hence, there will be a threshold value at which it will be optimal to exercise this option. Specifically, for any K there will exist a threshold, $Y(K)$, such that the option to build an additional unit of capacity will be exercised the first time that Y equals or exceeds $Y(K)$.

Following standard arguments ΔG_i satisfies the equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 \Delta G_i}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial \Delta G_i}{\partial Y}(K, Y) - r \Delta G_i(K, Y) = 0$$

The solution is subject to the following boundary conditions:

$$\Delta G_i(K, 0) = 0$$

$$\Delta G_i(K, Y(K)) = \Delta F_i(K, Y(K)) - I$$

$$\frac{\partial \Delta G_i}{\partial Y}(K, Y(K)) = \frac{\partial \Delta F_i}{\partial Y}(K, Y(K))$$

The first boundary condition arises because $Y = 0$ is an absorbing barrier for the process described in (2), and therefore the option to invest has no value at that point. This implies the following functional form for the option to invest in a marginal unit of capacity

$$\Delta G_i(K, Y) = \Delta D(K) Y^\lambda \quad \text{for } Y < Y(K).$$

where λ is the positive root of (A2). The last two boundary conditions form the system of equations that must be solved to get the values of $Y(K)$ and $D(K)$. They are the value-matching and the smooth-pasting condition respectively, and they imply that $Y(K)$ is the value of Y that maximizes the value of the option to invest. In solving this system of equations we notice that since the option to expand capacity will not be exercised when the current capacity is not fully used, that is when $Y < \gamma c K^{1/\gamma} / (\gamma - 1)$, because there is no reason to incur the investment cost to keep the additional capacity idle for some time. Therefore, using the expression for $\Delta F_i(K, Y)$ for $Y > \gamma c K^{1/\gamma} / (\gamma - 1)$ in equations (A3) and (A4) the system becomes

$$\begin{aligned} \Delta D(K) Y(K)^\lambda &= \Delta A(K) Y(K)^\alpha + \frac{n\gamma - 1}{n\gamma} \frac{Y(K) K^{-1/\gamma}}{\delta} - \frac{c}{r} - I \\ \lambda \Delta D(K) Y(K)^{\lambda-1} &= \alpha \Delta A(K) Y(K)^{\alpha-1} + \frac{n\gamma - 1}{n\gamma} \frac{K^{-1/\gamma}}{\delta} \end{aligned}$$

Eliminating $\Delta D(K)$ we are left with the following equation for the investment threshold.

$$(\lambda - \alpha) \Delta A(K) Y(K)^\alpha + (\lambda - 1) \frac{n\gamma - 1}{n\gamma} \frac{Y(K) K^{-1/\gamma}}{\delta} - \lambda \left(\frac{c}{r} + I \right) = 0 \quad (\text{A5})$$

Using the solution for $\Delta A(K)$ in (A4) we can write this equation as follows

$$c \left(\frac{n\gamma c}{n\gamma - 1} \right)^{-\alpha} \left(\frac{\lambda}{r} - \frac{\lambda - 1}{\delta} \right) v_n^\alpha + (\lambda - 1) \frac{n\gamma - 1}{n\gamma} \frac{v_n}{\delta} - \lambda \left(\frac{c}{r} + I \right) = 0 \quad (\text{A6})$$

Where $v_n = Y(K) K^{-1/\gamma}$ is the output price at which firms expand their capacity. It follows that for a given the number of firms in the industry n , v_n is a constant which is independent of the industry capacity K . This last equation cannot be solved analytically to get an expression for v_n , however it is easily solved numerically. Once we get v_n then the investment threshold is

$$Y(K) = v_n K^{1/\gamma} \quad (\text{A7})$$

Therefore, the equilibrium investment strategy is for each firm to invest in an additional unit of capacity whenever the demand factor Y rises to the trigger $Y(K)$. Given the number of firms in the industry n , the trigger function is an increasing function of K . We can also evaluate how the degree of competition affects the equilibrium investment strategies of firms. By totally differentiating equation (A5) we get

$$\frac{\partial Y(K)}{\partial n} = -\frac{Y(K)}{n(n\gamma - 1)} < 0 \quad (\text{A8})$$

Thus, increasing competition leads each firm to expand its capacity sooner to avoid losing the investment opportunity to its competitors.

As seen above, the function $Y(K)$ is the firms' optimal investment rule, if Y and K are such that $Y > Y(K)$, firms should add capacity, increasing K until $Y = Y(K)$. Equivalently, given the current level of the demand shock Y , we can determine the industry's optimal capacity by rewriting Equation (A5) in terms of $K(Y)$,

$$(\lambda - \alpha)\Delta A(K(Y))Y^\alpha + (\lambda - 1)\frac{n\gamma - 1}{n\gamma} \frac{YK(Y)^{-1/\gamma}}{\delta} - \lambda\left(\frac{c}{r} + I\right) = 0 \quad (\text{A9})$$

When the current level of demand is Y , the solution to equation (A9) gives the optimal capacity, $K(Y)$, for an industry that has no capacity. Alternatively, since investment is irreversible, at any time t , $K(Y(t))$ is the level of "desired capacity". Thus, the irreversibility constraint implies that $K(t) \geq K(Y(t))$ for all t , and $K(t) = K(Y(t))$ if the industry is expanding its capacity at time t .

Equation (A8) shows that the investment threshold for additional capacity is decreasing in the number of firms in the industry. This result is based on a given level of industry

capacity that is independent of the number of firms in the market. Equivalently, we can show how the degree of competition affects the industry optimal capacity. Let $K^n(Y)$ be the optimal capacity for an n -firm industry. Comparing equation (A1) for $n = 1$ to the same equation for any n it is easily verified that

$$K^n(Y) = \left[\frac{n\gamma - 1}{n(\gamma - 1)} \right]^\gamma K^1(Y) \quad (\text{A10})$$

The Value of the Assets in Place

First we derive the value of one unit of capacity for fixed K . Following standard arguments, the value of one unit of capacity $J(K, Y)$ satisfies the following differential equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 J}{\partial Y^2}(K, Y) + (r - \delta)Y \frac{\partial J}{\partial Y}(K, Y) - rJ(K, Y) + \pi U_i(K, Y) = 0$$

where πU_i is the profit per unit of installed capacity given by (13). In the region where $Y < n\gamma c K^{1/\gamma} / (n\gamma - 1)$, J satisfies the equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 J}{\partial Y^2}(K, Y) + (r - \delta)Y \frac{\partial J}{\partial Y}(K, Y) - rJ(K, Y) + \left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma} Y K^{-1/\gamma} \right)^\gamma = 0$$

Subject to the boundary condition

$$J(K, 0) = 0$$

Therefore, the solution is of the form

$$J^1(K, Y) = B(K)Y^\lambda + \frac{\left(\frac{c}{n\gamma - 1} \right)^{1-\gamma} \left(\frac{YK^{-1/\gamma}}{n\gamma} \right)^\gamma}{r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2 / 2}$$

We require $r > \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2 / 2$ to ensure that $J^1(K, Y)$ is well defined. Since $\gamma > 1$ and λ is the positive root of (A2) we must have $\gamma < \lambda$.

In the region where $Y > n\gamma c K^{1/\gamma}/(n\gamma - 1)$, J satisfies the equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 J}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial J}{\partial Y}(K, Y) - r \Delta J(K, Y) + Y K^{-1/\gamma} - c = 0$$

Subject to the boundary condition

$$\lim_{Y \rightarrow \infty} J(K, Y) = \frac{Y K^{-1/\gamma}}{\delta} - \frac{c}{r}$$

Which implies a solution of the form

$$J^2(K, Y) = A(K) Y^\alpha + \frac{Y K^{-1/\gamma}}{\delta} - \frac{c}{r}$$

To solve for $A(K)$ and $B(K)$ we consider the point $Y = n\gamma c K^{1/\gamma}/(n\gamma - 1)$, where the two regions meet. $J(K, Y)$ must be continually differentiable across $Y = n\gamma c K^{1/\gamma}/(n\gamma - 1)$, therefore $A(K)$ and $B(K)$ are the solutions to the system of equations

$$J^1\left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right) = J^2\left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right)$$

$$\frac{\partial J^1}{\partial Y}\left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right) = \frac{\partial J^2}{\partial Y}\left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right)$$

Solving this system we get

$$A(K) = \frac{c \left(\frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right)^{-\alpha} \left(\frac{\lambda - \gamma}{(n\gamma - 1)(r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2/2)} + \frac{\lambda}{r} - \frac{\lambda - 1}{\delta} \frac{n\gamma}{n\gamma - 1} \right)}{(\lambda - \alpha)} \quad (\text{A8})$$

$$B(K) = \frac{c \left(\frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right)^{-\lambda} \left(\frac{\alpha - \gamma}{(n\gamma - 1)(r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2/2)} + \frac{\alpha}{r} - \frac{\alpha - 1}{\delta} \frac{n\gamma}{n\gamma - 1} \right)}{(\lambda - \alpha)}$$

The value of one unit of capacity for fixed K , $J(K, Y)$, derived above does not take into consideration the effect of future investment on the value of the installed capacity. However the fact that firms can increase their capacity when the demand factor Y hits $Y(K)$ cuts off some of the upside potential for prices and profits, so the value of one unit of capacity must be less than $J(K, Y)$. Since $Y = 0$ is an absorbing barrier for the demand shock process the value of one unit of installed capital is of the form

$$H(K, Y) = E(K)Y^\lambda + J(K, Y) \quad (\text{A9})$$

Where $E(K)Y^\lambda$ represents the impact of future increases in industry capacity on the value of the firm current capacity. To find $E(K)$ we use the following boundary condition for the value of the K th unit of capacity $H(K, Y)$

$$\frac{\partial H}{\partial K}(K, Y(K)) = 0 \quad (\text{A10})$$

This condition ensures that when the trigger $Y(K)$ is reached, K increases by an infinitesimal amount dK , and J changes from $J(K, Y)$ to $J(K + dK, Y)$.

Combining (A9) and (A10) and the fact that firms invest when capacity is fully used we have

$$E'(K) = - \left[A'(K)Y(K)^\alpha - \frac{1}{\gamma} \frac{Y(K)K^{-1/\gamma-1}}{\delta} \right] Y(K)^{-\lambda}$$

Where $A'(K)$ and $E'(K)$ are the derivatives of the functions $A(K)$ and $E(K)$ with respect to K .

Using the expressions for $Y(K)$ and $A(K)$ in (A7) and (A8) we get

$$E'(K) = \left[\frac{\alpha}{\gamma} \theta_n v_n^\alpha + \frac{1}{\gamma} \frac{v_n}{\delta} \right] v_n^\lambda K^{-\lambda/\gamma-1}$$

Where

$$\theta_n = \frac{c \left(\frac{n\gamma c}{n\gamma - 1} \right)^{-\alpha} \left(\frac{\lambda - \gamma}{(n\gamma - 1)(r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2 / 2)} + \frac{\lambda}{r} - \frac{\lambda - 1}{\delta} \frac{n\gamma}{n\gamma - 1} \right)}{(\lambda - \alpha)}$$

Therefore

$$E(K) = -\frac{1}{\lambda} \left[\alpha \theta_n v_n^{\alpha - \lambda} + \frac{v_n^{1 - \lambda}}{\delta} \right] K^{-\lambda / \gamma}$$

The Value of the Growth Options

When the industry capacity is K the value of the option to invest in one unit of capacity $g(K, Y)$ satisfies the following boundary conditions

$$g(K, 0) = 0 \quad (\text{A11})$$

$$g(K, Y(K)) = H(K, Y(K)) - I \quad (\text{A12})$$

Condition (A11) implies that the value of the option has the functional form $g(K, Y) = c(K)Y^\lambda$

From boundary condition (A12)

$$c(K) = [H(K, Y(K)) - I]Y(K)^{-\lambda} = \left[\frac{\lambda - \alpha}{\lambda} \theta_n v_n^\alpha + \frac{\lambda - 1}{\lambda} \frac{v_n}{\delta} - \frac{c}{r} - I \right] v_n^{-\lambda} K^{-\lambda / \gamma}$$

To get the value of firm i 's growth options we sum the value of these unit options by integrating to get

$$G_i(K, Y) = C(K)Y^\lambda$$

where

$$C(K) = \frac{1}{n} \int_K^\infty c(z) dz = \frac{\gamma}{\lambda - \gamma} \left[\frac{\lambda - \alpha}{\lambda} \theta_n v_n^\alpha + \frac{\lambda - 1}{\lambda} \frac{v_n}{\delta} - \frac{c}{r} - I \right] v_n^{-\lambda} \frac{K^{\frac{\gamma - \lambda}{\gamma}}}{n}$$

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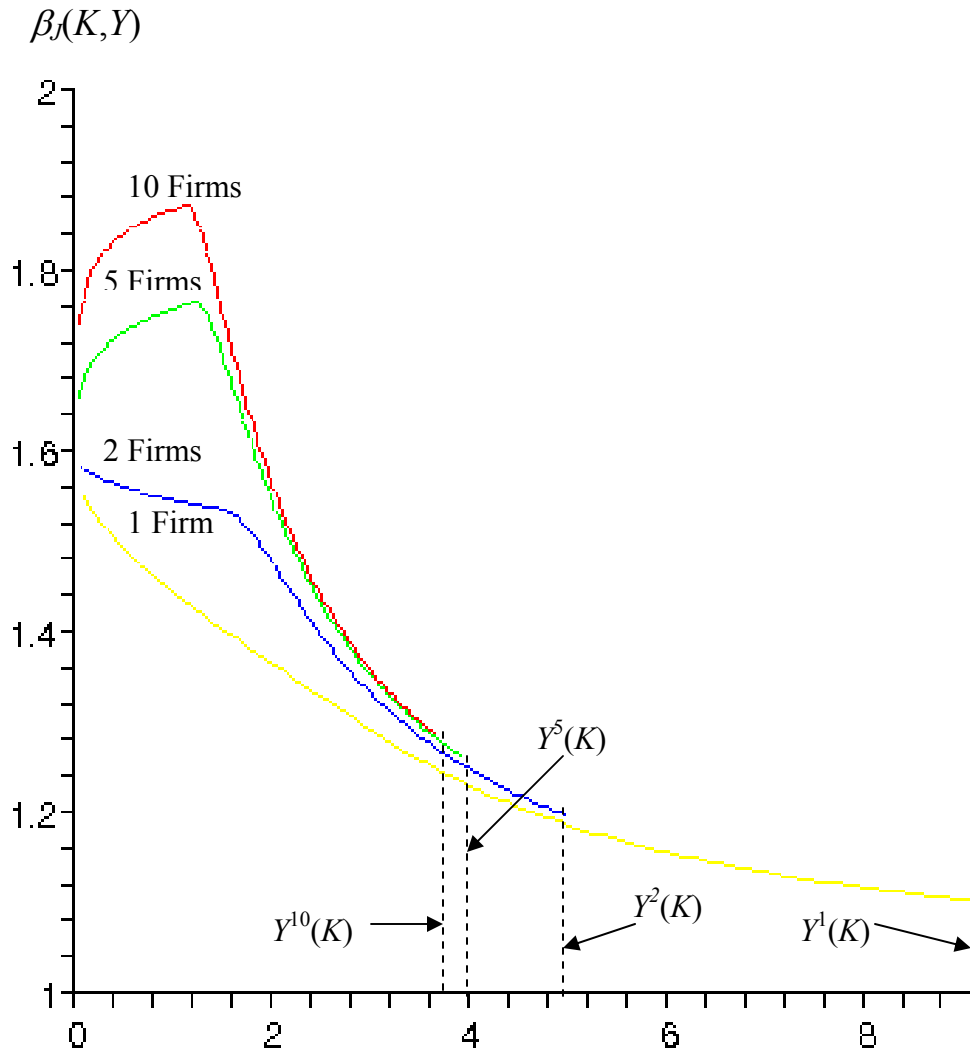
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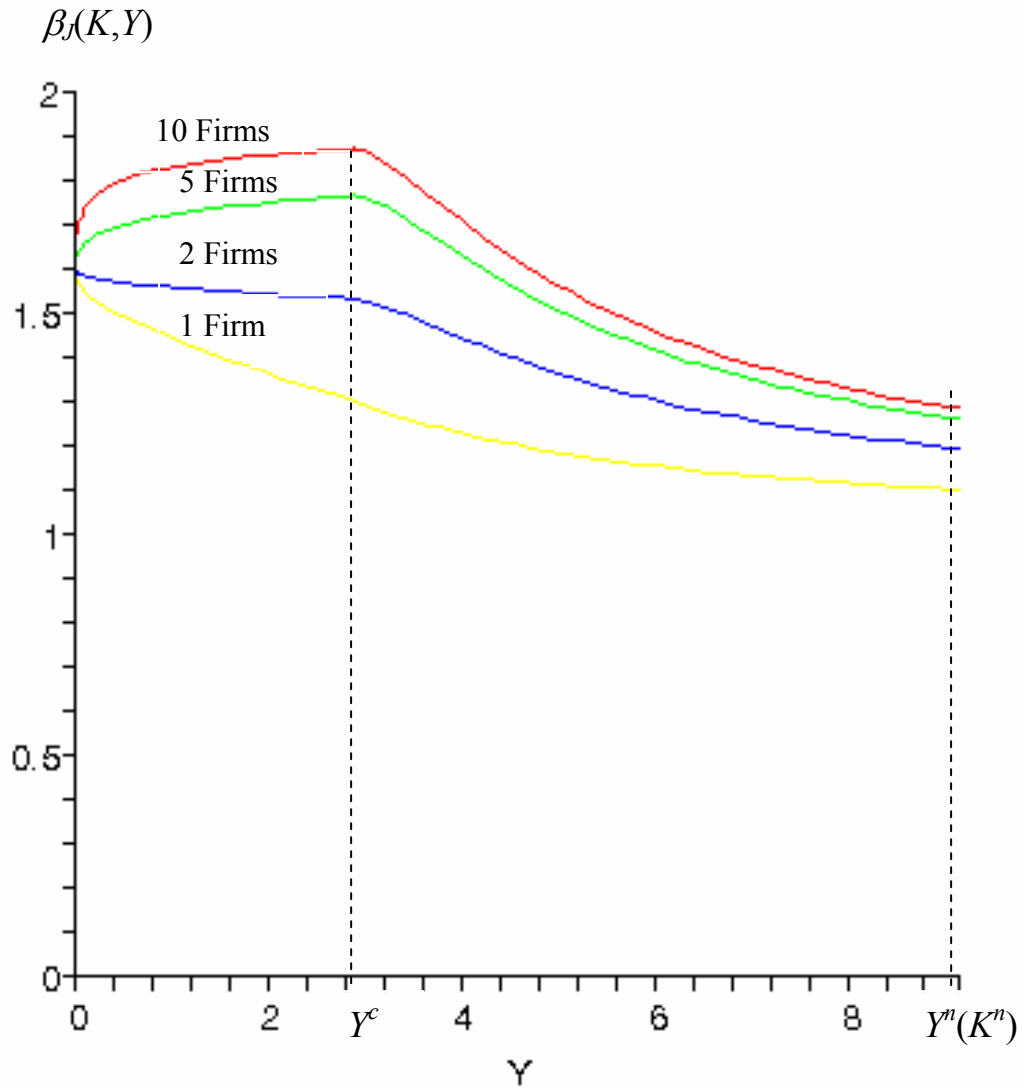
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Figure 1. The beta of the firm's capacity as a function of Y for different number of firms in the industry



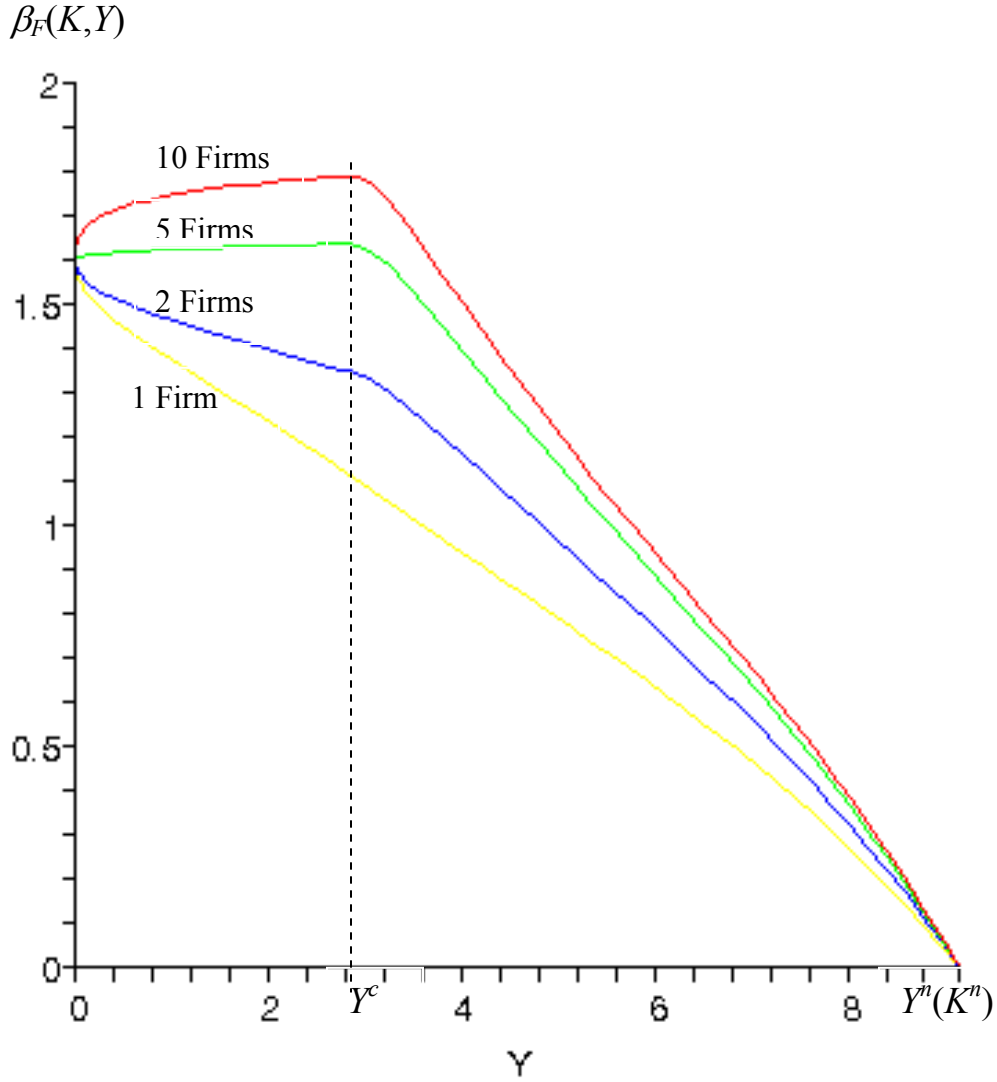
Notes. This figure shows the beta of the firm's capacity as a function of Y when $K = 100$ for 1 firm, 2 firms, 5 firms, and 10 firms. The assumed parameter values are $I = 1$, $c = 0.06$, $\gamma = 1.6$, $r = 0.06$, $\delta = 0.05$, and $\sigma = 0.2$.

Figure 2. The beta of the firm's capacity as a function of Y for different number of firms in the industry



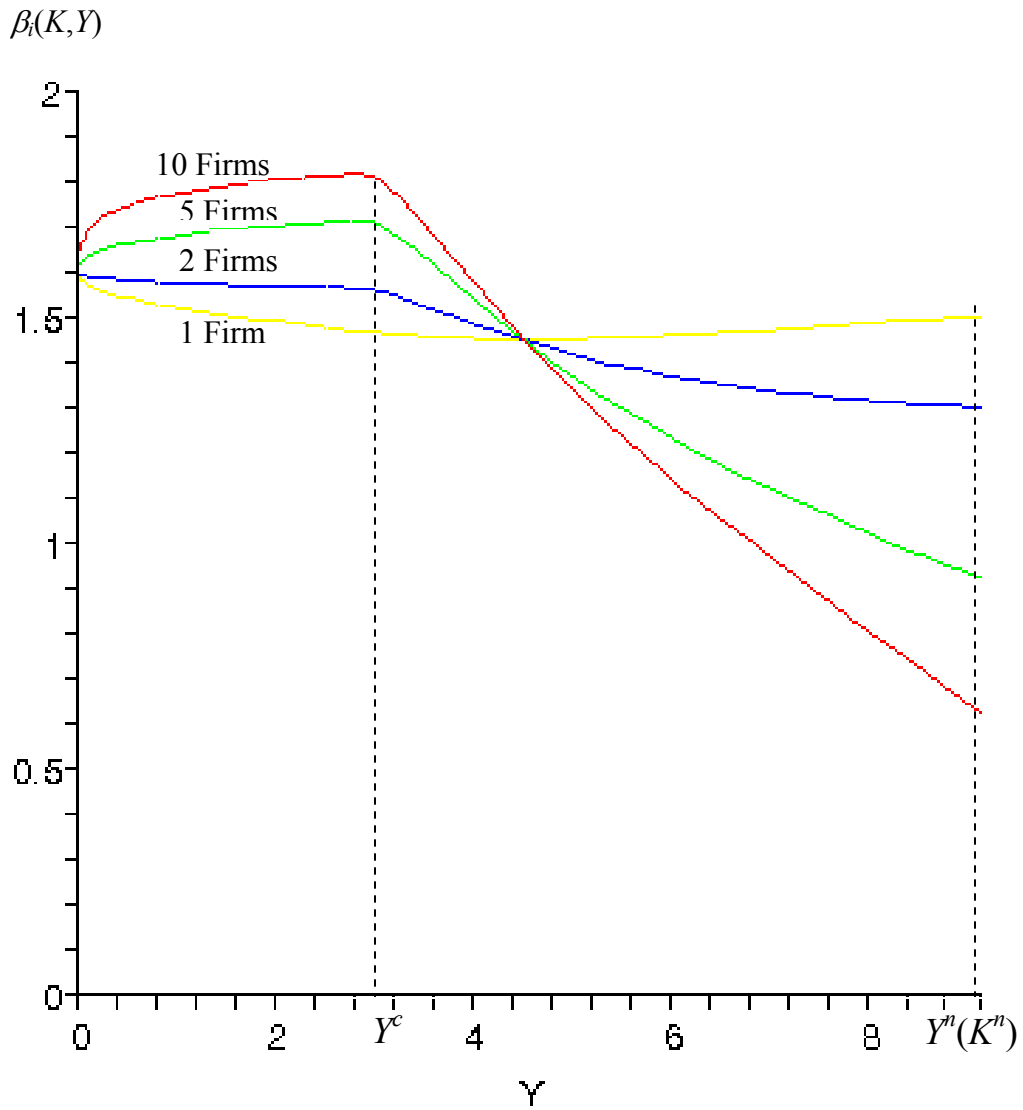
Notes. This figure shows the beta of the firm's capacity as a function of Y when K depends of the number of firms in the market for 1 firm, 2 firms, 5 firms, and 10 firms. The assumed parameter values are $I = 1$, $c = 0.06$, $\gamma = 1.6$, $r = 0.06$, $\delta = 0.05$, and $\sigma = 0.2$.

Figure 3. The beta of the assets in place as a function of Y for different number of firms in the industry



Notes. This figure shows the beta of the firm's assets in place as a function of Y when K depends of the number of firms in the market for 1 firm, 2 firms, 5 firms, and 10 firms. The assumed parameter values are $I = 1$, $c = 0.06$, $\gamma = 1.6$, $r = 0.06$, $\delta = 0.05$, and $\sigma = 0.2$.

Figure 4. The beta of the firm as a function of Y for different number of firms in the industry



Notes. This figure shows the beta of the firm as a function of Y for 1 firm, 2 firms, 5 firms, and 10 firms. The assumed parameter values are $I = 1$, $c = 0.06$, $\gamma = 1.6$, $r = 0.06$, $\delta = 0.05$, and $\sigma = 0.2$.