Revisiting the Asset Substitution Effect in a Contingent-Claims Analysis Setting

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Abstract

This article develops a contingent-claims model in which both the volatility and the return of the firm cash flows are altered by equityholders decisions. Our results contrast with the literature previously based on a reduction of asset substitution to a pure risk-shifting problem. We find larger agency costs and lower optimal leverages. Moreover we show that covenants that prevent equityholders from switching to an activity with high volatility and low return are highly value enhancing when the agency problem is severe. Our model highlights the tradeoff between ex-post inefficient behavior of equityholders and inefficient covenant restrictions.

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1. Introduction

Two competing (and complementary) theories try to explain observed firm's capital structures. The tradeoff theory emphasizes the tax shield value of debt, and compares it to the potential costs of financial distress induced by leverage (Modigliani and Miller (1958 and 1963)). The agency theory of financial structure analyzes information asymmetries and conflicts of interest between stakeholders (Jensen and Meckling (1976), Ross (1977), Leland and Pyle (1977) and Myers (1977)). While the role of both tradeoff and agency theories have been widely discussed, the magnitude of their effects remains poorly known, and we lack quantitative understanding of "the amount of debt a firm should issue in different environments" (Leland (1998)).

In this paper, we revisit in a contingent-claims analysis setting, a classical and important agency problem, namely the study of the conflict of interest between equityholders and debtholders due to the asset substitution effect. The asset substitution problem results from the incentives of equityholders to extract value from debtholders by avoiding safe positive net present value projects. This implies a decrease of the value of the firm, as a result of a decrease of the value of the debt and a smaller increase of the value of the equity. This opportunistic behavior of equityholders is incorporated into the price of debt and the ex ante solution to this agency problem is therefore to issue less debt. As a result, the optimal capital structure of the firm highlights the benefit of issuing debt because of tax benefits, and the cost of issuing debt because of both asset substitution problem and bankruptcy costs. Such a standard stockholder-bondholder conflict might be a key for understanding observed behavior of firms[1], but its relevance still depends on its magnitude.

Leland (1998) and Ericsson (2000) address the issue of modeling and quantifying the asset substitution effect in a contingent claim analysis setting. In both models equityholders can alter the volatility of the unlevered asset value of the firm and the asset substitution effect is therefore modeled as a pure risk shifting problem. In this paper we adopt the view that the asset substitution problem can be also explained by bad investments rather than by simply pure excessive risk taking. According to Bliss (2001) this agency problem may be fundamental: "Poor (apparently irrational) investments are as problematic as excessively risky projects (with positive risk-adjusted returns)". In particular Bliss (2001) reviews several empirical articles that conclude that bank failures are often provoked by bad investments rather than bad luck (and excessive risk taking). This leads us to consider a model in which equityholders can alter both the risk adjusted expected growth rate and the volatility of the cash flows generated by the firm's assets. Specifically, in our model, the firm's activity generates a lognormal cash flows process characterized by a given risk-adjusted expected growth rate and a given volatility. At any time equityholders have the opportunity to switch

[1] It is for instance well documented that firms tend to choose large amount of equity in their capital structure and set debt levels well below what would maximize the tax benefit of debt, see Graham (2000).

to a riskier activity (we will also say that equityholders adopt a *poor* project). The adoption of the poor project lowers the risk adjusted expected growth rate of the cash flows process and and increases its volatility. Therefore two problems jointly define asset substitution (i) a pure risk-shifting problem acting on the volatility of the growth rate of the cash flows, and (ii) a first order stochastic dominance problem acting on the risk adjusted expected growth rate of the cash flows. We show that, because of limited liability, for low values of the cash flows, equityholders can be tempted to change and to adopt the riskier activity.

We assume that the decision to switch to the poor activity is irreversible, that debt is a coupon bond with infinite maturity and that bankruptcy is triggered when equityholders strategically decide to cease paying the coupon to bondholders. Results thus obtained contrast with the previous literature where the asset substitution problem is reduced to a pure risk-shifting problem. For example, depending on the severity of the agency problem, the agency cost of debt at the optimal leverage can be very large (more than 7 %). Accordingly, optimal leverage when an agency problem exists is lower than that of a firm that cannot change its activity. Moreover, covenants that restrict equityholders from adopting the poor activity can exacerbate or mitigate agency costs. The model highlights the tradeoff between ex-post inefficient equityholders behavior and inefficient covenant restrictions.

The remainder of the paper is organized as follows. Section 2 discusses Leland (1998) and Ericsson (2000) and explain how our work complement their analysis. Section 3 presents the model. Section 4 presents optimal policies followed by equityholders. Section 5 defines and characterizes optimal capital structure and agency costs. Section 6 studies the role of covenants. section 7 concludes. Proofs are in Appendix.

2. Related literature.

In an influential paper Leland (1994) proposed a continuous time contingent-claims analysis framework to derive qualitative as well as quantitative insights into corporate finance decision making. Leland (1994) was mainly concerned with the optimal capital structure balancing the tax benefits and the default costs coming with debt. His work pioneered an impressive wealth of papers[2] that provided, for instance, quantitative results on optimal amount and maturity of debt (Leland and Toft (1996)), debt restructuring (Goldstein, Ju and Leland (2001)), credit spreads when bondholders have only imperfect information on the firm's cash flows (Duffie and Lando (2001)) or more recently the role of Warrant in solving agency costs in a setting with dynamic volatility choice (Henessy and Tserlukevich (2004)).

In the same line, Leland (1998) and Ericsson (2000) propose two contingent-claims models that aim at measuring and analyzing costs of debt due to the asset substitution effect. In

[2]Other papers have addressed related questions in settings close to Leland (1994): Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Mauer and Ott (2000) or Décamps and Faure-Grimaud (2002) among others.

both models, markets are complete and the agency problem is reduced to a pure risk shifting problem. In Leland (1998), equityholders can choose between two levels of volatility (σ_L and σ_H with $\sigma_L < \sigma_H$) for the firm's asset at each instant of time and without cost. The choice of the volatility is fully reversible and, in addition, equityholders strategically declare bankruptcy in order to maximize the value of their claim. Equityholders switch from the low volatility level (σ_L) to the high volatility level (σ_H) each time the value of the firm's asset goes below an endogenous switch point value V_S. The model is designed to analyze the role of the maturity of the debt, and allows for dynamic capital structure. It shows that the volatility parameter which is optimal ex ante is not the one chosen ex post, that is after debt has been issued. Striking conclusions are: 1) agency costs of debt due to the asset substitution effect are about 1.5% which is far less than the tax benefits of debt, 2) asset substitution will occur even in the ex ante case albeit to a lesser degree than in the ex post case, and 3) bond covenants that restrict equityholders from adopting the high volatility parameter are useless. Also stimulating in this approach is the higher optimal leverage when there is an agency problem in comparison with the optimal leverage of a firm that cannot increase risk.

We point out in our paper[3] that, the equityholders' strategy of choosing immediately and for ever the high volatility σ_H dominates the single point risk shifting strategy considered in the Leland's analysis. Consequently, in Leland (1998), equityholders do not behave optimally and thus agency costs are underestimated. The key reason is that, because bankruptcy is endogenous[4], equity value is convex in the firm's asset value. This implies in turn that the equity value increases with respect to the volatility of the firm, and therefore equityholders must choose strategically the highest possible volatility level even for large realizations of the cash flows. In our setting bankruptcy is endogenous and equity value increases with the volatility of the cash flows, but increasing the volatility of the firm's activity lowers its return and we show that, under some conditions on the deep parameters of the model, equityholders will not be tempted by the high risk activity for sufficiently large realizations of the cash flows.

In Ericsson (2000) contrary to Leland (1998), bankruptcy is exogenous. Liquidation occurs when the cash flows generated by the firm's activity is not sufficient to cover after-tax payments to debtholders[5]. In such a type of model, the equity value is concave in the firm's asset value and decreases with the volatility of the firm's asset. Equityholders should therefore choose immediately and for ever the low risk activity. Ericsson (2000) overcomes this difficulty assuming that, (contrary to Leland (1998)), the risk adjusted rate of growth of equities is negative (that is the riskless interest rate is lower than the total payout rate

[3] Lemma 4.2 section 4.

[4] That is equityholders have the option to cease paying interest to bondholders and to declare bankruptcy.

[5] Other modeling assumptions distinguish the two papers. The decision to increase the volatility is irreversible in Ericsson (2000) and there is no debt restructuring.

to security holders). This assumption restores the convexity of the equity in the firm's asset value and guarantees the existence of a threshold at which equityholders switch from the low risk activity to the high risk activity. In section 5 of the present paper we also focus on exogenous bankruptcy triggers, and interpret them as covenants that restrict the firm from switching to the high risk activity. We study their role in reducing or exacerbating agency costs.

3. The model

3.1. A simple model of the firm.

We start by reviewing a standard model of a firm. The ideas and the results that we present in this subsection are those of Leland (1994) and Goldstein, Ju and Leland (2001). Consider a firm whose "activity" generates cash flows ("EBIT") that follows the stochastic differential equation

$$\frac{dX_{t,G}}{X_{t,G}} = \mu_G dt + \sigma_G dW_t, \tag{1}$$

where dW is the increment of a Wiener process, μ_G is the instantaneous risk-adjusted expected growth rate of the cash flows and σ_G the volatility of the growth rate. The value of the unlevered firm for a current value x of the cash flows, after paying corporate income taxes, is

$$v_G(x) \;=\; (1-\theta)\mathbb{E}\left[\int_0^\infty e^{-rt} X^x_{t,G}\, dt\right] = \frac{x}{r-\mu_G}(1-\theta),$$

where θ is a tax rate on corporate income and $r > \mu_G$ is the risk free interest rate[6]. The total payout rate to all security holders is therefore

$$\delta_G = \frac{(1-\theta)x}{v_G(x)} = r - \mu_G \tag{2}$$

and consequently, the unlevered asset value V under the risk neutral measure follows the process

$$\frac{dV_{t,G}}{V_{t,G}} = (r-\delta_G)dt + \sigma_G dW_t. \tag{3}$$

Note that, because of relation (2), equation (3) and (1) are the same and we could consider as well for state variable the dynamics of the unlevered asset value of the firm. The firm chooses its initial capital structure consisting of perpetual coupon bond c that remains constant until equityholders endogenously default. In such a simple setting, the firm issues debt so as to take advantage of the tax shields offered for interest expense. Failure to pay the coupon c triggers immediate liquidation of the firm. At liquidation, a fraction γ of the unlevered firm value is lost as frictional cost. The liquidation value of the firm is therefore

$$\frac{(1-\theta)(1-\gamma)x}{r-\mu_G}. \tag{4}$$

[6]We assume that the present value of the cash flows is finite and therefore that $r > \mu_G$.

Taking into account tax benefits and bankruptcy cost, the value of the levered firm is

$$v_G(x) \;=\; \mathbb{E}\left[\int_0^{\tau_L^G} e^{-rt}((1-\theta)X_{t,G}^x + \theta c)\,dt \;+\; e^{-r\tau_L^G}\frac{(1-\theta)(1-\gamma)}{r-\mu_G}X_{\tau_L^G,G}\right].$$

where the stopping time τ_L^G defines the bankruptcy policy chosen by equityholders so as to maximize the value of their claim. Formally the problem of the equityholders is: Find the stopping time $\tau_L^G \in \mathcal{T}$ satisfying

$$E_G(x) \equiv \sup_{\tau\in\mathcal{T}} \mathbb{E}\left[\int_0^{\tau} e^{-rt}(1-\theta)\left(X_{t,G}^x - c\right)dt\right] = \mathbb{E}\left[\int_0^{\tau_L^G} e^{-rt}(1-\theta)\left(X_{t,G}^x - c\right)dt\right] \tag{5}$$

where $\mathcal{T}$ is the set of stopping times generated by the Brownian motion W. Standard computations show that the optimal bankruptcy policy is a trigger policy defined by the stopping time $\tau_L^G = \inf\{t \geq 0 \; s.t \; X_{t,G} = x_L^G\}$ with $x_L^G = -\dfrac{\alpha_G}{1-\alpha_G}\dfrac{c}{r}\dfrac{1}{\nu_G}$ where ν_G denotes the ratio $\dfrac{1}{r-\mu_G}$ and α_G denotes the negative root of the quadratic equation $y(y-1)\frac{\sigma_G^2}{2} + y\mu_G = r$. This implies the following expressions for the equity value $E_G(x)$ and for the firm value $v_G(x)$:

$$\begin{cases} E_G(x) = & (1-\theta)\left\{x\nu_G - \dfrac{c}{r} + \left(\dfrac{c}{r} - x_L^G\nu_G\right)\left(\dfrac{x}{x_L^G}\right)^{\alpha_G}\right\} \text{ if } x > x_L^G, \\ E_G(x) = & 0 \text{ if } x \leq x_L^G \end{cases} \tag{6}$$

and

$$\begin{cases} v_G(x) = & (1-\theta)x\nu_G + \dfrac{\theta c}{r} - \left(\dfrac{\theta c}{r} + x_L^G\gamma(1-\theta)\nu_G\right)\left(\dfrac{x}{x_L^G}\right)^{\alpha_G} \quad \text{if } x > x_L^G, \\ v_G(x) = & (1-\gamma)(1-\theta)x\nu_G \;\text{ if } x \leq x_L^G \end{cases}$$

The interpretation of (6) is standard. The equity value is equal to the net present value of the equities (if equityholders never declare bankruptcy) $(\nu_G x - \frac{c}{r})$ plus the option value associated to the irreversible closure decision at the trigger x_L^G.

We denote in the sequel by $x_{PV}^G = \frac{1}{\nu_G}\frac{c}{r}$, the trigger that equalizes to zero the present value of equities under perpetual continuation. Note that, in line with the real option theory, the bankruptcy trigger x_L^G chosen by the equityholders is smaller than the net present value trigger x_{PV}^G.

The optimal capital structure is characterized by the coupon c to be issued that maximizes the initial firm value.

3.2. A simple model of the firm with risk flexibility.

We now extend this standard model of capital structure by considering that, at any time, equityholders have the option to switch to a riskier activity (we will also say a *poor* activity) that lowers the drift and increases the volatility of the cash flows. There is no opportunity cost to change the activity but the decision to switch is irreversible.

Specifically the riskier activity generates cash flows ("EBIT") satisfying the stochastic differential equation

$$\frac{dX_{t,B}}{X_{t,B}} = \mu_B dt + \sigma_B dW_t, \tag{7}$$

with $\mu_B < \mu_G$ and $\sigma_B > \sigma_G$. Equivalently, the unlevered asset value V under the risk neutral measure follows the process

$$\frac{dV_{t,B}}{V_{t,B}} = (r - \delta_B)dt + \sigma_B dW_t \tag{8}$$

where

$$\delta_B = \frac{(1-\theta)x}{v_B(x)} = r - \mu_B \tag{9}$$

and where

$$v_B(x) \;=\; (1-\theta)\mathbb{E}\left[\int_0^\infty e^{-rt} X^x_{t,B}\, dt\right] = \frac{x}{r-\mu_B}(1-\theta).$$

The key inequalities $\mu_G > \mu_B$ and $\sigma_B > \sigma_G$ characterize the tradeoff that drives our model. Because of limited liability equityholders will be tempted to choose the high risk activity and thus to increase the volatility of the cash flows ($\sigma_B > \sigma_G$). However this choice has a cost since it induces a lower expected return ($\mu_B < \mu_G$). Intuitively, because of this cost, as long as the cash flows are large enough, changing the activity of the firm (that is switching to the riskier activity) is not attractive and equityholders run the firm under the low risk activity. However if the cash flows sharply drop, the lower expected return of the high risk activity may not dissuade equityholders to increase the riskiness of the cash flows. Saying it differently, the lower $\Delta\mu \equiv \mu_G - \mu_B$ with respect to $\Delta\sigma \equiv \sigma_B - \sigma_G$, the larger are the switching incentives of the equityholders. Accordingly, after switching, the liquidation value of the firm becomes

$$\frac{(1-\theta)(1-\gamma)x}{r-\mu_B}. \tag{10}$$

To sum up, in our model, equityholders have to decide (i) when to cease the activity in place and to switch to the riskier activity, (ii) when to liquidate. We refer these two irreversible decisions as the switching/liquidation policy.

4. Optimal switching/liquidation policy.

In order to study the optimal switching/liquidation policy, we first characterize situations where, whatever the initial value of the cash flows, (i) equityholders optimally decide to run always the firm under the low risk activity, and (ii) equityholders immediately adopt the high risk activity (and never switch to the low risk activity). We then study the more interesting case where choosing always the low risk activity or the high risk activity is not optimal.

In the previous section we derived $E_G(.)$, the equity value assuming equityholders run the firm under the low risk activity (and optimally liquidate at time τ_L^G). In the same vein we can obtain $E_B(.)$, the equity value when equityholders run always the firm under the high risk activity. We summarize this as follows.

LEMMA 4.1 *Assume equityholders choose the high risk activity, (that is the dynamics of the cash flows obeys the diffusion process (7)) then, the optimal liquidation policy is defined by the random time* τ_L^B *where* $\tau_L^B = \inf\{t \geq 0 \ s.t \ x_t = x_L^B\}$ *with* $x_L^B = -\frac{\alpha_B}{1-\alpha_B}\frac{c}{r}\frac{1}{\nu_B}$. *In this case, the value of the equities is defined by the equality*

$$E_B(x) = \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}(1-\theta)(X_{t,B}^x - c)dt\right]$$

or equivalently,

$$\begin{cases} E_B(x) = & (1-\theta)\left\{x\nu_B - \frac{c}{r} + \left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_L^B}\right)^{\alpha_B}\right\} \ \ \text{if } x > x_L^B, \\ E_B(x) = & 0 \ \ \text{if } x \leq x_L^B \end{cases}$$

where ν_B *denotes the ratio* $\frac{1}{r-\mu_B}$ *and* α_B *denotes the negative root of the quadratic equation* $y(y-1)\frac{\sigma_B^2}{2} + y\mu_B = r$.

The two following lemma identify the cases where equity value $E(x)$ is either $E_B(x)$ (lemma 4.2), or $E_G(x)$ (lemma 4.3).

LEMMA 4.2 *If* $\mu_G = \mu_B$ *and* $\sigma_G < \sigma_B$ *then, equityholders immediately choose the high risk activity and liquidate the firm at the trigger* x_L^B.

Here, the switching problem is reduced to a pure risk shifting problem. The equity value is increasing and convex with respect to the cash flows x. In turn, this implies that the equity value increases with the volatility of the cash flows. Formally, we have that for all $x \in (0, \infty)$, $E_G(x) < E_B(x)$ (see figure 1). Consequently, equityholders immediately choose the high risk activity[7] and liquidate at the trigger x_L^B. Note that the liquidation trigger is decreasing with the volatility and we have $x_L^B < x_L^G$. Since equityholders get nothing in the bankruptcy event, a necessary condition for never switching to the high-risk activity being always optimal is clearly $x_L^B > x_L^G$. The following lemma shows that it is also a sufficient condition.

[7]In Leland (1998), for a perpetual coupon bond (that is $m = 0$ with his notation), equityholders change the risk regime each time the firm's asset value crosses $V_S = V_0$ where V_0 is the initial firm's asset value normalized to 100 (see fig 4, p 1233). Explicit calculus similar those in the proof of our lemma shows however that equityholders should actually optimally switch to the high risk regime σ_L and then never change.

Lemma 4.3 *If $x_L^G < x_L^B$ then, equityholders optimally never choose the high-risk activity and liquidate at the trigger x_L^G.*

The condition $x_L^G < x_L^B$ ensures that $E_G(x) > E_B(x)$ for all values of x. Equityholders cannot enjoy the high risk activity because the gain from increasing the volatility does not compensate the loss in the expected return.

In these two polar cases the tradeoff between increasing riskiness and decreasing expected return that drives our model is extreme. On the one hand when increasing risk is costless (that is $\mu_G = \mu_B$) equityholders are better off choosing immediately the riskier activity and then never switch to the low risk activity. On the other hand when $\Delta\mu$ is large with respect to $\Delta\sigma$, the high risk activity throws down bankruptcy and equityholders optimally choose always the low risk activity. Note that these two results do not depend on the irreversibility assumption we made on the choice of the firm's activity.

We now study the more interesting case where neither choosing for ever the low risk activity or the high risk activity is optimal. According to the two previous lemma, a necessary condition for that is $x_L^B < x_L^G$ and $\mu_G > \mu_B$. Intuitively, switching to the high risk activity is optimal for low values of the cash flows (since for $x_L^B < x < x_L^G$ we have $E_B(x) > 0$ and $E_G(x) = 0$), whereas for sufficiently large values of the cash flows it may be optimal to postpone the switching decision in order to benefit from the larger expected return of the low risk activity.

Assuming equityholders start running the firm under the low risk activity, their problem is to decide when to switch to the high risk activity. Formally, equityholders solve the optimal stopping time problem: Find the stopping times $\tau_S^\star < \tau_L^\star \in \mathcal{T}$ satisfying

$$
\begin{aligned}
E(x) &\equiv (1-\theta) \sup_{\tau_S \in \mathcal{T}, \tau_L \in \mathcal{T}} \left\{ \mathbb{E}\left[\int_0^{\tau_S} e^{-rt}(X_{t,G}^x - c)dt + \mathbb{E}\left[\int_{\tau_S}^{\tau_L} e^{-rt}(X_{t,B}^{\tau_S, X_{\tau_S,G}^x} - c)dt | \mathcal{F}_{\tau_S} \right]\right]\right\} \\
&= (1-\theta) \left\{ \mathbb{E}\left[\int_0^{\tau_S^\star} e^{-rt}(X_{t,G}^x - c)dt + \mathbb{E}\left[\int_{\tau_S^\star}^{\tau_L^\star} e^{-rt}(X_{t,B}^{\tau_S^\star, X_{\tau_S^\star,G}^x} - c)dt | \mathcal{F}_{\tau_S^\star} \right]\right]\right\} \qquad (11)
\end{aligned}
$$

where $X_{t,B}^{\tau_S, X_{\tau_S,G}^x}$ denotes the process $X_{t,B}$ that takes value $X_{\tau_S,G}^x$ at time τ_S. We show the following:

Proposition 4.1 *If $x_L^B < x_L^G$ and $\mu_G > \mu_B$ then, equityholders strategically switch to the high risk activity at the random time $\tau_S^\star = \inf\{t \geq 0 \;\; s.t \;\; X_t = x_S\}$ and declare bankruptcy at the random time $\tau_L^B = \inf\{t \geq 0 \;\; s.t \;\; X_t = x_L^B\}$. The triggers x_S and x_L^B are defined by the relations*

$$
x_S = \left(\frac{(\alpha_B - \alpha_G)\nu_B}{(\nu_G - \nu_B)(1-\alpha_G)(-\alpha_B)} \right)^{\frac{1}{1-\alpha_B}} x_L^B, \quad \text{and} \quad x_L^B = -\frac{\alpha_B}{1-\alpha_B}\frac{c}{r}\frac{1}{\nu_B}.
$$

The value of equity is defined by the equalities

$$\begin{cases} E(x) = & (1-\theta)\left\{x\nu_G - \dfrac{c}{r} - x_S(\nu_G-\nu_B)\left(\dfrac{x}{x_S}\right)^{\alpha_G}\right\} \\ & +(1-\theta)\left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_S}\right)^{\alpha_G}\left(\frac{x_S}{x_L^B}\right)^{\alpha_B} \quad \textit{if}\, x > x_S, \\ E(x) = & (1-\theta)\left\{x\nu_B - \dfrac{c}{r} + \left(\dfrac{c}{r} - x_L^B\nu_B\right)\left(\dfrac{x}{x_L^B}\right)^{\alpha_B}\right\} \quad \textit{if}\, x_L^B < x \le x_S, \\ E(x) = & 0 \quad \textit{if}\, x < x_L^B \end{cases}$$

Our proposition deserves some comments. First, it shows that the conditions $x_L^B < x_L^G$ and $\mu_G > \mu_B$ are necessary and sufficient for switching from the low risk activity to the high risk activity being optimal. Second, it shows that the optimal switching policy is characterized by a switching trigger $x_S > x_L^G$ that we derive explicitly. Figure 3 illustrates our proposition. Once the cash flows go below the switching trigger x_S equityholders optimally switch to the high risk activity. Because this choice is by assumption irreversible, the equity value is then equal to E_B, the equity value under the high risk activity. As long as the cash flows are larger than x_S, the option value to switch is strictly positive and $E(x) > E_G(x)$.

In our setting, an approximate measure for the severity of the agency problem is the length of the interval $[x_L^B, x_L^G]$. Indeed the larger $\Delta\sigma$, the larger the length of the interval $[x_L^B, x_L^G]$ and the larger the switching trigger x_S. On the contrary the larger $\Delta\mu$, the lower the distance between x_L^B and x_L^G. Ultimately, when $\Delta\mu$ is too large with respect $\Delta\sigma$, the trigger x_L^B becomes larger than the trigger x_L^G, any incentive to choose the high risk activity disappear and, according to lemma 4.3, equityholders always choose the low risk activity. It is interesting to compare the switching trigger x_S to the triggers $x_{PV}^G = \frac{c}{r}\frac{1}{\nu_G}$ and $x_{PV}^B = \frac{c}{r}\frac{1}{\nu_B}$ that equalizes to 0 the net present value of the equities under perpetual continuation when the firm is run, respectively with the low risk activity and with the high risk activity. In particular, when $x_{PV}^G < x_S < x_{PV}^B$ the present value of the equities evaluated at the switching point x_S is positive under the low risk activity but negative under the high risk activity. Equityholders nevertheless strategically switch to the high risk activity at the trigger x_S because the increase in their option value to declare bankruptcy compensates the loss in the net present value defined by the difference $\nu_G - \nu_B$.

We now define the ex post firm value $v(x)$, that is the value of the firm when equityholders strategically switch at the trigger x_S. We have

$$v(x) \;=\; \mathbb{E}\left[\int_0^{\tau_S} e^{-rt}((1-\theta)X_{t,G}^x + \theta c)\,dt \;+\; e^{-r\tau_S} v_B(X_{\tau_S,G}^x)\right]$$

where

$$v_B(x) = \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}((1-\theta)X_{t,B}^x + \theta c)\,dt \;+\; e^{-r\tau_L^B}(1-\gamma)(1-\theta)\nu_B X_{\tau_L^B,B}^x\right].$$

Direct computations yield to

$$
\begin{cases}
v(x) = & (1-\theta)x\nu_G + \dfrac{\theta c}{r} - (1-\theta)x_S\left(\nu_G - \nu_B\right)\left(\dfrac{x}{x_S}\right)^{\alpha_G} \\
& - \left(\frac{\theta c}{r} + x_L^B\gamma(1-\theta)\nu_B\right)\left(\frac{x}{x_S}\right)^{\alpha_G}\left(\frac{x_S}{x_L^B}\right)^{\alpha_B} \quad \text{if } x > x_S, \\
v(x) = & (1-\theta)x\nu_B + \dfrac{\theta c}{r} - \left(\dfrac{\theta c}{r} + x_L^B\gamma(1-\theta)\nu_B\right)\left(\dfrac{x}{x_L^B}\right)^{\alpha_B} \quad \text{if } x_L^B < x \leq x_S, \\
v(x) = & (1-\gamma)(1-\theta)x\nu_B \ \text{ if } \ x \leq x_L^B
\end{cases}
\tag{12}
$$

Let us comment briefly equations (12). For $x \leq x_L^B$ the firm is all-equity financed and is run by the former debtholders and we have $v(x) = (1-\gamma)(1-\theta)\mathbb{E}\left[\int_0^\infty e^{-rt}X_{t,B}^x dt\right] = (1-\gamma)(1-\theta)x\nu_B$. For $x_L^B < x < x_S$, the firm value is equal to the present value of the cash flows when it is running under the high risk activity ($x\nu_B$) plus the present value of tax benefits ($\frac{\theta c}{r}$) minus the discounted expected loss in case of bankruptcy $\left(\left(\frac{\theta c}{r} + x_L^B\gamma(1-\theta)\nu_B\right)\right)\left(\frac{x}{x_L^B}\right)^{\alpha_B}$). The amount of this loss is equal at the bankruptcy trigger to the loss of the tax benefits ($\frac{\theta c}{r}$) plus the loss due to the bankruptcy cost ($x_L^B\gamma(1-\theta)\nu_B$). For $x > x_S$, the additional term $x_S(1-\theta)\left(\nu_G - \nu_B\right)\left(\frac{x}{x_S}\right)^{\alpha_G}$ represents the discounted expected loss in net present value that occurs at the switching trigger x_S.

5. Optimal Capital Structure and Agency costs

Equityholders' option to change the activity at the trigger x_S entails loss in value for debtholders and for the whole firm. If equityholders were able to commit to a certain management policy before debt is issued, this problem will disappear. Staying in the tradition of Leland (1998) we define agency costs as the difference between the optimal firm value when the switching policy can be contracted ex ante (before debt is in place) and the optimal firm value when the switching decision policy is taken ex post (that is after debt is in place). In each case the optimal capital structure is characterized by the coupon rate that maximizes the initial firm value. We comment in this section, through several examples, properties of the optimal capital structure and the magnitude of the agency costs. Table 1 lists the baseline parameters that support our analysis. Tables 2-3-4-5 report for different values of the couples (μ_G, σ_G) and (μ_B, σ_B) the optimal capital structure for the ex ante case and for the ex post case. The following observations can be made.

1. When the firm's activity policy can be committed ex ante to maximize firm value, equityholders will never switch to the high risk activity. Therefore the optimal ex ante firm value coincides in our setting with the optimal firm value when there is no risk flexibility. The agency costs, that can be very large[8], are highly sensitive to a change in $\Delta\mu$, the opportunity cost of choosing the high risk activity. Tables 3 and 4 illustrate this point with agency costs dropping from 13.24% to 1.92% for a 2.5% increase of

[8]In Leland (1998), agency costs are modest and around 1.5%.

$\Delta\mu$. Accordingly, agency costs increase with $\Delta\sigma$ (that is agency costs increase when equityholders have more incentives to choose the high risk activity). In tables 2 and 3 agency costs increase from 1.02% to 13.24% when $\Delta\sigma$ goes from 5% to 30%.

2. The model predicts that the larger the severity of the agency problem, the lower the optimal leverage ratios. Precisely, optimal leverages in presence of agency costs decrease relative to the ex ante case where there is no risk flexibility[9]. In table 3, leverages drop by more than 35% with respect to the ex ante case where there is no risk flexibility.

3. In our model, agency costs have no significant effect on yield spreads. The reason is that we focus on a pure switching problem between two activities. In particular, we do not consider an additional financing need at the switching trigger nor production costs for generating the cash flows. Remark however that the yield spreads are lower in the ex post case than in the ex ante case. This result can be explained noting that optimal leverage in the ex ante case is much more important than optimal leverage in the ex post case.

6. Covenants

Following Leland (1994), Leland (1998) or Duffie and Lando (2000), among many others, we have considered the case of endogenous bankruptcy (equityholders have the power to decide the time to go bankrupt). It is however also well documented that covenants written in the debt indenture can trigger bankruptcy. For instance, the so-called "cash flows based" covenant rule triggers bankruptcy as soon as the instantaneous cash flows x_t are not sufficient to cover payments c to debtholders. This is the line followed by Kim et al (1993), Anderson and Sundaresan (1996), Fan and Sundaresan (2000) or Ericsson (2000). Under this covenant rule, it can be easily shown that the equity value is concave in x, increasing in the rate of return μ and decreasing in the volatility σ. Thus, equityholders are never tempted by the high risk activity and the firm is liquidated at the exogenous trigger $x_L^{CF} = c$. Unfortunately, the fact that equityholders never switch to the high risk technology does not imply that the agency costs of debt is reduced. Quite on the contrary, numerical results show that rather than triggering bankruptcy very early at the trigger x_L^{CF}, it is socially optimal to let equityholders switching to the high risk activity and to let them liquidate at the threshold x_L^B lower than x_L^{CF}. This suggests that less strong covenants that restrict the firm from adopting the high risk activity may be useful to reduce agency costs. Based on these remarks we now introduce the "no-switching based" covenant rule that we define as the lowest liquidation trigger such that the unique optimal policy for the equityholders is never to switch to the

[9]Leland (1998) finds the opposite.

high risk activity. We show thereafter that depending on the severity of the agency problem such a covenant can reduce or increase the agency costs of debt.

PROPOSITION 6.2 *The smallest liquidation trigger such that the switching problem disappears is given by*

$$x_L^{NS} = \frac{c}{r}\frac{\alpha_B - \alpha_G}{\nu_G(1-\alpha_G) - \nu_B(1-\alpha_B)}.$$

First, note that $x_L^{NS} < x_L^{CF}$. In words, "cash flows based" covenant rule is not necessary to give equityholders the right incentives never to switch to the high risk activity. Triggering bankruptcy at the lower trigger x_L^{NS} is sufficient. Second, remark that $x_L^{NS} \geq x_L^G \Leftrightarrow x_L^G \geq x_L^B$. In words, the liquidation trigger x_L^{NS} is larger than x_L^G the optimal liquidation trigger when there is no switching if and only if equityholders have indeed incentives to switch. This last remark shows that deterring risk shifting incentives is costly for the firm and highlights the tradeoff between ex-post inefficient equityholders behavior and inefficient covenant restrictions. Third, note that the trigger x_L^{NS} is decreasing with the opportunity costs of switching ($\Delta\mu$). That is, when the difference in net present value of the two technologies increases, equityholders have less incentives to switch to the high risk activity, and consequently, there is less need to engage in costly covenant restrictions to make them never choose the high risk activity.

Under the "no-switching based" covenant rule, the ex post value of the firm is defined by the following expression:

$$\begin{cases} v(x) = & (1-\theta)x\nu_G + \dfrac{\theta c}{r} - \left(\dfrac{\theta c}{r} + (1-\theta)x_L^{NS}\gamma\nu_G\right)\left(\dfrac{x}{x_L^{NS}}\right)^{\alpha_G} \quad \text{if } x > x_L^{NS}, \\ v(x) = & (1-\gamma)(1-\theta)x\nu_G \;\; \text{if } \; x \leq x_L^{NS} \end{cases}$$

Tables 6-9 compare the optimal capital structure and the magnitude of the agency costs when bankruptcy is endogenous and when bankruptcy is triggered by our "no-switching based" covenant rule. It turns out that the covenant restriction restores some value to the firm as soon as the agency problem is severe enough. In Table 7 the covenant restriction allows to reduce agency costs by more than 9% (accordingly, optimal leverage increases from 43.84 % to 71.30%). This is also the case when the opportunity costs of risk shifting are large. On the contrary when the opportunity costs of risk shifting are low and the agency problem not important (that is when $\Delta\mu$ is small but large with respect to $\Delta\sigma$) , the covenant restriction may worsen the situation. In table 6 agency costs increase from 1.02% for the endogenous bankruptcy rule to 2.59% for the "no-switching based" covenant rule (however, optimal leverage do not drop; this is the effect of the lower risk environment). Table 9 illustrates that our covenants rule is a powerful tool to eliminate inefficient risk shifting. The fact that the "no-switching based" covenant rule worsens the situation when the agency problem is not enough severe suggests to study a less strong covenant restriction that may leave equityholders to switch to the risky activity, but still entails liquidation of the firm before equityholders will do (that is before the threshold x_L^B being reached). Precisely, consider a covenant that imposed liquidation at a threshold $x_L \in [x_L^B, x_L^{NS}]$, then equityholders react choosing a corresponding risk shifting trigger $x_S(x_L)$. The switching trigger $x_S(x_L)$ can be explicitly computed and shown to be decreasing in x_L on the interval $[x_L^B, x_L^{NS}]$ with $x_S(x_L^B) = x_S$ and $x_S(x_L^{NS}) = x_L^{NS}$. This last equality corroborates proposition 6.2 and states that equityholders never switch when liquidation is triggered at x_L^{NS}. We have then numerically compared agency costs when the liquidation policy is defined by the threshold $x_L = x_L^{NS}$ and when liquidation is triggered by $x_L \in [x_L^B, x_L^{NS})$. Our numerical results suggest that the optimal liquidation policy consists of a binary choice $x_L = x_L^B$ or $x_L = x_L^{NS}$. That is, covenant restrictions may be useful only to the extent that they can fully deter the switching problem. However, if the agency problem is not severe enough, covenants worsen the situation and it is preferable to let equityholders acting strategically.

7. Conclusion.

Most of the literature on agency problem in a contingent-claims analysis setting considers the case in which the drift μ of the firm's assets is unchanged and equal to the risk free interest rate r but the volatility σ increases by moral hazard. We adopt here the view that bankruptcy can also be explained by bad investments rather than by simply pure excessive risk taking. This leads us to consider a model in which both μ and σ are altered by the equityholders decisions. Our results drastically differ from contingent-claims models where equityholders only act on the volatility of the firm's cash flows. We find larger agency costs and lower optimal leverages. We show that covenants that restrict equityholders from switching to an activity with high volatility and low return are essentially value enhancing, although it may worsen the situation in less bad environment. The model highlights the tradeoff between ex-post inefficient behavior of equityholders and inefficient covenant restrictions.

The contingent-claims analysis is of course not the sole approach to examine agency problem in corporate finance. For instance Parrino and Weisbach (1999) using discounted cash flows analysis simulate equityholders/bonholders conflict when equityholders have a growth opportunity to invest in a project whose cash flows are correlated with the cash flows of the firm's existing asset. They conclude that the importance of the equityholders/bondholders conflict is small and cannot explain observed debt level. Biais, Bisière and Décamps (1999) estimated a structural model of financing choices in presence of moral hazard, default costs and tax shields. They found large agency costs and a non-significant role of tax shields in the financing decision. These mixed results suggest that analyzing and quantifying agency problems in corporate finance still remain to be solved. Models that allow disentangling and quantifying theories that could explain behavior of firms are needed. Contingent claim analysis, numerical based approach and structural econometric approach offer three complementary routes for this major challenge in financial economy.

8. Appendix

Proof of lemma 4.2 Let denote $\nu = \frac{1}{r-\mu}$, α_σ the negative root of the quadratic equation $\frac{1}{2}\sigma y^2 + (\mu - \frac{1}{2}\sigma^2)y - r = 0$ and $x_L^\sigma = -\frac{\alpha_\sigma}{1-\alpha_\sigma}\frac{c}{r}\frac{1}{\nu}$. A direct computation shows that the mapping $\sigma \longrightarrow x\nu - \frac{c}{r} + \left(\frac{c}{r} - x_L^\sigma \nu\right)\left(\frac{x}{x_L^\sigma}\right)^{\alpha_\sigma}$ is increasing on $(0, \infty)$. Lemma 4.2 is then deduced remarking that, if $\mu_G = \mu_B$, then $x_L^G > x_L^B$ and thus $E_B(x) > E_G(x) = 0 \;\; \forall x_L^B < x < x_L^G$.

Proof of lemma 4.3 A sufficient condition for obtaining our result is $E'_G(x) > E'_B(x)$ for all $x > x_L^B$. We have for all $x > x_L^B$:

$$\begin{aligned}
\frac{1}{1-\theta}\left(xE'_G(x) - xE'_B(x)\right) &= x(\nu_G - \nu_B) + \alpha_G(\frac{c}{r} - x_L^G\nu_G)\left(\frac{x}{x_L^G}\right)^{\alpha_G} \\
&\quad -\alpha_B(\frac{c}{r} - x_L^B\nu_B)\left(\frac{x}{x_L^B}\right)^{\alpha_B} \\
&> x_L^B(\nu_G - \nu_B) + \alpha_G(\frac{c}{r} - x_L^G\nu_G)\left(\frac{x}{x_L^B}\right)^{\alpha_G} \\
&\quad -\alpha_B(\frac{c}{r} - x_L^B\nu_B)\left(\frac{x}{x_L^B}\right)^{\alpha_B} \\
&> \{x_L^B(\nu_G - \nu_B) + \alpha_G(\frac{c}{r} - x_L^G\nu_G) \\
&\quad -\alpha_B(\frac{c}{r} - x_L^B\nu_B)\}\left(\frac{x}{x_L^B}\right)^{\alpha_G} \\
&> \{(\nu_G x_L^G - \frac{c}{r})(1-\alpha_G) \\
&\quad -(\nu_B x_L^B - \frac{c}{r})(1-\alpha_B)\}\left(\frac{x}{x_L^B}\right)^{\alpha_G} = 0
\end{aligned}$$

Proof of proposition 4.1

It follows from the strong Markov property that optimization problem (11) can be rewritten under the form

$$E(x) \equiv \sup_{\tau_S \in \mathcal{T}} \mathbb{E}\left[\int_0^{\tau_S} e^{-rt}(1-\theta)\left(X_{t,G}^x - c\right)dt \; + \; e^{-r\tau_S}E_B(X_{\tau_S,G}^x)\right].$$

The proof of our proposition relies then on the following lemma which shows that the optimal switching strategy is a trigger strategy.

Lemma 8.4 *If*

$$E(x) \,=\, \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}(1-\theta)\left(X_{t,B}^x - c\right)dt\right]$$

then

$$E(x-h) \;=\; \sup_{\tau_S \in \mathcal{T}} \mathbb{E}\left[\int_0^{\tau_S} e^{-rt}(1-\theta)\left(X_{t,G}^{x-h} - c\right)dt + e^{-r\tau_S}E_B(X_{\tau_S,G}^{x-h})\right]$$

$$= \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}(1-\theta)\left(X_{t,B}^{x-h} - c\right) dt\right]$$

Proof of the lemma 8.4: Taking advantage from the equalities $X_{t,G}^{x-h} = X_{t,G}^{x} - X_{t,G}^{h}$ and $X_{t,B}^{X_{\tau,G}^{x-h}} = X_{t,B}^{X_{\tau,G}^{x}} - X_{t,B}^{X_{\tau,G}^{h}}$, we deduce from the definitions of $E(x)$ and $E(x-h)$

$$E(x-h) \;\leq\; E(x) \;-\; \inf_{\tau\in\mathcal{T}}\left\{(1-\theta)\mathbb{E}\left[\int_0^{\tau} e^{-rt}X_{t,G}^{h}dt \;+\; e^{-r\tau}\mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}X_{t,B}^{X_{\tau,G}^{h}}dt \,|\, \mathcal{F}_\tau\right]\right]\right\}$$

Moreover,

$$\begin{aligned}
\mathbb{E}\left[\int_0^{\tau} e^{-rt}X_{t,G}^{h}dt\right] &= \nu_G\left(h - \mathbb{E}\left[e^{-r\tau}X_{\tau,G}^{h}\right]\right), \\
\mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}X_{t,B}^{X_{\tau,G}^{h}}dt \,|\, \mathcal{F}_\tau\right] &= \nu_B\left(X_{\tau,G}^{h} - x_L^B\mathbb{E}\left[e^{-r\tau_L^B} \,|\, \mathcal{F}_\tau\right]\right),
\end{aligned}$$

from which we deduce

$$\begin{aligned}
&\mathbb{E}\left[\int_0^{\tau} e^{-rt}X_{t,G}^{h}dt \;+\; e^{-r\tau}\mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}X_{t,B}^{X_{\tau,G}^{h}}dt \,|\, \mathcal{F}_\tau\right]\right] \\
&= \nu_G h - (\nu_G - \nu_B)\,\mathbb{E}\left[e^{-r\tau}X_{\tau,G}^{h}\right] - \nu_B x_L^B\mathbb{E}\left[e^{-r\tau}e^{-r\tau_L^B}\right]
\end{aligned}$$

Now, from a standard result in optimal stopping time theory, $\sup_{\tau\in\mathcal{T}}\mathbb{E}\left[e^{-r\tau}X_{\tau,G}^{h}\right] = h$ which implies that

$$\inf_{\tau\in\mathcal{T}}\left\{\mathbb{E}\left[\int_0^{\tau} e^{-rt}X_{t,G}^{h}dt \;+\; e^{-r\tau}\mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}X_{t,B}^{X_{\tau,G}^{h}}dt \,|\, \mathcal{F}_\tau\right]\right]\right\} \;=\; \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}X_{t,B}^{h}dt\right].$$

We thus obtain that

$$E(x-h) \leq \mathbb{E}\left[\int_0^{\tau_L^B} e^{-rt}(1-\theta)\left(X_{t,B}^{x-h} - c\right) dt\right].$$

As the converse inequality is always satisfied, lemma(8.4) is proved.

Thus, the optimal switching policy is a trigger policy. For a given switching trigger x_S, the equity value is given by standard computations

$$\begin{cases}
E(x) = & (1-\theta)\left\{x\nu_G - \dfrac{c}{r} - x_S(\nu_G - \nu_B)\left(\dfrac{x}{x_S}\right)^{\alpha_G}\right\} \\
& +(1-\theta)\left(\frac{c}{r} - x_L^B\nu_B\right)\left(\frac{x}{x_S}\right)^{\alpha_G}\left(\frac{x_S}{x_L^B}\right)^{\alpha_B} \quad \text{if } x > x_S, \\
E(x) = & (1-\theta)\left\{x\nu_B - \dfrac{c}{r} + \left(\dfrac{c}{r} - x_L^B\nu_B\right)\left(\dfrac{x}{x_L^B}\right)^{\alpha_B}\right\} \quad \text{if } x_L^B < x \leq x_S, \\
E(x) = & 0 \;\text{ if } x < x_L^B
\end{cases}$$

It is easy to see that this value function reaches its maximum for a value of x_S that does not depend on x, namely

$$x_S = \left(\frac{(\alpha_B - \alpha_G)\nu_B}{(\nu_G - \nu_B)(1-\alpha_G)(-\alpha_B)} \right)^{\frac{1}{1-\alpha_B}} x_L^B > x_L^B.$$

Proof of proposition 6.2

Since by construction $E_G(x_L^{NS}) = E_B(x_L^{NS}) = 0$, a necessary condition for equityholders being not tempted by switching is $E'_G(x_L) > E'_B(x_L)$ where x_L is a liquidation trigger. The minimum liquidation trigger that satisfies this condition is implicitly defined by the equation $x_L E'_G(x_L) = x_L E'_B(x_L)$. This leads to $x_L = x_L^{NS} = \frac{c}{r}\frac{\alpha_B - \alpha_G}{\nu_G(1-\alpha_G) - \nu_B(1-\alpha_B)}$. Conversely, reasoning as in the proof of lemma 4.3, we show that $E_G(x) - E_B(x) \geq 0$ for $x \geq x_L^{NS}$.

References

[1] Anderson, R. and S. Sundaresan, 1996, Design and Valuation of Debt contracts, *Review of Financial Studies* 9, 37-68.

[2] Biais, B., Bisière, C., and J.P. Décamps, J.P., 1999, A Structural Econometric Investigation of the Agency Theory of Financial Structure, Working Paper, University of Toulouse 1.

[3] Dangl T., and A. Lehar, 2004, Value-at-Risk vs. Building Block Regulation in Banking, *Journal of Financial Intermediation* 13, 96-131.

[4] Décamps, J.P., and A. Faure-Grimaud, 2002, Excessive continuation and Dynamic Agency Costs of Debt *European Economic Review* 46, 1623-1644.

[5] Décamps, J.P., Rochet, J.C. and Roger B., 2004, The Three Pillars of Basel II: Optimizing the Mix, *Journal of Financial Intermediation* 13, 132-155.

[6] Duffie, D., and D. Lando, 2001, Term Structures of Credit Spreads with Incomplete Accounting Information, *Econometrica* 69, 633-664.

[7] Ericsson, J., 2000, Asset substitution, Debt Pricing, Optimal Leverage and Maturity. *Finance* 21, 39-70.

[8] Fan, H., and S. Sundaresan, 2000, Debt Valuation, Renegotiation, and Optimal Dividend Policy, *Review of Financial Studies* 13, 1057-1099.

[9] Goldstein, R., Ju, N., and H. Leland, 2001, an EBIT Based Model of Dynamic Capital Structure, Journal of Business,74, 483-512.

[10] Graham, J., 2000, How Big Are the Tax Benefits of debt? *Journal of Finance*, 55, 1901-1941.

[11] Henessy, C.A., and Y. Tserlukevich, 2004, Dynamic Hedging Incentives, Debt and Warrants, Working Paper, Haas School of Business, Berkeley.

[12] Jensen, M., and W. Meckling, 1976, Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, *Journal of Financial Economics,* 3, 305-360.

[13] Leland,H., 1994, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance* 49, 1213-1252.

[14] Leland, H., and D. Pyle, 1977, Information Asymetries, Financial Structure and Financial Intermediation, *Journal of Finance*, 317-387.

[15] Leland, H., and K. Toft, 1996, Optimal Capital Struture, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, *Journal of Finance* 51, 987-1019.

[16] Leland, H., 1998, Agency Costs, Risk Management, and Capital Structure, *Journal of Finance* 53, 1213-1243.

[17] Mauer, D.C. and S.H. Ott, 2000, Agency Costs, underinvestment, and Optimal Capital Structure: The Effects of Growth Options to Expand. In M.J. Brennan and L. Trigeorgis, eds., Project Flexibility, Agency, and Competition, New-York, NY: Oxforf University Press, 151-180.

[18] Mella-Barral, P. and W. Perraudin, 1997, Strategic Debt Service, *Journal of Finance* 52, 531-556.

[19] Modigliani, F., and M. Miller, 1958, The Cost of Capital, Corporation Finance, and the Theory of Investment, *American Economic Review*, 48, 261-297.

[20] Modigliani, F., and M. Miller, 1963, Corporate Income Taxes and the Cost of Capital: A Correction, *American Economic Review*, 433-443.

[21] Myers, S., 1977, Determinants of Corporate Borrowing, *Journal of Financial Economics*, 147-175.

[22] Parrino, R., and Weisbach, 1999, Measuring Investment distorsion Arising from Stockholder-Bondholder Conflicts, *Journal of Financial Economics* 53, 3-42.

[23] Ross, S., 1977, The Determination of Financial Structure: The Incentive Signalling Approach, *Bell Journal of Economics*, 23-40.

Figures.

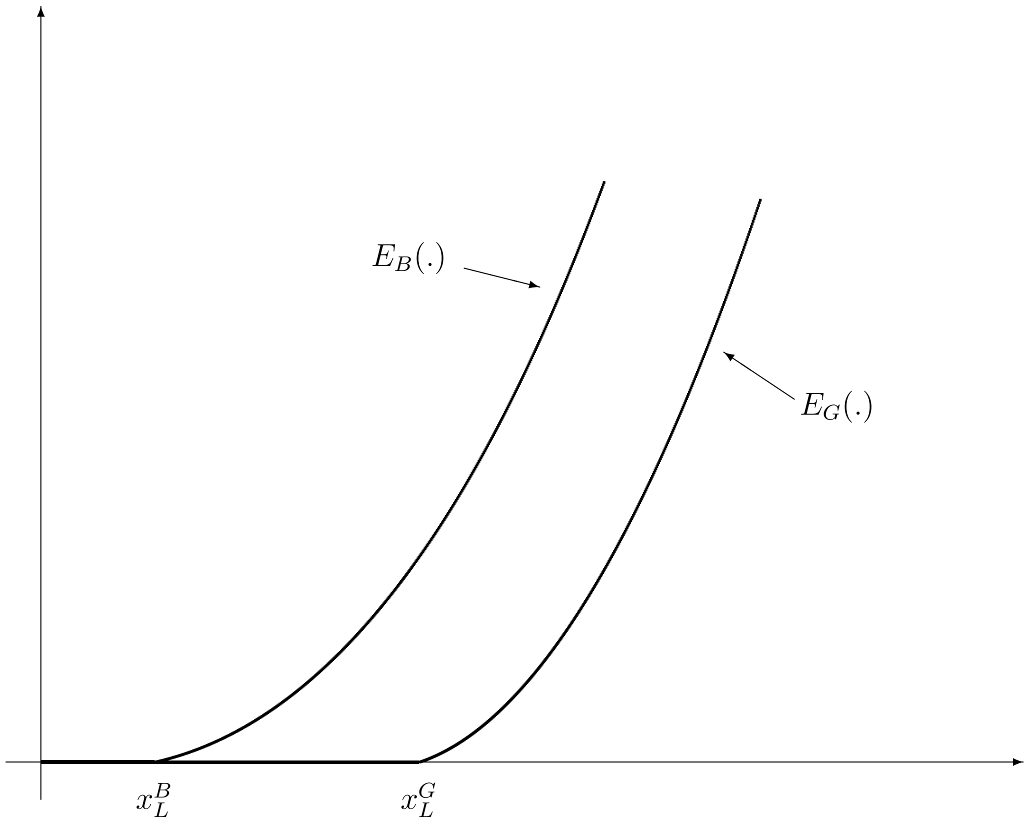

Figure 1 : $\mu_G = \mu_B$ and $\sigma_G < \sigma_B$

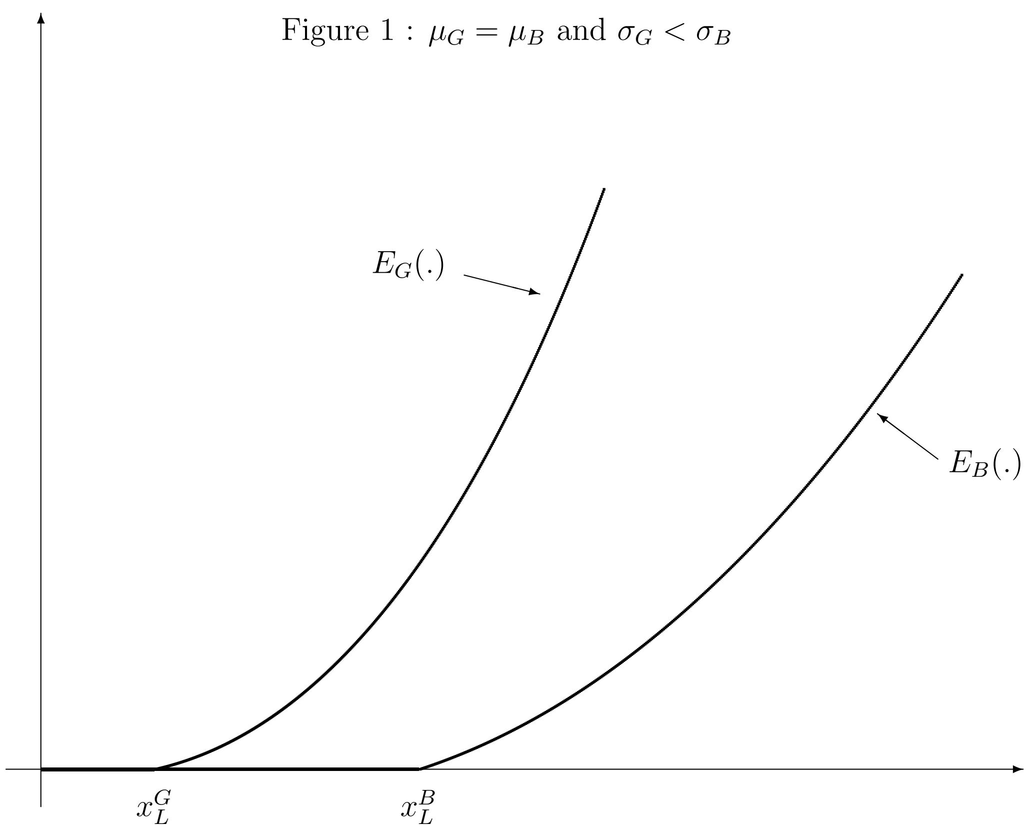

Figure 2 : $x_L^G < x_L^B,\ \mu_G > \mu_B,\ \sigma_G < \sigma_B$

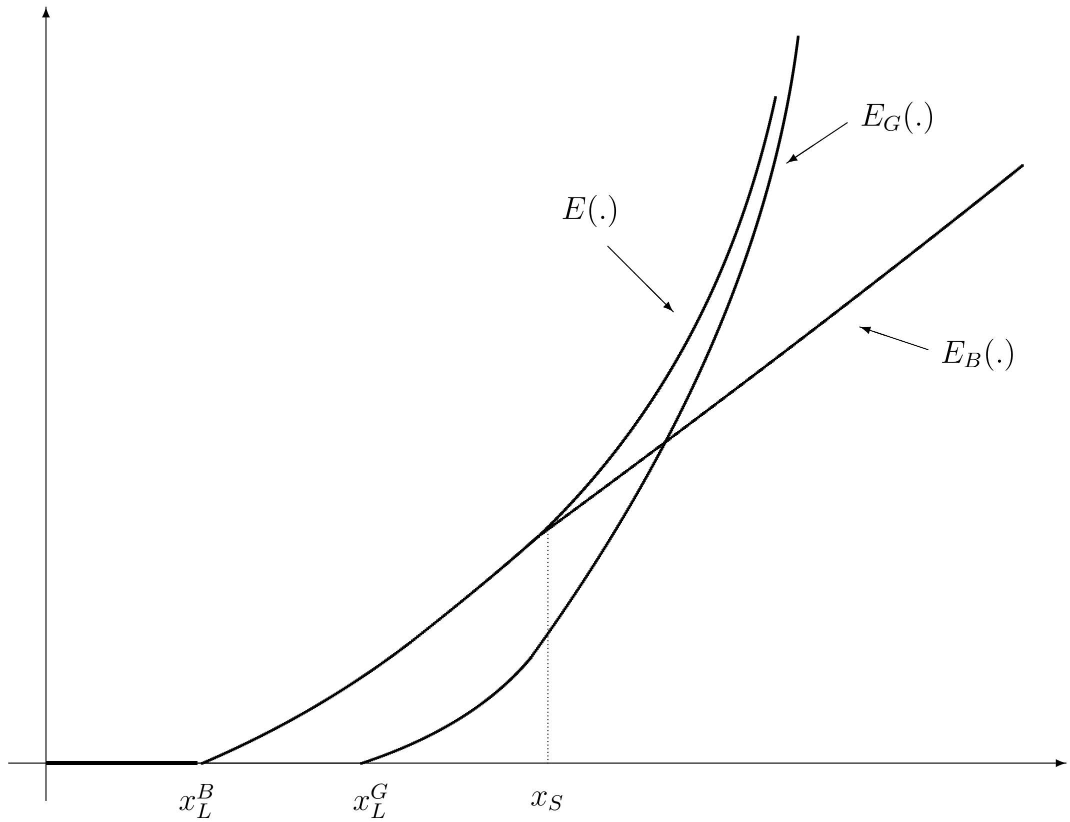

Figure 3 : $x_L^G > x_L^B,\ \mu_G > \mu_B,\ \sigma_G < \sigma_B$

Tables.

Table 1. Parameters for the base case: γ is the bankruptcy cost, θ the tax rate, r the fixed market interest rate and x the normalized initial cash flows value. Values are similar to Leland (1998), Duffie and Lando (2001) or Henessy and Tserlukevich (2004).

Table 1.

γ	θ	r	x
0.4	0.35	0.06	5

Tables 2-5. Optimal capital structure and magnitude of the agency costs for the ex ante case and for the ex post case, for different values of the couples (μ_G, σ_G) and (μ_B, σ_B). In these tables, $v(x)$ is the optimal firm value; $c^0(x)$ is the optimal coupon; L (in percentage of the firm value) is the optimal leverage (D/v) where the debt value D is equal to $v - E$; YS (in basis points) is the yield spread $(c/D - r)$ over the debt; AC (in percentage of the ex ante firm value) is the magnitude of the agency costs.

Table 2.

$\sigma_G = 0.15$	$\Delta\sigma = 5\%$	$\mu_G = 0.015$		$\Delta\mu = 0.5\%$	
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex ante	92.05	4.77	74.61	95	–
Ex post	91.11	4.54	72.07	91	1.02

Table 3.

$\sigma_G = 0.1$	$\Delta\sigma = 30\%$	$\mu_G = 0.015$		$\Delta\mu = 0.5\%$	
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex ante	96.34	5.03	80.51	49	–
Ex post	83.58	2.37	43.84	47	13.24

Table 4.

$\sigma_G = 0.1$	$\Delta\sigma = 30\%$	$\mu_G = 0.03$		$\Delta\mu = 3\%$	
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex ante	148.37	7.88	83.91	33	–
Ex post	145.52	7.32	79.62	32	1.92

Table 5.

$\sigma_G = 0.2$	$\Delta\sigma = 20\%$	$\mu_G = 0.03$		$\Delta\mu = 3\%$	
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex ante	135.88	7.08	72.54	118	–
Ex post	132.98	6.34	67.1	111	2.13

Tables 6-9. Optimal capital structure and magnitude of the agency costs when bankruptcy is endogenous and when bankruptcy is triggered by our "no-switching based" covenant rule. In these tables, $v(x)$ is the optimal firm value; $c^0(x)$ is the optimal coupon; L (in percentage of the firm value) is the optimal leverage (D/v) where the debt value D is equal to $v - E$; YS (in basis points) is the yield spread $(c/D - r)$ over the debt; AC (in percentage of the ex ante firm value) is the magnitude of the agency costs.

Table 6.

$\sigma_G = 0.15$ $\Delta\sigma = 5\%$	$\mu_G = 0.015$			$\Delta\mu = 0.5\%$	
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex post case with endogenous bankruptcy	91.11	4.54	72.07	91	1.02
Ex post case with no switching based covenant	89.64	4.19	73.22	38	2.59

Table 7.

$\sigma_G = 0.1$ $\Delta\sigma = 30\%$	$\mu_G = 0.015$		$\Delta\mu = 0.5\%$		
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex post case with endogenous bankruptcy	83.58	2.37	43.84	47	13.24
Ex post case with no switching based covenant	92.50	4.23	71.30	41	3.99

Table 8.

$\sigma_G = 0.1$ $\Delta\sigma = 30\%$	$\mu_G = 0.03$		$\Delta\mu = 3\%$		
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex post case with endogenous bankruptcy	145.52	7.32	79.62	32	1.92
Ex post case with no switching based covenant	147.33	7.61	82.41	27	0.70

Table 9.

$\sigma_G = 0.2$ $\Delta\sigma = 20\%$	$\mu_G = 0.03$		$\Delta\mu = 3\%$		
	$v(x)$	$c^o(x)$	$L(\%)$	$Y_S(bp)$	$AC(\%)$
Ex post case with endogenous bankruptcy	132.98	6.34	67.1	111	2.13
Ex post case with no switching based covenant	135.88	7.08	72.54	118	0