*A Two-stage Investment Game in Real Option Analysis†

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Abstract

This paper investigates an interaction between managerial flexibility and competition. We consider a two-stage game with two firms under demand uncertainty that follows a one-period binomial process. The cash flow generated from a project depends on both the demand and the firms' actions. We assume that the two firms make decisions sequentially at each stage whether they invest in the follow-up project. One firm called a leader primally make a decision, and the other firm called a follower decides secondly after observing the leader's decision. Namely, the leader has a competitive advantage over the follower. Both firms' managers can invest at either of the stages, hence they can defer their decisions for investment at the first stage. This means that they have flexibility to defer the project until the second stage. This flexibility can be considered a real option to defer the project.

Although the model developed here is very simple the implication from the model is plentiful. We fully characterize the equilibrium strategies for both firms which are classified by their investment costs. We consider several situations where either or both firms can invest only at the first stage. By the comparison among these situations, we can analyze the effects of flexibility and competition.

Our results indicate that under a monopolistic environment the existence of flexibility has a positive impact on the project value. However, under a competitive environment the effects of flexibility are not straightforward. For the follower obtaining the flexibility always increases the project value. On the other hand, the leader could decrease a project value by obtaining the flexibility on condition that the follower can invest only at the first stage. We call it *flexibility trap*, which can be interpreted as *commitment effects* in game theory.

*This is a second version of the research. We have changed many notations for consistency and simplicity.

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1 Introduction

In the traditional corporate finance it is usually recommended to use the net present value (NPV) method or the discounted cash flow (DCF) method for valuing a real project since they are consistent with maximizing shareholders value. In their methods an expected value is computed to evaluate a project under uncertainty. Its risk is adjusted by an appropriate discount rate in the valuation; Namely, high risk leads to a large discount rate.

It has been pointed out that one of the critical disadvantages of the NPV method is its inability to take managerial flexibility into consideration. Management can act differently against an uncertain environment and it should be reflected into am evaluation model. However, it is difficult for the NPV method to incorporate the management reaction since the method implicitly assumes the symmetric reaction of the management. Therefore, the NPV method could underestimate a project value when management has flexibility under uncertainty[1].

Myers (1984) proposes a real option approach to overcome this problem. In the real option approach the managerial flexibility is regarded as a real option. The generalized option pricing theory, which is originally developed for valuing financial options, provides us a powerful tool for evaluating real options quantitatively. Since the real option approach can naturally evaluate the managerial flexibility as a real option, some leading companies adopt this approach in their decision making[2].

In most studies of the real option it is assumed that underlying risk is exogenous and that the management cannot affect the underlying process. It is appropriate if we evaluate a project whose underlying risk is exposed to nearly perfect competition such as an oil refinery project or a project under the exchange rate uncertainty because both the price of the crude oil and the exchange rate cannot be controlled by a single company. Therefore, it is appropriate to assume it as a stochastic process.

However, there are many projects in which the management should take a competitive situation into consideration. A land development project of a restricted area and a research and development of similar drugs are typical examples. Brickley and Zimmerman (2000) show the airplane industry, soft drink, copy machines and the consumer film industry as other examples. In these cases there are small number of firms and a firm's decision significantly affects another firm's decision. Therefore, each firm should make a strategic decision which affects the other firms' decisions.

In order to incorporate the competition into the real option approach, Trigeorgis (1991) introduces competition by adding a random arrival of competitors to the diffusion process of the project value while Kulatilaka and Perotti (1998) introduce it by adjusting production costs. Note both researches incorporate competition exogenously.

[1] See Ross (1995), for example.

[2] Nichols (1994) reports the advantage of this approach in the case of pharmaceutical company.

The other approach to incorporate the competition into the real option analysis is to adopt a game theoretic idea into the model. The game theory enables us to analyze the effect of the competition in equilibrium. The optimal strategies are derived in the equilibrium. The importance of the competition for the firm's decision in the real option approach has been recognized. For example, Kester (1984) refers to the effect of competition. Ang and Dukas (1991), Grenadier (2000), Brickley and Zimmerman (2000) and Smit (2001) qualitatively analyze the effect of competition on the strategic decision of the management.

The chapter 4 of Dixit and Pindyck (1994) is one of the earliest studies that analyze the effects of competition under uncertainty. Grenadier (1996) considers two firms that compete in the land development business and analyzes the equilibrium price. In the model both firms can enter continuously but the model excludes the simultaneous entry, which is a theoretical disadvantage in the model. Huisman (2001) points out the disadvantage, and develops a rigorous model which is based on Fudenberg and Tirole (1985). This model can analyze the optimal entry strategy under both the demand uncertainty and competition between two firms[3]. Other researches that focus on the effects of competition in the real option approach are Huisman and Kort (2000) , Garlappi (2000), Murto and Keppo (2002), Pawlina and Kort (2002), Weeds (2002), Thijssen and Kort (2002), and Lambrecht and Perraudin (2003).

These researches above assume diffusion processes and continuous entries, which is easier to analyze. On the other hand, Smit and Ankum (1993) develop a simple binomial model and analyze two firms' decisions in a subgame perfect equilibrium. The analyses are done in two numerical examples. The first example assumes that both firms are symmetric and the second one assume that one firm dominates the other firm in the sense of market power. Smit and Trigeorgis (2001) analyze duopolistic competition which is typical in the area of the industrial organization of economics. The model has two stages and assumes that only one firm can access a strategic investment. In the second stages two firms compete the project under the demand uncertainty which follows a two-period binomial process. They show in numerical examples that there could emerge the Nash equilibrium, the existence of the leader and the follower in the Stackelberg sense, and the monopoly situation, which depends on the realized value of the demand and the investment size. They also discuss the optimal strategy for each firm. However, they do not derive conditions of these situations since their analysis is based on the numerical examples.

This paper analyzes an interaction between managerial flexibility and the existence of competition. The model developed in this paper is an extension of Imai and Watanabe (2003). We consider a two-stage game with two firms and a one-period binomial process of uncertainty. It assumes that the future cash flow depending on the demand is uncertain and follows a one-period binomial process. Two firms are introduced to analyze the competition. Both firms consider identical projects to invest under the competition at each stage. This

[3] Unfortunately, the model needs some technical constraints, which are difficult to interpret in the actual business.

paper assumes that one firm moves first and the other firm can move after observing the first firm's decision. While both firms' managers can invest in the project at the first stage they could have flexibility to defer the investment until the next stage. This flexibility can be considered a real option to defer the project.

We examine the impact of the competition on the strategic investments for the two firms. The idea is similar to those of Huisman (2001) and Huisman and Kort (2002) while Smit and Trigeorgis (2001) mainly focus on strategic investments and the effects on duopoly competition in price or quantity. Although our model is very simple, we can derive theoretical conditions of the equilibrium strategies for the firms in all situations.

In order to compute the project value two types of valuation methods are considered in our model, which reflects the presence of flexibility of the investment timing. The traditional net present value is adopted to compute the project value *without* flexibility, in which the firm can decide to invest only at the first stage. On the other hand, the real option approach is used for valuing the project *with* flexibility, in which the firm can wait to invest in the project until the second stage. By comparing these two types of values, we can analyze the effects of the real option. We can also analyze the effects of competition by comparing the project *with* competition and the project in monopoly. Moreover, by assuming that only one firm has the real option we can fully analyze the impacts of the interaction between flexibility and competition on the project values and the equilibrium strategies for both firms. Consequently, we can conclude the importance of the real option under a competitive environment and clarify the trade-off between flexibility and competition.

This paper is organized as follows. Section 2 sets up a valuation model that includes both uncertainty and competition and reviews a previous research, which will be the foundation of our analysis in this paper. Section 3 derives equilibrium strategies under competition when neither firm has flexibility and analyzes the effect of competition in case of no flexibility. Section 4 derives the equilibrium strategies under competition when both firms has the flexibility. The effects of flexibility and competition are analyzed in this section. Section 5 examines asymmetric situations where one of the two firms has flexibility while the other firm does not. Finally, concluding remarks are in Section 6.

2 A Valuation Model

This section sets up a basic valuation model of the project under uncertainty. Two firms are introduced, which denoted by firm L (Leader) and F (Follower), that consider to invest in a competing follow-up project. Both firms make decisions to maximize their projects values. The cash flow obtained by each firm depends on the current demand and actions of both firms. This model is analogous to that of the real estate development studied by Grenadier (1996), and it is applicable to many projects such as R&D competition, technology adoption and a pilot plant in a new market. We consider a two-stage game with two firms. In each stage both firms make their investment decisions when the demand follows a one-period binomial process. We assume that their decisions are made sequentially at each stage and that the demand does not change within the stages since the sequential decisions firms are made in short time with relative to the time to change of the demand. Hence, we can consider that the first stage is identical to time zero and the second stage is identical to time one in the model.

Let Y_0 denote the initial demand. The demand at time one, denoted by Y_1, could either move up to $Y_1 = uY_0$ or move down to $Y_1 = dY_0$ where u and d are rates of the demand in one period satisfying that $d < 1 < u$. Under this demand uncertainty, both firm L and firm F can either invest or defer the investment sequentially at each stage; i.e., the firms have a real option to defer.

If the underlying asset of the real option can be traded in the complete market we can apply the no-arbitrage principle to value the real option[4]. It is difficult, however, to apply the principle to our model since the demand of the merchandise cannot be observed in the market.

In this paper, we take an equilibrium approach that is a typical assumption for the real option analyses[5]. Especially, the demand risk in this paper can be considered private risk or unsystematic risk that is independent of the market risk. Since an investor pays no risk premium with respect to the unsystematic risk in equilibrium, we can assume that the investors are risk neutral in the valuation model. Let p denote a probability of the demand to move up and q denote a probability to move down that is equal to $1 - p$. Let r be the risk free rate for one period and let R define $R = 1 + r$. The expected demand at time one is

$$E\left[Y_1\right] = puY_0 + qdY_0 \equiv \mu Y_0. \tag{1}$$

Note that we define $\mu = pu + qd$.

The cash flow obtained by each firm depends on the current demand and actions of both firms. At time $i(i = 0, 1)$ when both firms invest in the project the cash flow obtained by each firms are given by $D_{11}Y_i$ while when neither firm invests the cash flow is given by $D_{00}Y_i$. When only one of the firms invests, the firm can obtain the cash flow $D_{10}Y_i$ and the other firm which does not invest in the project obtains $D_{01}Y_i$. Note $D_{ij}; i, j = 0, 1$ represents cash flow per unit of demand. We assume that

$$D_{10} > D_{11} > D_{00} > D_{01}, \tag{2}$$

which means that a firm prefer investing in the project if the investment cost is small enough and the other firms's strategy is fixed. Furthermore, we assume that

$$D_{10} - D_{00} > D_{11} - D_{01}. \tag{3}$$

[4]Mason and Merton (1985)insist that the value of real options can be evaluated with no-arbitrage principle if the underlying securities are observed in the markets which has the same risk profile as the real options .

[5]For example, Cox and Ross (1976), Constantinides (1978), and McDonald and Siegel (1984) propose the equilibrium approach for the real option pricing.

The term $D_{10} - D_{00}$ represents a marginal cash flow of the first mover, a firm that invests when the other firm does not invest, while term $D_{11} - D_{01}$ represents a marginal cash flow of the second mover, a firm that invests after the other firm has invested. Equation (3) means that the situation is preferable if the other firm do not invest. We also assume that the opportunity of investment is at most one, no firms can invest both time zero and time one.

While both firm L and firm F can choose the timing to invest, we assume that the order of decision is sequential. At each time firm L firstly makes a decision whether to invest or not and firm F makes an investment decision after observing the firm L's action. Therefore, firm L has a competitive advantage over firm F. We sometimes call firm L a leader and call firm F a follower in this sense. It is important to note that while the order of making a decision at each time is exogenously defined the order of the investment is determined endogenously. Thus, the follower firm F could invest in the project before firm L. For distinction, we call a firm that invests first a *first mover* and a firm that invests after the other firm a *second mover*. Note that equation (3) means that we assume the first mover has an advantage over the second mover, which is sometimes called first mover advantage[6].

The model used in this paper is really simple but it enables us to complete the full analysis for valuing the project, valuing the net value of the real option, and analyzing the effect of the competition. To examine the effect of the real option, we adopt two types of valuation. The first type is based on a net present value analysis in which a firm does not have an option to defer the project or the firm does not recognize the real option. Thus, the firm must decide whether the project should be started at time zero. The second type is based on the real option analysis which includes a value of the real option to defer the project until time one. Thus, the difference of these two values represents the net value of the real option. Accordingly, we can examine the impact of the real option by comparing these two values. On the other hand, to examine the effect of competition we compare a duopolistic situation with a monopolistic situation. By comparing these situations, the effect of competition can be analyzed.

Imai and Watanabe (2003) analyze the value of flexibility in the project under the demand uncertainty. Their analysis is in the monopolistic situation and does not incorporate the competition. It is necessary to understand their results because the effects of the competition can be obtained by comparing with their results. Thus, this section briefly reviews their findings.

They classify the firm's optimal strategy with the investment cost.

- The boundary investment cost at time zero can be given by $I^{\alpha} = (D_1 - D_0)\left(1 + \frac{\mu}{R}\right) Y_0$[7] when a firm has no real option to exercise, which is equivalent to the net present value approach[8]. It means that the firm invests when the investment cost is less than I^{α}.

[6] The terms leader and follower might be used in different way in other papers. For example, Grenadier (1996) call a firm leader that invests first, which is called a first mover in this paper..

[7] In their paper I^{α} is denoted by I^{R}.

[8] See Proposition 1 in Imai and Watanabe (2003).

- When the firm has the real option to defer the boundary investments at time one are given by I_0^u and I_0^d where

$$I_0^u = (D_1 - D_0)\, uY_0,$$

$$I_0^d = (D_1 - D_0)\, dY_0.$$

  The boundary I_0^u corresponds to the investment when the demand moves up at time one while I_0^d corresponds to that when the demand moves down[9].

- When the firm has the real option the boundary investment cost at time zero is given by $I^\beta = (D_1 - D_0)\,\frac{1+\frac{q}{R}}{1-\frac{p}{R}} Y_0$[10].

- When the firm has the real option there are two cases. In the first case where the volatility of the demand is rather small, the value of real option is equal to zero because the rational firm never exercises its option at time one[11].

- In the second case where the volatility of the demand is relatively large, there exist a real option value when the investment cost is between I^β and I_0^u[12]. It is important to note

  The firm without flexibility never invests when $I^\beta \le I \le I^\alpha$ while the firm with flexibility defers the investment and exercises the option only when the demand moves up at time one.

  The firm without flexibility always invests when $I^\alpha \le I \le I_0^u$ while the firm with flexibility defers the investment and exercises the option only when the demand moves up at time one.

  The value of the real option is maximized when the investment cost is equal to I^β.

3 Net Present Value of the Project under Competition

3.1 Derivation of the NPV for each firm

In this section we first derive equilibrium strategies for two competitive firms when they can invest in the project only at time zero. In this setting the project values for both of them can be evaluated by the net preset value method since they do not have any option. The net preset values will be compared with the real option values when both of the firms have flexibility, which is derived in the

[9] See proposition 2.
[10] See proposition 3.
[11] See proposition 4.
[12] See porposition 5.

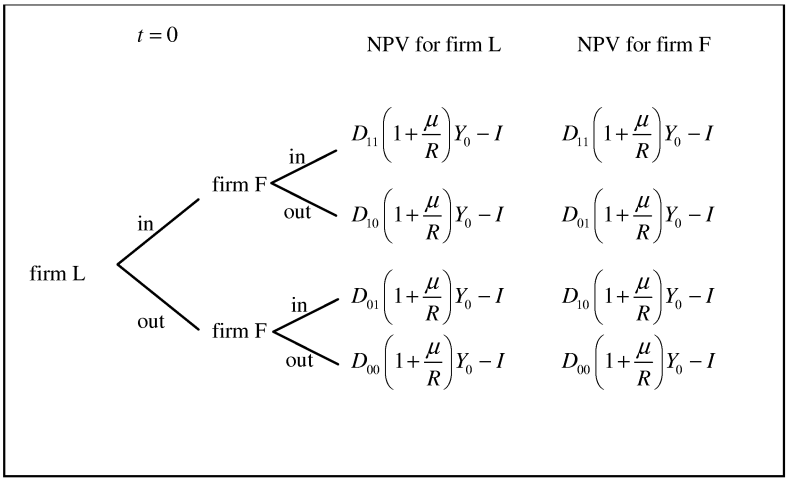

Figure 1: This figure illustrates a game tree when the projects of both firms are valued by the NPV method.

next section. The comparison enables us to analyze the net effect of the real option under competition.

At time zero firm L can make a decision first whether to invest. After observing the action of firm L firm F makes a decision for the investment. Since neither firm has a real option to defer the investment no decisions are made at time one. The decision tree is illustrated in Figure 1. The figure illustrates that firm L, a leader, can determine their strategy earlier than firm F. Four cases can be considered that depend on the decisions of the two firms.

By using the game theoretic approach we obtain the subgame perfect equilibrium. First, the optimal strategy for firm F is solved on the condition of firm L's action and then the optimal strategy for firm L is derived on the condition of the following optimal strategy for firm B.

Suppose that firm L decides to invest in the project. The optimal strategy for firm F is given by comparing the following two values,

$$V_F^{in} = D_{11}\left(1+\frac{\mu}{R}\right)Y_0 - I,$$

$$V_F^{out} = D_{01}\left(1+\frac{\mu}{R}\right)Y_0,$$

where V_F^{in} represents the net present value for firm F if the firm F decides to invest. On the other hand, V_F^{out} represents the net present value for firm F if the firm F decides not to invest. The investment cost is denoted by I. Then, the boundary investment cost for firm F is given by

$$I_2^{\alpha} = (D_{11} - D_{01})\left(1+\frac{\mu}{R}\right)Y_0. \tag{4}$$

Firm F should invest in the project if $I < I_2^\alpha$ and should not invest otherwise. Note that I_2^α can be interpreted as the boundary cost for the second mover because the first mover has already invested in the project.

Similarly, the boundary investment cost when firm L does not invest in the project is given by

$$I_1^\alpha = (D_{10} - D_{00}) \left(1 + \frac{\mu}{R}\right) Y_0, \tag{5}$$

which is also interpreted as the boundary cost for the first mover.

The optimal strategy for firm F can be characterized by these two boundary investment costs. For example, the firm F decides to invest if firm L does not invest and vice versa when the investment cost I satisfies $I_2^\alpha < I \leq I_1^\alpha$, which we denote (F_{out}, F_{in}). Note "*out*" represents that the firm does not invest while "*in*" represents the firm does invest, and the first term in parentheses indicates the case when the firm L invests in the project while the second term indicates the case when firm L does not invest.

Consequently, the optimal strategy for firm F is shown in Proposition 1.

Proposition 1 *The optimal strategy for firm F can be classified into three cases, which are written as follows:*

$$\begin{cases} (F_{in}, F_{in}) & \text{if } I \leq I_2^\alpha \\ (F_{out}, F_{in}) & \text{if } I_2^\alpha < I \leq I_1^\alpha \\ (F_{out}, F_{out}) & \text{if } I > I_1^\alpha. \end{cases}$$

The equilibrium strategy for firm L is driven on condition that firm F takes the optimal strategy after the firm L's behavior. When $I \leq I_2^\alpha$ the optimal strategy for firm F is (F_{in}, F_{in}) according to proposition 1. The strategy for firm L in equilibrium is solved by comparing the following two values.

$$V_L^{in} = D_{11} \left(1 + \frac{\mu}{R}\right) Y_0 - I,$$

$$V_L^{out} = D_{01} \left(1 + \frac{\mu}{R}\right) Y_0,$$

where V_L^{in} represents the net present value for firm L if the firm L decides to invest while V_L^{out} represents the net present value for firm L if the firm decides not to invest. Note that in both cases firm F always invests regardless of the firm L's decision. Since

$$\begin{aligned} V_L^{in} - V_L^{out} &= (D_{11} - D_{01}) \left(1 + \frac{\mu}{R}\right) Y_0 - I \\ &= I_2^\alpha - I > 0, \end{aligned}$$

it is optimal for firm L to invest in the project at time zero in the equilibrium, which we denote L_{in}. Similarly, the optimal decisions for the firm are given when $I_2^\alpha < I \leq I_1^\alpha$ and $I > I_1^\alpha$. The result is summarized in the next proposition.

Proposition 2 *The equilibrium strategy for firm L can be classified into three cases, which are written as follows.*

$$\begin{cases} L_{in} & \text{if } I \le I_2^\alpha \\ L_{in} & \text{if } I_2^\alpha < I \le I_1^\alpha \\ L_{out} & \text{if } I > I_1^\alpha \end{cases} \tag{6}$$

Accordingly, the equilibrium strategies for both firms can be classified into three cases. Both firms always invest if $I \le I_2^\alpha$. In this case the net present value for both firms is given by $D_{11}\left(1+\frac{\mu}{R}\right)Y_0 - I$. While firm L invests firm F never invests if $I_2^\alpha < I \le I_1^\alpha$ which reflects the fact that firm L is a leader and firm F is a follower. The net present value for firm L is $D_{11}\left(1+\frac{\mu}{R}\right)Y_0 - I$ while that for firm F is $D_{01}\left(1+\frac{\mu}{R}\right)Y_0$. It is easily confirmed that the project value of firm L is larger than that of firm B. Finally, neither firm enters the project if $I > I_1^\alpha$ and the net present values for both firms are given by $D_{00}\left(1+\frac{\mu}{R}\right)Y_0$. The boundary investment cost for the investment for firm L is I_1^α while that for firm F is I_2^α. Since $I_2^\alpha < I_1^\alpha$, firm L can invest in the project under a relatively larger investment cost than firm F. This indicates a competitive advantage for the leader over the follower.

3.2 Analysis of the NPV under duopoly and monopoly

This subsection investigates the effect of the competition when firms do not have real options. The analysis is done by comparing the net present values of the two firms shown in this subsection with the net present values without competition. The net present value in the monopolistic situation is reviewed in the previous section.

To compare the net present values the unit values of the demand in the monopolistic situation, denoted by D_0 and D_1, need to be adjusted. We assume that $D_0 = D_{10}$ and $D_1 = D_{11}$ which implies that the unit value of the demand for a monopolistic firm is equal to that of a first mover[13]. The result shows that strategies for firm L in the subgame perfect equilibrium are equivalent to that for the monopolistic firm. Thus, the net present value of firm L is also equivalent. This means that if a firm is a leader under the competition the firm can act as a monopolistic firm.

On the other hand the optimal strategy for firm F is different from that of the monopolistic firm. While the monopolistic firm can invest in the project when $I \le I^\alpha (= I_1^\alpha)$ the firm F can only invest when $I \le I_2^\alpha$. Namely, the firm F loses the project value when $I_2^\alpha < I \le I_1^\alpha$. The lost value is given by

$$(D_{10} - D_{01})\left(1+\frac{\mu}{R}\right)Y_0 - I,$$

[13] It is difficult to compare the value of the project under the different environment. Strictly speaking we must adjust the discount rate so that the degree of competition is reflected when we compute a net present value. In this paper we implicitly assume that the discount rate in the monopolic environment is equal to that in the competitive environment for comparing the effect of competition in the sense of comparative statics.

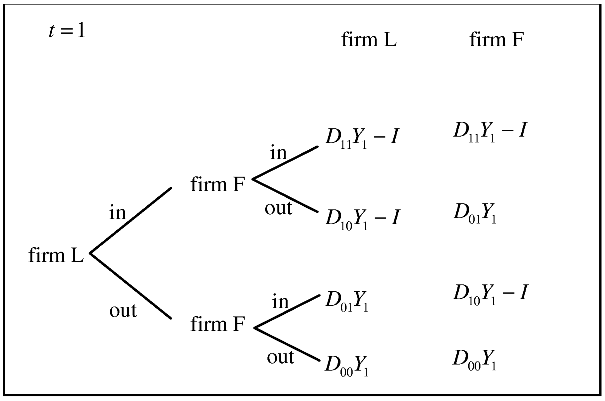

Figure 2: This figure illustrates the game tree at time one when both firms have real options.

assuming that $I_2^\alpha < I \leq I_1^\alpha$. In summary, when firms do not have flexibility to defer the projects the order of decision plays a critical role for the value of the project. The leader firm can act as a monopolist while the follower could lose their project value because the follower could not invest due to the preemption.

4 Real Option Value of the Project under Competition

4.1 The strategies in the equilibrium for competitive two firms

This section analyzes equilibrium investment strategies for both firm L and firm F when the firms has flexibility to defer the project until time one. This means that both firms evaluate their project with the real options to defer. To derive the project values the idea of dynamic programming is used;Namely, we first determine the equilibrium strategies for both firms at time one, and derive the equilibrium strategies at time zero. At each time it is assumed that firm L can make a decision first, and that firm F makes a decision after the firm L's decision.

At time one the equilibrium strategies are derived on condition that both firms do not invest at time zero since otherwise there is no option to exercise. Figure 2 illustrates the game tree at time one.

Since the demand at time one could be $Y_1 = uY_0$ or $Y_1 = dY_0$ the equilibrium strategies are derived, respectively. First, we derive the optimal strategy for firm F on the condition of the firm L's action at time one. On condition that firm L invests in the project the optimal strategy for firm F can be derived by comparing $D_{11}Y_1 - I$ with $D_{01}Y_1$ while it can be derived by comparing

$D_{10}Y_1 - I$ with $D_{00}Y_1$ on condition that firm L does not invest. Consequently, the equilibrium strategies for firm F can be characterized by the following four boundary investment costs.

$$I_1^u = (D_{10} - D_{00})\, uY_0, \;\; I_2^u = (D_{11} - D_{01})\, uY_0, \tag{7}$$

$$I_1^d = (D_{10} - D_{00})\, dY_0, \;\; I_2^d = (D_{11} - D_{01})\, dY_0. \tag{8}$$

Note that $I_2^u < I_1^u$ and $I_2^d < I_1^d$ are satisfied because of equation (3) and that $I_2^d < I_2^u$ and $I_1^d < I_1^u$ are satisfied because $d < u$. However, there is no apparent relations between I_2^u and I_1^d. Therefore, there are six areas that are analyzed respectively.

Taking firm F's optimal strategies into account the equilibrium strategy for firm L at time one can be also characterized by boundary investment costs. For example, consider the case if the investment cost is included in the area of $I < I_2^d$ where the optimal strategy for firm F is to invest in the project regardless of both the decision of firm L and the demand at time one. In this case the equilibrium strategy for firm L is derived by comparing the value of $D_{11}Y_1 - I$ with $D_{01}Y_1$, which leads to that firm L should invest as well. Similarly, all cases can be analyzed. The next proposition shows all the results of the equilibrium strategies for the two firms.

Proposition 3 *The equilibrium strategies for the two firms can be categorized into six areas from area (A) to area (F) that depend on the four boundary investment costs. The areas are illustrated in Figure 3.*

The figure illustrates the following.

- The optimal strategies for firm L in the equilibrium can be categorized into three areas and there are two boundary investment costs that are I_1^d and I_1^u to distinguish the firm L's strategy in the equilibrium. When $I < I_1^d$ firm L should invest regardless the demand at time one. When $I_1^d < I < I_1^u$ firm L should invest only when the demand moves up to $Y_1 = uY_0$ and should not invest otherwise. Finally, firm L should never invest when $I > I_1^u$.

- The optimal strategies for firm F in the equilibrium can be classified into six categories as illustrated in the figure. However, the realized strategy for firm F can be categorized into three areas and the boundary investment costs are I_2^d and I_2^u. When $I < I_2^d$ firm F should invest in the project regardless of the demand at time one. When $I_2^d < I < I_2^u$ firm F should invest only when the demand moves up to $Y_1 = uY_0$ and should not invest otherwise. Firm F should never invest when $I > I_2^u$.

The proposition indicates that when both firms have real options to exercise at time one firm L always has an advantage over firm F because firm L is

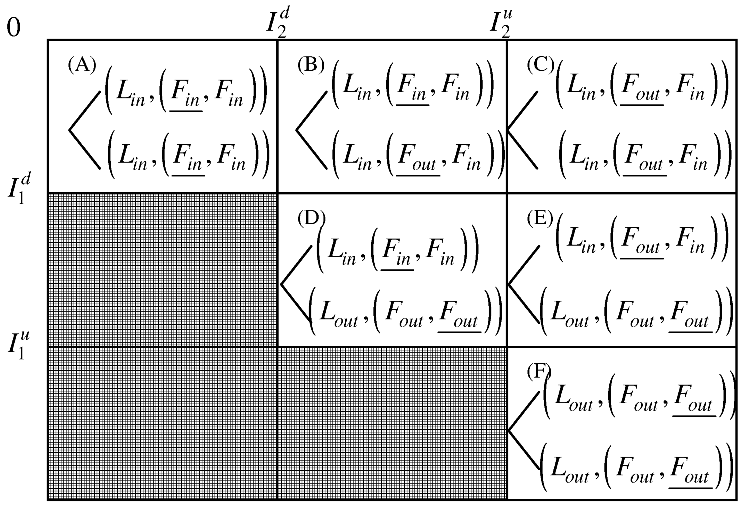

Figure 3: This figure illustrates the equilibrium strategies for both firm L and firm F at time one when both firms have real options to defer the projects.

a leader that can make a decision first. This result is similar to that in the previous section. In this case firm L can always act as a first mover.

The strategies in the equilibrium at time zero are analyzed using the optimal strategies at time one. The analysis is divided into two cases. The case 1 is analyzed when $1+\frac{\mu}{R}>u$ is satisfied and the case 2 is done when $1+\frac{\mu}{R}\leq u$. It is important to distinguish two cases since two cases lead to different results[14].

The game tree at time zero is illustrated in Figure 4. The project value denoted by v_M where both firms decide to invest in the projects at time zero can be explicitly computed as $v_M = D_{11}\left(1+\frac{\mu}{R}\right)Y_0 - I$ because they abandon their options. The value denoted by v_1 represents the project value for the first mover who invests at time zero while the other firm does not invest and will take the equilibrium strategy at time one. On the other hand, the value of v_2 represents the project value for the second mover who does not invest in the project at time zero and takes the equilibrium strategy at time one assuming that the first mover invests at time zero. The values of v_L and v_F represent the project values for firm L and firm F, respectively when neither firm invest at time zero and takes the equilibrium strategy at time one which is shown in proposition 3.

The values of v_1 and v_2 can be calculated explicitly on the condition of the investment costs.

[14]The case 1 corresponds to the situation when the volatility of the demand is relatively small. According to the standard option pricing theory we guess that the value of the real option is small. This procedure is same as that of Imai and Watanabe (2003).

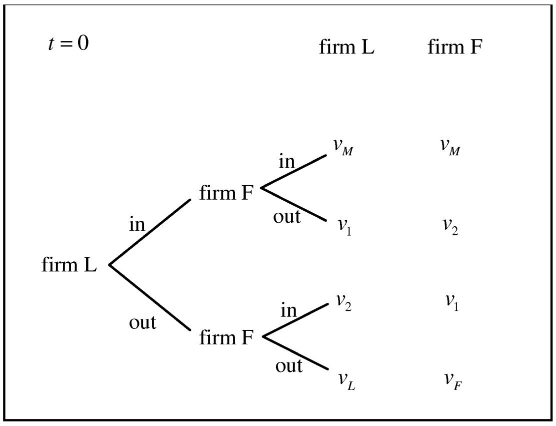

Figure 4: This figure illustrates a geme tree at time zoro when both firms have real otpions.

$$v_1 = \begin{cases} (D_{10}Y_0 - I) + \frac{\mu}{R} D_{11} Y_0 & if\ I \leq I_2^d \\ (D_{10}Y_0 - I) + \frac{1}{R}(pD_{11}uY_0 + qD_{10}dY_0) & if\ I_2^d < I \leq I_2^u \\ (D_{10}Y_0 - I) + \frac{\mu}{R} D_{10} Y_0 & if\ I > I_2^u, \end{cases} \tag{9}$$

$$v_2 = \begin{cases} D_{01}Y_0 + \frac{1}{R}(D_{11}\mu Y_0 - I) & if\ I \leq I_2^d \\ D_{01}Y_0 + \frac{1}{R}\{p(D_{11}uY_0 - I) + qD_{01}dY_0\} & if\ I_2^d < I \leq I_2^u \\ D_{01}Y_0 + \frac{\mu}{R} D_{01} Y_0 & if\ I > I_2^u. \end{cases} \tag{10}$$

The equilibrium strategies for firm F are derived by comparing v_M with v_2 and v_1 and v_F, respectively. If $v_M > v_2$ and $v_1 > v_F$ the equilibrium strategy for firm F at time zero is to invest in the project regardless of the firm L's decision. The equilibrium strategy for firm L in this case is to invest since $v_M > v_2$. We write the combination of the strategies in the equilibrium as $(L_{in}, (F_{in}, F_{in}))$. If $v_M \leq v_2$ and $v_1 \leq v_F$ the optimal strategy for firm F is not to invest in the project, which is written by (F_{out}, F_{out}). The optimal decision for firm L is derived by comparing v_L with v_1. Finally, if $v_M \leq v_2$ and $v_1 > v_F$ the optimal strategy for firm F depends on the firm L's decision at time zero. If firm L invests the optimal decision for firm F is not to invest while the optimal decision for firm F becomes to invest if firm L does not invest, which is written by (F_{out}, F_{in}). In this case the optimal decision for firm L is determined by comparing two values of v_1 and v_2[15].

To describe the solution of the game between the two firms two other boundary investment costs are introduced. Let I_1^β and I_2^β denote boundary investment costs that are defined by

[15]The possibility of the condition $v_M > v_2$ and $v_1 \leq v_F$ is excluded because of the preemption.

$$I_1^{\beta} = (D_{10} - D_{00})\, Y_0 \frac{1 + \frac{qd}{R}}{1 - \frac{p}{R}}, \tag{11}$$

$$I_2^{\beta} = (D_{11} - D_{01})\, Y_0 \frac{1 + \frac{qd}{R}}{1 - \frac{p}{R}}, \tag{12}$$

which reflects the fact a firm sometimes exercise a real option only when the demand moves up at time one. The following proposition shows the inequalities satisfied among the boundary investment costs.

Proposition 4 *The following inequalities are satisfied.*

1. *If* $1 + \frac{\mu}{R} > u$ *then* $I_1^d < I_1^u < I_1^{\alpha} < I_1^{\beta}$ *and* $I_2^d < I_2^u < I_2^{\alpha} < I_2^{\beta}$.
2. *If* $1 + \frac{\mu}{R} \leq u$ *then* $I_1^d < I_1^{\beta} < I_1^{\alpha} < I_1^u$ *and* $I_2^d < I_2^{\beta} < I_2^{\alpha} < I_2^u$.

A proof of this proposition is the same as in Imai and Watanabe (2003). According to the proposition we analyze the following two cases differently. Consider case 1 where $1 + \frac{\mu}{R} > u$ is satisfied. The next proposition summarizes the equilibrium strategies for both firm L and firm F.

Proposition 5 *The equilibrium strategies for firm L and firm F at time zero can be described as follows in case 1.*

$$\begin{cases} (L_{in,}\,(F_{in}, F_{in})) & if\ I < I_2^{\alpha} \\ (L_{in,}\,(F_{out}, F_{in})) & if\ I_2^{\alpha} < I < I_1^{\alpha} \\ (L_{out,}\,(F_{out}, F_{out})) & if\ I < I_1^{\alpha} \end{cases}$$

It is important to note that it is equivalent to the case when neither firm has a real option and the project values are based on the net present value method. Thus, there are no opportunities for both firms to exercise their options; Namely, the firms act as if they have no real option to exercise.

Next, we consider case 2 where $1 + \frac{\mu}{R} \leq u$ is satisfied, which implies that the volatility of the demand is relatively large. In this case both firms could defer the investment and exercise the option at time one. To accomplish a full analysis of case 2, we must classify the cases into 15 categories with the size of the investment cost which is illustrated in Figure 5. For example, the area (d) indicates the case when the investment cost satisfies both $I < I_1^d$ and $I_2^{\alpha} < I < I_2^u$. The area painted in black in the figure means that there is no possibility that the investment cost are contained in the area. Furthermore, we introduce another boundary investment cost denoted by I^{S_0} that is derived when we compare v_F and v_1 where

$$I^{S_0} = (D_{10} - D_{00})\, Y_0 + \frac{1}{R} \{puY_0\,(D_{10} - D_{01}) + qdY_0\,(D_{10} - D_{00})\}. \tag{13}$$

Since we can easily confirm that $I_1^{\alpha} < I^{S_0}$ the investment cost I^{S_0} could be included in (m), (n), or (o). Proposition 6 summarizes the result in case 2.

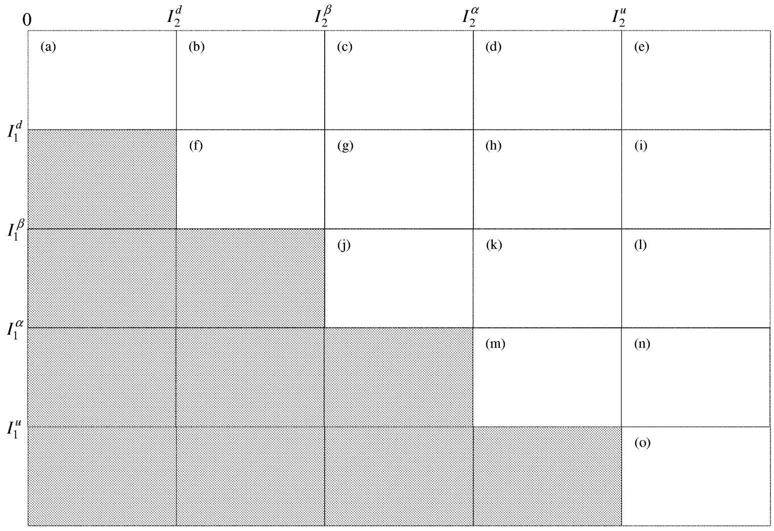

Figure 5: This figure illustrates 15 areas that are denoted from (a) to (o), which depends on the size of the investment cost I.

Proposition 6 *In case 2 the equilibrium strategies for firm L and firm F at time zero can be classified into three cases such as*

$$\begin{cases} (L_{in}, (F_{in}, F_{in})) & \textit{if } I \textit{ is in the area of (a),(b),(f)} \\ (L_{in}, (F_{out}, F_{in})) & \textit{if } I \textit{ is in the area of (c),(d),(e),(g),(h),(i),(l)} \\ (L_{out}, (F_{out}, F_{out})) & \textit{if } I \textit{ is in the area of (j),(k),(m),(o)} \end{cases} \tag{14}$$

As to area (n) if I^{S_0} is in (m) the equilibrium strategies becomes $(L_{out}, (F_{out}, F_{out}))$. If I^{S_0} is in (n) the area is further divided into two parts. When $I < I^{S_0}$ it is $(L_{in}, (F_{out}, F_{in}))$ while when $I > I^{S_0}$ it is $(L_{out}, (F_{out}, F_{out}))$. Finally, if I^{S_0} is in (o) the equilibrium strategies becomes $(L_{in}, (F_{out}, F_{in}))$. All the possibilities are illustrated in Figure 6, Figure 7, and Figure 8

The proposition 6 indicates the following.

- Unlike the case 1 it is possible for both firms to exercise the real option at time one; Namely, the real option to defer the project could be valuable in case 2.
- Firm L waits to invest and invests at time one when the demand moves up if the investment cost is in the area of (j), (k), (m)[16].

[16]The optimal strategy for firm L in area (n) depends on the location of I^{S_0}. If I^{S_0} is within (m) or (n) firm L waits to invest while firm L invests at time zero if I^{S_0} is in the area of (o).

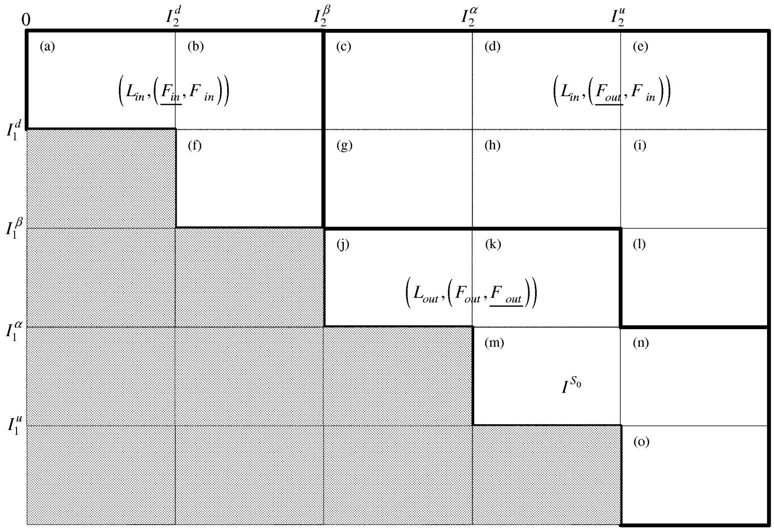

Figure 6: This figure shows the set of the equilibrium strategies when I^{S_0} is in the area (m).

- Firm F waits to invest if the investment cost is in the area of (g) or (h).

4.2 The comparison of the equilibrium strategies with vs. without real options

It is important to investigate the effects of the existence of flexibility under competition. The analysis is done by comparing the equilibrium strategies when neither firms have real option and evaluate the project values with the NPV method, and those when both firms have real option that is obtained in this section. As described in this section, there is no difference between two situations in case 1. This could be explained by the fact that case 1 implies that the volatility of the demand is not large enough to exercise the option, which is fully consistent with the standard option pricing theory in finance.

In case 2, on the other hand, the effects of the real option are emerged. In the areas of (c) and (g) firm F with the real option defers the investment and invests at time one while firm F without real option enters at time zero. The project value without the real option is given by

$$V_F^{NPV} = D_{11}\left(1+\frac{\mu}{R}\right)Y_0 - I$$

while the project value with the real option is given by

$$V_F^{RO} = D_{01}Y_0 + \frac{1}{R}\left\{p\left(uY_0D_{11} - I\right) + qdY_0D_{01}\right\}.$$

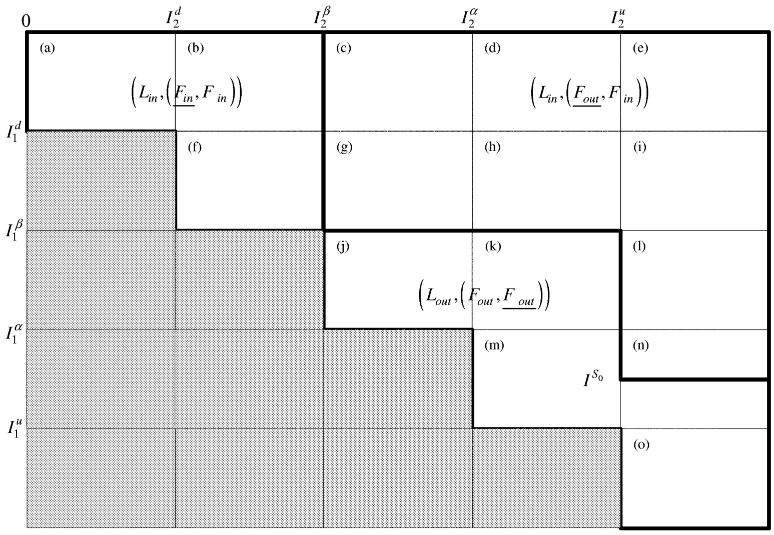

Figure 7: This figure shows the set of the equilibrium strategies when I^{S_0} is in the area (n).

Consequently, the net value of the real option gained by firm F can be given by

$$V_F^{RO} - V_F^{NPV} = \left(1 - \frac{p}{R}\right)\left(I - I_F^Q\right) > 0. \tag{15}$$

For firm L on the other hand, the project value without real option is

$$V_L^{NPV} = D_{11}\left(1 + \frac{\mu}{R}\right)Y_0 - I$$

while the project value with the real option is given by

$$V_L^{RO} = D_{10}Y_0 - I + \frac{1}{R}\left\{puY_0D_{11} + qdY_0D_{10}\right\}.$$

Consequently, the net value of the real option gained by firm L is given by

$$V_L^{RO} - V_L^{NPV} = \left(D_{10} - D_{11}\right)Y_0\left(1 + \frac{qd}{R}\right) > 0. \tag{16}$$

Note that firm L also gains a positive value in the presence of the real option although the optimal strategy for firm L is unchanged.

In the area of (j) when both firms have real options both firms decide not to invests at time zero and exercise their options at time one if the demand moves up while both firms invest at time zero when they do not have real options. In this case the net value of the real option for both firms is given by

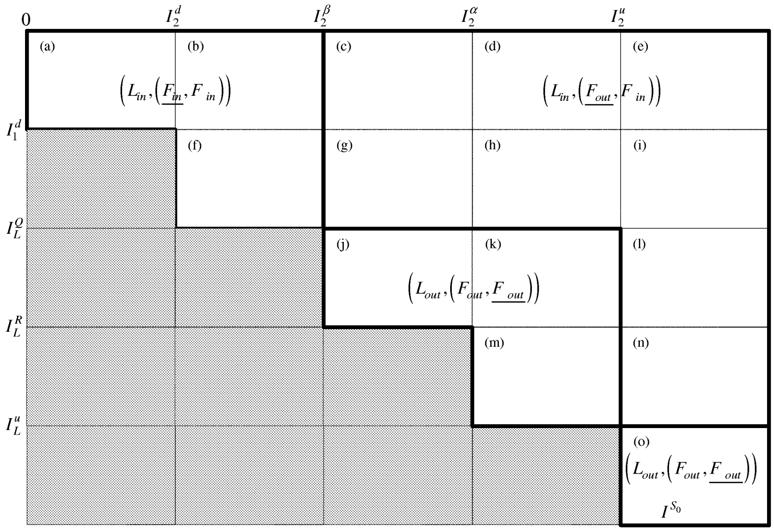

Figure 8: This figure shows the set of the equilibrium strategies when I^{S_0} is in the area (o).

$$V_L^{RO} - V_L^{NPV} = V_F^{RO} - V_F^{NPV} = \left(1 - \frac{p}{R}\right)\left\{I - (D_{11} - D_{00})\, Y_0 \frac{1 + \frac{qd}{R}}{1 - \frac{p}{R}}\right\} \tag{17}$$

$$> \left(1 - \frac{p}{R}\right)\left\{I_1^\beta - (D_{11} - D_{00})\, Y_0 \frac{1 + \frac{qd}{R}}{1 - \frac{p}{R}}\right\}$$

$$= \left(1 - \frac{p}{R}\right)(D_{10} - D_{11})\, Y_0 \frac{1 + \frac{qd}{R}}{1 - \frac{p}{R}} > 0$$

Thus, both firms obtain the benefit of flexibility by using the real options appropriately.

Finally, we consider the area of (n) where the optimal strategy could depends on the size of I^{S_0}. The equilibrium strategies are given by $(L_{out}, (F_{out}, F_{out}))$ without the real option while they are given by $(L_{in}, (F_{out}, F_{in}))$ with the real option if I^{S_0} is in the area of (n) or (o) and $I < I^{S_0}$. This means that the optimal strategy for L has changed. The firm L without the real option never invests while the firm L with the real option must invest at time zero. Otherwise firm F could invests to get benefit of the first mover advantage. As a result, the realized strategy for firm F is unchanged and never invests in the project. In this case firm F loses the project value when it possesses the real option that is given by

$$V_F^{RO} - V_F^{NPV} = -\left(D_{00} - D_{01}\right)\left(1 + \frac{\mu}{R}\right) Y_0 < 0. \tag{18}$$

The firm L also loses its value by possessing the real option. The difference between the net present value and the project value with flexibility is given by

$$\begin{aligned} V_L^{RO} - V_L^{NPV} &= \left(D_{10} - D_{00}\right)\left(1 + \frac{\mu}{R}\right) Y_0 - I \\ &= I_1^{\alpha} - I < 0. \end{aligned} \tag{19}$$

Consequently, both firms loses their project values due to the presence of the flexibility.

4.3 The comparison of the equilibrium strategies for a single firm vs. two firms under competition

In this subsection we compare the project value when two firms compete with each other to invest with and without competition. By comparing the equilibrium strategies for a firm with competition and those without competition, we can analyze the effect of competition when the firms have flexibility. A similar analysis in which the firms do not have real option was analyzed in the previous section.

In case 1 when $1 + \frac{\mu}{R} > u$ is satisfied the equilibrium strategies under the competition are equivalent to those without competition. Thus, we focus on case 2 when $1 + \frac{\mu}{R} \leq u$ holds. Consider first the effect of competition for firm L. Suppose $D_0 = D_{00}$ and $D_1 = D_{10}$ as we assumed in the previous section for the comparison purpose. The equilibrium strategies for both a single firm and firm L at time one are characterized by the same boundary investment costs I_1^u and I_1^d, which indicates that both strategies are equivalent at time one. At time zero, on the other hand, there is a slight difference between a single firm and firm L. The equilibrium strategy for a single firm is characterized by the two boundary investment costs of I_1^{β} and I_1^u. A single firm does not invest in the project when $I_1^{\beta} < I < I_1^u$ at time zero. On the other hand when the investment cost is in the area of (l)[17], which holds $I > I_1^{\beta}$, firm L invests at time zero. Thus, it can be recognized that this result comes from the presence of competition. The project value for a single firm is given by

$$D_{00} Y_0 + \frac{1}{R}\left\{p\left(uY_0 D_{10} - I\right) + qdY_0 D_{00}\right\},$$

while the project value for firm L at time zero is given by

$$D_{10} Y_0 \left(1 + \frac{\mu}{R}\right) - I.$$

[17]The same analysis is possible in the are of (n) if the investment cost I^{S_0} is located in (n) or (o).

Accordingly, the difference between the two values is given by $\left(1-\frac{p}{R}\right)\left(I_1^{\beta}-I\right) < 0$, which shows that firm L loses its project value because of the presence of competition in this area.

Next, the equilibrium strategies for a single firm and firm F are compared. At time one the optimal strategy for firm F is characterized by the boundary investment costs of I_2^u and I_2^d. Firm F loses some monopolistic benefit since firm L has advantage over firm F if we assume $D_0 = D_{00}$ and $D_1 = D_{10}$. If we assume that $D_0 = D_{01}$ and $D_1 = D_{11}$, on the other hand, which implies that the cash flow per unit of demand for a single firm is the same as that for a follower, firm F's strategies are unchanged which include the strategies at time zero. Consequently, firm F's strategy under the competition is equivalent to that of a single firm when $D_0 = D_{01}$ and $D_1 = D_{11}$.

5 The Effects of the Asymmetry of the Flexibility

In the previous section we assume that both firm L and firm F have flexibility to exercise as a real option. In this section we assume that one of the two firms has a real option to defer the project while the other firm does not have one and evaluates the project with the NPV method. By comparing the results in this section with those in the previous section we can examine the effect of flexibility and competition.

5.1 Firm L with flexibility vs. firm F without flexibility

In this subsection the equilibrium strategies are derived on condition that firm L has a real option while firm F does not have one. In this case firm L has a competitive advantage over firm F since firm L is a leader who can decide first. In addition, firm L has a real option to defer investing in the project until time one.

First, we consider the equilibrium strategy at time one. We consider those of firm L since it is impossible for firm F to invest at time one. The equilibrium strategies for firm L depend on both firm F's action at time zero and the demand at time one. It is easily confirmed that the equilibrium strategies for both firms are unchanged in case 1.

Proposition 7 *The equilibrium strategy for firm L at time one is summarized in Figure 9 assuming that only firm L has a real option to defer the project in case 2.*

Suppose that firm L does not invest at time zero. Then, the equilibrium strategies at time one for firm L are conditioned on the decision of firm F that is made at time zero, which is denoted by F_{in} and F_{out}. The strategies are also conditioned on the demand at time one. For example, if the investment cost I is

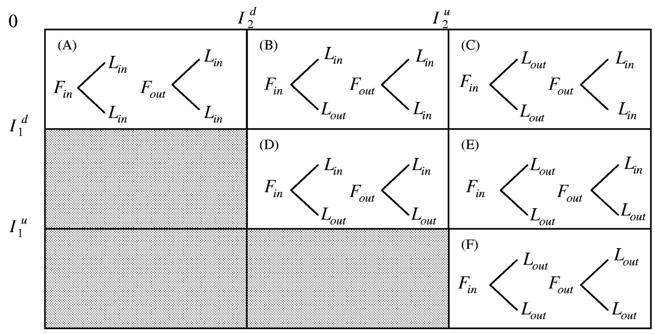

Figure 9: This figure illustrates the equilibrium strategy for firm L at time one conditioning on firm F's decision at time zero.

within the area of (A) firm L always invests in the project regardless of the firm F's action and the demand move. If I is within the area of (B) firm L invests except when firm F already enters and the demand moves down.

The equilibrium strategies for both firms at time zero can be analyzed in the similar way of those in the previous section. It is necessary to introduce other two boundary investment costs which are denoted by I^{S_1} and I^{S_2}. These values are given by

$$I^{S_1} = (D_{10} - D_{00})\, Y_0 + \frac{1}{R}\left\{puY_0\,(D_{11} - D_{01}) + qdY_0\,(D_{10} - D_{00})\right\}, \tag{20}$$

$$I^{S_2} = \frac{(D_{10} - D_{01})\, Y_0 + \frac{1}{R}\left\{puY_0\,(D_{10} - D_{11}) + qdY_0\,(D_{10} - D_{01})\right\}}{1 - \frac{p}{R}}. \tag{21}$$

The following proposition shows inequalities with respect to these boundary investment costs.

Proposition 8 *In case 2, the following inequalities are satisfied with regard to the size of* I^{S_1}.

$$I_2^{\alpha} < I^{S_1} < I_1^{\alpha} \tag{22}$$

$$I^{S_1} < I^{S_0} \tag{23}$$

The boundary investment cost I^{S_1} arises when analyzing the area of (h) and (k). Therefore, it is important to examine whether I^{S_1} could be contained in these areas. The following proposition shows the result.

Proposition 9 *The boundary investment cost* I^{S_1} *could be located within the area (i) and (k). It is never contained within the area of (h) and (l).*
Proof. *See the appendix.* ■

We examine the equilibrium strategies in the area (k). First, assume that firm L invests at time zero. The net present value denoted by V_F^{in} when firm F also invests is given by

$$V_F^{in} = D_{11}Y_0\left(1 + \frac{\mu}{R}\right) - I.$$

On the other hand, when firm F decides not to invest the net present value denoted by V_F^{out} is given by

$$V_F^{out} = D_{01}Y_0\left(1 + \frac{\mu}{R}\right).$$

Since

$$\begin{aligned} V_F^{in} - V_F^{out} &= \left(D_{11} - D_{01}\right)Y_0\left(1 + \frac{\mu}{R}\right) - I \\ &= I_2^{\alpha} - I < 0 \end{aligned}$$

the optimal strategy for firm F is never to invest. Next, assume that firm L does not invest in the project. The corresponding net present values are given by

$$V_F^{in} = D_{10}Y_0 - I + \frac{1}{R}\left\{puY_0D_{11} + qdY_0D_{10}\right\},$$

$$V_F^{out} = D_{00}Y_0 + \frac{1}{R}\left\{puY_0D_{01} + qdY_0D_{00}\right\}.$$

Hence,

$$V_F^{in} - V_F^{out} = I^{S_1} - I$$

is satisfied. Since we assume that the investment cost is within the area (k), this area should be divided into two subareas; Namely, the equilibrium strategies for firm F can be written by (F_{out}, F_{in}) when $I < I^{S_1}$ and by (F_{out}, F_{out}) when $I \geq I^{S_1}$.

Next, we consider the optimal strategy for firm L. Consider the project value if firm L invests in case of $I < I^{S_1}$. Since the optimal strategy for firm F is not to invest the project value for firm L, denoted by V_L^{in}, is given by

$$V_L^{in} = D_{10}Y_0\left(1 + +\frac{\mu}{R}\right) - I.$$

When the project value for firm L when firm L does not invest at time zero and thus firm F invests, which is denoted by V_L^{out}, is given by

$$V_L^{out} = D_{01}Y_0 + \frac{1}{R}\left\{p\left(uY_0D_{11} - I\right) + qdY_0D_{01}\right\}.$$

Then,

$$
\begin{aligned}
&V_L^{in} - V_L^{out} \\
&= (D_{10} - D_{01}) Y_0 + \frac{1}{R} \{puY_0 (D_{10} - D_{11}) + qdY_0 (D_{10} - D_{01})\} - I \left(1 - \frac{p}{R}\right) \\
&= \left(1 - \frac{p}{R}\right) \left(I^{S_2} - I\right).
\end{aligned}
$$

The above equation indicates that the equilibrium strategy for firm L depends on the location of the boundary cost I^{S_2}; Namely, it is optimal for firm L to invest if $I < I^{S_2}$ and it is optimal not to invest if $I \geq I^{S_2}$. By a numerical examination it is easily confirm that there could be a case where both I^{S_1} and I^{S_2} are located within the area (k) and $I^{S_2} < I^{S_1}$ is satisfied. In that case the combination of the equilibrium strategies is given by $(L_{in}, (F_{out}, F_{in}))$. In case of $I^{S_1} \geq I^{S_2}$ and I^{S_1} is located within the are (k)[18] it is $(L_{out}, (F_{out}, F_{in}))$. When $I \geq I^{S_1}$ is satisfied in this area it is $(L_{out}, (F_{out}, F_{out}))$. The next proposition summarizes all the results.

Proposition 10 *Suppose that only firm L has a real option to defer the project and that firm F does not have the flexibility. When $1 + \frac{\mu}{R} \leq u$ is satisfied the equilibrium strategies for both firms at time zero can be classified into Figure 10 and Figure 11 , which depends on the location of the boundary investment costs I^{S_1} and I^{S_2}.*

It is interesting to note that in the area of (c), (g) and (j) the pair of equilibrium strategies is given by $(L_{out}, (F_{in}, F_{in}))$, which means that firm F invests first and becomes a first mover, while firm L defers the investment and becomes a second mover. Thus, the project value for firm L, denoted by v_L, is

$$v_L = D_{01} Y_0 + \frac{1}{R} \{p (uY_0 D_{11} - I) + qdY_0 D_{01}\}, \tag{24}$$

while the project value for firm F denoted by v_F is

$$v_F = D_{10} Y_0 - I + \frac{1}{R} \{puY_0 D_{11} + qdY_0 D_{10}\}. \tag{25}$$

Hence, the difference of these values are

$$v_F - v_L = (D_{10} - D_{01}) Y_0 \left(1 + \frac{qd}{R}\right) - \left(1 - \frac{p}{R}\right) I. \tag{26}$$

Apparently, the project value of firm F is larger than that of firm L in the areas of (c) and (g). In the area of (j) the project value of firm L could be smaller than that of firm F as well. Namely, firm L loses its advantage over firm F by obtaining the flexibility. We call it *flexibility trap*. This is similar to a famous example of the commitment effects in game theory.

In addition, when both I^{S_1} and I^{S_2} are within the area of (k) and $I^{S_1} > I^{S_2}$ there exist a combination of strategies for $(L_{out}, (F_{out}, F_{in}))$. The realized

[18] In this case it does not matter whether I^{S_2} is within the area (k).

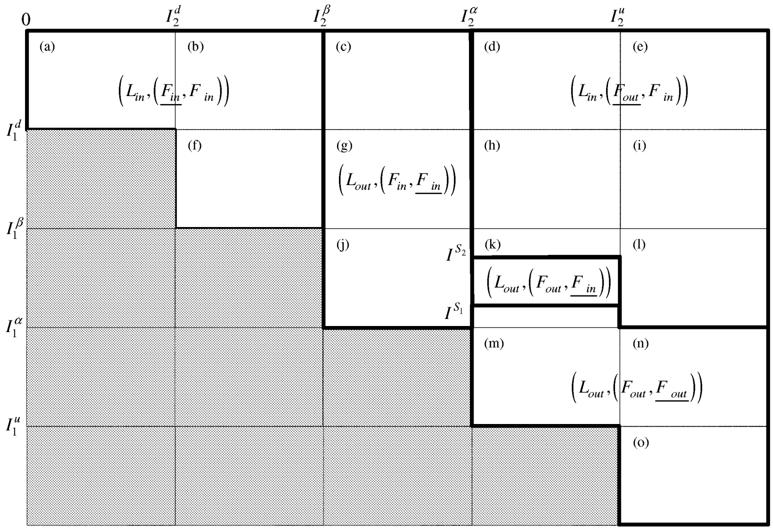

Figure 10: This figure illustrates the equilibrium strategies for both firms when only firm L has a real option. In the figure the boundary investment costs I^{S_1} and I^{S_2} are both located in the area (k) and $I^{S_2} < I^{S_1}$ holds.

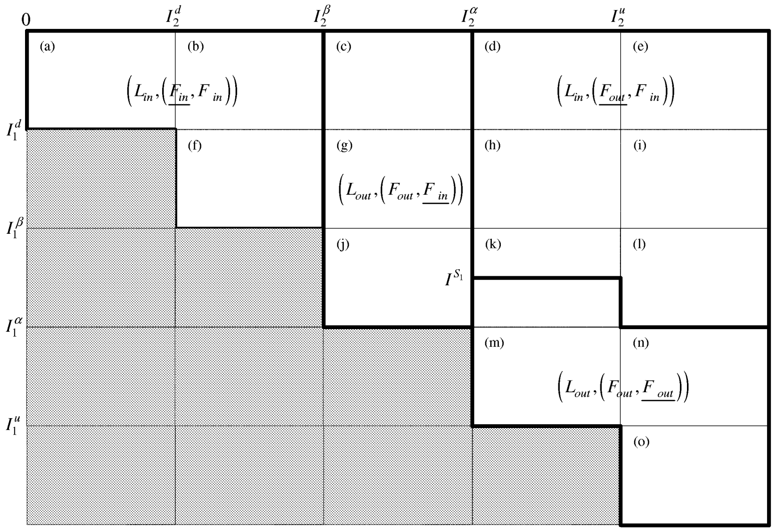

Figure 11: This figure illustrates the equilibrium strategies for both firms when only firm L has a real option. In the figure the boundary investment cost I^{S_1} is located in the area (k) and $I^{S_2} > I^{S_1}$ holds.

strategy in this case is that firm L does not invest while firm F invests and becomes a first mover. This case is also another example of the flexibility trap.

5.1.1 The comparison of the equilibrium strategies

It is useful to compare the pair of strategies derived in this section with the following two pairs of strategies in case 2. The first one is a pair of strategies when neither firm does not have a real option and hence the project value is calculated with the NPV method. By comparison of strategies we can extract the effects of acquiring flexibility on the project value of firm L. The other one is that when both firms have real options since we can extract the effect of losing flexibility on the project value of firm F.

Assume that firm F does not have a real option. When firm L does not have flexibility the optimal strategy for firm L at time zero is to invest in the project in the area of (c), (g), and (j) because of $I < I_1^{\alpha}$. On the other hand, when firm L obtains a real option the optimal strategy for firm L is to defer the investment at time zero and invest when the demand moves up. Hence, the net project value for firm L by acquiring a real option is given by $\left(1 - \frac{p}{R}\right)\left(I_2^{\beta} - I\right) > 0$. It is important to notice that firm F also gain a positive project value as a result of the change of firm L's strategy, which is given by $(D_{10} - D_{11})\, Y_0 \left(1 + \frac{qd}{R}\right) > 0$.

Next, the area (k) is considered when both I^{S_1} and I^{S_2} are within the area (k) and $I^{S_1} > I^{S_2}$ which is illustrated in Figure 10. If firm L does not have a real option a combination of strategies is given by $(L_{in}, (F_{out}, F_{in}))$. If firm L acquires a real option the combination is given by $(L_{out}, (F_{out}, F_{in}))$; Namely, firm F becomes a first mover in this area as sell. The analysis reveals that the project values of both firms could be increased when firm L acquires a real option.

We compare Figure 10 with Figure 8 to examine the effect of losing flexibility of firm F. Now consider the case when the investment cost is within the area of (c), (g). The pair of optimal strategies when both firms have real options is given by $(L_{in}, (F_{out}, F_{in}))$ while that when firm L has a real option and firm F does not is given by $(L_{out}, (F_{in}, F_{in}))$. Thus, the optimal strategy for firm F has been changed. It results from the fact that firm F loses a chance to defer the investment. Firm L also changes its strategy in response. The net project value gained by abandoning the flexibility of firm F is written by

$$
\begin{aligned}
&(D_{10} - D_{00})\, Y_0 \left(1 + \frac{qd}{R}\right) - \left(1 - \frac{p}{R}\right) I \\
&> \left(1 - \frac{p}{R}\right)\left(I_1^{\beta} - I\right) > 0.
\end{aligned}
$$

The equation means that the project values for firm F is increased when firm F abandons flexibility to defer the investment. For firm L on the other hand, the project value of is decreased when firm F throws away its real option. The net loss can be given by $(D_{10} - D_{00}) Y_0 \left(1 + \frac{qd}{R}\right) - \left(1 - \frac{p}{R}\right) I$,which is equal to the value gained by firm L.

5.2 Firm L with NPV vs. firm F with Real option

Finally, a set of the optimal strategies is derived on condition that only firm F has a real option. Note although firm L does not defer the project it can make a decision before firm F at time zero. Firm F, on the other hand, can defer the project and invest at time one but the decision at time zero must be made after the decision of firm L. In short, firm L has an competitive advantage of the decision at time zero while firm F has flexibility about the timing of the investment. It is important to analyze this trade-off between the competition and flexibility.

The optimal strategies for firm F at time one are considered. They are equivalent to the optimal strategies for firm L when only firm L has a real option which is analyzed in the previous subsection. Thus, the equilibrium strategies at time zero is analyzed. The next proposition summarizes the equilibrium strategies in case 2.

Proposition 11 *Suppose that only firm F has a real option to defer the project and that the project value for firm L is based on the net present value method. When* $1 + \frac{\mu}{R} \leq u$ *is satisfied the equilibrium strategies for both firms at time zero can be classified into Figure 12, Figure 13 and Figure 14, which depends on the location of the boundary investment costs* I^{S_0} [19].

In all figures, there are four sets of strategies and three sets of realized strategies. Consequently, the boundary investment cost for firm F is I_2^{β}, which is equivalent to the case when both firms have real options. For firm L the boundary investment cost depends on the location of I^{S_0} and I. These results are consistent with the previous analyses since firm F has a real option while firm L does not.

5.2.1 The comparison of the optimal strategies

The equilibrium strategies developed in this section is compared with other strategies. The net value of flexibility can be extracted by comparing with the case where neither firm has a real option. The analysis is especially interesting because we can examine the effect of acquiring the flexibility under the competitive disadvantage of firm F.

By acquiring flexibility the optimal strategy for firm F has changed in the area of (c), (g), (j). While firm F without flexibility invests at time zero firm F with the real option defers the investment and invests when the demand moves up at time one. Thus, the project value of firm F is also changed. The net

[19] There could be the case when the boundary investment cost I^{S_1} is outside the area (k). In that case a similar analysis is possible which is omitted in this paper.

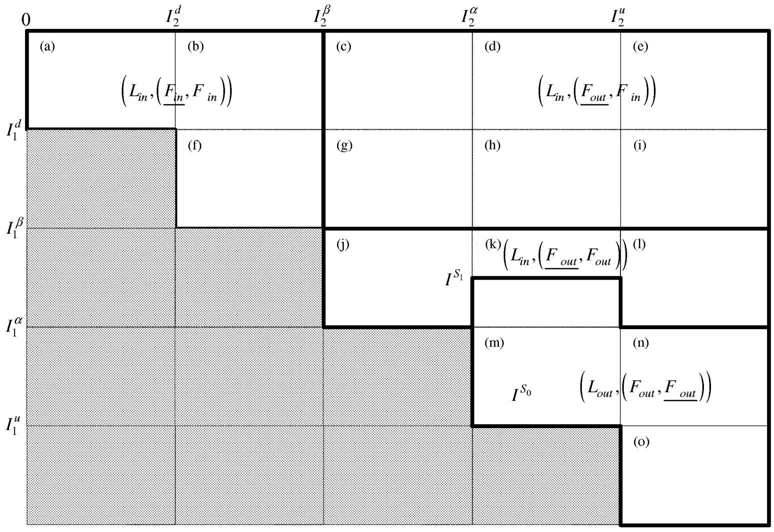

Figure 12: This figure illustrates the equilibrium strategies for both firms when only firm F has a real option. In the figure we assume that the boundary investment cost I^{S_1} is located in the area (k) and that I^{S_0} is located in the area (m).

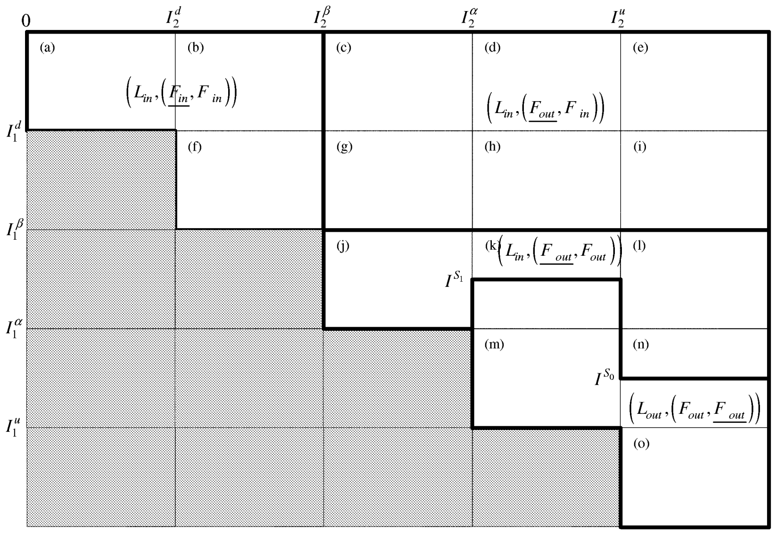

Figure 13: This figure illustrates the equilibrium strategies for both firms when only firm F has a real option. In the figure we assume that the boundary investment cost I^{S_1} is located in the area (k) and that I^{S_0} is located in the area (n).

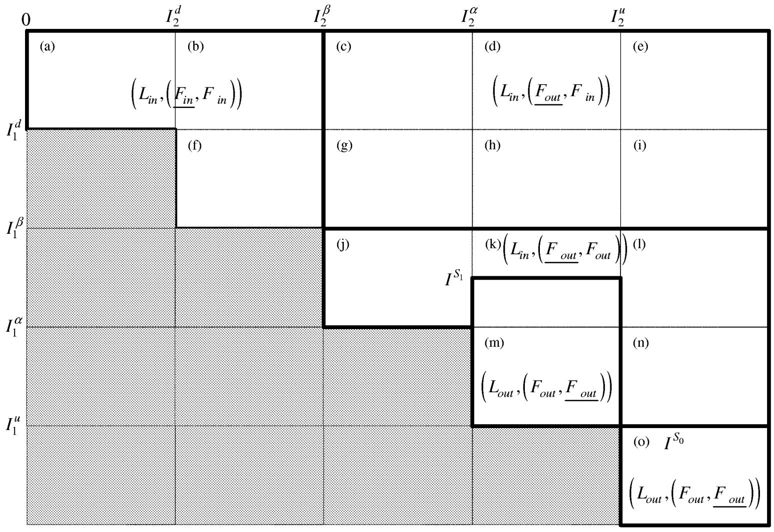

Figure 14: This figure illustrates the equilibrium strategies for both firms when only firm F has a real option. In the figure we assume that the boundary investment cost I^{S_1} is located in the area (k) and that I^{S_0} is located in the area (o).

present value of firm F without flexibility is given by

$$D_{11}\left(1+\frac{\mu}{R}\right)Y_0 - I,$$

while the project value with the real option can be given by

$$D_{01}Y_0 + \frac{1}{R}\left\{p\left(uY_0D_{11} - I\right) + qdY_0\right\}.$$

Accordingly, the net value of the flexibility obtained by firm F is

$$\begin{aligned}
&D_{01}Y_0 + \frac{1}{R}\left\{p\left(uY_0D_{11} - I\right) + qdY_0\right\} - \left\{D_{11}\left(1+\frac{\mu}{R}\right)Y_0 - I\right\} \\
&= \left(1-\frac{p}{R}\right)\left(I - I_2^{\beta}\right) > 0,
\end{aligned}$$

when the boundary investment cost is located in those areas. It is interesting to point out that the project value of firm L is also increased by firm F's flexibility which is equal to $\left(D_{10} - D_{11}\right)Y_0\left(1+\frac{qd}{R}\right)$.

In area (k) the optimal decision for firm L could be changed. The optimal strategy for firm L when neither firm has a real option is to invest at time zero. On the other hand, in case when $I > I^{S_1}$ in area (k) the optimal decision for firm L is never to invest. Apparently, the project value for firm L is decreased in this case. Firm F, on the other hand, could exercise the real option at time one in area (k). Similarly, firm F could also exercise the real option in area (m), which leads to increase the project value of firm F and decrease the project value of firm L.

It is also useful to compare this case with that when both firms have the real options since this comparison reveals the effect of losing flexibility of firm L. By comparing Figure 12 with Figure 6 we can examine the differences of the optimal strategies. When the investment cost is located in the area (j) or (k) the optimal strategy for firm L that loses flexibility changes to invest at time zero because there is no option for firm L to invest at time one. The change of the optimal strategy for firm L increases the project value of firm F although firm F's strategy does not change in that area.

6 Concluding Remarks

This paper investigates an interaction between the managerial flexibility and the competition. Our analysis clarifies the equilibrium strategies for two competitive firms, and derives theoretical conditions of the optimal decisions to make. We consider a two-stage game with two firms under demand uncertainty. It is assumed that the future demand follows a one-period binomial process. Two firms are introduced to analyze the competition. The cash flow generated from a project depends on both the demand and the firms' actions. We assume that the two firms make decisions sequentially at each stage. The cash flow generated from a project depends on both the demand and the firms' actions. While both firms' managers can invest in the identical project at the first stage they could have flexibility to defer the project until the second stage. This flexibility can be considered a real option to defer the project. One firm called a leader firstly makes a decision, and the other firm called a follower decides secondly after observing the leader's action. Namely, a leader has a competitive advantage over a follower.

We characterize the equilibrium strategies for both firms which are classified by investment costs of the firms. We examine several situations where either or both firms can invest only at the first stage. By comparison with each other, we can analyze the effects of flexibility and competition. In this paper we derive several boundary investment costs and show that the equilibrium strategies for both firms are classified by these boundaries.

Our results indicate that under a monopolistic environment the existence of flexibility has a positive impact on the project value. However, under the competitive environment the effects of flexibility are not straightforward. For a follower obtaining the flexibility always increases the project value. On the other hand, a leader could decrease a project value by obtaining the flexibility on condition that the follower can invest only at the first stage. We call it *flexibility trap* that can be interpreted as *commitment effects* in game theory.

7 Appendix: A proof of proposition 9

Proof. In this proof we show that the boundary investment cost I^{S_1} is never contained in the areas of (h) and (l). First, we can easily confirm that the following three inequalities are satisfied.

$$I^{S_1} < I_1^{\alpha},$$

$$I^{S_1} > I_2^{\alpha},$$

$$I^{S_1} > I_1^{d}.$$

All the inequalities can be proved by the result of equation (3). Then, these results indicate that the boundary investment cost I^{S_1} could be contained within the area of (h), (i), (k), and (l).

For the computational simplicity let

$$\begin{aligned} D_{00} &= D_{01} + x_1, \\ D_{11} &= D_{00} + x_2, \\ D_{10} &= D_{11} + x_3. \end{aligned}$$

Then, $x_i > 0; i = 1, 2, 3$ are satisfied because of equation (2), and $x_3 > x_1$ is satisfied because of equation (3).

We first prove that I^{S_1} is never contained in the area of (h). This is proven by showing that both $I^{S_1} < I_2^{u}$ and $I^{S_1} < I_1^{\beta}$ are not simultaneously satisfied. Since

$$\begin{aligned} I_2^{u} - I^{S_1} &= (x_1 + x_2)\, u - \left\{ \frac{pu}{R} x_1 + \alpha x_2 + \left(1 + \frac{qd}{R}\right) x_3 \right\} \\ &= u \left(1 - \frac{p}{R}\right) x_1 + (u - \alpha)\, x_2 - \left(1 + \frac{qd}{R}\right) x_3 \end{aligned}$$

where $\alpha = 1 + \frac{\mu}{R}$ then $I_2^{u} - I^{S_1}$ is monotonically increasing with respect to x_1 and x_2 and is monotonically decreasing with respect to x_3 because $1 - \frac{p}{R} > 0, u - \alpha > 0$, and $1 + \frac{qd}{R} > 0$. Similarly,

$$\begin{aligned} I_1^{\beta} - I^{S_1} &= (x_2 + x_3)\, \beta - \left\{ \frac{pu}{R} x_1 + \alpha x_2 + \left(1 + \frac{qd}{R}\right) x_3 \right\} \\ &= -\frac{pu}{R} x_1 + (\beta - \alpha)\, x_2 - \left\{ \beta - \left(1 + \frac{qd}{R}\right) \right\} x_3 \end{aligned}$$

where $\beta = \frac{1 + \frac{qd}{R}}{1 - \frac{p}{R}}$. $I_1^{\beta} - I^{S_1}$ is monotonically decreasing with respect to x_1 and x_2 and is monotonically increasing with respect to x_3. Note that we consider case 2 where $d < \beta < \alpha < u$ holds. Let x_1 and x_2 be fixed and let $x_3^{\max}$ denote the maximum value that satisfied $I_2^u - I^{S_1} \geq 0$. Then

$$u\left(1 - \frac{p}{R}\right)x_1 + (u - \alpha)\,x_2 - x_3^{\max} = 0.$$

$$\therefore x_3^{\max} = \frac{u\left(1 - \frac{p}{R}\right)x_1 + (u - \alpha)\,x_2}{\left(1 + \frac{qd}{R}\right)}.$$

In equation(), since $I_1^\beta - I^{S_1}$ is monotonically increasing with respect to x_3,

$$\begin{aligned}
I_1^\beta - I^{S_1} &\leq -\frac{pu}{R}x_1 + (\beta - \alpha)\,x_2 - \left\{\beta - \left(1 + \frac{qd}{R}\right)\right\}x_3^{\max} \\
&= -\frac{pu}{R}x_1 + (\beta - \alpha)\,x_2 - \left\{\beta - \left(1 + \frac{qd}{R}\right)\right\}\frac{u\left(1 - \frac{p}{R}\right)x_1 + (u - \alpha)\,x_2}{\left(1 + \frac{qd}{R}\right)} \\
&= 0
\end{aligned}$$

This result indicates that both $I^{S_1} < I_2^u$ and $I^{S_1} < I_1^\beta$ are not simultaneously satisfied, which proves that I^{S_1} is not contained in the area of (h).

The proof that I^{S_1} is not contained in the area of (l) can be also shown. To prove it we show that both $I^{S_1} > I_2^u$ and $I^{S_1} > I_1^\beta$ are not simultaneously satisfied, which can be proven in the same way. Consequently, we have proven that I^{S_1} is not contained in the area of (h) nor (l). ■

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