Vulnerable Options in Supply Chains:

Effects of Supplier Competition

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Abstract

This paper presents valuation of inventory-reorder options in a competitive environment with defaultable suppliers. Analysis of a single period model of a supply chain with two suppliers, a retailer, and exogenous sources of defaults, leads to a number of surprising observation on the effects of the supplier credit risk and competition on the value of the deferment option, retailer's procurement and production decisions, suppliers' pricing decisions, and firms profits. In particular, when wholesale prices are fixed, introduction of the deferment option may benefit the supplier with longer production lead-time at the expense of the supplier with shorter production lead-time and there are conditions for the retailer's profit to be increasing in default correlation. When wholesale prices are allowed to vary, analysis of the game between suppliers shows that introduction of the deferment option diminishes competition between suppliers and, thus, hurts the retailer if supplier defaults are highly correlated. On the other hand, retailer's profit is increasing in supplier default correlation if the level of correlation is low.

1 Introduction

Cost of purchased materials constitutes more than 50% of total sales for an average manufacturing firm [see Subramaniam (1998)], and, therefore, a significant portion of a firm's investment is exposed to supplier default risk. Even temporary supply disruptions may force a firm to incur additional costs of outsourcing and expediting and to pay penalties for defaulting on obligations to its customers.

Recent reports by credit rating agencies indicate that the corporate default rates have peaked at record high levels (10% default rate for high yield bonds in 2002) while recovery rates have sunk to record low levels (20% of par). Consequently, in the current state of the economy, the exposure to supplier credit risk is an important consideration for firms.

Concerned with supplier default risk, a firm (retailer) may profit by procuring from several suppliers, provided that diversification benefits surpass the losses from the higher wholesale prices and from the costs of transforming firm's production process to accept raw materials from multiple sources. Moreover, because of the discrepancy in the production lead-times among suppliers, the retailer, having ordered from the slowest supplier initially, has an option to continue increasing order size over time by contracting with other, faster suppliers. The option to defer ordering decisions could be very valuable because it gives the retailer the flexibility to respond to market events and to supplier defaults.

Intuitively, the benefits to the retailer from procuring from several suppliers should grow as the default correlation decreases (this is similar to the diversification of portfolio risk argument in finance) and, because of the option to defer, as the market volatility increases. However, according to Babich, Burnetas and Ritchken (2003), without deferment, the benefits to the retailer from procuring from several suppliers need not increase in the default correlation if the wholesale prices are endogenous to the problem (the problem is no longer a portfolio selection problem). If the suppliers have a significant bargaining power and compete in price for the retailer's business, Babich et al. (2003) demonstrate that the benefits of competition dominate the benefits of diversification and the retailer prefers suppliers whose defaults are highly positively correlated.

Answers to "What is the role of deferment when prices are set endogenously?" and "How do market volatility and the default correlation affect the deferment value when the suppliers compete in price for the retailer's business?" are not clear *a priori*.

Suppliers with the faster production technology have an option as well — an option to defer pricing decisions until some of the market uncertainty and default uncertainty is resolved. In some situations the retailer might find ex-ante contracts to be too expensive or impossible to enforce [see discussion on incomplete contracts in Van Mieghem (1999) and references therein]. Thus, if the powerful suppliers can change wholesale prices at will, the retailer will anticipate a renegotiation of contract terms. Assuming that this is the case, we define *deferment option* as the option held collectively by the fast suppliers and the retailer to postpone negotiations regarding wholesale prices and order quantities.

Besides prices, suppliers also decide whether to invest in a faster production technology in the first place. "What is the value of the deferment option for the suppliers?" and "Under which condition would suppliers profit from shorter production lead-times?" are interesting research questions.

The analysis of this problem is complicated by the fact that the deferment option is *vulnerable*, that is it could be rendered valueless by a supplier's default.

To address these and other research questions, we adopt a single period, multi-stage model of a two-echelon supply chain with two competing risky suppliers and a single retailer. The retailer, who is facing an uncertain future retail price and has a limited production capacity, decides on order quantities. The suppliers, whose defaults are correlated, control wholesale prices. The retailer and the suppliers may negotiate contracts stipulating order quantities and wholesale prices at time 0. Alternatively, the retailer and the supplier with the faster technology can postpone negotiations, both taking advantage of the deferment option. Contrasting the two cases, we can compute the effects of the deferment option on profits of the retailer, the suppliers, and the channel and study how these effects depend on model parameters (e.g. the default correlation and the retail price volatility).

We solve stochastic games between suppliers and the retailer (a Nash game in the scenario without deferment and a Stackelberg game in the scenario with deferment), derive the equilibrium retailer's ordering policies, the equilibrium suppliers' pricing policies, the equilibrium profits, and compute the deferment option values.

Among other results we find that even when wholesale prices are exogenous, but deferment option is present, the retailer's profit could be increasing in the default correlation. Furthermore, although the retailer always benefits from the deferment option, the fast supplier profits from the deferment option only under certain conditions. Surprisingly, the introduction of the deferment option could benefit the slow supplier, because the retailer would treat the fast supplier as a back-up facility and would increase the order quantity to the slow supplier. We identify conditions under which the introduction of the deferment option is a Pareto improving decision. The value of the deferment option to the retailer depends on the default correlation in a non-monotone way, and furthermore, the sensitivity of the option value to the default correlation depends on the level of the retail price volatility.

When wholesale price are endogenous and in the presence of the deferment option the outcome of the Stackelberg game between firms depends on the level of the default correlation. If supplier defaults are highly correlated the competition between suppliers breaks down and the slow supplier charges monopolist prices and makes monopolist profits. The retailer's profit in this case is 0. At the intermediate correlation levels, there is some competition between suppliers but the slow supplier charges even higher than monopolist price leaving 0 profits to the retailer. The profit of the slow supplier is increasing in the default correlation, while the profit of the fast supplier is the same as in no option case and is decreasing with the default correlation. Finally, if supplier defaults are negatively correlated (or slightly correlated), the profits with the deferment are the same as the profits without deferments and, therefore, suppliers' profits are decreasing in the default correlation while the retailer's profit is increasing in the default correlation. With endogenous wholesale prices the retailer never profits from the deferment option while the suppliers can never be worse off, with the slow supplier collecting the majority of the deferment option benefits.

Therefore, when wholesale prices are endogenous, the retailer prefers suppliers with identical lead times, because it enhances the competition between suppliers. However, ceteris paribus, if a faster technology is available to a supplier at a reasonable price, then retailer's preference will not be honored. The supplier will invest in the faster technology and profit from the deferment option.

There is a substantial body of literature that considers the value of options (or flexibility, in general) in supply chains as a hedge against demand uncertainty or channel coordinating mechanism. For example, Barnes-Schuster, Bassok and Anupindi (2002) study a two period, single-buyer, single-supplier model where demand for the buyer's product is correlated between periods. The buyer can purchase options that give her a right, after observing the demand in period 1, to adjust order quantity for period 2. The supplier, who can use either slow and cheap or fast and expensive production modes, makes production decisions trying to satisfy buyer's demand. Authors compute the equilibrium value of options for the buyer and study the effects of various model parameters (demand risk, demand correlation) on the option values. They also analyze the effect of options in channel coordination.

Burnetas and Ritchken (2000) investigate the role of call and put reorder options on the equilibrium solution of a Stackelberg game between a single supplier and a single retailer in a single period model with downward sloping demand curve. The authors show that the introduction of option could benefit all firms, however, there are circumstances (high variability of demand uncertainty) when the retailer is made worse off with options. The latter observation is similar to our conclusion that when suppliers, who are Stackelberg leaders, control wholesale prices the retailer could suffer from the introduction of deferment options.

The benefits of shorter lead-times that allow firms to defer decisions while learning about market uncertainty were studied by Iyer and Bergen (1997). They show that reducing lead times need not be good for the supplier, because it reduces expected retailer's order, thus reducing supplier's expected profit. We derive analogous results in our model with fixed wholesale prices.

Several papers assume that, similar to our model, there are several sources of raw materials for the retailer. Elmaghraby (2000) and Minner (2003) offer surveys of research on multi-sourcing in supply chain management.

Serel, Dada and Moskowitz (2001) consider a multi-period model where the retailer, besides reserving capacity of the long-term supplier, can aquire raw materials on the spot market. The presence of the spot market exerts competitive pressure on the long-term supplier and alters the equilibrium solution of the game between the supplier and the retailer.

In the multi-period discounted profit maximization model by Kouvelis and Milner (2002) a firm faces a stochastic demand and a stochastic supply. To meet its needs for non-core activities the firms can either produce products internally (using supplier owned by the firm) or rely on outside suppliers.

An excellent review of research papers dedicated to studying quantity flexibility, effects of shortening lead-times, buy-back and return policies, is offered by Tsay, Nahmias and Agrawal (1999), along with classification and review of other research on contracts in supply chain management.

However, to the best of our knowledge, this paper is the first one to study the value of vulnerable deferment options and their effect on supply chains in a stochastic game setting with multiple competing risky suppliers under both market and supply uncertainty.

Financial vulnerable options were first studied by Johnson and Stulz (1987) using Merton (1974) model of credit risk. The authors show numerically that premature exercise of the vulnerable American options may be optimal. Another model for valuation of vulnerable options in the general Merton's framework was offered by Hull and White (1995). Exogenously driven defaults are better described using intensity type models of credit risk. Papers by Jarrow and Turnbull (1995) and by Duffie and Singleton (1999a) are representative of research on valuation of vulnerable options under the intensity framework.

The mathematical analysis in this paper would still be valid if disruptions in supply were caused not by financial defaults but by other events such as natural disasters, labor strikes, machine failures. However, very active and fast growing financial market for credit risk securities[1] facilitates estimations of risk-neutral default distributions, whereas estimation of distributions due to "natural" events could be more problematic.

The rest of the paper has the following structure. Section 2 describes the model. Section 3 introduces the retailer's production option as a solution of the last stage in the retailer's three stage stochastic program. Section 4 computes the deferment option value and analyzes the effects of the default correlation and the retail price volatility on firm profits and the option value when wholesale prices are exogenously fixed. Section 5 repeats this analysis for the model with endogenous wholesale prices.

2 Model. Assumptions.

We consider a one period, multi-stage model of a two-echelon supply chain with two competing, risky suppliers and a retailer. The suppliers and the retailer are maximizing their expected (with respect to a risk-neutral measure) discounted profits. We assume that the conditions necessary for the existence of a risk-neutral pricing measure are satisfied [see, Harrison and Kreps (1979), Harrison and Pliska (1981)].

All firms have complete and symmetric information about the problem.

[1]The first credit derivatives appeared in 1993. In 1997, the notional amount of credit derivative securities was \$55 billion. In 2002, the notional amount of credit derivative securities was \$573 billion [source: OCC (2002)].

The retailer's variable production cost is c_R, its production lead-time is L_R, and its production capacity is D which, for simplicity, is assumed to be fixed. The product offered by the retailer is perishable and is in demand for a short while at time T. The retailer is a price taker and will sell the final product at time T at an exogenously given, random, retail price $S(T)$. We assume that the evolution of retail price $S(t)$ over time can be described by a geometric Wiener process (under a risk-neutral measure):

$$\frac{dS}{S} = \mu dt + \sigma dz, \tag{1}$$

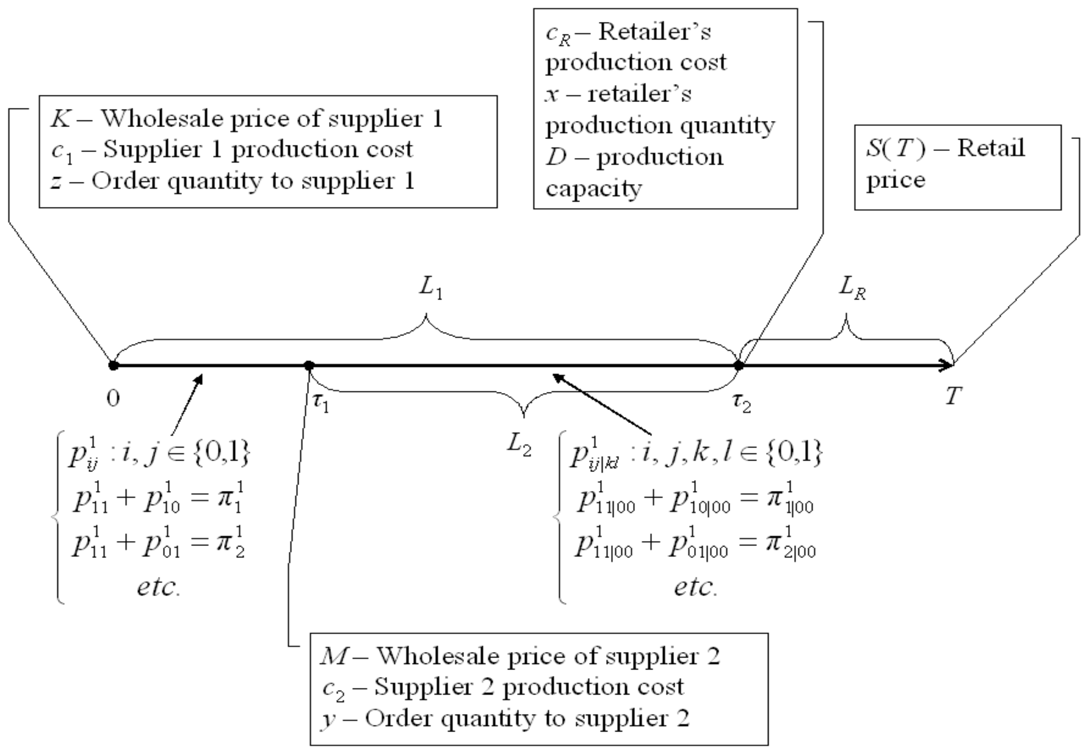
where μ need not be equal to the risk free rate r because of storage costs and convenience yield[2] to the users of the final product. Define $\delta = r - \mu$. Traditionally, the operations literature assumes that the prices are known and demand is stochastic [a notable recent exception is a paper by Li and Kouvelis (1999) who study supply contracts in an environment with deterministic demand and stochastic wholesale prices that follow a geometric Wiener process similar to (1)]. Conversely, the marketing literature usually assumes random prices and known demand. Because demand and prices are related through supply in micro-economic equilibrium, these two approaches are conceptually equivalent. We assume that the retailer's production capacity, D, is too small to affect total supply, so we introduce market uncertainty in retail prices rather than demand for analytical convenience.

Suppliers employ production technologies with lead times of $L_i, i = 1, 2$. Without loss of generality, assume that $L_1 > L_2$ and denote $\tau_2 = T - L_R$ and $\tau_1 = T - L_R - L_2$. Supplier's variable production costs are $c_i, i = 1, 2$ such that $c_1 \leq c_2$. See Figure 1 for the timeline and the notation of the model.

The disparity in production lead-times of the suppliers furnishes the retailer with a valuable option to delay the ordering decision and to observe the evolution of the retail price, S, and suppliers' defaults. Without credit risk, by analogy with the optimal exercise policy for financial American call options, the retailer should delay ordering from supplier 1 until time $T - L_R - L_1$ and from supplier 2 until time $T - L_R - L_2$. However, if an option is *vulnerable* (that is if a counterparty that has written this option can default) then, as Johnson and Stulz (1987) show for financial options, a premature exercise could be optimal. We assume that the retailer's storage costs are exorbitantly high. Therefore, the finished product has to be sold immediately and, in addition, the production must commence right after the raw materials have been provided by a supplier. Consequently, the

[2]For a discussion of convenience yields, see Dixit and Pindyck (1994) and Hull (2000).

Figure 1: Timeline and notation.



retailer will not order from supplier 1 prior to $T - L_R - L_1$ (designated as time 0) and from supplier 2 prior to time $\tau_1 = T - L_R - L_2$.

The exact sequence of events during planning horizon, $[0, T]$, depends on the type of contractual arrangements between the suppliers and the retailer. One possibility is for the firms to negotiate at time 0 contracts that stipulate wholesale prices and order quantities for both suppliers. Alternatively, the retailer and supplier 2 may defer the negotiation until time τ_1. The outcome of the negotiation depends on the bargaining power of the suppliers and the retailer. In Section 4 wholesale prices are fixed and assumed to be know to the retailer before time 0. In Section 5, which considers a model with endogenous wholesale prices, the suppliers are Stackelberg leaders that announce their prices before the retailer makes ordering decisions. Thus, supplier 1 chooses her price K at time 0. The retailer immediately responds by ordering z units from supplier 1. At time τ_1 supplier 2 announces her price M, the retailer responds by ordering y units from supplier 2. At time τ_2 the retailer decides on the size of the production batch x. The finished product is sold at time T at retail price $S(T)$. The units in the model are adjusted so that a unit of the retailer's finished product requires a unit of the supplier's product.

Exogenous events may cause a supplier's default at any time. The default process and the retail price process (1) are independent and doing business with the retailer does not affect the suppliers' default distribution. For tractability, we assume that, if a supplier defaults prior to completing retailer's order, the entire order is lost (recovery rate is 0). This could be the case when the entire production process has to be completed before the supplier's product becomes usable and defaults (or other disruptions) are so violent that they cause a shutdown of the supplier's operations. In random yield terminology, the yield function $\beta(\cdot)$ has a Bernoulli distribution [see Sobel (1995) for properties of yield functions and discussion of various yield functions]. Suppliers' defaults may be correlated. The finance literature models correlated defaults using various methods: though correlated default intensities [e.g. Jarrow and Yu (2001)], by introducing stochastic processes for joint defaults [e.g. Duffie and Singleton (1999b)], and using copula functions [e.g. Schönbucher and Schubert (2001)]. Because in our model the decisions are made at specific time points, the firms need to know only joint default probabilities over intervals between decision points. Therefore, modeling complexity and data requirements for correlated defaults in our model are significantly reduced compared to the financial models mentioned above. We will refer to the time interval $[0, \tau_1]$ as the first stage, to the time interval $[\tau_1, \tau_2]$ as the second stage, and to the time interval $[\tau_2, T]$ as the third stage. Let $\pi_k^1, k = 1, 2$ be the default probability of supplier k during the first stage (superindex indicates the stage). Furthermore, let p_{11}^1 be the probability that both suppliers will defaults during the first stage, p_{10}^1 be the probability that the first supplier will default and the second supplier will survive during the first stage, etc. The joint default probabilities, p_{ij}^1, and the marginals, π_k^1, satisfy the following relationships:

$$p_{00}^1, p_{01}^1, p_{10}^1, p_{11}^1 \geq 0; \quad p_{00}^1 + p_{01}^1 + p_{10}^1 + p_{11}^1 = 1; \tag{2a}$$

$$p_{00}^1 + p_{10}^1 = 1 - \pi_2^1; \quad p_{00}^1 + p_{01}^1 = 1 - \pi_1^1; \quad p_{11}^1 + p_{10}^1 = \pi_1^1; \quad p_{11}^1 + p_{01}^1 = \pi_2^1; \tag{2b}$$

Conditional on the events during the first stage we define joint default probabilities $p_{ij|mn}^2, i, j, m, n \in \{0, 1\}$ and the marginals $\pi_{k|mn}^2, k = 1, 2$ for the second stage (again, superindex indicates the stage). For example, $\pi_{1|00}^2$ is the probability that the first supplier will default in the second stage given that both suppliers survived the first stage, $\pi_{1|01}^2$ is the probability that first supplier will default in the second stage given that first supplier survived first stage and second supplier defaulted during the first stage, $\pi_{2|00}^2$ is the probability that the second supplier will default in the second stage given that both suppliers survived first stage, etc. Similarly, $p_{00|00}^2$ is the conditional probability that both suppliers will survive second stage given that neither supplier defaulted during the first stage, $p^2_{10|10}$ is conditional probability that the second supplier will survive second stage given that first supplier defaulted during the first stage and the second supplier survived the first stage, etc. Second stage joint default probabilities and the marginals satisfy relationships similar to (2). To denote default probabilities over first two stage we will omit superindex. For example, p_{00} is the probability that both suppliers survive the first two stages.

In the numerical examples we will assume that supplier default distributions are symmetric (that is $\pi^1_1 = \pi^1_2 = \pi^1$, $\pi^2_{1|00} = \pi^2_{2|0} = \pi^2_{|00}$, $\pi_1 = \pi_2 = \pi$) and that the following parameters are given: π^1 — probability of supplier's default in the first stage, π — probability of supplier's default in the first two stages, ξ — a magnifier of marginal default probability in the second stage if a default occured in the first stage ($p^2_{11|01} = \xi\pi^2_{1|00}$ and $p^2_{11|10} = \xi\pi^2_{2|00}$), p^1_{00} — probability that both suppliers will survive the first stage (represents default correlation in the first stage), p_{00} — probability that both suppliers survive the first two stages (represents default correlation in the first two stages). These parameters, which can be inferred from prices of financial securities, uniquely describe the symmetric joint default distribution of suppliers' defaults.

3 Stage three retailer's subproblem

Suppose that the retailer has X units of raw materials in inventory at time $\tau_2 \leq T$ and has to decide how many units x to produce to satisfy demand at time T. The retailer's problem is

$$\max_{0 \leq x \leq \min(X,D)} \left\{ e^{-r(T-\tau_2)} E_{\tau_2}[S(T)] - c_R \right\} x, \tag{3}$$

where $E_{\tau_2}[\cdot]$ is an expectation conditional on the information up to time τ_2. The retailer will commence production only if

$$e^{-r(T-\tau_2)} E_{\tau_2}[S(T)] \geq c_R. \tag{4}$$

Using the definition for the S process (1) we find that $e^{-r(T-\tau_2)} E_{\tau_2}[S(T)] = e^{-\delta(T-\tau_2)} S(\tau_2)$ and rewrite condition (4) as $S(\tau_2) e^{-\delta(T-\tau_2)} \geq c_R$ (recall $\delta = r - \mu$). Therefore, the retailer's expected profit at time τ_2 is

$$\left[S(\tau_2) e^{-\delta(T-\tau_2)} - c_R \right]^+ \min(X, D). \tag{5}$$

The expected profit of the retailer at time τ_2 [equation (5)] is equal to the payoff of $\min(X, D)$ call options on the process $\breve{S}(t) = S(t) e^{-\delta(T-t)}$ with exercise price c_R and expiration date τ_2. Let

$V(t) = e^{-r(\tau_2 - t)} E_t[\breve{S}(\tau_2) - c_R]^+$ denote the value of one such option at time $t \leq \tau_2$. Note that the evolution of $\breve{S}(t)$ under the risk-neutral measure is described by

$$\frac{d\breve{S}}{\breve{S}} = rdt + \sigma dz. \tag{6}$$

In some of the subsequent sections, to facilitate analysis of the model and computations, we will assume that $c_R \equiv 0$. In this case, the retailer's expected profit at time τ_2 [equation (5)] becomes $\breve{S}(\tau_2)\min(X, D)$ and option value $V(t) = \breve{S}(t)$.

4 Exogenous wholesale prices

Assume that wholesale prices $K \geq c_1$ and $M \geq c_2$ are given exogenously. For example, if the retailer wields great market power or if the channel is centralized, $K = c_1$ and $M = c_2$. The retailer is the only decision maker in this model.

4.1 Profits without deferment

Wholesale prices are fixed and at time 0 the retailer decides how much to order from each of the suppliers. The retailer's problem is:

$$\begin{aligned} \max_{z,y} \Big\{ & -Kz - (1-\pi_2^1)e^{-r\tau_1}My + \\ & + V_0 \left[p_{01}\min(D,z) + p_{10}\min(D,y) + p_{00}\min(D, z+y) \right] \Big\}, \end{aligned} \tag{7}$$

where $V_0 = V(0)$ is the value of option defined in Section 3. Supplier 2 receives M from the retailer and pays production cost, c_2, per unit of product at time τ_1. Denote by $\widetilde{M} = (1-\pi_2^1)e^{-r\tau_1}M$ the expected present value of M and by $\widetilde{c}_2 = (1-\pi_2^1)e^{-r\tau_1}c_2$ the expected present value of c_2. Then expected profits of the suppliers are

$$\begin{array}{cc} \text{Supplier 1} & \text{Supplier 2} \\ (K - c_1)z(K,M) & (\widetilde{M} - \widetilde{c}_2)y(K,M) \end{array} \tag{8}$$

The following proposition [which is similar to results derived by Babich et al. (2003)] introduces the optimal solution to problem (7). Let R^* denote the optimal expected profit of the retailer, S_1^* denote the optimal profit of supplier 1, S_2^* denote the optimal expected profit of supplier 2, and Q^* denote the optimal expected profit of the channel. For simplicity, assume that when the retailer is indifferent between ordering from supplier 1 and supplier 2, she will order from supplier 1.

Proposition 1. *Given exogenously fixed wholesale prices K and M such that $c_1 \leq K \leq (1-\pi_1)V_0$ and $\widetilde{c}_2 \leq \widetilde{M} \leq (1-\pi_2)V_0$, a solution to the retailer's problem (7) and profits at the optimum are given as follows:*

(i) *If $K \leq p_{01}V_0$ and $\widetilde{M} \leq p_{10}V_0$, then optimal order quantities are $(z^*, y^*) = (D, D)$ and optimal profits are*

$$\begin{aligned} &\textit{Retailer } (R^*) && \left[(1-p_{11})V_0 - K - \widetilde{M}\right] D, && \text{(9a)} \\ &\textit{Supplier 1 } (S_1^*) && (K - c_1)\, D, && \text{(9b)} \\ &\textit{Supplier 2 } (S_2^*) && \left(\widetilde{M} - \widetilde{c}_2\right) D, && \text{(9c)} \\ &\textit{Channel } (Q^*) && [(1-p_{11})V_0 - c_1 - \widetilde{c}_2]\, D. && \text{(9d)} \end{aligned}$$

(ii) *If $p_{01}V_0 < K$ and $\widetilde{M} < K + V_0(\pi_1 - \pi_2)$, then optimal order quantities are $(z^*, y^*) = (0, D)$ and optimal profits are*

$$\begin{aligned} &\textit{Retailer } (R^*) && \left[(1-\pi_2)V_0 - \widetilde{M}\right] D, && \text{(10a)} \\ &\textit{Supplier 1 } (S_1^*) && 0, && \text{(10b)} \\ &\textit{Supplier 2 } (S_2^*) && \left(\widetilde{M} - \widetilde{c}_2\right) D, && \text{(10c)} \\ &\textit{Channel } (Q^*) && [(1-\pi_2)V_0 - \widetilde{c}_2]\, D. && \text{(10d)} \end{aligned}$$

(iii) *If $p_{10}V_0 < \widetilde{M}$ and $\widetilde{M} \geq K + V_0(\pi_1 - \pi_2)$, then optimal order quantities are $(z^*, y^*) = (D, 0)$ and optimal profits are*

$$\begin{aligned} &\textit{Retailer } (R^*) && [(1-\pi_1)V_0 - K]\, D, && \text{(11a)} \\ &\textit{Supplier 1 } (S_1^*) && (K - c_1)\, D, && \text{(11b)} \\ &\textit{Supplier 2 } (S_2^*) && 0, && \text{(11c)} \\ &\textit{Channel } (Q^*) && [(1-\pi_1)V_0 - c_1]\, D. && \text{(11d)} \end{aligned}$$

Proposition 1 has several implications. First, profits of supplier 1, supplier 2, the retailer, and the channel are non-increasing in default probabilities and in the default correlation. Thus, as one's intuition might suggest, an increase in either probability of default or in the default correlation has detrimental effects on profits of firms and the channel.

Second, increasing retail price volatility σ could benefit all firms. Note that with the increase in σ, V_0 increases as well and, consequently, the region where the retailer orders from both suppliers,

$\mathcal{B} = [0, p_{01}V_0] \times [0, p_{10}V_0]$, expands. Thus, although initially it could be that $(K, \widetilde{M}) \notin \mathcal{B}$, and one of the suppliers makes zero profits, eventually both suppliers will have non-zero profits. To appreciate the logic behind this result, consider that at time τ_2 the retailer holds an option to commence production. The value of this option is increasing in the retail price volatility, σ, and decreasing in the retailer's variable production cost, c_R. However, the retailer can exercise her production option only if she receives raw materials by time τ_2. Therefore, as the value of the option increases the retailer is more likely to order from both suppliers to increase the probability of obtaining raw materials. Thus, suppliers benefit from the increase in σ as well. Note, however, that for the intermediate values of σ, supplier's profit could decrease. For example, suppose that $\pi_1 > \pi_2$. Then, as V_0 increases a point $(K, \widetilde{M})$ could move from region

$$\{(K, \widetilde{M}) : p_{10}V_0 < \widetilde{M} \text{ and } \widetilde{M} \geq K + V_0(\pi_1 - \pi_2)\}$$

to region

$$\{(K, \widetilde{M}) : p_{01}V_0 < K \text{ and } \widetilde{M} < K + V_0(\pi_1 - \pi_2)\}$$

before entering region $\mathcal{B}$. Profit of supplier 1 would change from positive, to 0, to positive. Profit of supplier 2 would change from 0, to positive and would remain positive. This happens, because as σ increases the sensitivity of the retailer to default risk increases and the retailer could find it profitable to switch to a lower risk supplier, in spite of her higher wholesale prices.

Third, the influence of the volatility, σ, on the retailer's (channel) profit depends on the level of the default correlation, p_{00}. Consider

$$\frac{\partial^2 R^*}{\partial p_{00} \partial \sigma} = \begin{cases} -\frac{\partial V_0}{\partial \sigma} D & \text{if } K \leq p_{01}V_0 \text{ and } \widetilde{M} \leq p_{10}V_0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

It follows from equation (12) that the marginal increase in profits of the retailer and the channel due an increase in the volatility, $\frac{\partial R^*}{\partial \sigma}$, is non-increasing in correlation (decreasing for small p_{00} and constant for large p_{00}). Equation (12) also implies that the marginal decrease in the profits of the retailer and the channel due to an increase in the default correlation, $-\frac{\partial R^*}{\partial p_{00}}$, is non-decreasing in the volatility (0 for small σ and increasing for large σ). Thus, the retailer and the channel are most concerned with the detrimental effects of the increase in the default correlation when retail prices are highly volatile.

4.2 Profits with deferment

Wholesale prices, K and M, are fixed. At time 0 the retailer places an order, z, with supplier 1 and has an option to place an order, y, with supplier 2 at time τ_1. Consider the retailer's problem:

$$\begin{aligned}
&\max_{z\le D}\Bigg\{-Kz + p_{01}V_0\min(z,D) + \\
&+ p^1_{10}e^{-r\tau_1}E_0\left(\max_{y\le D}\left\{-My + p^2_{10|10}V(\tau_1)\min(y,D)\right\}\right) + \\
&+ p^1_{00}e^{-r\tau_1}E_0\left(\max_{y\le D}\Big\{-My + V(\tau_1)\times\right. \\
&\qquad\qquad \left.\times\left[p^2_{00|00}\min(z+y,D) + p^2_{10|00}\min(y,D)\right]\Big\}\right)\Bigg\},
\end{aligned} \tag{13}$$

The first subproblem in problem (13) corresponds to the outcome where supplier 1 has defaulted and supplier 2 has not defaulted before time τ_1. The following lemma describes the optimal solution of this subproblem and shows how much this subproblem contributes to the expected profits of the firms.

Lemma 1. *If supplier 1 has defaulted before time τ_1 and supplier 2 has not then*

(i) *The optimal reorder quantity for the retailer is*

$$y^*_{10} = D\,1_{\{M\le p^2_{10|10}V(\tau_1)\}}. \tag{14}$$

(ii) *The time 0 expected present value of this subproblem to the retailer is*

$$\begin{aligned}
R^*_{10} &= p^1_{10}e^{-r\tau_1}E_0\left[p^2_{10|10}V(\tau_1) - M\right]^+ D = \\
&= p^1_{10}p^2_{10|10}DW_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right),
\end{aligned} \tag{15}$$

where $W_0(\tau_1, X)$ is the time 0 value of a compound call option[3] *with expiration date τ_1, a strike price X, and a payoff based on the value of option V.*

(iii) *Time 0 expected present value of this subproblem to supplier 2 is*

$$S^*_{2|10} = p^1_{10}e^{-r\tau_1}(M - c_2)D\Pr[M \le p^2_{10|10}V(\tau_1)]. \tag{16}$$

[3] For theory and valuation of compound options, see Geske (1979) and Rubinstein (1991).

The second subproblem in problem (13) correspond to an outcome where neither supplier has defaulted before time τ_1. Let's introduce the following notation:

$$M_1 := p^2_{10|00} V(\tau_1), \tag{17}$$

$$M_2 := \left(p^2_{10|00} + p^2_{00|00}\right) V(\tau_1). \tag{18}$$

Lemma 2. *If neither supplier has defaulted before time τ_1, then*

(i) *The optimal retailer's reorder quantity is*

$$y_{00} = \begin{cases} D & \text{if } M \le M_1 \\ D - z & \text{if } M_1 < M \le M_2 \\ 0 & \text{if } M > M_2 \end{cases} \tag{19}$$

(ii) *The time 0 expected present value of the retailer's profit from this subproblem is*

$$\begin{aligned} R_{00} = p_{00} V_0 z + p^1_{00}\left(1 - \pi^2_{2|00}\right) W_0\left(\tau_1, \frac{M}{1 - \pi^2_{2|00}}\right)(D - z) + \\ + p^1_{00} p^2_{10|00} W_0\left(\tau_1, \frac{M}{p^2_{10|00}}\right) z. \end{aligned} \tag{20}$$

Proof. Using expressions (19) in the last subproblem (13) we derive

$$\begin{aligned} R^*_{00} = \ & p^1_{00} e^{-r\tau_1} E_0 \left[(M_2 - M)(D \pm z) 1_{\{M \le M_1\}} + (M_2 - M)(D - z) 1_{\{M_1 < M \le M_2\}} + \right. \\ & \left. + p^2_{00|00} V(\tau_1) z 1_{\{M_1 < M \le M_2\}} + p^2_{00|00} V(\tau_1) z 1_{\{M > M_2\}}\right] = \\ = \ & p^1_{00} e^{-r\tau_1} E_0 \left[(M_2 - M)(D - z) 1_{\{M \le M_2\}} + (M_1 - M)\, z\, 1_{\{M \le M_1\}} + p^2_{00|00} V(\tau_1) z\right] \end{aligned}$$

■

Using expressions (15) and (20) we can rewrite the retailer's time 0 problem as follows

$$\begin{aligned} \max_{z \le D} \Bigg\{ & [(1 - \pi_1) V_0 - K]\, z + p^1_{10} p^2_{10|10} W_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right) D + \\ & + p^1_{00}\left(1 - \pi^2_{2|00}\right) W_0\left(\tau_1, \frac{M}{1 - \pi^2_{2|00}}\right)(D - z) + p^1_{00} p^2_{10|00} W_0\left(\tau_1, \frac{M}{p^2_{10|00}}\right) z \Bigg\}. \end{aligned} \tag{21}$$

Define:

$$\begin{aligned} \widehat{K}(M) :&= (1 - \pi_1) V_0 - \\ & - p^1_{00}\left[(1 - \pi^2_{2|00}) W_0\left(\tau_1, \frac{M}{1 - \pi^2_{2|00}}\right) - p^2_{10|00} W_0\left(\tau_1, \frac{M}{p^2_{10|00}}\right)\right]. \end{aligned} \tag{22}$$

The curve on Figure 2 is the graph of $\widehat{K}(M)$ when $c_R \equiv 0$ and $\delta = 0$.

An immediate consequence of the definition of $\widehat{K}(M)$ and option properties is that, for any M, $\widehat{K}(M)$ is non-increasing in the default correlation and, if $c_R \equiv 0$, then, $\widehat{K}(M)$ is non-increasing in σ.

The following proposition describes optimal profits of the firms and the channel when wholesale prices are exogenous and the retailer defers ordering decisions.

Proposition 2. *Given exogenous wholesale prices $K \geq c_1$ and $M \geq c_2$,*

The optimal order quantity to supplier 1 is

$$z^* = \begin{cases} D & \text{if } K \leq \widehat{K}(M) \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

The retailer's expected profit is

$$R^* = \begin{cases} [(1-\pi_1)V_0 - K]\,D + & \\ + p^1_{10} p^2_{10|10} W_0\left(\tau_1, \dfrac{M}{p^2_{10|10}}\right) D + & \text{if } K \leq \widehat{K}(M) \\ + p^1_{00} p^2_{10|00} W_0\left(\tau_1, \dfrac{M}{p^2_{10|00}}\right) D & \\ p^1_{10} p^2_{10|10} W_0\left(\tau_1, \dfrac{M}{p^2_{10|10}}\right) D + & \\ + p^1_{00}\left(1 - \pi^2_{2|00}\right) W_0\left(\tau_1, \dfrac{M}{1-\pi^2_{2|00}}\right) D & \text{otherwise} \end{cases} \tag{24}$$

The expected profit of supplier 1

$$S^*_1 = \begin{cases} (K - c_1)D & \text{if } K \leq \widehat{K}(M) \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

The expected profit of supplier 2

$$S^*_2 = \begin{cases} e^{-r\tau_1}(M - c_2)\Big\{p^1_{10} \Pr\left[M \leq p^2_{10|10} V(\tau_1)\right] + & \text{if } K \leq \widehat{K}(M) \\ \qquad + p^1_{00} \Pr\left[M \leq p^2_{10|00} V(\tau_1)\right]\Big\} D & \\ e^{-r\tau_1}(M - c_2)\Big\{p^1_{10} \Pr\left[M \leq p^2_{10|10} V(\tau_1)\right] + & \text{otherwise} \\ \quad + p^1_{00} \Pr\left[M \leq \left(p^2_{10|00} + p^2_{00|00}\right) V(\tau_1)\right]\Big\} D & \end{cases} \tag{26}$$

The expected profit of the channel is

If $K \le \widehat{K}(M)$

$$
\begin{aligned}
Q^* &= [(1-\pi_1)V_0 - c_1]\,D + \\
&+ p^1_{10} e^{-r\tau_1} E\left\{\left[p^2_{10|10} V(\tau_1) - c_2\right] 1_{\left\{p^2_{10|10} V(\tau_1) \ge M\right\}}\right\} D + \\
&+ p^1_{00} e^{-r\tau_1} E\left\{\left[p^2_{10|00} V(\tau_1) - c_2\right] 1_{\left\{p^2_{10|00} V(\tau_1) \ge M\right\}}\right\} D
\end{aligned}
\tag{27}
$$

Otherwise

$$
\begin{aligned}
Q^* = p^1_{10} e^{-r\tau_1} E&\left\{\left[p^2_{10|10} V(\tau_1) - c_2\right] 1_{\left\{p^2_{10|10} V(\tau_1) \ge M\right\}}\right\} D + \\
+ p^1_{00} e^{-r\tau_1} E&\left\{\left[(1-\pi^2_{2|00}) V(\tau_1) - c_2\right] 1_{\left\{(1-\pi^2_{2|00}) V(\tau_1) \ge M\right\}}\right\} D
\end{aligned}
\tag{28}
$$

Proof. Expression (21) is linear in z with a coefficient $\widehat{K}(M) - K$. Hence, the retailer should order from supplier 1 according to rule (23). Using formula (23), one can derive optimal profits of the firms and the channel. ■

Consider consequences of Proposition 2.

Corollary 1. *Given exogenous wholesale prices* $K \ge c_1$ *and* $M \ge c_2$*, and if marginal default probabilities,* $\pi^2_{k|00}, k = 1, 2$ *and* $p^2_{10|10}$*, do not change as the default correlation changes and* $p^2_{10|10} \ge p^2_{10|00}$*, then*

(i) *The optimal order quantity to supplier 1,* z^**, and the optimal profit of supplier 1,* S^*_1*, are non-increasing in the default correlation and in the volatility of retail prices,* σ *(if* $c_R \equiv 0$*).*

(ii) *If* $p^2_{10|10} \ge 1 - \pi^2_{2|00}$ *then the retailer's optimal profit,* R^**, is decreasing in the default correlation.*

If $p^2_{10|10} < 1 - \pi^2_{2|00}$ *the relationship between the retailer's optimal profit,* R^**, and the default correlation is non-monotone.*

(iii) *The retailer's optimal profit,* R^**, is increasing in* σ *(if* $c_R \equiv 0$*).*

(iv) *If* $p^2_{10|10} \ge 1 - \pi^2_{2|00}$ *then the optimal profit of supplier 2,* S^*_2*, is locally decreasing in the default correlation for* (K, M) *away from the boundary,* $K = \widehat{K}(M)$*.*

If $p^2_{10|10} < 1 - \pi^2_{2|00}$ *the relationship between the optimal profit of supplier 2,* S^*_2*, and the default correlation is non-monotone.*

(v) *The optimal profit of supplier 2,* S^*_2*, is locally increasing in* σ *(if* $c_R \equiv 0$*) for* (K, M) *away from the boundary,* $K = \widehat{K}(M)$*.*

Proof. Recall that, for any M, $\widehat{K}(M)$ is non-increasing in the default correlation and non-increasing in σ (if $c_R \equiv 0$). Therefore, formulae (23) and (25) imply that the optimal order quantity to supplier 1, z^*, and the optimal profit of supplier 1, S_1^*, are non-increasing in the default correlation and in σ (if $c_R \equiv 0$).

Next consider the effect of default correlation on the retailer's profit, R^*.

Then, for $K \leq \widehat{K}(M)$,

$$\begin{aligned} R^* = & \left[(1-\pi_1)V_0 - K\right] D + p^2_{10|10} W_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right) D - \\ & - p^1_{00}\left[p^2_{10|10} W_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right) - p^2_{10|00} W_0\left(\tau_1, \frac{M}{p^2_{10|00}}\right)\right] D. \end{aligned} \tag{29}$$

In (29) only the second line depends on the default correlation. As correlation increases, p^1_{00} increases and

$$\left[p^2_{10|10} W_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right) - p^2_{10|00} W_0\left(\tau_1, \frac{M}{p^2_{10|00}}\right)\right] \tag{30}$$

increases and remains positive. Therefore, R^* decreases.

For $K > \widehat{K}(M)$, if $p^2_{10|10} \geq 1 - \pi^2_{2|00}$

$$\begin{aligned} R^* = & - p^1_{00}\left[p^2_{10|10} W_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right) - \left(1-\pi^2_{2|00}\right) W_0\left(\tau_1, \frac{M}{1-\pi^2_{2|00}}\right)\right] D + \\ & + p^2_{10|10} W_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right) D, \end{aligned} \tag{31}$$

where

$$\left[p^2_{10|10} W_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right) - \left(1-\pi^2_{2|00}\right) W_0\left(\tau_1, \frac{M}{1-\pi^2_{2|00}}\right)\right] \geq 0 \tag{32}$$

It is easy to show that in this case R^* is decreasing in the default correlation. However, if $K > \widehat{K}(M)$ and $p^2_{10|10} < 1 - \pi^2_{2|00}$

$$\begin{aligned} R^* = & p^1_{00}\left[\left(1-\pi^2_{2|00}\right) W_0\left(\tau_1, \frac{M}{1-\pi^2_{2|00}}\right) - p^2_{10|10} W_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right)\right] D + \\ & + p^2_{10|10} W_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right) D, \end{aligned} \tag{33}$$

where

$$\left[\left(1-\pi^2_{2|00}\right) W_0\left(\tau_1, \frac{M}{1-\pi^2_{2|00}}\right) - p^2_{10|10} W_0\left(\tau_1, \frac{M}{p^2_{10|10}}\right)\right] \geq 0 \tag{34}$$

and, therefore, R^* is increasing in the default correlation.

To summarize, if $p^2_{10|10} \geq 1 - \pi^2_{2|00}$, then, because the retailer's profit in continuous at the boundary $K = \widehat{K}(M)$, the retailer's optimal profit, R^*, is decreasing in the default correlation. However, if $p^2_{10|10} < 1 - \pi^2_{2|00}$ the relationship may be non-monotone. Specifically, for negative correlation ($p_{00} \approx 0$), the retailer's profit is decreasing in the default correlation, but as the correlation continue to increase after certain threshold the retailer's profit starts growing.

If $c_R \equiv 0$, then each branch in the definition of R^* is increasing in σ. Because the retailer's profit is continuous at the boundary between branches, it follows that R^* is increasing in σ.

Analysis for supplier 2 is similar to the analysis for the retailer, except that profit of supplier 2 need not be continuous at the boundary, $K = \widehat{K}(M)$. Therefore, we can only make statements locally within each branch of the definition for S_2^*. ■

4.3 Deferment options, the retail price volatility, and the default correlation.

Define the value of the deferment option to the retailer as

$$\Phi = R^*_{defer} - R^*, \tag{35}$$

where R^*_{defer} is the retailer's profit when decisions are deferred and R^* is the retailer's profit when orders are placed at time 0. Similarly, the values of the deferment option to the suppliers and the channel are

$$\Psi_1 = S^*_{1,defer} - S^*_1, \tag{36}$$

$$\Psi_2 = S^*_{2,defer} - S^*_2, \tag{37}$$

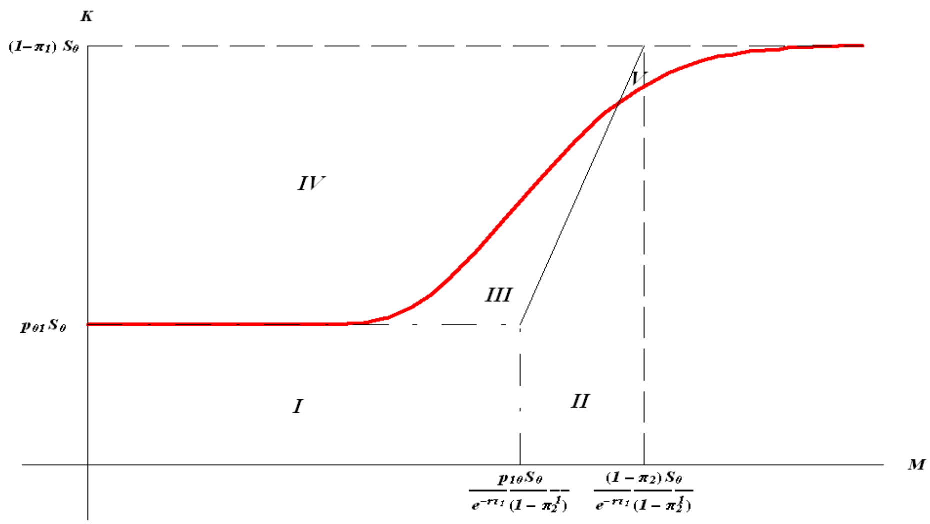
$$\Theta = Q^*_{defer} - Q^*. \tag{38}$$

Assume that $c_R \equiv 0$. Observe that $lim_{M \to +\infty} \widehat{K}(M) = (1 - \pi_1)\breve{S}_0$ and $\widehat{K}(0) = p_{01}\breve{S}_0$. Also, it follows from the Black-Scholes formula that $\frac{\partial V}{\partial X} = -e^{rT} \Pr[\breve{S} \geq X]$. Therefore, differentiating equation (22) with respect to M we obtain

$$\widehat{K}'(M) = -p^1_{00} \left[\frac{\partial V_0}{\partial X} \left(\tau_1, \frac{M}{1 - \pi^2_{2|00}} \right) - \frac{\partial V_0}{\partial X} \left(\tau_1, \frac{M}{p^2_{10|00}} \right) \right] \geq 0 \tag{39}$$

It follows that (K, M)-space is divided into 5 regions shown in Figure 2.

Figure 2: Regions in the definition of the deferment option. Regions are marked by roman numerals. $c_R \equiv 0$.



The option values are given in each of the regions as follows. For the retailer:

$$
\begin{aligned}
\Phi^I = \Bigg\{ & p_{10}^1 p_{10|10}^2 \left[V_0\left(\tau_1, \frac{M}{p_{10|10}^2}\right) - \breve{S}_0 \right] + \\
& + p_{00}^1 p_{10|00}^2 \left[V_0\left(\tau_1, \frac{M}{p_{10|00}^2}\right) - \breve{S}_0 \right] + \left(1-\pi_2^1\right) e^{-r\tau_1} M \Bigg\} D,
\end{aligned} \tag{40a}
$$

$$
\Phi^{II} = \left[p_{10}^1 p_{10|10}^2 V_0\left(\tau_1, \frac{M}{p_{10|10}^2}\right) + p_{00}^1 p_{10|00}^2 V_0\left(\tau_1, \frac{M}{p_{10|00}^2}\right) \right] D, \tag{40b}
$$

$$
\begin{aligned}
\Phi^{III} = \Bigg\{ & p_{10}^1 p_{10|10}^2 \left[V_0\left(\tau_1, \frac{M}{p_{10|10}^2}\right) - \breve{S}_0 \right] + \\
& + p_{00}^1 p_{10|00}^2 \left[V_0\left(\tau_1, \frac{M}{p_{10|00}^2}\right) - \breve{S}_0 \right] + \left(1-\pi_2^1\right) e^{-r\tau_1} M + p_{01}\breve{S}_0 - K \Bigg\} D,
\end{aligned} \tag{40c}
$$

$$
\begin{aligned}
\Phi^{IV} = \Bigg\{ & p_{10}^1 p_{10|10}^2 \left[V_0\left(\tau_1, \frac{M}{p_{10|10}^2}\right) - \breve{S}_0 \right] + \\
& + p_{00}^1 \left(1-\pi_{2|00}^2\right) \left[V_0\left(\tau_1, \frac{M}{1-\pi_{2|00}^2}\right) - \breve{S}_0 \right] + \left(1-\pi_2^1\right) e^{-r\tau_1} M \Bigg\} D,
\end{aligned} \tag{40d}
$$

$$
\begin{aligned}
\Phi^{V} = \Bigg\{ & p_{10}^1 p_{10|10}^2 V_0\left(\tau_1, \frac{M}{p_{10|10}^2}\right) + p_{00}^1 \left(1-\pi_{2|00}^2\right) V_0\left(\tau_1, \frac{M}{1-\pi_{2|00}^2}\right) - \\
& - \left[(1-\pi_1)\breve{S}_0 - K\right] \Bigg\} D.
\end{aligned} \tag{40e}
$$

For supplier 1:

$$\Psi_1^I = \Psi_1^{II} = \Psi_1^{IV} = 0, \tag{41a}$$

$$\Psi_1^{III} = (K - c_1)D, \tag{41b}$$

$$\Psi_1^V = -(K - c_1)D. \tag{41c}$$

For supplier 2:

$$\begin{aligned}\Psi_2^I = \Psi_2^{III} = e^{-r\tau_1}(M - c_2)\Big\{p_{10}^1 \Pr\left[M \le p_{10|10}^2 \breve{S}(\tau_1)\right] +\\ + p_{00}^1 \Pr\left[M \le p_{10|00}^2 \breve{S}(\tau_1)\right] - \left(1 - \pi_2^1\right)\Big\}D,\end{aligned} \tag{42a}$$

$$\begin{aligned}\Psi_2^{II} = e^{-r\tau_1}(M - c_2)\Big\{p_{10}^1 \Pr\left[M \le p_{10|10}^2 \breve{S}(\tau_1)\right] +\\ + p_{00}^1 \Pr\left[M \le p_{10|00}^2 \breve{S}(\tau_1)\right]\Big\}D,\end{aligned} \tag{42b}$$

$$\begin{aligned}\Psi_2^{IV} = e^{-r\tau_1}(M - c_2)\Big\{p_{10}^1 \Pr\left[M \le p_{10|10}^2 \breve{S}(\tau_1)\right] +\\ + p_{00}^1 \Pr\left[M \le \left(1 - \pi_{2|00}^2\right) \breve{S}(\tau_1)\right] - \left(1 - \pi_2^1\right)\Big\}D,\end{aligned} \tag{42c}$$

$$\begin{aligned}\Psi_2^V = e^{-r\tau_1}(M - c_2)\Big\{p_{10}^1 \Pr\left[M \le p_{10|10}^2 \breve{S}(\tau_1)\right] +\\ + p_{00}^1 \Pr\left[M \le \left(1 - \pi_{2|00}^2\right) \breve{S}(\tau_1)\right]\Big\}D.\end{aligned} \tag{42d}$$

For the channel:

$$\Theta = \Phi + \Psi_1 + \Psi_2. \tag{43}$$

The effect of deferment on profit of supplier 1 is clear from (41). Using put-call parity [$V_0 - \breve{S}_0 = U_0 - PV(strike)$, where U_0 is the put option on $\breve{S}$] we can show that $\Phi \ge 0$. For supplier 2, $\Psi_2^{II} > 0$, $\Psi_2^V > 0$ and because

$$p_{10}^1 \Pr\left[M \le p_{10|10}^2 \breve{S}(\tau_1)\right] + p_{00}^1 \Pr\left[M \le p_{10|00}^2 \breve{S}(\tau_1)\right] - \left(1 - \pi_2^1\right) \le 0, \tag{44}$$

$\Psi_2^I \le 0$, $\Psi_2^{III} \le 0$, and $\Psi_2^{IV} \le 0$.

Table 1 summarizes the effects of the deferment option on firms' profits. As you can see, the retailer always benefits from the deferment option.

Supplier 1 benefits from the option in region III because the retailer uses supplier 2 as a back-up facility placing larger order with supplier 1. Supplier 1 is hurt by the introduction of option in region V, because her prices are too high and the retailer prefers to wait and order from supplier 2.

Table 1: Effects of deferring ordering decisions on firms' profits when wholesale prices are exogenous and $c_R \equiv 0$. "+" indicates that profit increases, "−" indicates that profit decreases, "=" indicates that profit does not change.

Region	Supplier 1	Supplier 2	Retailer
I	$=$	$-$	$+$
II	$=$	$+$	$+$
III	$+$	$-$	$+$
IV	$=$	$-$	$+$
V	$-$	$+$	$+$

Supplier 2 benefits from the option in regions II and V because the options assures non-zero probability of the retailer placing an order with supplier 2 in those regions. On the other hand, the expected order quantity to supplier 2 in regions I, III, and IV is reduced by the deferment option.

Therefore, supplier 2 will invest in the faster technology only if prices, (K, M), are either in region II or in a region V. Note that in region II the introduction of the deferment option is a Pareto improving decision.

The following result summarizes the consequences for option values of changes in the default correlation and the retail price volatility.

Proposition 3. *For a given pair of wholesale prices, (K, M), the deferment option values depend on σ as follows*

$(K, M) \in$	Ψ_1	Ψ_2	Φ
I	*no change*	*increasing*	*increasing*
II	*non-increasing*	*locally increasing*	*increasing*
III	*non-increasing*	*locally increasing*	*increasing*
IV	*no change*	*increasing*	*increasing*
V	*no change*	*increasing*	*increasing*

where "local" property holds as long as (K, M) remains within specified region.

Proof. As σ increases, prices (K, M) may move from region III to region IV or from region II to region V but they will remain in regions I, IV, or V. Next, note that each branch of Φ and Ψ_2 is increasing in σ and none of the branches of Ψ_1 depends on σ. Finally, observe that Φ is continuous on the curve $K = \widehat{K}(M)$. Results follow. ■

Next consider the effect of the default correlation on the option value. Let's begin with the option value of the supplier 1, Ψ_1. For $\widetilde{M} > K + \breve{S}_0(\pi_1 - \pi_2)$, as correlation increases (starting from perfect negative correlation) a point (K, M) moves from region I to region II and possibly to region V. For $\widetilde{M} \leq K + \breve{S}_0(\pi_1 - \pi_2)$, a point (K, M) moves from region I to region III and possibly region IV. Within each of the regions, the value of the deferment option to supplier 1 does not depend on the default correlation. We have, therefore, the following proposition

Proposition 4. *Given wholesale prices (K, M),*

(i) *If $\widetilde{M} > K + \breve{S}_0(\pi_1 - \pi_2)$, then, Ψ_1, is non-increasing in the default correlation.*

(ii) *If $\widetilde{M} \leq K + \breve{S}_0(\pi_1 - \pi_2)$ then, Ψ_1, is non-monotone in the default correlation (non-decreasing initially and non-increasing after certain level of correlation).*

Continue with the option value for supplier 2, Ψ_2, assuming that marginal default probabilities $\pi_{k|00}, k = 1, 2$ and $p^2_{10|10}$ do not change as correlation changes.

If $p^2_{10|10} \geq p^2_{10|00}$, then

$$p^1_{00}\left\{\Pr\left[M \leq p^2_{10|10}\breve{S}(\tau_1)\right] - \Pr\left[M \leq p^2_{10|00})\breve{S}(\tau_1)\right]\right\} \tag{45}$$

is increasing in the default correlation.

If $p^2_{10|10} \geq 1 - \pi^2_{2|00}$, then

$$p^1_{00}\left\{\Pr\left[M \leq p^2_{10|10}\breve{S}(\tau_1)\right] - \Pr\left[M \leq (1 - \pi^2_{2|00})\breve{S}(\tau_1)\right]\right\} \tag{46}$$

is increasing in the default correlation.

If $p^2_{10|10} \leq 1 - \pi^2_{2|00}$, then

$$p^1_{00}\left\{\Pr\left[M \leq (1 - \pi^2_{2|00})\breve{S}(\tau_1)\right] - \Pr\left[M \leq p^2_{10|10}\breve{S}(\tau_1)\right]\right\} \tag{47}$$

is increasing in the default correlation. Properties of (45), (46), and (47) imply

Proposition 5. *If marginal default probabilities $\pi_{k|00}, k = 1, 2$ and $p^2_{10|10}$ do not change as correlation changes and $p^2_{10|00} \leq p^2_{10|10}$, then*

(i) *$\Psi^I_2 = \Psi^{III}_2$ and Ψ^{II}_2 are decreasing in the default correlation.*

(ii) *If* $p^2_{10|10} \geq 1 - \pi^2_{2|00}$ *then* Ψ_2^{IV} *and* Ψ_2^{V} *are decreasing in the default correlation.*

(iii) *If* $p^2_{10|10} \leq 1 - \pi^2_{2|00}$ *then* Ψ_2^{IV} *and* Ψ_2^{V} *are increasing in the default correlation.*

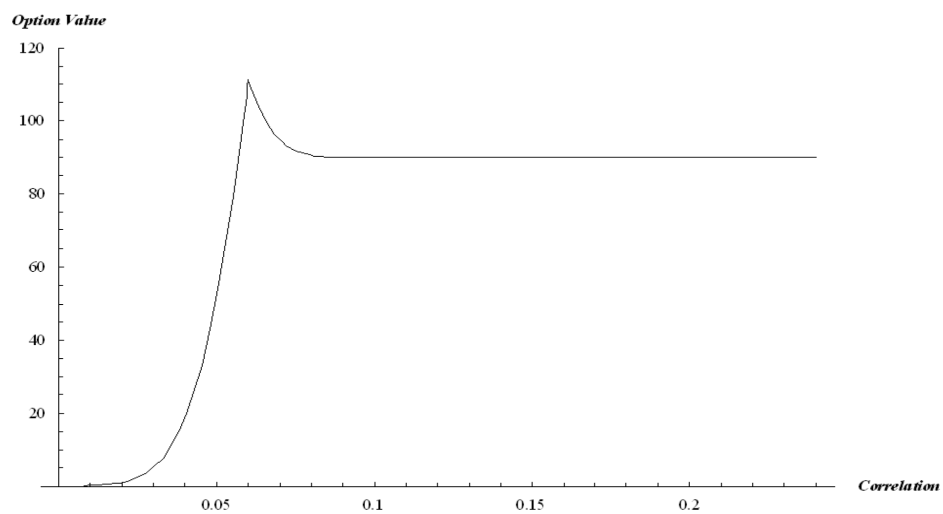
Therefore, if $p^2_{10|10} \leq 1 - \pi^2_{2|00}$*, as the default correlation increases* Ψ_2 *initially decreases and then increases.*

Proof. As the correlation increases a point (K, M) moves among regions either $I \rightarrow III \rightarrow IV$ or $I \rightarrow II \rightarrow V$. Ψ_2 is decreasing in correlation in regions I, II, and III and increasing in correlation in regions IV and V. ■

Figure 3 demonstrates that the value of the retailer's option Φ is non-monotone in correlation as well.

Figure 3: Non-monotone relationship between the retailer's option value, Φ, and the default correlation, p_{00}.

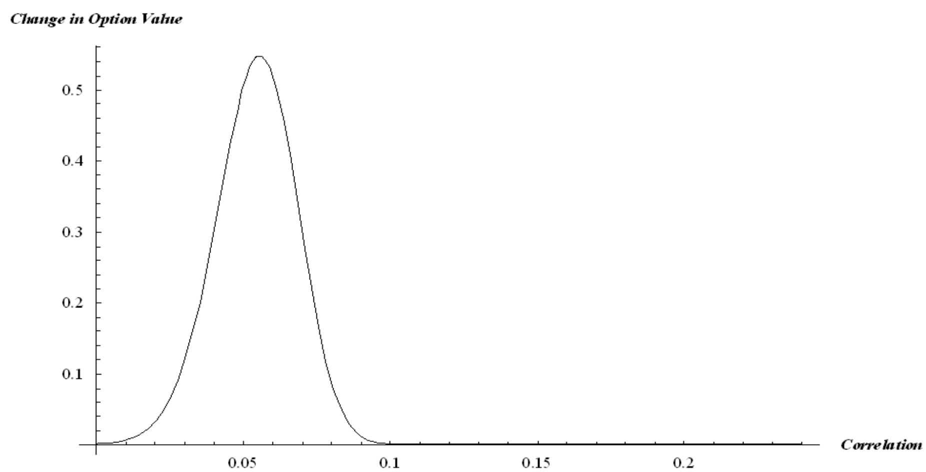
Parameters: $c_R \equiv 0$, $\delta = 0$, $K = 39$, $M = 40$, $\sigma = 0.1$, $S_0 = 100$, $D = 100$, $p^1_{00} = 0.5$, $1 - \pi^1 = 0.9$, $1 - \pi = 0.45$, $\xi = 1$, $p_{00} \in [0, 0.25]$.



The remaining issue is the interaction between effects on deferment options of the default correlation and the volatility of retail prices. Figure 4 shows dependence of $\frac{\Delta\Phi}{\Delta\sigma}$ on the default correlation. The sensitivity of option value Φ to change is σ depends in a non-monotone way on the default correlation. Conversely, the sensitivity of option value to a change in the default correlation depends in a non-monotone way on the level of the retail price volatility.

Figure 4: Non-monotone relationship between change in option value, $\Phi(\sigma) - \Phi(\sigma - \Delta\sigma)$, and the default correlation, p_{00}.

Parameters: $c_R \equiv 0$, $\delta = 0$, $K = 39$, $M = 40$, $\sigma = 0.1$, $\Delta\sigma = 0.001$, $S_0 = 100$, $D = 100$, $p^1_{00} = 0.5$, $1 - \pi^1 = 0.9$, $1 - \pi = 0.45$, $\xi = 1$, $p_{00} \in [0, 0.25]$.



5 Endogenous wholesale prices

It is more realistic to assume that the suppliers have at least some market power, the industry is not too heavily regulated and, therefore, wholesale prices are determined endogenously. In this case competition between suppliers plays an essential role in valuation of the deferment option and significantly contributes to the effects of the default correlation on profits of firms and the channel. This section investigates interaction between competition, default correlation and deferment option under assumption that market power belongs to the suppliers.

5.1 Profits without deferment

At time 0 suppliers simultaneously announce prices (K and M). Retailer [whose problem is given by (7)] responds with orders (z, y). The retailer's response to any pair of wholesale prices (K, M) is given in Proposition 1. Suppliers profit functions are given in (8). The following proposition specifies pure strategy equilibrium solution to the game between suppliers and equilibrium profits of the firms and the channel.

Proposition 6. *Assume that either $p_{01}V_0 > c_1$ or $p_{10}V_0 > \widetilde{c}_2$, then the unique pure strategy equilibrium solution to the game between suppliers is*

(i) *If $p_{01}V_0 > c_1$ and $p_{10}V_0 > \widetilde{c}_2$, then $(K^*, \widetilde{M^*}) = (p_{01}V_0, p_{10}V_0)$. Equilibrium order quantities are $(z, y) = (D, D)$. Equilibrium profits are*

$$\text{Supplier 1 } (S_1^*) \qquad (p_{01}V_0 - c_1)D, \tag{48a}$$

$$\text{Supplier 2 } (S_2^*) \qquad (p_{10}V_0 - \widetilde{c}_2)D, \tag{48b}$$

$$\text{Retailer } (R^*) \qquad p_{00}V_0D, \tag{48c}$$

$$\text{Channel } (Q_1^*) \qquad \left[(1 - p_{11})V_0 - c_1 - \widetilde{c}_2\right] D. \tag{48d}$$

(ii) *If $p_{01}V_0 > c_1$ and $p_{10}V_0 \leq \widetilde{c}_2$, then $(K^*, \widetilde{M^*}) = (\widetilde{c}_2 + V_0(\pi_2 - \pi_1) - \varepsilon, \widetilde{c}_2)$ for some small ε and equilibrium order quantities are $(z, y) = (D, 0)$. Equilibrium profits are*

$$\text{Supplier 1 } (S_1^*) \qquad \left[\widetilde{c}_2 + V_0(\pi_2 - \pi_1) - \varepsilon - c_1\right] D, \tag{49a}$$

$$\text{Supplier 2 } (S_2^*) \qquad 0, \tag{49b}$$

$$\text{Retailer } (R^*) \qquad \left[(1 - \pi_2)V_0 - \widetilde{c}_2 + \varepsilon\right] D, \tag{49c}$$

$$\text{Channel } (Q_1^*) \qquad \left[(1 - \pi_1)V_0 - c_1\right] D. \tag{49d}$$

(iii) *If $p_{01}V_0 \leq c_1$ and $p_{10}V_0 > \widetilde{c}_2$, then $(K^*, M^*) = (c_1, c_1 + V_0(\pi_1 - \pi_2) - \varepsilon)$ for some small ε and equilibrium order quantities are $(z, y) = (0, D)$. Equilibrium profits are*

$$\text{Supplier 1 } (S_1^*) \qquad 0, \tag{50a}$$

$$\text{Supplier 2 } (S_2^*) \qquad \left[c_1 + V_0(\pi_1 - \pi_2) - \varepsilon - \widetilde{c}_2\right] D, \tag{50b}$$

$$\text{Retailer } (R^*) \qquad \left[(1 - \pi_1)V_0 - c_1 + \varepsilon\right] D, \tag{50c}$$

$$\text{Channel } (Q_1^*) \qquad \left[(1 - \pi_2)V_0 - \widetilde{c}_2\right] D. \tag{50d}$$

Proof. Similar to Proposition **??**. ∎

Observe that equilibrium prices, equilibrium order quantities, and profits of firms and the channel are non-decreasing in the volatility of retail prices, σ.

In addition, if $p_{01}V_0 > c_1$ and $p_{10}V_0 > \widetilde{c}_2$, then the retailer's profit is increasing in the default correlation and profits of the suppliers and the channel are decreasing in the default correlation. Recall that with exogenous wholesale prices all firms suffered from the increasing default correlation. When prices are endogenous, increasing correlation intensifies the competition between suppliers. The competition lowers wholesale prices benefiting the retailer. Also note that, when prices are endogenous, the retailer's incentives are different from those of the channel and, therefore, the retailer should not be selected to coordinate the system.

Further, if $p_{01}V_0 > c_1$ and $p_{10}V_0 > \widetilde{c}_2$, then the marginal increase in the retailer's equilibrium profit due to the change in the volatility, $\frac{\partial R^*}{\partial \sigma}$, is increasing in correlation and the marginal increase in profits of suppliers and the channel due to the change in the volatility, $\frac{\partial S_k^*}{\partial \sigma}, k = 1, 2$ and $\frac{\partial Q^*}{\partial \sigma}$, are decreasing in the default correlation. The marginal increase in the retailer's equilibrium profit due to an increase in the default correlation, $\frac{\partial R^*}{\partial p_{00}}$, and the marginal decreases in profits of the suppliers and the channel, $-\frac{\partial S_k^*}{\partial p_{00}}, k = 1, 2$ and $-\frac{\partial Q^*}{\partial p_{00}}$, are increasing in the default correlation. Therefore, the consequences of increasing default correlation (positive consequences for the retailer and negative consequences for the suppliers and the channel) are most profound when retail prices are very volatile.

5.2 Profits for contracts with renegotiation

Supplier 1 announces her wholesale price, K, at time 0. The retailer responds by placing an order, z, with supplier 1. At time τ_1 supplier 2 and the retailer negotiate the wholesale price, M, and the order quantity, y. Thus, both supplier 2 and the retailer defer making decisions until time τ_1 and incorporate information about defaults and retail prices that has been revealed during $[0, \tau_1]$. The outcome of negotiation depends on bargaining powers of the retailer and supplier 2. If the retailer is the dominant firm then the price M will be set at the suppliers marginal production cost (to ensure the suppliers' participation). This would lead to the model considered in section 4. In this section we will assume that the supplier is the dominant firm so that the wholesale price is set at the highest level that does not violate retailer's participation constraints.

At time 0 the retailer solves

$$
\begin{aligned}
&\max_{z\le D}\Bigg\{-Kz + p_{01}V_0\min(z,D)+ \\
&p^1_{10}e^{-r\tau_1}E_0\left(\max_{y\le D}\Big\{-M(\tau_1)y + p^2_{10|10}V(\tau_1)\min(y,D)\Big\}\right)+ \\
&p^1_{00}e^{-r\tau_1}E_0\left(\max_{y\le D}\Big\{-M(\tau_1)y + V(\tau_1)\ \times\right. \\
&\qquad\qquad \left.\times\ \Big[p^2_{00|00}\min(z+y,D) + p^2_{10|00}\min(y,D)\Big]\Big\}\right)\Bigg\}.
\end{aligned} \tag{51}
$$

Note that, unlike problem (13), the price M in the subproblems of (51) depends on the information available by the time τ_1. The first optimization subproblem in equation (51), corresponds to the outcome when supplier 1 has defaulted before time τ_1 and supplier 2 has not. The optimal solutions to this subproblem for the retailer and supplier 2 are given in the following lemma:

Lemma 3. *If by time τ_1 supplier 1 has defaulted and supplier 2 has not, then*

(i) *Supplier 2 optimal price is*

$$M_{10}(\tau_1) = \max\left\{p^2_{10|10}V(\tau_1), c_2\right\}. \tag{52}$$

(ii) *The retailer's optimal order quantity is*

$$y_{10} = D1_{\left\{p^2_{10|10}V(\tau_1)\ge c_2\right\}}. \tag{53}$$

(iii) *Time 0 expected present value of this subproblem to the retailer is 0.*

(iv) *Time 0 expected present value of this subproblem to supplier 2 is*

$$p^1_{10}p^2_{10|10}W_0\left(\tau_1, \frac{c_2}{p^2_{10|10}}\right)D. \tag{54}$$

The second optimization subproblem in equation (51), corresponds to the outcome where neither supplier has defaulted before time τ_1. Given price $M(\tau_1)$ the retailer will order according to the rule in equation (19). Knowing the retailer's response, supplier 2 determines an optimal pricing policy. Define:

$$\widehat{z}(V) := \frac{p^2_{00|00}VD}{(1-\pi^2_{2|00})V - c_2}. \tag{55}$$

Lemma 4 summarizes the solution to the second optimization subproblem.

Lemma 4. *If neither supplier has defaulted before time τ_1 then the equilibrium wholesale price of supplier 2 and the retailer's order quantities are as follows:*

$$M^*(\tau_1) = \begin{cases} c_2 \\ M_2 \\ M_2 \\ M_1 \end{cases}, \quad y^*_{00} = \begin{cases} 0 & \textit{if } M_2(\tau_1) \leq c_2 \\ D - z & \textit{if } M_1(\tau_1) \leq c_2 < M_2(\tau_1) \\ D - z & \textit{if } M_1(\tau_1) > c_2 \textit{ and } z < \widehat{z}[V(\tau_1)] \\ D & \textit{if } M_1(\tau_1) > c_2 \textit{ and } z \geq \widehat{z}[V(\tau_1)] \end{cases} \tag{56}$$

The time 0 expected present value of this subproblem to the retailer is

$$p_{00} V_0 z + p_{00}(D - z) e^{-r\tau_1} E_0 \left[V(\tau_1) 1_{\{M_1 > c_2\}} 1_{\{z \geq \widehat{z}[V(\tau_1)]\}} \right]. \tag{57}$$

The time 0 expected present value of this subproblem to supplier 2 is

$$\begin{aligned} &p^1_{00}(1 - \pi^2_{2|00}) W_0 \left(\tau_1, \frac{c_2}{1 - \pi^2_{2|00}} \right) (D - z) + \\ &+ p^1_{00} \left[(1 - \pi^2_{2|00}) z - p^2_{00|00} D \right] W_0 \left(\tau_1, \frac{c_2 z}{(1 - \pi^2_{2|00}) z - p^2_{00|00} D} \right) 1_{\left\{ z > \frac{p^2_{00|00} D}{1 - \pi^2_{2|00}} \right\}}. \end{aligned} \tag{58}$$

Proof. Using the retailer's response rule (19) and profit function of the second supplier (8) we conclude that supplier 2 will charge either M_1 or M_2, if feasible. If $M_2 \leq c_2$, then supplier 2 is forced to set price at c_2 receiving no orders from the retailer. If $M_1 \leq c_2 < M_2$ the only alternative left for supplier 2 is M_2. If $M_1 > c_2$ then supplier 2 decides whether to charge M_1 and receive an order of D or charge M_2 and receive the order of $D - z$. If $z \leq \widehat{z}[V(\tau_1)]$ the supplier prefers the latter. Hence, the expected present value of the supplier's subproblem is

$$
\begin{aligned}
&p^1_{00}e^{-r\tau_1}E_0\Big\{(M_2-c_2)\,(D-z)\,1_{\{M_1\le c_2<M_2\}}+(M_2-c_2)(D-z)\,1_{\{c_2<M_1\}}1_{\{z<\widehat{z}(V)\}}+\\
&\qquad\qquad +\Big[M_1\pm p^2_{00|00}V(\tau_1)-c_2\Big]\,(D\pm z)\,1_{\{c_2<M_1\}}1_{\{z\ge\widehat{z}(V)\}}\Big\}=\\
&=p^1_{00}e^{-r\tau_1}E_0\Big\{(M_2-c_2)(D-z)\,1_{\{c_2\le M_2\}}+\\
&\qquad\qquad +\Big[(M_2-c_2)z-p^2_{00|00}V(\tau_1)D\Big]\,1_{\{c_2<M_1\}}1_{\{z\ge\widehat{z}(V)\}}\Big\}=\\
&=p^1_{00}(1-\pi^2_{2|00})W_0\left(\tau_1,\frac{c_2}{1-\pi^2_{2|00}}\right)(D-z)+p^1_{00}e^{-r\tau_1}\,1_{\left\{z>\frac{p^2_{00|00}D}{1-\pi^2_{2|00}}\right\}}\times\\
&\qquad\qquad\times E_0\left(\Big\{[(1-\pi^2_{2|00})z-p^2_{00|00}D]V(\tau_1)-c_2z\Big\}\,1_{\left\{V(\tau_1)\ge\frac{c_2z}{(1-\pi^2_{2|00})z-p^2_{00|00}D}\right\}}\right)=\\
&=p^1_{00}(1-\pi^2_{2|00})W_0\left(\tau_1,\frac{c_2}{1-\pi^2_{2|00}}\right)(D-z)+\\
&\qquad\qquad +p^1_{00}\Big[(1-\pi^2_{2|00})z-p^2_{00|00}D\Big]\,W_0\left(\tau_1,\frac{c_2z}{(1-\pi^2_{2|00})z-p^2_{00|00}D}\right)1_{\left\{z>\frac{p^2_{00|00}D}{1-\pi^2_{2|00}}\right\}}
\end{aligned}
$$

If the supplier's price is M_1 then the expected retailer's profit at time τ_1 is

$$-M_1D+V(\tau_1)\left(p^2_{00|00}D+p^2_{10|00}D\right)=p^2_{00|00}V(\tau_1)D.$$

If the supplier's price is M_2 then the expected retailer's profit at time τ_1 is

$$-M_2(D-z)+V(\tau_1)\left[p^2_{00|00}D+p^2_{10|00}(D-z)\right]=p^2_{00|00}V(\tau_1)z.$$

Expected present value of the retailer's subproblem is

$$
\begin{aligned}
&p^1_{00}e^{-r\tau_1}E_0\Big[p^2_{00|00}V(\tau_1)\,z\,1_{\{c_2>M_2\}}+p^2_{00|00}V(\tau_1)\,z\,1_{\{M_1\le c_2<M_2\}}+\\
&\qquad +p^2_{00|00}V(\tau_1)\,z\,1_{\{c_2\le M_1\}}1_{\{z<\widehat{z}(V)\}}+p^2_{00|00}V(\tau_1)\,(D\pm z)\,1_{\{c_2\le M_1\}}1_{\{z\ge\widehat{z}(S)\}}\Big]=\\
&=p_{00}V_0z+p_{00}(D-z)e^{-r\tau_1}E_0\Big[V(\tau_1)1_{\{c_2\le M_1\}}1_{\{z\ge\widehat{z}(V)\}}\Big].
\end{aligned}
$$ ■

Using Lemmas 3 and 4, we can rewrite the retailer's time 0 optimization problem (51) as follows:

$$
\begin{aligned}
\max_{z\le D}\Big\{&-Kz+(1-\pi_1)V_0z+\\
&+p_{00}(D-z)e^{-r\tau_1}E_0\left[V(\tau_1)1_{\{c_2\le M_1(\tau_1)\}}1_{\{z\ge\widehat{z}[V(\tau_1)]\}}\right]\Big\},
\end{aligned}
\tag{59}
$$

which is equivalent to

$$\max_{z\le D}\left\{[(1-\pi_1)V_0-K]\,z+ \right.$$
$$\left. +\,p_{00}(D-z)e^{-r\tau_1}E_0\left[V(\tau_1)1_{\left\{V(\tau_1)\ge\frac{c_2 z}{(1-\pi^2_{2|00})z-p^2_{00|00}D}\right\}}\right]1_{\left\{z>\frac{p^2_{00|00}D}{1-\pi^2_{2|00}}\right\}}\right\}. \tag{60}$$

Define

$$A(K):=(1-\pi_1)V_0-K, \tag{61}$$

and

$$f(z):=p_{00}(D-z)e^{-r\tau_1}E_0\left[V(\tau_1)1_{\left\{V(\tau_1)\ge\frac{c_2 z}{(1-\pi^2_{2|00})z-p^2_{00|00}D}\right\}}\right]. \tag{62}$$

Given process (1) for the evolution of retail prices and the definition of option V in Section 3 one can derive an explicit expression for f [the derivation is similar to the valuation of compound options by Rubinstein (1991)]

$$f(z)=p_{00}(D-z)\left\{\breve{S}_0 N_2\left[\alpha(z),\beta,\rho\right]-c_R e^{-r\tau_2}N_2\left[\alpha(z)-\sigma\sqrt{\tau_1},\beta-\sigma\sqrt{\tau_2},\rho\right]\right\} \tag{63}$$

where $N_2[\cdot,\cdot,\cdot]$ is the c.d.f. of the standard bivariate normal r.v. and

$$\alpha=\frac{\ln\left(\frac{\breve{S}_0}{S_{cr}(z)}\right)+\left(r+\frac{\sigma^2}{2}\right)\tau_1}{\sigma\sqrt{\tau_1}},\qquad \beta=\frac{\ln\left(\frac{\breve{S}_0}{c_R}\right)+\left(r+\frac{\sigma^2}{2}\right)\tau_2}{\sigma\sqrt{\tau_2}},\qquad \rho=\sqrt{\frac{\tau_1}{\tau_2}}.$$

$S_{cr}(z)$ satisfies the following equation:

$$S_{cr}N(\gamma)-c_R e^{-r(\tau_2-\tau_1)}N(\gamma-\sigma\sqrt{\tau_2-\tau_1})=\widehat{V}(z), \tag{64}$$

where $N(\cdot)$ is the cumulative standard normal distribution function,

$$\widehat{V}(z)=\frac{c_2 z}{(1-\pi^2_{2|00})z-p^2_{00|00}D}\qquad\text{and}\qquad \gamma=\frac{\ln\left(\frac{S_{cr}}{c_R}\right)+\left(r+\frac{\sigma^2}{2}\right)(\tau_2-\tau_1)}{\sigma\sqrt{\tau_2-\tau_1}}. \tag{65}$$

Note that one has to find a root of a non-linear equation (64) in order to compute $f(z)$. In numerical examples we will consider a simpler problem where $c_R\equiv 0$. In this case

$$f(z)=p_{00}(D-z)\breve{S}_0 N\{d_1[\widehat{V}(z)]\}, \tag{66}$$

where

$$d_1(X)=\frac{\ln\left(\frac{\breve{S}_0}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)\tau_1}{\sigma\sqrt{\tau_1}}.$$

The following lemma proves the existence of the solution to the retailer's time 0 problem

Lemma 5. *There exists a solution to the retailer's problem (60).*

Proof. Problem (60) is defined on a compact set $z \in [0, D]$. Therefore, to ensure the existence of a solution it is sufficient to show that the retailer's profit function is continuous in z.

To show continuity of the retailer's profit function it is sufficient to show that

$$g(z) := f(z)\, 1_{\left\{z > \frac{p^2_{00|00} D}{1-\pi^2_{2|00}}\right\}}$$

is continuous. Let $z_0 = \frac{p^2_{00|00} D}{1-\pi^2_{2|00}}$. If $z \le z_0$, then $g(z) \equiv 0$ and, thus, continuous. If $z > z_0$, then

$$\frac{c_2 z}{(1 - \pi^2_{2|00})z - p^2_{00|00} D}$$

is continuous (and, furthermore, monotone) function of z. $V(\tau_1)$ is integrable. Therefore, by the Dominated Convergence Theorem,

$$E_0 \left[V(\tau_1)\, 1_{\left\{V(\tau_1) \ge \frac{c_2 z}{(1-\pi^2_{2|00})z - p^2_{00|00} D}\right\}} \right]$$

is a continuous function of z. Consequently, $g(z)$ is continuous. To complete the proof we will show that $g(\cdot)$ is continuous at z_0. Observe that

$$\lim_{z \downarrow z_0} \frac{c_2 z}{(1 - \pi^2_{2|00})z - p^2_{00|00} D} = +\infty.$$

Therefore, as $z \downarrow z_0$

$$V(\tau_1)\, 1_{\left\{V(\tau_1) \ge \frac{c_2 z}{(1-\pi^2_{2|00})z - p^2_{00|00} D}\right\}} \to 0 \quad \text{a. s.}$$

and by the Dominated Convergence Theorem

$$\lim_{z \downarrow z_0} g(z) = 0 = g(z_0) = \lim_{z \uparrow z_0} g(z). \qquad \blacksquare$$

The following lemma describes some properties of the solution of (60) that may be used to increase efficiency of numerical solution procedures.

Lemma 6. *Let* $z_0 = \frac{p^2_{00|00} D}{1-\pi^2_{2|00}}$. *Then a solution of the optimization problem (60) is either* 0 *or it lies in the interval* $[z_0, D]$. *Furthermore, if the solution is unique and*

$$\begin{aligned} K \le & (1 - \pi_1)V_0 - \\ & - p_{00} \left\{ \breve{S}_0 N_2[\alpha(z), \beta, \rho] - c_R e^{-r\tau_2} N_2[\alpha(z) - \sigma\sqrt{\tau_1}, \beta - \sigma\sqrt{\tau_2}, \rho] \right\}, \end{aligned} \tag{67}$$

then the optimal order quantity $z^* = D$, *otherwise* z^* *satisfies*

$$f'(z) = -A(K). \tag{68}$$

Proof. Note that $f(z) \geq 0$ for $z > z_0$. Therefore, the maximum of function $A(K)z + f(z)$ is either 0 or is achieved on the interval $[z_0, D]$. For D to be an optimal order quantity it must be true that $A(K) + f'(D) \geq 0$. This implies condition (67). ■

Note that when $c_R \equiv 0$ condition (67) simplifies to

$$K \leq (1 - \pi_1)\breve{S}_0 - p_{00}\breve{S}_0 N \left\{ d_1 \left[\widehat{V}(D) \right] \right\}. \tag{69}$$

The following properties of the optimal retailer's order quantity are useful in the analysis of the supplier 1 problem.

Lemma 7. *For optimization problem (60),* $\arg\max A(K)z + f(z)$ *is non-increasing in* K*.*

Proof. Observe that for $H(u; K) = -A(K)u + f(-u)$, $\frac{\partial^2 H}{\partial u \partial K} \geq 0$. Therefore, function H, which is defined on a lattice $[-D, 0] \times [0, +\infty]$ and which is quasi-supermodular in u, satisfies single-crossing property. Theorem 4 in Milgrom and Shannon (1994) implies that $u^* = \arg\max H(u; K)$ is non-decreasing in K. Therefore, $z^* = -u^*$ is non-increasing in K. ■

Supplier 1 problem is

$$\max_K (K - c_1) z^*(K), \tag{70}$$

where $z^*(K)$ is the solution of the retailer's problem.

We will find a solution of (70) numerically and also will consider a simplified problem with deterministic retail prices ($\sigma = 0$).

5.3 Special case: deterministic prices

Assume that retail price S is deterministic and increases only due to the drift term. Thus, for any time t, $S(t) = S_0 e^{\mu t}$ (for adjusted process $\breve{S}$, $\breve{S}(t) = \breve{S}_0 e^{rt}$). For simplicity, assume that $c_R \equiv 0$.

5.3.1 Equilibrium profits with deferment

The retailer's time 0 optimization problem (59) becomes

$$\max_{z\le D}\left\{\left[(1-\pi_1)\breve{S}_0-K\right]z+\right.$$
$$\left.+\,p_{00}\breve{S}_0(D-z)1_{\left\{c_2e^{-r\tau_1}\le p^2_{10|00}\breve{S}_0\right\}}1_{\left\{z\ge\frac{p^2_{00|00}D\breve{S}_0e^{r\tau_1}}{(1-\pi^2_{2|00})\breve{S}_0e^{r\tau_1}-c_2}\right\}}\right\}. \tag{71}$$

Lemma 8. *A solution to the optimization problem (71) for a given K is*

If $c_2e^{-r\tau_1}>p^2_{10|00}\breve{S}_0$, then

$$z^*(K)=D\,1_{\left\{K\le(1-\pi_1)\breve{S}_0\right\}}. \tag{72}$$

If $c_2e^{-r\tau_1}\le p^2_{10|00}\breve{S}_0$, then

$$z^*(K)=\begin{cases}0 & \text{if } K>(1-\pi_1)\breve{S}_0+p^1_{00}\left(p^2_{10|00}\breve{S}_0-c_2e^{-r\tau_1}\right)\\ \frac{p^2_{00|00}D\breve{S}_0e^{r\tau_1}}{(1-\pi^2_{2|00})\breve{S}_0e^{r\tau_1}-c_2} & \text{if } \begin{array}{l}p_{01}\breve{S}_0<K\le\\ \le(1-\pi_1)\breve{S}_0+p^1_{00}\left(p^2_{10|00}\breve{S}_0-c_2e^{-r\tau_1}\right)\end{array}\\ D & \text{if } K\le p_{01}\breve{S}_0\end{cases} \tag{73}$$

Proof. If $c_2e^{-r\tau_1}\le p^2_{10|00}\breve{S}_0$, the function that we are optimizing is piece-wise linear. Let

$$z_0=\frac{p^2_{00|00}D\breve{S}_0e^{r\tau_1}}{(1-\pi^2_{2|00})\breve{S}_0e^{r\tau_1}-c_2}.$$

To the left of z_0 the slope of the retailer's profit function is $(1-\pi_1)\breve{S}_0-K$. To the right of z_0 the slope of the retailer's profit function is $p_{01}\breve{S}_0-K$. If $p_{01}\breve{S}_0-K\ge 0$ then the retailer will order D. Otherwise, the retailer will order z_0 if

$$\left[(1-\pi_1)\breve{S}_0-K\right]z_0+p_{00}\breve{S}_0(D-z_0)\ge 0$$

or equivalently

$$K\le\left(p_{00}\frac{D}{z_0}+p_{01}\right)\breve{S}_0=(1-\pi_1)\breve{S}_0+p^1_{00}\left(p^2_{10|00}\breve{S}_0-c_2e^{-r\tau_1}\right). \qquad \blacksquare$$

The following proposition describes solution of the suppliers' problem (70).

Proposition 7. *Assume that* $(1-\pi_1)\breve{S}_0 \geq c_1$ *and* $(1-\pi^2_{2|00})\breve{S}_0 e^{r\tau_1} \geq c_2$*. Then*

If

$$c_2 e^{-r\tau_1} > p^2_{10|00}\breve{S}_0, \tag{H}$$

then $K^* = (1-\pi_1)\breve{S}_0$, $z^* = D$ *and equilibrium profits are*

$$\begin{aligned}
&\textit{Supplier 1 } (S_1^*) && \left[(1-\pi_1)\breve{S}_0 - c_1\right] D, && \text{(74a)}\\
&\textit{Supplier 2 } (S_2^*) && p^1_{10}\left(p^2_{10|10}\breve{S}_0 - c_2 e^{-r\tau_1}\right)^+ D, && \text{(74b)}\\
&\textit{Retailer } (R^*) && 0, && \text{(74c)}\\
&\textit{Channel } (Q^*) && \left[(1-\pi_1)\breve{S}_0 - c_1 + p^1_{10}\left(p^2_{10|10}\breve{S}_0 - c_2 e^{-r\tau_1}\right)^+\right] D. && \text{(74d)}
\end{aligned}$$

If

$$c_2 e^{-r\tau_1} \leq p^2_{10|00}\breve{S}_0 \qquad \textit{and} \qquad p_{00}\breve{S}_0 \geq (p_{01}\breve{S}_0 - c_1)\frac{p^2_{10|00}\breve{S}_0 - c_2 e^{-r\tau_1}}{(1-\pi^2_{2|00})\breve{S}_0 - c_2 e^{-r\tau_1}}, \tag{M}$$

then $K^* = (1-\pi_1)\breve{S}_0 + p^1_{00}\left(p^2_{10|00}\breve{S}_0 - c_2 e^{-r\tau_1}\right)$, $z^* = \frac{p^2_{00|00} D \breve{S}_0 e^{r\tau_1}}{(1-\pi^2_{2|00})\breve{S}_0 e^{r\tau_1} - c_2}$ *and equilibrium profits are*

$$\begin{aligned}
&\textit{Supplier 1 } (S_1^*) && \left[(1-\pi_1)\breve{S}_0 + p^1_{00}\left(p^2_{10|00}\breve{S}_0 - c_2 e^{-r\tau_1}\right) - c_1\right] \times && \text{(75a)}\\
& && \times \frac{p^2_{00|00} D \breve{S}_0 e^{r\tau_1}}{(1-\pi^2_{2|00})\breve{S}_0 e^{r\tau_1} - c_2}, && \\
&\textit{Supplier 2 } (S_2^*) && \left(p_{10}\breve{S}_0 - \widetilde{c}_2\right) D, && \text{(75b)}\\
&\textit{Retailer } (R^*) && 0, && \text{(75c)}\\
&\textit{Channel } (Q^*) && S_1^* + S_2^* + R^*. && \text{(75d)}
\end{aligned}$$

If

$$c_2 e^{-r\tau_1} \leq p^2_{10|00}\breve{S}_0 \qquad \textit{and} \qquad p_{00}\breve{S}_0 < (p_{01}\breve{S}_0 - c_1)\frac{p^2_{10|00}\breve{S}_0 - c_2 e^{-r\tau_1}}{(1-\pi^2_{2|00})\breve{S}_0 - c_2 e^{-r\tau_1}}, \tag{L}$$

then $K^* = p_{01}\breve{S}_0$, $z^* = D$ *and equilibrium profits are*

$$\begin{aligned}
&\textit{Supplier 1 } (S_1^*) && \left(p_{01}\breve{S}_0 - c_1\right) D, && \text{(76a)}\\
&\textit{Supplier 2 } (S_2^*) && \left(p_{10}\breve{S}_0 - \widetilde{c}_2\right) D, && \text{(76b)}\\
&\textit{Retailer } (R^*) && p_{00}\breve{S}_0 D, && \text{(76c)}\\
&\textit{Channel } (Q^*) && \left[(1-p_{11})\breve{S}_0 - c_1 - \widetilde{c}_2\right] D. && \text{(76d)}
\end{aligned}$$

Proof. Suppose $c_2 e^{-r\tau_1} > p^2_{10|00} S_0$. Using Lemma 8 and supplier 1 problem (70) we conclude that $K^* = (1-\pi_1)\breve{S}_0$ and $z^* = D$. Expressions for profits of the retailer and supplier 1 follow from equations (71) and (70) respectively. To compute profit of supplier 2, invoke Lemmas 3 and 4, noting that condition (H) corresponds to $c_2 > M_1$

If $c_2 e^{-r\tau_1} \leq p^2_{10|00} S_0$, then from Lemma 8 it follows that supplier 1 has to choose between two values of profits

$$(p_{01}\breve{S}_0 - c_1)D$$

and

$$\left[(1-\pi_1)\breve{S}_0 + p^1_{00}\left(p^2_{10|00}\breve{S}_0 - c_2 e^{-r\tau_1}\right) - c_1\right] \frac{p^2_{00|00} D \breve{S}_0 e^{r\tau_1}}{(1-\pi^2_{2|00})\breve{S}_0 e^{r\tau_1} - c_2}$$

Supplier 1 prefers the latter, which corresponds to

$$K^* = (1-\pi_1)\breve{S}_0 + p^1_{00}\left(p^2_{10|00}\breve{S}_0 - c_2 e^{-r\tau_1}\right)$$

and

$$z^* = \frac{p^2_{00|00} D \breve{S}_0 e^{r\tau_1}}{(1-\pi^2_{2|00})\breve{S}_0 e^{r\tau_1} - c_2},$$

if condition (M) is satisfied.

Under condition (M) the retailer's profit is 0 [based on equation (71)].

Using Lemmas 3 and 4 and noting that $p^2_{10|10}\breve{S}(\tau_1) > p_{10|00}\breve{S}(\tau_1) > c_2$, supplier 2 profit is

$$\begin{aligned} &p^1_{00} e^{-r\tau_1}\left\{\left[(1-\pi^2_{2|00})\breve{S}(\tau_1) - c_2\right] D - p^2_{00|00}\breve{S}(\tau_1) D\right\} + p^1_{10} e^{-r\tau_1}\left[p^2_{10|10}\breve{S}(\tau_1) - c_2\right] D = \\ &= \left(p_{10}\breve{S}_0 - \widetilde{c}_2\right) D \end{aligned}$$

Similarly, we derive expressions for profits of firms and the channel under condition (L). ■

The order of conditions (H),(M), and (L) corresponds to the decreasing default correlation. Perfectly positive default correlation ($p_{00} = 1-\pi$, $p_{10} = 0$) satisfies condition (H). Perfectly negative default correlation ($p_{00} = 0$, $p_{10} = 1-\pi_2$) satisfies condition (L).

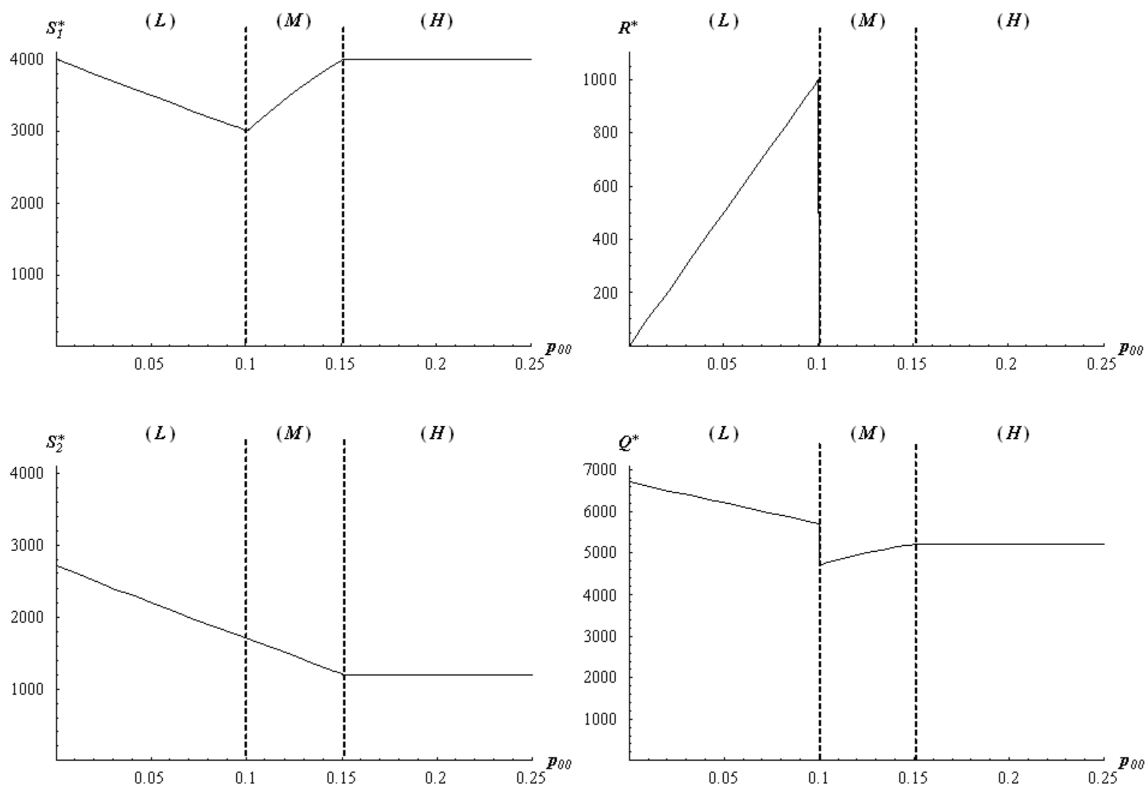
Figure 5 shows the dependence of the profits on the default correlation.

When the default correlation is high [condition (H)], supplier 1 charges the retailer the monopolist price and the retailer responds by ordering $z^* = D$ from supplier 1. Supplier 2 makes a profit only if supplier 1 defaults before time τ_1. The higher the correlation the lower the expected profit

Figure 5: Dependence of equilibrium profits of supplier 1, S_1^*, supplier 2, S_2^*, the retailer, R^*, and the channel, Q^*, on the default correlation, p_{00}.

Letters (L), (M), and (H) indicate regions described in Proposition 7.

Parameters: $c_R \equiv 0$, $\delta = 0$, $\sigma = 0$, $S_0 = 100$, $D = 100$, $p_{00}^1 = 0.5$, $1 - \pi^1 = 0.9$, $1 - \pi = 0.45$, $\xi = 1$, $p_{00} \in [0, 0.25]$.



of supplier 2. Profits of supplier 1 and the retailer do not depend on the default correlation in this case. Profit of the channel also decreases in the default correlation.

When the default correlation is medium [condition (M)], supplier 1 charges the retailer a price that is even higher than the monopolist price but the retailer now orders $z^* < D$ from supplier 1. The profit of supplier 1 depends on the default correlation in a non-linear fashion. As the correlation decreases, the expected profit of supplier 2 increases. The retailer's expected profit is always 0.

Finally, when the default correlation is low [condition (L)] the equilibrium solution is the same as the one in no-options case: supplier 1 charges a competitive price and the retailer places high orders with both suppliers. As the correlation decreases, profits of suppliers and the channel increase while the expected profit of the retailer decreases.

If there is no competition and all decisions are made at time 0, then according to section 4.1, as the default correlation increases, benefits of diversification decrease and profits of the suppliers, the retailer, and the channel decrease. However, as section 5.1 showed, when risky suppliers compete for the retailer's business, benefits of competition dominate benefits of diversification and as a result positive default correlation benefits the retailer while negative default correlation benefits the suppliers and the channel. However, if the discrepancy in production lead-times creates a deferment option for the suppliers and the retailer, then even in a competitive environment, positive default correlation could be bad for the retailer after all.

5.3.2 Deferment options and the default correlation

If $p_{01}S_0 \geq c_1$ and $p_{10}S_0 \geq \widetilde{c}_2$, then using expressions for firms' profits in Proposition 6 and Proposition 7, we can derive deferment option value for each of the firms. The results are presented in Table 2.

Table 2: Deferment option values when wholesale prices are endogenous and $\boldsymbol{\sigma = 0}$.

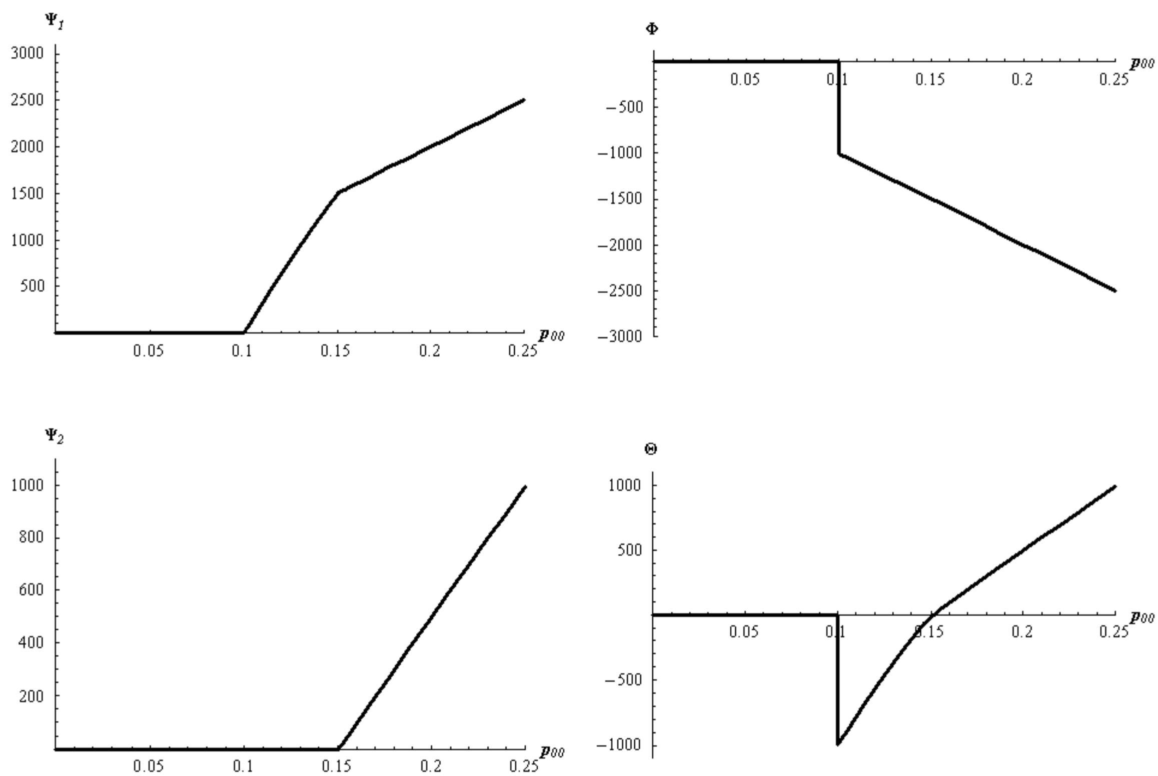
Firm	**(H)**	**(M)**	**(L)**
Supplier 1 (Ψ_1)	$p_{00}\breve{S}_0 D$	$\left[p_{10}\breve{S}_0 - c_1\right] D \times \left[\frac{c_2 - p_{10\|00}\breve{S}_0 e^{r\tau_1}}{(1-\pi^2_{2\|00})\breve{S}_0 e^{r\tau_1} - c_2}\right] + p_{00}\breve{S}_0 D$	0
Supplier 2 (Ψ_2)	$p^1_{00}\left(e^{-r\tau_1}c_2 - p^2_{10\|00}\breve{S}_0\right) D$	0	0
Retailer (Φ)	$-p_{00}\breve{S}_0 D$	$-p_{00}\breve{S}_0 D$	0
Channel (Θ)	$p^1_{00}\left(e^{-r\tau_1}c_2 - p^2_{10\|00}\breve{S}_0\right) D$	$\left[p_{10}\breve{S}_0 - c_1\right] D \frac{c_2 - p_{10\|00}\breve{S}_0 e^{r\tau_1}}{(1-\pi^2_{2\|00})\breve{S}_0 e^{r\tau_1} - c_2}$	0

As the default correlation increases, the value of the deferment option increases for the suppliers and decreases for the retailer (see Figure 6).

For low default correlation the value of the deferment option to the channel is 0. As correlation crosses boundary between (L) and (M) regions, the value of the option to the channel drops and

Figure 6: Dependence of the deferment option for supplier 1, Ψ_1, supplier 2, Ψ_2, the retailer, Φ, and the channel, Θ, on the default correlation, p_{00}.

Parameters: $c_R \equiv 0$, $\delta = 0$, $\sigma = 0$, $S_0 = 100$, $D = 100$, $p^1_{00} = 0.5$, $1 - \pi^1 = 0.9$, $1 - \pi = 0.45$, $\xi = 1$, $p_{00} \in [0, 0.25]$.



then increases as the default correlation continues to grow.

Table 3 presents the direction (positive or negative) in which firm's profits change after introduction of the deferment option.

Unlike the case of exogenous prices (Table 1), the retailer never benefits from the introduction of deferment option, because the introduction of the option changes the nature of competition between suppliers to the retailer's disadvantage. Furthermore, when prices are endogenous, both suppliers are better off with the option. Observe that when the default correlation is high the suppliers and the channel prefer model with the option. However, when correlation is at intermediate level both the retailer and the channel oppose introduction of the option. Therefore, none of the firms can be

Table 3: Effect of the deferment option introduction on firms' profits when wholesale prices are endogenous and $\sigma = 0$. "+" indicates that profit increases, "−" indicates that profit decreases, "=" indicates that profit does not change.

Firm \ Condition	**(H)**	**(M)**	**(L)**
Supplier 1 (Ψ_1)	+	+	=
Supplier 2 (Ψ_2)	+	=	=
Retailer (Φ)	−	−	=
Channel (Θ)	+	−	=

appointed to represent channel interests for all levels of the default correlation.

5.4 Numerical example: stochastic retail prices

When the volatility of retail prices $\sigma > 0$ finding a solution of supplier 1 problem (70) analytically is problematic, therefore, we resort to numerical study. To investigate effects of σ on equilibrium solution and interaction between σ and the default correlation, we will compute equilibrium prices, order quantities, profits, and option values for $\sigma = 0$ and $\sigma = 0.1$ (which is a moderate value of the volatility of retail prices) and contrast results. Figure 7 shows equilibrium prices of supplier 1 and equilibrium order quantities from the retailer to supplier 1. Note that the region (with respect to the default correlation) where supplier 1 charges higher than monopolist prices shift to the left if retail prices are uncertain. However, over this region the equilibrium wholesale prices of supplier 1 are lower when $\sigma > 0$. The uncertainty in retail prices has no effect on equilibrium prices and order quantities when correlation is either very high or very low. Figure 8 shows profits of the suppliers, the retailer, and the channel both with and without uncertainty in retail prices. Note that the retailer is worse off if there is an uncertainty about the retail prices.

Finally, Figure 9 presents deferment option values for the suppliers, the retailer and the channel. Note, again, that if the default correlation is very high or very low the effect from retail price uncertainty is small.

Figure 7: Dependence of equilibrium supplier 1 prices, K^*, and equilibrium order quantities to supplier 1, z^*, on the default correlation, p_{00}, and the volatility, σ.

Parameters: $c_R \equiv 0$, $\delta = 0$, $S_0 = 100$, $D = 100$, $p^1_{00} = 0.5$, $1 - \pi^1 = 0.9$, $1 - \pi = 0.45$, $\xi = 1$, $p_{00} \in [0, 0.25]$.

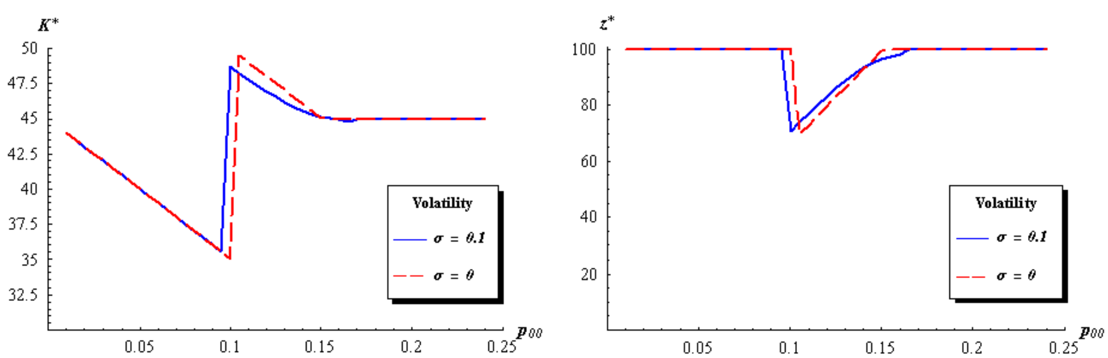


Figure 8: Dependence of equilibrium profits of supplier 1, S_1^*, supplier 2, S_2^*, the retailer, R^*, and the channel, Q^*, on the default correlation, p_{00}, and the volatility, σ.

Parameters: $c_R \equiv 0$, $\delta = 0$, $S_0 = 100$, $D = 100$, $p_{00}^1 = 0.5$, $1 - \pi^1 = 0.9$, $1 - \pi = 0.45$, $\xi = 1$, $p_{00} \in [0, 0.25]$.

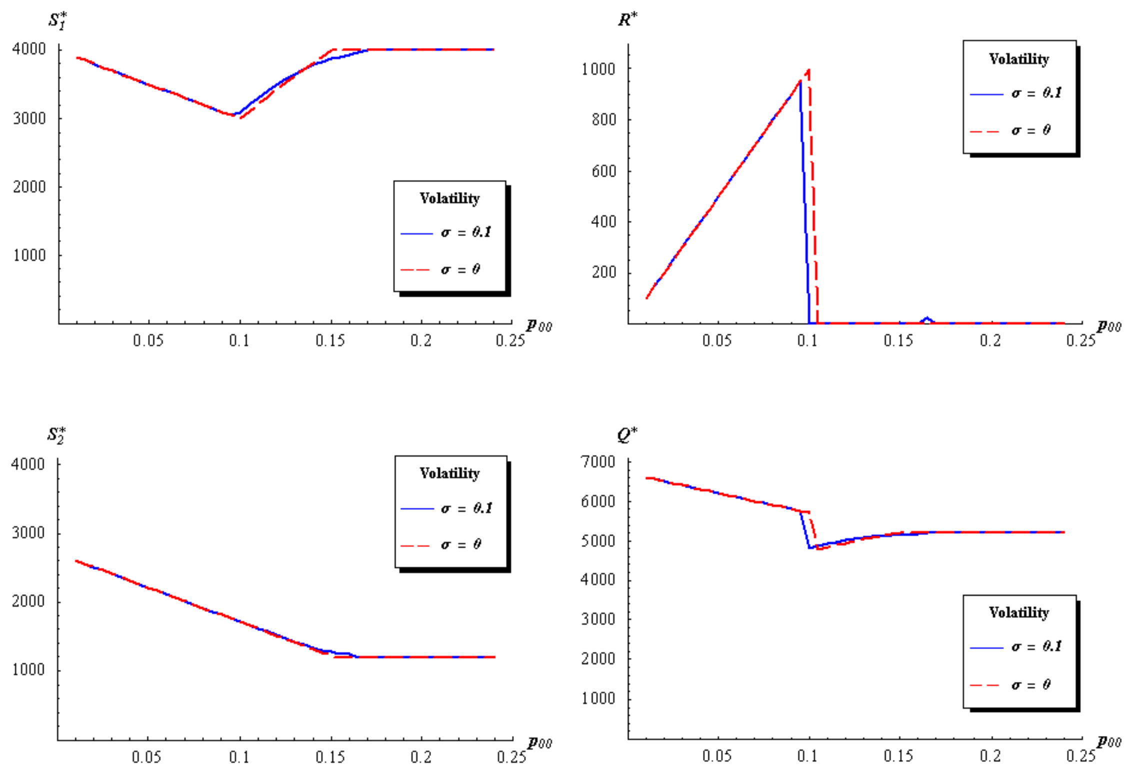
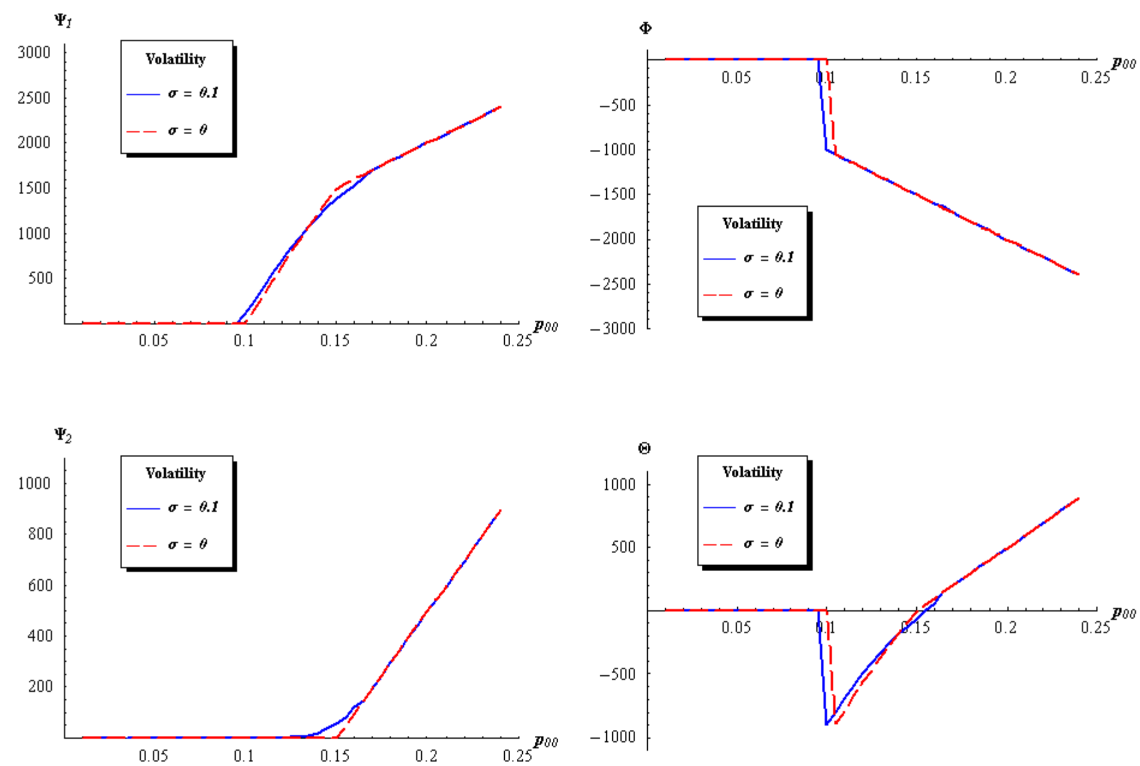


Figure 9: Dependence of option values for supplier 1, Ψ_1, supplier 2, Ψ_2, the retailer, Φ, and the channel, Θ, on the default correlation, p_{00}, and the volatility, σ.

Parameters: $c_R \equiv 0, \quad \delta = 0, \quad S_0 = 100, \quad D = 100, \quad p^1_{00} = 0.5, \quad 1 - \pi^1 = 0.9,$
$1 - \pi = 0.45, \; \xi = 1, \; p_{00} \in [0, 0.25].$



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