An Investment Criterion Incorporating Real Options

James Alleman, Hirofumi Suto, and Paul Rappoport
University of Colorado, Boulder, CO, USA and Columbia University, New York, NY, USA
NTT East, Tokyo, Japan
Temple University, Philadelphia, PA, USA
E-mails: ja703@columbia.edu, hs2012@columbia.edu, prapp4@comcast.net

Abstract: *This paper provides an investment decision-making criterion under uncertainty using real options methodology to evaluate if an investment should be made immediately, cautiously, deferred (wait-and-watch), or foregone. We develop a decision-making index d, which is equal to the expectation of net present value (NPV) normalized by its standard deviation. Under a lognormal assumption of the distribution of NPV discounted by risk-free rate, we find the "break-even point" at which the NPV equals the real option value (ROV): $d = D^* = 0.276$. Using the absolute value of D^*, one can make sophisticated decisions considering opportunity losses and costs of uncertainty. This new decision index, d, provides a criterion to make investment decisions under uncertainty. When making a decision, a manager only has to observe three parameters: expectation of future cash flow, its uncertainty as measured by its standard deviation, and the magnitude of investment. We discuss examples using this criterion and show its value. The criterion is particularly useful when NPV lies near zero or uncertainty is large.*

Keywords: *Real Options, Decision, Investment*

1. OPTION PRICE[1]

A financial option is the right to buy (a call) or sell (a put) a stock, but not the obligation, at a given price within a specific period of time. Option pricing theory determines the theoretical value of an option. There are several approaches to this problem, based on different assumptions concerning the market, the dynamics of stock price behavior, and individual preferences. The most important theories are based on the no arbitrage principle, which can be applied when the dynamics of the underlying stock take certain forms. The simplest of these theories is based on the multiplicative, binomial model of stock price fluctuations, which is often used for modeling stock behavior.

1.1 The Binomial Model[2]

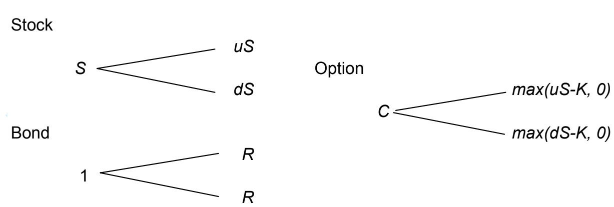
Assume a stock trades at a price S. Within one period, the price will be either uS or dS. Further assume we have a risk-free bond with return $R = 1+r_f$ per period. To avoid arbitrage opportunities, we must have

$$u > R > d \qquad (1.1)$$

If we have a stock option that allows us to buy the stock at the price K, called the exercise price or strike price one period later, the payoffs of the option are shown in equation (1.2) and Figure 1-1.

$$\begin{aligned} C_u &= \max(uS - K, 0) \\ C_d &= \max(dS - K, 0) \end{aligned} \qquad (1.2)$$

Figure 1.1 Three related lattices



To duplicate these two payoffs, we purchase x dollars worth of stocks and b dollars worth of the bond. One period later, this portfolio will be worth either $ux + Rb$ or $dx + Rb$, depending on the path. To match the option outcomes we therefore require

$$\begin{aligned} ux + Rb &= C_u \\ dx + Rb &= C_d \end{aligned} \qquad (1.3).$$

Solving this equation, we have

$$x = \frac{C_u - C_d}{u - d}$$

$$b = \frac{C_u - ux}{R} = \frac{uC_d - dC_u}{R(u - d)}$$

Combining these we find that the value of the portfolio is

$$\begin{aligned} x + b &= \frac{C_u - C_d}{u - d} + \frac{uC_d - dC_u}{R(u - d)} \\ &= \frac{1}{R}\left(\frac{R - d}{u - d} C_u + \frac{u - R}{u - d} C_d\right) \end{aligned}$$

Now we know that the value $x + b$ must be the value of the call option C because the payoffs of this portfolio are exactly the same as that of the stock option. (The one price principle) The portfolio made up of the stock and the bond that duplicates the payoff of the option is often referred to as a replicating portfolio.

$$C = \frac{1}{R}\left(\frac{R - d}{u - d} C_u + \frac{u - R}{u - d} C_d\right) \qquad (1.4)$$

[1] This section can be skipped by people who are familiar with option pricing theory.

[2] This interpretation is from [1]

There is a simplified way to view equation (1.4). Defining the quantity

$$q = \frac{R-d}{u-d} \qquad (1.5)$$

and from the relation $u > R > d$ assumed earlier, it follows that $0 < q < 1$. Hence q can be viewed as a probability. This q is referred to as the risk-neutral probability. Rewriting (1.4) yields (1.6):

Option pricing formula 1 *The value of a one-period call option on a stock governed by a binomial lattice process is*

$$C = \frac{1}{R}\left(qC_u + (1-q)C_d\right) \qquad (1.6).$$

Another way to obtain this risk-neutral probability is found by solving the equation,

$$S = \frac{1}{R}\left(quS + (1-q)dS\right) \cdot$$

As a suggestive notation, we write (1.6) as

$$C(T-1) = \frac{1}{R}\hat{E}[C(T)] \qquad (1.7)$$

Here $C(T)$ and $C(T$-1) are the option values at T and T-1, respectively, and $\hat{E}$ denotes expectation with respect to the risk-neutral probabilities.

We can extend this solution method to multi-period (T) options using the formula:

$$C = \frac{1}{R_T}\hat{E}[C(T)] \qquad (1.8)$$

where R_T is the risk-free return to the time to expiration.

The option price is calculated using payoffs for all cases, using the risk-neutral probability in the expectation function and discounting with the risk-free rate.

1.2 The Continuous Additive Model

Next, we set up the one period continuous additive model,

$$S_1 = RS_0 + Ru \qquad (1.9)$$

where S_t :stock price at time t

u : Normal distribution with mean 0, variance σ^2

R : Return of risk-free asset

This model always satisfies risk-neutral because

$$E[S_1] = E[RS_0 + Ru]$$

$$= E[RS_0] + E[Ru]$$

$$= RS_0$$

i.e. $S_0 = \frac{1}{R}E[S_1] \qquad (1.10)$

If we have a call option on this stock with exercise price = RK at time 1, payoff of option $C(1)$ is:

$$C(1) = [\max(S_1 - RK, 0)]$$

i.e. $C(1) = 0$ if $S_1 < RK$

$C(1) = S_1 - RK$ if $S_1 > RK \qquad (1.11)$

We calculate this option price using the general option pricing formula (1.8).

$$C = \frac{1}{R_T}\hat{E}[C(T)]$$

$$= \frac{1}{R}\hat{E}[C(1)]$$

$$= \frac{1}{R}\int_{-\infty}^{+\infty} C(1) f(S_1)\, dS_1$$

where $f(S_1)$ is probability density function of S_1, normally distributed.

$$= \frac{1}{R}\int_{RK}^{+\infty} (S_1 - RK) f(S_1)\, dS_1$$

substituting $S_1 = RS_0 + Ru$, $f(S_1) = (1/R) f(u)$, $dS_1 = Rdu$,

$$= \frac{1}{R}\int_{K-S_0}^{+\infty}\left[(RS_0 + Ru - RK)\frac{1}{R}f(u)\right]Rdu$$

$$= \int_{K-S_0}^{+\infty}[(S_0 + u - K) f(u)]du \qquad (1.12)$$

If we introduce $V = S_0 + u$, which is normally distributed with mean S_0, variance σ^2, (1.12) can be rewritten as a general option pricing formula for continuous outcome model.

Option pricing formula 2 *The value of a one-period call option on a stock governed by a continuous additive model is*

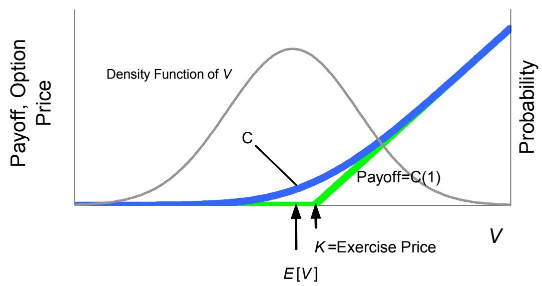
$$C = \int_K^{+\infty} (V - K) f(V)\, dV \qquad (1.13)$$

where V: the present value of the future random value discounted by risk-free rate

K is the present value of exercise price, discounted by the risk-free rate

We will use this option formula in section 3.

Figure 1.2 Payoff and Option Price



Note that when V takes a lognormal distribution, this formula is equivalent to Black-Scholes Call Option formula [2].

Following the discussion of the previous section, we extend this model to multi-period (T) options using the model:

$$S_t = RS_{t-1} + R^t u_t \qquad (1.14)$$

where u_i is the random variable normally distributed with mean 0, variance σ^2

$$\text{i.e. } S_T = R^T S_0 + R^T \sum_{i=1}^{T} u_i \qquad (1.15)$$

We can confirm risk neutral because:

$$E[S_T] = E[R^T S_0] + E\left[R^T \sum_{i=1}^{T} u_i\right] = R^T S_0$$

since $E[u_i] = 0$ for all i

$$\text{i.e. } S_0 = \frac{1}{R^T} E[S_T] = PV[E[S_T]] \cdot$$

And assuming u_i is not correlated with any other $u_{j \neq i}$, the variance of $PV[\ E[S_T]\ ]$ is:

$$\sigma_T^2 \equiv Var[PV[S_T]] = Var\left[\sum_{i=1}^{T} u_i\right] = T\sigma^2 \qquad (1.16)$$

We can write this as:

$$\sigma = \frac{\sigma_T}{\sqrt{T}} \qquad (1.17)$$

Then we can calculate one-term standard deviation from multi-term one. The standard deviation of the yearly return is called volatility.[3] When the return is defined as $PV[S_T / S_0]$, its volatility is given by (σ / S_0).

Although the option pricing theory has been developed in order to value financial options, it can be applied to the real asset or firm's project. The methodology is called "real options". In the next section, we show how it is applied.

2. Real Options

Real Options methodology is an approach used to evaluate alternative management strategies using traditional option-pricing theory applied to the real assets or projects. For example, when managers decide to assess a new project, they face several choices beyond simply accepting or rejecting the investment. Other choices include delaying the decision until the market is favorable, or deciding to start small and expanding later if the result seems to be superior. The traditional valuation method, DCF analysis, fails to account for these other choices. The list of these real options is shown in Table 2.1 [3].[4]

Table 2.1 : Description of Options

Option	Description
Defer	To wait to determine if a "good" state-of-nature obtains
Abandon	To obtain salvage value or opportunity cost of the asset
Shutdown & restart	To wait for a "good" state-of-nature and re-enter
Time-to-build	To delay or default on project - a compound option
Contract	To reduce operations if state-of-nature is worse than expected
Switch	To use alternative technologies depending on input prices
Expand	To expand if state-of-nature is better than expected
Growth	To take advantage of future, interrelated opportunities

Example: Value of the Option to Defer

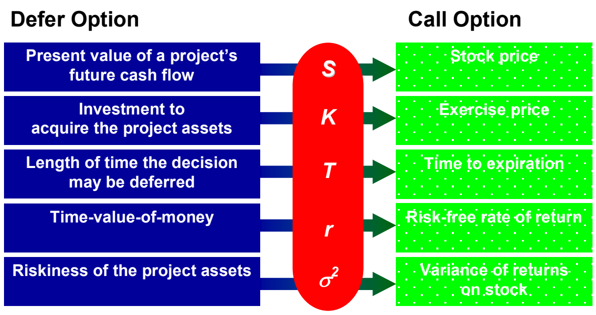
The simplest real option alternative is the deferral option which is based on the concept of the call option, as shown in Figure 2.1 [8].

Suppose you have a mining project, which is not profitable currently. A deferral option gives you the option to defer starting this project for one year to see if the price of gold rises high enough to make the investment worthwhile. We can interpret this right as a call option. The numerical example illustrates its value.

Table 2.2 displays the project's present value of future cash flow. V is assumed to be normally distributed with mean \$100 million (= S) and standard deviation \$30 million (= σ). The risk-free rate is 6% (R=1.06); the

[3] A precise definition of volatility is "the standard deviation of the return provided by the asset in one year when the return is expressed using continuous compounding." [13].

[4] Methods to valuate each kind of option are described in several books [4],[5],[6] and [7].



exercise price one year later is $110 million. With these assumptions, the project's present value is $103.8 or $110/1.06.

Table 2.2 A mining project

Defer Option	Variable	
Present value of operating future cash flow	S	$100 million
Investment in equipment	K	$103.8 million
Length of time the decision may be deferred	T	1 year
Risk-free rate	r_f	1.06
Riskiness	σ	$30 million

Conventional NPV is given by S-K = 100 – 103.8 = -3.8 million. This project would have been rejected under NPV criterion. However, applying call option formula in the pricing equation (1.13), the value of deferring the project one year is calculated as the *defer ROV* (Real Option Value):

$$ROV = C = \int_{K}^{+\infty} (V-K) f(V) dV$$

$$= \int_{K}^{+\infty} (V-K) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(V-S)^2}{\sigma^2}\right) dV$$

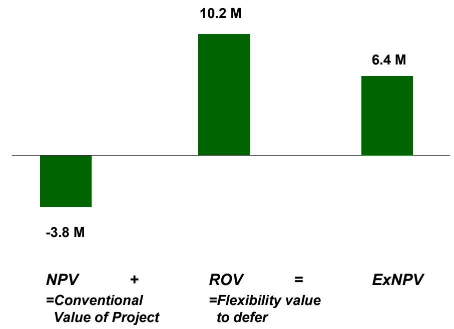
$$= \int_{103.8}^{+\infty} (V-103.8) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{(V-100)^2}{30^2}\right) dV$$

$$= 10.2 \qquad (2.1)$$

The flexibility that allows us to defer this project is valued at 10.2 million.

Adding NPV and ROV gives positive value 6.4 or –3.8 + 10.2. This is called Expanded NPV or *ExNPV*. *ExNPV* represents the value of this project including future flexibility [4]. Consequently, your optimum decision now is "defer", i.e. "wait and watch the gold market!"

Fig. 2.2 ExNPV (Expanded NPV)



3. THE NEW DECISION MAKING CRITERION

Decision under conditions of uncertainty should be made on the basis of the current state of information available to decision makers. If the expectation of the *NPV* were negative for the investment, the conventional approach would be to reject the investment. However, if one has the ability to delay this investment decision and wait for additional information, the option to invest later has value. This implies that the investment should not be undertaken at the present time. It leaves open the possibility of investing in future periods.

For the purpose of analyzing the relationship between *NPV* and the option value associated with the single investment, we assume that the random variable of interest is the present value of future cash flow V, which is assumed to be normally distributed $V \sim N(m', \sigma')$. The investment cost I is assumed to be a constant.

In the conventional method, *NPV* is expressed as:

$$\begin{aligned} NPV &= E\,[V - I] \\ &= E[V] - I \\ &= m' - I \qquad (3.1) \end{aligned}$$

We examine the two cases: $(a_1, a_2) \in A$ are defined as Act 1: (a_1) do not invest now when NPV < 0, and Act 2: (a_2) invest now when $NPV > 0$.

3.1 Case 1: Act 1 = do not invest as $NPV < 0$

Here, following Herath & Park [2], we introduce a loss function. When we do not invest, the cash flow is equal to 0. But imagine the situation that $V > I$, where the opportunity loss is recognized as V-I. Therefore the loss function of Act 1 is:

$$\begin{aligned} L(a_1, V) &= 0 && \text{if } V < I \\ &= V - I && \text{if } V > I \qquad (3.2) \end{aligned}$$

The expected opportunity loss can be calculated as:

$$E[L(a_1,V)] = \int_{-\infty}^{+\infty} L(a_1,V) f(V) dV$$

$$= \int_{I}^{+\infty} (V-I) f(V) dV \qquad (3.3)$$

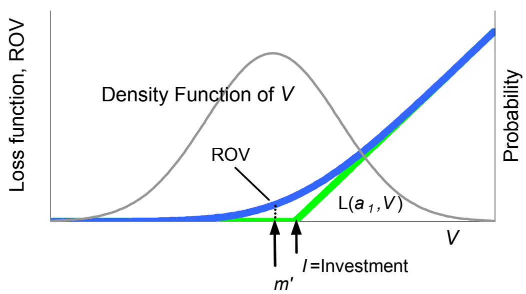
This function is the payoff of a call option, (1.11) using the pricing formula of a call option discussed in the previous section, (1.13).

Assuming we can defer this investment to obtain new information, we know this value is the same as the defer option for the investment. Moreover, the value is also equal to the expected value of perfect information (EVPI) for this investment opportunity [2].

$$ROV \text{ (Real Option Value)} = \int_{I}^{+\infty} (V-I) f(V) dV \qquad (3.4)$$

We can see the similar relationship in Figure 3.1 as we saw before in Figure 1.2.

Figure 3.1 Opportunity Loss function and ROV (NPV<0)



When the terminal distribution of V is normal, the real option value can be calculated using the unit normal linear loss integral:

$$L_N(D) = \int_{D}^{+\infty} (V-D) f_N(V) dV \qquad (3.5)$$

where $f_N(V)$ is the standard normal density function

$$ROV = \sigma' L_N(D) \qquad (3.6)^5$$

where $D = \frac{|m'-I|}{\sigma'}$

When the manager makes the decision to invest, her optimal decision is not to invest if $NPV = m'\text{-}I < 0$. Then she may compare the NPV and the defer option value. If she finds that the option value is larger than the absolute value of NPV (= | $m'\text{-}I$ |), she has the option to defer and watch for positive changes in the investment opportunity. If the option value is too small to compensate the NPV (< 0), she will abandon this investment proposal.

[5] This expression is only for the case NPV < 0 though general expression is possible.

Decision Criterion 1: (Case of NPV < 0)

$ROV > |NPV|$ → Wait and watch the opportunity carefully

$ROV < |NPV|$ → Do not invest

We can solve the equation,

$$ROV = |NPV| \qquad (3.7)$$

for $D = \frac{|m'-I|}{\sigma'}$.

From (3.1) and (3.6), (3.7) is expressed in

$$\sigma' L_N(D) = |m'-I| \qquad (3.8)$$

Divided by σ', we have: (because of $\sigma' > 0$)

$$L_N(D) = \frac{|m'-I|}{\sigma'}$$

i.e. $L_N(D) - D = 0 \qquad (3.9)$

Solving this equation, D^* is:

$$L_N(D) - D = 0$$

$$\int_{D}^{+\infty} (V-D) f_N(V) dV - D = 0$$

$$\int_{D}^{+\infty} V f_N(V) dV - D \int_{D}^{+\infty} f_N(V) dV - D = 0$$

$$\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}D^2\right) - D\Phi(-D) - D = 0$$

where $\Phi(a) = \int_{-\infty}^{a} f_N(x) dx$

$$\therefore \ D^* = 0.276 \qquad (3.10)$$

And also the left hand side of equation (3.9) is decreasing as D increases because:

$$\frac{d}{dD}(L_N(D) - D) < 0 \quad \text{for all } D > 0 \quad (3.11)$$

From (3.10) and (3.11), we can confirm that former Decision Criterion 1 can be written in;

Decision Criterion 1': (Case of NPV <0)

$D < D^*$ → Wait and watch the opportunity carefully

$D > D^*$ → Do not invest

3.2 Case 2: Act 2 = invest as $NPV > 0$

Similarly, in the opposite case we can discuss loss function as [2]:

$$L(a_2,V) = I - V \quad \text{if } V < I \qquad (3.12)$$

$$= 0 \quad \text{if } V > I$$

The expectation of loss is given by:

$$E[L(a_2,V)] = \int_{-\infty}^{+\infty} L(a_2,V) f(V) dV$$

$$= \int_{-\infty}^{I} (I - V) f(V) dV \qquad (3.13)$$

These equations are similar to the payoff and price of the put option (Figure 3.2). Assuming we can defer this investment, the option value is the same as the expectation of the loss.

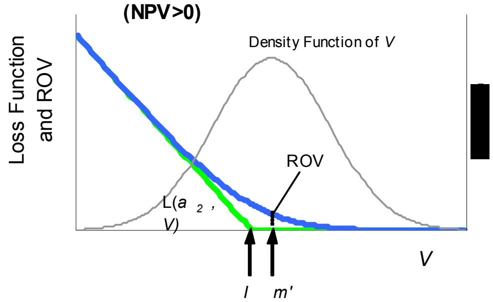
$$ROV = \int_{-\infty}^{I} (I - V) f(V) dV \qquad (3.14)$$

By symmetry, (3.14) can be written as:

$$ROV = \sigma' L_N(D) \qquad (3.15)$$

where $D = \dfrac{|m' - I|}{\sigma'}$

Figure 3.2 Loss Function and ROV (NPV>0)



In this case, optimal decision is invest as $NPV = m' - I > 0$. Comparing the NPV with the value of this option, which is same as the cost of uncertainty, we arrive at a similar criterion:

Decision Criterion 2: (Case of $NPV > 0$)

$ROV > |NPV|$ → Invest carefully

$ROV < |NPV|$ → Invest

Rewriting this as:

Decision Criterion 2': (Case of $NPV > 0$)

$D < D^*$ → Invest carefully

$D > D^*$ → Invest

Here we introduce decision-making index $d \equiv (m' - I)/\sigma'$, which is given from eliminating the absolute value sign from D. We can combine the two decision criteria as:

Combined Decision Criterion

Invest	if	$D^* < d$
Invest carefully	if	$0 < d < D^*$
Wait and watch	if	$-D^* < d < 0$
Do not invest	if	$d < -D^*$

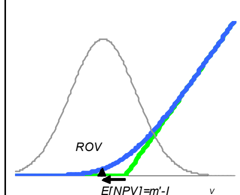
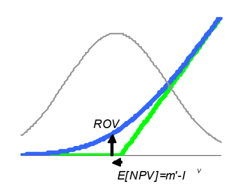
where $d = \dfrac{m' - I}{\sigma'}$, $\quad D^* = 0.276$

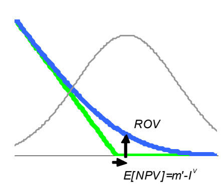
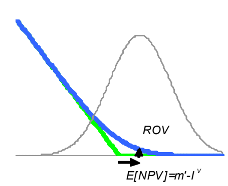
Consequently, we know that only observing three parameters, m', σ' and I gives us sufficient information to make more sophisticated decisions under uncertainty, expressing them in form of the new decision-making index, d.

3.3 What d and D^* mean?

What do d and D^* mean? First, d can be seen as NPV divided by its standard deviation. In other words, d is the ratio of NPV to its uncertainty. Because d does not depend on the size of the project, it can be called "uncertainty-adjusted NPV" or "risk-normalized NPV." We can easily compare several risky projects of which the sizes are different. Next, when $d = -D^*$, option value to defer is equal to expected loss of

Figure 3.3 Summary of the Criterion

NPV	NPV < 0	
ROV	|NPV| > ROV	|NPV| < ROV
d	d < -D*	-D* < d < 0
Decision	not Invest	wait and watch
		

NPV	NPV > 0	
ROV	|NPV| < ROV	|NPV| > ROV
d	0 < d < D*	D* < d
Decision	Invest carefully	Invest
		

NPV, namely, D^* is the break-even point of expectation of NPV and its option value, or the point where $ExNPV = 0$.

Let's calculate the probability that the payoff of this defer option is positive if $d = -D^*$ at time 0. The probability is calculated as follows,

$$
\begin{aligned}
P[V-I>0] &= P\left[\frac{V-I}{\sigma'}>0\right] \\
&= P\left[\frac{V-m'+m'-I}{\sigma'}>0\right] \\
&= P\left[\frac{V-m'}{\sigma'}>-\frac{m'-I}{\sigma'}\right] \\
&= P\left[N>-d\right] \\
&= P\left[N>D^*\right] \\
&= 0.39
\end{aligned}
$$

where N is standard normal distribution.

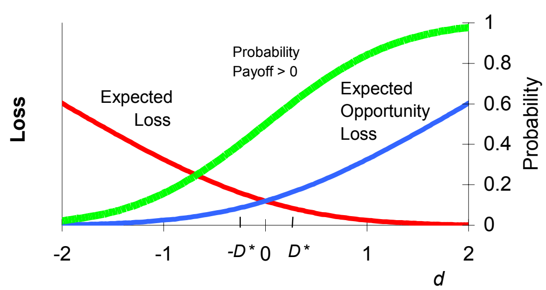
Because V is normally distributed with mean m' and standard deviation σ', $(V-m')/\sigma'$ is normally distributed mean 0, standard deviation 1.

Therefore, the probability that the payoff of the defer option is positive is 39%. Does it seem to be a high probability to abandon this option? Yes, it does! The criterion $d < -D^*$ means, "Do not invest now" but does not mean "Abandon the defer option." The defer option itself has value though the expectation of NPV is deeply negative. If holding the option does not require any cost, we do not have to throw it away! Just wait and watch what happens in the next period.

On the other hand, if $d = D^*$, the probability that the project will be out of the money is also 39%, by symmetry. When the manager makes her decision to invest as $d = D^*$, there is still 39% probability of losing money. If the manager wanted a positive NPV with probability 90%, d should be higher than 1.28. It might be the case that the manager could set a higher d for the decision criterion if she would not care about opportunity losses. The tradeoffs between opportunity loss and cost of uncertainty are shown in Figure 3.4

In the next section, we will show an example of this criterion.

Figure 3.4 Tradeoffs of Losses



4. SAMPLE CASE

Six Independent Projects

If we have projects shown in Table 4.1, how can we make decision using our new criterion? We have assets that have current value S, time to expiration T, exercise price K at time T, volatility σ, and risk-free rate r_f. To calculate d, we have to solve m', I and σ'. Assuming the value of S at time T is normally distributed with mean $S(1+r_f)^T$, $m' = S$ because m' is expressed in present value. After setting $I = PV(K) = K/(1+r_f)^T$, $\sigma' = S\sigma\sqrt{T}$, we can calculate $d = (m'-I)/\sigma'$. Therefore, we find d for each project and make decision to invest as shown in Table 4.1.

Luehrman [8], [9] defined "option space" having two axes value-to-cost ($S/PV(K)$) and volatility ($\sigma\sqrt{T}$) and showed that decision criterion depends on the region in the option space. Using his example, we find similar results with our criterion. Our new decision criterion can is an integrated, simplified version of Luhrman's method.

Furthermore, project "E" in Table 4.1 is what we illustrated before in section 2, the Mining Project, and the new criterion gives the same decision, "wait and watch"!

Variable	A	B	C	D	E	F
S	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00
K_T	\$90.00	\$90.00	\$110.00	\$110.00	\$110.00	\$110.00
T	0.0	2.0	0.0	0.5	1.0	2.0
σ	30%	30%	30%	20%	30%	40%
rf	6%	6%	6%	6%	6%	6%
	\$0.00	\$42.43	\$0.00	\$14.14	\$30.00	\$56.57
S-PV(K)	\$10.00	\$19.90	-\$10.00	-\$6.84	-\$3.77	\$2.10
d	+infinite	0.469	-infinite	-0.484	-0.126	0.037
Exercise decision	invest	invest	do not invest	do not invest	wait and watch	invest carefully

S	Current asset value
K_T	Exercise Price (at time = T)
T	Time to expiration (year)
σ	Standard deviation of return (per year)
r_f	Risk-free rate of return (% per year)
σ t.5	Standard deviation of V
S-PV(K)	Conventional NPV (=m'-I)
d	uncertainty adjusted NPV

5. CONCLUSION

Applying real option valuation methodology, we have shown that the new decision index d – the uncertainty adjusted NPV – and $D^* = 0.276$ – the break-even point of

NPV and ROV (real option value) – gives a clear solution to make a decision under uncertainty. When making decision, managers have to observe only three parameters: expectation of future cash flow, its uncertainty, and the amount of investment to acquire the project. And also we have discussed some examples using our new criterion and shown its usefulness.

REFERENCES

[1] David G. Luenberger, *Investment Science*, Oxford University Press, 1998

[2] Hemantha Herath and Chan Park, Real Options Valuation and Its Relationship to Bayesian Decision-Making Methods, *The Engineering Economist* 2001, Vol. 46, No.1

[3] James Alleman and Paul Rappoport, Modeling Regulatory Distortions with Real Options, *The Engineering Economist*, volume 47, number 4, 2002, pp. 390-417.

[4] Lenos Trigeorgis, *Real Options: Managerial flexibility and strategy in resource allocation*, MIT Press, 1996

[5] Martha Amram, Nalin Kulatilaka, *Real Options*, HBS Press, 1999

[6] Tom Copeland, Vladimir Antikarov, *Real Options: A Practitioner's Guide*, Texere LLC, 2001

[7] Tom Copeland, Tim Koller, and Jack Murrin, *Valuation: Measuring and Managing the Value of Companies*, McKinsey & Company Inc

[8] Timothy Luehrman, Investment Opportunities as Real Options: *Harvard Business Review*, 51-67, July-August 1998

[9] Timothy Luehrman, Strategy as a Portfolio of Real Options: *Harvard Business Review*, 89-99, September-October 1998

[10] James Smith and Robert Nau, Valuing Risky Projects: Option Pricing Theory and Decision Analysis, *Management Science* Vol. 41, No.5, May 1995

[11] James Alleman and Eli Noam (eds.), *The New Investment Theory of Real Options and Its Implication for Telecommunications Economics*, Kluwer Academic, 1999

[12] Avinash Dixit and Robert Pindyck, *Investment Under Uncertainty*, Princeton University Press, 1994

[13] John Hull, *Options, Futures and other Derivatives*, Prentice-Hall, 2000

[14] Robert Schlaifer, *Introduction to Statistics for Business Decisions*, McGraw Hill, 1961

[15] Peter Boer, *The Real Options Solution: Finding Total Value in a High-risk World*, John Wiley & Sons, Inc. 2002

[16] Aswath Damodaran, *Dark Side of Valuation*, Prentice-Hall, 2001

[17] Michael Mauboussin, *Get Real*, Credit Suisse Equity Research, June 1999

[18] Harriet Nembhard, Leyuan Shi, Chan Park, Real Option Models For Managing Manufacturing System Changes in The New Economy, *The Engineering Economist* 2000, Vol.45, No.3

[19] Daisuke Yamamoto, *Introduction to Real Options*, Toyo-Keizai, 2001 (in Japanese)