Real Options Effects on Employment: Does Exchange Rate Uncertainty Matter for Aggregation?

by Ansgar Belke und Matthias Göcke *

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Abstract: In a baseline micro model a band of inaction due to hiring- and firing-costs is widened by option value effects of exchange rate uncertainty. Based on this micro foundation an aggregation approach is presented. Under uncertainty, intervals of weak response to exchange rate reversals are introduced on the macro-level. 'Spurts' in new employment or firing may occur after an initially weak response. Since these mechanisms may apply to other "investment" cases where the aggregation of microeconomic real options effects under uncertainty are relevant, they may even be of a more general interest.

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1. Introduction

The usual presumption of EMU proponents has been that lowering exchange rate volatility promotes foreign trade. However, macroeconomic empirical evidence is far from conclusive in that respect (e.g. Côté, 1994). The possibility that exchange rate uncertainty could have an impact on labour market performance or at least *on the relation* between employment and its determinants has not been considered so far (Belke, Gros, 2001). The *lack of a theoretical and empirical literature* on the effect of exchange rate variability on employment is striking in view of the high policy relevance of the question (Dornbusch, 1987) and in view of the fact that there is considerable literature on the link between investment and uncertainty (Cushman, 1995, Darby et al., 1998,). Given the *high hiring and firing costs* affecting European labour markets (Bentolila, Bertola, 1990, Saint-Paul, 1996) one would assume that the same arguments that underpin the presumed effect of volatility on investment should also apply to exchange rate volatility and employment. But given the weak results concerning the link between exchange rate volatility and trade, the prior of many economists seemed to be that exchange rate variability is not important and that there should be no link between exchange rate variability and (un)employment.

Côté (1994) and de Grauwe and Skudelny (1997) show that the empirical puzzle concerning the impact of exchange rate uncertainty and foreign trade might have been due to the *methods* chosen so far. In the light of their arguments (but relying on a totally different theoretical approach) this paper argues that – *on a micro level* – the absence of evidence of a strong impact of exchange rate variability on the volume of trade is due to neglecting a "band of inaction" of export activity based on sunk costs. Its impacts are amplified by volatility-induced uncertainty. This export inactivity band spills over to labour markets leading to stronger persistence in (un)employment.[1] In order to solve the empirical puzzle, the micro approach has to be transferred to the *macro-level*.

A macro interpretation of micro hysteresis and of uncertainty impacts on 'investment' decisions cannot be performed in a straightforward way, since different firms have different 'investment' thresholds. For this reason, we pay special attention to the *problem of aggregation*.[2] The examination of the persistent effects of only temporary exchange rate shocks on employment behaviour of single firms and on overall employment is conducted based on an aggregation approach, developed by Krasnosel'skii, Pokrovskii (1989) and Mayergoyz (1986) and introduced to economics by Amable et al. (1991, 1995) and Cross (1994). This approach stresses the differences between hysteretic effects on the micro and on the macro level and the consequences of aggregation (*under certainty*): while at the micro level certain threshold values of the input variable have to be passed in order to produce branch-to-branch-transitions (i.e. shifts in the input-output relation), at the macro level small changes in the input variable can yield long-lasting effects.[3] What is up to now not well-known in the literature is how the introduction of *un*certainty influences the transition from micro to macro behaviour. In this contribution we show that under uncertainty areas of non-reaction – corresponding to mechanical play the so-called 'play'-area – have to be considered even at the macro level. Thus, similarities of macro relations ['play'] to micro behaviour [band of inaction] are enhanced by uncertainty.[4] The first result of the paper is that persistent aggregate (employment) effects do *not* necessarily result from small changes in the forcing

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1 For a theoretical treatment see Belke, Göcke (1999).

2 For aggregation problems in the presence of underlying non-linear micro relations see van Garderen, Lee, Pesaran (1997). Generally, the aggregation is regarded as especially complicated in this case. Since we impose very stringent restrictions on the shape of the non-linearity, we are able to derive a definite macro behaviour.

3 In this context, the terms '*input*' and '*output*' are used in a mathematical or physical sense ('input-output transducer'). The *macro* form is called '*strong*' hysteresis (Amable et al., 1991, 1994), Cross, 1994). Another recent strand of literature deals with models which are more general with respect to the non-linearities at the micro level than the one presented here. See e.g. Caballero, Engel and Haltiwanger (1997). These models explain aggregate employment dynamics, but dispense with an explicit derivation of an aggregation mechanism over heterogeneous firms.

4 For an overview of the variety of uses of the term 'hysteresis' in economics for different path-dependent phenomena ranging from 'band of inaction', 'play' to 'strong hysteresis' especially compared to the approximation of hysteresis via unit (zero) roots in difference (differential) equations see Amable et a. (1994) and Göcke (forthcoming).

(labour market) variables, as far as the changes occur inside the play-area. However, as far as changes go beyond the play-area, suddenly strong reactions (and persistence effects) will occur. With this, we theoretically capture the 'spurts' commonly observed in aggregate hiring / firing or investment decisions explicitly based on an aggregation approach for heterogeneous firms.[5]

The outline of the paper is as follows. Sections 2 to 4 give a repetitive introduction to aspects of hysteresis well known in the literature: In section 2 the baseline micro model of employment hysteresis under certainty is described. Section 3 deals with the aggregation approach under certainty. Modifications on the micro level necessitated by uncertainty are focused on in section 4. Section 5 "adds the beef" to our paper: As the main contribution a method to solve the problem of aggregation from micro to total economy employment under uncertainty is presented. Section 6 concludes.

2. The micro model under certainty

We consider an extremely simple model of an exporting firm which has been active or passive in a foreign market in the past. Depending on its past state of activity this firm might either retain activity or exit (if it has been active in the past) respectively remain passive or enter the foreign market (if it has been passive). If a previously inactive firm enters the market, it has to bear entry costs, i.e. costs of hiring new labour. These expenditures cannot be regained and are therefore ex-post treated as sunk costs. Thus entering (leaving) can be compared with an investment (disinvestment) project.

The firm j is assumed to be a price taker in the foreign and in the currency market. It produces one unit[6] of a final product using a_j units of labour, with the wage w being the price of labour, i.e. variable costs are $w \cdot a_j$. Selling on the foreign market, the firm receives the price p*. The

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5 For an (empirical) macro analysis of 'spurts' in investment implicitly based on *micro* (i.e. threshold) models see Darby et al. (1998), pp. 24 ff.

6 If an institutional firm is imagined to be divided into single production units, every unit is represented here individually. By this form of (fictitious) dis-aggregation a totally new erection as well as an enlargement of employment by an institutional firm can be included.

gross profit (in domestic currency) on sales in the foreign market, without consideration of hiring and firing costs, is:[7]

(1) $R_{j,a,t} = p^* \cdot e_t - w_t \cdot a_j$.

with:
- t : time index
- j : index for potential domestic exporters
- $R_{j,a,t}$: gross profit of firm j in period t (if active)
- e_t : exchange rate (home currency price of foreign exchange)
- w_t : wage rate
- a_j : input coefficient of labour of firm j
- p^* : price of the domestic export good (in foreign currency).

We feel justified to ascribe the revenue volatility solely to exchange rate volatility, since it is well-known that short-term volatility of exchange rates exceeds the variability of prices, wages and productivity by far.[8] We assume $w_t = w$ and a_j to be constant and normalise p* to unity. As a consequence, the exporters are forced to bear unit revenue changes (in their own currency) proportionally to the exchange rate changes. A weaker form of the results of our model stays valid, if p* changes under-proportionally compared with the exchange rate. The same logic can be applied to import-competing firms, whose revenues are influenced by the exchange rate as well. Using this simplification, the gross profit of firm j follows as:

(2) $R_{j,a,t} = e_t - w \cdot a_j$ (if active) , otherwise $R_{j,p,t} = 0$ (if passive) .

It is assumed that the hiring costs H_j (including training costs) must be spent at the moment the entry is executed, and the firm has to pay firing costs F_j at the time it leaves the market (e.g. severance pay). If it later decides to re-enter the market, the entire hiring costs must be repaid. If the firm is inactive for only one period, the staff must be completely re-set up and the hiring costs must be paid anew. Since switching the state of activity leads to a complete depreciation of hiring respectively firing costs, H_j and F_j have to be regarded as sunk costs ex post. The decision whether the firm should sell abroad or not, is reached by a comparison of the expected present values of the returns with or without being active in the decision period t.

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7 For a related model see Baldwin, Krugman (1989), p. 638 and Göcke (1994). Using an index j indicates heterogeneity between firms.

8 See e.g. Darby et al. (1998), pp. 1 f., Krugman (1989), p. 64.

A firm j which has been active in the preceding period and will continue its activity will gain the period t revenue $R_{j,a,t}$. The discount factor is defined as $\delta = 1/(1+i)$ with interest rate i. Since it expects the same revenue for the whole *infinite future* the present value of annuity due of continuing activity is $(e - w \cdot a_j)/(1-\delta)$, which has to be compared to the present value of an instantaneous exit, i.e. firing costs $(-F_j)$. Equating both leads to an exit trigger exchange rate value β_j for switching from employment to inactivity:

(3) $\beta_j = e^c_{j,exit} = w \cdot a_j - (1-\delta) \cdot F_j$ with: exit if $e_t < \beta_j$.

Therefore, the unit revenue e has to cover at least the wage costs $w \cdot a_j$ less the interest costs of exit. Hence, due to the sunk firing costs the price floor is below variable costs.

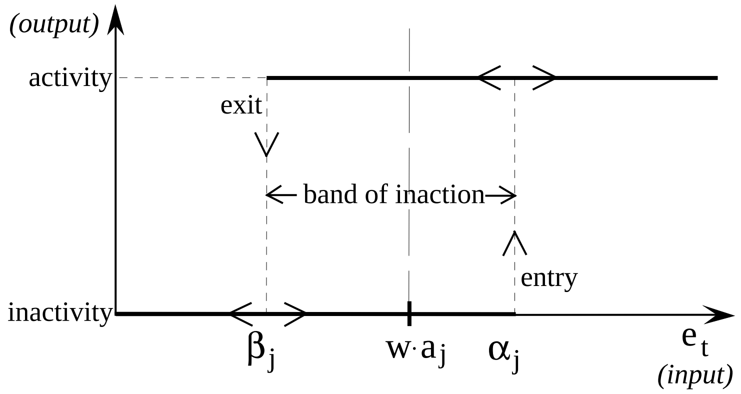
A previously non-active firm earns neither current nor future profits if it remains passive. If it enters the market and hires new employees it has to pay extra hiring costs H_j to be able to earn current and future profits: $-H_j + (e - w \cdot a_j)/(1-\delta)$. With this, the entry trigger α_j is:

(4) $\alpha_j = e^c_{j,entry} = w \cdot a_j + (1-\delta) \cdot H_j$ with: entry if $e_t > \alpha_j$ and $\alpha_j > \beta_j$.

Due to the sunk hiring costs, the necessary unit revenue is larger than the variable costs. Combining both trigger values, a 'band of inaction' results (Fig. 1). The latter implies that the current realisation of the exchange rate is not sufficient to determine the current state of the firm's activity. Depending on the past, the relationship between *input variable (exchange rate)* and *output variable (employment)* is represented by different curves.[9] In order to select one of the multiple equilibria, the history of activity has to be regarded. In the case of micro hysteresis nonlinearity has a very simple form: there exist only two 'branches' with a discontinuous branch-to-branch-transition when certain threshold-values are passed.

[9] Remember: the terms *'input'* and *'output'* are used in a mathematical or physical sense.

Fig. 1: Micro hysteresis for a single-unit firm j under certainty



3. Macro level: an aggregation approach

In this section we describe a procedure which allows an aggregation of heterogeneous single firms with an employment behaviour characterised by a micro hysteresis loop, resulting in a continuous macro loop of overall employment (see Amable et al., 1991, 1995, Cross, 1994, and Göcke, 1994). Every potentially active firm is characterised by its α_j/β_j-set. Since $\alpha_j \geq \beta_j$, all α/β-points are located in the triangle area T above the 45°-line. The aggregation is done without any serious restriction of heterogeneity in the cost structure (concerning H_j, F_j and a_j) between the firms. Points on the 45°-line describe non-hysteretic employers (H_j, $F_j = 0 \Rightarrow \alpha = \beta$), the distance from the origin given by $w \cdot a_j$. For points above the line there is micro hysteresis in employment, the distance from the ($\alpha = \beta$)-line measured in north west direction determined by H_j and F_j.

In order to prevent extensive assumptions concerning the past exchange rate development, the exchange rate $e = 0$ is assumed as an initial level. Therefore, no firm is active initially. Starting from $e = 0$ the exchange rate rises, resulting in hiring by firms with the lowest costs (and the lowest entry rate α_j).[10] Hence, overall employment increases, as traced in Fig. 2 (a).[11] The triangle T is divided into two shares: S_t^+ (including all active firms) and S_t^- (including all inactive firms). For increasing exchange rates, the share S_t^+ expands while S_t^- diminishes. The

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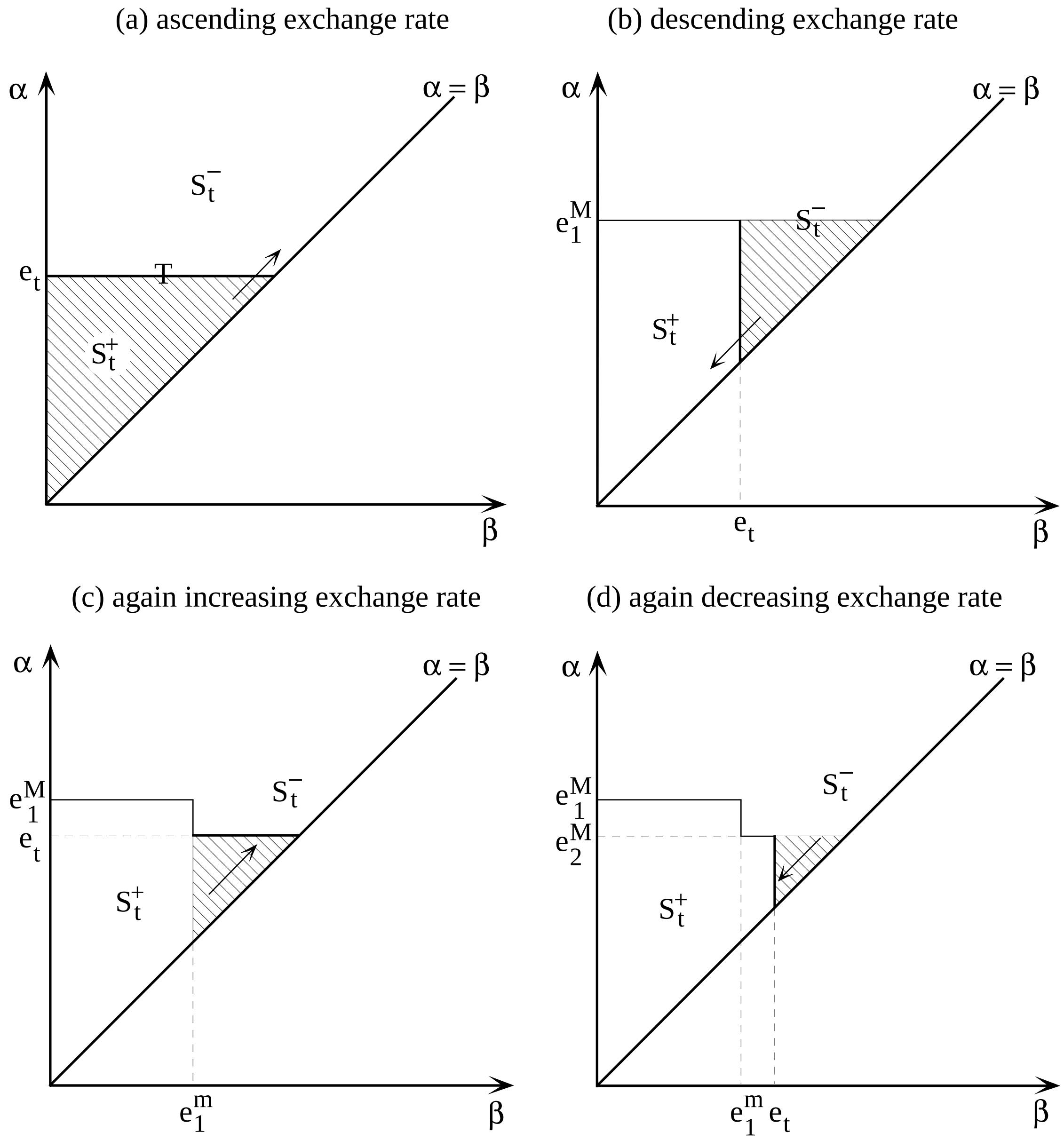
[10] The exchange rate $e = 0$ has not to be understood in a strict way: The exchange rates could also be stated in a standardised form with $e = 0$ as the rate that induces an exit of all firms. See Amable et al. (1991), p. 10.

[11] The increase in overall employment takes place on the lowest branch (path OAB in Fig. 4).

firms with a changing state of activity are described by the α/β-points in the hatched area. For rising rates, the S_t^+-expansion is indicated by an upward shift of the horizontal borderline.

Fig. 2: Active firms under a volatile exchange rate



Source: Following Amable et al. (1991), Figure 3 and 4.

In Fig. 2 (b) a subsequent decrease in the exchange rate is traced: e_t falls from the highest value, the (local) maximum e_1^M. Therefore, S_t^+ decreases, since firms, which recently hired new employees, now leave the market when the exchange rate falls below their exit rate β_j. For decreasing exchange rates, the activity changes (hatched area) are illustrated by a left vertical shift of the S_t^--S_t^+-borderline. (The corresponding macro reaction is path BC in Fig. 4.)

If the exchange rate rises again, after reaching the local minimum e_1^m (the lowest input level apart from the initial rate $e_0^m = 0$), S_t^+ expands again. This is depicted in Fig. 2 (c) once again by an upward shift of the right-horizontal part of the borderline.[12] The result of the subsequent shifts is a "staircase-shape" of the border between the two parts of T. If the recently reached (local) maximum is lower than the highest maximum e_1^M, a staircase step in the borderline remains characterised by the coordinates ($\alpha = e_1^M / \beta = e_1^m$). If the exchange rate would have continued to increase and would have passed the original maximum, the α-coordinate of the "e_1^M-step" would have been "erased" and would have been replaced by the "new" e_1^M.[13] However, if (as traced in Fig. 2 (c)) the new local maximum is lower than the "old" e_1^M, the maximum e_1^M remains and the new local maximum becomes the second highest, and is consequently labelled e_2^M.

Fig. 2 (d) illustrates a later decreasing exchange rate. The borderline is changed by a shift to the left of the lower vertical part (corresponding to DE in Fig. 4). If e_t does not fall below e_1^m with the new local minimum (disregarding the initial level) the second lowest minimum is given, labelled e_2^m. If the input were to fall under the "old" e_1^m, the β-coordinate of the corresponding staircase-step would be eliminated and the new local minimum would be (the "new") e_1^m.[14] If subsequent local maxima and minima are not as "extreme" as the preceding extrema, a new corner in the staircase border is created. On the other hand, local maxima which are higher than preceding maxima will erase the α-coordinate of the corresponding corners; subsequent local minima erase the β-coordinate of corners corresponding to higher preceding minima (Amable et al., 1991, pp. 11 ff.). For a given exchange rate path, the remaining maxima are ordered by descending size and the remaining minima by ascending size; a sequence of local maxima e_i^M and local minima e_i^m results (i=1,..., p). The α/β-coordinates of the outward corners of the staircase S_t^+-S_t^--borderline are given by the pairs ($\alpha = e_i^M$ and $\beta = e_i^m$). An illustration of the staircase borderline is drawn in Fig. 3 (a) for rising and in Fig. 3 (b) for falling exchange rates.

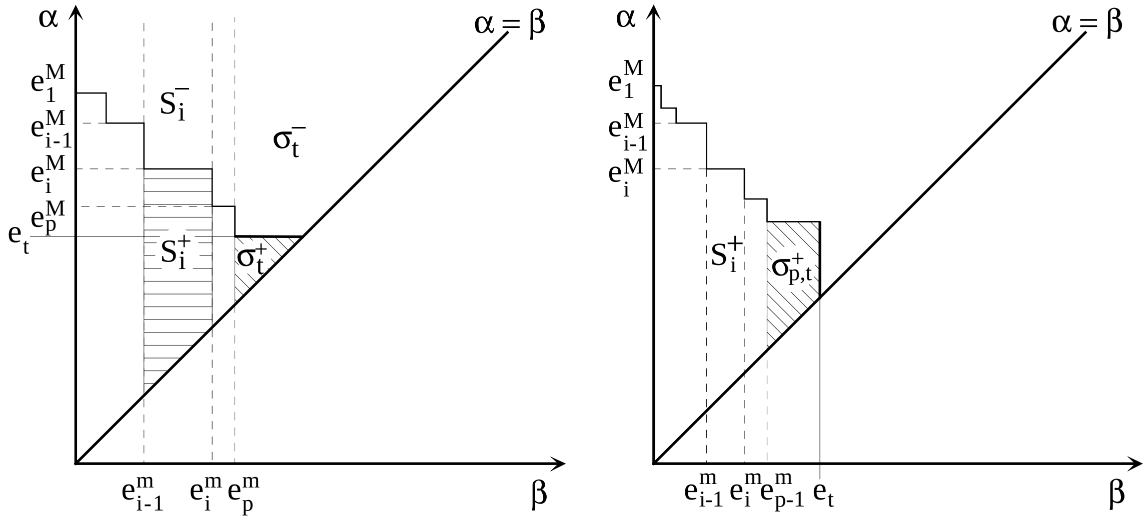
[12] The resulting macro reaction is depicted as path CD in Fig. 4.

[13] For the erasing property see Mayergoyz (1986), p. 605.

[14] Consequently, starting with $e = 0$ erases all steps and so the whole influence of the past.

Fig. 3: Active firms when the exchange rate

(a) rises (b) decreases



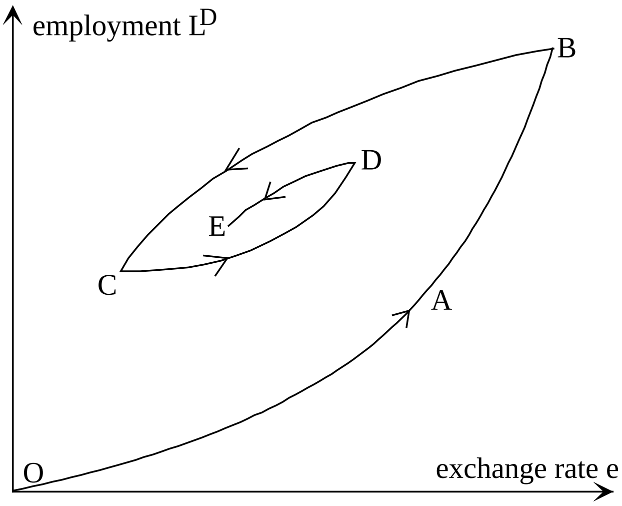
Source: Following Amable et al. (1991), Figures 5 and 8.

For ascending exchange rates the area S_t^+ (including all active firms) can be divided into two partial areas ($\sum_i^p S_i^+$ and σ_t^+). Path-dependence is captured by the influence of the past input extrema on the area of the active (i.e. employing) firms. When the exchange rate decreases, the area S_t^+ is changed by shifts to the left of the lower right vertical part of the border. Only $\sigma_{p,t}^+$ is directly influenced by e_t, while $\sum_i^{p-1} S_i^+$ depends on past extrema. $\sigma_{p,t}^+$ diminishes with decreasing e_t, until the stage corresponding to the (highest not yet not-erased) minimum e_{p-1}^m is erased and the order (p-1) of $\sum_i^{p-1} S_i^+$ is reduced by one.

Thus, the basic formal structure of the results for employment in the cases of rising and falling exchange rates is similar: an important constant term that is determined by the past extrema stands aside a term that is influenced by the current exchange rate e_t. The relations that are valid for the current exchange rate have been shifted by past input extrema. Consequently, every reversal in the *direction* of the exchange rate input path leads to a structural break in the exchange rate-employment (i.e. 'input-output') relation.

In the case of micro hysteresis only two branches are possible. A jump between these two (not continuously linked) 'branches' only occurs with the passing of certain threshold values. The continuous macro loop (Fig. 4) results from aggregation over a bulk of heterogeneous micro elements. Due to the lack of thresholds and to the fact that the path-dependence is captured by the local input variable extrema even small changes in the exchange rate can have durable effects on employment.[15]

Fig. 4: The macroeconomic exchange rate-employment hysteresis loop



The *distribution* of the heterogeneous firms in the α,β-space is important for the result of the analysis. A continuous distribution of the firms in the α,β-space results in *continuous* macroeconomic loops in Fig. 4. The exact density of the α_j,β_j-distribution will solely determine the curvature of the branches. The more all firms are clustered in a specific area (i.e. the more homogenous the firms are) the more curved are the branches. In the borderline case of a multiplicity of similar homogenous firms the macro-loop will degenerate to a loop of the form as depicted in Fig. 1 for a single firm.

4. The micro model under one-off uncertainty and the possibility of waiting

Uncertainty generates an option value of waiting, and therefore introduces a bias in favour of a "wait-and-see"-strategy. Since the firm's employment decision can be understood as an irreversible investment, we follow a real option approach. The firm's employment opportunity corresponds to a call option that gives the firm the right to employ (invest), hiring costs being the exercise price of the option, and to obtain a 'project'. The option itself is valuable, and

15 See Mayergoyz (1986) and Amable et al. (1991), p. 2, pp. 9 and p. 15. For a detailed mathematical analysis of various forms of hysteresis see Krasnosel'skii, Pokrovskii (1989) and Brokate, Sprekels (1996).

exercising the employment investment "kills" the option. Analogously, disinvestment (firing) can be interpreted as a put option.

We analyse the effects of an expected future stochastic one-time shock on the band of inaction. However, assuming a risk-neutral firm, we abstract from risk-aversion. Focusing on employment impacts of uncertainty, we take up an idea originally proposed by Dornbusch (1987), pp. 8 f., Dixit (1989), p. 624, fn. 3, Bentolila, Bertola (1990), and Pindyck (1991), p. 1111. Option price effects are modelled in a technically sophisticated way in these references. Since we focus on the aggregation problem, we model micro uncertainty effects as simple as possible (based on the model by Belke, Göcke, 1999). However, the basic structure and the main micro effects of uncertainty are left unchanged compared to the cited references.

We suppose a non-recurring single stochastic change in the exchange rate, which can be either positive (+ε) or negative (–ε) (with $\varepsilon \geq 0$, mean preserving spread). For simplicity reasons, both realisations of the change ε are presumed to have a probability of ½ : $e_{t+1} = e_t \pm \varepsilon$ and $E_t(e_{t+1}) = e_t$. From period t+1 on, the firm will decide under certainty again. Thus, the trigger exchange rate value under certainty ("c-trigger") as derived above will become valid again. The stochastic change between t and t+1 leads to a widening of the band of inaction due to the possibility of a "wait-and-see"-strategy. Under certainty, the relevant alternative strategies are to enter or not for a previously inactive firm respectively to exit or not for a formerly active firm. Under uncertainty and the feasibility to delay an investment, a third alternative has to be taken into account: the option to wait and to meet the respective decision (i.e. entry or exit) in the future. The option to employ in the future is valuable because the future value of the 'asset' obtained by employment is uncertain. If its value will decrease, the firm will not need to employ and will only lose what it will have spent to keep the employment opportunity. This limits the risk downwards and with this generates the inherent value of the option.

Under uncertainty a *previously active (i.e. employing) firm* has to decide whether to leave the market now or to stay active, including the option to leave later. It may appear advantageous for the firm to bear temporary period t losses, if there is a possibility of future gains. In this case the firm can avoid additional exit costs.[16] On the one hand, the firm anticipates the possibility of future gains if the future exchange rate turns out to be favourable (+ε). On the other hand, the firm foresees that it can avoid future losses if the exchange rate change will be negative (–ε) with a later exit in t+1.

The expected present value of the wait-and-see strategy in period t is defined as the probability-weighted average of the present values of both ε-realisations. Waiting implies a period t profit of $e - w \cdot a_j$. Conditional on the (–ε)-realisation, the firm will use its option to leave in t+1 causing discounted firing costs $\delta \cdot F_j$. In the case of the favourable (+ε)-realisation the firm will stay and earn annuity value $\delta \cdot (e + \varepsilon - w \cdot a_j)/(1-\delta)$. Combining both, the expected present value of the wait-and-see strategy is:

$$(5) \qquad E_t(V_{j,t}^{wait}) = e - w \cdot a_j - \frac{1}{2} \cdot \delta \cdot F_j + \frac{1}{2} \cdot \delta \cdot \frac{e + \varepsilon - w \cdot a_j}{1-\delta} \ .$$

The present value of an immediate exit is simply determined by firing costs: $E_t(V_{j,t}^{exit}) = -F_j$. Hence, the firm is indifferent between exit in t and 'wait-and-see' if $E_t(V_{j,t}^{exit}) = E_t(V_{j,t}^{wait})$. As a consequence, the u-exit-trigger follows (with exit for $e_t < e_{j,exit}^u$):

$$(6) \qquad e_{j,exit}^u = w \cdot a_j - (1-\delta) \cdot F_j - \frac{\delta \cdot \varepsilon}{2-\delta} = e_{j,exit}^c - \frac{\delta \cdot \varepsilon}{2-\delta} = e_{j,exit}^c - \frac{\varepsilon}{1+2i} \ .$$

A *previously inactive firm* has to decide whether to enter the market now or to stay passive, including the option to enter later. The firm anticipates the possibility of internalising future gains by an entry in t+1 if the future exchange rate turns out to be favourable (+ε). Besides, the firm foresees that it can avoid future losses if the exchange rate change will be negative (–ε) by staying passive. Waiting and staying inactive implies zero profits in t. Conditional on a (+ε)-realisation, the firm will use its option to enter in t+1 causing discounted hiring costs $\delta \cdot H_j$, gaining an annuity value of $\delta \cdot (e + \varepsilon - w \cdot a_j)/(1-\delta)$. For a (–ε)-realisation the firm will remain passive. Consequently, the expected present value of the wait-and-see strategy for a

[16] We abstract from a later re-entry (and analogously from a later re-exit by a previously inactive firm), and therefore neglect re-entry (re-exit) costs. For a more detailed presentation of the micro framework under uncertainty and for a simultaneous consideration of exit and re-entry (entry and re-exit) options see Belke, Göcke (1999).

previously inactive firm is given by $E_t(V_{j,t}^{wait})$ in eq. (7). The expected present value of an immediate entry (without a re-exit) is $E_t(V_{j,t}^{entry})$:

$$(7) \qquad E_t(V_{j,t}^{wait}) = -\frac{1}{2}\cdot\delta\cdot H_j + \frac{1}{2}\cdot\delta\cdot\frac{e+\varepsilon-w\cdot a_j}{1-\delta} \quad ; \qquad E_t(V_{j,t}^{entry}) = -H_j + \frac{e-w\cdot a_j}{1-\delta} \; .$$

The option value of having the flexibility to make the employment decision in the next period rather than to employ either now or never, can easily be calculated as the difference between the two expected net present values: $OV(e,\varepsilon) = E_t(V_{j,t}^{wait}) - E_t(V_{j,t}^{entry})$, with: $\partial OV/\partial e < 0$, $\partial OV/\partial\varepsilon > 0$. (An analogous logic can be applied to the disinvestment/firing put option case.) An increase in uncertainty enlarges the value of the option to employ later. The reason is that it enlarges the potential payoff of the option, leaving the downside payoff unchanged, since the firm will not exercise the option if the exchange rate falls. The firm is indifferent between entry in t and wait-and-see if $E_t(V_{j,t}^{entry}) = E_t(V_{j,t}^{wait})$, i.e. if $OV = 0$. The entry-trigger under uncertainty follows as (entry for $e_t > e_{j,entry}^{u}$):

$$(8) \qquad e_{j,entry}^{u} = w\cdot a_j + (1-\delta)\cdot H_j + \frac{\delta\cdot\varepsilon}{2-\delta} = e_{j,entry}^{c} + \frac{\delta\cdot\varepsilon}{2-\delta} = e_{j,entry}^{c} + \frac{\varepsilon}{1+2\,i} \; .$$

The entry (and exit) exchange rate trigger value under uncertainty is additively (subtractively) augmented by the term $\varepsilon/(1+2\,i)$. Thus *uncertainty leads to an expansion of the band of inaction*. . Uncertainty increases the probability that a firm stays active (passive) if the current level of the revenue e has descended (ascended) from a formerly high (low) level that had earlier induced entry (exit). In our specific simple model the expansion of the band of inaction is even linear with respect to ε. Furthermore, in this simple set up the expansion is even independent of any aspects of heterogeneity.

5. Aggregation to the macro level under uncertainty

The qualitative property of micro hysteresis, i.e. the entry trigger being larger than the exit trigger, has not changed, when switching to uncertainty. However, the distance between the trigger values has been enlarged *for every firm*. With this, the aggregation approach described in section 3 can principally be applied to the case of uncertainty as well. We now focus on the implications of a switch from certainty to a situation with uncertainty for the shape and the location of the macro hysteresis loop.

Uncertainty leads to an outward shift of the entry and the exit trigger by $\varepsilon/(1+2\,i)$. Taking the situation under certainty described by Fig. 3 as a starting point, α_j moves to the north, and β_j slips westward by this amount. Thus, every (α_j,β_j)-combination characterising firm j is projected to the north-west. Hence the orthogonal distance of each point to the $(\alpha=\beta)$-45°-line is increased by $\sqrt{2\,(\varepsilon/(1+2\,i))^2}$. However, the non-hysteretic firms with $H_j, F_j = 0$ (located on the $\alpha=\beta$-line) retain their position even under uncertainty. Expressed graphically, the whole S_t^+-block comprising the active firms (and analogously the S_t^--block describing the inactive firms) is shifted. A $\sqrt{2\,(\varepsilon/(1+2\,i))^2}$ -wide zone above and parallel to the $\alpha=\beta$-line free of firms emerges.[17] In Fig. 5 the impacts of an introduction of uncertainty to a situation described by Fig. 3 (a) and (b) are illustrated. Obviously, some firms with formerly positive exit rates may be shifted into the negative range. These firms would even bear negative revenues in order not to loose the sunk costs and the option value of waiting. Since in our simple model the unit revenue is expressed by the exchange rate which is expected to be positive, we will neglect the area on the LHS of the ordinate in our further analysis.

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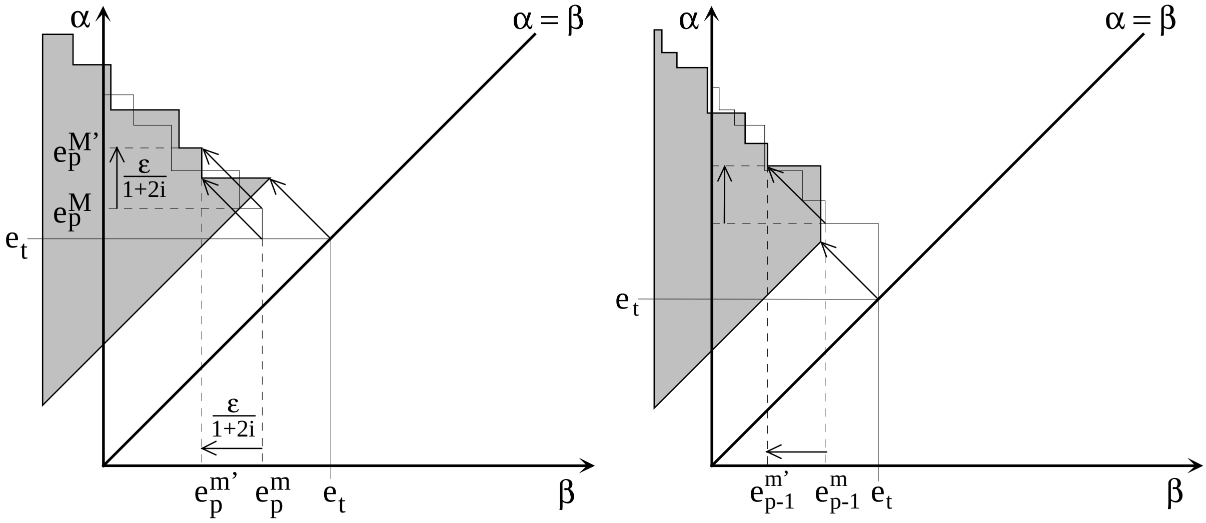
[17] For reasons of simplicity, in this paper we have neglected the non-trivial problems which may arise when the possibility of a re-entry after an exit in t (respectively a re-exit after an entry in t) is given. A re-entry or a re-exit-behaviour might become optimal if the sunk costs (H_j and F_j) are 'small' compared to the revenue change due to ε. As a consequence, these firms are not shifted by the full amount. This would result in a zone parallel to the $\alpha=\beta$-line which is not completely "depoputated" as derived above.

Fig. 5: Active firms after the introduction of uncertainty when exchange rate

(a) has risen before (b) has decreased before



For simplicity reasons, with respect to the certainty situation we assume a preceding time path of the exchange rate which has produced a 'staircase-borderline' with an infinite number of non-erased extrema resulting in infinitesimally small steps, resulting in a (–45°)-borderline with a negative slope equal to one. This procedure has the advantage to abstract from the differentiation concerning the actual direction of e underlying the parts (a) and (b) of Fig. 5 leaving the main effects of uncertainty on the aggregation procedure unaffected. All initially active firms are included in the triangle area S^+ depicted in Fig. 6 (a). The initial exchange rate is given as e_0. Introduction of uncertainty results in a north-western shift of S^+ to the triangle $S^{+'}$ as illustrated in Fig. 6 (b).

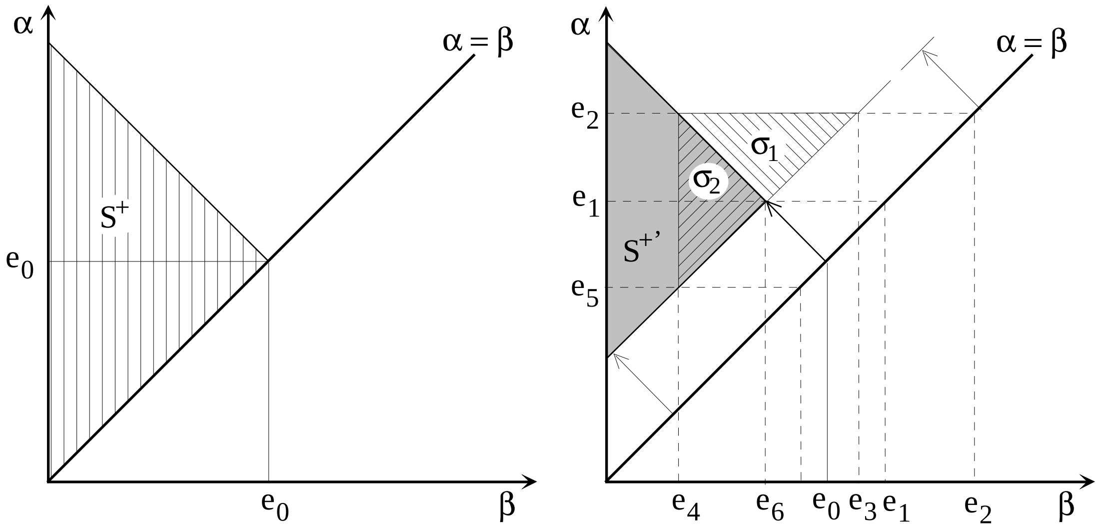
Taking e_0 as initial value, an increase in the exchange rate up to e_1 does not affect any 'hysteretic' firm (with $H_j, F_j > 0$) in the shifted triangle $S^{+'}$. Only 'non-hysteretic' firms (with $H_j, F_j = 0$) on the $\alpha = \beta$-line change from inactivity to activity. Thus, in this 'play' interval an increase in the exchange rate leads only to a weak reaction of employment, as it is depicted in Fig. 7 (as the movement from point A to B).[18] A further increase e.g. to the local maximum e_2 induces additional employment corresponding to the area σ_1 in Fig. 6 (b) (point B to C in Fig.

[18] For reasons of simplicity, this reaction is depicted in a linear way. I.e. non-hysteretic firms are assumed to be equally distributed on the $\alpha = \beta$-line.

7). While the initial weak reaction AB shows similarities to mechanical 'play',[19] the subsequent additional increase BC leads to a strong reaction which can be titled 'spurt' in employment. More concretely, after a reversal of the exchange rate the initially weak response of employment will evolve into a very strong response, once the thresholds of many firms are passed.[20] A reversal of the exchange rate towards e_3 will again cause a weak ('play') reaction (CD). A further decrease in the exchange rate to e_4 leads to an erasure of the formerly gained employment σ_1 as well as to new unemployment described by the area σ_2. Again a weak 'play'-reaction follows for a subsequent increase to e_5 (EF). An exchange rate movement $e_0 \rightarrow e_1 \rightarrow e_2 \rightarrow e_3 \rightarrow e_4 \rightarrow e_5$ will result in a hysteresis loop determined by ABCDEF. In contrast, the corresponding hysteresis loop under certainty (as already described in Fig. 4) is depicted by the dotted path AHKM in Fig. 7.

Fig. 6: Active firms dependent on the exchange rate

(a) initial situation without uncertainty (b) after introduction of uncertainty



Starting from the initial situation (e_0 respectively point A) again, now assume a decrease in the exchange rate from e_0 to e_6. As before, a 'play'-reaction results (AG). A further decrease towards e_4 analogously leads to a 'spurt' in unemployment described by σ_2 (GE).

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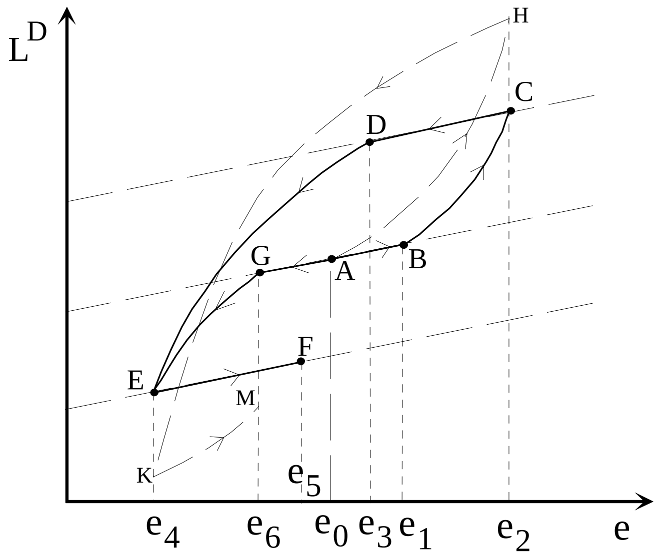
[19] For a description of original play hysteresis, see Krasnosel'skii, Pokrovskii (1989), pp. 6 ff., and Brokate, Sprekels (1996), pp. 24 f. and pp. 42 ff.

[20] For a non-formal description of 'spurts' in investment or in employment based on a microeconomic sunk cost mechanism under uncertainty see Pindyck (1988), pp. 980 f., and Belke, Göcke (2001a). Dixit, Pindyck (1994), pp. 15 f., vividly illustrate the non-linear reaction of US-employment after recovery in the mid 1993. However, in these contributions no explicit aggregation approach to derive macroeconomic behaviour is applied.

Fig. 7: Macro hysteresis loop under uncertainty (including 'play')



As a main result, the introduction of uncertainty implies additional 'play'-intervals in the macro loop with every reversal of the input direction. The width of the 'play'-interval, i.e. $(e_2 - e_3)$, $(e_5 - e_4)$ or $(e_1 - e_6)$ is given by the same amount: $(2\varepsilon)/(1+2i)$). This expression exactly equals the widening effect of uncertainty on the width of the microeconomic 'band of inaction' for the single firm $(e^u_{j,entry} - e^u_{j,exit})$ as implied by eq. (8).[21] Due to our special initial situation, e_0 not being an extremum, the initial play intervals (AB or AG) amount to the half (i.e.: $\varepsilon/(1+2i)$).

The occurrence of 'play'-intervals – implying a weak reaction with every reversal of the input (exchange rate) – results in a weaker reaction of the dependent variable (employment) compared to the situation without uncertainty. This weaker reaction corresponds to the fact that (a) the 'play'-interval has to be passed in order to lead to permanent employment impacts, and (b) a smaller number of firms is affected by exchange rate changes, since all 'hysteretic' firms are shifted to the north-west by uncertainty. These mechanisms finally result in a 'flatter' shape of the hysteresis loop under exchange rate uncertainty.

[21] However, this simple and clear result is due to our extremely simple micro model, where the expansion of the band of inaction is linear with respect to uncertainty ε and even independent of the heterogeneity between firms. But even in a more general formulation the widening of band of inaction will occur due to uncertainty and, thus a kind of a "depopulated" area parallel to the $\alpha = \beta$-line and play-hysteresis will result.

Our micro model is based on a risk-neutral single-unit (dis)employment decision under revenue uncertainty caused by exchange rate (step) volatility and fixed sunk (i.e. irreversible) hiring and firing costs. It is comparable to models incorporating irreversible investment decisions. However, we do not rely on the asymmetry of adjustment costs (Caballero, 1991) and on scrapping values (Darby et al., 1998), since we analyse 'investments' in employment and not in real capital. In addition, the degree of competition in the output market and economies of scale (Caballero, 1991) do not play a predominant role since we analyse a single-unit decision.

Strictly speaking, our results can only be applied to the *tradables* sector of an open economy, with L^D representing employment in the tradables sector. To the extent that employment in the non-tradables sector is not directly influenced by exchange rate movements, the macro hysteresis loops depicted in Fig. 7 will be *shifted vertically* by the amount of employment in the *non-tradables* sector, resulting in the loops for total economy employment. However, our results have to be modified if our partial equilibrium approach would be generalised, e.g. exports could be increased by redirecting resources (including labour) away from the non-tradables to the tradables sector. The importance of this effect depends on the substitutability of labour between both sectors. In the case of perfect substitution, the loop in Fig. 7 would mainly describe the distribution of labour between both sectors. If substitutability is not perfect, the quality of the relationship ceteris paribus continues to hold. However, the loop would be compressed vertically. Limited labour mobility and factor specifity implies that in industrialised countries substitutability should not expected to be perfect. Admittedly, analysing these problems in a general equilibrium context would probably allow us to tackle them more adequately. However, this is beyond the scope of this paper.

Of course, our theoretical results concerning macroeconomic play-areas as a consequence of sunk cost and uncertainty on a microeconomic level demand an empirical corroboration. Belke, Göcke (2001) develop an algorithm in order to approximate the play-shape of the relationship between exchange rate and employment via linear partial sections. Implementing this approximation of play into a switching-regression approach they find empirical support for play-effects based on exchange rate variability on West-German employment.

6. Conclusion

In this contribution, non-linearities in the relation between total economy employment and the exchange rate are emphasised. A potential mechanism based on a band of inaction that is induced by sunk costs and could account for a 'weaker' relationship between employment and its determinants has been augmented by exchange rate uncertainty. As a result of option value effects the band of inaction is widened and, thus, hysteresis effects are strongly amplified by uncertainty. Special attention has been paid to the problem of aggregation under uncertainty. It has been shown that under uncertainty 'play'-areas have to be considered at the macro level. Thus, similarities of macro relations ['play'] to micro behaviour [band of inaction] are enhanced by uncertainty.[22]

We ascribe the revenue volatility to exchange rate volatility. However, since uncertainty effects on the revenue ε is additively included in the revenue function, an interpretation of ε as an all comprising expression of uncertainty in exchange rates, foreign prices, wages and productivity is straightforward. Moreover, the relation between employment and all its driving forces (e.g. the wages) is affected by uncertainty. We focus on the calculation of exchange rate triggers, holding other determinants of employment constant. Similarly, one could calculate trigger values for e.g. wages resulting in a hysteresis-band concerning wages. An application of our aggregation approach based on wages is straightforward. According to our results, due to uncertainty, total economy employment is expected to be less sensitive than previously expected to movements in the exchange rate, foreign prices, wages and productivity.

Moreover, our findings may be of a more general interest, since the aggregation mechanisms detected here may apply to other cases where the aggregation of microeconomic real option effects under uncertainty becomes necessary.

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[22] Thus, additional empirical studies in the manner of Parsley, Wei (1993) introducing uncertainty related zones of inaction may be adequate even on the macro level.

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