

Using Real Options to Analyze the Effect of Contractual Investment Restrictions: The Case of Construction Guarantees

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Abstract:

Non-equity project participants, such as creditors and the host government, have an important effect on the attractiveness of investing in natural resource projects. An obvious cause of this effect is through the distribution of a portion of project cash flows as taxes, mineral royalties, and interest payments. Non-equity participants also influence project value in a less direct manner through non-monetary contract terms that are intended to protect their interests. Unfortunately, it is difficult to quantitatively assess the impact of such terms on the distribution of project value so that equity owners, government officials, and project financiers are left with qualitative or even rhetorical analysis to negotiate the terms of their participation.

In this paper, we use the real option valuation method to look at the effect of construction (completion) guarantees on project attractiveness when there is mineral price and foreign exchange rate uncertainty. Construction guarantees are used by project financiers and some host countries as one method of ensuring that project development is completed instead of being suspended in the event of downside outcomes such as low mineral prices or extreme foreign exchange rate movements. We show that these guarantees can result in a direct reduction of value in the equity owner's claim to project cash flows that is not accounted for by conventional discounted cash flow valuation methods. There are good reasons for project creditors and host governments to desire construction guarantees such as protecting downside risk exposure and ensuring economic development. However, it is equally important for the mining company, project financiers, and the host country to understand the full implications that non-monetary contractual terms have on mining investment.

1.0 Introduction

The mining industry, like any other business sector, is ultimately founded on its ability to create economic value and benefits for all its participants. Mining industry participation is not limited to just equity or ownership interests but also includes creditors that provide project financing, governments at the local, regional and national level, employees, suppliers, customers, and other sections of society that are affected by mining activity. This is an extremely “broad tent” in which the interests of each participant must be compared to those of other participants and the tradeoffs created by mining investment determined. A mining project can only begin once it is shown that it is reasonable to expect that the net effect of project development is positive economic value. However, calculating and accounting for the economic benefits or losses accruing to each participant in a mining project is difficult. For some participants, such as environmentalists, this is currently conducted with qualitative assessments carried out through public debate and the resulting government legislation. For others, such as equity owners and project creditors, the economic benefits and losses are more easily quantified by calculating the value of their claims to mine cash flows.

The economic interests of many of these different project participants are protected through the legal system which provides public regulatory provisions, such as mining codes and bankruptcy laws, to broadly control mining activity and the recognition of private legal contracts, such as project financing agreements, to define the manner in which individuals, corporations, non-governmental organizations and other entities participate in a mining project. This protection arises in the form of restrictions on some of the actions of other participants and the guarantee of particular actions by other project participants. Some of these protective measures have direct economic consequences such as worker health provisions that increase operating or development costs. Other measures, such as financing terms controlling how additional project capital may be raised, have less obvious consequences and at present are only assessed in a more qualitative manner.

This paper considers the impact of construction guarantees on the value of equity participation in a mining project. Construction guarantees in one form or another may be used by project creditors, non-governmental organizations, and host governments to ensure that development of a project is actually completed once it is started. These guarantees protect the interests of non-equity participants by ensuring that a mine is fully built since the equity owners are the participants with direct management control. Project creditors have a direct interest in ensuring that a project is built once started since they often fund a large portion of the development costs but are restricted to being repaid out of the production cash flows that are generated by the project (called non-recourse financing). Host governments also have an interest in project completion since royalties and local economic development are linked to a completed mine.

Further, the host government may fund infrastructure development to assist in the sale of mine output on world markets and the import of production inputs that would otherwise not be justified unless the mine is developed.

An example of a construction guarantee is found in the financing details of the Los Pelambras project in Chile that was developed by Antofagasta Holdings plc and a consortium of Japanese industrial corporations (Antofagasta, 1998). The total development costs were projected to be US\$1.36 billion of which US\$410 million would be financed by equity and the remainder from a loan provided by a project financing syndicate. This loan is repayable in installments over 10 to 12 years after project development is completed and carries an interest rate of approximately 1% over the London Inter-Bank Overnight Rate (LIBOR). To ensure project completion, during November 1997, Antofagasta Holdings placed US\$316 million into an escrow account to meet its share of the development cost of Los Pelambras. In addition, Antofagasta guaranteed 60% of the borrowings by pledging as security its investment in Quiñenco S.A., a Chilean company, which would lapse once project development is completed.

The combination of pledging other investments as security and placing their share of the development funds into an escrow fund is effectively a construction guarantee. Antofagasta Holdings had no incentive to stop development of the project if adverse business conditions occurred since its full funding obligations had been placed in an escrow account before development started and its investment in Quiñenco S.A. was forfeit if the project was not developed. The cost of this construction guarantee is offset by lower financing costs in the form of low interest rates.

1.1 Analysis of non-equity project participation with the real option valuation method

This paper uses the real option valuation method to assess the impact of a construction guarantee on the equity values of two copper projects. The real option method has been noted for its ability to determine the value of management flexibility and to provide valuations that differentiate projects by risk and uncertainty characteristics. Real options has also been used assess impact of the interaction between project participants on the value of each participant's claim to project cash flows.

Government taxation and royalty policy in the natural resources industries has been extensively considered with real options. Mackie-Mason (1990) demonstrates how non-linear tax rules in the mining industry may discourage some investments and encourage the early shut down of others. Jacoby and Laughton (1992) consider the risk underlying tax flows from petroleum projects and show that taxes paid early in a project's life are much more risky than those received later in the project. Bradley (1998)

investigates how different royalties and taxes alter the risk profile and value distribution of a natural resource project.

Project financing has also been discussed within the real option framework. Merton and Mason (1985) provide an overview of using real options to value equity claims, loans and loan guarantees. They setup (but do not solve) an example where the development and operation of an oil pipeline is financed by equity, senior and subordinated debt, and loan guarantees. Samis (1995) uses real options to analyse the capital structure and value distribution of a marginal South African gold mine that is financed by equity, preferred shares, several loans ranked by seniority in the event of default, a government loan guarantee and various government subsidies.

2.0 A valuation comparison between two copper projects in a foreign country

The value effect of construction guarantees are demonstrated with two copper projects, the Long-Life Mine and Short-Life Mine, which are similar except for the size of reserve. These projects are located in a country different to that of the equity owners. The currency in the host country of the two projects is called a Foreign Monetary Unit (FMU) and the currency of the equity's home country is the Domestic Monetary Unit (DMU). Over the life of both projects, the nominal riskless interest rate in the host country is assumed to be a constant 12.5% and the inflation rate is assumed to be a constant 7.5%. The nominal riskless interest rate in home country of the project owners is a constant 3.0% and the inflation rate a constant 1.5%.

Currently, the FMU : DMU foreign exchange (FOREX) rate is two DMU to one FMU. This FOREX rate is expected to depreciate at an annual rate of 9.5%. The FOREX exchange rate is an integral part of this valuation example because the project output is sold on world markets in DMU while project costs are incurred in both DMU and FMU. **Figure 1** delineates the expected FOREX rates for both DMU and FMU.

2.1 Project uncertainty

The projects are exposed to both copper price uncertainty and FOREX rate uncertainty.

Mineral price uncertainty follows a reverting log-normal stochastic process for which the annual standard deviation (volatility) is 20% and the price median growth rate is zero. The annual price of mineral risk, or

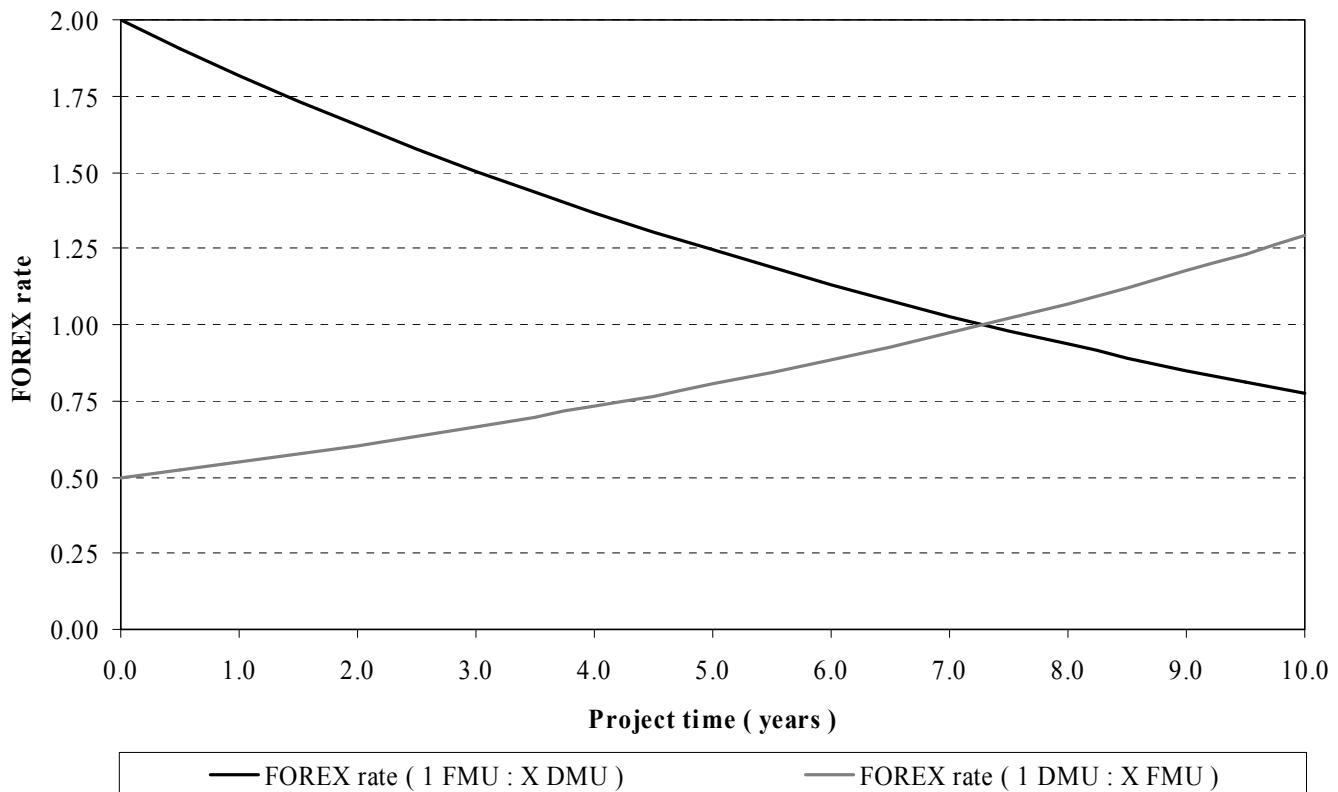


Figure 1. Foreign exchange rates over the first ten years of the projects.

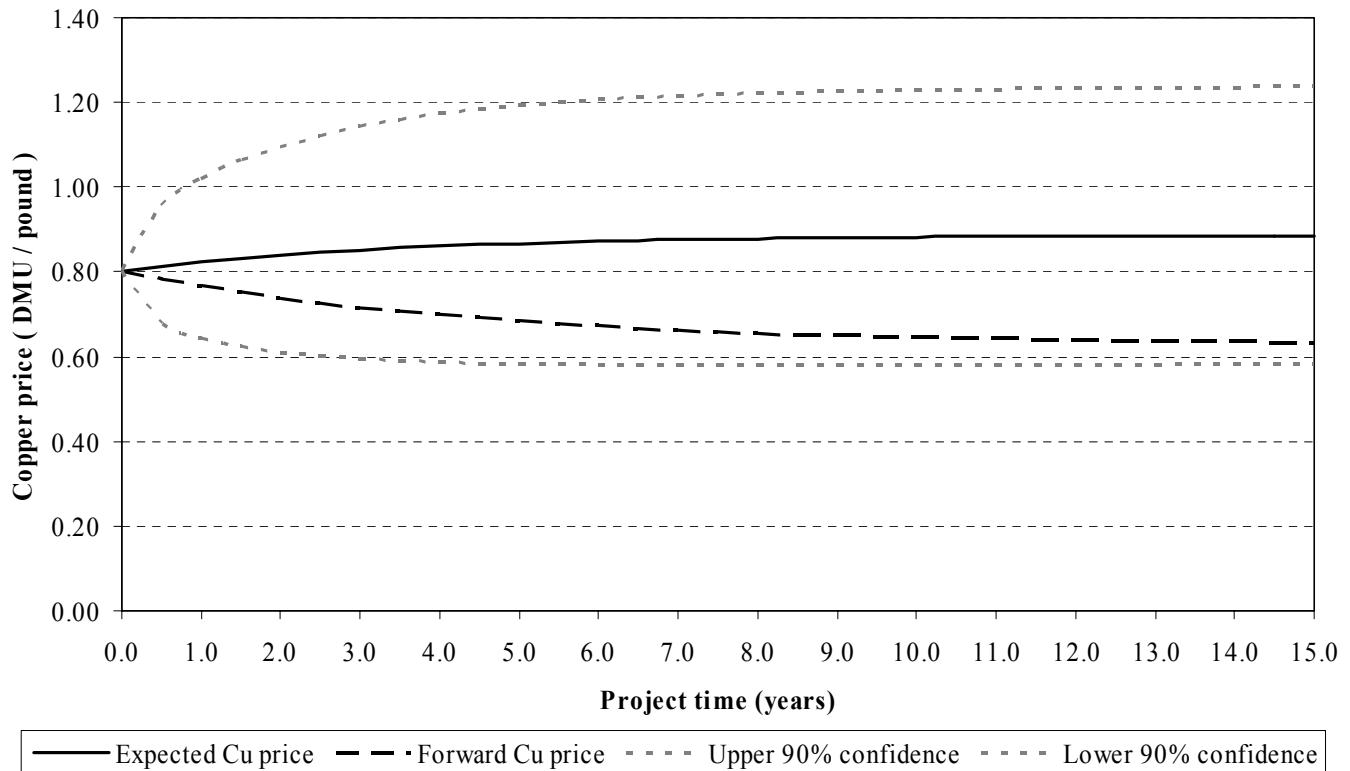


Figure 2. Expected price, forward price and 90% confidence price boundaries for copper.

the additional return for each unit of price volatility, is 0.40 (*i.e.* an investor needs an additional 0.4% return over the riskfree rate for each 1% of cash flow uncertainty stemming from the copper price).¹ A defining characteristic of a reverting mineral price process is that uncertainty increases at a decreasing rate due to market forces pulling the mineral price towards long-term supply-demand equilibrium. Long-term expected and forward mineral prices change less over given time period than short-term expected and forward prices when the mineral price follows a REV process (called the Samuelson (1965) effect).² **Figure 2** outlines the expected price, forward (risk-adjusted) price and the 90% confidence price boundaries for the copper over the project time horizons.

FOREX rate uncertainty follows a non-reverting stochastic process in which the annual standard deviation is 40%. The stochastic process is risk-neutral (risk-adjusted) in which the expected depreciation rate of the FMU with respect to the DMU is 9.5%. This is consistent with FOREX rate processes outlined in Hull (2000) and Hakala and Wystup (2000) in which the expected FOREX depreciation / appreciation rate is equal to the difference between the nominal riskfree rates in the domestic and the foreign economies. A fundamental characteristic of this process is that uncertainty grows at a constant rate over the lives of both projects.

2.2 Project development and production parameters

The two projects are differentiated by reserve size. Their unit operating costs, production rates, and development costs are the same so that a comparison of a construction guarantee's value effect is not distorted by project differences. The Long-Life Mine has 400 million tonnes of reserves with an average grade of 0.5% copper. The Short-Life Mine has reserves of 265 million tonnes also with a grade 0.5% copper.

Both projects have a total development cost in current monetary terms of FMU 224.8 million and DMU 150.5 million (totaling DMU 600 million at current FOREX rates). The projects require 4 years to develop and capital expenditures are incurred at half year intervals. **Table 1** outlines the uninflated capital expenditure schedule.

The annual production rate at both mines is 20.4 million tonnes (224.7 million pounds of copper) so that the Long-Life Mine has approximately 20 years of production while the Short-Life Mine has 13 years. At current FOREX rates, the uninflated unit operating costs are DMU 0.40 per pound of copper which, given

¹ See Salahor (1998), Laughton and Jacoby (1993), or Samis (2002) for closed-form reverting price process equations and a detailed discussion of the characteristics of each process.

² A forward contract is an agreement between two parties for the delivery on a specific future date of a predetermined amount of a commodity. The forward price is the price at which the delivery is made. Note that no funds change hands until the delivery date.

Project time (year)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Domestic CAPEX (DMU millions)	8.1	16.2	20.3	24.3	24.3	24.3	20.3	8.1	4.6
Foreign CAPEX (FMU millions)	12.2	24.3	30.4	36.5	36.5	36.5	30.4	12.2	6.1

Table 1. Uninflated capital expenditures for Long-Life and Short-Life Mines.

that 75% of the operating costs are incurred in FMU and 25% are in DMU, is equivalent to FMU 33.7 million and DMU 22.5 million annually.

The only form of flexibility available to management is early project closure after paying a rehabilitation cost. This option is continually available during the production phase. It is only available during the development phase when there is no construction guarantee in effect. Management can not close the project to limit downside risk if there is a construction guarantee since project development must be completed regardless of movements in the copper price or FOREX rates.

3.0 Valuation results

Both projects are valued using the conventional DCF method with a 10% risk-adjusted discount rate and no early closure option available. The projects are then valued using real options given no early closure option (no flexibility), early closure available at any time (full flexibility), and a construction guarantee that limits early closure to the production phase of each project (restricted flexibility). When there is some project flexibility, the real options valuation considers the influence of each uncertainty separately to simplify the numerical calculations. This means that risk-neutral (risk-adjusted) FOREX rates are used in the value calculation when the effects of mineral price uncertainty are studied and risk-neutral (risk-adjusted or forward) mineral prices are used when the effects of FOREX rate uncertainty are assessed. This simplification is permissible when there is no correlation between FOREX rate and mineral price uncertainties.

3.1 No flexibility project value results: A DCF versus real options value comparison

Given a current copper price of DMU 0.80/lb and a FOREX rate of 1 FMU : 2 DMU, the DCF method calculates a value of **DMU 160.7 million** for the Long-Life Mine and a value of **DMU 51.3 million** for the Short-Life Mine. With the real options method, the Long-Life Mine has a value of **DMU 275.4 million** and the Short-Life Mine has a value of **DMU 38.1 million**. The DCF and real option values of each mine are presented in **Table 2** while the project cash flows underlying these calculations are

Valuation method	Mine value (DMU million)
Long-Life Mine (no flexibility)	
DCF method	160.677
Real option method	275.402
Short-Life Mine (no flexibility)	
DCF method	51.307
Real option method	38.116

Table 2. DCF and real option mine values with no flexibility
 (copper price: DMU 0.80/lb; FOREX rate : 1 FMU : 2 DMU).

delineated in **Tables 4** and **5** (located at the end of the paper). Note that capital expenditures at the start of a year and at the half year point are consolidated to the beginning of the year in these cash flows.

The large difference between DCF and real option Long-Life Mine values is the result of risk-and-time discounting. The DCF method uses an aggregate risk-and-time adjustment that is applied to the net cash flow stream. Real options uses differential discounting in which the a risk adjustment is applied directly to the source of uncertainty (in this case the copper price through the revenue stream) and the time adjustment is applied to the net cash flow stream. The effect of each discounting approach on the value of each cash flow can be appreciated by calculating a Net Cash Flow Discount Factors (NCFDF) for each project cash flow. This factor is the ratio of the present value of a project cash flow to the expected cash flow value. It indicates the magnitude of the overall risk-and-time adjustment that is applied to each cash flow.³

Figure 3 outlines the NCFDFs for a value calculation using DCF and real options. These factors apply equally to the Long-Life Mine and Short-Life Mine since they both have the same production parameters and are affected by the same mineral and price uncertainty. The reason the Long-Life Mine has greater value using the real options method is that, with real options, the long-term cash flows have much greater present value (*i.e.* are subject to a smaller risk adjustment) than when DCF method is used. This is the result of real options recognizing that the copper price fluctuates around a long-term equilibrium price so that copper price uncertainty stops growing after ten years (at least with this model). This recognition

³ See Samis (2003) for an extended discussion on the discounting differences between real options and DCF. This discussion includes section on Net Cash Flow Discount Factors.

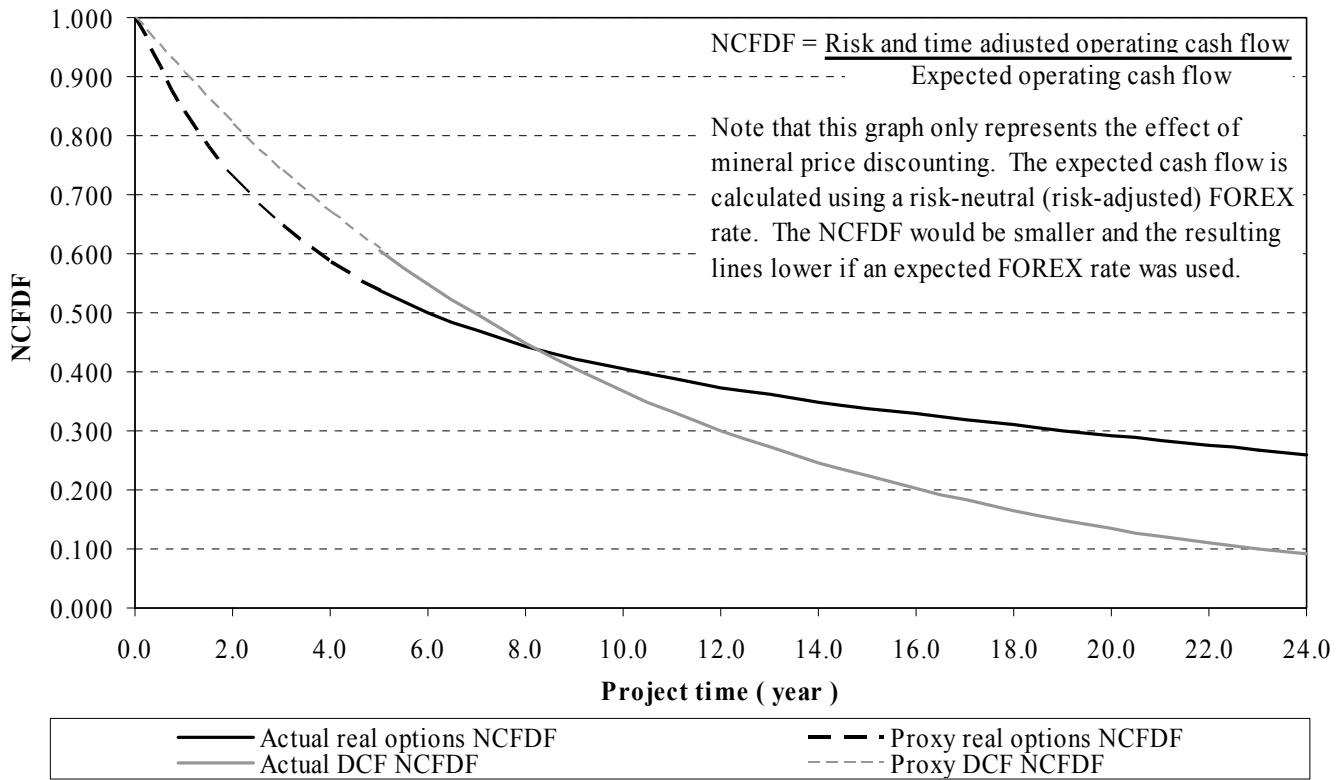


Figure 3. DCF and real options net cash flow discount factors.⁴

results in a constant risk-adjustment being applied to long-term cash flows and causes any changes in the long-term real option NCFDF to be the result of adjustments for the time value of money. The aggregate risk-adjusted discount rate used by the DCF method implies that project risk is increasing at a constant rate for both projects and results in a NCFDF that decreases rapidly over time.

3.2 Early closure flexibility value results: Full flexibility and construction guarantees values

The real option valuation is noted for its ability to determine the value of flexibility. This approach is used here to calculate the value of early closure for the Long-Life and Short-Life Mines. **Table 3** provides the value of each mine for various combinations of flexibility and uncertainty types, given a current copper price of DMU 0.80/lb and a current FOREX rate of 1 FMU : 2 DMU.

The value columns “Copper price uncertainty” and “FOREX rate uncertainty” provide the values of the two mines when management can respond to the resolution of the indicated uncertainty source but cannot

⁴ Proxy NCFDFs are calculated for each mine during its development phase. These are the NCFDF that mine cash flows would have been subject to if operating cash flows were generated and no capital expenditures were incurred during this phase.

Mine flexibility / value of flexibility	Mine value (DMU million)	
	Copper price uncertainty	FOREX rate uncertainty
Long-Life Mine		
No flexibility	275.402	275.402
Full early closure	275.710	391.043
Construction guarantee + early closure during production	275.710	369.909
Value of full early closure	0.308	115.641
Value effect of construction guarantee	0.000	-21.134
Short-Life Mine		
No flexibility	38.116	38.116
Full early closure	38.487	172.840
Construction guarantee + early closure during production	38.458	95.822
Value of full early closure	0.371	134.724
Value effect of construction guarantee	-0.029	-77.018

Table 3. Real option mine values (copper price: DMU 0.80/lb; FOREX rate: 1 FMU : 2 DMU).

respond to resolution of the other source of uncertainty. These value calculations use the current risk-neutral (risk-adjusted) expectations of the uncertainty source which management cannot respond to changes in. This approximation underestimates mine value and is acceptable in this example because mineral price uncertainty and FOREX rate uncertainty are assumed to be uncorrelated and the value of early closure when there is mineral price uncertainty is very small for both mines. **Table 3** highlights that the value of early closure is very small when the value calculation only considers copper price risk. The ability to close early in response to low mineral price adds **DMU 0.3 million** to the Long-Life Mine and **DMU 0.4 million** to the Short-Life Mine. Restricting this flexibility with a construction guarantee has very little impact on overall value of either mine since both are low-cost producers. Early closure has little value at either mine because the copper price is modeled to fluctuate around a long-term equilibrium price. When prices are low, managers expect the copper price to improve over the next few years so they are unwilling to close the mine early unless the copper price is much lower than the unit operating cost.

The option to close has very little value when the current copper price is DMU 0.80/lb since it is very unlikely that this option will be exercised.

The early closure option has more value when mine management can respond to the resolution of FOREX rate uncertainty. In this situation, the Long-Life Mine has a value of **DMU 391.0 million** when there is full early closure while the value of the Short-Life Mine is **DMU 172.8 million**. Early closure becomes a valuable option with FOREX rate uncertainty because this uncertainty continues to grow at a constant rate over the life both projects. When the exchange rate moves adversely, there are no economic forces pulling the current FOREX rate back to some long-term equilibrium so project managers are more likely to make use of this option when exposed to non-reverting FOREX uncertainty.

Construction guarantees have a notable impact on mine value in this example when considered in the presence of FOREX rate uncertainty. The value of the Long-Life Mine decreases to **DMU 369.9 million** and the Short-Life Mine value decreases to **DMU 95.2 million**. The Short-Life Mine is greatly affected by the construction guarantee since its value is highly sensitive to changes in the FOREX rate. This sensitivity is due to its smaller reserve size which results in fewer production cash flows to justify the development costs.

The sensitivity of mine value to changes in the current FOREX rate is outlined in **Figures 4 and 5**. Mine values increase as the DMU appreciates against the FMU since operating and development costs that are incurred in FMU decrease in DMU terms. **Figure 4** shows that the value of the Long-Life Mine is insensitive to the presence of a construction guarantee because of its large ore reserve. In **Figure 5**, the Short-Life Mine is shown to be more sensitive to the presence of a construction guarantee because of its smaller reserve.

Note that a full early closure option allows the Short-Life Mine to retain positive value in the event that the FMU appreciates against the DMU. The project still has positive value when the FMU appreciates by over 40% and an unrestricted early closure option is available. The project's value becomes negative when there is a construction guarantee is present and the FMU appreciates by approximately 10%.

3.3 Future work

The valuation results presented here are preliminary and consider the effect of restricting early mine closure through the use of construction guarantees by non-equity project participants. Mineral price and FOREX rate uncertainty is assumed to be uncorrelated and the value of early closure is assessed given management's ability to respond to the resolution of one of these uncertainties. There will be situations where these uncertainties are correlated. An obvious example is a host country in which a large portion

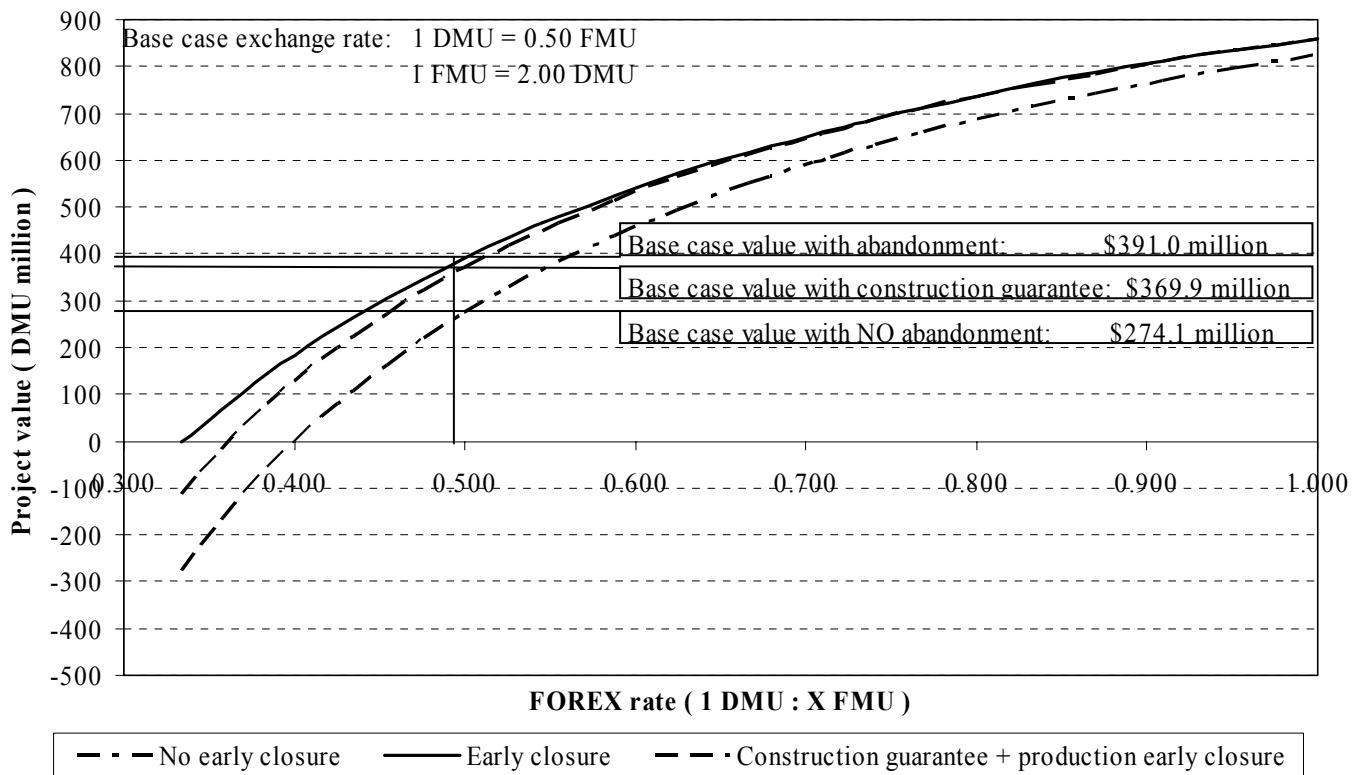


Figure 4. Long-Life Mine values over a range of current FOREX rates.

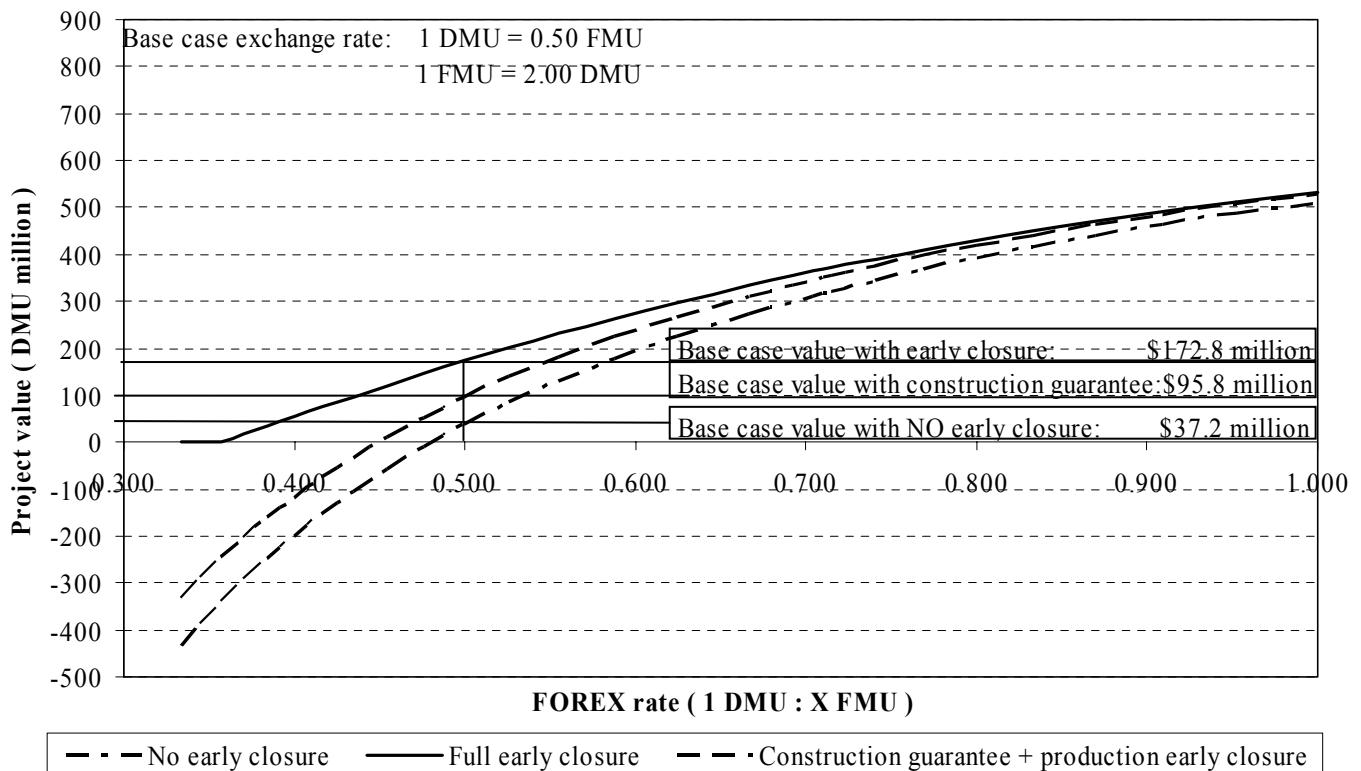


Figure 5. Short-Life Mine values over a range of current FOREX rates.

of foreign exchange earnings are due to the export of a particular mineral. In this case, it is reasonable to expect that there would be positive correlation between FOREX rate movements and changes in the mineral price. This would reduce the value of early closure in the event of adverse FOREX rate or mineral price movements since price decreases would be offset by related depreciation of the host country's currency. A two dimensional numerical model is required to assess the impact of correlation between mineral price and FOREX rate uncertainty.

The model discussed also holds inflation constant over the life of both projects. However, it is likely that there is some relationship between inflation and currency movements in a project's host country which would produce variable inflation rates for project costs denominated in FMU. This could lead to unexpected declines in the host country currency being offset by increasing project cost inflation. Such behavior could be incorporated assuming a constant real riskfree rate and by modeling the uncertainty in the nominal riskfree rates in economies of both the domestic and the host country. A FOREX rate can be derived from the relative movements of the nominal riskfree rates while inflation is determined as the difference between nominal and real interest rates. Such an approach would be consistent with Davveta *et al* (2002) who suggest that long-term FOREX rate movements should be modeled using the nominal riskfree interest rates in each country.

Finally, in the model presented in this paper, the only form of flexibility considered is irrevocable early closure. This can be a valuable but extreme form of flexibility whose value may be severely affected by exercise restrictions such as construction guarantees. An alternative form of flexibility that may offset the effect of the construction guarantee could be an option to allow project development to be delayed, but not abandoned, by paying a penalty until better business conditions occur. This type of option would allow management to avoid project completion until conditions are favorable and may dilute the value effect of the construction guarantee.

4.0 Conclusion

This paper considers the effect of construction guarantees on the value of a project owner's claim to project cash flows when there is mineral price and FOREX rate uncertainty. These guarantees may be used by project financiers and host governments to ensure that a project is developed. Construction guarantees can restrict a project owner's ability to manage the development phase of a mine by forcing development in adverse business conditions. Such restrictions may lead to a significant reduction in the equity value of a project.

Two mining projects, the Long-Life Mine and the Short-Life Mine, were valued using real options to assess the impact of a construction guarantee when there is an early closure option and the projects are exposed to (reverting) mineral price and FOREX rate uncertainty. An unrestricted early closure option was shown in both cases to have only negligible value at both projects when there is mineral price uncertainty. A construction guarantee which restricts the horizon over which an early closure option can be exercised has little impact on project value in this environment since an unrestricted early closure option has little value to begin with. This result will most likely not hold when a project is producing a mineral, such as gold, that does not exhibit price reversion.

However, a construction guarantee did have an effect on equity value when there was a large degree of FOREX rate uncertainty. FOREX uncertainty is modeled in derivative markets without a long-term equilibrium rate so that early closure options can have significant value. An unrestricted early closure option had a large amount of value at both mines. At the Long-Life Mine, the construction guarantee had a moderate effect on the value of early closure since its long production life reduces the importance of early closure during the project development. However, at the Short-Life Mine, the construction guarantee had a large impact on the value of early closure since its shorter production life results in the early closure option being an important method of managing downside risk.

This paper has also shown more generally that non-monetary contractual terms that arise through negotiating the financing and development of a mine can have a significant effect on the equity value of a mine. Such terms may have offsetting benefits that justify their acceptance such as reduced financing costs. However, the valuation methods often used in the mining industry have difficulty assessing the impact of non-monetary contract terms so that the value effect of these terms is considered qualitatively. This paper has used real options to calculate the impact of a construction guarantee and demonstrates that this method may be useful for determining the costs and benefits of non-monetary contract terms.

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Project time (years)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	...	22.0	23.0	24.0		
Time index	0	2	4	6	8	10	12	14	16	...	44	46	48	
Nominal mineral price														
Expected price	0.800	0.823	0.840	0.852	0.861	0.867	0.872	0.875	0.878	...	0.887	0.887	0.887	
Risk discount factor	1.0000	0.9311	0.8797	0.8410	0.8115	0.7888	0.7713	0.7576	0.7470	...	0.7089	0.7085	0.7083	
Forward price	0.800	0.766	0.739	0.716	0.698	0.684	0.672	0.663	0.656	...	0.629	0.629	0.629	
FOREX information (1 Foreign : X Domestic)														
Risk-neutral FOREX rate	2.000	1.819	1.654	1.504	1.368	1.244	1.131	1.029	0.935	...	0.247	0.225	0.205	
Production statistics														
Mineral production (million units)						224.668	224.668	224.668	224.668	...	224.668	224.668	136.563	
Uninflated foreign project costs (million monetary units)														
Operating costs						33.700	33.700	33.700	33.700	...	33.700	33.700	20.500	
CAPEX	36.450	66.825	72.900	42.525	6.075									
Closure costs													18.750	
Inflated cash flow calculation (domestic currency; million monetary units)														
Expected operating revenue						194.804	195.884	196.694	197.307	...	199.339	199.355	121.184	
Risk-adjusted operating revenue						153.670	151.084	149.024	147.385	...	141.303	141.252	85.834	
Foreign-linked operating costs						60.986	59.778	58.595	57.434	...	43.408	42.549	25.370	
Operating cost						24.216	24.582	24.953	25.330	...	31.249	31.722	19.574	
Risk discounted operating profit						68.468	66.724	65.476	64.620	...	66.646	66.981	40.890	
CAPEX	24.300	45.274	50.080	29.707	4.884									
Foreign-linked CAPEX	72.900	131.004	140.083	80.097	11.216								17.917	
Closure costs													23.204	
Foreign-linked closure costs														
Risk discounted net cash flow	-97.200	-176.278	-190.163	-109.804	-16.100	68.468	66.724	65.476	64.620	...	66.646	66.981	-0.231	
Expected operating cash flow	-97.200	-176.278	-190.163	-109.804	-16.100	109.602	111.524	113.146	114.543	...	124.681	125.084	35.119	
RO NPV calculation (domestic currency; million monetary units)														
Time and risk discounted cash flow	-97.200	-171.068	-179.089	-100.354	-14.280	58.931	55.733	53.074	50.832	...	34.446	33.596	-0.112	
DCF NPV calculation (domestic currency; million monetary units)														
RADR (%):	10.0%	-97.200	-159.503	-155.692	-81.345	-10.792	66.477	61.206	56.187	51.467	...	13.815	12.541	3.186

Table 4. Inflated cash flow calculation for the Long-Life Mine.

Project time (years)	0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	...	15.0	16.0	17.0	
Time index	0	2	4	6	8	10	12	14	16	...	30	32	34	
Nominal mineral price														
Expected price	0.800	0.823	0.840	0.852	0.861	0.867	0.872	0.875	0.878	...	0.886	0.886	0.887	
Risk discount factor	1.0000	0.9311	0.8797	0.8410	0.8115	0.7888	0.7713	0.7576	0.7470	...	0.7150	0.7134	0.7122	
Forward price	0.800	0.766	0.739	0.716	0.698	0.684	0.672	0.663	0.656	...	0.633	0.632	0.631	
FOREX information (1 foreign : X domestic)														
Risk-neutral FOREX rate	2,000	1.819	1.654	1.504	1.368	1.244	1.131	1.029	0.935	...	0.481	0.437	0.398	
Production statistics														
Mineral production (million units)						224.668	224.668	224.668	224.668	...	224.668	224.668	224.668	
Uninflated foreign project costs (million monetary units)														
Operating costs						33.700	33.700	33.700	33.700	...	33.700	33.700	33.700	
CAPEX	36.450	66.825	72.900	42.525	6.075									
Closure costs													15.469	
Inflated cash flow calculation (domestic currency; million monetary units)														
Expected operating revenue						194.804	195.884	196.694	197.307	...	199.028	199.109	199.173	
Risk-adjusted operating revenue						153.670	151.084	149.024	147.385	...	142.312	142.052	141.846	
Foreign-linked operating costs						60.986	59.778	58.595	57.434	...	49.931	48.942	47.973	
Operating cost						24.216	24.582	24.953	25.330	...	28.135	28.560	28.992	
Risk discounted operating profit						68.468	66.724	65.476	64.620	...	64.246	64.550	64.881	
CAPEX	24.300	45.274	50.080	29.707	4.884									
Foreign-linked CAPEX	72.900	131.004	140.083	80.097	11.216								13.299	
Closure costs													22.021	
Foreign-linked closure costs														
Risk discounted net cash flow	-97.200	-176.278	-190.163	-109.804	-16.100	68.468	66.724	65.476	64.620	...	64.246	64.550	29.561	
Expected operating cash flow	-97.200	-176.278	-190.163	-109.804	-16.100	109.602	111.524	113.146	114.543	...	120.963	121.607	86.888	
RO NPV calculation (domestic currency; million monetary units)														
Time and risk discounted cash flow	-97.200	-171.068	-179.089	-100.354	-14.280	58.931	55.733	53.074	50.832	...	40.965	39.942	17.751	
DCF NPV calculation (domestic currency; million monetary units)														
RADR (%):	10.0%	-97.200	-159.503	-155.692	-81.345	-10.792	66.477	61.206	56.187	51.467		26.990	24.552	15.873

Table 5. Inflated cash flow calculation for the Short-Life Mine