"PROFIT SHARING AND ADJUSTMENT OPTIONS IN SUPPLY CONTRACTS"

BARDIA KAMRAD

AKHTAR SIDDIQUE

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Georgetown University
McDonough School of Business
Washington, D.C. 20057

Phone: (202) 687-4112
Fax: (202) 687-4031

KAMRADB@MSB.EDU
SIDDIQUA@MSB.EDU

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ABSTRACT

A common theme in the studies of flexible supply contracts has been the producer's profit maximization problem without regard for the suppliers' reactions to the producer's operating policies. However, suppliers do react and protect their downside against producer's operating policies by revising their strategies in a manner consistent with their profit maximization objectives. This fact motivates our work. Using a real options (contingent claims) approach, we analyze and value supply contracts in a setting characterized by exchange rate uncertainty, supplier-switching options, order quantity flexibility, profit sharing, and supplier reaction-options. We also use basic diversification concepts, from portfolio theory, to provide a unique framework for risk reduction. Given this set up, we explicitly model how flexibility can be mutually beneficial to the producer and his suppliers. We also analyze what induces the producer and the suppliers to accept flexibility in their contracts.

I. INTRODUCTION

A common theme in the studies of (flexible) supply contracts is the profit-maximization problem for the producer without regard for the suppliers' reactions to the producer's optimal operating policies. Yet, suppliers react through counteroffers, or by imposing scenario specific contract clauses with penalty premiums attached. These protective measures by suppliers can be viewed as an *option to react* to the producer's policies. In this respect, suppliers' counteroffers reflect the flexibility to revise actions. It also represents the suppliers' preferences and aversions toward reward and risk, reduce the producer's initial expected profits, and are likely to induce a revised offer by the producer. Repeated offers and counteroffers constitute a sequence of bargaining games over time that ensure no one party's benefits may be obtained at the expense of the other's loss. In this context, the bargaining games are cooperative. The notion of benefit reassessment, policy revision, and bargaining define our *profit sharing* view to the problem of analyzing supply contracts in operations. This perspective, in the context of a portfolio theory based real options valuation approach, characterizes a major contribution of this paper. It is also a significant departure from the traditional approach where the suppliers' reactions are, at best, exogenously fixed and state independent policies. To the best of our knowledge, the existing literature has not addressed the impact of producer's decisions on the suppliers' actions and contract values, nor has it considered the profit sharing implications of the suppliers' *reaction option*.

In this paper, we analyze and value supply contracts in an environment characterized by exchange rate uncertainty, order quantity flexibility, supplier-switching options, reaction options, and profit sharing. Given the aforementioned, we adopt a real options approach to the analysis and valuation of such contracts. Also, by viewing the producer's supplier selection set as a portfolio of risky assets, we employ basic diversification concepts from portfolio theory and provide a unique framework for risk reduction.

McDonald and Siegel (1985), Dixit (1989) and Pindyck (1991) have applied a real options methodology for valuing real assets with discretionary decisions in the presence of uncertainty. Kogut and Kulatilaka (1994) use real option pricing methodology to value the flexibility induced by exchange rate movements. They show that in the case of two plants in two countries, the value of the aggregate investment with an option to switch production between plants could be positive even though each investment by itself has negative net present value. Kamrad and Ritchken (1994) establish an optimal production and (finished goods) inventory policy for valuing supply contracts in the presence of input price uncertainty. Huchzermeir and Cohen (1996) also analyze the impact of exchange rate uncertainty on profit maximization problem for the producer in the presence of product options and switching costs. In this study, we explicitly incorporate the suppliers' reactions to the producer's policies along with that of the producer's, in a real options, optimization based valuation framework. In particular, given the opportunity to revise policies, we analyze what induces the producer and the suppliers to accept flexibility in their policies and how that flexibility affects their contracts' values. Thus, we explicitly model how a contract in an environment characterized by uncertainty, flexibility, and competition among the suppliers can provide for circumstances with neither the producer nor the suppliers being worse off as compared to inflexible contracts.[1] At one level, this implies that decisions to switch between suppliers are not costless, but result in the producer having to compensate the suppliers in manner conducive to their perception of relative risk. That is, the producer's flexibility to revise the composition of the supplier portfolio comes at an expense. Finally, the exogenous sources of risk can be viewed as value drivers for both the producer and the suppliers. We also examine how changes in the value drivers affect the profit-maximization, and the choice of the optimal policies, for the producer and the suppliers.

Related studies have resulted in development of various approaches to evaluating and analyzing supply contracts with multiple sources of uncertainty. Using an options approach, Li and Kouvelis (1999) analyze risk sharing in supply contracts in the presence of uncertainty and with respect to "time", and "quantity" flexibility. In their setting, the producer enters a contract with each supplier to purchase a certain amount of a material in order to satisfy its customers' future demand. Specifically, the producer must obtain D units (known demand quantity) of the material from its suppliers at or before time T (single period horizon) to satisfy the demand. In their setup, risk sharing occurs through a process of input price readjustment. This readjustment takes place in the face of stochastic prices for the product and in light of an exogenously fixed constant that effectively determines the degree of risk sharing. However, this eloquent approach ignores the reality that costs are not set unilaterally. The same also holds true when considering the level of risk sharing. That is, the degree of risk sharing is not the producer's unilateral perception and choice. Additionally, the degree of risk sharing, which is assumed to remain constant, is likely to be dependent on time, state or both. In a different setting, Dasu and Li (1997) examine operating policy ramifications in the presence of exchange rate uncertainty. They highlight the advantages of having a portfolio of suppliers whose exchange rates are not perfectly correlated. However, they do not probe the supplier portfolio properties and characteristics as we have, herein. Furthermore, their approach also ignores the risk or profit sharing implications that typically exist in such settings.

Our setup accommodates the general notion of profit sharing as captured by the producer's and the suppliers' reactions toward each others' policies. Our approach also endogenizes the degree and extent of profit sharing through operating policies. Thus, the relative gains to the producer from a flexible contract (i.e. order level flexibility) are smaller as compared to a contract wherein the suppliers do not react. A lowering of the contract value to the producer corresponds to a concurrent gain in the suppliers' contract values. The gain to the suppliers can alternatively be viewed as the value of a compound "*reaction*" option available to the suppliers. This setup is a more realistic representation, since suppliers typically require compensation for agreeing to a flexible contract. That is, in our approach, the "quantity risk" generated by the producer's preferred order level flexibility is priced. Also, the contracts considered here are not fixed price supply agreements.

Our modeling approach to risk minimization echoes the portfolio optimization problem in a risk/return tradeoff sense. From the producer's perspective, there is a set of potential suppliers whose exchange rates are not perfectly correlated. The producer can form a portfolio from this set such that the portfolio of suppliers has a lower total volatility than any single supplier's exchange rate volatility. The suppliers also face risk. Theirs, in addition to exchange rate uncertainty, also results from the volatility of the order levels

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[1] That is, the contract is Pareto Superior (or dominant). However, we note that the approach taken is **not** a game theoretic one.

over time, the “quantity risk”. That is, it results from the producer having the flexibility to revise periodic order levels in response to exchange rate uncertainty. In our model, the suppliers also exercise flexibility by having an option to react to the producer’s order size modifications and also by responding to exchange rate volatility. In reaction to quantity risk, the suppliers impose order quantity restrictions and by adjusting their sales prices. Sales price adjustments reflect competition among the suppliers as well as quantity discounts. The model we develop permits a tradeoff between the volatility of the supply schedule over time versus the prices (i.e. the supplier’s revenues) that the producer pays.[2] Within this setting and in light of uncertainty in the exchange rates the concurrent optimization/valuation problem of the producer and the suppliers’ is modeled. Here, the producer and the suppliers are not modeled as traditional profit maximizers. Instead, the profit maximization also includes a preference for lower exchange rate volatility by the producer and a preference for lower supply schedule volatility by the suppliers.

The paper is organized as follows. In the next section, we set out the assumptions, notation and the necessary framework. In section III, we develop the valuation models for the producer and the suppliers. These models reflect a sequence of Bellman valuation equations. Since these classes of models typically do not yield closed form solutions, we use numerical procedures for results. To that end, section IV develops a set of stochastic dynamic programs (SDP) defined on a multinomial lattice. Solving the SDPs recursively obtains the value maximizing policies for both the producer and the suppliers. By offering a stylized example, these results are illustrated and presented in section V. We also examine how changes in the properties of exchange rates and model parameters affect the optimal policies and profit sharing results. This is through comparative statics. The last section presents the conclusion and possible future extensions.

II. ASSUMPTIONS, NOTATION AND THE MODEL

A firm manufactures a product requiring a distinct raw material, component or sub-component as input. The firm furnishes this product to a client according to a predetermined rate, at a fixed price and delivery schedule. Let $P(t),\, t\in[0,T]$ define the time t sales price and $D(t),\ t\in[0,T]$ the demand rate over the production horizon $[0,T]$ where T depicts the contract's termination time. The firm produces at rate $q(t)$ and if needed maintains an inventory of its finished goods. Both production and inventory are assumed capacitated.

M suppliers, located in different countries, can supply the needed material for production. The producer selects from this pool over the production horizon, $[0,T]$. Given the pool of potential suppliers, the producer optimally creates a portfolio of suppliers by separately entering into a contract with each supplier. Let $\sim$ (tilde) denote the vector notation. The portfolio weights, $\tilde{u}(t)\in\mathbb{R}^M$, $t\in[0,T]$ identify a fraction of the

[2] Note that our set up does not involve a portfolio of producers for each supplier. Naturally, such a consideration would result in an intractable problem.

total needed input supplied by a particular supplier.[3] Let $I(t)$ define the input rate with $u_j(t)$. $I(t)$ as the fraction of the input supplied by supplier j at time t, $j = 1,2, \ldots, M$ where $0 \le u_j(t) \le 1.0$, with,

$$\sum_{j=1}^{M} u_j(t) = 1.0$$

(1)

The production rate, $q = \{q(t), t \in [0,T]\}$, which is a control variable, is taken as an adapted positive real valued process and is assumed capacitated: that is, $q(t) \in (0, \bar{q})$, with $\bar{q}$ as the maximum production capacity. The rate of production is related to the input rate via the production function,

$$q(t) = \left(I(t)\right)^b = \left(\sum_{j=1}^{M} I(t) u_j\right)^b$$

(2)

with $0 < b \le 1.0$. This provides for a convenient characterization of technology in terms of return to scale as depicted by b. The production cost is defined by the function, $A(q(t))$. We assume that periodic adjustments to the rate of production are costly, thus resulting in a production switching cost given by the function $f(q(s), q(t)), t > s \in [0,T]$. Since production is capacitated, when necessary the producer stockpiles output to meet its delivery obligations. Let $R(t)$ define the finished goods inventory level at time $t \in [0,T]$ with $\bar{R}$ as the inventory capacity. The holding cost function is $H(R(t))$. Given that the producer's demand schedule is known, it follows that:

$$dR(t) = \{q(t) - D(t)\}\, dt$$

(3)

The supply contract between the producer and its suppliers identifies an order-purchasing schedule for the parties involved. The producer adjusts its order quantities over time in response to exchange rate fluctuations.[4] The suppliers, in response, impose a penalty as a protective measure against unanticipated shifts in order quantities. These penalties curb the order size variability exposure. In our model, the penalties are defined as an additional surcharge to the producer if the change in the periodic order levels exceeds a supplier established penalty band, $e_j(t)$, $j = 1,2,...,M$. Since the change in periodic order quantities depend on the producer's demand schedule and the exchange rate dynamics, for analysis purposes $u_j = \{u_j(t), t \in [0,T]\}$ are also taken as an adapted positive real valued process with $0 \le u_j(t) \le 1.0$, where for any $t \in [0,T]$, condition (1) holds. Therefore, in addition to supplier switching options, adjustments to the rate of production and to the purchase weights also constitute a set of nested options available to the producer. To define stochastic evolution of the exchange rates, let $Z(t) \in \mathbb{R}^M$ represent a standard Brownian motion that is a martingale with respect to the probability space $(\Omega, \ddot{o}, \Im, -)$. The filtered probability space $(\Omega, \ddot{o}, \Im, -)$ is defined over $[0,T]$ where the augmented filtration,

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[3] Note that the extremities of these weights imply switching.

[4] This essentially implies that the portfolio weights (fraction to be purchased from a particular supplier), $u_j(t)$ are implicitly a function of the exchange rate process. Thus, in addition to the output rate, $q(t)$, we also define $u_j(t)$ as a control variable.

$\Im = \{ \ddot{o}_t : t \hat{I} [0,T]\}$, is right continuous and increasing. $\{X_j(t); t \geq 0\}$ is the spot exchange rate between the producer and supplier j = 1, 2, …, M where,

$$dX_j(t) = X_j(t)\{a_j dt + s_j dZ_j(t)\}$$

(4)

$$E\left(dZ_k(t) \cdot dZ_l(t)\right) = r_{kl} dt \qquad\qquad k, l = 1,2,...,M$$

(5)

The constant drift, a_j define the local trend of the process while the constant and instantaneous standard deviations, s_j characterize the volatility for each (real) exchange process. $dZ_j(t)$ represent an increment to the standard Gauss-Weiner process: as the exchange rates are assumed correlated, their instantaneous and constant correlation coefficient is given by r .[5] We also assume that the constant risk-free rate of interest in the supplier's market is r_j, the producer's is r_p, and that there is a futures currency market. To that end, let $F_j(X_j, T-t)$ represent the current price (in supplier j's currency) for delivery of one unit of the producer's currency at time T with, $F_j(X_j, 0) = X_j$ as the terminal boundary condition. We use the following no-arbitrage restriction (Ross, (1976)),

$$F_j(X_j, T-t) = X_j(t)e^{(r_p - r_j)(T-t)}$$

(6)

This is the covered interest rate parity (IRP) relationship, and establishes a deterministic relationship between the spot and futures exchange rates. For the most part, empirical tests of this no-arbitrage relationship have shown significance [6].

To value of the producer's supply contract, $V(.)$ the cash flows accrued must be defined. Let the net cash flow rate be defined by $f(t)$. This cash flow process reflects the revenue from meeting the demand schedule, less the cost of purchasing, holding, production, switching, and periodic penalties. To minimize its costs and to ensure a smooth flow of the needed input, the producer changes periodic order levels in response to exchange rate fluctuations. This objective is achieved by changing the portfolio weights, $u_j(t)$. Unlike standard financial options with contractually fixed exercise prices, the option to switch suppliers is quite complex as the exercise price for such an option is both state and policy dependent.

To the supplier(s), periodic order level changes imply unnecessary demand variability in terms of their schedule(s). To protect against their implicit costs, the suppliers tolerate order level changes so long as the change in the level of orders is within a supplier defined "penalty band". Otherwise, a penalty relative to

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5 The purchasing power parity (PPP) would imply mean reversion in the (real) exchange rate behavior. Yet, what stochastic process exactly depicts real exchange rates remains an open question. A mean reverting depiction can be accommodated in a restricted manner by allowing, $a_j X_j = \overline{X} - X_j$, where ? defines the mean reversion constant; pulling back the X_j to its long run mean value $\overline{X}$.

6 See Cornell (1977) and Giddy and Dufey (1975).

the extent of the departure from the band is charged. Therefore, supplier j, requires a penalty charge of magnitude, $p_j(\varepsilon_j(t))$ at time t, if for all $t > s$, with $t, s \in [0,T]$,

$$\left| u_j(t)I(t) - u_j(s)I(s) \right| > \varepsilon_j(t)$$

The penalty band, $\varepsilon_j(t)$, constrains the producer's flexibility for the obvious reason: larger bands imply a more convenient ordering policy for the producer, albeit at the suppliers' expense. In effect, the penalty bands, $\varepsilon_j(t)$ are the suppliers' "physical" reaction to the exchange rates fluctuations as the portfolio weights, $u_j(t)$ are that of the producer's. There is an implicit economic tradeoff between the choice of portfolio weights and order level flexibility. The producer's desired order level flexibility will be realized at a corresponding larger penalty so that,

$$\frac{dp_j(\varepsilon_j(t))}{d\varepsilon_j(t)} > 0 \qquad j = 1, 2, \ldots, M \qquad (7)$$

The producer's response to the market conditions, $u_j(t)$ and the suppliers' corresponding reaction, $\varepsilon_j(t)$ reflect a particular aspect of the tradeoff considered. A different aspect concerns the periodic sales prices established by the suppliers in response not only to the exchange rate fluctuations but also to the producer's purchase weight adjustments, resulting in a "quantity risk" to suppliers. We also assume that the supplier established sales prices include a quantity discount feature, but not at the expense of nullifying the penalty bands, $\varepsilon_j(t)$. Let $C_j(\tilde{u}, \tilde{e}, t) \in \mathbb{R}$ define the per unit gross margin rate for supplier j at time $t \in [0,T]$ and $j = 1, \ldots, M$.[9] We constrain the sales price function, $C_j(.)$ such that:

$$\frac{\partial C_j(.)}{\partial u_j(t)} < 0 \quad \text{and,} \quad \frac{\partial C_j(.)}{\partial \varepsilon_j(t)} > 0 \qquad j = 1, \ldots, M$$

(8)

The first term above reflects a quantity discount feature. The latter term indicating the fact higher desired flexibility by the producer (i.e. wider penalty bands) are achieved at a corresponding higher cost. Also, due to competition among the suppliers, supplier established sales prices depend on the exchange rates. Given the exchange rate process, $\tilde{X}(t) \in \mathbb{R}^M$, $t \in [0,T]$ let,

$$Y_{jk}(t) = \frac{X_j(t)}{X_k(t)} \qquad j \neq k; \; j, k = 1, 2, \ldots, M.$$

(9)

Expression (9) defines the exchange rate process between any pair of suppliers. Let $\tilde{Y}_j(t) \in \mathbb{R}^{M-1}$ represent time $t \in [0,T]$ vector of exchange rates between supplier j and other suppliers. That is, $\tilde{Y}_j(t) = \{Y_{jk}(t)\}$ with $j \neq k$; $j, k = 1, 2, \ldots, M$. Given expression (4), it is well established that $Y_{jk}(t)$ is lognormally distributed.[10] Supposing the existence of a futures currency market for all feasible currency

[9] Implicit is the assumption that fixed costs do not enter the analysis: that is, gross margin = price – variable cost.

[10] See Karlin and Taylor (1981).

pairs, let $F_{jk}(Y_{jk}, T-t)$ define the current price (in supplier j's currency) for delivering of a single unit of supplier k's currency at time T with, $F_{jk}(Y_{jk},0)=Y_{jk}$. Analogous to equation (6) we have for all $j \neq k$; $j,k=1,2,\ldots,M$,

$$F_{jk}(Y_{jk}, T-t) = Y_{jk}(t)e^{(r_j - r_k)(T-t)}$$

(10)

We derive a sequence of interrelated models for evaluating M supply contracts. Our approach requires construction of a requisite dynamic trading strategy that does not allow riskless opportunities. This continuous time arbitrage-based approach results in M+1 Bellman equations whose solutions define the set of optimal policies that maximize the contracts' values. The optimal policies for the producer and the suppliers are, $\{u^*(t), q^*(t)\}$ and $\{C^*(t), e^*(t)\}$, respectively. To that end, let $W_j(\tilde{Y}_j, t; C_j, e_j) \in \mathbb{R}$ represent the time $t \in [0,T]$ value of the contract for supplier j given that the supplier exchange rate vector is $\tilde{Y}_j(t) \in \mathbb{R}^{M-1}$, the sales price is $C_j(t)$ and the critical order level change threshold is set at $e_j(t)$, $j=1,2,\ldots,M$. Analogously, let $V(\tilde{X}, R, t; q\ \tilde{u}) \in \mathbb{R}$ represent the value of the supply contract to the producer at time $t \in [0,T]$ given that the current level of exchange rate process is $\tilde{X}(t) \in \mathbb{R}^M$, the finished goods inventory is, $R(t)$ and that the current policy in place is $\{q(t), u(t)\}$. We assume further that the functions $V(.)$ and $W_j(.)$ are Ito differentiable, for all $t \in [0,T]$ and $j=1,2,\ldots,M$.

III. The Valuation Models

Our approach requires the construction of a dynamic trading strategy. To avoid riskless arbitrage opportunities, the replicating portfolio positions are chosen so that the total expected return on the portfolio is the (local) riskless rate of return. We demonstrate how this dynamic trading strategy obtains the valuation model for the producer. Without loss of generality, a similar argument is also used in the follow-up subsection to develop the corresponding valuation models for the suppliers.

III.1 The Producer's Model

Let $y_p(t)$ represent the value of a portfolio at time $t \in [0,T]$ consisting of a long position in $V(t,.)$, together with d_j units short in the futures contracts on supplier j's currency with $j=1,2,\ldots,M$. Over an instant of time, dt we have after adjusting for the cash flows:

$$dy_p(t) = dV(t,.) - \{\sum_{j=1}^{M} d_j\, dF_j(.)\} + f(t)dt$$

(11)

while requiring;

$$E(dy_p(t)) = r_p V(t,.)dt$$

(12)

For equation (12) to hold, we choose d_j such that[11],

$$d_j = \frac{\partial V(t,.)/\partial X_j}{\partial F_j(.)/\partial X_j} \qquad j = 1,\ldots, M$$

(13)

Using equations (11), (12) and (13), we have from Ito's lemma:

$$\sum_{j=1}^{M}\frac{\partial V}{\partial X_j}X_j(r_p - r_j) + \frac{\partial V}{\partial R}(D-q) + \frac{1}{2}\sum_{j=1}^{M}\frac{\partial^2 V}{\partial X_j^2}X_j^2 s_j^2 + \frac{\partial V}{\partial t} + \sum_{\substack{j=1\\(j\neq i)}}^{M}\sum_{i=1}^{M}\frac{\partial^2 V}{\partial X_i X_j}X_i X_j s_i s_j r_{ij} + f(t) - r_p V = 0 \tag{14}$$

The net cash flow rate, $f(t)$ is given by;

$$f(t) = P(t)D(t) - \{A(q(t)) + H(R(t)) + \sum_{j=1}^{M} C_j(\tilde{u},\tilde{e},t) + \sum_{j=1}^{M} p_j(e_j(t))\} \tag{15}$$

The above expressions (14) and (15) fully characterize the producer's contract value. Conditional on a predetermined supplier policy, $\{C_j(\tilde{u},\tilde{e},t),\ e_j(t)\}$ and a predefined $p_j(e_j(t))$ the optimal value of the contract and the corresponding value maximizing policy $(q^*(t),\ \tilde{u}^*(t))$ is obtained from the following Bellman valuation equation for all $t, s \in [0,T]$ and $s < t$, $j = 1,2,\ldots,M$:

$$\underset{(q,u)}{Max}\left\{ \sum_{j=1}^{M}\frac{\partial V}{\partial X_j}X_j(r_p - r_j) + \frac{\partial V}{\partial R}(D-q) + \frac{1}{2}\sum_{j=1}^{M}\frac{\partial^2 V}{\partial X_j^2}X_j^2 s_j^2 + \frac{\partial V}{\partial t} + \sum_{\substack{j=1\\(j\neq i)}}^{M}\sum_{i=1}^{M}\frac{\partial^2 V}{\partial X_i X_j}X_i X_j s_i s_j r_{ij} \right.$$
$$\left. + P(t)D(t) - \left\{A(q(t)) + H(R(t)) + \sum_{j=1}^{M} C_j(\tilde{u},\tilde{e},t) + \sum_{j=1}^{M} p_j(e_j(t))\right\} - r_p V \right\} = 0$$

(16)

s.t.

$$0 \le R(t) \le \overline{R}$$

(16a)

$$0 \le q(t) \le \overline{q}$$

(16b)

$$0 \le \tilde{u}(t) \le 1.0 \tag{16c}$$

$$p_j(e_j(t)) = \begin{cases} h(e) & if \quad |u_j(t)I(t) - u_j(s)I(s)| > e_j(t) \\ 0 & \text{otherwise} \end{cases} \tag{16d}$$

with

$$\lim_{\tilde{X}\to\infty}\frac{V(\tilde{X},R,t;q,\tilde{u})}{\tilde{X}} = 0$$

(16e)

[11] The derivation of d_j is straightforward, though tedious. For brevity sake, we have excluded it form the paper.

$$\lim_{\tilde{X}\to 0}\frac{V(\tilde{X},R,t;q,\tilde{u})}{\tilde{X}} < \infty$$

(16f)

Expressions (16a-c) are self-explanatory. In (16d), the supplier imposed penalty functions and penalty conditions are defined. We will define $h(.)$ specifically when the model is illustrated through a stylized numerical example. Conditions (16e,f) ensure that the valuation function is well behaved. The above Bellman equation (16) does not yield a closed form solution and must be solved numerically to obtain the optimal contract value $V^*(t,.)$. However, in light of known functional forms that characterize the above equation, it is possible to derive closed form expressions for the optimal policies in place: that is $\{q^*(t),\ u^*(t)\}$.

III.2 <u>The Suppliers' Model</u>

Using a similar approach, we can also obtain the suppliers' valuation models in the form of a sequence of Bellman equation. Here, suppliers' policies are defined by their sales price and penalty bands, $C_j\left(\tilde{u},\tilde{e},\ t\right)$ and $e_j(t),\ j=1,2,\ldots M$, respectively. Furthermore, the cash flow accrued to the each supplier over an instant, dt is defined by;

$$C_j(.)+p_j(e_j(t)) \tag{17}$$

Given equations (9), (10) and (17) we have, without loss of generality, for $j=1,2,...,M$:

$$\underset{(C_j,e_j)}{Max}\left\{\sum_{\substack{k=1\\k\neq j}}^{M}\frac{\partial W_j}{\partial Y_{jk}}Y_{jk}(r_j-r_k)+\frac{1}{2}\sum_{\substack{k=1\\k\neq j}}^{M}\frac{\partial^2 W_j}{\partial Y_{jk}}Y_{jk}(s_j^2+s_k^2-2s_j\,s_k\,r_{jk})+\frac{\partial W_j}{\partial t}+p_j(e_j(t))+C_j\left(\tilde{u},\tilde{e},t\right)-r_jW_j\right\}=0 \tag{18}$$

s.t.

$$0\le u_j(t)\le 1.0 \tag{18a}$$

$$\lim_{Y_j\to\infty}\frac{W_j(\tilde{Y}_j,t;C_j,e_j)}{Y_j}=0 \tag{18b}$$

$$\lim_{Y_j\to 0}\frac{W_j(\tilde{Y}_j,t;C_j,e_j)}{Y_j}<\infty \tag{18c}$$

$$p_j(e_j(t))=\begin{cases} h(e) & if\quad \left|u_j(t)I(t)-u_j(s)I(s)\right|>e_j(t)\\ 0 & \text{otherwise}\end{cases} \tag{18d}$$

We omit elaborating on the supplier's model constraints: they are analogous to the producer's.

IV. <u>The Solution Procedure</u>

The Bellman Valuation equations (16) through (18) fully characterize each party's optimal policies and contract value. For the producer, $\left\{q^*(t),u_j^*(t)\right\}$ signify the optimal production rate and purchase weights, respectively. For the suppliers, $\left\{C_j^*(t),\ e_j^*(t)\right\}$ maximize their respective net revenues, and penalty bands.

Equations (16) through (18) do not yield closed form analytic solutions and must be solved numerically to obtain results. To that end, we develop a backward stochastic dynamic programming procedure to numerically solve the concurrent models characterized by expressions (16) – (19). We assume that the number of suppliers, M = 2 and employ the Feynmann-Kac results to accommodate the recursive procedure (algorithm). This result essentially states that for evaluation purposes, the solution to certain partial differential equations are known to be equivalent to the solution of an appropriately adjusted *expectation*. Such an adjustment is with respect to the drift term, a_j of the exchange rate process as shown in equation (4). In particular, as a consequence of the arbitrage-free valuation approach employed herein, this drift maybe proxied by an equivalent martingale measure: the risk neutralized expected growth rate, $(r_p - r_j),\ j = 1,2\ldots,M$. Hence, for valuation purposes only, equation (4) can be replaced by,

$$dX_j(t) = X_j(t)\left\{(r_p - r_j)dt + s_j dZ_j(t)\right\} \tag{19}$$

That is, the risk neutralized exchange rate process. To implement our recursive procedure, we adapt a *multinomial lattice* framework to approximate the stochastic evolution of the above equation (19). We use this approximating procedure together with the backward stochastic dynamic programming (SDP) algorithm to obtain the required *expectations*, which are "state" and "action" dependent. This recursive (SDP) algorithm is essentially comprised of three components. First, partitioning of the planning horizon $[0,T]$ into n equi-width time intervals. Second, defining the approximating lattice upon this time partition. Third, transposing equations (16) – (18) as a set of backward dynamic programming recursions relative to the same time partition and by superimposing them on the lattice to obtain expected values. To that end, let $\mathbb{P}_n = \{0 \le t_0 < t_1 < t_2 < \ldots t_n = T\}$ represent an equi-width partition of the planning horizon $[0,T]$, such that during each production period $[t_i, t_{i+1}] = \Delta t$, the exchange rate pair $\left(X_1(t_i), X_2(t_i)\right),\ i = 0,1,2,\ldots,n-1$ can take on any one of the following five jump values.

Jump Event	Corresponding Values	Probability
(up, up)	$X_1(t_0)w_1$, $X_2(t_0)w_2$	p_1
(up, down)	$X_1(t_0)w_1$, $X_2(t_0)d_2$	p_2
(down, down)	$X_1(t_0)d_1$, $X_2(t_0)d_2$	p_3
(down, up)	$X_1(t_0)d_1$, $X_2(t_0)w_2$	p_4
(none, none)	$X_1(t_0)$, $X_2(t_0)$	p_5

With $\sum_{j=1}^{5} p_j = 1$. In the above table, $w_k = e^{l s_k \sqrt{\Delta t}}$ defines the sign of an up jump while $d_k = (w_k)^{-1}$ is the down jump size with s_k and Δt as defined earlier, $k = 1,2$. The stretch parameter $l \ge 1$ ensures the opportunity for a horizontal jump (i.e., a no-change state) in each period. This lattice specification is due to Kamrad and Ritchken (1991) and its computational accuracy in approximating the lognormal distribution of the exchange rate pair is well documented. For valuation purposes the risk neutralized probability terms are needed and are expressed below:

$$p_1 = \frac{1}{4}\left\{\frac{1}{l^2} + \frac{\sqrt{\Delta t}}{l}\left(\frac{m_1}{s_1} + \frac{m_2}{s_2}\right) + \frac{r}{l^2}\right\} \tag{20a}$$

$$p_2 = \frac{1}{4}\left\{\frac{1}{l^2} + \frac{\sqrt{\Delta t}}{l}\left(\frac{m_1}{s_1} - \frac{m_2}{s_2}\right) - \frac{r}{l^2}\right\} \tag{20b}$$

$$p_3 = \frac{1}{4}\left\{\frac{1}{l^2} + \frac{\sqrt{\Delta t}}{l}\left(\frac{-m_1}{s_1} - \frac{m_2}{s_2}\right) + \frac{r}{l^2}\right\} \tag{20c}$$

$$p_4 = \frac{1}{4}\left\{\frac{1}{l^2} + \frac{\sqrt{\Delta t}}{l}\left(\frac{-m_1}{s_1} + \frac{m_2}{s_2}\right) - \frac{r}{l^2}\right\}$$

(20d)

$$p_5 = 1 - \frac{1}{l^2} \tag{20e}$$

$$m_k = r_p - r_k - \frac{s_k^2}{2} \qquad k = 1,2$$

(21)

Expression (21) defines the drift of the approximating process. Toward developing the SDP, let $V_i(\tilde{X}, R; \tilde{u}_{i-1}, q_{i-1})$ represent the time t_i value of the contract to the producer given that the current exchange rate is $(X_1(t_i), X_2(t_i))$, the finished goods level of inventory is R_i, during production period $[t_{t-1}, t_i]$ the raw material purchase weights were $(u_1(t_{i-1}), u_2(t_{i-1}))$ and that q_{i-1} units were produced. Given the partition, $\mathbb{P}_n$ consider the time epoch t_n :

$$V_n(\tilde{X}, R; \tilde{u}_{n-1}, q_{n-1}) = P_n D_n - \{A(q_{n-1}) + f(q_{n-1}, q_n)\}$$

(22)

$$A(q_n) = 0$$

(22a)

$$R_n = q_n = 0 \tag{22b}$$

$$q_{n-1} = (I_{n-1})^b = \{I_{n-1}\ u_1(t_{n-1}) + I_{n-1}\ u_2(t_{n-1})\}^b$$

(22c)

$$u_1(t_{n-1}) + u_2(t_{n-1}) = 1.0$$

(22d)

$$f(q_{n-1}, q_n) = f(q_{n-1}, 0) \tag{22e}$$

Expressions (22a,b) define the terminal conditions: at time t_n the level of finished goods inventory and production rate are set to zero and that the cost of production is zero. Equation (22c) relates the production quantity decision to total raw material purchase at time t_n. Equation (22d) identifies purchase weight's constraint, while (22e) is the production-switch cost function at expiration: shutdown cost. The production cost $A(q_{n-1})$ is incurred at time t_n.

Let $C_j(\tilde{u},\tilde{e},t) \equiv C_j(t_i)$ and $p_j(e_j(t_i)) \equiv p_j(t_i)$ define the per unit gross margin and penalty induced revenue to supplier j at time t_i, $j = 1,2$ and $i = 0,1,\dots,n$. Consider time t_{n-1}, marking the period $[t_{n-1}, t_n]$ where

$$V_{n-1}(\tilde{X},R;\tilde{u}_{n-2},q_{n-2}) = \underset{(\tilde{u}_{n-1},q_{n-1})}{Max} \{\tilde{E}\{P_{n-1}D_{n-1} - \{A(q_{n-2}) + H(R_{n-1}) + \sum_{j=1}^{2} C_j(t_{n-1}) + \sum_{j=1}^{2} p_j(t_{n-1})$$

$$+ f(q_{n-2},q_{n-1})\} + e^{-r_p(t_n - t_{n-1})} V_n(\tilde{X},R;\tilde{u}_{n-1},q_{n-1})\}\} \tag{23}$$

s.t.

$$R_{n-1} + q_{n-1} = D_n$$

(23a)

$$q_{n-1} = \{I_{n-1}\, u_1(t_{n-1}) + I_{n-1}\, u_2(t_{n-1})\}^b \tag{23b}$$

$$q_{n-2} = \{I_{n-2}\, u_1(t_{n-2}) + I_{n-2}\, u_2(t_{n-2})\}^b \tag{23c}$$

$$u_1(t_{n-1}) + u_2(t_{n-1}) = 1.0 \tag{23d}$$

and for $j = 1,2$

$$p_j(t_{n-1}) = \begin{cases} 0 & if \quad |I_{n-2}u_j(t_{n-2}) - I_{n-1}\, u_j(t_{n-1})| \le e_j(t_{n-1}) \\ h(e_j(t_{n-1})) & if \quad o/w \end{cases} \tag{23e}$$

$$0 \le q_{n-2} \le \overline{q} \tag{23f}$$

$$0 \le q_{n-1} \le \overline{q} \tag{23g}$$

$$0 \le R_{n-1} \le \overline{R} \tag{23h}$$

In equation (23), $\tilde{E}(\cdot)$ defines the expectation operator with respect to equation (19). Furthermore, the total cost incurred at time t_{n-1} is comprised of production, inventory, purchase, switching and penalty costs. Constraint (23a) ensures that the level of inventory on hand together with next period's production quantity satisfies the demand during the next period. The penalty conditions and surcharge are given by expression (23e). Other constraints, (23 f, g, h) account for production and inventory capacity. In general, we have for every period $[t_i, t_{i+1}]$, $i = 0,1,\dots,n-1$ the following Bellman recursion:

$$V_{i-1}(\tilde{X},R;\tilde{u}_{i-2},q_{i-2}) = \underset{(\tilde{u}_{i-1},q_{i-1})}{Max} \{\tilde{E}\{P_{i-1}D_{i-1} - \{A(q_{i-2}) + H(R_{i-1}) + \sum_{j=1}^{2} C_j(t_{i-1}) + \sum_{j=1}^{2} p_j(t_{i-1})$$

$$+ f(q_{i-2},q_{i-1})\} + e^{-r_p(t_i - t_{i-1})} V_i(\tilde{X},R;\tilde{u}_{i-1},q_{i-1})\}\} \tag{24}$$

For brevity's sake, we omit restating constraints (24b-h) as they echo previously stated constraints (23b-h), which must be modified to the appropriate time increments. Given this SDP-based solution approach to the producer's problem, we can now turn to the suppliers'. To this end, the net revenue accrued during each purchase period is defined first. This revenue is comprised of cash flows generated through the sale of raw materials, $C_j(\tilde{u},\tilde{e},t) \equiv C_j(t_i)$ in addition to the penalties paid by the producer, $p_j(e_j(t_i)) \equiv p_j(t_i)$.

Let $W_i(Y_i;C_j,e_j)$ represent the time t_i value of the contract to supplier j given that the supplier's exchange rate is $Y(t_i) = Y_{jk}(t_i) = X_j(t_i)(X_k(t_i))^{-1}$, $j = 1,2$ and $k \neq j$, that the sales proceeds are set to be $C_j(t_i)$ and the "penalty band" is established to be $e_j(t_i)$. The value of the contract to supplier j (in supplier j's currency) at time t_n is

$$W_n(Y;C_j,e_j) = 0 \qquad j = 1,2 \tag{25}$$

For all other time epochs t_i, $i = 0,1,\ldots,n-1$ and the corresponding Bellman recursion equation representing the suppliers' contract values expressed in each supplier's home currency, is given by,

$$W_i(Y;C_j,e_j) = \underset{(c_j,e_j)}{Max}\left(E\left(\left(\frac{C_j(t_i)}{X_j(t_i)} + \frac{p_j(t_i)}{X_j(t_i)}\right) + e^{-r_j(t_{i+1}-t_i)}W_{i+1}(Y;C_j,e_j)\right)\right)$$

(26)

with $p_j(t_i) = h(e_j(t_i))$. Equations (22) through (26) define our SDP. In light of equation (26), it should be clear that each party maximizes its' contract value in its own currency. To obtain results, we superimpose the above equations on the approximating lattice introduced earlier where expectations are taken with respect to the probability terms, (20 a-e). A given lattice node, such as node, (i,k,l) represents the state of the exchange rate vector at time t_i, $i = 0,1,2,\ldots n$. That is,

$$(X_1(t_0), X_2(t_0)) = (X_1(t_0)w_1^k, X_2(t_0)w_2^l)$$

with w_1 and w_2 defined earlier. Here, k and l define the number of up jumps each underlying variable has taken at the time t_i. Furthermore, the set of feasible realizations, for k and l in approximating the stochastic evolution of exchange rates is given by $k,l \in (B(i),B'(i))$ with,[12]

$$B(i) = \{-i,-i+2,\ldots,i-2,i\} \quad \& \quad B'(i) = \{-i+1,-i+3,\ldots,i-3,i-1\}$$

In the following section, we illustrate this set up through a stylized example where the expectations as seen in equation (22) through (26) are computed using the lattice probability terms (20a-e).

[12] The sets *B(i)* and *B'(i)* define the time t_i feasible realizations (nodes) for the exchange rates on the lattice. At $i = 2$ we have *B(2)*=(-2, 0, 2) and *B'(2)*=(-1,1). The feasible nodes that approximate the stochastic evolution of the exchange rate pair are: {(2,2), (2,0), (2,-2), (0,2), (0,0), (0,-2), (-2,2), (-2,0), (-2,-2), (1,1), (1,-1), (-1,-1), (-1,1)}. That is, for every *k* in *B(i)*, every *l* in *B'(i)* is exhausted. In this context, as an example, node (2,-2) defines the situation where the first exchange rate has had two consecutive up jumps while the second exchange rate has had two consecutive down jumps. Likewise, at time t_2, node (0,2) defines the state wherein the first exchange rate has made no change in its value while the second exchange rate has had two consecutive up jumps. See also Kamrad and Ritchken (1991) for further detail.

V. Illustration and Results

We illustrate the models numerically in the case of two suppliers (M = 2) and the producer. We consider a planning horizon of T= 1.0 year which, for valuation purposes has been partitioned into $n=4$ production periods. The producer's demand schedule and other relevant input parameters are provided in Table 1. The production cost function, the purchase cost function, and the penalty cost function for each supplier $j=1,2$ at time t_i is also defined:

$$A(q_i) = a_0 + a_1 q_i + a_2 q_i^2 \tag{27}$$

$$C_j(t_i) = \left(C_{0j} - C_{1j}(t_i) I(t_i) u_j(t_i)\right) e_j(t_i)\, u_j(t_i) \tag{28}$$

$$p_j(t_i) = h(e_j(t_i)) = (C_{0j}^2 e_j(t_i) + C_{1j}^2(t_i) e_j(t_i) - 2C_{0j}^2 C_{1j}^2(t_i) e_j^2(t_i)) \tag{29}$$

Equation (27) reflects the quadratic nature of the production cost function. In equation (28), we rate the quantity discount feature of the purchase cost function. Specifically, the higher the purchase quantity (Iu_j) from a supplier, the lower the variable cost of the purchased quantity. Equation (29) defines the penalty cost to the producer: violating contract terms are effectively penalized as a function of current purchase costs and "penalty bands". It is important to recognize that we are faced with two separate problems. The first problem is to establish, at each node of the lattice, through an optimization based bargaining approach, the optimal operating policies for each player. The optimal policies result in the corresponding optimal contract values. At a given node, this optimization occurs repeatedly in the context of the offers/counteroffers betwixt the producer and the suppliers until an agreement, as specified below, is reached. The second problem is to compute the value of the contracts to the players. This value, at a given lattice node, is the expected discounted present value of the appropriate "future" optimal node values. Appendix A, provides further implementation-related details on the optimization methods and bargaining rounds we deploy at each of the node.

At each node of the approximating lattice at time $t_i,\ i = 0,1,2\ldots,n-1$ the producer must establish the optimal output quantity and the proportion of raw material to purchase (i.e. q, u) from each supplier. These policies are then passed onto the suppliers whom in response optimally establish their policies (i.e. e_j, C_{1j}). One set of sequential optimizations by the producer and the suppliers' amount to a single bargaining round. At the end of the round, the producer starts a new round by treating the previous round's optimal e_j and C_{1j} as given. The producer then carries out an optimization that produces new optimal policies u and q. If the optimum contract value reached by the producer in this round is higher than the previous round's then the producer's offers are passed onto the suppliers as described before. However, if the producer's value is lower than the previous round's, we are faced with a conundrum. To prevent the players from in effect "going home with the ball", we need to impose auxiliary conditions on the bargaining. It is well established that in any bargaining game the offers and counteroffers can diverge unless an additional mechanism is imposed to ensure convergence. Such mechanisms can include (lump sum) side payments between the players, maximum limits on the number of bargaining rounds of offers and counter-offers, or other forms of limits. With side payments, a player may accept an inferior offer if it is accompanied by a sufficiently large lump sum cash payment. However, side payments do not make economic sense in our setting since to establish the size of the requisite cash payments we need to know the reservation prices as well as the full spectrum of preferences (utility functions). In a similar vein, constraining the maximum number of rounds can result in offers being widely apart in our setting, during the last round. When offers are widely apart, results can be economically nonsensical.

To ensure convergence, we impose a 10% limit on the contract value changes. In particular, if a counter-offer lowers the contract value for a player, the bargaining can continue as long as the value reduction is 10% of the previous round's value or less. This mechanism, in effect, assures that the players do not abandon the game if a particular round temporarily lowers the contract values. Thus, our solutions are Pareto superior without a burden of formal proof. We have also required a minimum of four bargaining rounds to avoid locally optimal solutions in the process. At time $t=0$, we relax this 10% constraint. We do not impose an upper limit on the number of rounds but permit the rounds to go on as long as there are improvements in the players' contract values. We then compute discounted expectations recursively backward to time zero. Thus, at each time and node, we have effectively a vector of the producer's policies and the corresponding policies made by the suppliers. To provide some intuition, it is important to note that the optimization implements the risk/return tradeoffs we have identified before. In particular, for the producer, the weights on the suppliers, u_1 and u_2, parallel the security weights. Commensurately, for the suppliers, the problems faced by each supplier involve conflicting choices. If C_{1j} were to be increased, the revenues also increase. However, the producer can then purchase fewer units. Concurrently, the choice of a smaller penalty band, e_j results in increased penalties paid. But here too, the producer is also likely to purchase smaller quantities. Table 1, shown below, summarizes our base case parameter values used to generate results.

Table 1: Base Case Parameters and Initial Values for Variables

Producer's Demand Schedule	D_i	$D_1 = 120;\quad D_2 = 120;\quad D_3 = 170;\quad D_4 = 120$
Producer's unit Sales Price	$P(t_i)$	$P(t_i) = P = \$80$ per unit
Production Capacity	$\bar{q}$	$q_i \in (0,300)$
Inventory Capacity	$\bar{R}$	$R_i \in (0,60)$
Production Cost Function	$A(q_i)$	$a_0 = 1.2;\ a_1 = 0.8;\ a_2 = 0.2$
Elasticity	b	$b = 1.0$
Initial Exchange Rate	$\tilde{X}_0$	$(X_1(t_0), X_2(t_0)) = (1.0,1.0)$
Exchange Rate Volatilities and Correlations	s	$s_1 = .80;\ s_2 = 1.00;\ r_{12} = .80$ (per annum)
Local riskfree interest rates	r	$r_p = .08;\ r_1 = .10;\ r_2 = .15$ (per annum)
	Supplier 1	$(C_{01} = 3.0;\ C_{11} = .15;\ e_1 = 20)$

	Supplier 2	

To focus discussion, our findings and results have been classified into three distinct subsections. They reflect the supplier switching option value for the producer and related parametric results; contract values to the producer / suppliers and related comparative static results; and results on the optimal policies for the producer and the suppliers. In Figure 1 we show three different sample paths on the lattice where for each path considered, Table 2 provides the corresponding optimal policies and contract values.

[FIGURE 1] [TABLE 2]

V.1 <u>The Value of Option to Switch</u>

The value of the (producer's) option to switch between the suppliers is the defined as the producer's contract value with the option to switch, less the maximum of the contract values arising from each supplier alone. That is, the option value is the value of the flexible contract less the best of the dedicated contract values. As such, the switch option value is given by,

$$V^*(u_1,u_2) - Max\left(V^*(u_1=1,\ u_2=0),V^*(u_1=0,\ u_2=1),0\right)$$

As long as this value is positive, the option to switch is valuable for the producer even if the suppliers react to the producer's policies by altering their decisions. The sensitivity of this option value with respect to the correlation coefficient between the two exchange rates as well the relative volatilities are insightful and of interest. For the latter, the ratio of the volatility terms σ_1/σ_2 is used, where the second supplier's exchange rate volatility is fixed. An alternative would have been to reflect on the difference between the volatility terms. Yet, relative comparisons make logical sense and have intuitive appeal.

To that end, note that in traditional financial option pricing theory the value of an option (put or call) increases as the volatility of the underlying asset increases. This intuition also underlies the results we obtain on the switch option value as a function of the parameters that characterize it. However, a very important difference between standard financial options and the real option in our setup is that the parameters for the underlying asset in a standard financial option do not change as a result of exercising the option. However, in this (compound) real option setting the producer and suppliers can, and indeed, do alter their actions in response to actions undertaken by the others. Here, the value of the option is higher when the total volatility increases: which can increase either when the correlation increases or (at a given positive correlation) when the volatility of one or both of the suppliers' exchange rates increases. The volatility of the portfolio of suppliers can be defined as:

$$\sigma_{portfolio} = \sqrt{u_1^2\sigma_1^2 + u_2\sigma_2^2 + 2\rho_{12}u_1u_2\sigma_1^2\sigma_2^2}$$

The option to switch is worth more when the total volatility increases. Thus, as the correlation increases, all being equal, the total volatility faced by the producer increases and the option to switch becomes more valuable. However, it is important to note that when the correlation is +1, there is no option to switch, since notwithstanding individual volatilities, the two exchange rates tend to behave identically. As the total variability (defined above) increases due to $\rho \to 1$, the downside to the producer's contract value is truncated because there is a switching option while the upside potential remains intact until, $\rho = +1$. In this case, the producer will, correctly, choose the lowest volatility supplier. In a similar manner, when the correlation declines, there is less need to switch since there is a "natural diversification" effect in progress with very low correlations. That is, at very low correlations, an increase in one supplier's exchange rate is almost perfectly offset by a corresponding decrease in the other. When the correlation is –1, the exchange rate movements perfectly offset each other and the value of the option to switch becomes zero. In this case, the producer will optimally choose her supply quantities (i.e., u_1, u_2) in a manner that minimizes total volatility of the portfolio of suppliers. In Figure 2, we present the value of the option to switch as a function of the correlation between the exchange rates. This explanation provides the basic insight for Figure 2, where the option value increases as the correlation increases.

[FIGURE 2]

Figure 3 shows the value of the option to switch and the producer's contract value, as a function of the ratio of the volatilities, s_1 / s_2. Here, the volatility of the second supplier is fixed at 85% (per annum) while the volatility of the first supplier is subject to change. In comparing the option value to that of the contract value, it becomes evident that for the parameter values considered, the value of the contract to the producer, without the option to switch is highly negative. In other words, the switch option, given these particular parameter values, makes the supply contract lucrative to the producer. This particular issue becomes even more obvious when we consider the results shown in Figure 4.

[FIGURE 3]

V.2 <u>The Contracts' Values</u>

To understand how the models' parameters affect the profit sharing nature of the contracts, we consider their impact on both the producer and the suppliers' contract values. To that end, we examine the effect of the correlation coefficient, as well as the volatility terms. As before, the ratio of the volatilities s_1 / s_2 is used with $s_2 = 0.85$ and s_1 changing. Figure 4 presents the producer's contract value across different volatility ratios for a fixed correlation coefficient. The four value curves considered reflect: **(I)** producer's contract value with an optimally established purchasing policy (i.e. portfolio weights are optimally established) where the suppliers have no opportunity to react to the producer's purchasing policy or the market; **(II)** producer's contract value with an optimally established purchasing policy where the suppliers have the opportunity to react to the producer's policy and the market; **(III)** producer's contract value when fully supplied (i.e., 100%) by the second supplier and the supplier having no opportunity to react and; **(IV)** producer's contract value when fully supplied (i.e., 100%) by the first supplier and the supplier having no opportunity to react . The area between the first and the second value curves defines the total loss of value to the producer as a result of profit sharing. This benefit, which is gained by the suppliers, effectively arises from the suppliers having the (nested) option to react to the producers' purchasing policy and to the market over time. In essence, this collective gain to the suppliers can be viewed as the value of a "*reaction*" option available to the suppliers. The area between the first and third value curves defines the benefit arising from supplier diversification. Specifically, this is the value accrued to the producer by adding the first supplier to a portfolio that already contains the second supplier. Likewise, the gap between the first and the fourth value curves defines the value obtained from diversification by including the second supplier. When viewed across different correlations, the value of diversification to the producer (as well as the suppliers) is higher for higher correlations. The same is true for relative volatilities. These issues are reflected upon in Figures 7 and 8, and will be discussed, accordingly, in more detail.

[FIGURE 4]

In Figure 5, the producer's contract value sensitivity to variations in the correlation coefficient and across different relative volatilities is considered. For increased levels of correlation, we observe that the producer's gains, which result from a corresponding increase in switch option values, dominate the losses. These losses result from lower diversification due to lower correlation coefficient values. These results are consistent with our earlier observations in that increased correlation levels tend to elevate the switch option value to the producer. Analogously, for a fixed correlation, higher volatility ratios result in higher contract values to the producer. The basic essence of this observation also holds true for the suppliers' contract values. The key issue is that with increased volatility, the suppliers also tend to gain. The source of this gain is, however, from penalty based revenues rather than increased sales revenues due to higher prices. This is subtle. Specifically, as a result of increased exchange rate volatilities, order level changes by the producer become more likely. Viewed from the suppliers' perspective, this results in a more volatile supply schedule and thereby a higher propensity for the producer to violate penalty bands, e and therefore, trigger penalties. Figure 6 represents this phenomenon for the first supplier. The results for the second supplier also look similar but have not been included.

[FIGURE 5] & [FIGURE 6]

As stated earlier, Figures 7 and 8 provide a sense of how the value of diversification to the producer (and the suppliers) changes across different volatility ratios and correlations. For the producer, increased contract values due to increased volatilities and correlations result from a corresponding increase in the value of the option to switch. On the other hand, the corresponding increase in the supplier's contract value (Figure 8) results from higher penalty revenues. In particular, for increased correlations, across different volatility ratios, the propensity for the producer to violate the supplier imposed penalty bands increases. This triggers higher penalty revenues, which accrue to the suppliers. This increased benefit to the supplier(s') contract value(s) is precisely the loss that the producer observes as a result of profit sharing and as stated earlier, when Figure 4 was considered. The concurrent ramification of these simultaneous parametric changes on the producer's and the suppliers' contract values is furnished by Figures 9, 10, and 11, respectively. In the following subsection, the operating policy results, their impact and their ramification to the models risk parameters provides a deeper insight to the results considered thus far.

[FIGURE 7] [FIGURE 8]

&

[FIGURE 9] [FIGURE 10] [FIGURE 11]

V.3 The Operating Policies

We also investigate the economic impact of the optimal policies. To that end, Figures 12, 13, 14, 15, and 16 provide a graphic representation the optimal policies and contract values. In this context, Table 3 summarizes these results by considering the impact of changing the volatility ratio, s_1 / s_2 and the correlation coefficient, r , respectively. In Figure 12, an increase in the level of the volatility ratio or in the correlation coefficient results in a corresponding increase in the producer's purchase weight for the first supplier (i.e., u_1), and a corresponding decrease for the second (i.e., $u_2 = u_1 - 1$). Given this observation, an increase in the level of r results in a corresponding increase in the level of optimal policies for the suppliers: that is, $(e_1, C_{11});(e_2, C_{12})$, as shown in Figures 13,14, 15, and 16. To provide insight, recall that as r increases, so will the total risk to the producer. As explained earlier, an increase in the level of risk exposure to the producer increases her likelihood of supplier switching. The suppliers, on the other hand, try to dampen the producer's propensity to switch, or shift purchase weights drastically, by increasing their penalty bands (i.e.,e_1, e_2 in Figures 13, 14), thus allowing the producer more flexibility to change order levels in the face of increased total risk. This, however, comes at the expense of loosing penalty triggered revenues. To compensate for this loss, sales prices, C_{1j} are increased enough to recover lost revenues, optimally without triggering the producer to switch. That said, we note that the suppliers' optimal policies,$(e_1, C_{11});(e_2, C_{12})$ are essentially invariant relative to change in the level of the volatility ratio,s_1/s_2. That is, all being the same, the resulting optimal policies as well as the contract values are much more sensitive to the dynamics of exchange rate fluctuations when exchange rates tend to be highly correlated versus when their relative volatilities is high. The sensitivity of the suppliers' optimal policies across different volatility ratios and correlation coefficient values are shown in Figures 13 – 16, respectively.

[TABLE 3]

[FIGURE 12] & [FIGURE 13]

Given these policy implications, the resulting optimal contract values as furnished by Figures 9, 10, and 11 can be explored even further. In particular, as s_1/s_2 increases, the switching option value to the producer increases, thus increasing the value of the producer's contract. Recall that with an increasing volatility ratio, the purchase weight of the first supplier increases while that of the second supplier decreases. In the former case, an increase in u_1 increases revenues to the first supplier. In the latter case, exactly the opposite takes place. This explains, in part, the reason for an increasing W_1 and a decreasing W_2 in Figures 10 and 11, respectively. The implication of an increasing correlation coefficient, in light of these policy ramifications, is also insightful. Here too, we note that with an increasing r, the purchase weight for the first supplier, u_1 increases while u_2 decreases. With an increasing correlation, all being the same, the total volatility of the portfolio is also increased thus increasing the switch option value to the producer. This increase in the option value increases the contract value to the producer. Recall that as the correlation increases, the suppliers' optimally react by increasing their sales prices, C_{1j}, as seen in Figures 15 and 16. Therefore, as a result of increased sales prices and therefore revenues, on an average basis the contract values for both suppliers increases, as shown in Figures 10 and 11. However, this increase is at a more rapid progression for the first supplier due to a correspondingly higher purchase weight relative to the second supplier.

[FIGURE 14] [FIGURE 15] [FIGURE 16]

VI. CONCLUSIONS

Multinational enterprises have become aware that their sourcing and procurement decisions impact their corporate functions in numerous ways. At the same time, multiple sourcing, in particular, across international borders, exposes the enterprise to new dimensions of risk. Simultaneously, multiple sourcing (or international sourcing) also presents new opportunities that allow the firm to introduce flexibility in its cost structure and to hedge other sources of risk for the firm. However, multiple sourcing alternatives can also be instrumental in providing for flexibilities that help reduce the need for financial hedging efforts without compromising the downside protection that is furnished by them. Therefore, with multiple sourcing, in addition to the possibility of lower costs, designed flexibility in supply contracts can be an important component of the competitiveness of a multinational enterprise.

Previous studies in this arena have developed various approaches for evaluating and analyzing supply contracts under multiple sources of uncertainty. Given the contingent nature of the decision making process, both operationally and financially, and the notion of designed flexibility in the face of exchange rate uncertainty, in this paper we adopt a real options approach to the problem. We complement this approach further by employing basic concepts from portfolio theory to provide a unique framework for risk reduction in a dynamic and contingent decision making environment. Real options have come to be widely used in analyzing contingent decision processes and in evaluating risky opportunities. We concurrently examine the dual optimization problems for the suppliers along with that of the producer, in the context of flexibility valuation.

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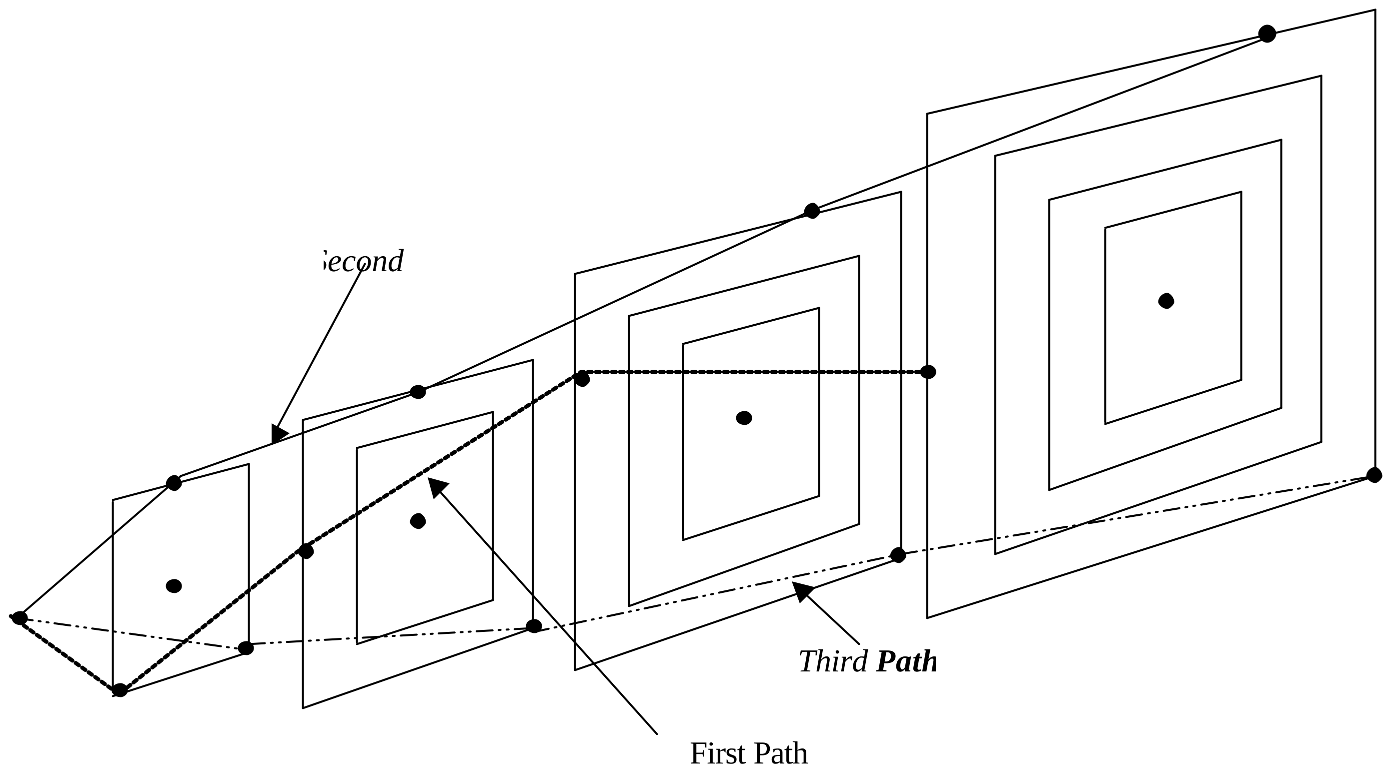
Figure 1: Lattice Paths For a 5 Jumps Lattice Each



<u>**Table 2: Policies for Three Paths Along the Lattice**</u>

	i,j,k	V	q	R_i	u_1,u_2	W_1	C_{11}	e_1	W_2	C_{21}	e_2
First Path	0,0,0	1771.8	165	0	0.60 0.40	706.3	0.41	15.0	949.8	0.41	15.0
	1,-1,-1	2190.3	105	45	0.08 0.92	763.9	0.25	12.5	1690	0.25	12.5
	2,-2,0	2592.6	170	30	0.93 0.07	402.5	0.25	12.5	91.385	0.25	12.5
	3,-3,+1	2908.4	90	30	1.00 0.00	615.6	0.25	12.5	0	0.25	12.5
	4,-4,0	7906.8	0	0		2.2	0.25		844.37	0.25	
Second Path	0,0,0	1771.8	120	0	0.60 0.40	706.3	0.30	15.0	949.9	0.30	15.0
	1,0,+1	1356.2	135	15	0.68 0.32	1618.3	0.25	12.5	169.8	0.20	12.0
	2,0,+2	2238.1	185	30	0.97 0.03	430.5	0.25	12.5	2.1	0.25	12.5
	3,+1,+3	1326.5	90	30	0.82 0.18	416.2	0.25	12.5	28.6	0.25	12.0
	4,+2,+4	7906.8	0	0		1224.8			12.0		
Third Path	0,0,0	1771.8	165	0	0.60 0.40	706.3	0.30	15.0	949.9	0.30	15.0
	1,1,-1	2033.2	120	15	0.02 0.98	195.6	0.25	12.5	1709.5	0.20	12.0
	2,2,-2	3463.1	170	15	0.02 0.98	0.7	0.25	12.5	411.8	0.25	12.5
	3,3,-3	4062.4	75	15	0.02 0.98	0.8	0.25	12.5	562.9	0.25	12.5
	4,4,-4	8413.8	165	0		10.9			5931.1		

Table 3: Impact of Changes in Parameters on the Objects of Interest in the Model

Parameters / Objects of Interest	An Increase in $\frac{s_1}{s_2}$	An Increase in r
Purchase Weight, u_1	Increase	Increase
Purchase Weight, u_2	Decrease	Decrease
Penalty Band, e_1	Steady	Increase
Penalty Band, e_2	Steady	Increase
Supplier Sales Price, C_{11}	Steady	Increase
Supplier Sales Price, C_{12}	Steady	Increase
Contract Value, V_0	Increase	Increase
Contract Value, W_1	Increase	Increase
Contract Value, W_2	Decrease	Increase

Figure 2: Producer's Option Value As Correlation Between Supplier's Changes

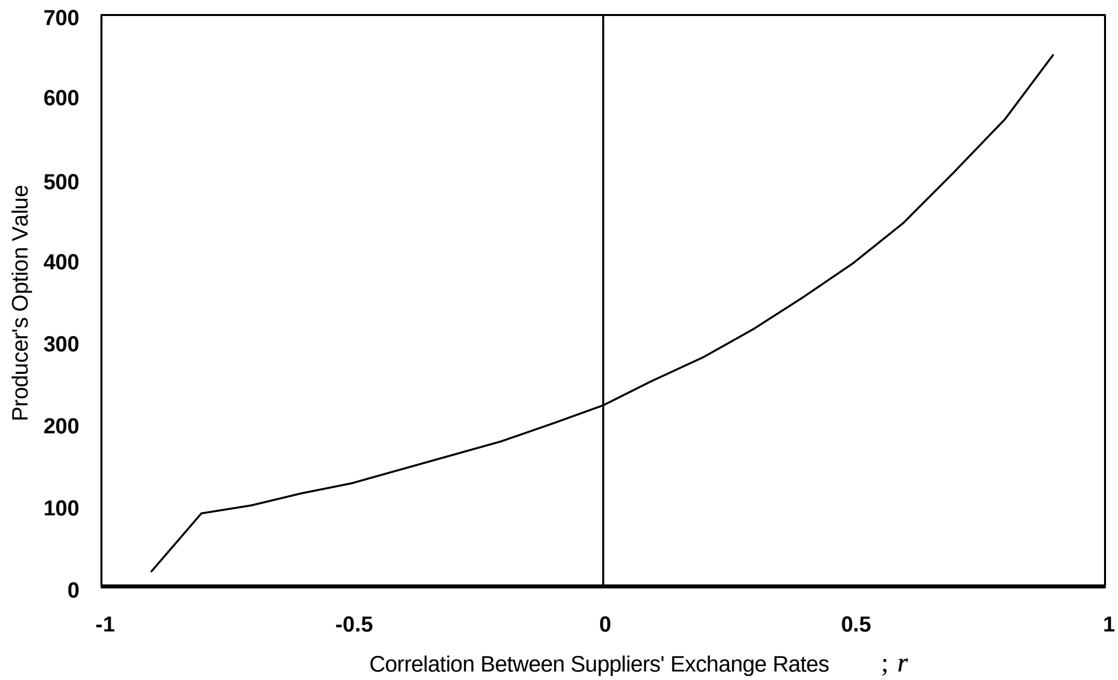


Figure 3: Producer's Option Value As Volatility Ratio Between Supplier's Change

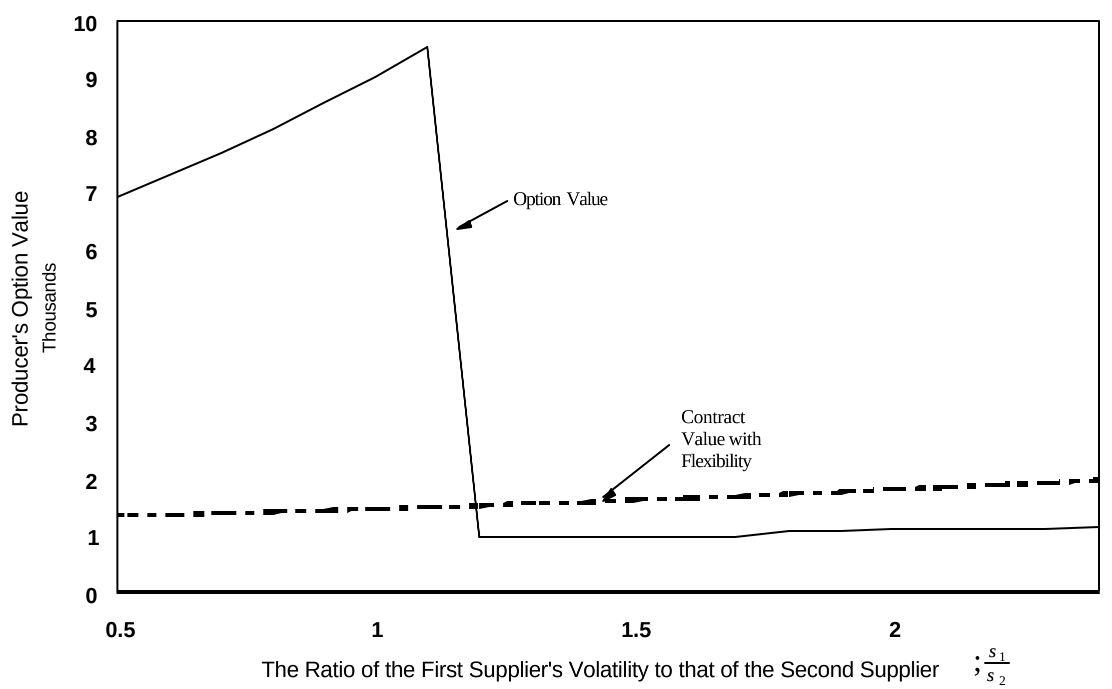


Figure 4: Producer's Contract Value As Volatility Ratio Between Supplier's Change

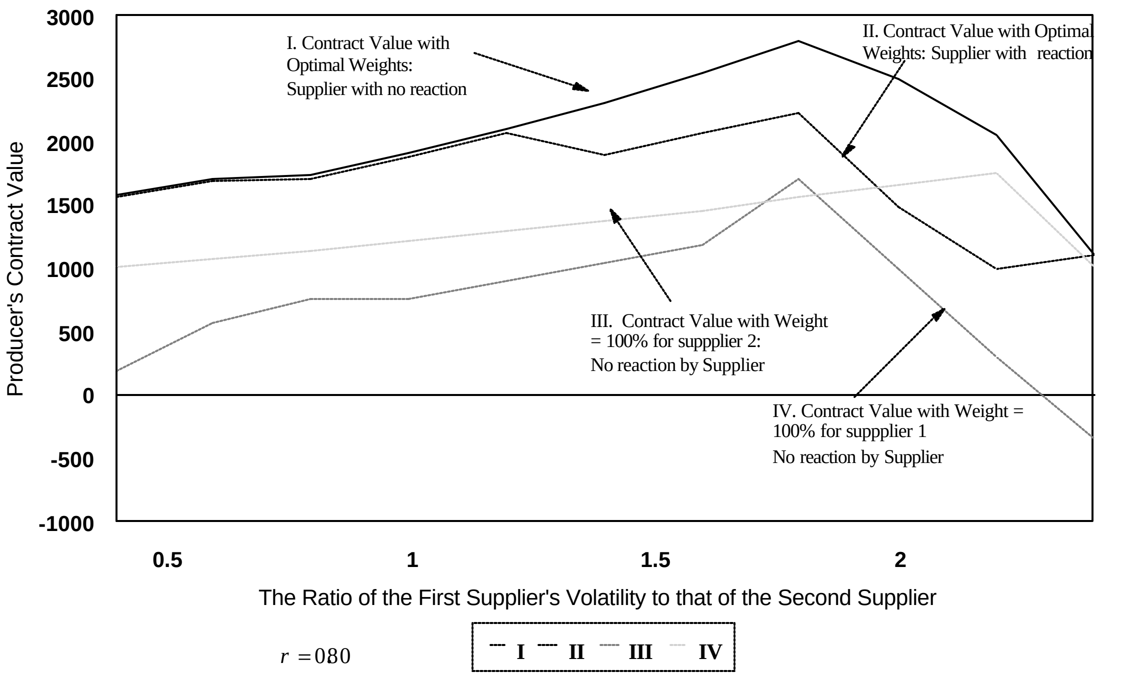


Figure 5: Producer's Contract Value as the Correlation Changes

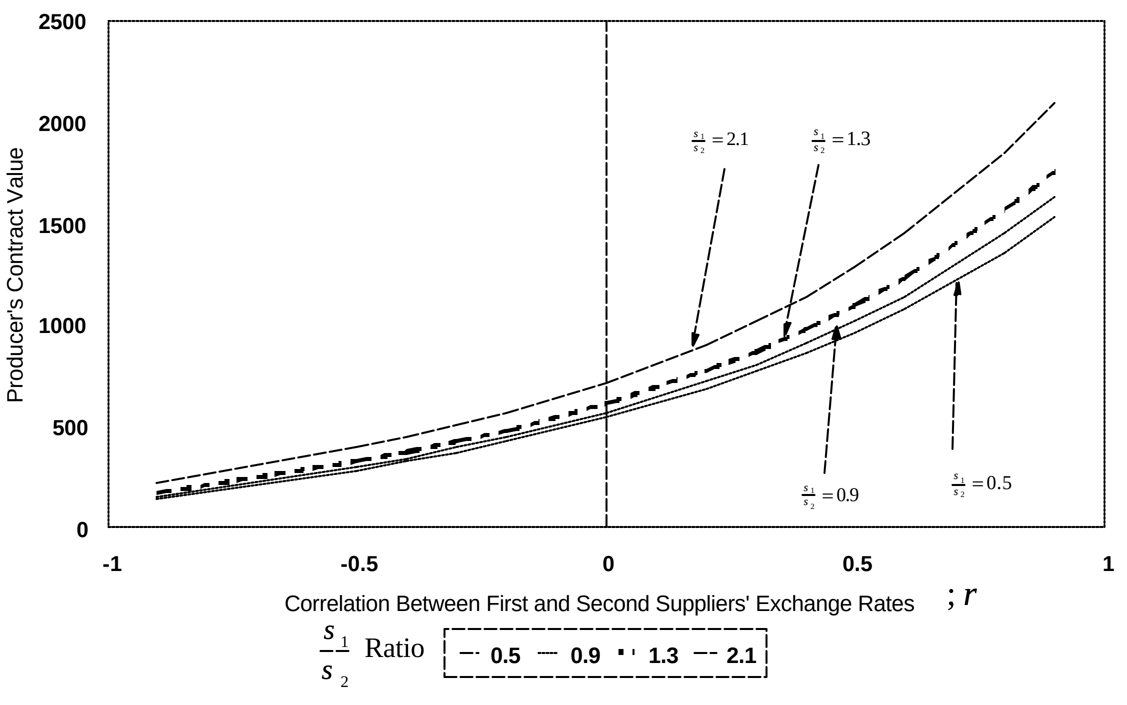


Figure 6: First Supplier's Contract Value as the Correlation Changes

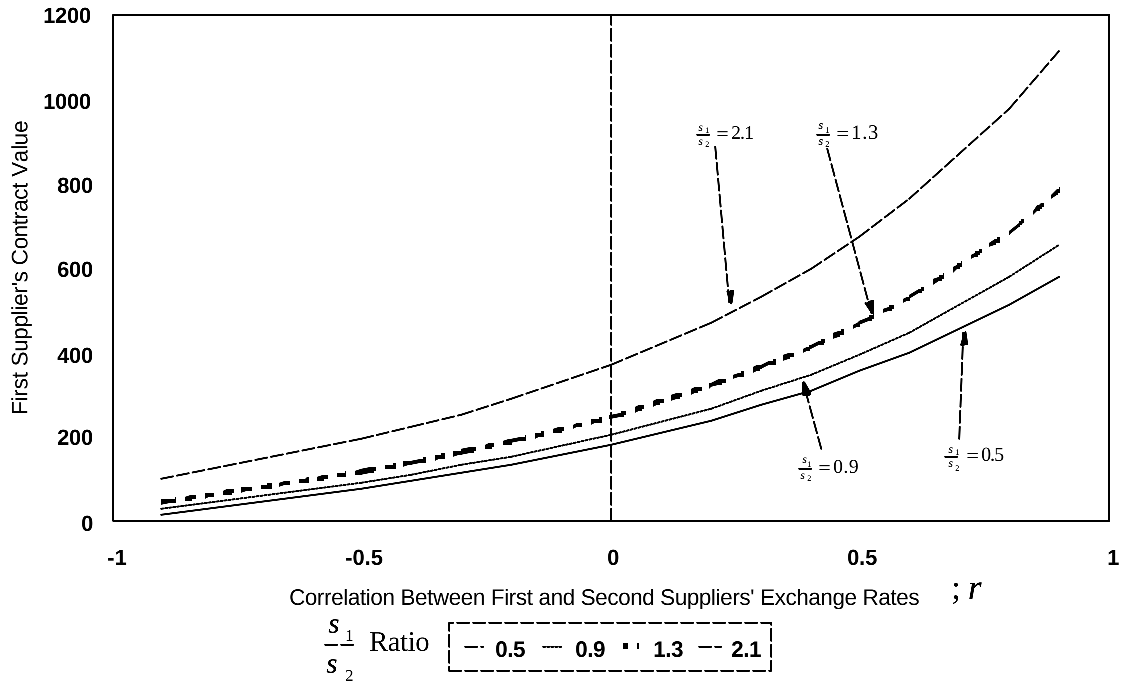


Figure 7: Producer's Contract Value as Volatility Ratio Between Supplier's Change

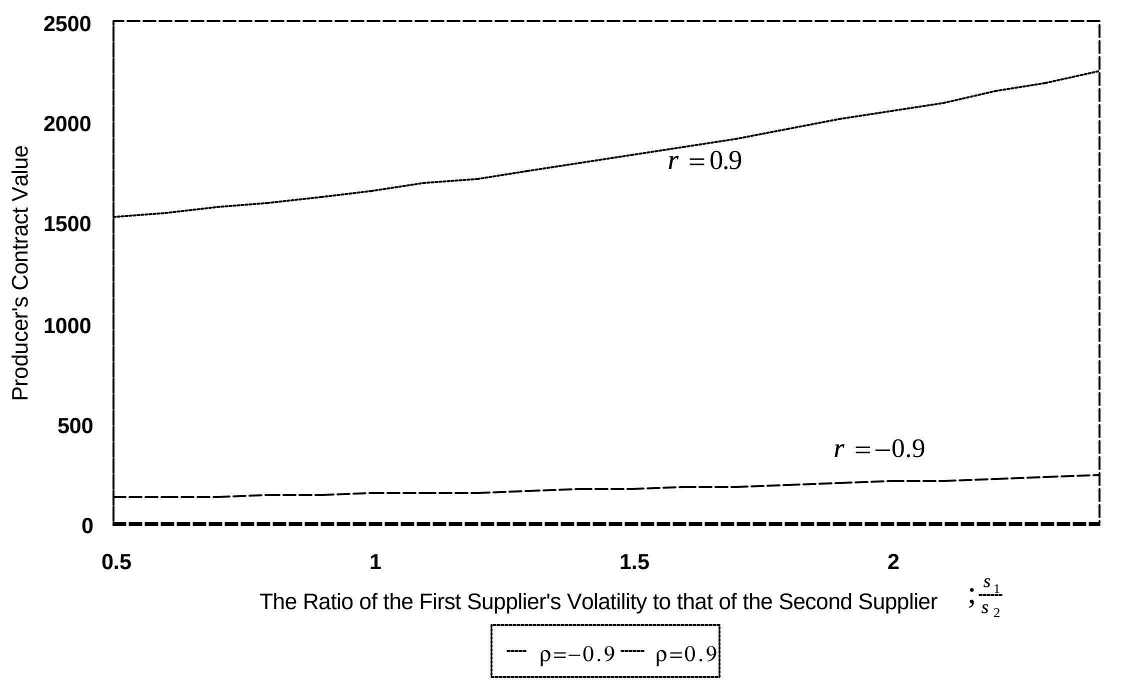


Figure 8: First Supplier's Contract Value As Volatility Ratio Between Supplier's Change

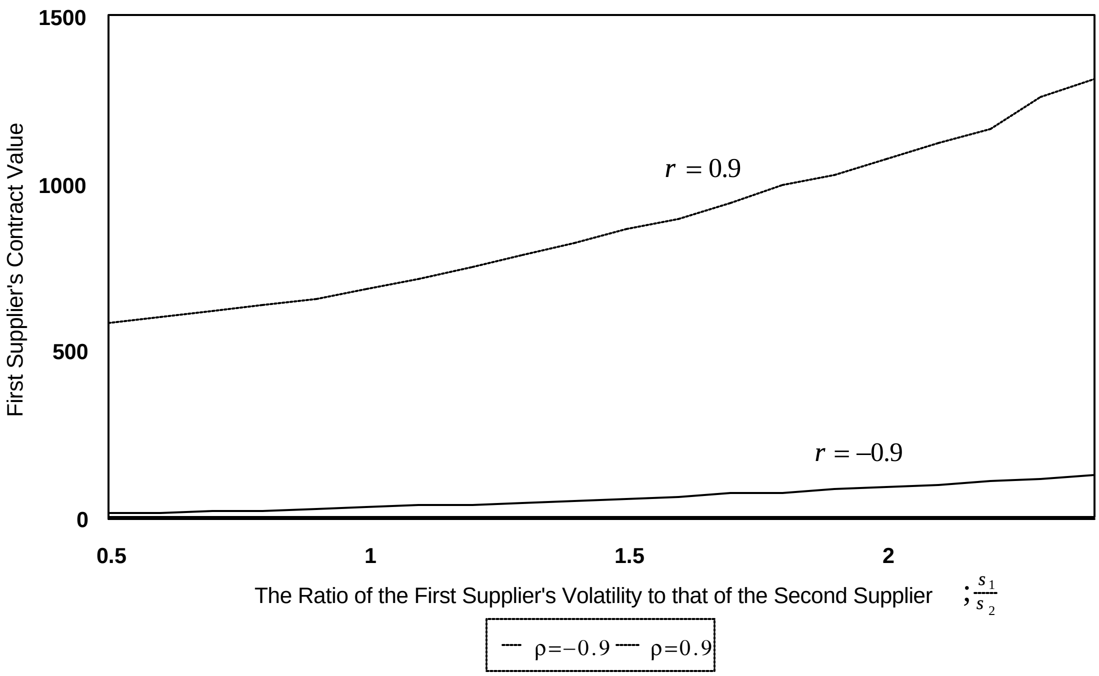


Figure 9: Producer's Contract Value as ρ and $\frac{\sigma_1}{\sigma_2}$ Change

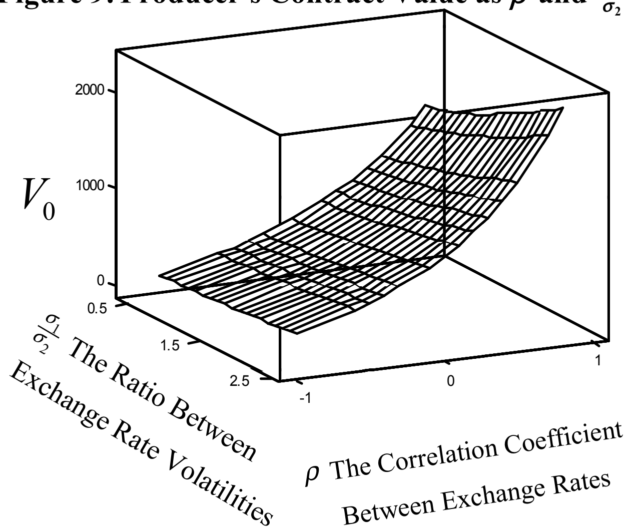


Figure 10: First Supplier's Contract Value as ρ and $\frac{\sigma_1}{\sigma_2}$ Change

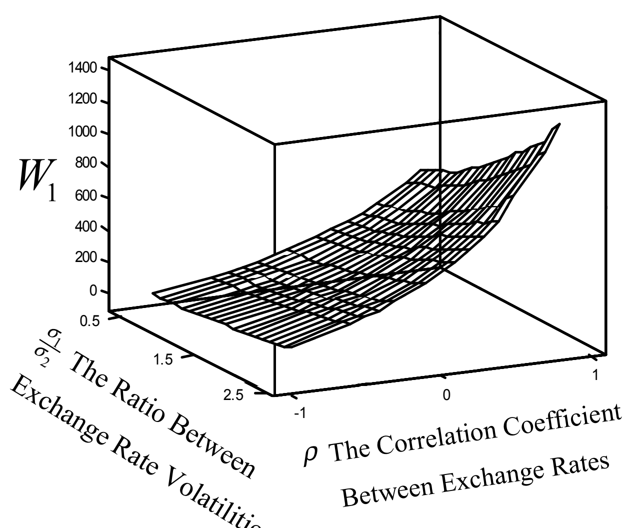


Figure 11: Second Supplier's Contract Value as ρ and $\frac{\sigma_1}{\sigma_2}$ Change

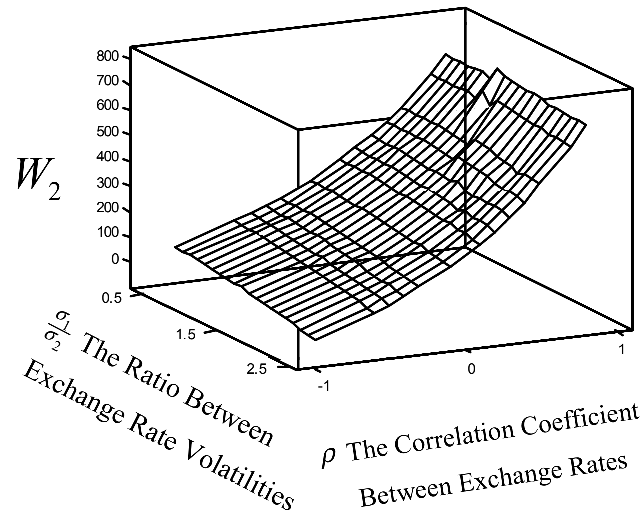


Figure 12: Purchase Weight Change by Producer as ρ and $\frac{\sigma_1}{\sigma_2}$ Change

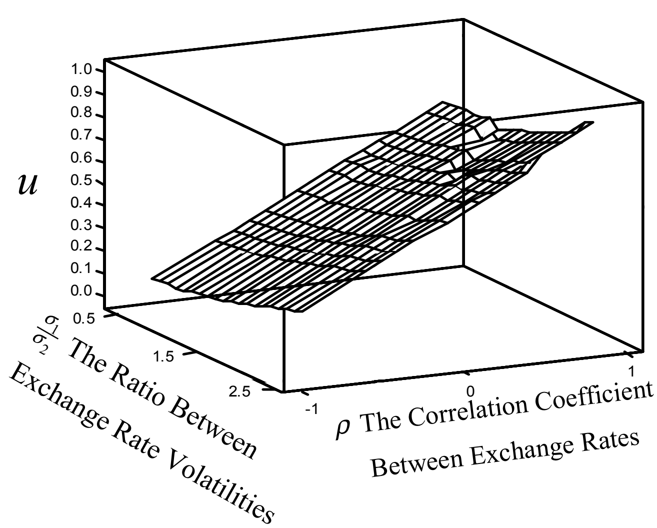


Figure 13: Penalty Band Change by First Supplier as ρ and $\frac{\sigma_1}{\sigma_2}$ Change

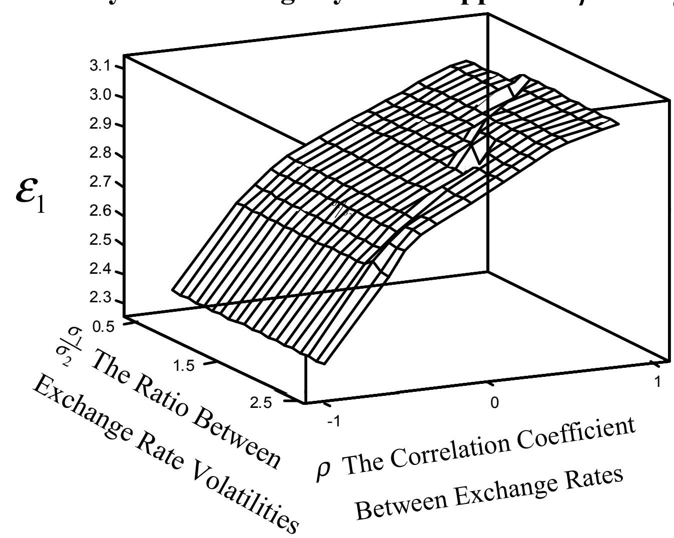


Figure 14: Penalty Band Change by Second Supplier as ρ and $\frac{\sigma_1}{\sigma_2}$ Change

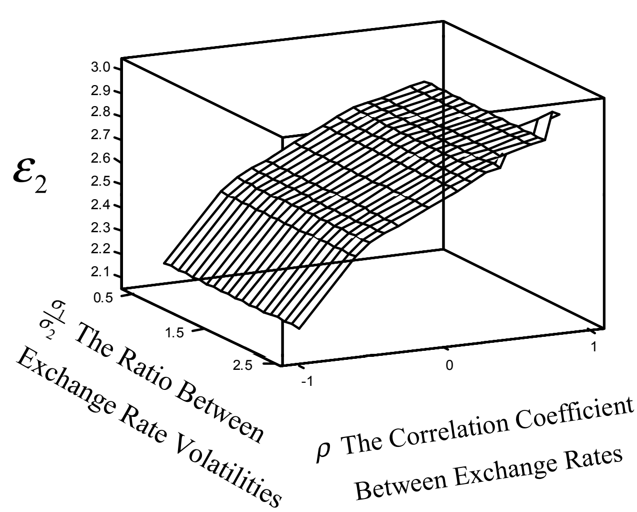


Figure 15: First Supplier's Sales Price (C_{11})as ρ and $\frac{\sigma_1}{\sigma_2}$ Change

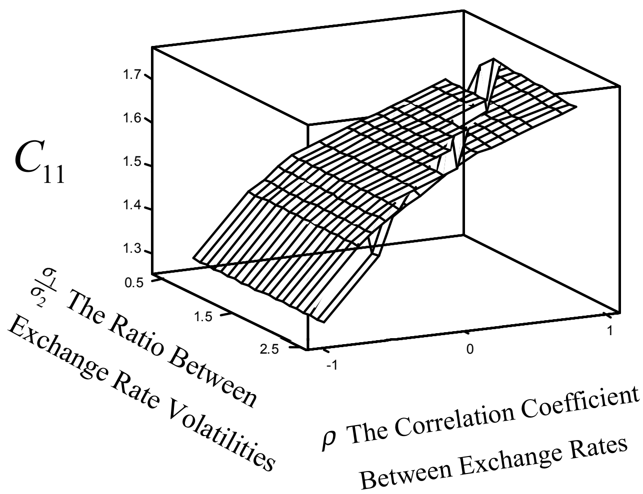


Figure 16: Second Supplier's Sales Price (C_{12})as ρ and $\frac{\sigma_1}{\sigma_2}$ Change

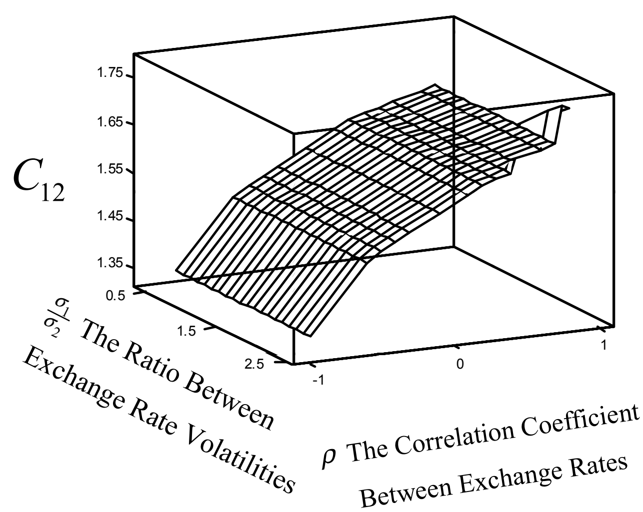


Figure 17: First Supplier's Fixed Price $\left(C_{01}\right)$ as ρ and $\frac{\sigma_1}{\sigma_2}$ Change

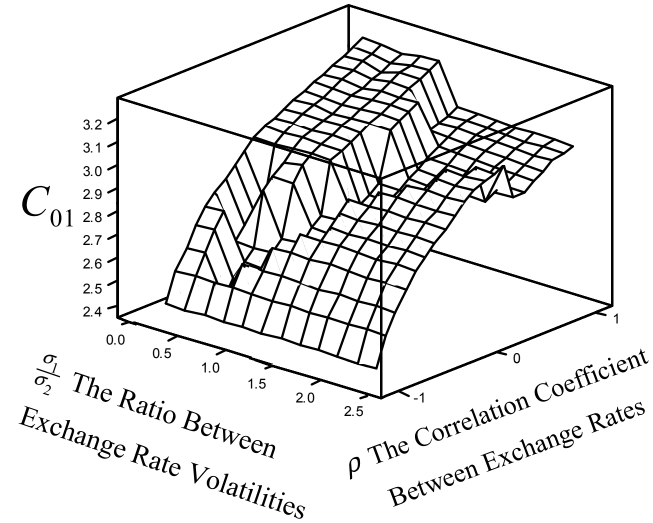
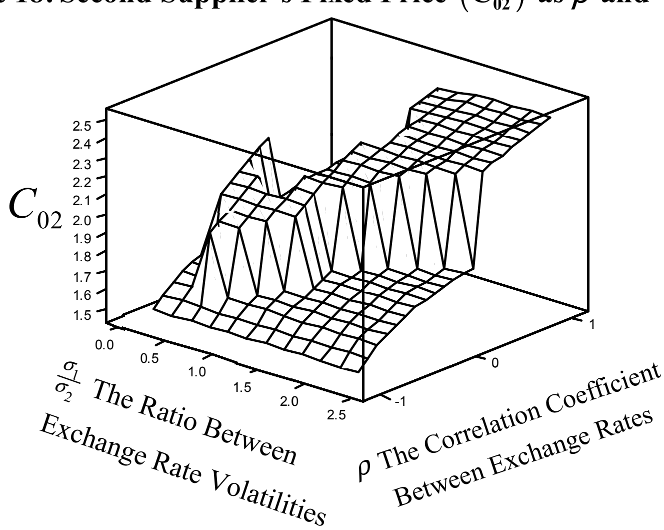


Figure 18: Second Supplier's Fixed Price $\left(C_{02}\right)$ as ρ and $\frac{\sigma_1}{\sigma_2}$ Change



Appendix A:

Numerical Optimization Methodology

At each node of the lattice, we need to optimize the contract values for the producer and the suppliers and to obtain the corresponding optimal policies. To perform these optimizations, which are highly nonlinear due to the constraints and the functional forms (e.g. equations (27), (28), and (29)) we adapt the most suitable methodology. In particular, the high degree of nonlinearities makes it impractical to compute derivatives for the objective functions. Hence, the gradient based optimization methods such as the quasi-Newton method of Davidon-Fletcher-Powell (DFP) or the Broyden-Fletcher-Goldfarb-Shanno (BFGS) are not suitable. These nonlinearities also create a second pitfall in that the optimization can end at the locally optimal solutions. Hence, we rely on derivative free methods from the grid-search family. We adapt the golden section method and perform rather time-consuming function evaluations to optimize. Along with the grid-search approach, we have also used numerically computed derivatives to corroborate some of the results.

Our grid search adapts the golden section methods to perform two sequences of optimizations. The first sequence in effect is "coarse" with broad grids and the second sequence is "fine" with closely spaced grids. Our reasoning for the two separate sets of optimizations is that the coarse optimization can rule out the problems of ending up with local optima. A third solution, to avoid the problems of local optima, is to use different starting values and we have relied upon that strategy as well. We carry out our optimization in FORTRAN and search over 1200 steps of the decision variable at each node and using the golden section method, the steps are smaller at each step. More mechanically, at each node of the approximating lattice at time t_i, the manufacturer must establish the optimal output quantity and the proportion of raw material to purchase from each supplier: essentially the "security weights" in a portfolio maximization context. This is carried through the grid-search method and results in the "offers" of q, u_j.

These offers are then passed on to the suppliers with the first supplier being the second player to optimize. This, in effect, commences the bargaining round. Given the manufacturer's operating decisions, the first supplier uses the producers' policies as an input to her corresponding optimization problem to maximize over her control variables, namely C_{11} and e_1. Implicit is the assumption that the fixed purchasing cost, C_{01} is not a controlled decision. The choices made by the producer, and the first supplier is then passed on to the second supplier. In a similar manner, the second supplier takes the producer and the first supplier's policies as given and arrives at her optimal policies i.e. C_{12} and e_2. This ends the first bargaining round and the producer carries out the optimization again using the supplier policies, C_{1j} and e_j, $j = 1, 2$ as given. The supplier policies are effectively counteroffers. The optimization by the producer is in effect, the beginning of the second bargaining round and results in optimal "offers" of q, u_j that are likely to be different from the previous rounds'. If the optimum contract value reached by the producer in this round is higher than the previous round's then the producer's offers are passed onto the suppliers as described before. However, if the producer's value is lower than the previous round's, we are faced with a conundrum. To prevent the players from in effect "going home with the ball", we impose a 10% band. This in effect says that even if the value to the producer is lower, the offers to the suppliers are passed on (the bargaining continues) as long as the value reduction is lower than 10%. These relaxations are often not required but are needed to prevent the bargaining from collapsing. We use similar rules for the suppliers. At the beginning, i.e. time period 0, and nodes 0,0, we effectively relax the value reduction to 100%.

We carry out similar bargaining rounds at each node on the lattice. We impose a minimum of 4 bargaining rounds at each node to ensure that we do not end up with inferior solutions. We do not impose an upper limit on the number of rounds but permit the rounds to go on as long as there are improvements in the players' contract values. We then compute discounted expectations recursively backward on the lattice to time zero. Thus, at time and node, we have effectively a vector of the producer's policies and the corresponding and compatible choices made by the suppliers.