The Option to Switch Scheduling Priority

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Consider a group of product devices such as DRAM's of different specifications while such products can be purchased from the market at the spot price that fluctuates over time. Because of this, a final product-maker often makes a contract with a supplier where delivery dates and prices of such devices are pre-determined. In this situation, the supplier typically ignores the price volatilities and organizes its production activities based on the delivery dates. A prevalent software package using MRP for example cannot take the price volatilities into account explicitly. In this paper, a theoretical framework is exhibited where the supplier determines its production schedule based on not only the delivery date and the pre-determined prices but also the price volatilities of the devices.

Introduction

In this paper, the volatility of the final product's price process is modeled with the use of real options. The model interprets the permutation of the production priority of the products currently in-process as a real option. Real options have been applied to manufacturing systems by Trigeorgis [10], Kamrad and Ernst[9]. The key to the real option implementation is to characterize the manufacturing environment as a system with fluctuating inputs and outputs. Trigeorgis [10] examined the case when the manufacturing plant possesses multiple projects and determines the optimal manufacturing product mix through real option evaluation. The analysis of the demand uncertainty has been investigated for decades in the past, Kamrad and Ernst [9] focused on the yield uncertainty in the production scheduling pointed out by Lee and Yano [8], Yano and Lee [17]. They obtain the results that enable the evaluation of the flexibility of the manufacturing environments. Kutakila [6] has modeled the value of flexible systems with fixed supply contracts.

In this paper, we focus on the relation between the production due dates and the fluctuation of the price processes. MRP (material resource planning) has been widely implemented in the production planning area, such as Leachman [7]. The order of the production priority has been examined by Shanthikumar and Yao [14], and generalized to cover a wide class of scheduling problems by Bertimas [2]. The proposed model considers a special case of two products waiting in a queueing system (for queueing systems refer to Buzacott and Shanthikumar [1]). This is like considering two assets like Boyle [4], but in the context of exchanging the production schedule for one product for the other like Margrabe [11] has modeled.

In the past, most production scheduling models only considered the expected value of the products with holding costs and the increase in the market value of delivered products. Takezawa [3] has argued about a special case when the increase in market value modeled as a benefit associated with the service time of the product at the final inspection station. In this analysis, the variability of the market price is only a function of the service time with no external effects. Takezawa [3] further uses a heuristic to improve the total running cost of the system by mixing the service priority of products with different market values.

Instead of using such a measure like the average costs and average prices, we attempt to dynamically set the relative value of the product by linking the market price to the products that are currently in-process. This allows us to mark-to-market the work-in-process and thus set the scheduling priority accordingly to its market price. The scheduling priority schemes introduced in Bertimas [2], Shanthikumar and Yao [14] may be considered as a static optimization of the future price process, where the new approach takes into account the dynamic value of price volatility. This value is modeled as the price of an “option to switch the scheduling priority”. In other words, it interprets as an alternative production schedule linked to its market value and volatility. This dynamic evaluation will be very effective when the product’s market price fluctuates significantly given that the product can be purchased and sold at the market price at any time epoch.

Model

Definitions

$\prod(t)$	The set of all admissible policies at time t.
$\pi \in \prod(t)$	Let π be any admissible policy at time t. $\pi \in \prod(t)$
$\pi_{opt}(t)$	The optimal policy at time t.
$P_{\pi}(t)$	The price process of products for policy π
$X_{\pi}(t)$	The quantity of the products for policy π

The admissible polices may change over time, and because we are considering dynamic price processes, the optimal policy becomes a function of time t. Thus, we now have the following.

$$\pi_{opt}(t) = \underset{\pi\in\prod(t)}{Max}\{X_{\pi}(t)P_{\pi}(t)\}$$

or equivalently

$$\pi_{opt}(t) = \underset{\pi\in\prod(t)}{Max}\{X_{\pi}(t)P_{\pi}(t) - \pi_{opt}(t), 0\} + \pi_{opt}(t)$$

The first term corresponds to the option to deviate from the current schedule. At time *t*, it is clear that this option has no value. But as time changes, this option may have a value such that the optimal policy may change in the future, i.e.

$$\pi_{opt}(t') = \underset{\pi\in\prod}{Max}\{X_{\pi}(t')P_{\pi}(t') - \pi_{opt}(t), 0\} + \pi_{opt}(t)$$

The option emerges from the asymmetry in production priority, i.e. the schedule may not be changed if the current priority is optimal and may be altered accordingly if the schedule is not optimal. The payoff of this option is the difference of serving with priority $\pi_{opt}(t)$ and $\pi_{opt}(t')$. This is equivalent to an option to exchange One-Asset-for-Another Option introduced by Margrabe [11] (1978).

If we assume that the price process $P_{\pi}, P_{\pi_{opt}}$ follow Log Normal distributions,

$$dP_{\pi}(t) = P_{\pi}\mu_{\pi}dt + P_{\pi}\sigma_{\pi}dW$$

and

$$dP_{\pi_{opt}}(t) = P_{\pi_{opt}}\mu_{_{opt}\pi}dt + P_{\pi_{opt}}\sigma_{\pi_{opt}}dW$$

ρ is the correlation between the two price processes $P_\pi, P_{\pi_{opt}}$.

σ_π is the standard deviation of P_π

$\sigma_{\pi_{opt}}$ is the standard deviation of $P_{\pi_{opt}}$

μ_π is the drift deviation of P_π

$\mu_{\pi_{opt}}$ is the drift deviation of $P_{\pi_{opt}}$

$dW_\pi, dW_{\pi_{opt}}$ are wiener processes.

Now, we can apply the option pricing formula from Margrabe [11] (1978).

$$c(P_\pi, P_{\pi_{opt}}, T) = \max(X_\pi P_\pi - X_{\pi_{opt}} P_{\pi_{opt}}, 0)$$

$$c = X_\pi P_\pi e^{(b_\pi - r)T} N(d_1) - X_{\pi_{opt}} P_{\pi_{opt}} e^{(b_{\pi_{opt}} - r)T} N(d_2)$$

where

$$d_1 = \frac{\ln(X_\pi P_{\pi_{opt}} / X_\pi P_{\pi_{opt}}) + (b_\pi - b_{\pi_{opt}} + \hat{\sigma}^2 / 2)T}{\hat{\sigma}\sqrt{T}},$$

$$d_2 = d_1 - \hat{\sigma}\sqrt{T}$$

The volatility is approximated by

$$\hat{\sigma} = \sqrt{\sigma_\pi{}^2 + \sigma_{\pi_{opt}}{}^2 - 2\rho\sigma_\pi\sigma_{\pi_{opt}}}$$

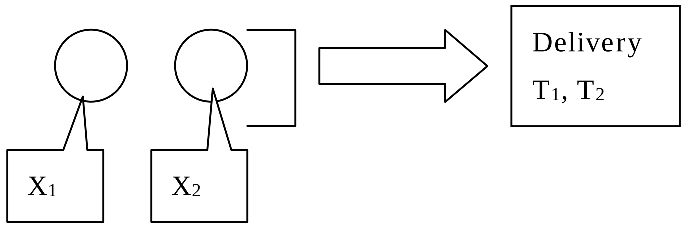
$b_\pi, b_{\pi_{opt}}$ are the carry of the assets $P_\pi, P_{\pi_{opt}}$.

$N(\cdot)$ is the cumulative normal distribution function.

T is the time to the two assets will be exchanged.

Example

We will consider the simplest example the single server single class queue. Refer to Buzacott and Shanthikumar [2] for details on queueing systems.



Suppose we only have two batches waiting in the system with the number of customers in the batch X_1 and X_2 with deliver dates T_1, T_2 and service times S_1 and S_2. Let us assume that the price for X_1 and X_2 are $P_1(t)$ and $P_2(t)$ where $t = T_1$ or T_2. In addition, let us assume that there exists a feasible schedule that meets both due dates. This eliminates any constraints related to the service times. Under this assumption, the profit function will be defined according to the price uncertainties. Recall that the set of indices π is {1,2}. The example will be examined for two cases. The first Case 1 assumes that the price processes are deterministic, and thus corresponds to the standard static schedule optimization. The second Case 2 assumes that the two prices processes $P_1(t)$,$P_2(t)$ follow geometric Brownian motions, and the existence of an option is shown.

Case 1 : Deterministic price and service time (Static Optimization)
Because the price processes are deterministic, the optimal scheduling policy is to serve in order that yields the highest payoff which corresponds to choosing the maximum of the following equation, i.e. the best profit function for X_1, X_2 becomes,
$Max\{ X_1 * P_1(T_1) + X_2 * P_2(T_2), X_1 * P_1(T_2) + X_2 * P_2(T_1)\}$

Case 2 Uncertainty in Price and deterministic service time
The profit function is the same as in Case 1 except for the fact that now the prices are random variables, and we need to take the expectation of the profit function.
$E[Max\{ X_1 * P_1(T_1) + X_2 * P_2(T_2), X_1 * P_1(T_2) + X_2 * P_2(T_1)\}]$

If we assume that the price processes are lognormally distributed,

$$E[Max\{ X_1 *(P_1(T_1)-P_1(T_2)) - X_2 *(P_2(T_1)-P_2(T_2)), 0\} + X_1 * P_1(T_2) + X_2 * P_2(T_1)]$$
$$= E[X_1 * Max\{(P_1(T_1)-P_1(T_2)) - (X_2/X_1)*(P_2(T_1)-P_2(T_2)), 0\}]$$

$$+E[X_1 * P_1(T_2) + X_2 * P_2(T_1)]$$

$$=E[c(P_\pi, P_{\pi_{opt}}, T_2 - T_1)] + E[X_1 * P_1(T_2) + X_2 * P_2(T_1)]$$

where the last equation comes from the Margrabe's formula.

If we assume that the initial schedule is optimal, this characterizes the option to switch the production schedule under the lognormal price process.

Numerical Examples

In the following, we will present some numerical examples that illustrate the results. The data we have used was obtained from "Handoutai Sangyo Shinbun (The Semiconductor Industry News) 1999 April 7"[16].

The numerical example for the case of the 16M(SOJ) DRAM and 16M synchronous DRAM is presented in Table 1.

Definition of parameters

1. The carries $b_\pi, b_{\pi_{opt}}$ were defined as the weekly carry price of the DRAM, i.e. the price of the DRAM divided by the production lead time 1 (assumed to be 1 week).
2. The price prices $P_\pi, P_{\pi_{opt}}$ are defined as the mid price of the High, Low prices on the weekly price data.
3. The Volatilities $\sigma_\pi, \sigma_{\pi_{opt}}$ are defined by the High, Low of the weekly price quotes, using the formula in Parkinson [17].

$$\sigma^2 = \frac{(\sigma_{high} - \sigma_{low})^2}{4\ln 2}$$

4. The risk free rate r was assumed to be 0.
5. We assume that the production schedule difference $T_2 - T_1$ is 1 day. One year is assumed to have 360 days.

The value of the option has a value of 29.80 where the difference of the two prices at current calendar time is 265-255=10. This interprets as the following. There is a value (265-255) = 10 to produce the synchronous DRAM over the SOJ DRAM at the current price level and in addition there is a 29.80-10 =19.8 value that accounts for the product's price volatility. The optimal schedule obtained from this analysis differs with the ordinary deterministic scheduling priority by the value that accounts for the price volatility.

Conclusion

In this paper, we have attempted to price inventory dynamically according to the market price of the work-in-process. This idea necessitates the production schedule to be altered according to the market demand dynamically. In other words, the inventory is marked-to-market. This may be equivalent to the standard MRP arguments when only the expectation of the inventory is considered. However, when the volatility of the price process is taken into account the schedule changes dynamically. We have empirically shown the impact for the DRAM case, one of the many potential applications. Future research would consider the relation of production variability that may affect the delivery date of the products.

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16MDRAM (syncronous)	P_1	265
16M DRAM (SOJ)	P_2	255
Quantity 4M DRAM	X_1	1
Quantity 1M DRAM	X_2	1
Time to Maturity	T	0.002778
Risk Free Rate	r	0
Carry 16MDRAM (syncronous)	b_1	14.29%
Carry 16M DRAM (SOJ)	b_2	14.29%
Vol. 16MDRAM (syncronous)	σ_1	353%
Vol. 16M DRAM (SOJ)	σ_2	277%
Correlation	ρ	0.00%
European Value		29.801

Table 1 Option Price to Exchange (reschedule) a 16MDRAM (syncronous) for a 16M DRAM (SOJ)

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