# Optimal entry deterrence under uncertainty

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**Abstract**

The objective of this paper is to study an incumbent-entrant model under uncertainty. The entrant knows the realization of the random variable(s) before it makes its decision on entry and eventual capacity choice. So all the uncertainty is on the incumbent's side. The sources of uncertainty consider the characteristics of the entrant's product and the entry cost the entrant needs to incur before becoming active. We know from the literature that the incumbent-entrant setup could result in three different outcomes: *blockaded entry*, i.e., the incumbent behave like a monopolist and the entrant does not enter, *deterred entry*, i.e., the incumbent overinvests to make the market unprofitable for the entrant, and *accommodated entry.* The main result from our work is that under uncertainty there can be four outcomes: apart from *blockaded entry and accommodated entry*, it can be either *100% entry deterrence* or *entry deterrence with a certain probability.*

## 1 Introduction

The objective of this paper is to study an incumbent-entrant model under uncertainty. Starting point of our analysis is Maskin [12] and Dixit [6]. In Maskin [12] the framework is such that the incumbent moves first and chooses its capacity level; the entrant then moves and either chooses to stay out of the market or else selects a level of capacity; after capacity is installed, firms observe the realization of a random variable which affects demand and costs and then choose output levels simultaneously.

Our approach differs from Maskin [12] in that the entrant knows the realization of the random variable(s) before it makes its decision on entry and eventual capacity choice. So all the uncertainty is on the incumbent's side. The sources of uncertainty consider the characteristics of the entrant's product and the entry cost the entrant needs to incur before becoming active. This means that, where Maskin [12] analyzes a homogeneous product market, we depart from a heterogeneous product market, where the demand system is taken from Dixit [6].

We know from the literature that the incumbent-entrant setup could result in three different outcomes: *blockaded entry*, i.e., the incumbent behaves like a monopolist and the entrant does not enter, *deterred entry*,

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i.e., the incumbent overinvests to make the market unprofitable for the entrant, and *accommodated entry*. The main result from our work is that under uncertainty there can be four outcomes: apart from *blockaded entry and accommodated entry*, it can be either *100% entry deterrence* or *entry deterrence with a certain probability*.

The literature on entry deterrence starts off with Spence [14], Dixit [6], and Dixit [7]. Maskin [12] extends this analysis by adding uncertainty. In Maskin [12] the incumbent chooses its capacity level. The entrant then chooses to stay out of the market or decides about a level of capacity. After capacity is installed, firms observe the realization of a random variable which affects demand and costs and then choose output levels simultaneously. Our paper also considers an incumbent-entrant model with uncertainty but our approach differs from Maskin [12] in that the entrant knows the realization of the random variable before it makes its decision on entry and eventual capacity choice. In Bonanno [3] the incumbent also has some uncertainty about the potential entrant's behavior when making its decision, but our model is different in that we consider a heterogenous good market.

Planer-Friedrich and Sahm [13] study the role of CSR in entry deterrence. A dynamic analysis of an incumbent-entrant model in a deterministic setting is provided by Dockner and Mosburger [8] and Kort and Wrzaczek [11]. Huisman and Kort [10] and Huberts et al. [9] consider entry deterrence in a duopoly model with demand uncertainty. Cookson [4] takes a more empirical approach in his focus on the casino industry where there is uncertainty over the strength of the incumbent's response. Ambrose et al. [1] consider entry deterring incentives under demand uncertainty applied to the real estate market. All these models have in common that the goods market is homogenous.

Our approach is different in that the goods market is heterogenous. We analyze the situation that at the moment it has to make its investment decision, the incumbent does not know all characteristics of the product that the entrant will offer for sale. These characteristics refer to what extent the entrant's product is horizontally or vertically separated from the incumbent's product, or there is uncertainty about the entry cost of the entrant.

The role of entry deterrence in heterogenous good markets is considered by other contributions as well. At first there was Dixit [6] from which we adopt our demand system. We extend Dixit's analysis by introducing uncertainty. By realizing that technology transfers may facilitate imitation by local competitors, Sun et al. [15] also depart from any uncertainty and instead analyze the effect of technology transfers of global firms. Besanko et al. [2] study a dynamic duopoly model where there is uncertainty about the rival's exact cost/benefit of capacity addition/withdrawal. Creane and Miyagiwa [5] consider technology choice under cost uncertainty, putting forward the argument that the incumbent may not want to develop an efficient technology if a new technology is distinct from the one of the potential entrant when uncertainty is technology-specific.

## 2 Preliminaries

As stated in the Introduction, the purpose of this paper is to contribute to the incumbent-entrant literature, with our extension being that the incumbent is uncertain about certain characteristics of the entrant. Before we discuss this in the next section, the current section provides an overview of the deterministic incumbent-entrant models that we build on. Section 2.1 presents the analysis of the incumbent-entrant model in the case of a homogeneous product market. A heterogeneous product market is discussed in Section 2.2.

## 2.1 Incumbent-entrant model with a homogenous product market

The model is taken from Tirole [16] (Section 8.2). Consider a two-firm industry. Both firms have the option to invest in capacity. The investment is irreversible. Firm 1 is the existing firm, the incumbent, which chooses $K_1$, the level of its capacity. Firm 2 is the (potential) entrant, which observes $K_1$,and then chooses its capacity level $K_2$. Upon entry, Firm 2 incurs an entry cost equal to $F \geq 0$.

Firms produce up to capacity so that both firms' output levels are given by $K_i$ $(i = 1, 2)$. Denoting the output price by $p$, we introduce a linear inverse demand function of the form

$$p = 1 - K_1 - K_2. \tag{1}$$

If Firm 2 enters, it chooses a capacity level that maximizes its profit

$$\pi_2 (K_1, K_2) = K_2 (1 - K_1 - K_2) - F, \tag{2}$$

which gives

$$K_2 = \frac{1 - K_1}{2}. \tag{3}$$

Firm 1, given that it takes entry for granted, takes Firm 2's choice into account and maximizes

$$\begin{aligned} \pi_1 (K_1, K_2) &= K_1 (1 - K_1 - K_2) \\ &= K_1 \frac{1 - K_1}{2}. \end{aligned} \tag{4}$$

The result is that

$$K_1 = \frac{1}{2}, \quad K_2 = \frac{1}{4}, \quad \pi_1 = \frac{1}{8}, \quad \pi_2 = \frac{1}{16} - F. \tag{5}$$

Necessary (but not sufficient) for this equilibrium to occur, is that Firm 2's profit is positive, i.e.,

$$F < \frac{1}{16}. \tag{6}$$

Note that in case of a monopoly for Firm 1, it will also choose a capacity level $K_1 = \frac{1}{2}$, implying that a *natural monopoly* will occur, if

$$F > \frac{1}{16}. \tag{7}$$

If $F < \frac{1}{16}$, next to allowing entry of Firm 2 and obtaining a profit of $\frac{1}{8}$, Firm 1 can also choose to deter entry. To do so it has to choose a capacity level $K_1$ such that Firm 2's profit equals zero, i.e.,

$$\pi_2 (K_1, K_2) = K_2 (1 - K_1 - K_2) - F = \left( \frac{1 - K_1}{2} \right)^2 - F = 0, \tag{8}$$

which gives

$$K_1 = 1 - 2\sqrt{F}, \pi_1 = 2\sqrt{F} \left( 1 - 2\sqrt{F} \right). \tag{9}$$

So, for $F < \frac{1}{16}$ Firm 1 will choose for *entry deterrence* if

$$2\sqrt{F} \left( 1 - 2\sqrt{F} \right) \geq \frac{1}{8}. \tag{10}$$

This leads to the following proposition:

**Proposition 1** *Consider the incumbent-entrant model with the homogenous product market. Depending on the level of the entry cost $F$, the following equilibrium outcomes occur:*

- $F \in \left[0, \frac{1}{32}\left(3-2\sqrt{2}\right)\right)$*: entry accommodation with* $K_1 = \frac{1}{2}$, $K_2 = \frac{1}{4}$
- $F \in \left[\frac{1}{32}\left(3-2\sqrt{2}\right), \frac{1}{16}\right)$*: entry deterrence with* $K_1 = 1 - 2\sqrt{F}$*, firm 2 does not enter*
- $F \in \left[\frac{1}{16}, \infty\right)$*: natural monopoly with* $K_1 = \frac{1}{2}$*, firm 2 does not enter*

## 2.2 Incumbent-entrant model with a heterogenous product market

Some of the main steps of this analysis appeared in Dixit [6]. The incumbent irreversibly invests in production capacity denoted by $K_1$. The incumbent produces up to capacity and sells its product on the market against a price $p_1$. Upon entry the entrant incurs an entry cost equal to $F$, irreversibly invests in production capacity denoted by $K_2$, and the market price of its product is $p_2$. The products of the incumbent and entrant are heterogeneous but substitutes. The inverse demand system is given by

$$p_1 = 1 - \kappa K_2 - K_1, \tag{11}$$

$$p_2 = \theta - \kappa K_1 - K_2. \tag{12}$$

The parameter $\theta$ is the vertical differentiation parameter. If $\theta > (<)\, 1$, it means that the entrant's product is of higher (lower) quality than the incumbent's product. The parameter $\kappa$, which is less than one because the effect of the quantity of the other product on its own product price can never be greater than the effect of its own quantity, is the horizontal differentiation parameter. The smaller $\kappa$ is, the more the products are differentiated and the less the firms are competing.

If Firm 2 enters, it chooses a capacity level that maximizes its profit

$$\pi_2(K_1, K_2) = K_2(\theta - \kappa K_1 - K_2) - F, \tag{13}$$

which gives

$$K_2 = \frac{\theta - \kappa K_1}{2}. \tag{14}$$

Firm 1 takes this choice into account and maximizes

$$\pi_1(K_1, K_2) = K_1\left(1 - \kappa\left(\frac{\theta - \kappa K_1}{2}\right) - K_1\right). \tag{15}$$

This gives the following *entry accommodation* quantities and profits:

$$K_1 = \frac{2-\theta\kappa}{4-2\kappa^2}, \quad K_2 = \frac{4\theta - \kappa^2\theta - 2\kappa}{8-4\kappa^2}, \quad \pi_1 = \frac{(2-\theta\kappa)^2\left(2-\kappa^2\right)}{2\left(4-2\kappa^2\right)^2}, \quad \pi_2 = \frac{\left(4\theta - \theta\kappa^2 - 2\kappa\right)^2}{\left(8-4\kappa^2\right)^2} - F. \tag{16}$$

If the vertical differentiation parameter $\theta$ is larger, Firm 2 enters with a stronger product, which is more competitive to Firm 1. This is reflected in larger (smaller) quantity and profit for Firm 2 (1). For $\theta \geq \frac{2}{\kappa}$, Firm 2's product is that strong that Firm 1 exits ($K_1 \leq 0$). As for the horizontal differentiation parameter $\kappa$, there are opposite effects. *First*, a larger $\kappa$ increases competition among firms, because the cross-effect of quantity on price is greater. This reduces profits so that, according to this effect, both quantities and profits in $\kappa$ decrease.

*Second*, there is the effect that a larger $\kappa$ results in a larger negative effect of the firm's quantity on the other firm's quantity, so that an increase of $K_i$ has a larger positive effect on Firm $i$'s market share and thus on the profit. For Firm 1 the second effect is the dominant effect, implying that $K_1$ increases with $\kappa$, if $\theta$ is sufficiently small and $\kappa$ sufficiently large. For Firm 2 it holds that $K_2$ increases with $\kappa$ if its product is much stronger than Firm 1's product, i.e. if $\theta$ is significantly greater than one.

Apart from accommodating entry, Firm 1 has the option to perform a policy of *entry deterrence.* To do so it has to choose its quantity $K_1$ such that Firm 2's profit is equal to zero, i.e.,

$$\pi_2(K_1, K_2) = K_2(\theta - \kappa K_1 - K_2) - F = \left(\frac{\theta - \kappa K_1}{2}\right)^2 - F = 0, \tag{17}$$

which gives

$$K_1 = \frac{\theta - 2\sqrt{F}}{\kappa}, \quad \pi_1 = \frac{\left(\kappa - \theta + 2\sqrt{F}\right)\left(\theta - 2\sqrt{F}\right)}{\kappa^2}. \tag{18}$$

If Firm 2 is a strong competitor, thus when $\theta$ is large, Firm 1 needs to overinvest more, leading to a larger $K_1$, to prevent Firm 2's entry. If the market is more competitive, thus when $\kappa$ is large, Firm 1's quantity reduces Firm 2's output price a lot. For that reason Firm 1 needs to overinvest less to prevent Firm 2's entry, so that $K_1$ decreases with $\kappa$.

A *natural monopoly* arises if Firm 1's monopoly quantity, $K_1 = \frac{1}{2}$, would give a non-positive profit for Firm 2, which is the case if

$$F \geq \left(\frac{\theta - \frac{1}{2}\kappa}{2}\right)^2 := F_{DM}, \tag{19}$$

in which $F_{DM}$ is the lowest level of the entry cost for which we have a *natural monopoly.*

For $F < F_{DM}$, Firm 1 will choose for *entry deterrence* if the corresponding profit is greater or equal than the *entry-accommodation* profit:

$$\frac{\left(\kappa - \theta + 2\sqrt{F}\right)\left(\theta - 2\sqrt{F}\right)}{\kappa^2} \geq \frac{(2 - \theta\kappa)^2 (2 - \kappa^2)}{2(4 - 2\kappa^2)^2}. \tag{20}$$

Straightforward calculations result in the following proposition.

**Proposition 2** *Consider the incumbent-entrant model with the heterogeneous product market. Depending on the level of the entry cost $F$, the following equilibrium outcomes occur:*

- $\theta \geq \frac{2}{\kappa}$, $F \in \left[0, \frac{1}{4}\theta^2\right)$*: exit of firm 1 with* $K_2 = \frac{\theta}{2}$
- $\theta < \frac{2}{\kappa}$, $F \in [0, F_{AD})$*: entry accommodation with* $K_1 = \frac{2-\theta\kappa}{4-2\kappa^2}$, $K_2 = \frac{4\theta - \kappa^2\theta - 2\kappa}{8-4\kappa^2}$
- $\theta < \frac{2}{\kappa}$, $F \in [F_{AD}, F_{DM})$*: entry deterrence with* $K_1 = \frac{\theta - 2\sqrt{F}}{\kappa}$*, firm 2 does not enter*
- $\theta < \frac{2}{\kappa}$, $F \in [F_{DM}, \infty)$*: natural monopoly with* $K_1 = \frac{1}{2}$*, firm 2 does not enter*

*in which $F_{DM}$ is given by (19), and*

$$F_{AD} := \frac{\left((2 - \kappa^2)(4\theta - 2\kappa) - \sqrt{D}\right)^2}{64(2 - \kappa^2)^2}, \tag{21a}$$

$$D := (2 - \kappa^2)^2 (4\theta - 2\kappa)^2 + 2(2 - \kappa^2)\left(8(2 - \kappa^2)(\theta\kappa - \theta^2) - \kappa^2(2 - \theta\kappa)^2\right). \tag{21b}$$

Given that $\theta < \frac{2}{\kappa}$, for larger $\theta$ it is more difficult for the incumbent to prevent entry, so then the *entry-accommodation* region, $[0, F_{AD})$, is larger and the lower boundary of the *natural-monopoly* region, $F_{DM}$, is also larger. As we concluded earlier, a larger $\kappa$ reduces the overinvestment level in the *entry-deterrence* region. Therefore, it passes into the *natural-monopoly region* for a lower value of $F$, i.e., $F_{DM}$ decreases with $\kappa$. A lower level of overinvestment implies that profits in the *entry-deterrence* region increase in $\kappa$. For *entry accommodation,* on the one hand, competition between firms increases for larger $\kappa$, resulting in lower profits for Firm 1. On the other hand, Firm 1, as the incumbent, can benefit from the first-mover effect: if $\kappa$ is larger, announcing a larger $K_1$ lowers $K_2$, which increases Firm 1's profits. These opposing effects make the overall effect of $\kappa$ on $F_{AD}$ ambiguous.

Due to the clear monotonic effects of $\theta$, we can easily translate the $F$-regions from Proposition 2 into $\theta$-regions for the different equilibria modes. This is useful for the next section, where we consider that Firm 1 does not know the exact values of parameters such as $F$, $\theta$, and $\kappa$.

**Corollary 3** *Consider the incumbent-entrant model with the heterogeneous product market. Depending on the level of the vertical differentiation parameter $\theta$, the following equilibrium outcomes occur:*

- $\theta \geq \frac{2}{\kappa}$, $F \in \left[0, \frac{1}{4}\theta^2\right)$*: exit of firm 1 with* $K_2 = \frac{\theta}{2}$
- $\theta \in \left[\theta_{AD}, \frac{2}{\kappa}\right)$*: entry accommodation with* $K_1 = \frac{2-\theta\kappa}{4-2\kappa^2}$, $K_2 = \frac{4\theta-\kappa^2\theta-2\kappa}{8-4\kappa^2}$
- $\theta \in [\theta_{DM}, \theta_{AD})$*: entry deterrence with* $K_1 = \frac{\theta-2\sqrt{F}}{\kappa}$*, firm 2 does not enter*
- $\theta \in [0, \theta_{DM})$*: natural monopoly with* $K_1 = \frac{1}{2}$*, firm 2 does not enter*

*in which*

$$\theta_{DM} := \frac{\kappa}{2} + 2\sqrt{F}, \tag{22}$$

$$\theta_{AD} := \frac{16\left(2-\kappa^2\right)\kappa + 8\kappa^3 + 64\left(2-\kappa^2\right)\sqrt{F} - \sqrt{D}}{4\left(2-\kappa\right)^2\left(2+\kappa\right)^2}, \tag{23}$$

$$\begin{aligned} D = {} & \left(16\left(2-\kappa^2\right)\left(\kappa + 4\sqrt{F}\right) + 8\kappa^3\right)^2 \\ & -64\theta^2\left(2-\kappa\right)^2\left(2+\kappa\right)^2\left(\kappa^2 + 4\kappa\left(2-\kappa^2\right)\sqrt{F} + 8\left(2-\kappa^2\right)F\right). \end{aligned} \tag{24}$$

# 3 Incumbent entrant behavior under uncertainty

In the previous section the incumbent knew exactly the situation of the (potential) entrant. In reality this assumption can be quite strong, especially when the products of the incumbent and the entrant are not the same, and therefore when the product market is heterogeneous. This makes it relevant to study the situation where the incumbent does not know the entrant's product characteristics, such as the level of horizontal and vertical differentiation, or the entry costs. Not knowing what to expect from its potential competitor it becomes a difficult task for the incumbent to choose its optimal investment level $K_1$. Note that in this sense the entrant's situation is much simpler. Because the incumbent is by definition already active, the entrant is already aware of the product characteristics of the incumbent. The entrant can therefore decide whether or not to enter, and if so, how much to invest in $K_2$, in a situation where it has perfect information. We conclude that where the incumbent's decision problem is stochastic, the entrant's decision problem is deterministic. Exactly this scenario is the subject of research in this section.

This section consecutively examines the effects of uncertainty about the levels of the entry cost $F$, the vertical differentiation parameter $\theta$, and the horizontal differentiation parameter $\kappa$, on the incumbent's investment decision. In general, the problem of the incumbent, Firm 1, is as follows. Denoting the uncertain parameter by $X = \{F, \theta, \kappa\}$, and the realization $x \in \Gamma_X$, Firm 1 maximizes its expected profit, i.e., it solves

$$\begin{aligned}\max_{K_1} E_X[\pi_1] &= \max_{K_1} E_X[p_1] K_1 \\ &= \max_{K_1} E_X[1 - \bar{p}(X) - K_1] K_1 \\ &= \max_{K_1} \left[1 - \int_{\Gamma_X} f_X(x)\, \bar{p}(x)\, \mathrm{d}x - K_1\right] K_1, \end{aligned} \tag{25}$$

where

$$\bar{p}(x) := \kappa K_2 (F, \theta, \kappa, K_1)|_{x \in \Gamma_X}, \tag{26}$$

and $f_X(x)$ is the probability density function of the parameter whose level is uncertain. In our analysis we resort to the uniform distribution so that

$$\begin{aligned}\Gamma_X &= [x_{\min}, x_{\max}], \\ f_X(x) &= \frac{1}{x_{\max} - x_{\min}} \quad \text{for } x \in [x_{\min}, x_{\max}], \end{aligned} \tag{27}$$

and thus

$$\max_{K_1} E_X[\pi_1] = \max_{K_1} \left[1 - \frac{1}{x_{\max} - x_{\min}} \int_{x_{\min}}^{x_{\max}} \bar{p}(x)\, \mathrm{d}x - K_1\right] K_1. \tag{28}$$

## 3.1 Uncertain entry cost

Here we consider the situation in which Firm 1 does not know what entry cost Firm 2 will incur upon entry. Therefore, Firm 1 is not sure in advance how much incentive Firm 2 has to enter the market. This makes it difficult to determine the capacity level $K_1$, because Firm 1 does not know the minimum level of $K_1$ that induces Firm 2 not to enter. The information available to Firm 1 is that $F$ is uniformly distributed with $F \in [F_{\min}, F_{\max}]$. On the other hand Firm 2 knows the amount of the entry cost $F$. It follows that Firm 1 determines its capacity level by solving

$$\max_{K_1} E_F[\pi_1] = \max_{K_1} \left[1 - \frac{1}{F_{\max} - F_{\min}} \int_{F_{\min}}^{F_{\max}} \kappa K_2 (F, \theta, \kappa, K_1)\, \mathrm{d}F - K_1\right] K_1. \tag{29}$$

Figure 1 shows, as a function of the vertical differentiation parameter $\theta$, which ranges of $F$ are considered against the background of equilibrium patterns under certainty (top). The effect of, for Firm 1, uncertain entry cost on Firm 1's capacity level $K_1$ is shown in the middle panels. The resulting entry behavior of Firm 2 is shown at the bottom of Figure 1. The panels on the left vary the mean of $F$, while the panels on the right show results for different variances of $F$.

Both top panels of Figure 1 show three equilibrium configurations that occur under certainty for different levels of $\theta$ and $F$. The fourth, *exit of Firm 1*, occurs for $\theta \geq 2/\kappa$ (see Corollary 3), and is not visible. The market

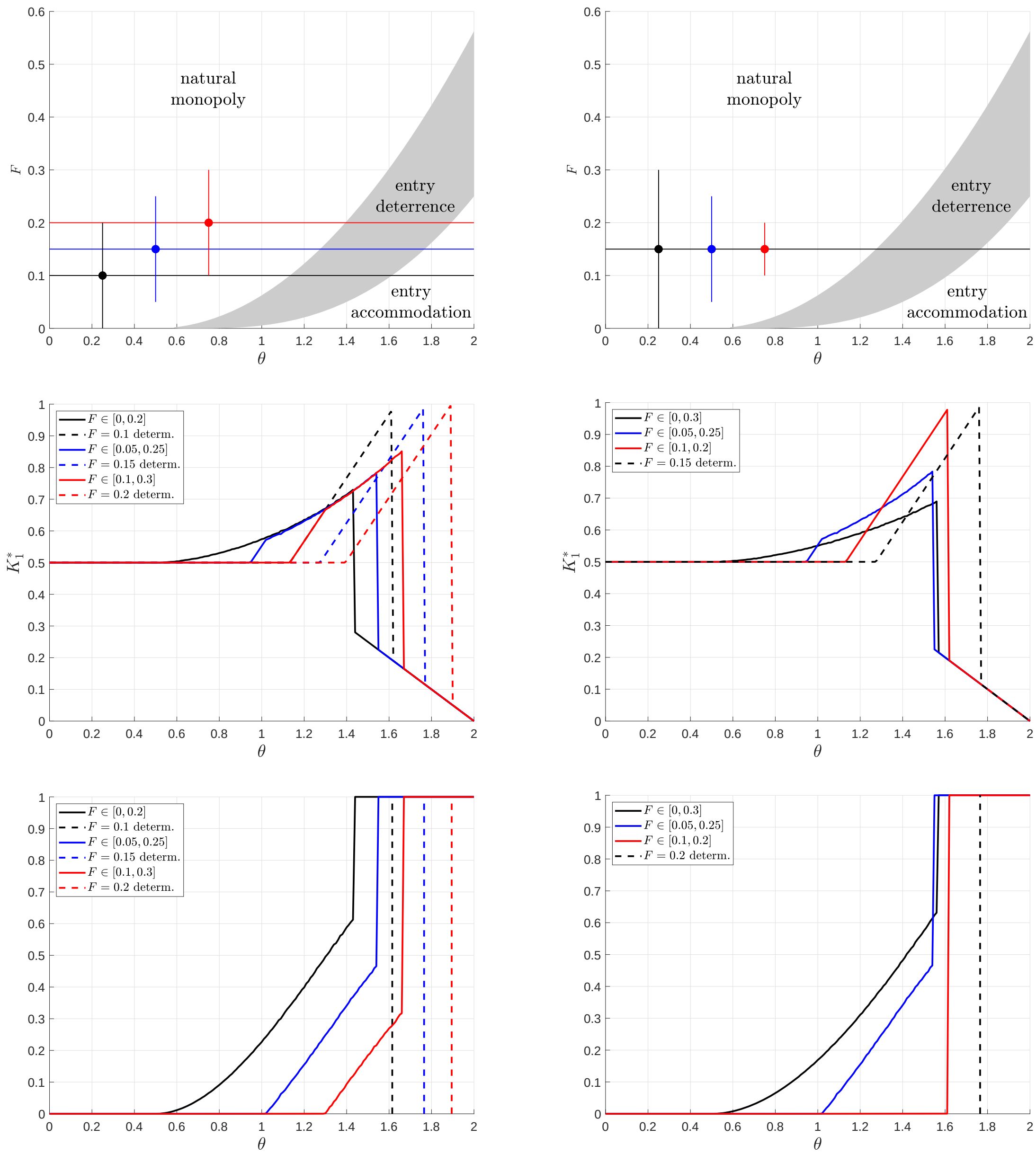


Figure 1: Stochastic $F$ and $\kappa = 1$.

Left panels: varying mean, constant range.

Right panels: constant mean, varying range.

is unprofitable for the entrant if the quality of its product, $\theta$, is low and the entry cost $F$ is high. Then Firm 2 abstains from entering anyway and Firm 1 is a *natural monopoly.* If $\theta$ is slightly larger and/or $F$ is slightly smaller, so that we are in the shaded area, the market situation is in principle profitable enough for Firm 2 to enter. However, Firm 1 pursues an *entry deterrence* policy. This occurs through overinvestment in $K_1$,which reduces the output price sufficiently to make entry suboptimal for Firm 2. For a sufficiently large value of $\theta$, and $F$ small enough, the market is so attractive to Firm 2 that preventing entry would require a too large and too expensive overinvestment in production capacity $K_1$ for Firm 1. Therefore, Firm 2 will enter so that we have *entry accommodation.* Note that the boundary between *entry deterrence* and *accommodation* is given by (21), while expression (19) is the boundary between the *entry-deterrence* and the *natural-monopoly* region.

Let us first look at the left panels of Figure 1, thus where the mean of $F$ is varied, starting with the upper-left panel. To understand the effect of an uncertain $F$, first look at the red part, where the vertical line represents the probability mass of $F$ around the mean of 0.2. For $\theta = 0$ consumers do not value the entrant's product, so it stands to reason that we are in the *natural-monopoly* region. Increasing $\theta$ gradually implies a move towards the *entry-deterrence* region, followed by the *entry-accommodation* region. If the mean of $F$ equals 0.15, the blue part prevails. Then the market is more attractive to the entrant, which is reflected by the blue probability mass leaving the *natural-monopoly* region for lower values of $\theta$, and also moving from the *entry-deterrence* to the *entry-accommodation* region for lower values of $\theta$. For an even smaller value of the mean of $F$, namely $F = 0.1$ (the black part), the market is even more attractive for the entrant. Then the *entry-deterrence* region is already partly covered for $\theta = 0.6$, and for $\theta$ close to 2 the entire probability mass is embedded in the *entry-accommodation* region.

In the middle-left panel of Figure 1 we see the implications for the optimal production capacity level. Let us look at the red part first. The red dashed curve shows the optimal deterministic capacity level for varying $\theta$, when $F = 0.2$. For $\theta$ small Firm 1 is a natural monopoly and the capacity level is the monopoly quantity, implying that $K_1 = 1/2$. For $\theta$ larger, i.e., from the point it reaches $\theta_{DM} = 1.39$ (see Corollary 3), the entrant's product is so attractive to consumers that Firm 1 must overinvest to prevent entry. We are in the *entry-deterrence* region and we see the firm overinvesting there. Note that $K_1$ increases in $\theta$ because it becomes more difficult to keep the entrant out when $\theta$ is larger, which requires a higher level of overinvestment. The fact that the entrant does not enter in this region is confirmed in the bottom-left panel of Figure 1 where we see that the probability of entry is zero. When $\theta$ reaches a level of about 1.9 ($\theta = \theta_{AD}$), the level of overinvestment required to deter entry has become too high. Therefore, Firm 1 switches to the *entry-accommodation* strategy, which explains the drop in capacity investment. The bottom-left panel of Figure 1 confirms this, where we can see that Firm 2 will enter with probability one for $\theta \geq \theta_{AD}$. In the entry-accommodation region Firm 2's product gains demand as $\theta$ increases. This at the same time reduces Firm 1's market share, and therefore $K_1$ decreases with $\theta$.

To study the impact of uncertainty, look at the red solid curve. Again we have a *natural monopoly* for $\theta$ small. For $\theta$ larger the firm overinvests necessary to deter entry, so that the firm implements the *entry-deterrence* policy. The $\theta-$level for which overinvestment begins corresponds to the point at which the red probability mass begins to enter the *entry-deterrence* region in the upper-left panel. For about $\theta = 1.3$, the red solid curve shows a kink. From then on Firm 1's investment still increases with $\theta$, but the level of $K_1$ is less sensitive to an increase in $\theta$. The implication is that, according to the bottom-left panel of Figure 1, Firm 2's entry probability is no longer zero, but instead begins to increase. At this stage it is too expensive for Firm 1 to deter entry with certainty. The current region is a new region, which does not appear in the deterministic case. We refer to this as the region of *entry deterrence with a certain probability.* The overinvestment stops once $\theta$ reaches a level of about 1.65. Then overinvestment is no longer efficient. Therefore, Firm 1 fully accommodates entry, as we can see in the bottom-left panel of Figure 1, which shows that Firm 2 enters with probability one. The following proposition provides an analytical existence result of the region of *entry deterrence with a certain probability.*

**Proposition 4** *Consider the problem where, according to Firm 1's information, Firm 2's entry cost is uniformly distributed such that*

$$\Gamma_F = [F_{\min}, F_{\max}], \tag{30a}$$

$$f_F(F) = \frac{1}{F_{\max} - F_{\min}} \quad \text{for } F \in [F_{\min}, F_{\max}]. \tag{30b}$$

*Firm 2 knows its entry cost. Consider the case where*

$$F_{AD} < F_{\min} < F_{\max} < F_{DM}. \tag{31}$$

*Then Firm 1*

*(i) overinvests to deter entry with certainty if*

$$\frac{\kappa + 4\sqrt{F_{\min}}}{2 - \kappa^2 \frac{F_{\min}\left(1-2\sqrt{F_{\min}}\right)}{F_{\max}-F_{\min}}} > \theta, \tag{32}$$

*(ii) overinvests to deter entry with a certain probability in the complementary case.*

**Proof.** If $F_{AD} < F_{\min} < F_{\max} < F_{DM}$ firm 1 has two options: Either (i) increase the capital stock such that firm 2 never enters, or (ii) increase it such that firm 2 does not enter for some realizations of $F$. In case of (i) firm 1 has to increase the capital stock to $\frac{\theta - 2\sqrt{F}}{\kappa}$ and collects

$$\pi_1^D := \frac{\theta - 2\sqrt{F_{\min}}}{\kappa}\left(1 - \frac{\theta - 2\sqrt{F_{\min}}}{\kappa}\right) \tag{33}$$

as profit independently of $F$'s realization. If, on the contrary, firm 1 has to increase the capital stock to $\bar{K}_1 \leq \frac{\theta-2\sqrt{F}}{\kappa}$ follows strategy (ii) the profit can only be evaluated in expected terms as

$$\begin{aligned}
\pi_1^E\left(\bar{F}\right) := \mathbb{E}_F\left[\pi(\bar{K}_1)\right] &= \left(1 - \bar{K}_1\right)\bar{K}_1 - \frac{\kappa}{F_{\max} - F_{\min}} \int_{F_{\min}}^{\bar{F}} K_2^*(\kappa, \theta, F, \bar{K}_1)\,\mathrm{d}F \cdot \bar{K}_1 \\
&= \left(1 - \bar{K}_1\right)\bar{K}_1 - \kappa \frac{\left(\bar{F} - F_{\min}\right)}{F_{\max} - F_{\min}} \frac{\theta - \kappa\bar{K}_1}{2}\bar{K}_1 \\
&= \left(1 - \bar{K}_1 - \kappa \frac{\left(\bar{F} - F_{\min}\right)}{F_{\max} - F_{\min}} \frac{\theta - \kappa\bar{K}_1}{2}\right)\bar{K}_1 \\
&= \left(1 - \frac{\theta - 2\sqrt{\bar{F}}}{\kappa} - \kappa \frac{\left(\bar{F} - F_{\min}\right)}{F_{\max} - F_{\min}} \frac{\theta - \kappa\frac{\theta-2\sqrt{\bar{F}}}{\kappa}}{2}\right)\frac{\theta - 2\sqrt{\bar{F}}}{\kappa}.
\end{aligned} \tag{34}$$

where $\bar{F}$ marks the entry cost level (corresponding to $\bar{K}_1 = \frac{\theta-2\sqrt{\bar{F}}}{\kappa}$) where firm 2 does not make profit (implying that firm 2 makes profit for $F < \bar{F}$ and no profit for $F > \bar{F}$). If $\bar{K}_1 = \frac{\theta-2\sqrt{F_{\min}}}{\kappa}$ both cases, of course, coincide and $\pi_1^D = \pi_1^E(F_{\min})$ holds.

Firm 1 will chose strategy (i) if $\pi_1^E\left(\bar{F}\right)$ would not increase in $\bar{F}$ at $\bar{F} = F_{\min}$, i.e., $\left.\frac{\partial \pi_1^E\left(\bar{F}\right)}{\partial \bar{F}}\right|_{\bar{F}=F_{\min}} < 0$. Evaluating the derivative

$$\begin{aligned}\frac{\partial \pi_1^E\left(\bar{F}\right)}{\partial \bar{F}} &= -\left(1 - \frac{\theta - 2\sqrt{\bar{F}}}{\kappa} - \kappa \frac{\left(\bar{F} - F_{\min}\right)}{F_{\max} - F_{\min}}\sqrt{\bar{F}}\right)\frac{1}{\kappa\sqrt{\bar{F}}} \\ &\quad + \left(\frac{1}{\kappa\sqrt{\bar{F}}} - \kappa\frac{1}{F_{\max} - F_{\min}}\sqrt{\bar{F}} - \frac{\kappa}{2\sqrt{\bar{F}}}\frac{\left(\bar{F} - F_{\min}\right)}{F_{\max} - F_{\min}}\right)\frac{\theta - 2\sqrt{\bar{F}}}{\kappa} \end{aligned} \tag{35}$$

at $\bar{F} = F_{\min}$ yields (32) and proves (i).

If (32) does not hold firm 1 profits from following (ii). ■

Now if we look at the blue solid curve, we see the same regions, but Firm 1 starts to overinvest for a smaller $\theta$. The reason is that the entry cost is likely to be lower, implying that Firm 2 will already consider entry at a lower value of $\theta$. As a result, all regions shift to the left, which is reflected in the entry probability curve in the bottom-left panel of Figure 1. This effect is magnified by the black solid curve, which covers a situation where the entry cost is very likely to be low. Note that if Firm 1 now overinvests, this only has the effect of deterring entry with a certain probability that remains below one. The *entry-deterrence* region therefore does not appear under the black probability mass.

Comparing the solid curves with the dashed curves of the same color of the middle-left panel, we can analyze the effect of uncertainty on Firm 1's optimal investment behavior. First, since it does not know exactly what the level of the entry cost is, Firm 1 has to begin with overinvestment for a lower level of $\theta$ to prevent entry with certainty. Second, in the region of *entry deterrence with a certain probability* the firm overinvests a lot compared to the deterministic case, while the effect is that it can only prevent entry with a certain probability. Therefore, the firm gives up on entry deterrence already for a lower value of $\theta$ and instead goes for an entry accommodation policy. The following proposition proves these two results formally.

**Proposition 5** *Consider the problem where, according to Firm 1's information, Firm 2's entry cost is uniformly distributed such that*

$$\Gamma_F = \left[F_{\min}, F_{\max}\right], \tag{36a}$$

$$f_F\left(F\right) = \frac{1}{F_{\max} - F_{\min}} \quad \textit{for } F \in \left[F_{\min}, F_{\max}\right]. \tag{36b}$$

*Firm 2 knows its entry cost. Then, Firm 1's uncertainty regarding the entry cost has the following implications for its optimal investment policy:*

- *In order to keep the entry probability of Firm 2 equal to zero, Firm 1 begins with overinvesting for a lower level of $\theta$ if*

$$\kappa\left(\kappa + 1 - \theta\right)\left(F_{DM} - F_{\min}\right) + \kappa F_{\min} > 0. \tag{37}$$

- *Firm 1 begins with accommodating entry for a lower level of $\theta$.*

**Proof.** For the proof of assertion 1 we look at increasing $\theta$ starting from the value $\theta = 0$. For this case $\pi_2 < 0$ for all possible $F$ (the whole range of $F$ is in the 'natural monopoly'-region) and all possible $K_1$. Thus, in the deterministic case firm 1 will behave as a monopoly for any $F$ as

$$\max_{K_1} \mathbb{E}_F[\pi_1] = \max_{K_1}\left[(1-K_1)K_1 - \frac{\kappa}{F_{\max}-F_{\min}}\int_{F_{\min}}^{F_{\max}} \underbrace{K_2^*(\kappa,\theta,F,K_1)}_{=0}\,\mathrm{d}F\cdot K_1\right]. \tag{38}$$

Therefore, $K_1 = \frac{1}{2}$ turns out to be optimal for the stochastic case as well. The same behavior is optimal as long as $F_{\text{min}}$ remains in the natural monopoly region, i.e., $\frac{\kappa}{2} + 2\sqrt{F_{\text{min}}} > \theta > 0$.

The situation changes if $F_{\text{min}}$ enters the deterrence region ($F_{\text{mean}}$ remaining in the natural monopoly region), i.e., $\frac{\kappa}{2} + 2\sqrt{F_{\text{min}}} < \theta < \frac{\kappa}{2} + 2\sqrt{F_{\text{mean}}}$: According to (19) firm 2 will enter if $F$ is larger than $F_{DM} = \left(\frac{\theta-\frac{1}{2}\kappa}{2}\right)^2$. Thus firm 1 considers

$$\begin{aligned}\max_{K_1} \mathbb{E}_F[\pi_1] &= \max_{K_1}\left[(1-K_1)K_1 - \frac{\kappa}{F_{\max}-F_{\min}}\left(\int_{F_{\min}}^{\left(\frac{\theta-\kappa K_1}{2}\right)^2} K_2^*\,\mathrm{d}F + \int_{\left(\frac{\theta-\kappa K_1}{2}\right)^2}^{F_{\max}} \underbrace{K_2^*}_{=0}\,\mathrm{d}F\right)\cdot K_1\right]\\ &= \max_{K_1}\left[(1-K_1)K_1 - \frac{\kappa}{F_{\max}-F_{\min}}\int_{F_{\min}}^{\left(\frac{\theta-\kappa K_1}{2}\right)^2} \frac{\theta-\kappa K_1}{2}\,\mathrm{d}F\cdot K_1\right].\end{aligned} \tag{39}$$

Taking the first derivative (Leibnitz rule) gives

$$1-2K_1-\kappa\frac{\left(\frac{\theta-\kappa K_1}{2}\right)^2-F_{\min}}{F_{\max}-F_{\min}}\frac{\theta-2\kappa K_1}{2} - \frac{\kappa}{F_{\max}-F_{\min}}\frac{\theta K_1-\kappa K_1^2}{2}\cdot 2\left(\frac{\theta-\kappa K_1}{2}\right)\frac{-\kappa}{2}, \tag{40}$$

which is a polynomial of third order without the possibility of an analytic root. However, as $F_{\text{mean}}$ still lies in the natural monopoly region, firm 1 would always chose $K_1 = \frac{1}{2}$ in the deterministic case with $F = F_{\text{mean}}$. Thus, plugging $K_1 = \frac{1}{2}$ into the derivative (40) yields

$$\frac{\kappa(\kappa+1-\theta)}{2}\frac{F_{DM}-F_{\min}}{F_{\max}-F_{\min}} + \frac{\kappa}{2}\frac{F_{\min}}{F_{\max}-F_{\min}} \tag{41}$$

which means that it is profitable for firm 1 to increase its capital stock above $\frac{1}{2}$ if (41) is positive. This proves the first assertion.

For the second assertion we vary $\theta$ from the opposite direction. For very high values $\theta$ is such that $F_{\max} < F_{AD}$ (see (21)), i.e., the entire range of $F$ lies in the region of entry accommodation. Here entry deterrence would be deterministically optimal for any $F \in [F_{\min}, F_{\max}]$ and would give the same optimal value of $K_1$. Therefore also in the stochastic case this value of $K_1$ is optimal.

Decreasing $\theta$ means that $F_{\max}$ crosses $F_{AD}$ and part of the range of $F$ lies in the deterrence region, i.e., $F_{AD} < F_{\max} < F_{MD}$ (see (19). Firm 1 again faces

$$\max_{K_1} \mathbb{E}_F[\pi_1] = \max_{K_1}\left[(1-K_1)K_1 - \frac{\kappa}{F_{\max}-F_{\min}}\int_{F_{AD}}^{F_{\max}} \frac{\theta-\kappa K_1}{2}\,\mathrm{d}F\cdot K_1\right]. \tag{42}$$

Consider now the $\theta$ value such that $F_{\max}$ is slightly above $F_{AD}$ (keeping $F_{\text{mean}}$ exactly at $F_{AD}$). From the deterministic case we know that $F = F_{AD}$ means that firm 1 gets the same profit if it goes for entry deterrence and for accommodation. This is equivalent to the limit case of making the range of $F$ smaller around $F_{mean} = F_{AD}$. If $F_{\max}$ is marginally increased above $F_{AD}$ and $F_{\text{mean}}$ accordingly (not necessarily by the same value), firm 1 is still indifferent between deterrence and accommodation. As this argument still holds for values $F_{\min} > 0$, it is possible to keep $F_{\max}$ constant and further decreasing the $F_{\min}$, which implies that entry accommodation dominates entry deterrence for $F_{\min} = 0$ and a sufficiently higher $F_{\max}$ as compared to $F_{AD}$. ■

Next, consider the bottom-left panel of Figure 1 and compare the deterministic dashed curves with the stochastic solid curves of the same color. The conclusion is then that the introduced uncertainty makes it more difficult for the incumbent to prevent entry. The reason for this is that the incumbent has less information about the level of the entry cost, making it more difficult to determine the investment amount $K_1$ in such a way that entry becomes unprofitable for the entrant. This is in fact confirmed by the right panels of Figure 1. Firm 1 has the least information there in the black part, where the variance is largest, and this results in the highest entry probabilities. In the spirit of Proposition 2, we also see that when uncertainty is higher the firm starts overinvesting for a lower level of $\theta$. Furthermore, the size of the region of *entry deterrence with a certain probability* increases with the variance.

### 3.2 Uncertain vertical differentiation parameter

Here we consider the situation where Firm 1 does not know how good the potential entrant's product is, i.e., it is about the uncertainty of the entrant's reservation price $\theta$. This is problematic because, first, Firm 1 does not know how profitable the market is for Firm 2, and thus it cannot infer the effect of $K_1$ on Firm 2's investment decision. Second, Firm 1 does not know what level of $K_1$ is required that induces Firm 2 not to enter. The information available to Firm 1 is that $\theta$ is uniformly distributed with $\theta \in [\theta_{\min}, \theta_{\max}]$ . On the other hand Firm 2 knows its own reservation price $\theta$. It follows that Firm 1 determines its capacity level by solving

$$\max_{K_1} E_\theta [\pi_1] \quad = \quad \max_{K_1} \left[1 - \frac{1}{\theta_{\max} - \theta_{\min}} \int_{\theta_{\min}}^{\theta_{\max}} \kappa K_2 (F, \theta, \kappa, K_1) \, \mathrm{d}\theta - K_1\right] K_1. \tag{43}$$

Figure 2 illustrates the effect of the uncertain vertical differentiation parameter $\theta$. The upper-left panel shows the three equilibrium configurations under certainty that occur for different levels of the entry cost $F$ and $\theta$. If the entry cost is small and the entrant's product is of high quality, reflected by a high value of $\theta$, it is optimal for Firm 1 to *accommodate entry.* For Firm 1 it is impossible to prevent entry in an affordable way. The latter is possible in the *entry deterrence* region, i.e., where $\theta$ is smaller or $F$ is larger. The *natural monopoly* region occurs where $F$ is large and/or $\theta$ small, i.e., when the market is not profitable for Firm 2. In addition, the upper-left panel shows the probability mass of the different cases considered, with the red part having the highest mean of $\theta$, followed by blue and black, where the variance is the same for all cases.

The middle-left panel of Figure 2 shows the implications for the investment size of Firm 1. The result of Proposition 2 from the previous section about the uncertainty of the entry cost also applies here, namely that uncertainty reduces the *natural-monopoly* region and increases the *entry-accommodation* region. The four different regions are clearly visible in the blue solid curve. For small values of $F$ Firm 2 always enters and we are in the *entry-accommodation* region. A gradual increase of $F$ will at some point result in the occurrence of the region of *entry deterrence with a certain probability*, generalizing the existence result of Proposition 1 of the previous section to the case with the uncertain vertical differentiation parameter $\theta$. Then it is worthwhile for Firm 1 to overinvest to reduce the entry probability of Firm 2. The larger $F$ is, the more it is worth reducing the entry probability of Firm 2, and this is why $K_1$ increases with $F$ in this region. Indeed, the bottom-left panel of Figure 2 shows that Firm 2's entry probability in this region decreases significantly. At the moment that the probability of entry of Firm 2 equals zero, the *entry-deterrence* region begins. In this region Firm 1 invests in such a way that the entry probability of Firm 2 remains zero. The middle-left panel of Figure 2 shows

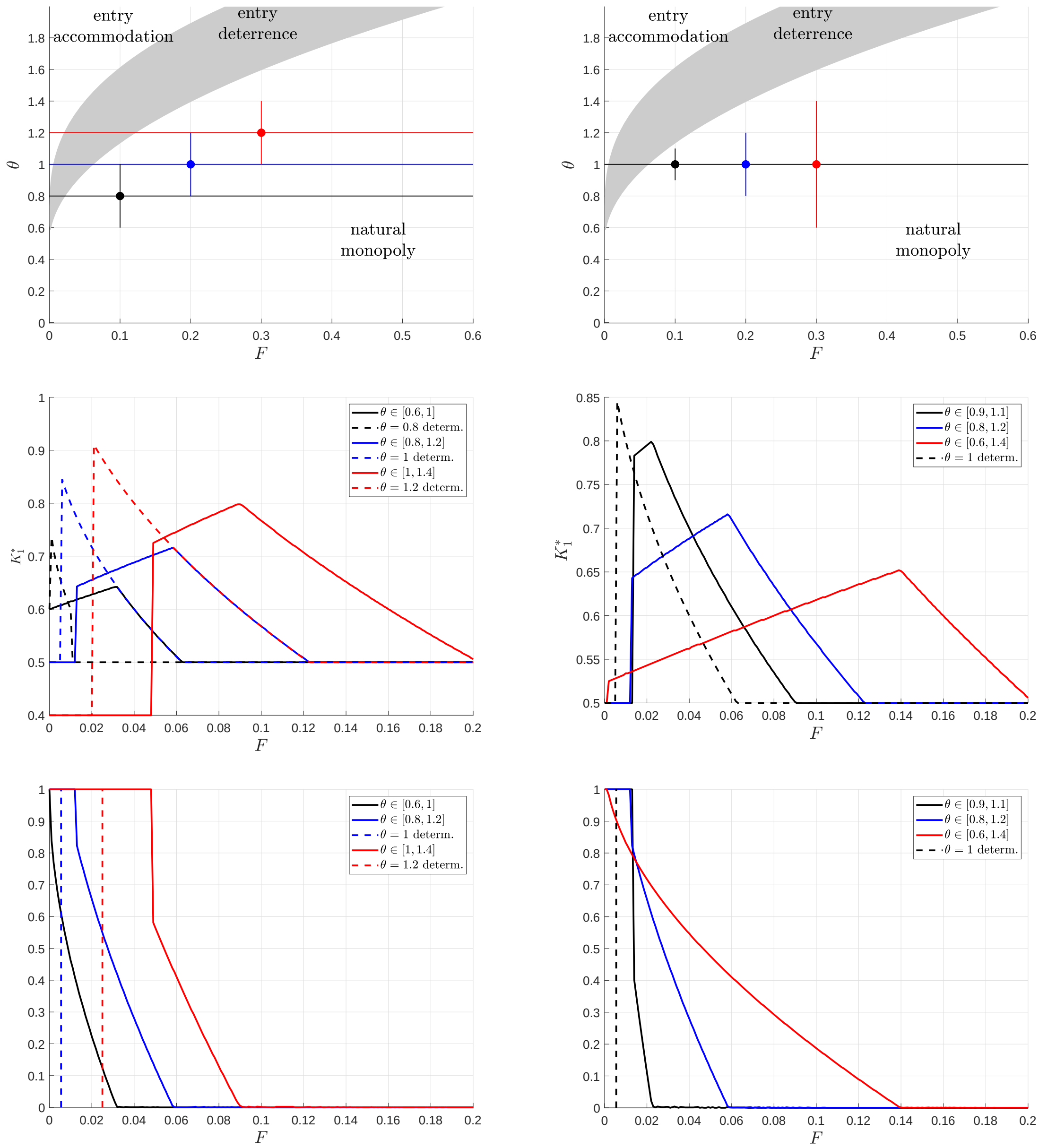


Figure 2: Schematic figure for stochastic $\theta$ and $\kappa = 1$.

Left panel: varying mean, constant range.

Right panel: constant mean, varying range.

the obvious result that Firm 1 needs to invest less as $F$ increases. At the moment $K_1$ reaches its monopoly level of 1/2, the *entry-deterrence* region becomes the *natural-monopoly* region. Now the entry cost $F$ has become so high that market entry is no longer an affordable option for Firm 2.

The red solid curve represents the case where Firm 2 has on average a better product that is more competitive to Firm 1. Therefore, Firm 1 invests less in the *entry accommodation* region, and starts overinvesting for a higher level of $F$ to reduce the entry probability of Firm 2. As the middle panel of Figure 2 shows, Firm 1 must overinvest much more in both the region of *entry deterrence with a certain probability* and the *entry-deterrence* region. If Firm 2's product is worse on average, i.e., the black solid curve, the effects are opposite. As the middle left panel of Figure 2 shows, Firm 1 overinvests less than in the other cases, and yet, as we see in the bottom left panel of Figure 2, the probability that Firm 2 is also less likely to enter than in the other cases.

The right panels of Figure 2 consider cases with different uncertainty levels, keeping the mean fixed. The red part has the largest variance, and Firm 1 has more reliable information about $\theta$ in the black scenario where the variance is the lowest. As the middle and the bottom panels of Figure 2 show, the region of *entry deterrence with a certain probability* is largest in the red scenario. In fact, it starts for very low values of $F$, because there is a certain probability that $\theta$ is quite low and then overinvestment will prevent the entry of Firm 2. However, in the red scenario $\theta$ can also be very large, which makes it difficult to rule out entry in all cases. Therefore, there is also a positive entry probability if $F$ is reasonable large. In general, the region of *entry deterrence with a certain probability* becomes more prominent as there is more uncertainty, which was also found in the previous section where the entry cost is uncertain.

### 3.3 Uncertain horizontal differentiation parameter

Here we consider the situation that Firm 1 is uncertain about the extent to which Firm 2's product appeals to the same consumers as Firm 1's product, i.e., it does not know the value of $\kappa$ as long as Firm 2 does not enter. When $\kappa$ is close to unity, the product market is almost homogeneous and there is a lot of competition. On the other hand, if $\kappa$ is close to zero, both firms can serve their own consumers without much interference. Again, it is problematic for Firm 1 to set the appropriate capacity level before the eventual entry of Firm 2, because it depends on $\kappa$ how much effect $K_1$ has on Firm 2's behavior, and it also depends on $\kappa$ how much Firm 2's behavior affects Firm 1's profitability. The information available to Firm 1 is that $\kappa$ is uniformly distributed with $\kappa \in [\kappa_{\min}, \kappa_{\max}]$. On the other hand, Firm 2 knows its own product and therefore also knows $\kappa$. It follows that Firm 1 determines its capacity level by solving

$$\max_{K_1} E_\kappa [\pi_1] \quad = \quad \max_{K_1} \left[1 - \frac{1}{\kappa_{\max} - \kappa_{\min}} \int_{\kappa_{\min}}^{\kappa_{\max}} \kappa K_2 (F, \theta, \kappa, K_1) \, \mathrm{d}\kappa - K_1\right] K_1. \tag{44}$$

The left part of Figure 3 compares scenarios with a given variance, but where the mean of $\kappa$ differs. The upper-left panel shows that the red scenario has the largest mean, followed by blue and then black. In addition, the upper-left panel shows the deterministic equilibrium configurations. For low values of $\kappa$ the products are quite different. Therefore, each firm serves its own consumers and there is not much reason for Firm 2 not to enter, as long as the cost of entry is not too high. Hence, an *entry accommodation* policy results. If $\kappa$ is large, after entry Firm 2 would put a similar product on the market as Firm 1 does, implying that Firm 2's entry would significantly reduce Firm 1's market share. Then Firm 1 has a high incentive to deter entry. Note that the *natural monopoly* region is also large, because upon entry Firm 2 faces strong competition from Firm 1, making entry less attractive.

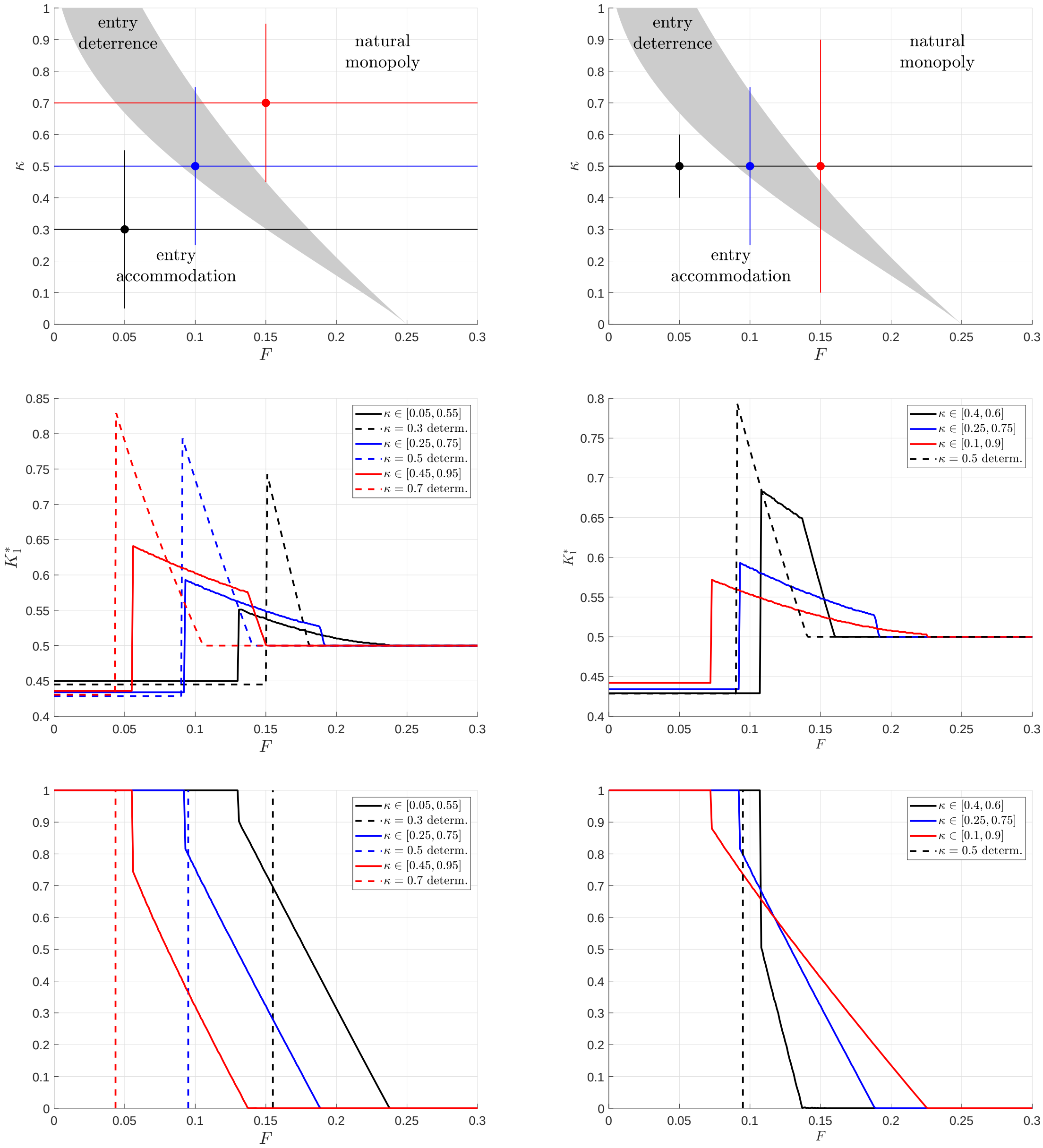


Figure 3: Schematic figure for stochastic $\kappa$ and $\theta = 1$.

Left panel: varying mean, constant range.

Right panel: constant mean, varying range.

The middle left panel generalizes Proposition 2, that is, under uncertainty the *entry accommodation* region is larger and the *natural monopoly* region is smaller. Proposition 1 is also generalized in the sense that uncertainty about $\kappa$ rather than $F$ also generates the region of *entry deterrence with a certain probability*. For Firm 1 a higher value of $\kappa$ has a negative and a positive effect. The positive effect is that an increase in $K_1$ has a stronger negative effect on $K_2$ and also on Firm 2's profits, so that entry is more easily deterred. The negative effect is that a given level of $K_2$ has a greater negative effect on the output price of Firm 1's product. So, under *entry accommodation,* it is therefore not clear in advance what the effect of $\kappa$ is on $K_1$, which explains why for small $F$ the $K_1$ under the red and the blue scenarios are virtually the same. The fact that an increase of $K_1$ has a stronger negative effect on Firm 2's profit when $\kappa$ is large, makes the policy of *entry deterrence* through overinvestment more powerful. Therefore, in the red scenario Firm 1 already overinvests for a fairly low level $F$ level. First it pursues the policy of *entry deterrence with a certain probability.* As the entry cost increases, the probability that Firm 2 enters decreases (see the bottom left panel of Figure 3), and at the point when it reaches zero, the *entry deterrence* region begins. If $F$ is large enough no overinvestment is needed to prevent the entry of Firm 2. Then Firm 1 is a natural monopoly, and $K_1$ admits the monopoly quantity level of 1/2. The bottom left panel of Figure 3 confirms that when the product market is more homogeneous, i.e., $\kappa$ is larger, entry of Firm 2 is less likely, which is then also the case if $\kappa$ is uncertain for Firm 1.

The right part of Figure 3 considers $\kappa-$scenarios that all have the same mean but where the variance is different. The red scenario has the largest variance, and, as before, here the region of *entry deterrence with a certain probability* is the largest. We see that, where normally uncertainty increases the region of *entry accommodation*, like also in the blue and the black scenario, this is not the case in the red scenario. This is because in the latter case there is a positive probability that $\kappa$ can be quite large, which makes overinvesting attractive because, first, entry of Firm 2 leads to a lot of competition if $\kappa$ is large, and, second, increasing $K_1$ reduces the profit of Firm 2 a lot when $\kappa$ is large, making *entry deterrence* an attractive policy. It follows that Proposition 2 does not apply to this situation.

## 4 Conclusions

This paper analyzes the problem of an incumbent firm facing the threat of potential market entry. We extend the corresponding literature by considering that at the time the incumbent has to decide on its investment in production capacity, it is unaware of certain parameters of the entrant's product or the cost of entry, while at the same time the entrant can take its decisions in a fully deterministic setting. In particular, we take into account uncertainty about the entry cost, and to what extent the entrant's product is vertically and horizontally differentiated. The implication is that the incumbent does not know the exact effect its investment on the entrant's behavior. In this sense, the incumbent does not know the size of the capacity that deters entry, nor does it know the effect of its investment size on the entrant's investment, given that the entrant enters.

Our main result is that the uncertainty generates a new incumbent policy. In case there is no uncertainty, the standard industrial organization literature learns that there are three incumbent policies: blockaded entry in the case of a natural monopoly, overinvestment to deter entry, and entry accommodation. Adding the uncertainty component to the incumbent's problem creates a new policy, namely that the incumbent overinvests to create entry deterrence with a certain probability. Another effect of adding uncertainty to the incumbent's problem with respect to the entry cost and the vertical differentiation parameter is that the lack of information makes overinvesting less effective so that the incumbent accommodates entry to a larger extent. With the horizontal differentiation parameter there are opposite effects. In addition to the fact that lack of information reduces the incentive to overinvest, increased uncertainty can lead to products either being very different or very similar. In the latter case, the incumbent is willing to deter entry through overinvestment, because if the products are very similar, entry will lead to a lot of competition in the output market. At the same time, this makes it worthwhile for the incumbent to deter entry, and overinvestment will be more effective in encouraging the potential entrant to refrain from entry. Therefore, in such a case, the incumbent prefers to pursue a policy of ”entry deterrence with a certain probability” above ”entry accommodation”.

We would like to mention two avenues as interesting topics for future research. First, based on the analysis in this paper, the potential entrant could consider whether it would be wise to disclose certain product features or not. Second, since an investment problem is inherently a dynamic problem, introducing this uncertainty about certain parameters of the entrant’s problem in a dynamic framework would be a very relevant topic.

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