

1 **The pay-for-success contract: A valuation note**

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19 **Acknowledgement:** We thank Ioannis Baltas, Thanassis Fassas, Konstantinos Kostaris, Mania  
20 Mavri, Giorgos Papayiannis, Fani Vasilikou and Thanassis Yannacopoulos, participants at the  
21 EBEEC 2023 Conference that was held in Chios, participants at the 29<sup>th</sup> MFS Conference  
22 Conference that was held in Paphos in 2023, participants at the 2023 ICBE-HOU Conference  
23 that was held in Athens and participants at the 13<sup>th</sup> FEBS Conference that was held in Panteion  
24 University of Social and Political Sciences for their comments on earlier versions of the paper.

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## The pay-for-success contract: A valuation note

### 4 **Abstract**

5 Pay-for-success contracts are social and financial innovations in social policy and capital  
6 markets, respectively. This paper argues that they exhibit option-like payoffs and implements  
7 standard option-pricing arguments in assessing the value of investing in pay-for-success  
8 contracts. Sensitivities vis-à-vis contract specifications are reflected in the valuation formula  
9 and help reach investment and social policy decisions. These sensitivities are demonstrated  
10 via a numerical application that uses parameters drawn from the Massachusetts Juvenile  
11 Justice Pay for Success Initiative, the largest pay-for-success initiative in the U.S. at the time  
12 of its launch.

13 **JEL:** G13; I30; Z19

14 **Keywords:** pay-for-success contracts; social impact bonds; pay-for-performance contracts;  
15 social finance; option pricing

16

### 17 **Introduction**

18 Pay-for-success contracts are joint projects of public sector, private sector, and civil society to  
19 address social or environmental problems in a way that lowers the costs and exceeds the  
20 performance of similar activities that are implemented by the public sector exclusively. Pay-  
21 for-success contracts are also known as social impact bonds or pay-for-performance contracts  
22 and funded projects are part of the growing impact-investment ecosystem (OECD, 2016;  
23 Andrikopoulos, 2021). In these contracts investors provide funds for a project that tackles a  
24 social problem. Tackling the social problem involves the articulation of a network of  
25 collaborating nodes, namely a) government authorities who have undertaken the political  
26 responsibility for addressing a social problem such as homelessness, unemployment,  
27 recidivism of former prisoners, early school leavers etc. b) suppliers of capital that include for-  
28 profit investors, impact investors, civil society organizations and organizations affiliated with  
29 the government (e.g., the National Community Lottery Fund in the UK), c) private-sector or  
30 civil-society social service providers that consume the largest part of the project's budget, d)  
31 service users, which are citizens, e) independent evaluators who identify and measure project  
32 outcomes, and f) intermediaries who coordinate all involved nodes in this network (for an  
33 introduction to the rationale and structure of these contracts, see, e.g., Gustafsson et al.  
34 (2015), Maduro et al. (2015), Edmiston and Nicholls (2018) and Millner and Meyer (2022)).

35 If the problem is successfully resolved, then the investors receive a payoff. The payoff is  
36 financed by the government that achieves cost savings since it is the investors who pay to  
37 implement a social policy, provided that the amount of accomplished savings exceeds the  
38 payoffs to investors. The payoff to investors has an upper bound, reflecting, in principle, that  
39 investments in social policy are not purely speculative. The first social impact bond was  
40 launched in 2010 and aimed at reducing the recidivism of young former prisoners in HMP  
41 Peterborough in United Kingdom (Nicholls and Tomkinson, 2015). Subsequent similar projects  
42 in the US include the social impact bond for addressing recidivism in the Rikers Island prison

1 in 2012 (Olson and Phillips, 2013), the Massachusetts Juvenile Justice Pay for Success Initiative  
2 in 2014 (Pandey et al., 2018), the Integration SIB Project in Finland in 2017, the Inclusive Youth  
3 Employment Pay For Performance Platform in South Africa in 2018, and the AiLSi jail diversion  
4 program for homeless people suffering from mental illness in Marseille in 2021. Critically,  
5 payoff depends on causally associating the funded social intervention with observed social  
6 change. For example, if the pay-for-success contract addresses recidivism of former prisoners,  
7 the reduction of recidivism helps the government save expenses on the penitentiary system  
8 (e.g., less bed-days for prisons) and these savings can trigger payments to investors. Likewise,  
9 in social inclusion projects, successful project implementation may imply steady employment  
10 and residence for citizens that have formerly been socially excluded, thereby helping the  
11 government save expenses on the respective welfare services and make payments to  
12 investors. The causal link between the implemented social policy and observed social changes  
13 is usually assessed by an independent validator that employs methodologies largely based on  
14 randomized control trials.

15 As an emerging policy instrument, pay-for-success contracts have spawned an ongoing debate  
16 on the limitations, the span and the synergies between the public sector, the private sector,  
17 and the civil society as systems for shaping the content and prospect of social welfare (e.g.,  
18 Schinckus, 2017; Harvey and Ogman, 2019; Andrikopoulos, 2020). The limitations of pay-for-  
19 success contracts hinder their way to becoming mainstream (Walker et al. 2022), even though  
20 there have been 292 contracts initiated since 2010 and their population keeps growing.<sup>1</sup> First,  
21 sustainable social outcomes may require stakeholder involvement long after these contracts  
22 have expired (Sinclair et al., 2021). Second, significant transaction costs are involved in  
23 designing a contract that is uniquely context-specific, bringing together counterparties with  
24 substantially diverse priorities and identities, thereby raising agency problems and challenging  
25 the possibility of pursued social and monetary benefits (Pauly and Swanson, 2017; Del Giudice  
26 and Magliavacca, 2019; Muñoz and Kimmit, 2020; Millner and Meyer 2021). Third, an  
27 evidence-informed and reward-based social policy that accommodates a profit-making  
28 private sector is exposed to making the provision of commodified social services contingent  
29 upon the attainment of profit, as opposed to the provision of social services by the  
30 government that is accountable to citizens whose problems are being addressed (Harvey and  
31 Ogman 2019; Sinclair et al. 2021). Despite all these reasonable concerns, pay-for-success  
32 contracts are important and can bear a positive impact on policy design since they expand the  
33 range of contractual possibilities and can therefore provide solutions to social problems where  
34 conventional policy designs and financial contracts fail to do so.

35 Admittedly, social policies cannot be exhaustively contained in valuation formulas and the  
36 epistemic foundations of financial economics are substantially challenged in the field of social  
37 finance (e.g., Lagoarde-Segot, 2019; Paraque and Revelli, 2019). Nevertheless, the discussion  
38 on investment problems can be richer when properly quantified, and, when discussions on  
39 impact investments grow, the range on available responses to social challenges widens. The  
40 following section derives a valuation result, and the subsequent section provides a numerical  
41 application of the valuation formula, using parameters drawn from the Massachusetts  
42 Juvenile Justice Pay for Success Initiative, the largest pay-for-success initiative in the US at the  
43 time of its launch in 2014. The last section concludes this note.

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<sup>1</sup> <https://golab.bsg.ox.ac.uk/knowledge-bank/indigo/impact-bond-dataset-v2/> .

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2 **Pay-for-success contracts: valuation**

3 Pay for success contracts exhibit immense variety in their contractual arrangement because  
 4 they reflect the diverse characteristics of the social problems being addressed. For simplicity,  
 5 we assume all contract cash flows take place at contract expiry. The payoff to an investment  
 6 in a pay-for-success contract is

$$7 \qquad \qquad \qquad \min (\alpha \times \max(K - S_T, 0), B), \qquad \qquad \qquad (1)$$

8 where:

9  $K$  is the cost currently incurred by the public sector in the provision of the social service within  
 10 a  $[0, T]$  time span, where  $T$  is the time when the contract expires. If the implemented contract  
 11 does not deliver cost savings for the public sector, then there is no payoff to the investor.

12  $S_T$  is the cost incurred by the public sector in the provision of the social service under the pay-  
 13 for-success contract, at contract expiry; e.g.,  $S_T$  stands for the cost of welfare services that are  
 14 provided by the government in the context of a pay-for-success contract that pursues  
 15 inclusion for the socially excluded, whereas  $K$  stands for the cost of welfare services if no pay-  
 16 for-success contract is implemented.

17 Parameter  $\alpha$  is the fraction of public-sector cost savings,  $\max(K - S_T, 0)$ , that is the paid to  
 18 the investor,  $0 \leq \alpha \leq 1$ . Parameter  $\alpha$  could be called a *profit sharing constant*.

19  $B$  is the maximum amount that can be paid to the investor according to the contract.  $B$  could  
 20 be called a *surplus cap*. Contract specifications for  $\alpha$  and  $B$  must be such that  $\alpha K - B > 0$ .

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22 Figure 1 shows the payoff to the impact investor, demonstrating the similarity between the  
 23 pay-for-success contract and a put spread.

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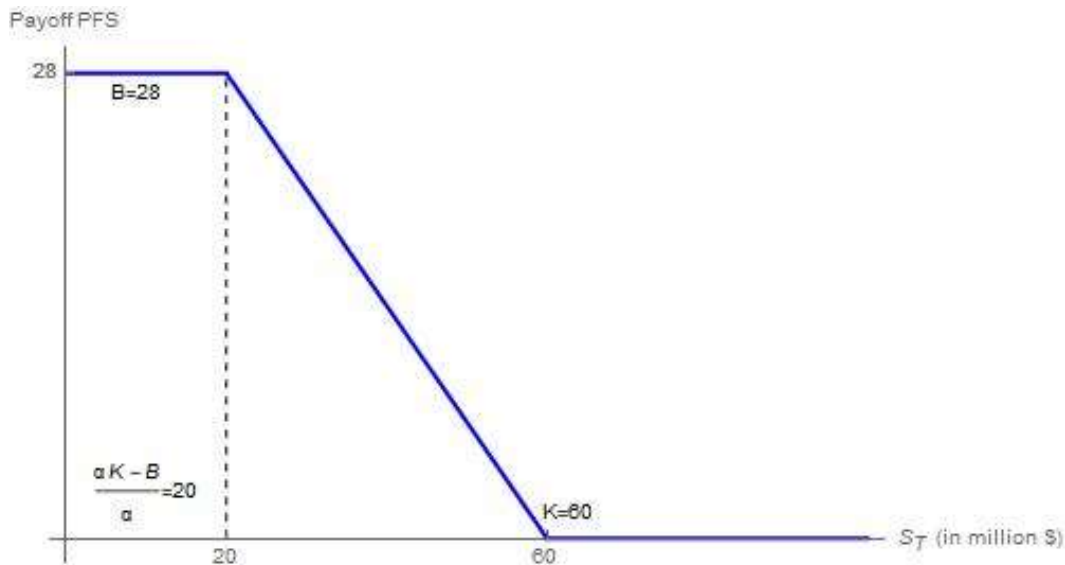
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1 **Figure 1.** Pay-for-success payoff at maturity

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3 Note:  $K = 60$ ,  $a = 0.70$ ,  $B = 28$ .

4 The payoff to this capped option resembles that of a put spread where the underlying asset is  
 5  $Y = \alpha S$ , the strike price of the long put position is  $\alpha K$  and the strike price of the short put  
 6 position is  $\alpha K - B$ . Therefore, (1) can be rewritten as  $\max(\alpha K - Y_T, 0) - \max(\alpha K - B -$   
 7  $Y_T, 0)$ , i.e., the difference of two put options. However, a pay-for-success contract is different  
 8 from a put spread in two ways. First, there is no optional exercise involved, in the sense that,  
 9 provided investors are rational, the option is automatically exercised when in-the-money at  
 10 the points in time when payments to investors are contractually arranged to take place.  
 11 Second, there are no traded European options and traded underlying assets involved, thereby  
 12 rendering the pay-for-success contract more similar to a real option on a non-traded asset  
 13 rather than a financial option on a traded asset.

14 The problem of pricing (European) options on non-traded assets was first tackled in Smith and  
 15 Nau (1995) in the context of real options and by Detemple and Sundaresan (1999) in the  
 16 context of portfolio constraints. However, our treatment below follows the subsequent work  
 17 of Davis (1999). Assume that the cost for the provision of the social service under the pay-for-  
 18 success contract is a stochastic process that follows a Geometric Brownian Motion under the  
 19 risk-adjusted measure  $\mathbb{P}$

$$20 \quad dS_t = vS_t dt + \eta S_t dW_t, \quad (2)$$

21 where  $\eta$  is the diffusion (volatility) parameter and  $dW_t$  is the increment of a Wiener process.  
 22 Observe that the drift parameter  $v$  does not include a dividend yield because pay-for-success  
 23 contracts do not include payments resembling dividend payments. A simple application of  
 24 Ito's lemma yields  $dY_t = vY_t dt + \eta Y_t dW_t$ .

25 Although  $S_t$  is not traded, assume there is a correlated asset,  $P_t$ , that is traded and whose  
 26 dynamics under the risk-adjusted measure  $\mathbb{P}$  are given by the Geometric Brownian Motion

$$27 \quad dP_t = \mu P_t dt + \sigma P_t dB_t, \quad (3)$$

28 where  $\sigma$  is the diffusion (volatility) parameter and  $dB_t$  is the increment of a correlated Wiener  
 29 process, with

$$\mathbb{E}^{\mathbb{P}}[dW_t dB_t] = \rho dt$$

and  $\rho$  the (constant) correlation coefficient. For simplicity, we assume that the traded asset pays no dividend, although it would be straightforward to relax this assumption.

Davis (1999) provides an approach, based on utility maximization, for pricing options in such an incomplete market setting. An option is fairly priced (for a particular decision-maker) in this setting, if going long or short a small amount of it has a neutral effect on the decision maker's achievable utility. Such fair prices are also known as utility indifference prices (see, e.g., Henderson and Hobson, 2008) or reservation prices (see, e.g., Munk, 1999).

Let's assume that the investor has a logarithmic utility function,  $U(X_t) = \log X_t$ , where  $X_t$  the value of the investor position at time  $t$ . The investor forms a portfolio that invests a fraction  $\pi_t$  in the traded asset  $P_t$  and  $(1 - \pi_t)$  in cash, with a view towards maximizing the expected terminal value of the position,  $\mathbb{E}^{\mathbb{P}}[U(X_T)]$ . In this setting the fair price  $\hat{p}$  of a European put option with underlying asset current value  $Y_0$  and strike price  $K$  is given by

$$\hat{p}(Y_0, K) = p(F(T), K, r, r, \eta, T), \quad (4)$$

where

$$F(T) = Y_0 \exp \left[ \left( v - \frac{\eta \rho}{\sigma} (\mu - r) \right) T \right] \quad (5)$$

and  $p(x, K, r, \delta, \eta, T)$  is the dividend-inclusive, Black and Scholes (1973) European put option pricing formula, for an underlying price  $x$ , risk-free interest rate  $r$ , dividend yield  $\delta$ , volatility  $\eta$  and maturity  $T$  (see Davis, 1999, Theorem 18.2 for the proof). Observe that in (4), one needs to set  $x = F(T)$  and  $\delta = r$  to get the fair price  $\hat{p}$ .<sup>2</sup> Given that the pay-for-success contract (PFS) is

$$PFS = \hat{p}(Y_0, aK) - \hat{p}(Y_0, aK - B), \quad (6)$$

we can use equations (4)-(6) to estimate its fair price.

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## 25 Numerical application: Drawing on the Massachusetts Juvenile Justice Pay for Success 26 Initiative

Injecting theory to practice, this section delivers a calibration of our model that is based on the Massachusetts Juvenile Justice Pay for Success Initiative (JJ-PFS hereafter), a program that was launched in 2014 and amended in 2016. While our model design is different than the Massachusetts Initiative in that it involves a single final payoff instead of many intermediate ones, the rationale of our model reflects the architecture of JJ-PFS and all our parameters are calibrated on the basis of the prison system of Massachusetts at the time of the project's implementation.

JJ-PFS aimed at reducing crime rates among former inmates in Chelsea, Springfield and Boston, Massachusetts. The program was planned to apply to up to 1,036 young men between the ages of 17 and 24. The time span of JJ-PFS was seven years and it was extended for two additional years in 2021, completing operations in 2023, outcome reports being expected in

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<sup>2</sup> The Black and Scholes (1973) formula  $p(x, K, r, \delta, \eta, T)$ , with  $x = F(T)$  and  $\delta = r$  is equivalent to the Black (1976) formula for pricing futures options, with a current futures price  $F(T)$  given in (3).

1 early 2024. The main parties were the Commonwealth of Massachusetts, Third Sector Capital  
 2 Partners, and Roca. Third Sector Capital Partners coordinated the implementation of the  
 3 project via its subsidiary Youth Services Inc. and Roca provided the social services in the  
 4 context of its own cognitive-behavioral intervention model. The State of Massachusetts would  
 5 not pay more than \$28 million, would pay nothing if the reduction of recidivism was less than  
 6 5% and would make savings of \$22 million if recidivism was reduced by 40% (reflecting a  
 7 reduction of 223,577 bed-days in prison).

8 Financial resources were provided by lenders of diverse capital outlay, seniority, and interest  
 9 rate (Goldman Sachs Social Impact Fund, Kedge Foundation and Living Cities), as well as by  
 10 donors, such as the Laura and John Arnold Foundation, New Profit Inc and Boston Foundation.  
 11 The US Department of Labor supplied a grant up to \$10.77 million, which was contingent on  
 12 early-stage success. The assessment of JJ-PFS outcomes would rely on quarterly reporting by  
 13 Roca, independent evaluation by Sibalytics and the Urban Institute, independent validation  
 14 by Public Consulting Group, and pro bono technical assistance by the Kennedy School Social  
 15 Impact Bond Technical Assistance Lab.

16 To apply our valuation model to the JJ-PFS, we need to estimate the maximum amount that  
 17 can be paid to the investor according to the contract ( $B$ ), the cost of the service (incarceration  
 18 costs) to the Commonwealth of Massachusetts if no pay-for-success contract is implemented  
 19 (i.e., the exercise price  $K$ ), the cost incurred by the Commonwealth of Massachusetts in the  
 20 provision of the social service if the pay-for-success contract is successfully implemented (i.e.,  
 21 the (non-traded) underlying asset,  $S$ ), the drift ( $\nu$ ) and volatility ( $\eta$ ) of the dynamics of the  
 22 (non-traded) underlying asset,  $S$  (see equation 2), and the fraction of public-sector cost  
 23 savings that is the paid to the investor, i.e. the profit sharing constant  $0 \leq \alpha \leq 1$ .<sup>3</sup>

24 From the data and information that are publicly available for the JJ-PFS, we can infer that the  
 25 incarceration costs to the Commonwealth of Massachusetts if no pay-for-success contract is  
 26 implemented are close to  $K = \$60$  million and that the profit sharing constant  $\alpha$  ranges from  
 27 0.6 to 0.73.<sup>4</sup> We set  $\alpha = 0.70$ . From the same source, we know that the Commonwealth of  
 28 Massachusetts will make up to at most \$28 million in success payments for this seven-year  
 29 project, which provides us of an estimate of the maximum paid amount  $B = \$28$  million.  
 30 Finally, from the same source, we know that the project's target impact is a 40% decrease in  
 31 days of incarceration, which translates to gross savings for the Commonwealth of  
 32 Massachusetts of \$22 million, making the cost incurred if the pay-for-success contract is  
 33 successfully implemented equal to  $\$60 - \$22 = \$38$  million. This is the value we employ for  
 34 the current value,  $S_0$ , of the (non-traded) underlying asset.

35 Unfortunately, we do not have a time-series of the (non-traded) underlying asset, the cost  
 36 incurred by the Commonwealth of Massachusetts in the provision of the social service if the  
 37 pay-for-success contract is successfully implemented, from which to estimate the drift ( $\nu$ ) and  
 38 volatility ( $\eta$ ) of its dynamics. However, we can proxy these parameters from an alternative  
 39 source. As the pay-for-success contract provides intervention to at-risk young men who are in  
 40 the probation or parole system or are leaving the custody of the Suffolk, Essex, Hampden, and  
 41 Middlesex Houses of Correction, we hand-collected the costs for the operation of the Suffolk,

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<sup>3</sup> We also need to estimate the dynamics of the correlated traded asset  $P$  (see equation 3), along with the correlation coefficient. We discuss their estimation further down in this section.

<sup>4</sup> See <https://www.thirdsectorcap.org/wp-content/uploads/2021/10/MA-JJ-PFS-Fact-Sheet-Revised-210101.pdf>, and more specifically the Table of the incarceration-based payment terms on p. 3.

1 Essex, Hampden, and Middlesex Sheriff's offices from the annual budget of the  
 2 Commonwealth of Massachusetts, for the years 2012-2019.<sup>5</sup> Under the assumption that the  
 3 costs associated with recidivism in Suffolk, Essex, Hampden, and Middlesex are, every year, a  
 4 constant fraction of the total operating costs of the Sheriff's offices in these areas, we can  
 5 proxy the drift ( $v$ ) and volatility ( $\eta$ ) of our underlying asset with the average return and  
 6 standard deviation of returns of the total operating costs of the four Sheriff's offices. This  
 7 yields estimates of  $v = 5\%$  and  $\eta = 10\%$ .

8 Finally, for the correlated, traded asset, we select the FTSE US Broad Investment-Grade Social  
 9 Impact Bond Index, which is a multi-sector social bond benchmark of investment-grade, US  
 10 Dollar denominated debt issued by US Treasury, government-sponsored organizations, and  
 11 corporations.<sup>6</sup> From the time-series of the index over the life of the JJ-PFS, we get estimates  
 12 of  $\mu = 5.5\%$  and  $\sigma = 15\%$  for the drift and volatility of the correlated asset (see equation 3).  
 13 The correlation between the two-time series (our proxies for  $S$  and  $P$ ) is  $\rho = 0.6$ .

14 For these parameter values, and a risk-free rate of 4% ( $r$ ), the value of the Massachusetts  
 15 Juvenile Justice Pay for Success contract is estimated at \$5.7786 million ( $\hat{p}(Y_0, aK) =$   
 16 \$5.7788 million and  $\hat{p}(Y_0, aK - B) = \$0.0002$  million). Valuation of the PFS contracts  
 17 requires estimation of the drifts ( $\mu, v$ ) and volatilities/correlation ( $\sigma, \eta, \rho$ ) of both  $S$  and  $P$ .  
 18 Difference of opinion on these parameters between suppliers and users of impact capital is  
 19 what makes pay-for-success investments plausible. The rest of the parameters can either be  
 20 directly identified in the contract or calibrated thereof, as we demonstrate with our numerical  
 21 application.

22 Figure 2 plots the value of the JJ-PFS as a function of the social service cost  $S$  for different  
 23 correlation coefficients between changes in the traded asset and the social service cost  $S$ . The  
 24 value of the project decreases in the social service cost  $S$  and increases in the correlation  $\rho$ .  
 25 For  $\mu - r > 0$ , as in our numerical application, a higher correlation  $\rho$  reduces the underlying  
 26 price  $F(T)$  in equation (5), making the put options  $\hat{p}(Y_0, aK)$  and  $\hat{p}(Y_0, aK - B)$  more  
 27 valuable, and overall increasing the value of the JJ-PFS. Moreover, the value of the investment  
 28 is increasing in the maximum amount that can be paid to the investor, namely the surplus cap  
 29  $B$  (see Figure 3). Interestingly, the increasing effect is more pronounced, the higher the  
 30 correlation of the traded asset with the cost of social service  $S$ . In Figure 3, as  $B$  increases the  
 31 term  $\frac{\alpha K - B}{\alpha}$  decreases (see Figure 1), making the  $\hat{p}(Y_0, aK - B)$  put option deep-out-of-the-  
 32 money, and hence the investment value almost insensitive to further increases of  $B$ .

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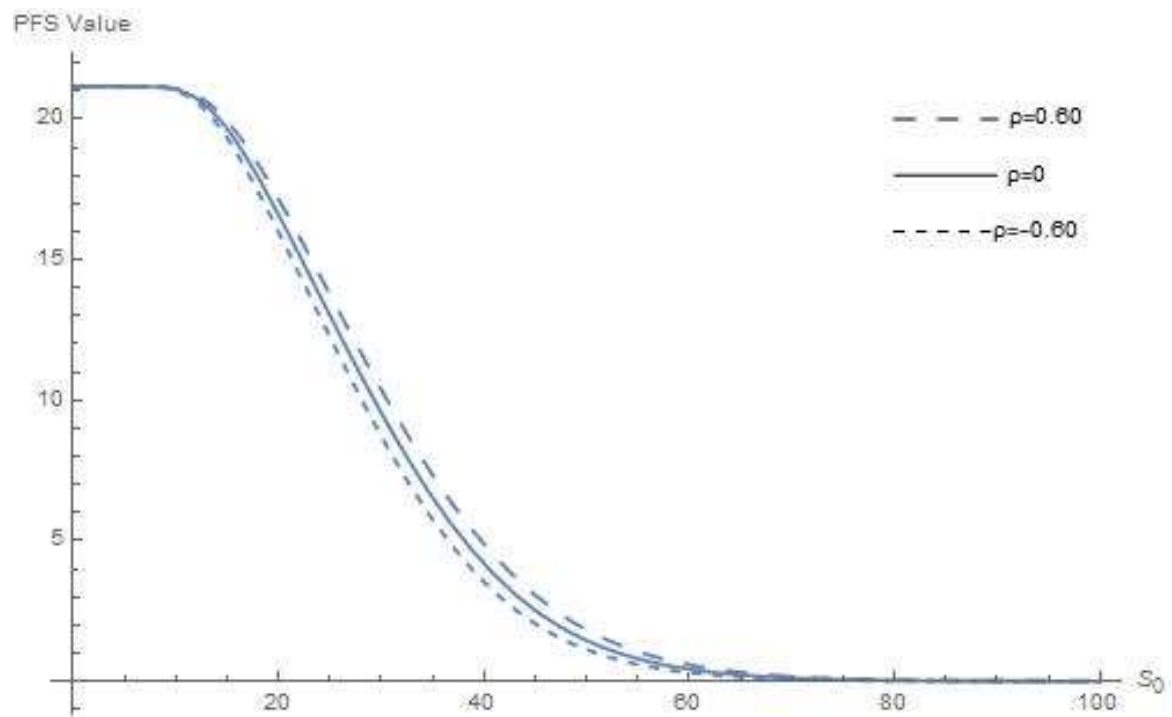
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<sup>5</sup> These are items 8910-0102, 8910-0107, 8910-0619 and 8910-8800 from the Commonwealth's annual budgets, that are available at <https://www.mass.gov/lists/budget-archives>.

<sup>6</sup> The constituents of the index are eligible Social and Sustainability Bonds that are in line with the core components of International Capital Markets Association (ICMA) Social Bond Principles (SBP). See <https://www.lseg.com/en/ftse-russell> for details.

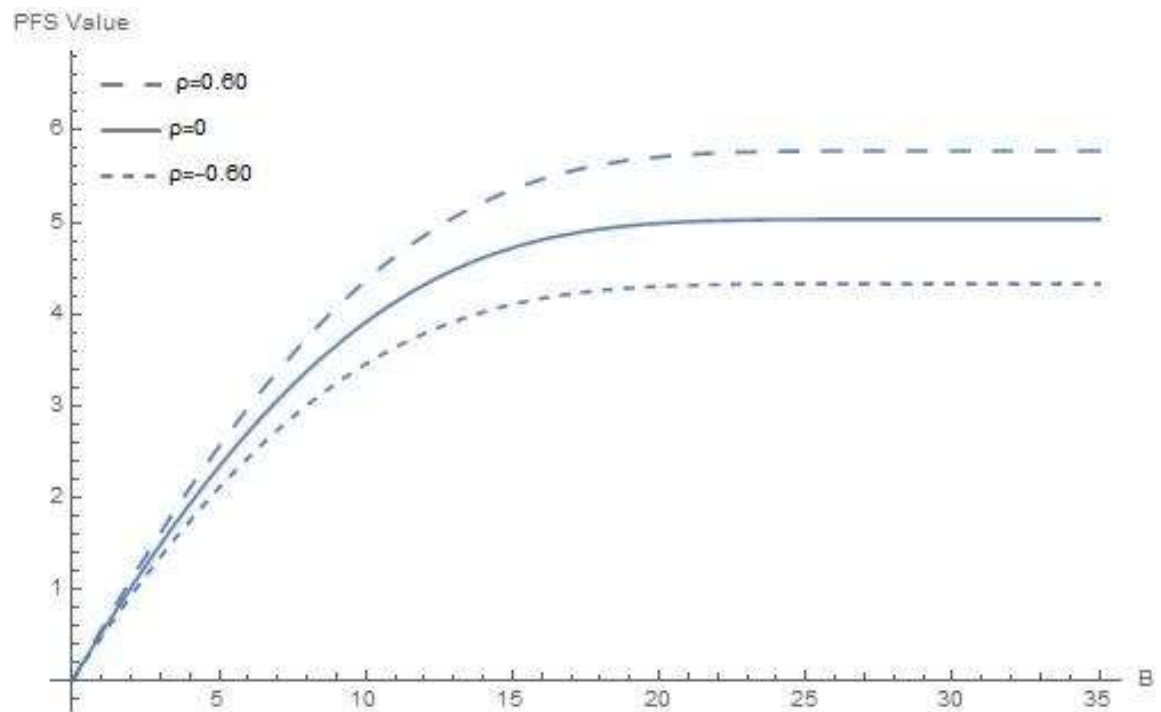


1 **Figure 2** Pay-for-success value and the social service cost

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3 Note:  $K = 60, \alpha = 0.70, r = 4\%, \mu = 5.5\%, v = 5\%, \sigma = 15\%, \eta = 10\%, T = 7, B = 28$ .

4

5 **Figure 3** Pay-for-success value and the surplus cap

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7 Note:  $K = 60, S_0 = 38, \alpha = 0.70, r = 4\%, \mu = 5.5\%, v = 5\%, \sigma = 15\%, \eta = 10\%, T = 7$ .

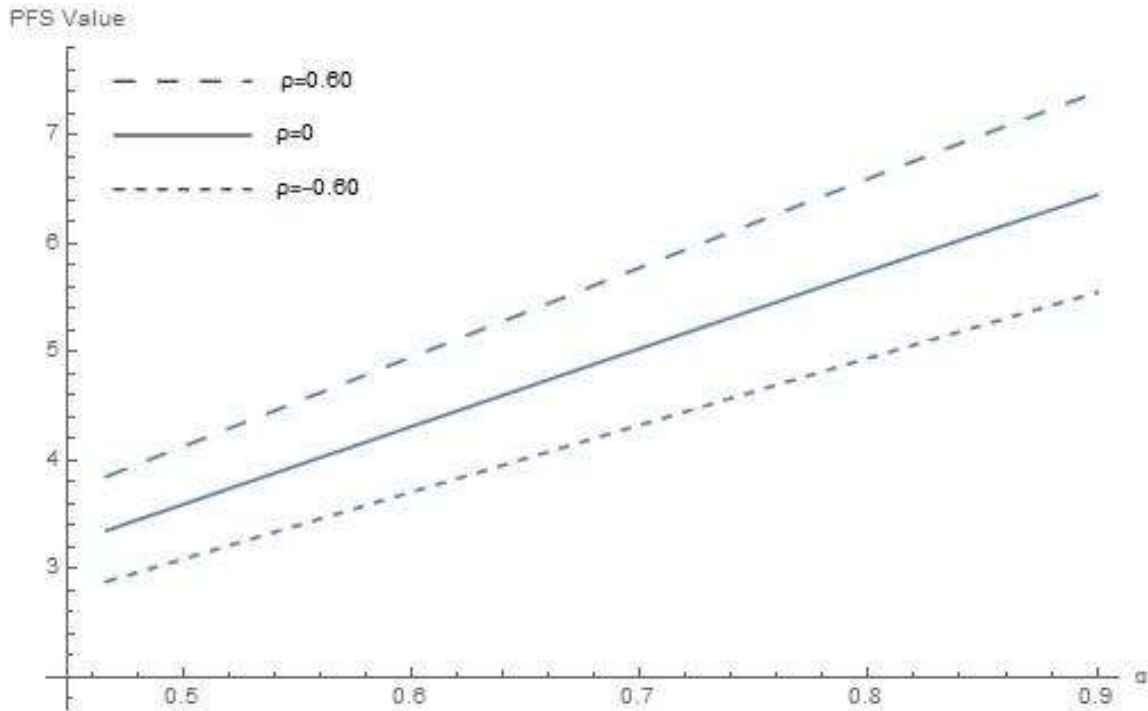
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1 In a similar vein, the value of the investment is increasing in the profit sharing constant  $\alpha$ ,  
 2 since the higher the portion of surplus accumulating to the investor, the more valuable the  
 3 investment will be (see Figure 4).

4

5 **Figure 4** Pay-for-success value and the surplus profit sharing constant



6

7 Note:  $K = 60, S_0 = 38, r = 4\%, \mu = 5.5\%, v = 5\%, \sigma = 15\%, \eta = 10\%, T = 7, B = 28$ .

8

9 Figures 5 plots the pay-for-success value with respect to the volatility  $\eta$  for different  
 10 correlation coefficients, while Figure 6 plots the sensitivity of PFS with respect to  $\eta$  (i.e. the  
 11 PFS vega), as a function of the cost of social service  $S$ . In Figure 5, the PFS value is increasing  
 12 in the volatility  $\eta$ , with the effect being more pronounced the more positive the correlation of  
 13 the traded asset with the cost of social service  $S$ . However, as Figure 6 demonstrates, the vega  
 14 of the PFS contract (i.e. the sensitivity  $\partial PFS / \partial \eta$ ) can be an increasing or decreasing  
 15 function of the social service cost  $S$  and of the correlation coefficient  $\rho$  (observe that the dependence  
 16 of vega on  $\rho$  reverses sign to the left and to the right of the point where vega attains its  
 17 maximum in Figure 6).

18 Finally, Figure 7 demonstrates that the PFS contract value could be increasing or decreasing  
 19 in the volatility of the traded asset, depending on (a) the correlation of the social service cost  
 20 with the traded asset  $\rho$ , and (b) the sign of  $\mu - r$ . Observe from (5) that  $F(T)$ , and thus the  
 21 investment value, is insensitive to  $\sigma$  if  $\rho = 0$ , i.e. if the traded asset is uncorrelated with the  
 22 cost of social service  $S$ .

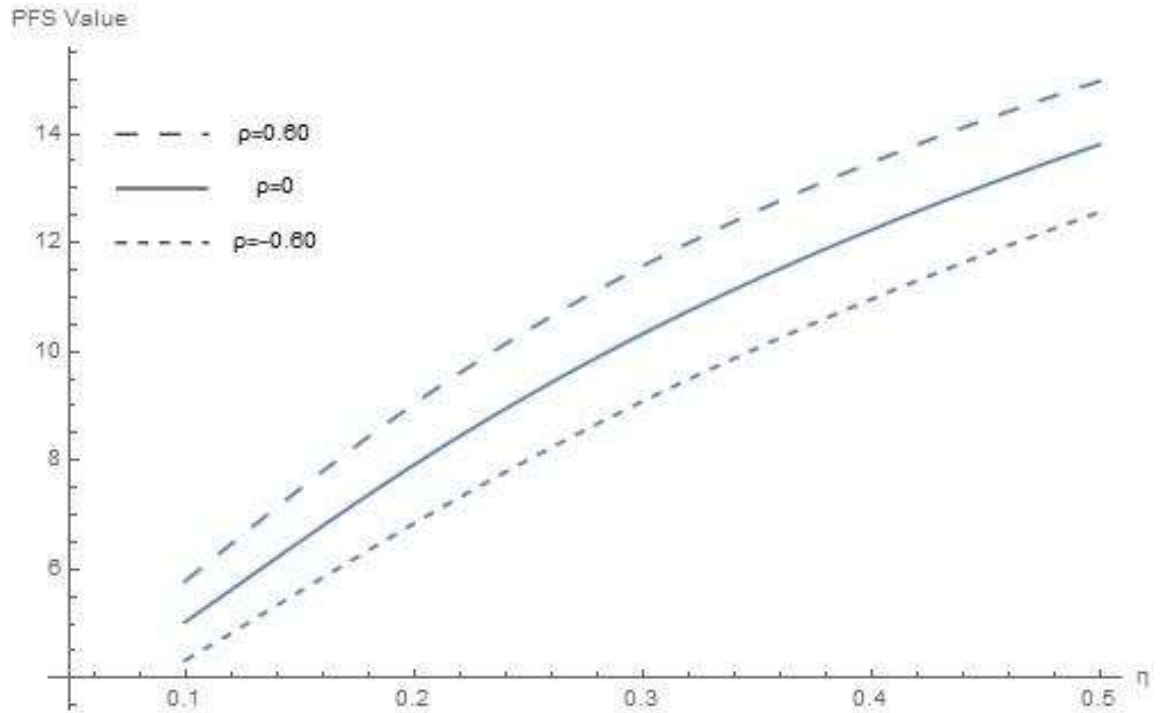
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2 **Figure 5** Pay-for-success value and the volatility of the social service cost

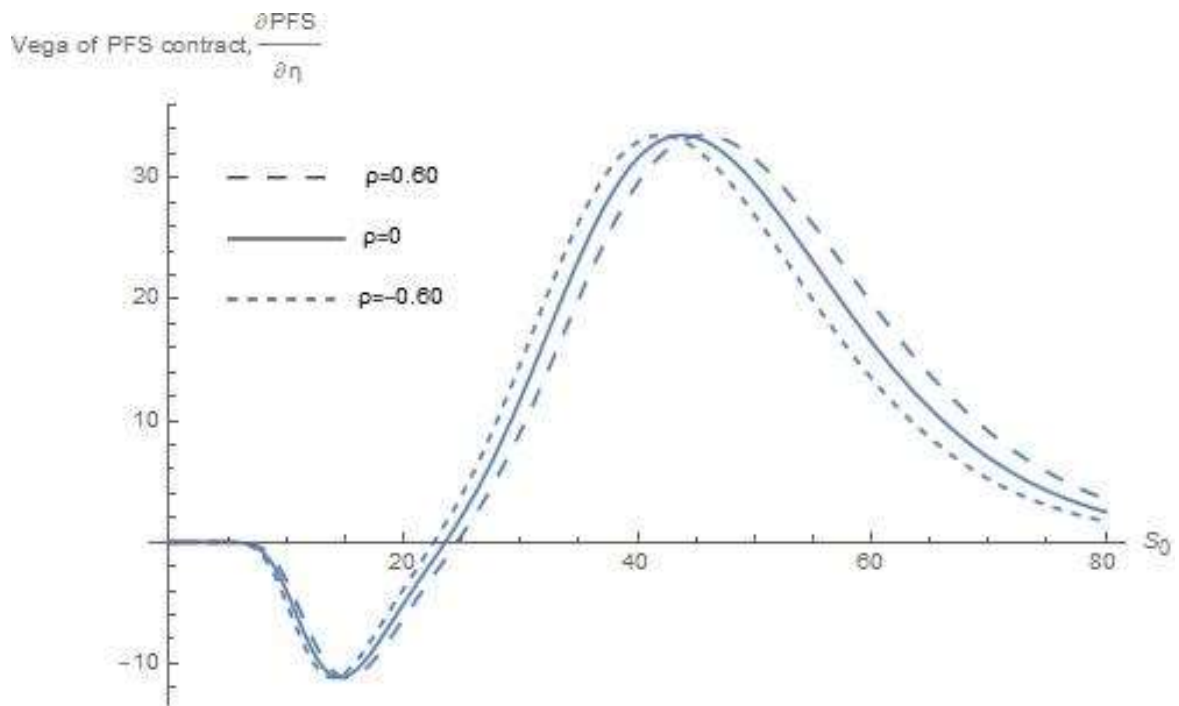


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4 Note:  $K = 60, S_0 = 38, \alpha = 0.70, r = 4\%, \mu = 5.5\%, v = 5\%, \sigma = 15\%, T = 7, B = 28$ .

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6 **Figure 6** The vega of the Pay-for-success contract as a function of the social service cost

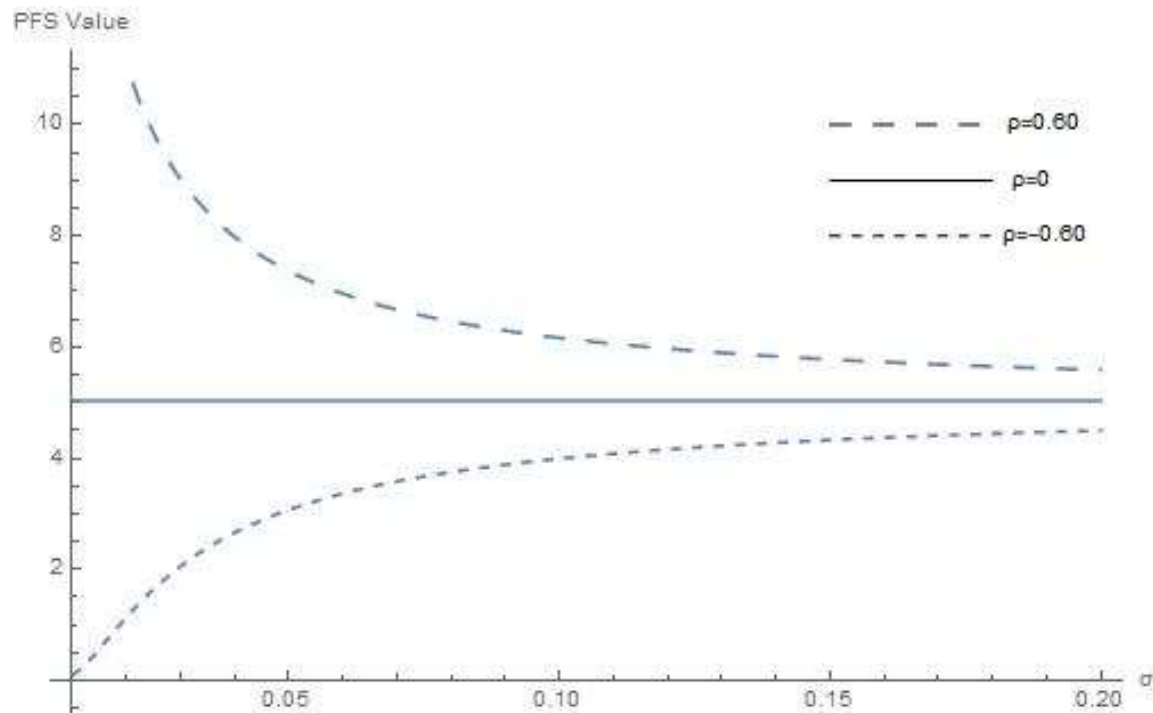


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8 Note:  $K = 60, \alpha = 0.70, r = 4\%, \mu = 5.5\%, v = 5\%, \sigma = 15\%, \eta = 10\%, T = 7, B = 28$ .

9

1

2 **Figure 7** Pay-for-success value and the volatility of the correlated traded asset

3

4 Note:  $K = 60, S_0 = 38, \alpha = 0.70, r = 4\%, \mu = 5.5\%, v = 5\%, \eta = 10\%, T = 7, B = 28$ .

5

6 **Conclusion**

7 Pay-for-success contracts (or pay-for-performance contracts or social impact bonds) are  
 8 emerging tripartite fusions of the public sector, the private sector, and the civil society, set to  
 9 tackle social and environmental problems. This note shows that investment payoffs exhibit  
 10 option-like characteristics, accommodating stochastic calculus arguments in investment  
 11 appraisal, and yielding familiar (from option pricing) intuition on the importance of investment  
 12 parameters. The calibration of our contingent claims model -a put spread in partially complete  
 13 markets- was based on the Massachusetts Juvenile Justice Pay for Success Initiative, a project  
 14 on the recidivism of formerly incarcerated young men.

15 Admittedly, option pricing formulas cannot capture all essential aspects of a social policy  
 16 instrument, like the pay-for-success contract. Therefore, a comprehensive assessment will  
 17 have to complement the valuation formula in this note with an analysis of the social, political,  
 18 and historical context that is unique for each pay-for-success contract. Furthermore, at the  
 19 cost of closed-form solutions, future research could incorporate more realistic contract  
 20 specifications, such as a piecewise contract implementation bearing similarities with  
 21 compound options, pay-for-success underwriting that minimizes investor risk and partial  
 22 funding of the pay-for-success investment with donations.

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