# The timing and terms of mergers between organically growing firms

## (*Preliminary and incomplete*)

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**Abstract**

We study how competitive firms accumulate capital and undertake a productivity-raising merger. We derive optimal investment and acquisition policies for standalone firms and with a merger option. Industry development generally involves an initial phase of catch-up growth to reach a long-run expansion path. On the long-run path highly capitalized firms wait to merge their current assets before pursuing further growth, whereas undercapitalized firms grow their assets first and merge at a higher threshold. If decisions are decentralized and firms commit to future deal terms, relative productivity-based shares induce efficient investment and merger decisions.

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# 1 Introduction

Corporate asset transactions are generally driven by expected productivity gains and tend to occur during market expansions (Maksimovic and Phillips 2002). A natural explanation of these regularities is that productivity gains accrue through uncertain cash flows whereas asset reallocation involves irreversibilities., so standard investment theory implies these transactions take place when market demand reaches sufficient thresholds. Existing analyses of mergers and acquisitions (M&A) exemplified by Lambrecht (2004) highlight the role of irreversibility in this process by holding industry conditions constant while firms contemplate whether and on what terms to transact. But over the horizon where firms decide mergers, capital stocks and hence the productivity increases achievable by reallocation are also liable to evolve. When determining whether to engage in corporate asset transactions therefore, firms must balance the productivity gains they can achieve externally against those they obtain by pursuing their own internal growth.

To shed light on such decisions, we study an industry consisting of two firms that regularly weigh furthering their capital accumulation against a one-off merger transaction that reshapes the industry. Capital investment is irreversible and stocks of marketable capital adjust upward instantaneously at a constant unit cost. Decreasing returns to scale lead firms to invest gradually. As a result the industry's capital grows incrementally and is eventually allocated efficiently between the firms. Merging the two firms brings synergies due to increased productivity, but is also costly. Moreover, for growing firms merging has a local effect similar to increasing returns. In this setting, we derive efficient investment policies and merger thresholds, as well as merger terms that support this outcome.

We allow for two forms of capital and therefore two kinds of misallocation in our model. The first kind of asset is marketable capital, which firms can accumulate by making irreversible investments. Firms resolve inefficient levels of such capital through internal growth. Notably, the industry can experience an initial phase of catch-up growth where only a single firm accumulates capital up, until reaching its long-run expansion path. Another possible source of misallocation is the fixed asset, and this is resolved by firms externally, through merger. The second kind of asset is fixed capital, which is invariable but can be combined across firms (with some possible irreversibility). Misallocations in this kind of capital are generally resolved externally, through mergers once market conditions are sufficiently favorable.

Our first main finding is to characterize the different forms that industry growth and mergers can assume. If firms are initially well capitalized, they generally wait to merge at a threshold reminiscent of Lambrecht (2004) but subsequently grow as a merged entity. If firms start with smaller capital levels however, then they must first grow internally before merging, and mergers occur at a higher threshold than for highly capitalized firms. In addition to this, mergers disrupt the path of incremental capital accumulation that firms would otherwise follow, with a surge in investment at the moment of the merger.[1]

We derive explicit solutions for the optimal merger threshold along with the barrier policies firms follow before and after merging. These solutions have natural interpretations in terms of markups of incremental profits over installation costs. Although market uncertainty determines the timing of mergers. Greater merger synergies lead to earlier mergers, and to firms being smaller at the time of the merger. In our framework, synergies are more important when firms are more symmetric all else equal, so our theory predicts that "mergers of equals" will occur earlier than if firms are more asymmetric.

Our second main finding concerns the terms of the merger. When we decentralize

[1]For a real-world example, mergers of EU telecoms firms are expected to raise infrastructure investment (see "FT wire: mergers necessary to stimulate investment in a fragmented industry", https://www.ft.com/content/cc674303-157e-4c96-9e4a-393899c3e4ab).

decisions and allow firms to commit to deal terms early on, we show that having each firm gets a share of the merger surplus that depends only on its relative productivity induces efficient investment and merger behavior. Moreover because firms follow their optimal growth paths up until the merger in our model, these surplus shares have an even simpler expression as they correspond to the pre-merger equilibrium capital shares.

## 2 Literature

Mergers and acquisitions are a fundamental topic in corporate finance which has been extensively studied from both empirical and theoretical perspectives. Maksimovic and Phillips (2001) identify the main empirical regularities surrounding corporate transactions and has shaped much of subsequent research in this area. Eckbo (2014) is an influential survey of subsequent research corroborating earlier findings regarding post-merger productivity improvements.

Productivity differences play a central role in our model, and Syverson (2011) surveys empirical work on total factor productivity, highlighting robust evidence of wide productivity variations. Our theory of mergers suggests that less capitalized firms have an incentive to raise investment significantly around the time of a transaction, which is consistent with evidence regarding R&D spending ahead of takeovers in Phillips and Zhdanov (2013). Kim (2018) highlights the importance of industry-specific assets, but focuses on transactions occurring at times of financial distress rather than growth. Recent work in corporate finance, Frésard et al. (2023), shows how firms alternate between phases of growth and productivity through instantaneous effort choice, whereas we highlight the role internal and external asset accumulation plays in such a process, with firms committed to total factor productivity levels from the onset.

Our work contributes to the literature on mergers and acquisitions under uncertainty going back to Lambrecht (2004)'s model of mergers motivated by economies of scale. Subsequent work in this area has highlighted the role of strategic behavior (Hackbarth and Miao 2012), as well as incorporating financial constraints (Gorbenko and Malenko 2018). By comparison we develop a complementary motivation for mergers, based on productivity differences, and above all we innovate by incorporating ongoing capital accumulation ahead of and after the transaction. By doing this, we offer new predictions on the terms and timing of mergers, together with their efficiency.

Margsiri et al. (2008) study firms choosing between internal growth and growth by acquisition, whereas these choices are not mutually exclusive in our model. In their paper, the amount of internal investment is exogenously given and influences the timing and terms of the acquisition only insofar it affects the acquisition price. In our model firms choose optimally their investment policy at any point of time, and this impacts both timing and terms of the merger even if the division of the synergies is considered as exogenous.

# 3 Model

Two firms in a competitive market operate a similar production technology in continuous time. They are endowed with a firm-specific asset which cannot be accumulated, and make irreversible decisions regarding the growth of the assets they can accumulate and external growth. The first kind of decision firms make is their ongoing investment choice, i.e. to accumulate productive capital as standalone firms. The second kind of decision they make is whether and when to merge their fixed assets and operate as a single entity thereafter.

We denote the two firms by $I$ and $J$. We refer here to firms for clarity, but our model applies just as well to individual assets, tangible or intangible, or firm divisions.

The firms use the same technology to produce a homogeneous output. Their production technology has three inputs: an invariant and firm-specific asset which we refer to as managerial or entrepreneurial ability, $H$; marketable capital, $K$, which firms accumulate irreversibly over time; and a fully variable input, $L$, which can be adjusted upward or downward at any moment and which we refer to as labor.[2] We suppose for simplicity that the stocks $H$ and $K$ do not depreciate. The variable input $L$ is not critical for our results, but comes at little additional computational burden and facilitates comparison with other work, particularly with Lambrecht (2004).

We assume the following technology specification:

**Assumption 1.** *Each firm produces a single output via the Cobb-Douglas production function*

$$q\left(L_i, K_i, H_i\right) = L_i^a K_i^b H_i^c,\ i \in \{I, J\}\,, \tag{1}$$

*where $L_i$ is firm $i$'s level of the variable output (labor), $K_i$ its level of the accumulated input (capital), and $H_i$ its level the fixed input (entrepreneurship). The input shares satisfy*

*i)* $a, b, c \in (0, 1)$,

*ii)* $a + b < 1$, *and*

*iii)* $a + b + c > 1$.

Assumption 1 thus posits that the returns to capital and labor are decreasing for a given level of the fixed input. This property of the technology ensures firms have the

[2]In other words, labor can be bought and sold instantaneously at the same price $w_L$, capital can be bought at a positive price but has a zero resale price, and entrepreneurial ability has an infinite purchase price and a zero resale price.

incentive to accumulate capital gradually rather than all at once. If all three inputs are allowed to vary however, the production function exhibits increasing returns to scale. This second property of the technology implies that there are potential synergies from reallocating the industry's fixed inputs.[3,4,5]

On the demand side, we suppose that firms operate under competitive conditions which evolve over time. We embody this idea by supposing that the two firms face an exogenous price process for their output. Let uncertainty be modeled by a filtered probability space $(\Omega, \mathfrak{F}, P)$. Then:

**Assumption 2.** *At any time $t \geq 0$, firms face an exogenous output price $p_t$ which evolves according to a geometric Brownian motion*

$$dp_t = \mu p_t dt + \sigma p_t dZ_t \tag{2}$$

*with initial state $p_0 = p$, where $\mu$ is the drift, $\sigma > 0$ is the volatility, and $dZ_t$ is the increment of a standard Wiener process.*

Firm cash flows are subject therefore to ongoing market uncertainty. The most straightforward situation that fits this assumption is if both firms are price-takers.[6]

Together Assumptions 1 and 2 imply that each firm $i \in \{I, J\}$ has an instantaneous

[3]The motivation for mergers in our model is therefore similar to the motivation developed in Lambrecht (2004), whose model is a limiting case of the present one as $b \to 0$ (so firms do not accumulate capital) provided that $a + c > 1$.

[4]Equivalently the firm can be viewed as holding a single composite asset $\widehat{K}_i = K_i^\alpha H_i^{1-\alpha}$ with $\alpha = \frac{b}{b+c}$, so $K_i$ and $H_i$ respectively represent those of the firm's assets which can and cannot be traded on a market (see footnote 2).

[5]The production function (Eq. 1) is also consistent with an alternative interpretation in situations where the fixed input cannot be directly measured. We would then just observe a total factor productivity term $A_i = H_i^c$ for each firm, and our analysis is otherwise unchanged.

[6]Eventually, the two firms may be price-takers in a broader market, whose other participants are not modeled explicitly. If capacity usage is constant (if we fix $L_{i,t}$s), then Assumption 2 is also consistent with cartel behavior.

profit function $p_t L_{i,t}^a K_{i,t}^b H_i^c - w_L L_{i,t}$, where $w_L$ is the wage which we take to be constant. Given the current price, each firm sets an optimal level of the variable input at any time $t$, $L_{i,t}^* = \left(\frac{aK_{i,t}^b H_i^c p_t}{w_L}\right)^{\frac{1}{1-a}}$. The instantaneous profit of each firm is therefore $\Pi K_{i,t}^\theta H_i^\eta p_t^\gamma$ where $\Pi = \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right) w_L^{-\frac{a}{1-a}}$, $\gamma = \frac{1}{1-a}$, $\theta = \frac{b}{1-a} < 1$, and $\eta = \frac{c}{1-a}$. Assumption 1 implies $\theta + \eta > 1$, so the returns to the two input stocks are increasing. Hereafter, we drop time subscripts when they are not necessary.

Both firms are assumed to have the same constant discount rate, which we denote by $r$. Suppose that firm $i$'s current capital were maintained at a constant level $K_i$ indefinitely. Ito's lemma implies that $p_t^\gamma$ follows a geometric Brownian motion with drift $g(\gamma) = \mu\gamma + \frac{1}{2}\sigma^2\gamma(\gamma - 1)$ so the expected value of the firm's optimal profit stream can be written as

$$E\left(\int_0^\infty \Pi K_i^\theta H_i^\eta p_t^\gamma e^{-rt} dt \middle| p_0 = p\right) = \Pi K_i^\theta H_i^\eta \frac{p^\gamma}{r - g(\gamma)}. \tag{3}$$

Eq. (3) is the expected present value of the firm's optimal profit stream, without accounting for the capital investments the firm makes after time zero. As will become apparent in Equation (5) below, the value of each firm considered as a standalone entity is the sum of this expected present value component and of the value of firm's expansion options, which we determine further below.

We next describe the irreversible decisions that firms make. The first of these is the decision to grow their capital stock. In contrast with previous work like Lambrecht (2004) and Hackbarth and Miao (2012) where capital stocks are invariant and exogenous, firms in our model bring an endogenous level of assets to a merger that results from ongoing managerial decisions.[7]

**Assumption 3.** *At any moment, firms determine whether to irreversibly increase their*

[7] Differently from Margsiri et al. (2008), in our model firms choose the optimal investment at every point of time, and therefore can combine periods of internal growth with external growth (i.e. mergers).

*capital stock, at a unit cost of capital $r_K$.*

Since there are no market imperfections, optimal capital accumulation by each firm implies that capital will eventually be allocated efficiently from the perspective of the industry. Any value of merging cannot then reside in gains from reallocating capital and must instead be due to another form of synergy. In our framework, this synergy comes from reallocating the fixed asset.

We suppose that a merger of the two firms is governed by the following set of conditions:

**Assumption 4.** *When the two firms merge*

*i) a single firm ($M$) operates afterwards,*

*ii) there is a fixed cost $X > 0$ associated with combining their assets,*

*iii) the merged firm's assets are $H_M = \lambda (H_I + H_J)$ and $K_M = K_I + K_J$, where $1 - \lambda \in [0, 1]$ is the degree of specificity of the fixed input,*

*iv) the post-merger production function $q(L_M, K_M, H_M)$ is given by Eq. (1).*

The fixed cost of the acquisition described in Assumption 4 above may be due both to financial and legal expenses associated with corporate transactions or to restructuring costs. As is standard in the literature, we suppose that such costs are independent of the level of cash flows.

Without specificity of the fixed assets (if $\lambda = 1$), merging is always profitable at a sufficiently high demand state. Otherwise, the degree of asset specificity must be bounded in order to ensure that merging is eventually profitable:

**Assumption 5.** *The degree of specificity of the fixed input satisfies $1-\lambda < 1-\underline{\lambda}(H_I, H_J)$, where $\underline{\lambda}(H_I, H_J) = \left( \left( \frac{H_I}{H_I+H_J} \right)^{\frac{\eta}{1-\theta}} + \left( \frac{H_J}{H_I+H_J} \right)^{\frac{\eta}{1-\theta}} \right)^{\frac{1-\theta}{\eta}}$.*

Finally, we make a technical restriction that ensures the profit streams and growth option values converge:

**Assumption 6.** *The parameters* $(\gamma, \theta, \mu, \sigma\ )$ *satisfy* $g\left(\frac{\gamma}{1-\theta}\right) < r$.

# 4 First-best

In this section we describe the efficient policy if the two firms grow first along separate paths and eventually merge their assets to function as a single entity, starting with an intuitive overview of the first-best solution.[8]

With incremental and irreversible investments the optimal policy generally involves regulation. This means that the inaction region is bounded above by an increasing barrier, with just enough investment occurring whenever a new price threshold is hit so as to maintain the capital stock inside the barrier. The timing of investment in this framework is driven by the productivity of a firm's current capital stock, and not by any future productivity levels that the firm can expect to eventually achieve. The same logic holds true whether future productivity increases result from increases in the firm's capital stock or from merging. Because merging does not alter prior investment incentives, we can therefore start our account of the first-best by considering the firms as standalone entities, and then determine the exercise of the merger option.[9]

When the firms operate separately, the capital stock of each firm is regulated by an increasing barrier which is inversely related to its marginal productivity. Depending on initial endowments, one firm will be generally be relatively undercapitalized and therefore

[8]The formal proofs are collected in Appendix 6.1.

[9]As it will become clear below in Proposition 2, this property holds because the efficient merger happens at a price threshold which is independent of individual capital stocks. As long as the output price is such that a merger is not profitable, the two separate firms operate with a technology exhibiting decreasing returns on capital (see Assumption 1 (ii)) where incremental investments follow the optimal policy in Abel and Eberly (1996).

more productive. In that case there is an initial adjustment phase where only the under-capitalized firm accumulates capital along its barrier, up until a high enough price state is reached that both firms subsequently accumulate capital along their respective barriers. Thereafter, the efficient policies of each firm equate their productivities which allows us to aggregate the solutions. We thus obtain a barrier for the industry which corresponds to individual capital stocks lying along the industry's long-run expansion path, and an expression for industry value which depends only on total capital.

Merging raises the expected discounted value of the industry profit stream but requires a constant fixed cost. It is better therefore to wait before merging if price and capital levels are initially low. After the merger, the same reasoning applies for the merged entity as for the pre-merger standalone firms, with incremental investment regulated by a similar increasing barrier. Because productivity is higher after the merger, the post-merger barrier lies strictly below the pre-merger level. At the moment of the merger therefore, the immediate effect is to shift the industry's barrier downward. This downward shift implies that a one-time discrete investment is needed to align the industry's capital stock with its new barrier.[10] Aside from determining incremental investment before and after the merger, the first-best solution also requires therefore us to characterize the merger in terms of both its timing and structure, which involves determining both a threshold and the discrete investment level associated with it.

We first describe the industry's expansion with standalone firms, i.e. leaving the option to merge aside (Section 4.1), and then incorporate the merger option (Section 4.2).

[10] An alternative explanation of this discrete investment involves viewing the industry as a single firm. The upward jump in productivity from merging is analogous to locally increasing returns to scale for industry production. Whereas it is still optimal to accumulate capital gradually at sufficiently low or high capital levels, there is therefore an intermediate range where investment is lumpy, as shown by Dixit (1995). One difference in our model is that the timing of the merger and the size of the discrete investment are endogenous, and another is our addition of a fixed cost of merging.

## 4.1 Pre-merger industry dynamics (separately growing firms)

We first characterize each firm's individual investment and then the aggregate solution. As noted above, because each firm's incremental investment is driven by its instantaneous productivity rather than future productivity gains, we can momentarily leave the effect of merging aside and treat firms as standalone entities in this part of the section.

Because the technology exhibits decreasing returns in the individual capital stocks $K_i$, each firm $i$ solves a standard incremental investment problem.[11] The solution involves regulation at a barrier which describes the upper bound of each firm's inaction region in its state space $(K_i, p)$. We denote this barrier by $p_i(K_i)$, $i \in \{I, J\}$. It has an explicit expression which satisfies

$$\theta \Pi K_i^{\theta-1} H_i^{\eta} \frac{(p_i(K_i))^{\gamma}}{r - g(\gamma)} = \frac{\beta_1}{\beta_1 - \gamma} r_K, \tag{4}$$

where $\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$ is a standard parametric expression (it is the upper root of the characteristic equation associated with the firm's ODE, Eq. 27), which satisfies $\beta_1 > \gamma$ (by Assumption 6). Eq. (4) is written so as to highlight how the barrier represents a an optimal mark-up, where the left-hand side is the (perpetualized) marginal value product of the firm's current capital and the right hand side is a multiple of its cost of capital, by the factor $\frac{\beta_1}{\beta_1 - \gamma}$.[12]

Firm $i$'s standalone value, i.e. the value from regulation along its barrier without

[11] See Abel and Eberly (1996). One intuition for the incremental investment problem is to consider that the firm has a succession of investment options of infinitesimal size $dK_j$. Because productivity decreases, the exercise thresholds of these successive options increase with the firm's capital stock, tracing out the barrier in Eq. (4).

[12] Increases in uncertainty in the form of higher volatility have the effect of raising both the mark-up $\left(\frac{\beta_1}{\beta_1 - \gamma}\right)$ and the expected perpetual value (through a lower adjusted discount rate $r - g(\gamma)$), so the overall effect on the barrier is generally ambiguous.

accounting for the merger option, is

$$V^i(K_i,p)=\begin{cases}\Pi K_i^\theta H_i^\eta \frac{p^\gamma}{r-g(\gamma)}+\frac{\gamma^2}{(\beta_1(1-\theta)-\gamma)(\beta_1-\gamma)}r_K K_i\left(\frac{p}{p_i(K_i)}\right)^{\beta_1}, \\ p\leq p_i\left(K_i\right) \\ \Pi\left(K_i+\Delta_K^i\right)^\theta H_i^\eta\frac{p^\gamma}{r-g(\gamma)}+\frac{\gamma^2}{(\beta_1(1-\theta)-\gamma)(\beta_1-\gamma)}r_K\left(K_i+\Delta_K^i\right)-r_K\Delta_K^i, \\ p>p_i\left(K_i\right)\end{cases} \tag{5}$$

where $\Delta_K^i$ is a discrete investment defined by $p_i\left(K_i+\Delta_K^i\right)=p$ which is required to immediately bring the firm back to its barrier if it lies outside of it initially, i.e. if $p>p_i\left(K_i\right)$. This value function has two pieces, the first piece being the along and below the barrier whereas the second is the value outside of it where firm $i$ makes an immediate one-time investment. In each piece of Eq. (5), the first set of terms is the expected present value of the firm's profit stream with either its current $(K_i)$ or its target $(K_i+\Delta_K^i)$ capital stock respectively. Firm $i$'s capital stock is expected to increase over time however, so the second term incorporates the additional value from the firm's exercise of subsequent growth options. Finally, the last term in the second piece is the cost of the firm's one-time discrete investment.

Figure 1 plots the investment barriers $p_i\left(K_i\right)$, $i=I,J$, for the two firms. Both of the barriers are increasing because the marginal productivity of $K_i$ decreases (see Eq. 4). In the figure, we assume $H_I>H_J$ so firm $I$ is more productive and has a lower barrier. The specific forms allow us to determine the efficient long-run capital ratio between the two firms. When the two firms accumulate capital along their respective barriers, $p_I\left(K_I\right)=p_J\left(K_J\right)$, and taking the ratio of the respective threshold conditions (Eq. 4 for $i=I,J$) gives

$$\frac{K_I}{K_J}=\left(\frac{H_I}{H_J}\right)^{\frac{\eta}{1-\theta}}. \tag{6}$$

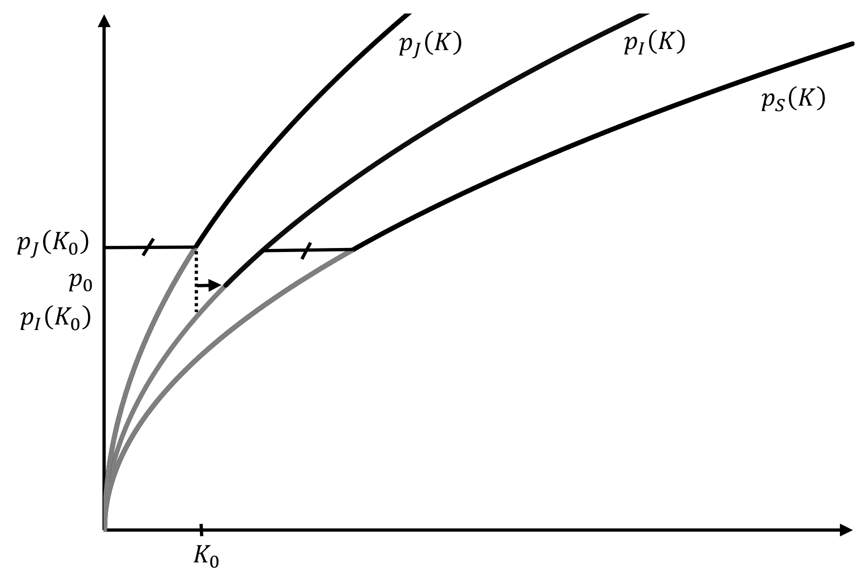


Figure 1: (Growth of standalone firms). Each firm's inaction region is bounded above by an increasing barrier $p_i\left(K_i\right)$, $i = I, J$. We assume $H_I > H_J$ so firm $I$'s barrier is lower. Firms have the same initial capital level $K_0$, so firm $I$ is undercapitalized. At the initial price $p_0 \in \left(p_I\left(K_0\right), p_J\left(K_0\right)\right)$, firm $I$ makes an immediate discrete investment to attain its barrier (black arrow) and accumulates capital incrementally thereafter while firm $J$ waits. The efficient capital ratio is attained once the price reaches $p_J\left(K_0\right)$. Thereafter both firms invest incrementally and industry capital follows the aggregate barrier $p_S(K)$.

From the perspective of the industry, the efficient ratio corresponds to the long-run expansion path. Along this path, because firm $I$'s barrier is lower it ultimately grows more than firm $J$, achieving a consistently higher capital stock. Moreover, Eq. (4) implies that this ratio is also the ratio of firm profits along the expansion path. The actual pattern of growth in the industry is governed by initial conditions. If firms start with an efficient capital ratio or if the initial price state is sufficiently high, both firms immediately begin accumulating capital along their respective barriers (after possible immediate discrete investments if they start outside the inaction region). Letting $K = K_I + K_J$ denote total industry capital, their individual capital stocks then follow

$$K_I(K) = \frac{H_I^{\frac{\eta}{1-\theta}}}{H_I^{\frac{\eta}{1-\theta}} + H_J^{\frac{\eta}{1-\theta}}} K \tag{7}$$

and

$$K_J(K) = \frac{H_J^{\frac{\eta}{1-\theta}}}{H_I^{\frac{\eta}{1-\theta}} + H_J^{\frac{\eta}{1-\theta}}} K \tag{8}$$

respectively.

The expressions $K_I(K)$ and $K_J(K)$ allow us to construct an aggregated or industry profit function along the industry's expansion path. Using the subscript $S$ to refer to standalone or separately operating firms, the expected discounted value of the industry profit stream is then obtained by substituting the expressions (7) and (8) into each firm's production function. The expected present value of the industry's profit stream at its current capital stock is then

$$\Pi K^{\theta} H_S^{\eta} \frac{p^{\gamma}}{r - g(\gamma)}, \tag{9}$$

where $H_S$ is the industry's effective level of the fixed input, which satisfies $H_S = \left(H_I^{\frac{\eta}{1-\theta}} + H_J^{\frac{\eta}{1-\theta}}\right)^{\frac{1-\theta}{\eta}}$. Treating the industry as a single firm, we can define an aggregate barrier by $p_S(K) = p_I(K_I(K)) = p_J(K_J(K))$. This barrier is plotted in Figure 1, as the horizontal sum of the individual barriers $p_I(K)$ and $p_J(K)$.

On the other hand, if the initial capital ratio is inefficient and the initial price state is not too high, then one of the firms, say $I$, is undercapitalized in the sense that its current capital stock has a higher marginal productivity, or equivalently $p_I(K_I(K)) < p_J(K_J(K))$. In this case the industry experiences an initial phase of catch-up growth, where only the undercapitalized firm accumulates capital along its barrier (after a possible discrete investment) up until the capital ratio reaches the efficient level (Eq. 6), whereupon both firms grow incrementally so industry progresses along its long-run growth path. This situation is illustrated in Figure 1. Both firms hold identical capital stocks $K_I = K_J = K_0$ initially, but firm $I$ has a greater endowment of the fixed input and therefore a lower barrier. At the initial price state $p_0$, firm $I$ makes a discrete investment to position itself at its barrier and subsequently accumulates along this barrier, whereas firm $J$ waits. Once the price state reaches $p_J(K_0)$, firm $I$ has accumulated enough so the capital ratio is efficient. Thereafter, firm $J$ accumulates along its barrier as well and the industry progresses along its long-term expansion path (black curve).

Proposition 1 sets out these results.

**Proposition 1** *For a given initial state $(p, K_I, K_J)$, the industry growth path with standalone firms is either:*

1. *If $p \geq \max\{p_I(K_I), p_J(K_J)\}$ or if $\frac{K_I}{K_J} = \left(\frac{H_I}{H_J}\right)^{\frac{\eta}{1-\theta}}$, then after a possible discrete investment $\Delta_K^I + \Delta_K^J$ such that $p_i(K_i + \Delta_K^i) = p$, $i = I, J$, industry capital grows incrementally along the barrier $p_S(K)$ with individual capital levels given by Eqs. (7) and (8);*

2. *If $p < \max\{p_I(K_I), p_J(K_J)\}$ and $\frac{K_I}{K_J} \neq \left(\frac{H_I}{H_J}\right)^{\frac{\eta}{1-\theta}}$, denote the undercapitalized firm by $i$, i.e. $p_i(K_i) = \min\{p_I(K_I), p_J(K_J)\}$. Then after a possible discrete investment $\Delta_K^i$ such that $p_i(K_i + \Delta_K^i) = p$, firm $i$'s capital grows incrementally along the individual barrier $p_i(K_i)$ until $p$ first reaches $\max\{p_I(K_I), p_J(K_J)\}$. Thereafter industry capital grows incrementally along the barrier $p_S(K)$ with individual capital levels given by Eqs. (7) and (8).*

One implication of Proposition 1 which is useful for our subsequent analysis of the merger option is to gives a lower bound on the price state, $\max\left\{p_I\left(K_I\right), p_J\left(K_J\right)\right\}$, beyond which the capital stocks can be aggregated and the industry viewed as a single firm. For such price states, industry value is $V_S(K,p) = V_I(K_I(K),p) + V_J(K_J(K),p)$ has a straightforward expression resulting from Eq. (5),

$$V^S(K,p) = \begin{cases} \Pi K^{\theta} H_S^{\eta} \frac{p^{\gamma}}{r-g(\gamma)} + \frac{\gamma^2}{(\beta_1(1-\theta)-\gamma)(\beta_1-\gamma)} r_K K \left(\frac{p}{p_S(K)}\right)^{\beta_1}, \\ p \leq p_S\left(K\right) \\ \Pi\left(K+\Delta_K^S\right)^{\theta} H_S^{\eta} \frac{p^{\gamma}}{r-g(\gamma)} + \frac{\gamma^2}{(\beta_1(1-\theta)-\gamma)(\beta_1-\gamma)} r_K\left(K+\Delta_K^S\right) - r_K \Delta_K^S, \\ p > p_S\left(K\right) \end{cases} \tag{10}$$

where $\Delta_K^S = \Delta_K^I + \Delta_K^J$ with $p_i\left(K_i + \Delta_K^i\right) = p$, $i = I, J$.

Proposition 1 also makes broader predictions regarding firm growth in industries depending on their structural conditions. The more asymmetric is the distribution of the invariant and firm-specific factor for example, the more asymmetric the long-run size distribution of firms in the industry. In other words, if input $H$ consists of intangible assets whose accumulation is extremely slow and costly, equation (6) predicts that differences in fixed assets across firms lead to an amplified difference in their growth paths.[13] This effect of initial endowments is stronger the greater is the productivity of the unobserved input ($c$) or the closer measured returns ($a+b$) are to constant returns to scale. In addition, if we consider that firm assets, particularly intangible assets, are either easier ($K$) or harder ($H$) to accumulate, our model implies that holding total returns ($a+b+c$) constant the firm size dispersion increases in industries where the unobservable capital (that cannot be accumulated) is more productive.

[13] I.e. as $\frac{\eta}{1-\theta} = \frac{c}{1-a-b} > 1$, small differences between $H_I$ and $H_J$ leads to magnified difference between $K_I$ and $K_J$.

## 4.2 The timing and structure of mergers on the long-run path

Suppose that the allocation of marketable capital is initially efficient, so the conditions of Proposition 1 (1) are met. The two firms will eventually find it profitable to engage in a merger whose timing and structural characteristics depend on the industry's level of capitalization and are formally stated below in Proposition 2.

When the assets of both firms are combined the profit stream of the merged entity has the same form as the industry profit stream of separately run firms (Eq. 9), i.e. $\Pi K^{\theta} H_M^{\eta} \frac{p^{\gamma}}{r-g(\gamma)}$. After the merger, the industry faces a standard incremental investment problem. The solution if this problem is characterized by the barrier $p_M(K) = \left(\frac{\beta_1 r_K (r-g(\gamma))}{(\beta_1-\gamma)\theta\Pi}\right)^{\frac{1}{\gamma}} \frac{K^{\frac{1-\theta}{\gamma}}}{H_M^{\frac{\eta}{\gamma}}}$, which follows a mark-up rule similar to Eq. (4) above. This policy results in a value $V^M(K,p)$ similar to Eq. (10) above, with $H_i = H_M$, i.e.

$$V^M(K,p) = \begin{cases} \Pi K^{\theta} H_M^{\eta} \frac{p^{\gamma}}{r-g(\gamma)} + \frac{\gamma^2}{(\beta_1(1-\theta)-\gamma)(\beta_1-\gamma)} r_K K \left(\frac{p}{p_M(K)}\right)^{\beta_1}, \\ p \leq p_M(K) \\ \Pi \left(K+\Delta_K^M\right)^{\theta} H_M^{\eta} \frac{p^{\gamma}}{r-g(\gamma)} + \frac{\gamma^2}{(\beta_1(1-\theta)-\gamma)(\beta_1-\gamma)} r_K \left(K+\Delta_K^M\right) - r_K \Delta_K^M, \\ p > p_M(K) \end{cases} \tag{11}$$

where $\Delta_K^M > 0$ solves $p\left(K+\Delta_K^M\right) = p$.

Figure 2 illustrates pre- and post-merger barriers in the industry state space. Like in Figure 1, both barriers are increasing. Because merging raises marginal productivity upward for all capital levels, the post-merger barrier lies below the pre-merger barrier. The positioning of these barriers has different implications for how firms merge, depending on the industry's initial capital level.

At low initial capital levels ($K < \overline{K}$ in Figure 2), the industry invests incrementally

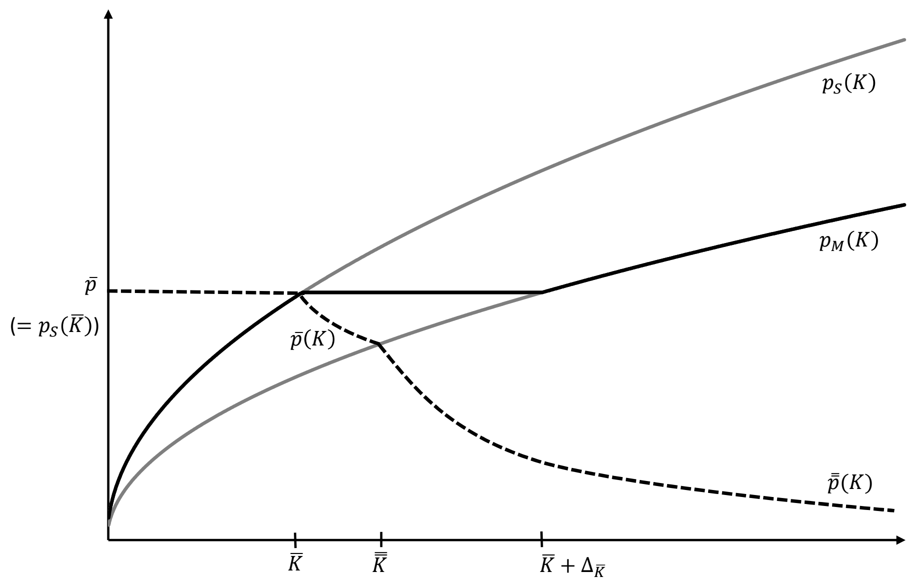


Figure 2: (Long-run growth path and merger thresholds). At low capital levels ($K < \overline{K}$), firms invest incrementally along $p_S(K)$ up to the threshold $\overline{p}$ where they make a discrete investment $\Delta_{\overline{K}}$ and merge. At high capital levels ($K \geq \overline{\overline{K}}$), firms wait without accumulating until the threshold $\overline{\overline{p}}(K)$ is reached and then merge their existing assets. At intermediate capital levels ($\overline{K} \leq K < \overline{\overline{K}}$), firms wait until the threshold $\overline{p}(K)$ is reached to make a discrete investment and merge.

along the barrier $p_S(K)$ ahead of the merger if the initial price state is not too high. There is a fixed merger threshold $\overline{p}$. When this threshold is reached, the industry's capital level is $\overline{K}$ and the firms subsequently merge. The merged entity finds itself outside the post-merger barrier, as $p_M\left(\overline{K}\right) < p_S\left(\overline{K}\right)$. An immediate mass of investment $\Delta_{\overline{K}}$ such that $p_M\left(\overline{K} + \Delta_{\overline{K}}\right) = \overline{p}$ is therefore required to bring the industry back onto the long-run path.

At high initial capital levels ($K \geq \overline{\overline{K}}$ in Figure 2), if the initial price state is not too high the industry initially waits ahead of the merger without accumulating further capital. In this case the merger threshold is a decreasing function $\overline{\overline{p}}(K)$ of the industry's capital stock, which lies below the standalone barrier $p_S(K)$. Firms therefore wait to merge, deferring further growth until they have reallocated the fixed input. When the merger threshold is reached, the firms merge by combining their existing assets only, since the merger threshold also lies below the post-merger barrier. Relative to the less capitalized case, the firms merger earlier as $\overline{\overline{p}}(K) < \overline{p}$. Observationally this case comes closest to the analysis in Lambrecht (2004), except the firms in our model resume growth after the merger instead of remaining indefinitely at their starting capital levels.

Finally there is an intermediate case that combines features of the two preceding ones. Firms with such intermediate capital levels wait to merge if the initial price state is not too high without accumulating further capital, set a lower merger threshold than they would if they were less capitalized, but also need to make a discrete investment at the time of the merger in order to position themselves along the post-merger barrier.

The industry value when firms have the ability to merge is the sum of their standalone value $V^S(K,p)$ and the merger option, which we denote $OM(K,p)$. Letting $\tau$ denote the time at which the merger option is exercised, the instantaneous gain from the merger is the difference between the post-merger industry value and the current standalone value, $V^M(K_\tau,p_\tau) - V^S(K_\tau,p_\tau)$, net of the direct merger cost $X$. As discussed above, the exact form that the merger option takes is sensitive however to the industry's initial capital level.

We start with the "low capital" case, $K < \overline{K}$.[14] If the initial price is not too high, in this case the industry grows along the standalone path $p_S(K)$ up until the merger threshold $\overline{p}$ is first reached, whereupon it has accumulated a capital stock defined by $p_S\left(\overline{K}\right) = \overline{p}$. At this point the firms merge and make a discrete investment, defined by $p_M(\overline{K} + \Delta_{\overline{K}}) = \overline{p}$, so as to reach the post-merger barrier.

[14]The specific expression of $\overline{K}$ is given below, in Proposition 2.

The value of the option to merge in the low capital case is

$$OM\left(p,K\right)=\left(\frac{p}{\overline{p}}\right)^{\beta_{1}}\left(V^{M}(\overline{K}+\Delta_{\overline{K}},\overline{p})-V^{S}(\overline{K},\overline{p})-r_{K}\Delta_{\overline{K}}-X\right),\ p\leq\overline{p}\text{ and }K\leq\overline{K}. \tag{12}$$

We can obtain more insight into the nature of the merger gains by substituting Eqs. (10) and (11), which gives

$$\begin{aligned}OM\left(p,K\right)=\left(\frac{p}{\overline{p}}\right)^{\beta_{1}}\Bigg(\Pi\left(\left(\overline{K}+\Delta_{\overline{K}}\right)^{\theta}H_{M}^{\eta}-\overline{K}^{\theta}H_{S}^{\eta}\right)&\frac{\overline{p}^{\gamma}}{r-g\left(\gamma\right)}\\&-\frac{\beta_{1}\left(\left(\beta_{1}-\gamma\right)\left(1-\theta\right)-\gamma\right)}{\left(\beta_{1}\left(1-\theta\right)-\gamma\right)\left(\beta_{1}-\gamma\right)}r_{K}\Delta_{\overline{K}}-X\Bigg).\end{aligned}$$

The value of the merger option therefore consists first of all of the instantaneous productivity increase, which can be further broken down as the sum of gains from reallocating the fixed input, $\Pi\overline{K}^{\theta}\left(H_{M}^{\eta}-H_{S}^{\eta}\right)$, and from scaling up industry capital, $\Pi\left(\left(\overline{K}+\Delta_{\overline{K}}\right)^{\theta}-\overline{K}^{\theta}\right)H_{M}^{\eta}$. The second part of the merger option value is the net effect of reallocating the fixed input and scaling up capital on the industry's growth options, and the last term is the direct cost of the merger.

In the low capital case, there is a constant merger threshold which is determined by requiring that the productivity gain involve a similar mark-up over incremental cost as in Eq. (4),

$$\Pi\left(\left(\overline{K}+\Delta_{\overline{K}}\right)^{\theta}H_{M}^{\eta}-\overline{K}^{\theta}H_{S}^{\eta}\right)\frac{\overline{p}^{\gamma}}{r-g\left(\gamma\right)}=\frac{\beta_{1}}{\beta_{1}-\gamma}\left(X+r_{K}\Delta_{\overline{K}}\right). \tag{13}$$

Substituting for $\overline{K}$ and $\Delta_{\overline{K}}$ then yields the closed expression (Eq. 18) in Proposition 2 below.

In the "high capital" case ($K\geq\overline{\overline{K}}$ in Figure 2), firms wait without accumulating further if the initial price is not too high and merge their initial assets when the capital-sensitive threshold $\overline{\overline{p}}(K)$ is reached. In this case the value of the option to merge is

$$OM(p,K) = \left(\frac{p}{\overline{\overline{p}}(K)}\right)^{\beta_1} \left(V^M(K, \overline{\overline{p}}(K)) - V^S(K, \overline{\overline{p}}(K)) - X\right),\ p \leq \overline{\overline{p}}(K) \text{ and } K \geq \overline{\overline{\overline{K}}}. \tag{14}$$

Substituting for industry value expressions gives

$$\begin{aligned} OM(p,K) = &\left(\frac{p}{\overline{\overline{p}}(K)}\right)^{\beta_1} \left(\Pi K^{\theta}\left(H_M^{\eta} - H_S^{\eta}\right)\frac{\left(\overline{\overline{p}}(K)\right)^{\gamma}}{r - g(\gamma)} - X\right) \\ &+ \left(\left(\frac{p}{p_M(K)}\right)^{\beta_1} - \left(\frac{p}{p_S(K)}\right)^{\beta_1}\right)\frac{\gamma^2}{\left(\beta_1(1-\theta)-\gamma\right)\left(\beta_1-\gamma\right)} r_K K. \end{aligned} \tag{15}$$

With highly capitalized firms, the first line of Eq. 15 indicates that there is an instantaneous value of merging which is due to increased productivity from reallocating the fixed input, net of the merger cost. The second line of Eq. 15 indicates that the industry also benefits from increased growth options. However the discounted value of this second component of the merger gain is independent of merger timing. Because industry capital is stationary while the firms wait to merge and there is no discrete investment at the time of the merger, the merger threshold $\overline{\overline{p}}(K)$ results from straightforward arguments (see Eq. 21).

The intermediate capitalization case ($\overline{K} \leq K < \overline{\overline{\overline{K}}}$ in Figure 2) has elements of each of the preceding cases. If the initial price state is not too high the firms wait until the merger threshold $\overline{p}(K)$ is reached without accumulating capital gradually. Once the merger threshold is reached, these less capitalized firms need to make a discrete investment $\Delta_K(K)$in order to reposition themselves along the post-merger barrier, such that $p_M(K + \Delta_K) = \overline{p}(K)$.

The merger option value corresponding to this case is

$$\begin{aligned} OM\left(p,K\right) &= \left(\frac{p}{\overline{p}\left(K\right)}\right)^{\beta_1}\left(V^M(K+\Delta_K,\overline{p}\left(K\right))-V^S(K,\overline{p}\left(K\right))-r_K\Delta_K-X\right), \\ p &\leq \overline{p}\left(K\right) \text{ and } \overline{K}\leq K<\overline{\overline{K}}. \end{aligned} \tag{16}$$

Substituting gives the specific form

$$\begin{aligned} OM\left(p,K\right) = \left(\frac{p}{\overline{p}\left(K\right)}\right)^{\beta_1} & \left(\Pi\left(\left(K+\Delta_K\right)^{\theta}H_M^{\eta}-K^{\theta}H_S^{\eta}\right)\frac{\left(\overline{p}\left(K\right)\right)^{\gamma}}{r-g\left(\gamma\right)}\right. \\ & \left.+\frac{\gamma^2}{\left(\beta_1\left(1-\theta\right)-\gamma\right)\left(\beta_1-\gamma\right)}r_K\left(K+\Delta_K\right)-r_K\Delta_K-X\right) \\ & -\left(\frac{p}{p_S\left(K\right)}\right)^{\beta_1}\frac{\gamma^2}{\left(\beta_1\left(1-\theta\right)-\gamma\right)\left(\beta_1-\gamma\right)}r_K K. \end{aligned} \tag{17}$$

The next proposition states our results regarding optimal growth and merging formally:

**Proposition 2** *Suppose the initial industry capital allocation is efficient (i.e.* $\frac{K_I}{K_J}=\left(\frac{H_I}{H_J}\right)^{\frac{\eta}{1-\theta}}$*) and let* $\overline{K}:=\frac{\theta}{1-\theta}\frac{H_S^{\frac{\eta}{1-\theta}}}{H_M^{\frac{\eta}{1-\theta}}-H_S^{\frac{\eta}{1-\theta}}}\frac{X}{r_K}$ *and* $\overline{\overline{K}}:=\theta\frac{H_M^{\eta}}{H_M^{\eta}-H_S^{\eta}}\frac{X}{r_K}$*. Then the first-best is:*

1. *(incremental growth before merger) If* $K\leq\overline{K}$*, the merger threshold is*

$$\overline{p}=\left(\frac{\beta_1}{\beta_1-\gamma}\frac{r-g\left(\gamma\right)}{\Pi}\frac{r_K^{\theta}}{\theta^{\theta}\left(1-\theta\right)^{1-\theta}}\right)^{\frac{1}{\gamma}}\left(\frac{X}{H_M^{\frac{\eta}{1-\theta}}-H_S^{\frac{\eta}{1-\theta}}}\right)^{\frac{1-\theta}{\gamma}}. \tag{18}$$

*If* $p<\overline{p}$*, firms initially invest incrementally along the barrier* $p_S(K)$*. When the threshold* $\overline{p}$ *is reached, they make a discrete investment* $\Delta_{\overline{K}}=\frac{\theta}{1-\theta}\frac{X}{r_K}$ *and merge.*

2. *(jump to merger) If $\overline{K} < K < \overline{\overline{K}}$, the merger threshold is*

$$\overline{p}\left(K\right) = p_M\left(K + \Delta_K\right), \tag{19}$$

*where $\Delta_K$ is the lower root of*

$$-\left(\frac{H_S}{H_M}\right)^{\eta} K^{\theta}\left(K + \Delta_K\right)^{1-\theta} + \left(1 - \theta\right)\Delta_K + K = \theta\frac{X}{r_K}. \tag{20}$$

*If $p < \overline{p}(K)$, firms initially wait. When the threshold $\overline{p}(K)$ is reached, they make a discrete investment $\Delta_K$ and merge.*

3. *(merger with initial assets) If $K \geq \overline{\overline{K}}$, the merger threshold is*

$$\overline{\overline{p}}\left(K\right) = \left(\frac{\beta_1}{\beta_1 - \gamma}\frac{r - g\left(\gamma\right)}{\Pi}\frac{X}{K^{\theta}\left(H_M^{\eta} - H_S^{\eta}\right)}\right)^{\frac{1}{\gamma}}. \tag{21}$$

*If $p < \overline{\overline{p}}\left(K\right)$ firms initially wait. When the threshold $\overline{\overline{p}}\left(K\right)$ is first reached, they merge without any discrete investment.*

*In all cases, if the initial price is at or above the relevant merger threshold ($\overline{p}$, $\overline{p}(K)$ or $\overline{\overline{p}}\left(K\right)$), firms immediately make a discrete investment $\Delta_K$ such that $p_M\left(K + \Delta_K\right) = p$ and merge.*

*After the merger, the combined entity invests incrementally along the barrier $p_M\left(K\right)$.*

Proposition 2 shows how internal and external growth are not mutually exclusive, but instead complementary if firms are not too highly capitalized initially. When the output price is low, firms with little capital stock (i.e. small, or young, firms) grow internally whereas firms with larger capital stock endowed with the same technology would not invest. When the output price increases and eventually reaches the threshold $\overline{p}$ two firms that can generate positive synergies by combining their fixed inputs choose to grow externally with a merger. Therefore, mergers of small or young firms occur after a phase of investment.

The initial capital stocks $K_I$ and $K_J$ determine the industry's position along its growth path, but do not affect the merger timing ($\overline{p}$). For young firms therefore both merger timing ($\overline{p}$) and structure ($\overline{K}$ and $\Delta_{\overline{K}}$) are jointly determined by the magnitude of synergies ($H_M^{\frac{\eta}{1-\theta}} - H_S^{\frac{\eta}{1-\theta}}$), the uncertainty of the price process (through $\beta_1$ and $\gamma$), the cost of capital $r_K$ and the direct costs of merging $X$, in addition to the underlying technological parameters $(a, b, c)$. The magnitude of synergies is larger when initial firm productivities are more similar, in which case the current capital stocks of each firm are also closer to one another, leading to a "merger of equals." The larger the synergies, i.e. the greater is the productivity increase $H_M^{\frac{\eta}{1-\theta}} - H_S^{\frac{\eta}{1-\theta}}$, the sooner the merger takes place (i.e. the lower the merger threshold $\overline{p}$), and the higher the "mass" investment at the moment the deal is concluded (i.e. the lower the sizes $\overline{K}$ that firms reach when they decide to merge).

Because labor and output adjust to reflect the market price, the firm faces operational risk in addition to market risk so market drift ($\mu$) and volatility ($\sigma^2$) have a complex effect on the timing of mergers. The cost of capital on the other hand is specific to our model and has an unambiguous effect, as a higher cost of capital $r_K$ induces later mergers. An increase in the cost of capital raises both the merger cost and the value of the option to grow for the conglomerate firm, and the former effect prevails over the latter. Both these effects are proportional to the mass investment $\Delta_{\overline{K}}$ that occurs when firms merge. The lower $\Delta_{\overline{K}}$ therefore, the smaller the impact of the cost of capital on the merger timing.

**Corollary 3** *Greater productivity increases lead to earlier mergers, and to mergers of smaller firms. Mergers of more symmetric firms lead to greater synergies happen at an* *earlier stage in firm development and at a lower price threshold. A greater cost of capital delays mergers.*

A remarkable feature of the optimal solution is that the pre- and post-merger capital levels are entirely determined by technological parameters. The mass investment $\Delta_{\overline{K}}$ depends on the degree of the specificity of the fixed input, which determines the ratio $\frac{H_M}{H_S}$, and on the production technology ($a, b, c$, i.e. $\eta$ and $\theta$). The more specific to each firm is the fixed asset $H$, the closer $H_M$ is to $H_S$, and the lower the investment observed when firms merge. The higher the factors' productivities $a, b, c$, the higher the parameters $\eta$ and $\theta$, so that the investment $\Delta_{\overline{K}}$ is larger.

To provide further context for Proposition 2, we can compare with the timing of mergers in our model with the situation where firms do not accumulate capital at all. By letting $b = \theta = 0$, there is no capital accumulation and merging is optimal provided that $a + c > 1$. Taking the limit of Eq. (18), we get $\overline{p}|_{\theta=0} = \left(\frac{\beta_1}{\beta_1-\gamma}\frac{r-g(\gamma)}{\Pi}\frac{X}{H_M^\eta - H_S^\eta}\right)^{\frac{1}{\gamma}}$ which is the efficient threshold in Lambrecht (2004).

**Corollary 4** *For given initial capital levels on the long-run path, as $r_K$ increases (e.g. due to more external financing frictions) or $X$ decreases (e.g. due to lower corporate transaction frictions) both $\overline{K}$ and $\overline{\overline{K}}$ increase, implying that firms progressively move from pre-merger growth (case* (1) *of Proposition 2) to waiting (case* (3) *of Proposition 2).*

# 5 Decentralized decisions

We now turn to the merger decision if the firms are run independently. This requires to identify how the merger surplus is divided and a consistent set of investment and merging decisions. We maintain the same assumption as in the preceding section and suppose that firms start on the industry's long-run growth path, and we consider two different situations. In the first, firms negotiate early on and have the ability to commit to surplus shares. In this case, because bargaining happens ahead of investment decisions, subsequent investment and merger timing turn out to be efficient. The second situation involves firms that cannot make commitments ahead of the corporate asset transaction, which leads to inefficient investment decisions even if merger timing is ex-post efficient.[15]

## 5.1 Commitment case

Suppose that the two firms set the terms of a possible future deal an early stage in the industry's development. The terms of the deal are the shares of the merged entity $s_I$ and $s_J$ attributed to each party, with $s_I + s_J = 1$. These terms are set cooperatively at $t = 0$. Once the terms are set, they cannot be renegotiated. The fixed cost of the transaction, $X$, is paid by the merged entity. Bargaining is therefore over the net surplus from the merger.[16] Once the deal terms are set, each of the firms pursues its own investment strategy and decides independently when to accept the deal, i.e. the threshold at which it is ready to accept to merge on these terms. The merger takes place once both of the firms have accepted, at which time the new entity is formed and the two firms split the surplus according to the terms they initially bargained.

Given the initial capital stock, the efficient bargaining outcome is to merge at the relevant threshold ($\overline{p}$, $\overline{p}(K)$ or $\overline{\overline{p}}(K)$) defined in Proposition 2. The shares of each firm are then determined by the requiring that both firms accept the deal at this threshold.

[15] A third alternative is that firms agree on the timing of the merger first, as in Lambrecht (2004), then proceed to make investment decisions and determine the sharing rule at the time of the merger.

[16] We could also assume that each firm bears an exogenous share of the merger cost $X_j$ (so $X = X_I + X_J$). In our framework though, this requirement is unnecessary to pin down merger shares, because firms already face idiosyncratic opportunity costs of merging in our framework due to the private value of the growth options that they forego.

We can reason by backward induction to determine these shares by first studying the timing decision of each firm for given deal terms. We focus on the low capital case (the reasoning for the other cases is similar). The private value of merging at a threshold $\overline{p}_i$ in this case is[17]

$$OM^i\left(p,K,s_i\right)=\left(\frac{p}{\overline{p}_i}\right)^{\beta_1}\left(s_i\left(V^M(\overline{K}+\Delta_{\overline{K}},\overline{p}_i)--r_K\Delta_{\overline{K}}-X\right)-V^i(K_i\left(\overline{K}\right),\overline{p}_i)\right),\ p\leq\overline{p}_i\text{ and }K\leq\overline{K}. \tag{22}$$

Reexpress the above as

$$OM^i\left(p,K,s_i\right)=s_i\,OM^i\left(p,K\right)\big|_{\overline{p}=\overline{p}_i}+\left(\frac{p}{\overline{p}_i}\right)^{\beta_1}\left(s_iV^S(\overline{K},\overline{p}_i)-V^i(K_i\left(\overline{K}\right),\overline{p}_i)\right),\ p\leq\overline{p}_i\text{ and }K\leq\overline{K}. \tag{23}$$

Eq. (23) expresses the private merger option as the sum of two terms, where the first of which is the firm's share of the first-best option and the second depends on the firms share relative to its contribution to the standalone value at the time of the merger. Maximization with respect to firm $i$'s exercise threshold results in an optimal exercise policy $\overline{p}_i(s_i)$ which is monotonically decreasing in the firm's share. Comparing with the first-best, the first-order condition shows that firm $i$ accepts the merger inefficiently late if $s_i<\frac{V^i(K_i\left(\overline{K}\right),\overline{p}_i)}{V^S(\overline{K},\overline{p}_i)}$ and inefficiently early if $s_i>\frac{V^i(K_i\left(\overline{K}\right),\overline{p}_i)}{V^S(\overline{K},\overline{p}_i)}$. In order to induce merger at the efficient threshold therefore, is necessary that the initial shares reflect the relative productivity of each firm, i.e. $\overline{p}_I^*=\overline{p}_J^*=\overline{p}$ if and only if $s_i=\frac{V^i(K_i\left(\overline{K}\right),\overline{p}_i)}{V^S(\overline{K},\overline{p}_i)}$, which depends only on each firm's relative endowment of the fixed asset.

**Proposition 5** *If firms negotiate initially can commit to merger terms, the shares that induce both firms to accept a merger at the efficient threshold* $\overline{p}$ *correspond to their relative*

[17] This is true because it remains privately optimal for firms to accumulate capital efficiently ahead of the merger threshold if subsequent shares are fixed, see Appendix.

*productivities, i.e.*

$$s_i^* = \left(\frac{H_i}{H_S}\right)^{\frac{\eta}{1-\theta}}, \ i = I, J. \tag{24}$$

Because firms grow efficiently up until the merger, the proposition implies that the shares in the merged firm just correspond to the pre-merger capital shares, so the requirement that shares reflect relative productivity simplifies to ensuring they reflect capital shares at the time of the merger. A consequence of Proposition 5 is that if any other split is imposed on the firms, for example if one firm has a different level of bargaining power, then the deal is invariably delayed as the earliest acceptance threshold $\max\left\{\overline{p}_I^*\left(s_I\right), \overline{p}_J^*\left(s_J\right)\right\}$ increases.

### 5.2 No-commitment case

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# 6 Appendix

## 6.1 First-best problem

The first-best problem is to determine the industry value $V^*$ for a given initial state $(K_I, K_J, p)$, i.e.

$$\begin{aligned}
V^*(K_I, K_J, p) = & \sup_{\tau, \{K_{I,t}\}_{t\geq 0}, \{K_{Jt}\}_{t\geq 0}} E\left[\int_0^\tau e^{-rt}\left(\Pi K_{I,t}^\theta H_I^\eta p_t^\gamma dt - r_K dK_{I,t}\right)\right. \\
& + \int_0^\tau e^{-rt}\left(\Pi K_{J,t}^\theta H_J^\eta p_t^\gamma dt - r_K dK_{J,t}\right) \\
& + \int_\tau^\infty e^{-rt}\left(\Pi\left(K_{I,t} + K_{J,t}\right)^\theta H_M^\eta p_t^\gamma dt - r_K d\left(K_{I,t} + K_{J,t}\right)\right) \\
& \left. | K_{I,0} = K_I, K_{J,0} = K_J, p_0 = p\right], \qquad (25)
\end{aligned}$$

where $\tau$ is a stopping time and $\{K_{I,t}\}_{t\geq 0}, \{K_{J,t}\}_{t\geq 0}$ are nondecreasing stochastic processes.

Our presentation of the solution follows the two steps outlined in the text by first characterizing optimal investment for standalone firms and then incorporating the option to merge.

### 6.1.1 Proof of Proposition 1

If the firms are viewed as standalone entities the first-best value (Eq. 25) is

$$\begin{aligned}
V^S(K_I, K_J, p) = & \sup_{\{K_{I,t}\}_{t\geq 0}, \{K_{Jt}\}_{t\geq 0}} E\left[\int_0^\infty e^{-rt}\Pi\left(K_{I,t}^\theta H_I^\eta + K_{J,t}^\theta H_J^\eta\right) p_t^\gamma dt\right. \\
& \left. - \int_0^\infty r_K e^{-rt}\left(dK_{I,t} + dK_{J,t}\right)\right| K_{I,0} = K_I, K_{J,0} = K_J, p_0 = p\Bigg]. \qquad (26)
\end{aligned}$$

Eq. (26) is separable in the two capital processes so we can consider each firm's investment problem separately.

Let $V^i(K_i,p)$ denote firm $i$'s value and $p_i(K_i)$ the upper boundary of its inaction region. In the inaction region, the firm's value satisfies the Hamilton-Jacobi-Bellman (HJB) equation

$$rV^i(K_i,p) = \Pi K_i^{\theta} H_i^{\eta} p^{\gamma} + \mu p V_p^i(K_i,p) + \frac{1}{2}\sigma^2 p^2 V_{pp}^i(K_i,p). \tag{27}$$

The particular solution is $\frac{\Pi K_i^{\theta} H_i^{\eta}}{r-\mu\gamma-\frac{1}{2}\sigma^2\gamma(\gamma-1)}p^{\gamma}$, and we hereafter abbreviate the denominator as $r-g(\gamma)$. The general solution is then $V^i(K_i,p) = \Pi K_i^{\theta} H_i^{\eta}\frac{p^{\gamma}}{r-g(\gamma)}+B_1(K_i)p^{\beta_1}+B_2(K_i)p^{\beta_2}$ where $\beta_1>1$ and $\beta_2<0$ are roots of the fundamental quadratic $g(\beta)-r=0$. The lower boundary condition is $V^i(K_i,0)=0$, implying $B_2(K_i)=0$. The general solution therefore reduces to

$$V^i(K_i,p) = \Pi K_i^{\theta} H_i^{\eta}\frac{p^{\gamma}}{r-g(\gamma)} + B_1(K_i)p^{\beta_1}. \tag{28}$$

The barrier $p_i(K_i)$ is defined implicitly by the upper bound on marginal profitability $V_K^i(K_i,p_i(K_i)) = r_K$. Differentiating Eq. (28) with respect to $K_i$ and substituting in gives

$$\theta\Pi K_i^{\theta-1} H_i^{\eta}\frac{(p_i(K_i))^{\gamma}}{r-g(\gamma)} + B_1'(K_i)(p_i(K_i))^{\beta_1} = r_K. \tag{29}$$

Optimal timing at the barrier requires $V_{Kp}^i(K_i,p_i(K_i)) = 0$. Differentiating Eq. (29) with respect to $p$ gives

$$\gamma\theta\Pi K_i^{\theta-1} H_i^{\eta}\frac{(p_i(K_i))^{\gamma-1}}{r-g(\gamma)} + \beta_1 B_1'(K_i)(p_i(K_i))^{\beta_1-1} = 0. \tag{30}$$

Together, Eqs. (29) and (30) imply

$$p_i(K_i) = \left(\frac{\beta_1}{\beta_1-\gamma}\frac{r-g(\gamma)}{\Pi}\frac{r_K}{\theta}\frac{K_i^{1-\theta}}{H_i^{\eta}}\right)^{\frac{1}{\gamma}} \tag{31}$$

and

$$B_1'(K_i) = -\frac{\gamma}{\beta_1 - \gamma} r_K \left(p_i\left(K_i\right)\right)^{-\beta_1}. \tag{32}$$

The coefficient $B_1(K_i)$ is found by integrating the above expression, which yields

$$\begin{aligned} B_1(K_i) &= \frac{\gamma}{\beta_1 - \gamma} r_K \int_{K_i}^{\infty} \left(p_i\left(k\right)\right)^{-\beta_1} dk \\ &= \frac{\gamma^2}{\left(\beta_1\left(1-\theta\right)-\gamma\right)\left(\beta_1-\gamma\right)} r_K \frac{K_i}{\left(p_i\left(K_i\right)\right)^{\beta_1}}. \end{aligned} \tag{33}$$

The barrier and coefficient (Eqs. 31 and 33) then result in the individual firm value in the text (Eq. 10).

Proposition 1 follows from observing that $p_I\left(K_I\right) = p_J\left(K_J\right)$ when both firms accumulate along their respective barriers, which implies that $\frac{K_I}{K_J} = \left(\frac{H_I}{H_J}\right)^{\frac{\eta}{1-\theta}}$ in such states. If capital levels satisfy this ratio then both firms immediately accumulate along their respective barriers, whereas if $p \geq \max\left\{p_I\left(K_I\right), p_J\left(K_J\right)\right\}$ then they make immediate discrete investments so as to accumulate along their respective barriers thereafter. Noting that $K = K_I + K_J$, the individual barriers imply Eqs. (7) and (8) in the text. Substituting these expressions into individual profit flows and summing gives the industry profit flow on the long-run expansion path (Eq. 9), as well as the industry barrier $p_S\left(K\right)$ and value (Eq. 10). If $p < \max\left\{p_I\left(K_I\right), p_J\left(K_J\right)\right\} = p_J\left(K_J\right)$ and $\frac{K_I}{K_J} \neq \left(\frac{H_I}{H_J}\right)^{\frac{\eta}{1-\theta}}$, then only firm $I$'s policy calls for incremental investment (or an immediate discrete investment followed by incremental investment if $p > p_I\left(K_I\right)$), up until firm $J$'s threshold $p_J\left(K_J\right)$ is reached, whereupon industry investment proceeds as in the preceding case.

#### 6.1.2 Proof of Proposition 2

We derive the merger threshold in two steps. First of all, we verify that the investment policy of standalone firms is unchanged by the prospect of a future merger, provided that the net value of merging is positive. We then determine the value of merging at an arbitrary threshold, verify that it meets this condition, and solve for the optimal threshold at which to exercise the merger option.

**Standalone investment with exogenous termination** Consider the first-best value (Eq. **??**), evaluated at a stopping time $\widehat{T} = \inf\{t \geq 0 \,| p_t \geq \widehat{p}\}$ where $\widehat{p}$ is an arbitrary merger threshold. Because the industry starts on a balanced growth path and the post-merger value depends only on total industry capital, we can restrict attention to aggregate industry value.

We first find the optimal policy for such an industry, which holds an initial capital stock $K = K_I(K) + K_J(K)$, receives a profit flow $\Pi K^\theta H_S^\eta p^\gamma$, and can accumulate capital up until a given price threshold $\widehat{p} > p$ is reached. We suppose that once $\widehat{p}$ is reached, the industry receives a terminal payoff $\Omega\left(\widehat{K}, \widehat{p}\right) = V^S\left(\widehat{K}, \widehat{p}\right) + M\left(\widehat{p}\right)$. The first part of the termination payoff,

$$V^S\left(\widehat{K}, \widehat{p}\right) = \Pi\left(\widehat{K} + \Delta\widehat{K}\right)^\theta H_S^\eta \frac{\widehat{p}^\gamma}{r - g\left(\gamma\right)} + \left(\frac{\gamma}{\beta_1\left(1-\theta\right)-\gamma}\right)\left(\frac{\gamma}{\beta_1 - \gamma}\right) r_K\left(\widehat{K} + \Delta\widehat{K}\right) - r_K\Delta\widehat{K}, \tag{34}$$

is the ongoing standalone value of current industry assets and technology. Because the industry lies above the barrier that it would have as a standalone concern if capital accumulation stops early so $\widehat{p} > p_S\left(\widehat{K}\right)$, the term $V^S\left(\widehat{K}, \widehat{p}\right)$ incorporates a discrete investment $\Delta\widehat{K}$ such that $\widehat{p} = p_S\left(\widehat{K} + \Delta\widehat{K}\right)$. Observe that $\frac{\partial V^S\left(\widehat{K}, \widehat{p}\right)}{\partial \widehat{K}} = r_K$ because $\widehat{K} + \Delta\widehat{K}$ is fixed. The second part of the termination payoff, $M\left(\widehat{p}\right)$, is the net termination payoff or merger gain, which measures what the industry obtains above its standalone value. We assume for now that the net termination payoff is positive. Under these conditions, we show that it is optimal for the industry to accumulate capital along the barrier $p_S\left(K\right)$ until the exogenous threshold $\widehat{p}$ is reached, at which point it attains a terminal level of capital $\widehat{K}$ such that $p_S\left(\widehat{K}\right) = \widehat{p}$.

Under the conditions described above, industry value is

$$\widehat{V}^S(K,p|\,\widehat{K},\widehat{p}) = \sup_{\{K_t\}_{t\geq 0}} E\left[\int_0^{\widehat{T}} e^{-rt}\left(\Pi K_t^\theta H_S^\eta p_t^\gamma dt - r_K dK_t\right) + e^{-r\widehat{T}}\Omega\left(K_{\widehat{T}},\widehat{p}\right)\middle|\, K_0 = K, p_0 = p\right]. \tag{35}$$

Denote the barrier for this truncated accumulation problem by $\widehat{p}_S(K)$, and omit the arguments $\widehat{K},\widehat{p}$ which are given for now. In the inaction region, firm value satisfies the HJB equation

$$r\widehat{V}^S(K,p) = \Pi K^\theta H_S^\eta p^\gamma + \mu p \widehat{V}_p^S(K,p) + \frac{1}{2}\sigma^2 p^2 \widehat{V}_{pp}^S(K,p). \tag{36}$$

Similar reasoning to the preceding section yields a candidate general solution

$$\widehat{V}^S(K,p) = \Pi K^\theta H_S^\eta \frac{p^\gamma}{r - g(\gamma)} + B_1(K)p^{\beta_1}. \tag{37}$$

The barrier $\widehat{p}_S(K)$ is defined implicitly by the upper bound on marginal profitability $\widehat{V}_K^S(K,\widehat{p}_S(K)) = r_K$ and optimal timing requires $\widehat{V}_{Kp}^S(K,\widehat{p}_S(K)) = 0$. Substituting into the solution gives

$$\theta\Pi K^{\theta-1} H_S^\eta \frac{\left(\widehat{p}_S(K)\right)^\gamma}{r - g(\gamma)} + B_1'(K)\left(\widehat{p}_S(K)\right)^{\beta_1} = r_K \tag{38}$$

and

$$\gamma\theta\Pi K^{\theta-1} H_S^\eta \frac{\left(\widehat{p}_S(K)\right)^{\gamma-1}}{r - g(\gamma)} + \beta_1 B_1'(K)\left(\widehat{p}_S(K)\right)^{\beta_1-1} = 0 \tag{39}$$

which together imply $\widehat{p}_S(K) = p_S(K)$. The barrier in the truncated problem is therefore the same as with standalone firms over the range $\left[K_0,\widehat{K}\right]$ where capital accumulation occurs. It is necessary still to verify that the industry continues accumulating capital until the threshold $\widehat{p}$ is first hit. Eqs. (38) and (39) imply

$$B_1'(K) = -\frac{\gamma}{\beta_1 - \gamma} r_K \left(\widehat{p}_S\left(K\right)\right)^{-\beta_1} \tag{40}$$

and the coefficient $B_1(K)$ is found by integrating from $K$ to $\widehat{K}$, which yields

$$\begin{aligned} B_1(K) &= -\frac{\gamma}{\beta_1 - \gamma} r_K \int_K^{\widehat{K}} \left(\widehat{p}_S\left(k\right)\right)^{-\beta_1} dk \\ &= \left(\frac{\gamma}{\beta_1 - \gamma}\right)\left(\frac{\gamma}{\beta_1\left(1-\theta\right) - \gamma}\right) r_K \left(\frac{K}{\left(\widehat{p}_S\left(K\right)\right)^{\beta_1}} - \frac{\widehat{K}}{\left(\widehat{p}_S\left(\widehat{K}\right)\right)^{\beta_1}}\right) + C \end{aligned} \tag{41}$$

where $C$ is a nonzero constant of integration. This constant of integration is determined by value matching at $\widehat{p}$, i.e. $\widehat{V}^S(\widehat{K}, \widehat{p}) = \Omega\left(\widehat{K}, \widehat{p}\right)$, so

$$C = \left(\Omega\left(\widehat{K}, \widehat{p}\right) - \Pi \widehat{K}^\theta H_S^\eta \frac{\widehat{p}^\gamma}{r - g\left(\gamma\right)}\right) \widehat{p}^{-\beta_1}. \tag{42}$$

Putting these elements together,

$$\begin{aligned} \widehat{V}^S(K,p) = \Pi K^\theta H_S^\eta \frac{p^\gamma}{r - g\left(\gamma\right)} + \\ + \left(\frac{\gamma}{\beta_1 - \gamma}\right)\left(\frac{\gamma}{\beta_1\left(1-\theta\right) - \gamma}\right) r_K \left(K\left(\frac{p}{\widehat{p}_S\left(K\right)}\right)^{\beta_1} - \widehat{K}\left(\frac{p}{\widehat{p}_S\left(\widehat{K}\right)}\right)^{\beta_1}\right) \\ + \left(\frac{p}{\widehat{p}}\right)^{\beta_1}\left(\Omega\left(\widehat{K}, \widehat{p}\right) - \Pi \widehat{K}^\theta H_S^\eta \frac{\widehat{p}^\gamma}{r - g\left(\gamma\right)}\right). \end{aligned} \tag{43}$$

Differentiating with respect to $\widehat{K}$,

$$\frac{\partial \widehat{V}^S}{\partial \widehat{K}}\left(K,p\right) = \frac{\gamma}{\beta_1 - \gamma} r_K \left(\frac{p}{\widehat{p}_S\left(\widehat{K}\right)}\right)^{\beta_1} + \left(r_K - \theta \Pi \widehat{K}^{\theta - 1} H_S^\eta \frac{\widehat{p}^\gamma}{r - g\left(\gamma\right)}\right)\left(\frac{p}{\widehat{p}}\right)^{\beta_1}. \tag{44}$$

Using the mark-up condition to substitute for $\frac{\theta\Pi\widehat{K}^{\theta-1}H_S^\eta}{r-g(\gamma)}$, the above expression is positive if $\frac{\gamma}{\beta_1-\gamma}\left(\frac{\widehat{p}}{\widehat{p}_S(\widehat{K})}\right)^{\beta_1} - \frac{\beta_1}{\beta_1-\gamma}\left(\frac{\widehat{p}}{\widehat{p}_S(\widehat{K})}\right)^{\gamma} + 1 > 0$, which holds for all $\widehat{p} \geq \widehat{p}_S\left(\widehat{K}\right)$ because $\beta_1 > \gamma$. It is optimal for the industry to accumulate along the barrier $p_S(K)$ up until $\widehat{p}$ is reached.

**Merger option exercise** The instantaneous value of merging at a given threshold $\widehat{p}$ is the difference between the value of the merged entity and the standalone industry value. If the industry accumulates along the barrier $p_S(K)$, its capital stock when the merger threshold is hit is $\widetilde{K}$ such that $p_S(\widetilde{K}) = \widehat{p}$, which is lower than the post-merger capital stock $\overline{K}$ such that $p_M(\overline{K}) = \widehat{p}$.

The net value of merging is therefore determined by $\widehat{p}$ only, as

$$M(\widehat{p}) = V^M\left(\overline{K}, \widehat{p}\right) - V^S\left(\widetilde{K}, \widehat{p}\right) - r_K\left(\overline{K} - \widetilde{K}\right), \tag{45}$$

or developing,

$$\begin{aligned} M(\widehat{p}) = &\left(\frac{\gamma}{\beta_1(1-\theta)-\gamma}\right)\left(\frac{\gamma}{\beta_1-\gamma}\right) r_K\left(\overline{K} - \widetilde{K}\right) \\ &+ \Pi\left(\overline{K}^\theta H_M^\eta - \widetilde{K}^\theta H_S^\eta\right)\frac{\widehat{p}^\gamma}{r-g(\gamma)} - \left(X + r_K\left(\overline{K} - \widetilde{K}\right)\right). \end{aligned}$$

Substituting specific forms and using Eq. (**??**) gives a more compact expression for the merger option, which we still evaluate here for an arbitrary exercise threshold $\widehat{p}$,

$$M(\widehat{p}) = \left(\left(H_M^{\frac{\eta}{1-\theta}} - H_S^{\frac{\eta}{1-\theta}}\right)\left(\frac{(\beta_1-\gamma)\Pi}{r-g(\gamma)}\right)^{\frac{1}{1-\theta}}\left(\frac{\theta}{\beta_1 r_K}\right)^{\frac{\theta}{1-\theta}}\frac{(1-\theta)^2}{\beta_1(1-\theta)-\gamma}\widehat{p}^{\frac{\gamma}{1-\theta}} - X\right). \tag{46}$$

Merger gains are positive for sufficiently high $\widehat{p}$, implying capital accumulation along the barrier $p_S(K)$ ahead of the merger.

Maximization of the option value $M(\widehat{p})\left(\frac{p}{\widehat{p}}\right)^{\beta_1}$ with respect to the threshold $\widehat{p}$ leads to the optimal exercise threshold

$$\overline{p}=\left(\frac{\beta_1}{\beta_1-\gamma}\frac{r-g\left(\gamma\right)}{\Pi}\frac{r_K^{\theta}}{\theta^{\theta}\left(1-\theta\right)^{1-\theta}}\right)^{\frac{1}{\gamma}}\left(\frac{X}{H_M^{\frac{\eta}{1-\theta}}-H_S^{\frac{\eta}{1-\theta}}}\right)^{\frac{1-\theta}{\gamma}}, \tag{47}$$

and hence the values of $\widetilde{K}$ and $\overline{K}$ in the text. $\square$