Financial engineering for sustainable infrastructure projects

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Abstract

The economic development of emerging countries necessitates an efficient funding mechanism. This paper models the cultural perspectives of blended infrastructure financing in developing Muslim economies by examining Islamic bonds (*sukuk*). We identify the structural imperfections of this value-based instrument and offer a novel mathematical solution to remediate opacity and agency costs of debt. Relying on a mix of Participating and Continuous Workout financing, we mathematically model a financially engineered *sukuk* for impact financing by incorporating charitable endowment (*waqf*) into the public-private partnership. This allows us to develop a fragile-free blended finance facility to overcome the impact of financialization and inequality in infrastructure development.

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Inadequate infrastructure...has been recognized as a major barrier to economic advancement...

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1 Introduction

Efficient project finance is among the critical enabling factors in sustainable development (Subramanian and Tung, 2016). McHugh (2023) warns that countries face years of pent-up deficits without a suitable infrastructure funding scheme. A recent study estimates that annual financing gap for achieving net-zero by 2050 to be USD 6.3 trillion (Climate Policy Initiative, 2023). This gap is widening in developing economies as the region's needs of adaptation finance is between 10 and 18 times larger than that of its developed counterparts (United Nations Environment Programme, 2023). Refinitiv (2023) further estimates that between 2016 and 2040, the emerging Muslim-majority countries, represented by the Organisation of Islamic Cooperation (OIC), necessitate a substantial USD 2.7 trillion financing gap for infrastructure alone, with an additional yearly shortage of USD 1 trillion for Sustainable Development Goals (SDGs). As a result, the world is currently off-course in achieving SDGs, such as eradicating poverty and hunger, reducing inequalities, developing renewable energy and sustainable infrastructure, and combating climate change (i.e., People, Profit and Planet) (Geczy et al., 2021).

Although a substantial amount of research addresses this impact investment issue, the general context of a "developing country" is typically deployed. Little work, if any, examines the specific cultural aspects of infrastructure financing, and especially in relation to the emerging Muslim economies. This is despite the significant challenges faced by the region in achieving sustainable finance and development (Jatmiko et al., 2023). Considerable number of studies also document the role of culture in financial markets and development (Raz, 2023; Stulz and Williamson, 2003). This paper, therefore, seeks to fill this gap by proposing a novel financing vehicle which is specially crafted for these cultural areas.

For this purpose we start by noting that, from a financial engineering's point of view, Muslim regions of the world have some intriguing idiosyncratic characteristics, ensuing from cultural prohibitions of both agency cost of debt (*riba*) and asymmetric information (*gharar*) in financial contracts (Jatmiko et al., 2024). One would therefore expect emergence of locally originated financing vehicles that incorporate these features by default. Surprisingly, this is not the case. For the last three decades the means of dealing with infrastructure development in these geographies have been mobilizing resources from the private sector via an off-the-shelf Public-Private Partnership (PPP) (Refinitiv, 2023).

While many praise the success of this method, it also confers several disadvantages (Leviäkangas et al., 2016). PPP incentivizes the investor to focus solely on positive NPV projects, impeding the equitable distribution of infrastructure (Tan, 2012). This exacerbates spatial inequality in developing countries and within underdeveloped regions of countries, disempowering many local infrastructure initiatives as a result (Demirgüç-Kunt and Levine, 2009). More importantly, PPP's reliance on default-prone bonds also runs counter to the Islamic value system, which forbids financialization involving agency cost of debt and asymmetric information (Ebrahim et al., 2016). By avoiding the infrastructure financialization, and without losing emphasis on economic aspects (Millner, 2020), we therefore seek to incorporate cultural and legal dimensions into the analysis, consistent with Stulz and Williamson (2003) and Graham et al. (2022).

Consequently, in contrast to the standard PPP approach, we incorporate the cultural values under the patronage of a charitable endowment (*waqf*), funded by a novel quasi-equity facility through securities-based crowdfunding platform. To this end we adapt the *sukuk* (plural of Arabic term *sakk*, meaning certificate) as an ingenious culture-led financing mechanism for the above scheme.[1] Unlike the PPP-linked conventional bonds, our implementation of *sukuk* is structured in the form of Asset-Backed Securities (ABS), where the income for its holders stems from the underlying project's performance (see Figure 1). As it links the payoffs of the proposed quasi-equity with that of the infrastructure, our financing solution is consistent with the empirical findings of Sarno and Taylor (1999). Moreover, our method yields to positive outcomes for both investors and communities. Effectively, it results in a 'blended' finance by strategically incorporating philanthropic funds "to mobilize private capital flows to emerging and frontier markets," as advocated in World Economic Forum (2016). The securities-based *waqf* crowdfunding further eliminates financing fractions and facilitates the integration of donor-investors' social preferences (Brown and Davies, 2020).

[Figure 1 about here.]

Our paper makes significant contributions to operations research by analyzing a financially engineered product aimed at achieving a greater societal impact. We engage with key asset pricing theories, refined and tested over many years in established market economies, and which achieve accurate pricing. The three areas of financial theory which enable pricing under uncertainty and lead to consistent investment choices involve: 1) Utility theory which goes beyond expected value to describe decisions; 2) Equilibrium asset pricing, including mean-variance analysis and the Capital Asset Pricing Model (CAPM); and 3) No arbitrage asset pricing, including derivatives pricing.

The utility theory can be applied to characterize simple decisions faced by risk-averse individuals independently of the nature of the economy.

As Zenios et al. (2021) rightly put it, incorporating the trade-off amongst agents involved in the pricing of sustainable finance is intricate. This complexity arises, in part, from the significant disparity between environmental science and financial indicators, which results in imperfect price discovery within this emerging asset class (Ginglinger and Moreau, 2023; Klusak et al., 2023). Not without difficulties, the mean-variance analysis and the CAPM have found their ways into development finance, including environments of different cultural background, such as Sustainable and Islamic Finance (Zerbib, 2022). In this particular case the technical difficulty of forbidden positive interest has been circumvented by using non-market correlated assets.

However, the literature establishing the link and applying arbitrage pricing in impact investing environments is much scarce. To our knowledge, no research progress has been made to approach *sukuk* with option pricing techniques. Akin to contributions which found many applications, such as real options and embedded options in mortgage contracts, our paper makes this step. Our model pioneers the application of risk-neutral pricing to the various options embedded in *sukuk* financing vehicles. While often discussed in general and in descriptive terms, these instruments were not given a more in-deep and mathematical treatment in the literature. Our paper identifies and fills this gap.

In this paper we therefore propose and mathematically model a robust infrastructure financing scheme by reengineering the *sukuk* structure, while integrating the same with the charitable endowment and advanced financial innovations. This leads us to contribute to the strands of literature in operations research as follows:

First, we contribute to impact financing literature (Dye and Hsieh, 2024; Zenios et al., 2021) by proposing a novel mathematical model to value the *sukuk* and *waqf* structures. In doing so, we combine the Continuous Workout of Shiller et al. (2019) and Participating Financing (PF) of Ebrahim et al. (2011) to serve as an efficient funding scheme alleviating agency costs of debt (*riba*), and thereby endowing viability to our social project.

Second, we incorporate the cultural analysis by instituting a charitable endowment (*waqf*) to alleviate one of the acute drawbacks of PPPs and *Sukuk*, i.e., the inequality problem.[2] This also enables us to incorporate the intricate interaction between the social preferences of charitable donors and the monetary incentives of private sectors into our pricing strategy, as discussed by Papier and Thonemann (2021) and Brânzei et al. (2022).

Third, we integrate the *sukuk* structure with the blended finance literature (Brown and Davies, 2020; Lee and Parlour, 2021) to illustrate how its asset centrality and the donor-investors' direct participation in funding the projects can mitigate inequality arising from market frictions (i.e., asymmetric information or *gharar*) in the contract. In contrast to the majority of contemporary *sukuk* which deviate from the ideal structure,[3] our new proposed vehicle does away with the adverse effects of financialization. This is achieved by excluding the use of fragile debt, and thus precludes the associated endemic agency issues that heighten uneven development. Consequently, our approach yields a sustainable financial solution.[4]

The rest of this paper is structured as follows. Section 2 proposes a way forward by elaborating the theoretical underpinnings of our innovative *sukuk* structure by alleviating agency costs of debt, asymmetric information and uneven geographies of development. Next, we mathematically model and numerically test our innovative blended financing structure by amalgamating the Continuous Workout Mortgage with the Participating one in Section 3. In Section 4 we simulate our *sukuk* numerically and discuss the outcomes and implications. Finally, Section 5 concludes.

2 Theoretical underpinnings of our blended finance model

Our approach overcomes the three major problems of contemporary *sukuk* financing. These include: (i) agency costs of debt, (ii) asymmetric information, and (iii) development inequalities.

In this section we introduce and explain our main contribution i.e. how our improved *sukuk* alleviates agency problems arising from asymmetric information between borrowers and lenders. This highlights the connections between our contributions and our formal model of the next Section. For the sake of brevity, we relegate the discussion of the remaining issues, i.e. asymmetries (ii) and inequalities (iii), to the Appendix A.

2.1 Mitigating agency issues (*riba*) through Participating Preferred Financing (PPF)

While we incorporate our *sukuk* into the PPP framework, its proposed structure is incomplete without a robust financing model. In this respect, the product innovation literature offers a range of choices for mitigating agency costs of debt. The Continuous Workout (CW) of Shiller et al. (2019) and Participating Financing (PF) of Ebrahim et al. (2011) are simple and powerful candidates for this endeavor. These facilities emerged in response to the current financial architecture's failure from insulating the global macroeconomy during the recent financial crisis. Blending these two innovations allows infrastructure *sukuk* to preserve the long-term fixed-income feature along with a built-in stabilizer with embedded compound options on the sharing of income and/or capital appreciation as follows.

First, the CW feature ensures that our quasi-equity infrastructure facility undertakes a preferred stock profile, as illustrated in Figure 2. Compared to plain vanilla debt, it delivers a slightly increased coupon $R_{CW} > c_L$. This is obtained in exchange for paying bondholders proportionally less when net income decreases below R_{CW}. As a result, the lenders bear part of the downside risk of the infrastructure's net operating income and the terminal value. Figure 3 depicts that the financier is subject to receiving lower payoffs in the economy's poor states (between t_1 and t_2) when the value of the collateralized asset and/or the infrastructure project's revenue goes down. It thus prevents the equity holder from defaulting when the equity goes 'underwater.'

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

Second, the CW allows both the charitable endowment and the *sukuk* holders to share the project risk, thus moderating the risk-shifting (or fragility) problem, which becomes pervasive when equity goes 'underwater' (Shiller et al., 2019).[5] However, this risk-sharing requires a higher coupon rate ($R_{CW} > c_L$)[6] to compensate for the *sukuk* holders. This exacerbates the underinvestment issue highlighted in Myers (1977).[7] The charitable endowment needs to trade-off the higher cost of the CW financing, by attaching *call options* (Black and Scholes, 1973) to share in the income and/or appreciation of the project (above a threshold $b > R_{CW}$, see Figure 5) to alleviate the underinvestment issue. In other words, the CW financing needs to be amalgamated with a PF facility.

Third, as illustrated in Figure 5, the PF encompasses a fixed payoff in conjunction with sharing in the income above a certain threshold and/or capital appreciation.[8] Unlike CW, PF offers upside potential for investors in the rich states of the economy. They allow lower fixed payments than the CW contract, i.e. $\xi < R_{CW}$, in return for extraneous remuneration emanating from excess operating income and/or appreciation in the value of the underlying project in the rich states of the economy.[9]

[Figure 5 about here.]

[Figure 6 about here.]

Coalescing the CW with PF cures our hybrid PPF facility of risk-shifting at the micro-level (and thus systemic risk at the macro-level) as well as underinvestment.[10] That is, our PPF structure, therefore, offers downside risk-sharing (below a) along with a reasonable cost of funds ensuing from the upside payoff-sharing (above b), as depicted in Figure 5. This mitigates the counter-effects of the incomplete PPP contract mentioned in Hart (2003) as the structure hedges its payoffs towards the infinite possibilities of the future economic conditions. In other words, our revenue-sharing PPF contract encompasses all states of the economy and is 'iron-clad.'

Finally, it is possible to derive closed-form formulae, within the combination of CW and PF variants, using cap and floor on continuous flows of Shackleton and Wojakowski (2007), as illustrated in the following Section. This allows us to tackle the twin agency issues and to trade-off current income with future capital appreciation of the project. This links the *sukuk* payoffs to the performance of the underlying assets/projects in the real sector of the economy. The closed-form solutions corroborate the model to be not only theoretically appealing but also practically feasible.

3 Modeling the blended financing of infrastructure

This section proposes a new approach to develop an economically viable infrastructure project by a charitable endowment (*waqf*). We assume the *waqf* to finance a social project[11] partially using its own resources and by employing a Shariah-compliant financing vehicle called a *sukuk*. This infrastructure can, for example, constitute an airport, a hospital, a toll road, etc. There are four parties involved in this development: (i) the government, which provides the land; (ii) the developer, who builds the infrastructure and operates it until relinquishing it at time $T > 0$ (see Figure 7); (iii) the *sukuk* holders, who partially fund the same; and (iv) the *waqf* that ultimately owns the infrastructure and operates it from time T until the end of timeline. Our modelling of the infrastructure appreciation and financing is flexible and can accommodate a variety of specifications encountered in practice.

[Figure 7 about here.]

3.1 Infrastructure implementation timeline

We assume that the construction starts at time $t_C < 0$ and ends upon time 0. The infrastructure costs $C > 0$ at time t_C and requires prior purchase of land costing L and accomplished at time $t_L < t_C < 0$. The cost C includes a premium payment for a high-quality facility built on time as discussed below. The managing phase starts as soon as the construction is finished at time 0 and the developer operates the facility until time $T > 0$.

The total costs K to acquire land and build the infrastructure for time 0 are reimbursed by: 1) a *sukuk* funding K_s; and 2) a charitable endowment (*waqf*) K_w, both raised at time t_L. Assuming a risk-free rate of inflation $\pi > 0$, the present value of the required costs at time 0 is[12,13]

$$K = e^{-\pi t_L} \left(K_s + K_w\right) = e^{-\pi t_L} L + e^{-\pi t_C} C \; . \tag{1}$$

We assume the private *sukuk* holders do not get any payoffs in the project's gestation period (i.e., from time t_L to time 0). They, however, get paid from time 0 to time T. To make the model tractable, the *sukuk* tenure period also coincides with the end time T of the developer's managing stage.

The continuous grade of infrastructure can be measured by the variable q, reflecting the quality from low $q = 0$ to high $q = 1$. More importantly, this grade is certified ex-ante by an independent organization. We assume that this external body is fair and objective and accurately assigns a penalty $\alpha \in [0, 1]$ for surrendering a lower quality structure as follows

$$\alpha = 1 - q \; . \tag{2}$$

This means that e.g. when the quality fully corresponds to the specifications, there is no penalty imposed and $\alpha = 0$.[14] In terms of present value at time 0, the developer-operator expends $K_C(\alpha) \leq K$ to build the infrastructure, where $0 \leq \alpha \leq 1$. Furthermore, we assume that when the quality is high ($\alpha = 0$), the builder matches the present value of the total costs, i.e. $K_C(0) = K$.

Moreover, there is a mechanism to incentivize quality by way of a compensation package. The developer-operator receives a fixed rate of payment $\gamma_C > 0$ during the construction phase (from time t_C to time 0) and a stochastic payoff during the managing phase (from time 0 to time T). This organization also has the authority to control the payoffs and thus to discipline the developer. This means that the compensation rate g during the latter phase depends on the grade/quality of the construction via penalty parameter α, as well as on the observed benchmark income x_t which follows a stochastic process

$$g(\alpha, x, t) = \gamma_C 1_{t \in [t_C, 0]} + G(\alpha, x, t)\, 1_{t \in [0, T]} \; , \tag{3}$$

where $G(\alpha, x, t)$ is a function decreasing in α and increasing in x. Consequently, during the developer's managing stage of the infrastructure, the payoffs to the *sukuk* and *waqf* holders are also contingent on both the penalty parameter α, as well as on the random income levels x_t. The Net Operating Income (NOI) y is

$$y_t = x_t - g(\alpha, x_t, t) \; . \tag{4}$$

The *sukuk* and the *waqf* get a fraction of the residual income y each, that is $s(y)$ and $w(y) = y - s(y)$ respectively, where $s(y)$ is the payoff to the *sukuk* holders.

3.2 Revenue dynamics

We assume that the infrastructure revenue, x, follows a Geometric Brownian Motion (GBM) under the pricing measure

$$dx_t = x_t \left((\pi - \delta)\, dt + \sigma dz_t \right) \ , \tag{5}$$

where z_t is a Wiener process and σ is the revenue volatility. This is a simplification, but it has its appeal as it allows for clarity and results in intuitive and meaningful closed form solutions which are easy to interpret. For using the pricing measure rather than the actual measure to be possible, the underlying market must trade assets which are correlated to both the income uncertainty x as well as bonds paying exactly the rate of inflation π assumed known and constant. Parameter $\delta > 0$ is assumed to be constant. It can be interpreted as reflecting the expected constant physical rate of decay affecting the structure of the facility[15] and thus lowering the revenue rate x. In particular, note that the stochastic differential equation (5) can be solved to yield

$$x_t = x_0 e^{\left(\pi - \delta - \frac{\sigma^2}{2}\right)t + \sigma z_t}, \tag{6}$$

where x_0 is the level of revenue flow at the beginning of the managing phase.

3.3 Economic benefits of the infrastructure to society

We consider social benefits from the infrastructure revenue to accrue to the community at large as a positive externality of development. This can either be described as: 1) a function $H(V)$ of the value V of the infrastructure; or 2) a function $h(x)$ of the infrastructure revenue flow x; or 3) as a mixture of both. In what follows we adopt the first approach and we assume that H is a simple multiplication function i.e. $H(V) = \Omega V$, where $\Omega > 0$. These social benefits further propagate and act as a positive feedback loop to reinforce economic development, thereby boosting the endowment's (*waqf*'s) charitable contributions by $\Pi H(V)$, where $0 \leq \Pi < 1$. To model the impact on *waqf*, here too we assumed a simple multiplicative relation.

3.4 Motivating the developer-operator via payoff pricing

As described earlier in equation (3), the developer-operator is paid at the constant rate γ_C during the construction phase. After completion, this is followed by the operating phase, during which the company is also training the *waqf* employees to run the infrastructure. As also described in equation (3), the developer's payoffs during the operating stage occur at the rate $G(\alpha, x_t, t)$ i.e. they are contingent on both the independently observed and certified quality grade q, and thus the corresponding penalty $\alpha \in [0, 1]$, as well as the current level of the revenue stream x_t. We specify the function G as follows

$$G(\alpha, x, t) = \underbrace{\gamma \min\left\{\frac{x}{a}, 1\right\}}_{\text{risk sharing}} + \underbrace{\beta (x-b)^{+} 1_{t\in[\alpha T, T]}}_{\text{profit sharing (bonus)}}, \tag{7}$$

where $\gamma > 0$ is the base contractual constant payment rate during the managing phase. We require $\gamma \leq a$, so that $\frac{\gamma}{a} \leq 1$, i.e. the developer cannot be promised to be paid at a rate higher than the income rate x. The $\beta \leq 1$ is the agreed supplementary bonus rate rewarding the quality of construction.[16,17]

The first term in this equation represents the project *risk sharing* by the developer-operator. That is, it gets paid less if the revenue stream x is weak, below a. Moreover, the risk-sharing element can be interpreted (in the context of mathematical finance) as a series of positions in two components: 1) long positions in γ inflation-free zero-coupon bonds continuously paying at a *unit* "face" rate of 1; 2) short positions in $\frac{\gamma}{a}$ default put options on the stream of revenues x struck at $a > 0$

$$\gamma \min\left\{\frac{x}{a}, 1\right\} = \gamma \times \underbrace{1}_{\text{inflation bond}} - \frac{\gamma}{a}\underbrace{(a-x)^{+}}_{\text{default put}}. \tag{8}$$

The second term in equation (7) represents *profit sharing* or bonuses paid to the developer. If the quality is high, the firm is eligible for bonuses straight away from completion at time 0, i.e. when $t \in [0, T]$. In contrast, a medium-quality grade, e.g. $\alpha = \frac{1}{2}$, means that the operator is eligible for bonuses later and thus for shorter periods of time, i.e. when $t \in \left[\frac{T}{2}, T\right]$. Finally, for a builder who submitted a bad quality project ($q = 0$, $\alpha = 1$), this time window disappears, meaning they will never be able to collect any bonuses. Bonuses are only paid when the revenue flow x exceeds the upper threshold $b > a$. Likewise, the profit sharing element can be interpreted as a series of long positions in β call options on the stream of revenues x struck at $b > a$.[18]

Equation (7) can alternatively be expressed as

$$G(\alpha, x, t) = \underbrace{\gamma \frac{x}{a} 1_{x<a}}_{\text{poor states}} + \underbrace{\gamma 1_{x\in[a,b]}}_{\text{normal states}} + \underbrace{\left(\gamma + \beta (x-b)^{+} 1_{t\in[\alpha T, T]}\right) 1_{x>b}}_{\text{good states}}. \tag{9}$$

In poor states of the economy, when revenues are weak and below the lower threshold a, the payoffs to the operator become state-contingent and are damped by the factor $\frac{x}{a}$. In normal times the company receives payoffs at the constant rate γ. In good times, in addition to the constant rate payoffs, the operator receives bonus payoffs when the revenue flow x exceeds b. Better quality projects are eligible for bonuses *earlier*. A low quality project will attract the highest penalty $\alpha = 1$ and will *never* pay bonuses.[19]

3.5 Project value to the developer operator

The expected present value of all cash flows after completing the structure, when the developer starts operations, can be calculated for time 0 as follows

$$V_C = E^Q \int_{-\infty}^{\infty} e^{-\pi t} g(\alpha, x_t, t)\, dt\ . \tag{10}$$

Using (3), (7) and (8) this can further be transformed into

$$\begin{aligned} V_C(\alpha) &= \gamma_C \int_{t_C}^{0} e^{-\pi t} dt + \gamma \int_0^T e^{-\pi t} dt - \frac{\gamma}{a} \int_0^T \underbrace{e^{-\pi t} E^Q (a - x_t)^+ dt}_{\text{put options}} \\ &\quad + \beta \int_{\alpha T}^{T} \underbrace{e^{-\pi t} E^Q (x_t - b)^+ dt}_{\text{call options}}\ . \end{aligned} \tag{11}$$

In particular, the third and fourth integrals involve time integrals of European put and call options struck at a and b, respectively. Integration in the fourth term starts at $\alpha T \geq 0$. This means that the higher the penalty α is and the closer it is to 1, the *later* the entitlement to bonuses vests. As a result, the integration in the fourth term is delayed from time 0 to $\alpha T > 0$, penalizing the developer for coming up with a lower quality infrastructure. We use Shackleton and Wojakowski (2007) results on flow options to represent these terms in closed form. Integrating the first and second terms is straightforward. We obtain

$$\begin{aligned} V_C(\alpha) &= \frac{\gamma_C}{\pi}\left(e^{-\pi t_C} - 1\right) + \frac{\gamma}{\pi}\left(1 - e^{-\pi T}\right) - \frac{\gamma}{a} F(x_0, a, 0, T, \pi, \delta, \sigma) \\ &\quad + \beta \left[U(x_0, b, 0, T, \pi, \delta, \sigma) - U(x_0, b, 0, \alpha T, \pi, \delta, \sigma)\right]\ . \end{aligned} \tag{12}$$

We have the following components: 1) a long position in the forward value of annuity for the period $[t_C, 0]$ paying at rate γ_C; 2) a long position in the present value of annuity covering operations during the period $[0, T]$ and paying γ; 3) a short position in $\frac{\gamma}{a}$ floors F on revenue flow x struck at a for maturity T; 4) a long position in β caps U on revenue flow x struck at $b > a$ for maturity T; and 5) a short position in β caps U on revenue flow x struck at $b > a$ for maturity $\alpha T \geq 0$.

The first term represents the construction reward as a future value of an annuity. The second annuity is the regular, risk-free and inflation-free reward during the operations phase. The third term represents risk sharing and can be interpreted as the cost of insurance against bad states of the economy the operator has to pay for ex ante, in order to enjoy undisturbed the benefits of the regular stream of income included in the second term. The fourth term represents the *full* state-contingent completion stream of bonuses continuously accruing to the developer during the terminal stage. This requires the revenue flow x_t to be boosted above the threshold b prior to the handover T of the infrastructure to the *waqf*. Finally, the fifth term represents the quality-contingent penalty. The lower the quality, the longer the maturity αT, and also the bigger the cumulative value of the floor to be subtracted from the full bonus of maturity T.

3.6 Instigating the Developer-Operator to build a high-quality product

The developer-operator will only accept to build the project if the present value of the stream of its cash-inflows (12) is greater than the deployed cost $K_C(\alpha)$ (see Section 3.1). Consequently, the expected Net Present Value (NPV) must be non-negative

$$NPV(\alpha) = V_C(\alpha) - K_C(\alpha) \geq 0 \,. \tag{13}$$

Note that when the costs are proportional to the quality grade q, they are decreasing in the penalty α, but the expected gross income is also decreasing in α because of the delayed bonuses stream

$$\frac{\partial K_C(\alpha)}{\partial \alpha} < 0 \qquad \text{and} \qquad \frac{\partial V_C(\alpha)}{\partial \alpha} < 0 \,. \tag{14}$$

This means that while the operator can make cost savings by decreasing quality, ultimately this may not pay out because of higher applied penalty α and foregone bonuses. Therefore, to encourage the developer-operator to build a high-quality structure, we need to ensure that the following *incentive-compatible* condition holds

$$\frac{\partial NPV(\alpha)}{\partial \alpha} < 0 \,, \tag{15}$$

i.e. lowering the quality level q increases the penalty α and thus lowers the NPV which is not what the developer ultimately wants. Therefore, the above condition (15) will motivate the developer-operator to instill high quality in its project. This condition is always satisfied when the cost is independent of the quality and constant, and in particular when it also matches the funding i.e. when $K_C(\alpha) = K$ for all $\alpha \in [0, 1]$.

3.7 The payoffs to the *sukuk*

The payoffs to the *sukuk* are a function of the Net Operating Income $y = x - g$ of the infrastructure defined in equation (4). The *sukuk* promises a non-guaranteed coupon payoff at the rate $\xi > \pi$. When introducing equation (7) we already required $\gamma \leq a$. Here, we assume that the *sukuk* payoffs change regime at the levels a', b' of the NOI which are in sync with the developer-operator thresholds a, b so that

$$a' = a - \gamma \qquad \text{and} \qquad b' = b - \gamma \,. \tag{16}$$

We also assume that *sukuk* benefits from a bonus scheme set at the corresponding threshold b' at the rate $\beta_s > 0$, akin to developer's bonuses. Consequently, when $x < a$, implying $y < a'$, the *sukuk* holders are paid at the rate $\frac{y}{a'}\xi < \xi$. When $x \in [a, b]$, *sukuk* obtains the promised rate ξ. Finally, when $x > b$, implying $y > b'$, in addition to the nominal rate ξ, the *sukuk* holders get the extra bonus rate $\beta_s (y - b')^+$. The payoff *rate* to the *sukuk* can be expressed as

$$s(y) = \frac{y}{a'} \xi 1_{y<a'} + \xi 1_{y \in [a',b']} + \left[\xi + \beta_s (y - b')^+\right] 1_{y>b'} \ . \tag{17}$$

Comparing (17) to (9) we conclude that the *sukuk* payments are structured in a similar way to the builder-operator. That is, in poor states of the economy *sukuk* holders share risk responsibly and they are rewarded with extra surpluses in good times. Similarly like the operator's profits in (7), it is possible to express (17) as

$$s(y) = \underbrace{\xi \min\left\{\frac{y}{a'}, 1\right\}}_{\text{risk sharing}} + \underbrace{\beta_s (y - b')^+}_{\text{profit sharing (bonus)}} \ . \tag{18}$$

The role of those terms is risk and profit sharing. This is the same as the role of the analogous terms in the operator payoff in (7) and (8). Note that there is no additional provision penalizing *sukuk* holders for bad quality infrastructure. However, because the NOI variable y depends on α, x and time t, so is the *sukuk* payoff rate s. Therefore, we can write

$$s(\alpha, x, t) = \xi \min\left\{\frac{x - g(\alpha, x, t)}{a'}, 1\right\} + \beta_s (x - g(\alpha, x, t) - b')^+ \ . \tag{19}$$

Going further, we can expand the functional terms $g(\alpha, x, t)$ which change their form in known ways upon thresholds a, b. For $t \in [0, T]$ we have

$$s(\alpha, x, t) = \xi \min\left\{\frac{x - G(\alpha, x, t)}{a'}, 1\right\} + \beta_s (x - G(\alpha, x, t) - b')^+ \ . \tag{20}$$

Using (9) we then obtain 3 cases:[20]

1. When $x < a$ we have $G(\alpha, x, t) = \gamma \frac{x}{a}$ and

$$s(\alpha, x, t) = \xi \frac{x}{a} \ . \tag{21}$$

2. When $x \in [a, b]$ we have $G(\alpha, x, t) = \gamma$ and

$$s(\alpha, x, t) = \xi \ . \tag{22}$$

3. When $x > b$ we have $G(\alpha, x, t) = \gamma + \beta (x - b)^+ 1_{t \in [\alpha T, T]}$ and

$$s(\alpha, x, t) = \xi + \beta_s (x - b)\left(1 - \beta 1_{t \in [\alpha T, T]}\right) \ . \tag{23}$$

This can be written compactly as

$$s(\alpha, x, t) = \underbrace{\xi \frac{x}{a} 1_{x<a}}_{\text{poor states}} + \underbrace{\xi 1_{x \in [a,b]}}_{\text{normal states}} + \underbrace{\left[\xi + \beta_s (x - b)\left(1 - \beta 1_{t \in [\alpha T, T]}\right)\right] 1_{x>b}}_{\text{good states}} , \tag{24}$$

or

$$s(\alpha, x, t) = \xi - \frac{\xi}{a}(a-x)^{+} + \beta_s (x-b)^{+} \left(1 - \beta 1_{t\in[\alpha T, T]}\right) . \tag{25}$$

That is, the *sukuk* holders get paid at the constant rate ξ but are short $\frac{\xi}{a}$ of default put options struck at the lower threshold a which they have to pay for in poor states $x < a$. These put options represent the insurance for the *waqf* and expire worthless when $a \leq x$. In good states, *sukuk* holders benefit from the residual fraction $1-\beta$ of the developer-operator's appreciation call above the upper threshold b. These call option allow the *sukuk* holders to participate in the NOI increase. Moreover, when the developer provides a structure of lower quality, the penalty is positive $\alpha > 0$ which means that before time $t = \alpha T$, while the residual fraction is equal to 100%, the *sukuk* holders get paid the full payoff $x - b$ of the appreciation call, which they don't have to share with the developer.

Overall, the payoff of the *sukuk* holder is structurally similar to the payoff of the developer, except that the former gets paid at the rate ξ, while the latter at the rate γ. Also, in states when the developer is penalised and $\alpha > 0$, the sharing fraction β is retained and an extra compensation is provided to the *sukuk* holders instead.

3.8 Project value to *sukuk* holders

Following the logic of computing the present value, we have

$$V_S = E^Q \int_{-\infty}^{\infty} e^{-\pi t} s(\alpha, x_t, t)\, dt . \tag{26}$$

Using (25) we obtain

$$\begin{aligned} V_S &= \xi \int_0^T e^{-\pi t} dt - \frac{\xi}{a} \int_0^T e^{-\pi t} (a - x_t)^{+} dt + \beta_s \int_0^T e^{-\pi t} (x_t - b)^{+} dt \\ &\quad - \beta_s \beta \left[\int_0^T e^{-\pi t} (x_t - b)^{+} dt - \int_0^{\alpha T} e^{-\pi t} (x_t - b)^{+} dt \right] . \end{aligned}$$

Identifying puts and calls as in (11) and time-integrating into the corresponding floors and caps, we finally obtain the closed form for the value of *sukuk*

$$\begin{aligned} V_S &= \frac{\xi}{\pi}\left(1 - e^{-\pi T}\right) - \frac{\xi}{a} F(x_0, a, 0, T, \pi, \delta, \sigma) \\ &\quad + \beta_s \left[(1-\beta) U(x_0, b, 0, T, \pi, \delta, \sigma) + \beta U(x_0, b, 0, \alpha T, \pi, \delta, \sigma)\right] . \end{aligned} \tag{27}$$

This expression is very useful to the willing *sukuk* holder, who can estimate whether this project is likely to be profitable before committing. The first term to the RHS shows the value of a riskless annuity i.e. *sukuk*'s promised payment, akin to a conventional bond. To incorporate the impact of the bad states of the economy, the *sukuk* payment is offset by the value of a (short) position in the put options, struck at the lower threshold a. This correction is done by the second term on the RHS which is a weighted floor function. In good states of the economy, the value of investment increases by the two long positions in call options that give opportunities to the *sukuk* holders to gain more if the NOI increases. This is given by the last term on the RHS, composed of two parts, weighted by the *sukuk* holder's fraction β_s (the ultimate residual fraction, $1-\beta_s$, goes to the *waqf*). The first part grants the residual fraction, $1-\beta$, of the developer-operator's appreciation when the NOI increases above b. The second part gives the remaining fraction, β, and thus completes $1-\beta$ to 100% (before $t=\alpha T$) in occurrences when the developer provides a lower quality structure. The *sukuk* holder will consider participating in this project if the expected value, adjusted for the additional short and long positions in the related options, compares well enough to alternative investment opportunities.

3.9 The payoffs to the *waqf*

This is simply the residual amount left, if any, after all due payments to the developer and the *sukuk* have been made. Combining all the equations derived so far it is easy to see that for $t\in[t_C,0]$

$$w(\alpha,x,t)=-\gamma_C\ ,$$

then for $t\in[0,T]$

$$\begin{aligned} w(\alpha,x,t) &= x-\underbrace{(\xi+\gamma)\frac{x}{a}1_{x<a}}_{\text{poor states}}-\underbrace{(\xi+\gamma)\,1_{x\in[a,b]}}_{\text{normal states}} \\ &\quad -\underbrace{\left[\xi+\gamma+(x-b)\left[\beta_s+\beta(1-\beta_s)\,1_{t\in[\alpha T,T]}\right]\right]1_{x>b}}_{\text{good states}}\ , \end{aligned} \tag{28}$$

and for $t\geq T$

$$w(\alpha,x,t)=x\ .$$

Equivalently, the equation (28) can be transformed into

$$\begin{aligned} w(\alpha,x,t) &= x-\underbrace{\left\{\gamma-\frac{\gamma}{a}(a-x)^+ +\beta(x-b)^+1_{t\in[\alpha T,T]}\right\}}_{\text{developer}} \\ &\quad -\underbrace{\left\{\xi-\frac{\xi}{a}(a-x)^+ +\beta_s(x-b)^+\left(1-\beta 1_{t\in[\alpha T,T]}\right)\right\}}_{\textit{sukuk}}\ . \end{aligned}$$

That is, as in a structured product, the *waqf* gets the third "tranche" after developer and *sukuk* holders are paid first. Grouping terms this gives

$$w(\alpha,x,t)=x-(\gamma+\xi)+\frac{\gamma+\xi}{a}(a-x)^+-\left\{\beta_s+\beta(1-\beta_s)\,1_{t\in[\alpha T,T]}\right\}(x-b)^+\ .$$

3.10 Project value to the *waqf*

At time $t = 0$ it is simply equal to the time integral of the expected cashflows net of any disbursements

$$V_W = E^Q \int_{-\infty}^{\infty} e^{-\pi t} w(\alpha, x_t, t)\, dt \, .$$

We obtain

$$\begin{aligned} V_W \quad = \quad & -\gamma_C \int_{t_C}^{0} e^{-\pi t} dt + E^Q \int_0^{\infty} e^{-\pi t} x_t dt - (\gamma + \xi) \int_0^T e^{-\pi t} dt \\ & + \frac{\gamma + \xi}{a} E^Q \int_0^T e^{-\pi t} (a - x_t)^+ dt - \beta_s E^Q \int_0^T e^{-\pi t} (x_t - b)^+ dt \\ & - \beta (1 - \beta_s) \left\{ E^Q \int_0^T e^{-\pi t} (x_t - b)^+ dt - E^Q \int_0^{\alpha T} e^{-\pi t} (x_t - b)^+ dt \right\} \, . \end{aligned}$$

Therefore the closed form is

$$\begin{aligned} V_W \quad = \quad & \frac{x_0}{\delta} - \frac{\gamma_C}{\pi} \left(e^{-\pi t_C} - 1 \right) - \frac{\gamma + \xi}{\pi} \left(1 - e^{-\pi T} \right) \\ & + \frac{\gamma + \xi}{a} F(x_0, a, 0, T, \pi, \delta, \sigma) \\ & - [\beta + (1 - \beta) \beta_s] U(x_0, b, 0, T, \pi, \delta, \sigma) \\ & + \beta (1 - \beta_s) U(x_0, b, 0, \alpha T, \pi, \delta, \sigma) \, . \end{aligned} \tag{29}$$

So the value to the *waqf* is equal to: a) the present value of the perpetual project paying off at initial rate x_0 and decaying at rate δ; minus b) the present value of annuity cost paid at constant rate γ_C to the developer from time $t_C < 0$ to time 0 when operation begins; minus c) the present value of the riskless annuity paid to both developer and *sukuk* at rates γ, ξ respectively; plus d) the present value of "insurance" payoff which can be represented by the floor function F;[21] minus e) the present value of the participation payments rewarding both the developer and *sukuk* holders at rates β, β_s respectively, in case of the project yielding high outcomes. This last element is represented by the two weighted cap functions U which also take into account any penalty parametrized by α. Where there is no penalty i.e. $\alpha = 0$, then the last cap function U is equal to zero and the payments to both *sukuk* and developer start immediately. The developer gets the proportion β of the surplus represented by the first cap function U. Of the $1 - \beta$ left, a proportion β_s is then paid to *sukuk* holders. When a penalty is applied, then $\alpha > 0$ and, before time αT, only *sukuk* holders get paid a proportion β_s of the surplus U. That means more money goes to the *waqf* i.e. an additional proportion β of the $1 - \beta_s$ left after payment of β_s to the *sukuk* goes to the *waqf*. This is represented by the last term, which is positive, involving the second surplus function U over the time horizon $[0, \alpha T]$.

4 Computational simulation

We require the expected net payouts to the developer, *sukuk* and the *waqf* to be non-negative, i.e.

$$0 \le V_C \;, \qquad 0 \le V_S - e^{-\pi t_L} K_s \qquad \text{and} \qquad 0 \le V_W - e^{-\pi t_L} K_w \;.$$

This will insure that the value of the project's expected payouts to stakeholders equals or exceeds its costs

$$K \le V_C + V_S + V_W \;.$$

For the purposes of the numerical illustration, we need to parametrize our model.

[Table 1 about here.]

[Table 2 about here.]

Table 1 summarizes our base case calibration and notation. We assume that to build the infrastructure project, a developer is offered an assured payment of $\gamma_c = 30$ millions for two years, followed by a stream of $\gamma = 50$ millions per annum and an extra participation of $\beta = 50\%$ if the revenue stream x exceeds the threshold b of 200 millions a year. However, if the revenue drops below $a = 100$ millions a year, the developer will proportionally cover any losses, so that if the revenue stream stops and $x = 0$, any payments to the developer must stop too.

In this example we discuss differences between projects of three different qualities: good, medium and bad, so that the corresponding penalties are $\alpha = 0$, 0.5 and 1, respectively. We also consider high, medium and low risk scenarios, corresponding to the volatility parameter σ equal to 50%, 20% and 1% respectively. We focus on the values to the construction company V_C, to the *sukuk* V_S and to the *waqf* V_W. They must sum to the value expected from the project in the long run, i.e. to the present value of the perpetuity, which in the zero-interest environment of Islamic economies must be equal to

$$V = V_C + V_S + V_W = \frac{x_0}{\delta} \;, \tag{30}$$

where x_0 is the expected initial annual revenue of the project and δ is the rate of decay of the infrastructure.

In the base case scenario we assume an inflation rate of $\pi = 10\%$, as well as the decay rate of the project, δ, equal to 5% per annum. The volatility which determines the riskiness of the project is set to be $\sigma = 20\%$ and the maturity for the developer and the *sukuk* are both $T = 10$ years.

With our benchmark calibration we can now demonstrate the impact of the various characteristics of our structured product. As the illustrative operating points we consider five initial yield estimates, $x_0 = 50, 100, 150, 200$ and 250 which cover a range including both thresholds a and b. Table 2 shows the expected present values to: a) the construction company, V_C; b) the *sukuk*, V_S; and c) the *waqf*, V_W, for low, medium and high risk scenarios σ, equal to 1%, 20%

and 50%; and three project qualities α, equal to 0 (good), 0.5 (medium) and 1 (bad), respectively. Clearly, the higher the initial yield estimate x_0, the higher any of the expected present values V_C, V_S and V_W. This can be seen in further detail in Figures 8 and 9 for the construction company, Figures 10 and 11 for the *sukuk* as well as in Figures 12 and 13 in the case of *waqf*.

[Figure 8 about here.]

[Figure 9 about here.]

[Figure 10 about here.]

[Figure 11 about here.]

[Figure 12 about here.]

[Figure 13 about here.]

Furthermore, the collection comprising Figures 8, 10 and 12 illustrates the impact of the quality parameter α. As can be seen, keeping the risk σ of the project constant and improving the quality α, increases the contract values for both the developer and the *sukuk*. However, improving quality means decreasing α, which also acts as the penalty parameter. This lowers the sums received by waqf as it has less influence on the other participants. Therefore, improved project quality α actually *decreases* the expected value of the waqf (see Figure 12), because they are then entitled to get less or no compensation.

Finally, the cross-section comprising Figures 9, 11 and 13 reveals the endogenous reversal obtained by varying the exogenous parameters: risk σ (volatility) and the project initial value x_0. Riskier projects with higher initial values are more valuable to the developer-operator and the *sukuk*, and less valuable to the *waqf*. When the initial values x_0 decrease towards the loss participation threshold a, these same risk-class projects become less valuable for the developer-operator and the *sukuk* and relatively more valuable for the *waqf*, in contrast to the low risk projects.

5 Conclusion

Developing Muslim countries are in dire need of an innovative infrastructure financing that not only satisfies their religious boundary but also improves the economic efficiency of conventional financing, thereby unlocking greater societal impact. Islamic finance has the potential to address this issue, as highlighted in a World Bank (2017) report, p.iv:

> *Given the potential of Islamic finance to support infrastructure development in emerging and developing countries, it is critical to address how to best deploy Islamic project finance in PPP delivery frameworks…*

However, empirical evidence suggests the failure of these countries to implement a more just and sustainable financing (Jatmiko et al., 2023). Ill-suited financialization can drag these emerging economies into an infrastructure trap, where no investment goes to social welfare enhancing projects (Ghosh and Meagher, 2015). Healthy green growth is difficult to implement due to the limitation of an efficient mechanism containing the adverse issues of financialization and spatial inequality (De Angelis et al., 2023; Klusak et al., 2023). We seek to overcome these issues by incorporating cultural and legal aspects into our analysis, without losing sight of economic aspects.

This paper proposes and mathematically models an improved structure of *sukuk*, integrating the literature on financial engineering, charitable endowments, and development finance. We argue that the PPP framework offers the potential to rejuvenate the structure of *sukuk* as it does not separate the financing of the project with the development, operation, maintenance, and transfer mechanism. This allows our *sukuk* to mitigate the asset-linked centrality issue as well as the opacity problem of adverse selection and moral hazard (Knight and Sharma, 2016). However, PPPs are not a satisfactory solution for the *sukuk* structure due to their drawbacks of (i) sharpening the spatial inequality (Demirgüç-Kunt and Levine, 2009); (ii) escalating the cost of funds (Trebilcock and Rosenstock, 2015); and (iii) delinking their payoffs from the economic performance of the project in the future states of the economy (Hart, 2003).

To address these shortcomings, we further integrate the blended finance, ensuing from *waqf* crowdfunding, into the structure of our *sukuk*. We anticipate the charitable endowment's social dimension to address *sukuk*'s and PPPs' inequality issue. This is done by incorporating the positive externalities and social preference of donor-investors into the project's valuation (Armitage, 2017; Brown and Davies, 2020; Lee and Parlour, 2021). Furthermore, the charitable endowment also reduces the cost of capital of *sukuk* and eliminates the inefficiency of *Shari'a* certification. The latter allows reconfiguration of our *sukuk* into sustainable infrastructure financing, consistent with the objectives of Islamic law (Starks, 2023).

We finally design our cost-effective fragile-free *sukuk* termed as 'Participating Preferred Financing' (PPF) by coalescing the Continuous Workout feature with the Participating Facilities (Ebrahim et al., 2011; Shiller et al., 2019). This allows our PPF to be malleable in the income-growth dimensions and robust to external shocks in the future states of the economy. It thus mitigates the negative impact of financialization by reducing the fragility (risk-shifting) and underinvestment. The adoption of PPF can vitalize the financial sector, boost its resilience, and further regional economic and spatial development.

The innovative framework of PFF is not without its limitations. To address the growing demand for such blended finance, it necessitates, among other factors, the availability of transparent public information concerning the time-varying performance and values of projects over time. While achieving this necessary condition is not straightforward, it is certainly feasible.

Notes

[1]The robust demand of *Sukuk* in the market is well documented (Jatmiko et al., 2023). The need for integrating *waqf* in development finance is also escalating, as evidenced by several initiatives like the Cash-Waqf Linked *Sukuk* (CWLS) by the Government of Indonesia and the Global Islamic Fund for Refugees (GIFR) by the Islamic Development Bank. However, the pricing employed in these facilities relies on interest-based references, thereby giving rise to the agency cost of debt (*riba*) and asymmetric information (*gharar*) in their structures (Jatmiko et al., 2024).

[2]Examples include the PPP water privatization fiasco in developing countries, see Tan (2012) and prevalence of road projects over public transit, see Siemiatycki (2011). Asiyanbi (2018) explores a related problem of environmental financialization and marketisation in the making. See also Jatmiko et al. (2023) for positive nexus between *Sukuk* and inequality.

[3]At present, the industry (i) employs asset-based instead of asset-backed *sukuk*, see Jatmiko et al. (2023); (ii) de-links pricing from the performance of the underlying infrastructure project, see Kuran (2018); (iii) institutionalizes inefficient Shari'ah (Islamic legal) certification, see Hayat et al. (2013) and Gozubuyuk et al. (2020); and (iv) adopts the medieval Islamic contracts at the expense of economic efficiency, see Jobst (2007).

[4]To the best of our knowledge, our paper is the first to study the sustainability of debt of a charitable endowment by creatively mitigating the agency costs of debt and asymmetric information. The literature in contrast is focused on sovereign debt sustainability/capacity and venture capital investment in innovative projects (Croce et al., 2021; Crosignani, 2021; Mayer, 2022).

[5]This feature makes the CW in conformity with the Islamic values by 'giving respite' to the users of funds in the poor states of the economy.

[6]Here, we contrast the cost of financing in Figure 2 with that in Figure 4.

[7]The aggravation of the underinvestment problem also makes the CW facility to run afoul of the expropriation aspect of the *riba* injunction.

[8]In addition to sharing *income*, the PF can also include sharing *appreciation* and/or sharing *equity*. See Figure 6. For a discussion of these facilities see Ebrahim et al. (2011). They all have potential to offset the underinvestment problem of the CW but do not relieve the financial system's fragility.

[9]While we require $\xi \leq a$, so that when $\xi = a$ the PPF payments can initially be set the same as CW payments, Figure 5 actually shows a *sukuk* payment ξ strictly lower than a. This is to allow the net income x_t to be also shared with the *developer* and *waqf*, as implemented in our mathematical model in Section 3.

[10]These two attributes of PPF comply with the Quranic injunctions of *riba*, see *Quran* 2:280 and 4:161.

[11]Investments in social projects alleviate inequalities, enhance inclusiveness, labor relations, investment in human capital and communities, as well as human rights issues.

[12]While the values of times t_L and t_C are negative, the mathematical formulas for computing the present and future values at time 0 are structurally the same.

[13]The Islamic law restricts payments over the rate of inflation for a risk-free debt contract. It, however, allows risk-sharing, i.e. state-contingent hybrid contracts, as discussed in our paper.

[14]Our setup can be expanded to model whether the costs are on target. This can be achieved by e.g. adding an extra binary variable $\zeta \in \{0, 1\}$ and replace $K_C(\alpha)$ with $K_C(\alpha, \zeta)$. When $\alpha = 0$ and $\zeta = 0$ the infrastructure is of high quality and costs

are not overrun or underrun. To incentivize the builder, cost overruns can be penalized based on the value of ζ. Alternatively, the same can be achieved by raising the penalty α. Please note in particular that we do not differentiate the quality level of service of the developer-operator because a poor or medium grade, i.e. $q < 1$ implying $\alpha > 0$, impacts the overall revenue and, ultimately, that of its cash flow stream over the managing phase.

[15]Note that in our paper the variable x is a flow. In applications where x is a stock, the parameter δ can be interpreted as a rate (e.g. a dividend rate when x is a stock price) or, otherwise, as a rental revenue rate paid to landlords or as a "service flow" rate received by owners-occupiers (when x is the value of a property).

[16]Note in particular that we cannot have $\beta > 1$ as it would mean that for some very high revenue levels x the regular reward plus bonus stream together $\gamma + \beta(x - b)$ would have to exceed the revenue stream for some $x > b$, triggering *waqf*'s bankruptcy.

[17]We assumed there is no completion bonus lump sum at the time of the handover of the project to the *waqf*.

[18]It will become clear in the sequel that the interpretations in terms of the put and call options on the stream of revenues x are important because these continuous series of positions form the *flow options* which have closed form solutions. See Shackleton and Wojakowski (2007). This will greatly facilitate our analysis.

[19]Mathematically, this is because the Lebesgue measure of the interval $[T, T]$ is zero.

[20]See the Online Appendix for proofs.

[21]Here the "insurance" is implicitly provided by both the developer and *sukuk* holders, who both agree beforehand to proportionally reduce their incomes in case of the project yielding lower outcomes.

A Appendix: Reducing asymmetric information & uneven development

A.1 Reducing asymmetric information (*gharar*) by integrating *sukuk* with PPP

We argue that *sukuk* should be structured within the PPP framework (see Figure 14). While the PPP can alleviate the asymmetric information problem, the uneven geographies of development and agency cost of debt issues remain (see Table 3).

[Figure 14 about here.]

[Table 3 about here.]

We conceptualize *sukuk* as a real asset securitization bundled not only for financing purposes but also for the infrastructure projects' design, development, operation, maintenance, and transfer mechanism. Figure 1 shows the simplified mechanics of a *sukuk*, employing a PPP framework along the lines described above. The contract consists of three sequential phases, namely (i) construction; (ii) private operation; and (iii) charitable endowment (*waqf*) acquisition. Four main parties are involved in this arrangement, including the (i) government; (ii) charitable endowment; (iii) private investors; and (iv) developer-operator. By doing so, the existence of underlying real assets, asset-backed structures, and the apparent transfer of ownership is, by definition, guaranteed, and thus moderating the opacity problem of adverse selection and moral hazard. This is discussed in what follows.

The government plays a crucial role in incentivising infrastructure development and mitigating the inherent problem of opacity. First, the government is responsible for providing land for the infrastructure project. This is because the majority of delays in infrastructure delivery stem from the intricate land acquisition process. The government may use its eminent domain (land acquisition power) to ascertain the availability of the land (Peterson, 2008). It then sells the land at a discount to the charitable endowment, the project's equity owner. Second, the government encourages the involvement of private parties by granting tax incentives. This includes not only income tax exemption for the charitable endowment but also private *sukuk* investors[1] and developer-operator. The government can also mitigate uneven geographies of development by offering preferential tax treatment to infrastructure procurement in economically distressed regions (Rajan, 2019). Finally, the contract may also include government guarantees for the pricing of infrastructure services to mitigate the project's income risk (Ramamurti, 2003).

The financier co-sponsors the construction of the infrastructure project along with the charitable endowment. In return, the financier has a first claim to the infrastructure payoffs until the facility's tenure. The infrastructure's title is pledged against the financiers' lien to restrain the endowment from selling it, hence moderating the moral hazard problem. They provide funds on a quasi-equity basis. They are entitled to regular payments contingent on

[1] This practice is widely observed in municipal bonds, see Pan et al. (2017).

the performance of the underlying infrastructure project, which is malleable in the income-growth dimensions.[2] The financiers have recourse (a lien) on the infrastructure assets in the event of default. They strengthen their claims by mandating the developer and the charitable organization to maintain the value of the underlying infrastructure collateral through meticulous maintenance and adequate insurance coverage (Smith and Warner, 1979). This mitigates moral hazard in the financial arrangement.

Finally, the developer-operator's central position as the Build, Operate, and Transfer (BOT) entity makes its selection of import. The government, along with the capital contributors (charitable endowment and private investors), should conduct a thorough due diligence process to eliminate adverse selection issues (Manove et al., 2001). This can be achieved by splitting into stages the disbursement of funds, where payment of one construction stage is contingent on completing the prior one (Webb, 1991). An electronic procurement can be performed to alleviate the adverse selection, reduce the delays of project delivery, and improve the quality of infrastructure (Lewis-Faupel et al., 2016). The developer is incentivized to build quality by setting additional rewards for timely project completion and high-quality infrastructure. After completing the project construction, the developer-operator is responsible for the daily operation and maintenance of the infrastructure assets. In our setup, the operator is entitled to a fixed fee with a performance bonus. At the end of the operation stage, the developer-operator must deliver all the technology and knowledge of the infrastructure to the charitable endowment, the project's ultimate owner. This can also be encouraged by a completion bonus.

A.2 Reducing development inequalities through charitable endowments

A distinct feature of our paper is introducing Islamic endowments as a provider of infrastructure services. Apart from the public and private sector entities, charitable endowments are a third source of entities whose goal is to accelerate the development of emerging economies. As its name indicates, the charitable endowment offers a sustainable disbursement of benefits from the return of the invested capital. While the scheme has flexibility in terms of the beneficiaries, purposes and forms, it ensures that its capital remains to guarantee the perpetuity feature.

The recent development of Islamic social finance addresses this by proposing a cash endowment fund that receives capital from donors invested in a portfolio of real assets from which the payoffs are disbursed to the charity. The cash endowment is set up to serve society through infrastructure investment. The government facilitates a crowdfunding platform to allow public donations for infrastructure projects listed in the system funded by *sukuk*. The endowment fund buys the land at a discount from the government. This blended finance enables the cash endowment to help develop infrastructure by accepting lower returns to achieve its social goals and addresses the high cost of funds, as stated in Trebilcock and Rosenstock (2015).[3] This allows it to pay a higher return to *sukuk* holders and could lead to

[2]That is, the facility provides a trade-off between payoffs which are a function of income and those depending on capital appreciation.

[3]The government of Indonesia became the first country linking the cash endowment and *sukuk*. Nonetheless, the model, launched

an abnormal situation where the total returns to the residual (i.e., equity) claimants are lower than the senior *sukuk* ones. Even though the payoffs received by the charitable endowment may be lower than that of financiers, it is entitled to full ownership of the infrastructure assets at the end of the contract.

The charitable endowment is also expected to reduce the uneven geographies of development issue in the contemporary PPPs. The problem arises as the profitability-based investment decision-making encourages private investors to centralize their infrastructure investments in selected prosperous regions instead of peripheral or poor ones (Siemiatycki, 2011; Tan, 2012). Table 4 illustrates this by showing the distribution of commutative PPP investments across the developing Muslim world between 1990 and the first half of 2022.[4] The charitable endowment can play a redistribution role by incorporating a social dimension in infrastructure development (Pollard et al., 2016). It allows the project valuation to account for positive externalities by integrating the Net Present Social Value (NPSV) with the traditional NPV (Armitage, 2017).[5]

[Table 4 about here.]

in March 2020 (for the private placement) and in October 2020 (for the retail investors), employs a temporary medium-term *waqf* that hinders the long-term investment benefits, see Chambers et al. (2020). This arrangement is also prone to agency issues as the payoffs rely on the government budget and are detached from the project's performance. A robust model alleviating the problems of risk-shifting and underinvestment and thus fulfilling the spirit of the religious law is the need of the hour. This is discussed in Subsection 2.1.

[4]It should be noted that 99% of PPP investments were conducted from 1990 onwards.

[5]The NPSV can be calculated by employing ranges of non-market pricing methods. These include (i) the Revealed Preference, where the value is assessed based upon the market price of directly or indirectly related products (Samuelson, 1938, 1948); (ii) the Stated Preference, where the survey-based technique is conducted to discover the willingness to pay for the project's benefits (Ciriacy-Wantrup, 1947); and (iii) the Life Satisfaction, where the value is elicited by evaluating people's subjective well-being towards the development (Frey et al., 2004).

B Appendix: Proofs

Proof. (a contrario) We have $y < a' \Leftrightarrow x - g(\alpha, x, t) < a - \gamma$. Assume $b > x \geq a$, then $g(\alpha, x, t) = \gamma$ implying

$$y < a' \Leftrightarrow x - \gamma < a - \gamma \Leftrightarrow x < a \;, \tag{31}$$

which is a contradiction, therefore we must have $x < a$ which implies $g(\alpha, x, t) = \gamma \frac{x}{a}$ and so

$$y < a' \Leftrightarrow x - \gamma \frac{x}{a} < a - \gamma \Leftrightarrow x\left(1 - \frac{\gamma}{a}\right) < a\left(1 - \frac{\gamma}{a}\right) \Leftrightarrow x < a \;, \tag{32}$$

which is not a contradiction. QED ■

Proof. The first two cases are straightforward. In the 3rd case because $\beta \leq 1$ and $a < b$ we must have

$$x - a > x - b \geq \beta(x - b) \;\; . \tag{33}$$

This implies

$$x - \gamma - \beta(x - b) > a - \gamma \;. \tag{34}$$

As $a > \gamma$ we then have

$$\frac{x - \gamma - \beta(x - b)}{a - \gamma} > 1 \;. \tag{35}$$

For $x > b$ this implies

$$\min\left\{\frac{x - \gamma - \beta(x - b)^+ 1_{t \in [\alpha T, T]}}{a - \gamma}, 1\right\} = 1 \;, \tag{36}$$

so that the first term is ξ. The second term when $x > b$ becomes

$$\begin{aligned} \beta_s\left(x - \gamma - \beta(x - b)^+ 1_{t \in [\alpha T, T]} - b'\right)^+ &= \beta_s\left(x - \beta(x - b)\, 1_{t \in [\alpha T, T]} - b\right)^+ & (37) \\ &= \beta_s(x - b)\left(1 - \beta 1_{t \in [\alpha T, T]}\right) \;. & (38) \end{aligned}$$

QED ■

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Table 1: Base case parameter values.

PHASE	DESCRIPTION	PARAMETER	VALUE	UNIT
	inflation rate	π	10%	1/yr
	infrastructure decay rate	δ	5%	1/yr
	initial yield estimate	x_0	50 to 250	$mil/yr
Land	acquisition time	$t_L \leq t_C$	n/a	yrs
Construction	start time	t_C	-2	yrs
	assured payment	γ_c	30	$mil/yr
Managing	start time	t_0	0	years
& *Sukuk*	assured payment	γ	50	$mil/yr
	loss participation threshold	a	100	$mil/yr
	gains participation threshold	b	200	$mil/yr
	gains participation fraction	β	50%	-
	project quality (good, medium, bad)	α	$0, 0.5, 1$	-
	project risk (volatility)	σ	$50\%, 20\%, 1\%$	-
	revenue stream	x	$[0, \infty]$	$mil/yr
Waqf	start time	T	10	yrs

Table 2: Expected present values to the construction company V_C, the *sukuk* V_S and the *waqf* V_W in \$ millions, for high, medium and low risk scenarios.

		V	$x_0 = 50$ 1000. in \$ millions	$x_0 = 100$ 2000. in \$ millions	$x_0 = 150$ 3000. in \$ millions	$x_0 = 200$ 4000. in \$ millions	$x_0 = 250$ 5000. in \$ millions
σ	α						
1%	0	C	263.16	382.48	403.39	537.30	734.03
		S	196.73	316.06	326.51	393.47	491.84
		W	540.11	1301.46	2270.10	3069.23	3774.13
	$\frac{1}{2}$	C	263.16	382.48	403.38	488.37	574.51
		S	196.73	316.06	326.51	369.00	412.07
		W	540.11	1301.46	2270.10	3142.63	4013.42
	1	C	263.16	382.48	382.48	382.48	382.48
		S	196.73	316.06	316.06	316.06	316.06
		W	540.11	1301.46	2301.46	3301.46	4301.46
20%	0	C	253.72	378.73	463.27	585.85	753.75
		S	186.73	301.74	354.16	417.14	501.49
		W	559.55	1319.53	2182.56	2997.01	3744.76
	$\frac{1}{2}$	C	253.70	376.20	441.51	509.61	585.82
		S	186.72	300.48	343.28	379.02	417.53
		W	559.58	1323.32	2215.21	3111.37	3996.65
	1	C	252.59	357.60	377.89	381.27	382.08
		S	186.17	291.18	311.47	314.85	315.66
		W	561.24	1351.22	2310.64	3303.89	4302.25
50%	0	C	239.69	388.53	530.20	682.97	850.33
		S	159.69	271.48	359.00	443.94	532.78
		W	600.61	1339.99	2110.80	2873.09	3616.89
	$\frac{1}{2}$	C	234.85	359.39	454.81	540.77	624.15
		S	157.27	256.91	321.30	372.84	419.69
		W	607.88	1383.70	2223.89	3086.39	3956.16
	1	C	212.53	287.28	320.64	337.75	348.08
		S	146.11	220.86	254.21	271.33	281.66
		W	641.36	1491.86	2425.15	3390.91	4370.27

Table 3: Pros and Cons of PPP

Advantages	Disadvantages
• Enlarge the short and medium fiscal space. • Bundling of finance, organizational, and operational contract can reduce agency conflict experienced in traditional infrastructure investment. • Reduce the probability of having low bids, high contract verification, and changes in the government objective during the construction period (political risk). • Assets backed: recourse to SPV's assets and its cash flow. • Reduce coordination costs that would otherwise be faced by intergovernmental departments. • Speedy infrastructure development. • Lower cost overruns, hence better project budgeting. • Higher relative efficiency, including (i) quality of service; (ii) satisfaction of consumers; and (iii) employment productivity. • Transfer of risks of design, construction, operation, and maintenance of the project to the private sector.	• Financing costs might be higher. • The incomplete contract may lead to the private party cutting corners in the project's quality and delivery. • The recourse of lenders is limited as the project's assets are generally public infrastructure that is difficult to liquidate. • Promotes uneven geographies of infrastructure development. • Opportunity cost from the loss of project revenue during the contract period. • High government monitoring cost of the private sector. • Huge long-term obligation to buy-back the assets. • Infrastructure clients may face higher charges without government support. • The bilateral contract may lead to a higher bargaining power of the project sponsor over the government.

Table 4: Total PPP investments across developing Muslim countries ($ millions) (January 1990 – June 2022)

No.	ECA		%	EAPR		%	MENA		%	SSA		%	South Asia		%
1	Albania	3,639	0.79%	**Indonesia**	**78,198**	**16.94%**	Algeria	9,186	1.99%	Benin	1,141	0.25%	Afghanistan	259	0.06%
2	Azerbaijan	889	0.19%	**Malaysia**	**53,766**	**11.65%**	Djibouti	728	0.16%	Burkina Faso	1,055	0.23%	Bangladesh	10,090	2.19%
3	Kazakhstan	5,809	1.26%				Egypt	15,547	3.37%	Cameroon	3,320	0.72%	Maldives	518	0.11%
4	Kyrgyzstan	140	0.03%				Iran	1,881	0.41%	Chad	524	0.11%	**Pakistan**	**33,325**	**7.22%**
5	Tajikistan	966	0.21%				Iraq	3,400	0.74%	Comoros	75	0.02%			
6	**Turkey**	**155,978**	**33.79%**				Jordan	10,563	2.29%	Ivory Coast	5,741	1.24%			
7							Lebanon	383	0.08%	Gabon	2,971	0.64%			
8							**Morocco**	**22,502**	**4.87%**	Gambia	742	0.16%			
9							Syria	519	0.11%	Guinea	1,271	0.28%			
10							Tunisia	4,665	1.01%	Mali	863	0.19%			
11							Palestine	239	0.05%	Mauritania	1,058	0.23%			
12							Yemen	677	0.15%	Mozambique	3,771	0.82%			
13										Niger	19	0.00%			
14										Nigeria	14,385	3.12%			
15										Senegal	5,071	1.10%			
16										Sierra Leone	201	0.04%			
17										Somalia	530	0.11%			
18										Sudan	1,091	0.24%			
19										Togo	1,552	0.34%			
20										Uganda	2,287	0.50%			
Investment		**167,421**	**36.27%**		**131,964**	**28.59%**		**70,290**	**15.23%**		**47,668**	**10.33%**		**44,192**	**9.57%**
Project		**355**			**274**			**470**			**290**			**208**	
Avg. / country		**27,904**			**65,982**			**5,858**			**2,383**			**11,048**	
Avg. / project		**472**			**482**			**150**			**164**			**212**	

This table depicts total PPP investments in developing Muslim countries based on the World Bank Private Participation in Infrastructure (PPI) database. We exclude canceled projects as well as those who reported no total investments. ECA, EAPR, MENA, and SSA respectively stand for Europe and Central Asia, East Asia and Pacific Region, the Middle East and North Africa, and Sub-Saharan Africa.

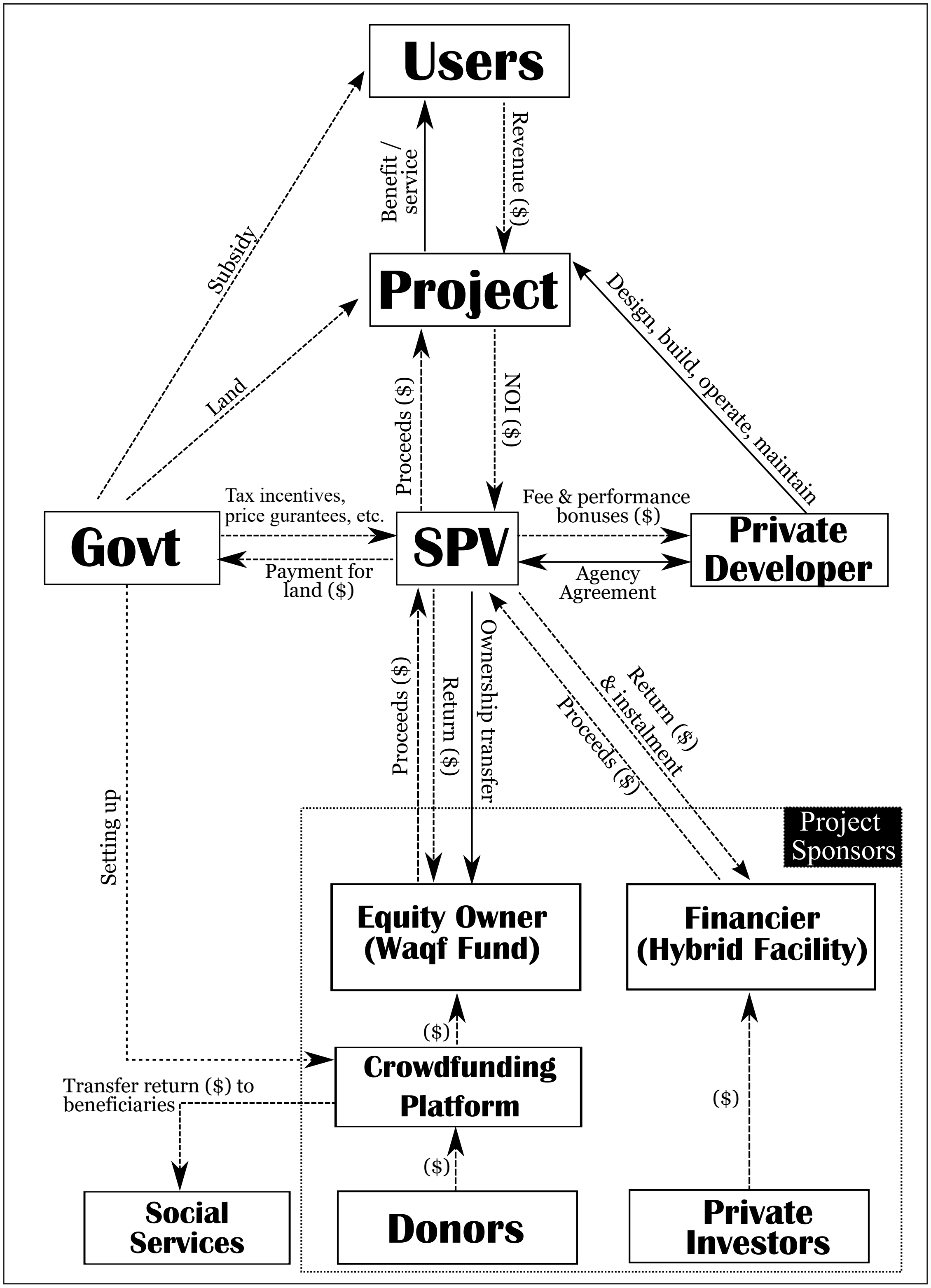

Figure 1: Simplified structure of the blended *sukuk* financing proposed in this paper.

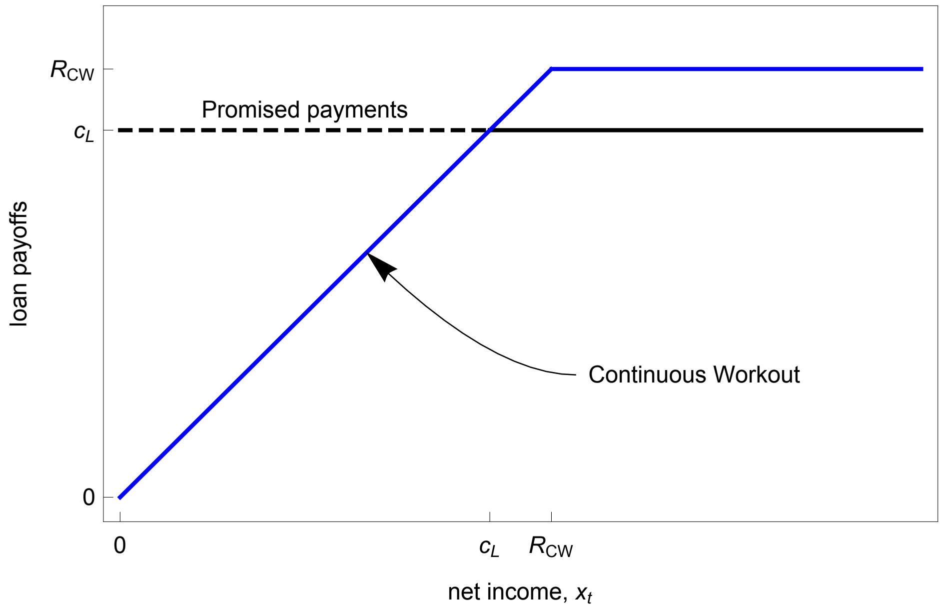

Figure 2: Continuous Workout financing (CW) facility payoffs with no risk-shifting.

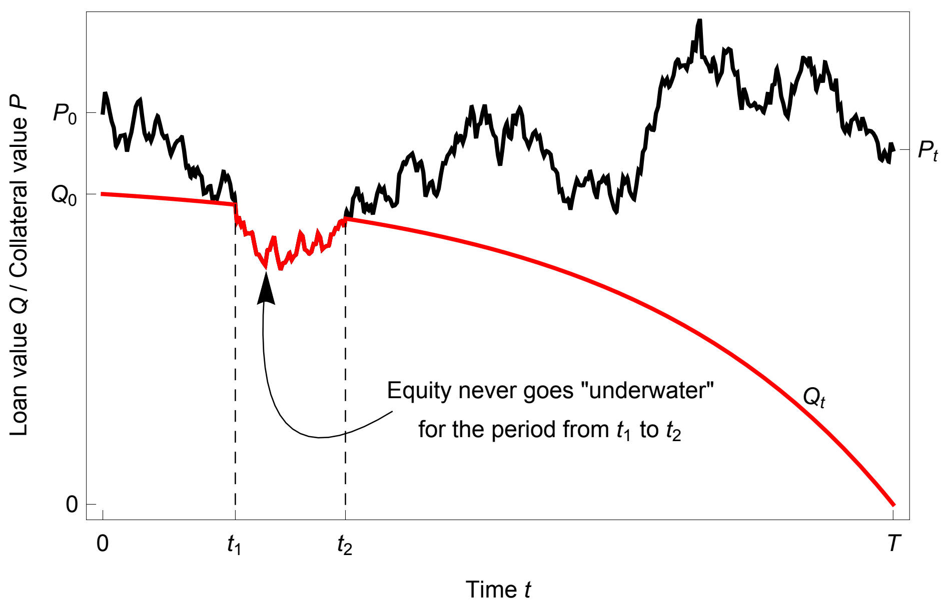

Figure 3: Risk-sharing feature of the Continuous Workout financing advocated by Shiller et al. (2019).

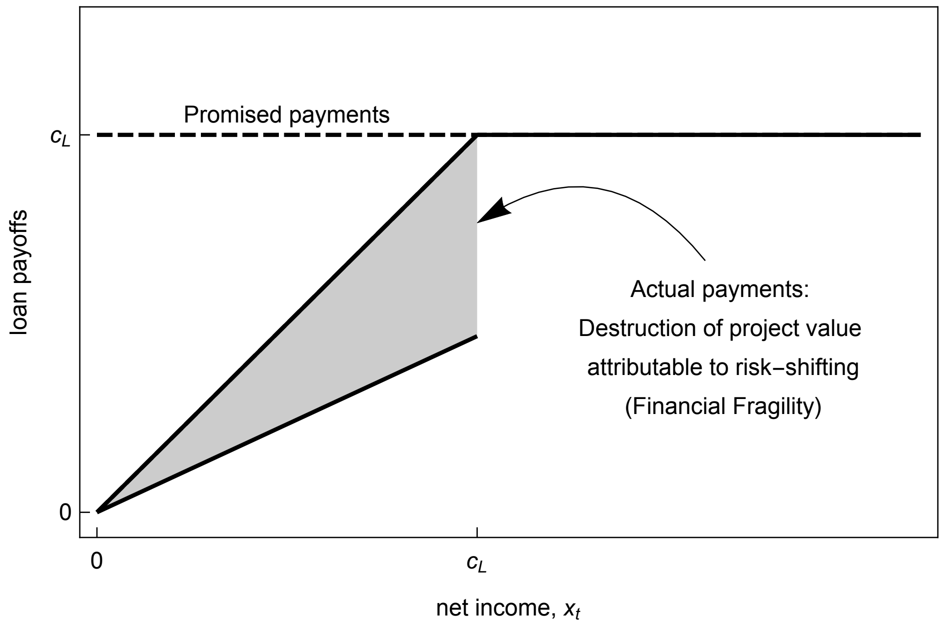

Figure 4: Payoffs for fragile i.e. plain-vanilla, risky loan.

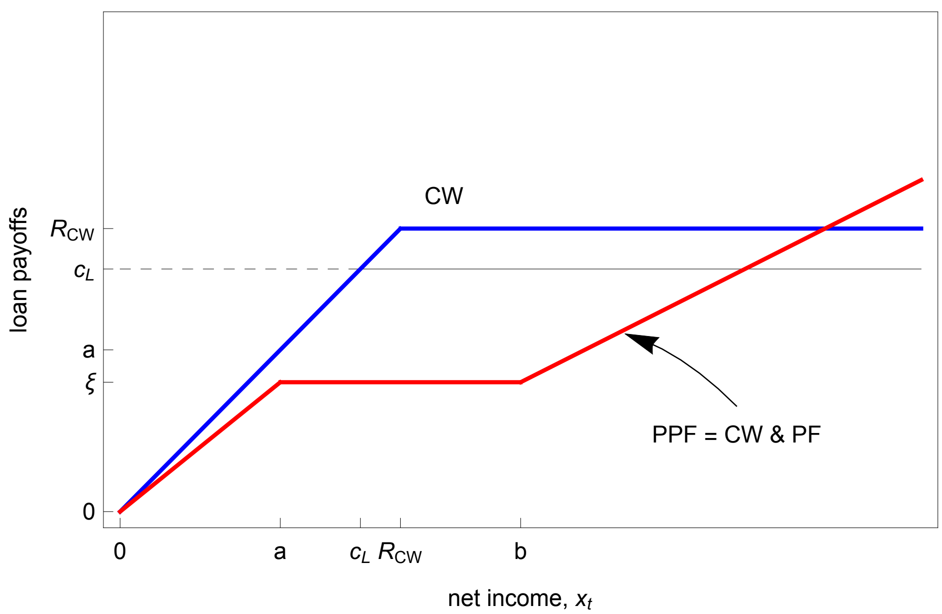

Figure 5: Comparing a PPF facility with a CW.

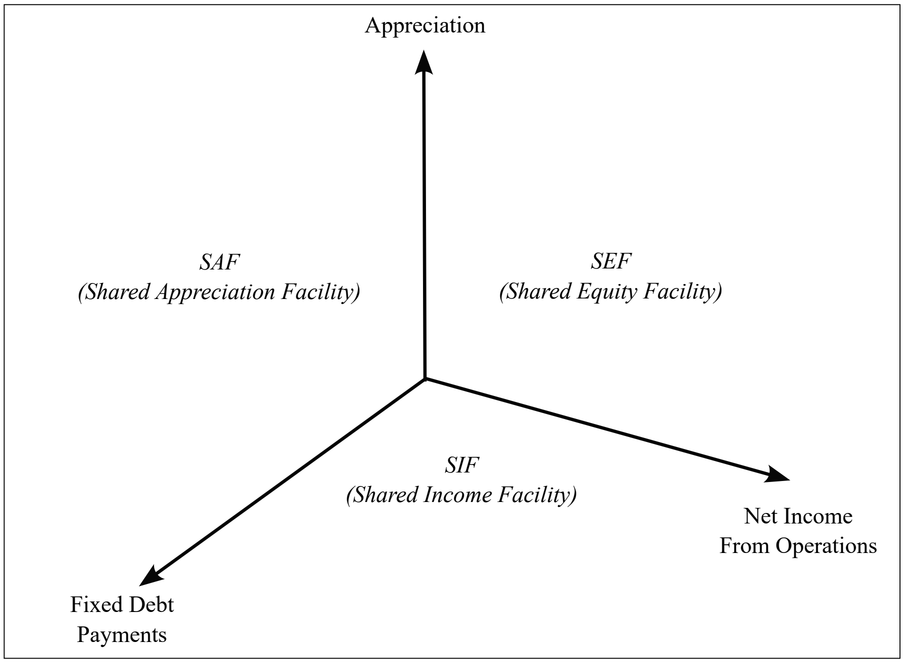

Figure 6: Participating Financing (PF) of Ebrahim et al. (2011): sharing income, appreciation and equity.

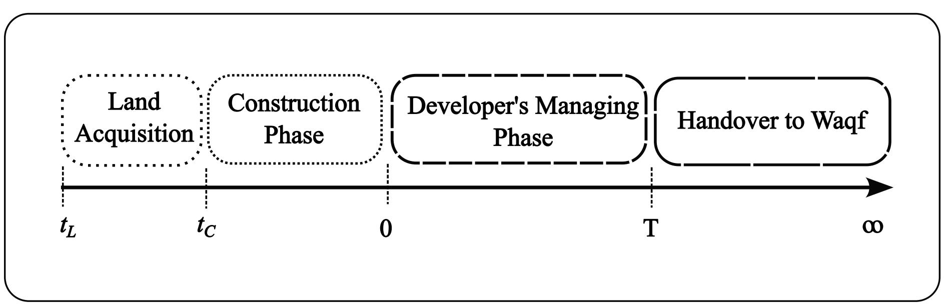

Figure 7: Infrastructure timeline

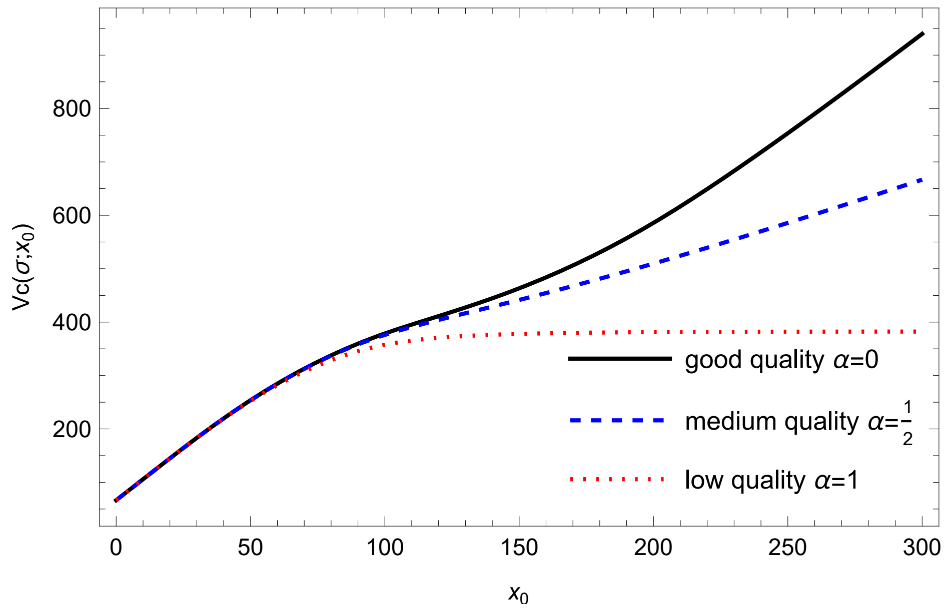

Figure 8: Value of the contract to the developer for a medium risk $\sigma = 20\%$ project with high, medium and low quality of execution i.e. low, medium and high penalty $\alpha = 0, \frac{1}{2}, 1$.

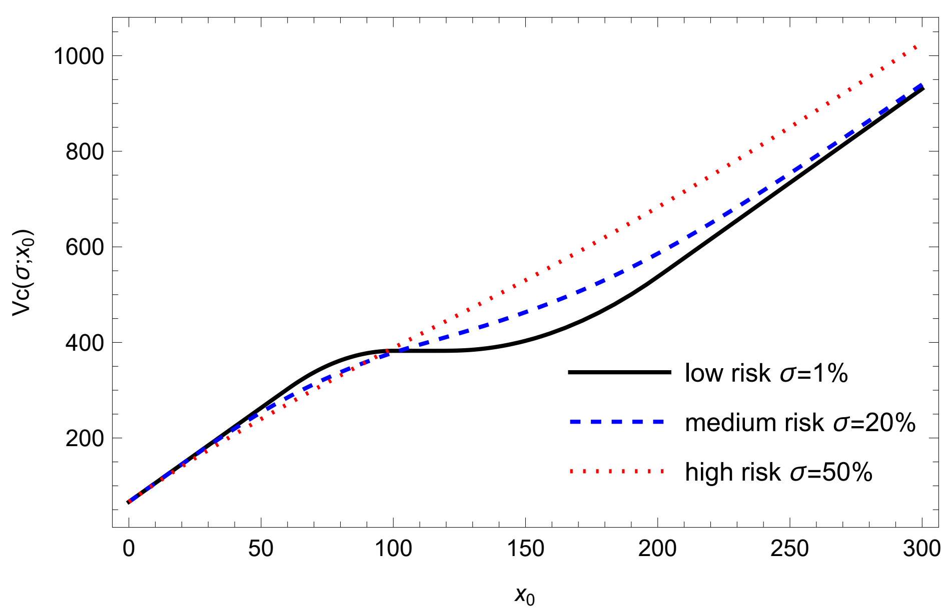

Figure 9: Value of a medium quality project to the developer-operator, i.e. medium penalty $\alpha = \frac{1}{2}$, for low, medium and high risks: $\sigma = 1\%, 20\%$ and 50%, respectively.

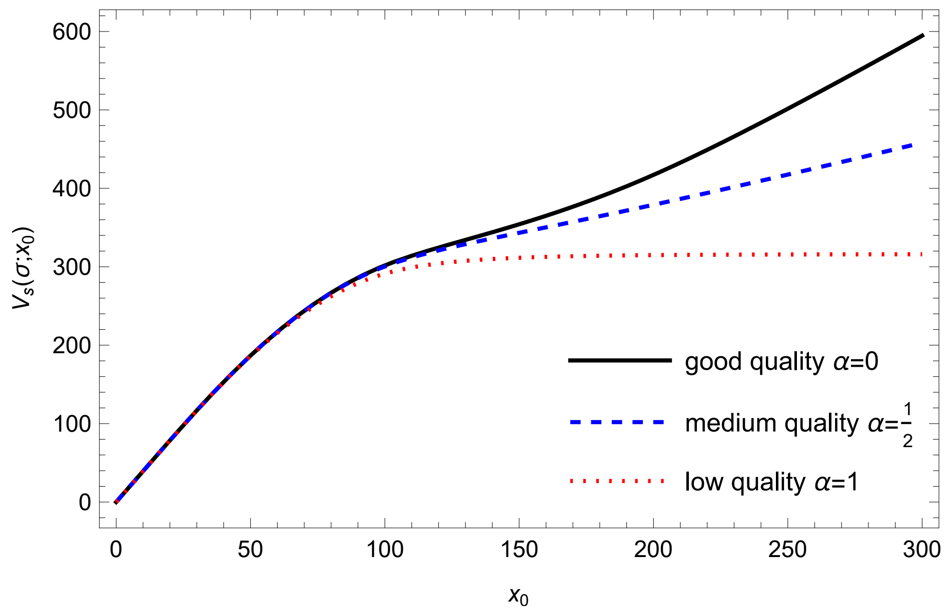

Figure 10: Value of the contract to the *sukuk* for a medium risk $\sigma = 20\%$ project with high, medium and low quality of execution i.e. low, medium and high penalty $\alpha = 0, \frac{1}{2}, 1$ served upon the developer.

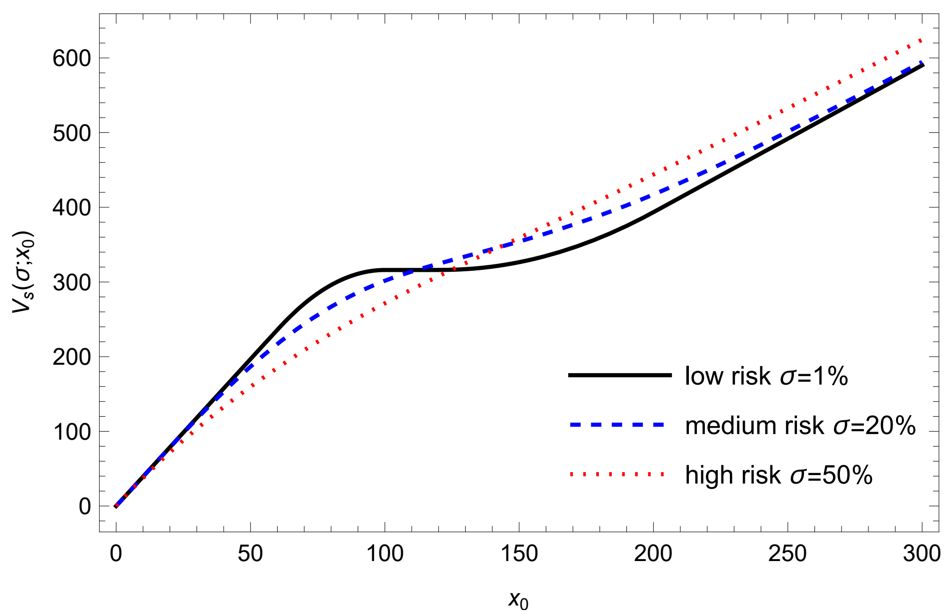

Figure 11: Value of a medium quality project to the *sukuk*, i.e. medium penalty $\alpha = \frac{1}{2}$, for low, medium and high risks: $\sigma = 1\%, 20\%$ and 50%, respectively.

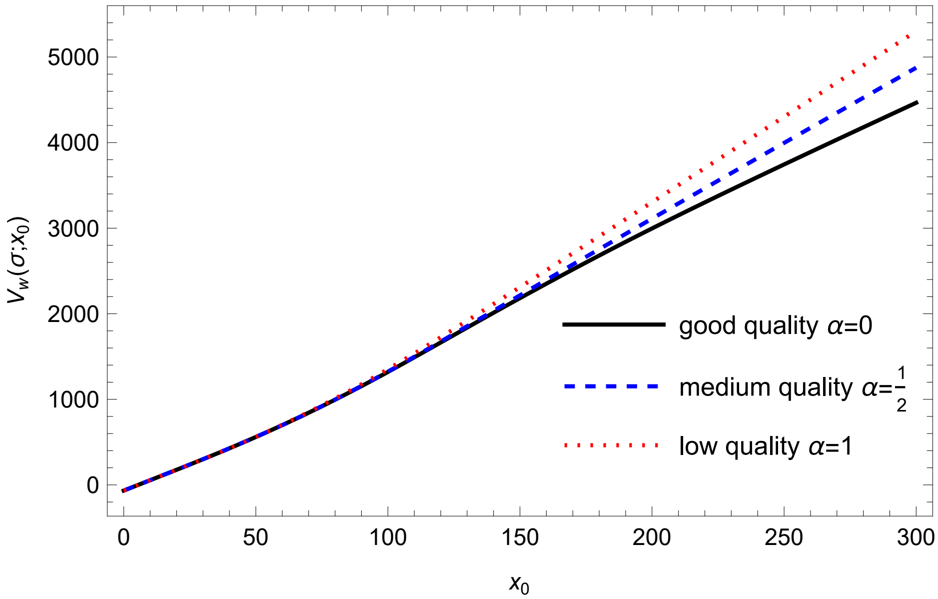

Figure 12: Value of the contract to the *waqf* for a medium risk $\sigma = 20\%$ project with high, medium and low quality of execution i.e. low, medium and high penalty $\alpha = 0, \frac{1}{2}, 1$ served upon the developer.

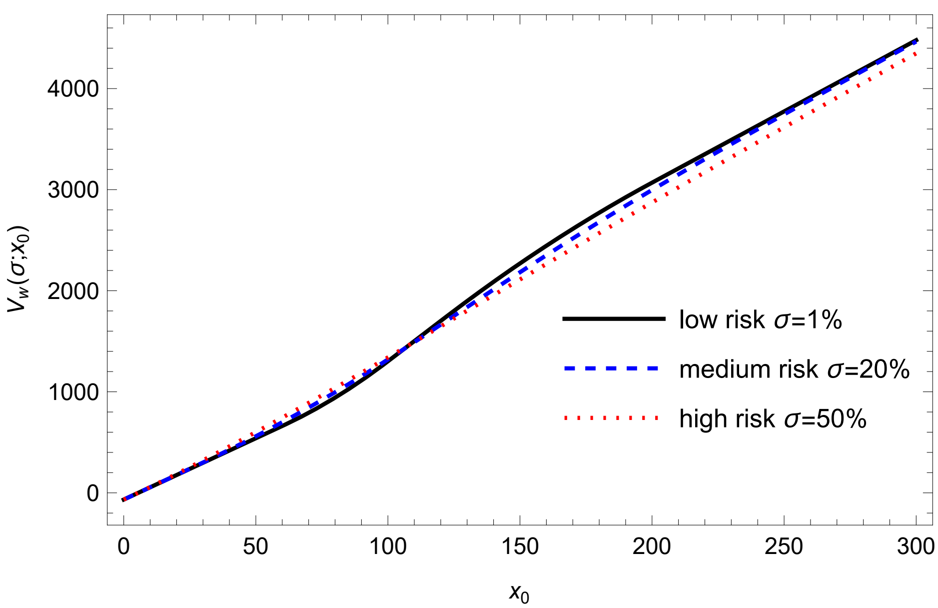

Figure 13: Value of a medium quality project to the *waqf*, i.e. medium penalty $\alpha = \frac{1}{2}$, for low, medium and high risks: $\sigma = 1\%, 20\%$ and 50%, respectively.

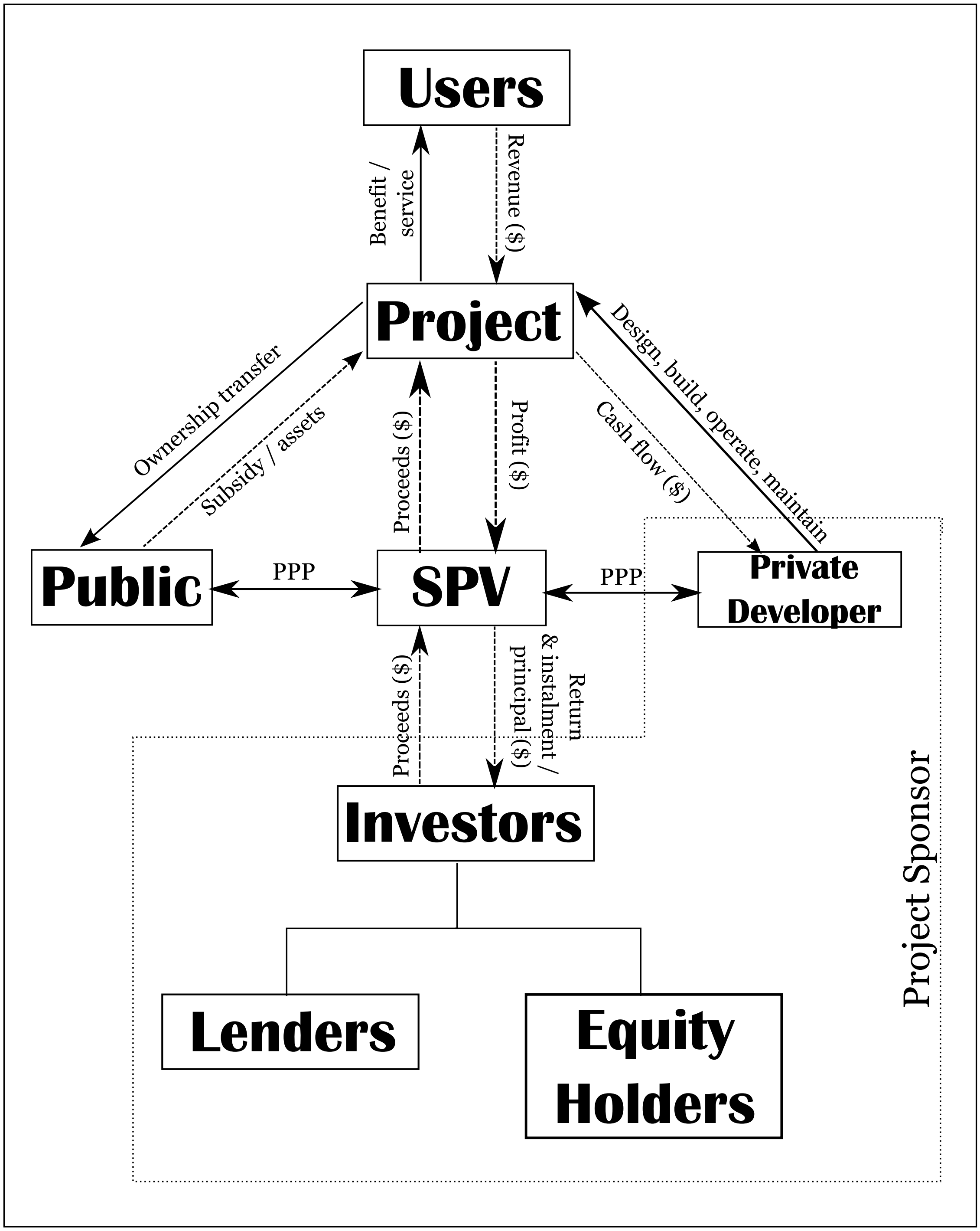

Figure 14: Simplified structure of a Private-Public Partnership (PPP).