Add-On Service Quality and Preemption with Non-Savvy Consumers: A Real Options Approach*

Kohei Daido
School of Economics, Kwansei Gakuin University

Keizo Mizuno†
School of Business Administration, Kwansei Gakuin University

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Abstract

This paper examines the impact of the presence of non-savvy consumers on firms' entry incentives in durable goods markets. In the durable goods markets, firms provide vertically-differentiated add-on services to consumers. We firstly show that the price of a durable good with high quality add-on services is lower (resp. higher) than that with low quality add-on services when the proportion of non-savvy consumers in the population is high (resp. low). We then show that a firm that supplies a durable good with high quality add-on services enters earlier than that with low quality add-on services, irrespective of the proportion of non-savvy consumers. In addition, when the proportion of non-savvy consumers is high, each firm has a preemptive incentive such that the entry timing of a firm with high quality add-on services into the market becomes earlier than that when savvy consumers are prevalent.

Keywords: durable goods, add-on service, non-savvy consumers, entry timing, preemption.

JEL classification: L22, L24, M21, O31.

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†Corresponding author: Keizo Mizuno, School of Business Administration, Kwansei Gakuin University, 1-1-155 Uegahara, Nishinomiya, Hyogo 662-8501, Japan. Phone: +81-798-54-6181. E-mail: kmizuno@kwansei.ac.jp

1 Introduction

In durable goods markets such as cars, copy machines, and computers, consumers obtain the benefits not only from the durable products themselves but also from the add-on services such as maintenance or repair services and spare parts provision. It is then well-known that a part of consumers in the population do not fully recognize the benefits from the add-on services when purchasing the durable products. In the literature of behavioral economics, this type of consumers are called *non-savvy* (or *naive*) consumers, whereas the other type of consumers are called *savvy* (or *sophisticated*) consumers. This paper examines the impact of the presence of non-savvy consumers on firms' entry incentives in durable goods markets when the add-on services are vertically differentiated.

In our model, two firms consider entering a market for durable goods. These firms supply core goods and add-on services such as maintenance services and spare parts provision. While the core goods are horizontally differentiated a la Hotelling market, the add-on services are vertically differentiated between the two firms. We consider two types of consumers: savvy and non-savvy. Savvy consumers recognize the necessity for add-on services after purchasing a core good and also understand the quality of those services. On the other hand, non-savvy consumers neither understand the need for such services nor know their quality. After examining the impact of existing non-savvy consumers on price setting by both firms, we analyze the firms' timing decisions for entry into the durable goods market in a real options model a la Dixit and Pindyck (1994).

The main findings obtained in this paper are the followings. First of all, as a preliminary result, we describe the prices of a durable good and its add-on service set by firms. Specifically, we show that the price of a durable good with high quality add-on services is lower (resp. higher) than that with low quality add-on services when the proportion of non-savvy consumers in the population is high (resp. low). In addition, we also verify that the price of add-on services is an increasing function of the proportion of non-savvy consumers in the population, which is a well-known result called "exploitation by firms" in the behavioral economics literature.

We then characterize the entry equilibria in the durable goods market. At first, we verify that a firm that supplies a durable good with high quality add-on services enters earlier than that with low quality add-on services, irrespective of the proportion of non-savvy consumers. Then, two types of equilibrium are distinguished: One type of equilibrium is a *preemptive equilibrium*, while the other is a *non-preemptive equilibrium*. It is shown that when the proportion of non-savvy consumers is high, a preemptive equilibrium occurs in which both firms have a preemptive incentive such that the entry timing of a leader (i.e., a firm with high quality add-on services) into the market becomes earlier than that when savvy consumers are prevalent. On the contrary, when the proportion of non-savvy consumers is low, a non-preemptive equilibrium occurs in which one of the firms does not have an preemptive incentive into the market. A remarkable feature of this equilibrium configulation is that the occurence of some type of entry equilibrium is affected by the proportion of non-savvy consumers in the population.

Our findings provide a policy implication regarding the entry delay in durable goods markets. There are several factors that delay entry into durable goods markets. A well-known technological factor is a large sunk investment cost for producing durable products. However, from a policy viewpoint, a more serious factor that delays (or deters) entry is an incumbent's anticompeitive practices. In durable goods markets, a prevalent anticompetitive practice is tying (or bundling) between a durable product and its after-service such as maintenance or repair service and spare parts provision. However, our main finding suggests that consumers' attitude towards the add-on (or after) services is also a crucial factor that affects a firm's entry incentive in the markets. Specifically, when the add-on services are vertically differentiated, consumers' inattention to add-on services can encourage firms' entry. On the contrary, as consumers get acquainted with the quality difference of add-on services between firms, a firm's entry tends to be delayed. Therefore, consumers education for learning the importance of add-on services has a trade-off in the sense that consumers can realize their benefits from the add-on services of durable products, whereas firms' entry into the markets may be discouraged.

This study belongs to two strands of literature, i.e., behavioral industrial organization and real options analysis. How the presence of non-savvy consumers in the population yields an externality to savvy consumers is one of the main issues in the behavioral industrial organization literature. Armstrong (2015) and Heidhues and Köszegi (2018) give an extensive survey on the interaction between savvy consumers and non-savvy consumers. Shulman and Geng (2013) consider the competitive environment where two firms provide both horizontally and vertically differentiated products to boundedly rational consumers and examine the effect of the presence of boundedly rational consumers on the firms' profits. Li et al. (2014) examines firms' incentive for information disclosure in a vetically differentiated duopoly with unaware consumers. The impact of the presence of non-savvy consumers on firms' entry incentives, however, is not addressed in these studies.

The entry or investment timing by firms in various stochastic environments is analyzed in the real options literature. See Dixit and Pindyck (1994) for a systematic treatment of the real options approach and Smit and Trigeorgis (2004) for real options analyses in the game-theoretic environments. Our study focuses on the impact of the interaction between savvy and non-savvy consumers on firms' entry timing in durable goods markets. In other words, we apply the real options approach to the behavioral industrial organization literature and show that the behavioral aspects of consumers matter for the industry studies from a dynamic perspective.

Needless to say, the anticompetitive literature on durable goods markets also relates to our study. In particular, Whinston (1990) shows in his seminal paper that tying (or bundling) is an effective tool to deter entry, called the *leverage theory*. Carlton and Waldman (2002) extend the leverage theory of tying to a dynamic perspective, which is relevant to the relationship between a durable good and its after services. Choi and Stefanadis (2001) show that tying is effective for entry deterrence when the practice of research and development (R&D) investment affects the probability of success in a sequential R&D investment in a related market. These existing literature, however, does not pay attention to consumers' attitudes towards the after services in durable goods markets.

The remainder of this paper is structured as follows. Section 2 presents the model. Section 3 derives firms' profit flows in a duopoly and in a monopoly. Section 4 describes the entry equilibria and shows how the proportion of non-savvy consumers affects the characterization of entry equilibria. Section 5 offers concluding remarks.

2 The Model

Two firms, firm l and firm h, want to enter into a durable goods market. Durable goods provided by two firms, called *core goods* hereafter, are horizontally differentiated. Due to the durability of goods, consumers need an *add-on service* (or product) such as maintenance or repair services and spare parts provision. We assume that the add-on services are vertically differentiated between the two firms: firm l provides low quality services, while firm h provides high quality services.

We consider a continuous time model. We assume that core goods provided by two firms are horizontally differentiated à la Hotelling market. At each time, consumers are generated and distributed uniformly on the unit interval of $[0,\ 1]$. After entering the durable goods market at any time, firm l is located at the extreme point 0, whereas firm h is located at the extreme point 1 of the unit interval. Each consumer is assumed to buy one unit of core goods. A consumer located at x ($\in [0,\ 1]$) obtains the basic willingness to pay, R, from a core good and incurs a disutility from travelling to buy the core good of firm l (resp. firm h), denoted by tx (resp. $t\,(1-x)$).

After the purchase of a core good, a consumer obtains an add-on service from the firm that he/she buys that core good. At the price p of the add-on services provided by firm l (resp. firm h), a consumer demands $q_l\,(p)$ (resp. $q_h\,(p)$, respectively) and gains the consumer surplus of $s_l(p) \equiv \int_p q_l(x)\,dx$ (resp. $s_h(p) \equiv \int_p q_h(x)\,dx$) from the add-on services. Because the add-on services are vertically differentiated, we assume that for any p, $q_l(p) < q_h(p)$ and $s_l(p) < s_h(p)$. To derive a firm's profit flow explicitly in the analysis below, we use a linear demand representation for add-on services, i.e., $q_l(p) = a_l - p$ and $q_h(p) = a_h - p$, where $a_l < a_h$.

There are two kinds of consumers, *savvy* and *non-savvy*. The proportion of savvy (resp. non-savvy) consumers in the population is θ (resp. $1-\theta$).[1] We assume that a savvy consumer recognizes the existence and quality of add-on services, whereas a non-savvy consumer does not do so. On the other hand, all consumers recognize the core goods and their basic willingness to pay, R.

Denoting the price of firm i's core good by P_i $(i = l,\ h)$, the indirect utility of a consumer located at x is represented as follows.

$$\begin{aligned} U_l(x) &= R - P_l + s_l(p_l) - tx \text{ when buying from firm } l, \\ U_h(x) &= R - P_h + s_h(p_h) - t(1-x) \text{ when buying from firm } h. \end{aligned}$$

We should note that a non-savvy consumer makes a purchase decision of a core good without recognizing the existence and quality of the add-on services derived from that core good.

Each firm's unit production cost for core goods is constant and identical between the two firms, denoted by C. For analytical simplicity, we assume that the unit production cost for add-on services is also identical, i.e., $c_l = c_h \equiv c$, irrespective of their quality difference.[2]

Firm i's profit flow at each time, φ_{it}, consists of two parts, i.e., $\varphi_{it} \equiv Y_t \Pi_i$ $(i = l,\ h)$, where Y_t represents a stochastic part of profit, while Π_i represents its deterministic part.

[1] Two firms do not know which consumers are savvy or non-savvy in the unit line. Thus, θ is interpreted as the probability that each consumer is savvy.

[2] Even when we introduce cost differentiation between the two firms such as $c_l < c_h$, the qualitative results derived from our analysis does not change as long as $a_l - c_l < a_h - c_h$ holds.

Π_i depends on the market structure, which takes Π_i^m (resp. Π_i^d) when firm i is a monopoly (resp. a duopoly).

We interpret Y_t as the industry-wide shock and it follows a geometric Brownian motion:

$$dY_t = \alpha Y_t dt + \sigma Y_t dZ_t, \quad \text{where } dZ_t \sim N(0,\ dt).$$

Here, α (> 0) is a drift parameter that represents the industry-wide growth, σ is an instantaneous standard deviation parameter, and Z_t is the Wiener increment that is normally distributed with mean zero and variance dt.

For analytical simplicity, we assume that the entry cost is also identical between the two firms, denoted by I.

3 Derivation of Profit Flows

In this section, we derive the deterministic part of a firm's profit flow Π_i.

3.1 Duopoly

First of all, we obtain the profit flow when the market is duopoly. A savvy consumer (resp. a non-savvy consumer) that is indifferent between firms l and h, denoted by $\widehat{x}$ (resp. $\widehat{\widehat{x}}$), is represented by

$$\begin{aligned} \widehat{x} &= \frac{1}{2} + \frac{1}{2t}\left[(s_l(p_l) - P_l) - (s_h(p_h) - P_h)\right], \\ \widehat{\widehat{x}} &= \frac{1}{2} + \frac{1}{2t}(P_h - P_l). \end{aligned}$$

Then, firm l's price-decision problem is the following.

$$\max_{P_l,\ p_l} \Pi_l^d \equiv (P_l - C + \pi_l(p_l))\left[\theta\widehat{x} + (1-\theta)\widehat{\widehat{x}}\right],$$

where $\pi_l(p_l) \equiv (p_l - c) q_l(p_l)$. From the two first-order conditions, we obtain the followings.

$$(p_l - c) q_l'(p_l) + (1-\theta) q_l(p_l) = 0, \quad (1)$$

$$P_l - C + \pi_l(p_l) = 2t\left[\theta \widehat{x} + (1-\theta)\widehat{\widehat{x}}\right] \quad (2)$$

From (1), we realize that the price of add-on services p_l depends only on the proportion of savvy consumers θ and the quality of add-on services by firm l, despite the price competition in duopoly. In particular, as θ increases, p_l monotonically decreases. More specifically, at $\theta = 0$, the price of add-on services becomes a monopoly price, i.e., $p_l = p_i^M (\equiv \arg\max \pi_l(p_l))$. Remembering the linear demand specification for add-on services, $q_l(p) = a_l - p$, we obtain $p_l^M = (a_l + c)/2$. On the contrary, at $\theta = 1$, it is a competitive (or efficient) price, i.e., $p_l = c$. This property is generated due to the fact that non-savvy consumers' utility from the add-on services are exploited by the firm that provides a core good they originally purchase ("*exploitation*" by a firm). On the other hand, the price of the core good P_l is affected by the rival firm's pricing decision, P_h and p_h, through the competition for grabbing market demands.

Similarly, firm h's price decision problem is represented by

$$\max_{P_h,\, p_h} \Pi_h^d \equiv (P_h - C + \pi_h(p_h))\left[\theta(1-\widehat{x}) + (1-\theta)\left(1-\widehat{\widehat{x}}\right)\right],$$

where $\pi_h(p_h) \equiv (p_h - c) q_h(p_h)$. From the two first-order conditions, we obtain the followings.

$$(p_h - c) q_h'(p_h) + (1-\theta) q_h(p_h) = 0, \quad (3)$$

$$P_h - C + \pi_h(p_h) = 2t\left[\theta(1-\widehat{x}) + (1-\theta)\left(1-\widehat{\widehat{x}}\right)\right] \quad (4)$$

(3) and (4) have exactly the same characteristics as those in (1) and (2), respectively.

From (1) and (3), we firstly obtain the equilibrium prices of add-on services as follows.

$$p_l^* = \frac{(1-\theta)\,a_l + c}{2-\theta}, \;\; p_h^* = \frac{(1-\theta)\,a_h + c}{2-\theta}.$$

Because $a_l < a_h$, we verify that $p_l^* < p_h^*$. That is, the price of high quality add-on services is higher than that of low quality add-on services, irrespective of the proportion of savvy consumers in the population.

We next derive the prices of core goods. Applying p_l^* and p_h^* to (2) and (4) and arranging them, we obtain the equilibrium prices of core goods as follows.

$$\begin{aligned} P_l^* &= t + C - \frac{1}{6}\left((4-5\theta)\,A_l(\theta) + (2-\theta)\,A_h(\theta)\right), \\ P_h^* &= t + C - \frac{1}{6}\left((2-\theta)\,A_l(\theta) + (4-5\theta)\,A_h(\theta)\right), \\ \text{where } A_j(\theta) &\equiv \left(\frac{a_j - c}{2-\theta}\right)^2, \;\; j = l,\ h. \end{aligned}$$

Substituting the equilibrium prices into a firm's profit, we obtain the equilibrium profits of firm j $(= l,\ h)$, Π_j^{d*}, as follows.

$$\Pi_l^{d*} = \frac{1}{2t}\left(t - \frac{(a_h - c)^2 - (a_l - c)^2}{6(2-\theta)}\right)^2 \equiv \Pi_l^{d*}(\theta), \tag{5}$$

$$\Pi_h^{d*} = \frac{1}{2t}\left(t + \frac{(a_h - c)^2 - (a_l - c)^2}{6(2-\theta)}\right)^2 \equiv \Pi_h^{d*}(\theta). \tag{6}$$

Differentiating (5) and (6) with respect to θ, we obtain the effect of the change in the proportion of savvy consumers on the profit flows of firms l and h in the followings. See also Figure 1.

$$\frac{d\Pi_l^{d*}(\theta)}{d\theta} < 0, \text{ and } \frac{d\Pi_h^{d*}(\theta)}{d\theta} > 0.$$

(Insert Figure 1 around here.)

To understand the effect of the change in the proportion of savvy consumers on firms' profit flows, we report its effect on the prices of add-on services and core goods.

$$
\begin{aligned}
\frac{dp_l^*}{d\theta} &< 0, \ \frac{dp_h^*}{d\theta} < 0, \\
\frac{dP_l^*}{d\theta} &= -\frac{1}{6(2-\theta)^3}\left[-(2+5\theta)(a_l-c)^2+(2-\theta)(a_h-c)^2\right] \gtreqless 0, \\
\frac{dP_h^*}{d\theta} &= -\frac{1}{6(2-\theta)^3}\left[(2-\theta)(a_l-c)^2-(2+5\theta)(a_h-c)^2\right] > 0.
\end{aligned}
$$

We also report the equilibrium demands for firm l's good.

$$
\begin{aligned}
\widehat{x}^* &= \frac{1}{2}-\frac{5-4\theta}{12t}\left(A_h(\theta)-A_l(\theta)\right), \\
\widehat{\widehat{x}}^* &= \frac{1}{2}+\frac{2\theta-1}{12t}\left(A_h(\theta)-A_l(\theta)\right), \\
D_l^* &\equiv \theta\widehat{x}^*+(1-\theta)\widehat{\widehat{x}}^* \\
&= \frac{1}{2}-\frac{2-\theta}{12t}\left(A_h(\theta)-A_l(\theta)\right) \qquad (7)
\end{aligned}
$$

From (7), we verify that $D_l^* < \frac{1}{2}$ for any $\theta \in [0, 1]$. That is, the market demand for firm h's good is larger than that for firm l's good, irrespective of the proportion of savvy consumers θ. In addition, we obtain

$$
\frac{\partial D_l^*}{\partial\theta} = -\frac{1}{12t}\left(A_h(\theta)-A_l(\theta)\right) < 0.
$$

Thus, as the proportion of savvy consumers increases, the market demand for firm l's good decreases.

Using these properties, we obtain the intuitive explanation of the results in Figure 1. To begin with, as a benchmark, we consider the case in which all consumers are savvy, i.e., $\theta = 1$. In this case, all consumers recognize that each firm's durable good be considered as a combination of a core good and its add-on services, and that firm h's add-on services are better than firm l's. Hence, due to the provision of high quality of add-on services, firm h can attract more savvy consumers than firm l (i.e., $D_l^* < \frac{1}{2}$) even though it sets a higher core price than firm l ($P_h^* < P_l^*$), with the competitive prices of two firms' add-on services (i.e., $p_h^* = p_l^* = c$).

On the contrary, suppose $\theta = 0$, i.e., the case in which all consumers are non-savvy. In this case, all consumers do not recognize the existence and quality of add-on services. From a firm's point of view, this is a chance to set a monopoly price on its add-on services (see (1) and (3)). That is, firms can exploit consumers' benefits by providing their add-on services. Then, firm h can set a higher monopoly price and obtain higher monopoly profit from its add-on services than firm l. Due to this higher monopoly profit from add-on services, firm h has a higher incentive to attract non-savvy consumers. Hence, firm h sets a lower price of its core good and obtains a larger demand and a higher profit than firm l; $P_h^* < P_l^*$, $D_l^* < \frac{1}{2}$, and $\Pi_h^{d*}(0) > \Pi_l^{d*}(0)$ at $\theta = 0$.

As θ increases from $\theta = 0$, a part of consumers recognize the existence and quality of add-on services. In that situation, firm h's profitable strategy is to increase the price of its core good and to decrease the price of its add-on services, because the price of its core good is lower than that of firm l and the core good is recognized by all consumers. At the same time, the price of firm l's core good increase due to the property of strategic complements.

As θ increases further, savvy consumers still accept a higher price of firm h's core good than that of firm l's, because its core good exhibits high quality and the price of firm h's add-on services is low enough to compensate for the high price of its core good. In fact, as mentioned earlier, at $\theta = 1$, although $P_h^* > P_l^*$, firm h can still attract a larger share of savvy consumers than firm l (i.e., $D_l^* < \frac{1}{2}$) with the competitive prices of two firms' add-on services (i.e., $p_h^* = p_l^* = c$).

An interesting point is that the relative magnitude of the equilibrium prices of core goods depends on the proportion of savvy (or non-savvy) consumers in the population. As a proposition, we report this property in addition to that of add-on services.

Proposition 1 *(i) When the proportion of non-savvy consumers in the population is high*

(resp. low), the price of core good provided by a firm with high quality add-on services is lower (resp. higher) than that provided by a firm with low quality add-on services.

(ii) As the proportion of savvy consumers in the population increases, the price of add-on services decreases. Specifically, the add-on price is the monopoly price when all consumers are non-savvy, whereas it is the competitive (or efficient) price when all consumers are savvy.

3.2 Monopoly

We assume that in the case of monopoly, a firm that enters the durable goods market locates at the extreme point 0 of the unit interval despite the quality of add-on services.

When a consumer located at x buys firm i's core good, his/her indirect utility is represented by

$$U_i(x) = R - P_i + s_i(p_i) - tx, \quad i = l,\ h.$$

Assuming that the firm can maximize its profit when it offers prices such that all consumers buy its good, its profit-maximization problem is formulated as follows.

$$\begin{aligned} \max_{P_i,\ p_i} \Pi_i^m &\equiv P_i - C + \pi_i(p_i) \\ s.t.\ \ R - P_i + s_i(p_i) - t &\geq 0, \\ R - P_i - t &\geq 0, \quad i = l,\ h. \end{aligned}$$

It is apparent that only the participation constraint for non-savvy consumers is binding. Then, the prices set by a firm, $\{P_i^{m*},\ p_i^{m*}\}$, are derived as follows.

$$\begin{aligned} P_i^{m*} &= R - t, \\ p_i^{m*} &= \frac{a_i + c}{2} \equiv p_i^M \left(= \arg\max \pi_i(p_i)\right). \end{aligned}$$

That is, the price of core goods are the same between the firms despite the quality difference of their add-on services.

Substituting these prices into firm i's monopoly profit, the equilibrium profit Π_i^{m*} is represented as follows.

$$\Pi_i^{m*} = R - t - C + \pi_i \left(p_i^M\right). \tag{8}$$

From (8), we understand that the equilibrium monopoly profit Π_i^{m*} is not affected by the proportion of savvy consumers θ. In addition, $\Pi_l^{m*} < \Pi_h^{m*}$.

4 Entry Equilibria

We now turn to the analysis of entry-timing decisions by the two firms.[3]

4.1 Value functions

As preliminary results, we derive value functions when each of the two firms becomes a follower or a leader.

4.1.1 Follower

When firm l becomes a follower, its value function is represented as follows.

$$V_l^F(Y) = \begin{cases} \Psi_l Y^{\beta} & \text{if } Y \leq Y_l^{F*} \\ \frac{Y \Pi_l^{d*}(\theta)}{r-\alpha} - I & \text{if } Y > Y_l^{F*} \end{cases} \tag{9}$$

where

$$\begin{aligned} \Psi_l &\equiv \left(Y_l^{F*}\right)^{-\beta} \left[\frac{Y_l^{F*} \Pi_l^{d*}(\theta)}{r-\alpha} - I\right], \\ \beta &= \frac{1}{2}\left(1 - \frac{2\alpha}{\sigma^2} + \sqrt{\left(1 - \frac{2\alpha}{\sigma^2}\right)^2 + \frac{8r}{\sigma^2}}\right). \end{aligned}$$

[3] The analysis of this section is based on Dixit and Pindyck (1994) and Pawlina and Kort (2006).

Here, Y_l^{F*} is the threshold of entry timing represented by the level of industry-wide shock Y. Indeed, Y_l^{F*} is characterized by

$$Y_l^{F*} = \frac{\beta}{\beta - 1} \frac{r - \alpha}{\Pi_l^{d*}(\theta)} I \tag{10}$$

Similarly, when firm l becomes a follower, its value function is the following.

$$V_h^F(Y) = \begin{cases} \Psi_h Y^\beta & \text{if } Y \leq Y_h^{F*} \\ \frac{Y \Pi_h^{d*}(\theta)}{r-\alpha} - I & \text{if } Y > Y_h^{F*} \end{cases} \tag{11}$$

where

$$\Psi_h \equiv \left(Y_h^{F*}\right)^{-\beta} \left[\frac{Y_h^{F*} \Pi_h^{d*}(\theta)}{r - \alpha} - I\right].$$

Also, Y_h^{F*} is the threshold of firm h's entry timing as a follower, and it is characterized by

$$Y_h^{F*} = \frac{\beta}{\beta - 1} \frac{r - \alpha}{\Pi_h^{d*}(\theta)} I \tag{12}$$

From (10) and (12), we verify that for any $\theta \in [0,\ 1]$, $Y_h^{F*} < Y_l^{F*}$ because $\Pi_l^{d*}(\theta) < \Pi_h^{d*}(\theta)$. That is, as a follower, firm h always enters the market earlier than firm l.

4.1.2 Leader

Next, we characterize a value function when each of the two firms becomes a leader.

Suppose that a firm has an incentive to preempt the market. Then, firm l's value function as a leader is the following.

$$V_l^L(Y) = \begin{cases} \frac{Y \Pi_l^{m*}}{r-\alpha} - I - \frac{Y_h^{F*}\left(\Pi_l^{m*} - \Pi_l^{d*}(\theta)\right)}{r-\alpha} \left(\frac{Y}{Y_h^{F*}}\right)^\beta & \text{if } Y \leq Y_h^{F*} \\ \frac{Y \Pi_l^{d*}(\theta)}{r-\alpha} - I & \text{if } Y > Y_h^{F*} \end{cases} \tag{13}$$

Note that when $Y \leq Y_h^{F*}$, $V_l^L(Y)$ is a concave function of Y. The entry timing as a preemptive leader is discussed in the analysis of the next section.

Similarly, firm h's value function as a leader is the following.

$$V_h^L(Y) = \begin{cases} \frac{Y\Pi_h^{m*}}{r-\alpha} - I - \frac{Y_l^{F*}\left(\Pi_h^{m*}-\Pi_h^{d*}(\theta)\right)}{r-\alpha}\left(\frac{Y}{Y_l^{F*}}\right)^{\beta} & \text{if } Y \leq Y_l^{F*} \\ \frac{Y\Pi_h^{d*}(\theta)}{r-\alpha} - I \quad \text{if} \quad Y > Y_l^{F*} & \end{cases} \tag{14}$$

As in the case of firm l, $V_h^L(Y)$ is a concave function of Y when $Y \leq Y_l^{F*}$. Also, the entry timing as a preemptive leader is discussed later.

As shown in the next section, in our analysis, it is plausible that a rival firm does not have an incentive to preempt the market. This situation is the same as the one where a firm's role (as a leader or a follower) is predetermined. In that case, the entry timing as a leader $\widetilde{Y}_i^{L*}$ $(i = l,\ h)$ is characterized by

$$\widetilde{Y}_i^{L*} = \frac{\beta}{\beta-1}\frac{r-\alpha}{\Pi_i^{m*}}I, \quad i = l,\ h.$$

4.2 Two types of equilibrium

In our model, two types of entry equilibrium can occur, depending on the level of value functions. One type of equilibrium is called a *preemptive equilibrium*, while the other is called a *non-preemptive equilibrium*. We characterize each of the two equilibria.

4.2.1 Preemptive equilibrium

The preemptive equilibrium occurs when both firms have an incentive to preempt the market. This situation is characterized by the stipulation that firm i $(= l,\ h)$ has some range of Y that satisfies the following condition before its rival enters the market as a follower.

$$\xi_i(Y) \equiv V_i^L(Y) - V_i^F(Y) > 0, \quad i = l,\ h. \tag{15}$$

The condition (15) implies that firm i actually has an incentive to become a leader rather than a follower. We already verify that firm h enters the market earlier than firm l if each of the firms would become a follower ($Y_h^{F*} < Y_l^{F*}$). In addition, we also verify the following condition, because $\Pi_l^{d*}(\theta) < \Pi_h^{d*}(\theta)$ for any θ, $\Pi_l^{m*} < \Pi_h^{m*}$, and $V_i^L(Y)$ is concave in Y.

$$\begin{aligned} \text{At } Y &= Y_l^P, \text{ where } Y_l^P \text{ is the smallest solution of } \xi_l(Y) = 0, \\ \xi_h\left(Y_l^P\right) &\equiv V_h^L\left(Y_l^P\right) - V_h^F\left(Y_l^P\right) > 0. \end{aligned}$$

This implies that firm h becomes a leader if both firms have an incentive to preempt the market. See Figure 2. Then, firm h's entry timing as a leader, Y_h^{L*}, is characterized by

$$Y_h^{L*} = \min\left\{Y_l^P,\ \widetilde{Y}_i^{L*}\right\}.$$

After firm h's entry as a leader, firm l enters the market at Y_l^{F*} as a follower.

(Insert Figure 2 around here.)

4.2.2 Non-preemptive equilibrium

Non-preemptive equilibrium occurs when one of the two firms does not have an incentive to preempt the market. Specifically, the condition that firm i $(= l,\ h)$ does not have an incentive to preempt the market is characterized by

$$\xi_i(Y) < 0 \ \text{ for } \forall Y \in \left[Y_0,\ Y_j^{F*}\right],\ i,\ j = l,\ h, \text{ and } i \neq j,$$

where Y_0 is the initial value of Y.

Then, we verify that only firm l does not have an incentive to preempt the market by the results that $\Pi_l^{d*}(\theta) < \Pi_h^{d*}(\theta)$ for any θ, $\Pi_l^{m*} < \Pi_h^{m*}$, and $V_i^L(Y)$ is concave in Y. In other words, there exists only the sequential equilibrium in which firm h is the leader and firm l is the follower. That is, in the non-preemptive equilibrium, we have

$$\xi_l(Y) < 0 \text{ for } \forall Y \in \left[Y_0,\ Y_h^{F*}\right].$$

Then, in the equilibrium, firm h's entry timing as a leader is $\widetilde{Y}_i^{L*}$, whereas firm l's entry timing as a follower is Y_l^{F*}.

From Figure 3, we conjecture that the non-preemptive equilibrium is likely to occur when the difference between $\Pi_l^{d*}(\theta)$ and $\Pi_h^{d*}(\theta)$ is large, i.e., when θ is large. This conjecture is verified by the analysis in the next subsection.

(Insert Figure 3 around here.)

4.3 Conditions for the occurrence of equilibria

We examine the conditions under which each of the entry equilibria occurs. To do so, we define *the relative magnitude of duopoly profits*, κ^d, as follows.

$$\kappa^d \left(= \kappa^d(\theta)\right) \equiv \frac{\Pi_l^{d*}(\theta)}{\Pi_h^{d*}(\theta)} (< 1).$$

Using the fact that $\Pi_l^{d*}(\theta) = \kappa^d \Pi_h^{d*}(\theta)$, we obtain

$$\frac{d\kappa^d}{d\theta} = \frac{\Pi_l^{d*\prime}(\theta) - \Pi_h^{d*\prime}(\theta)}{\Pi_h^{d*}(\theta)} < 0.$$

This property is also verified by Figure 1.

We re-state the conditions under which the non-preemptive equilibrium occurs.

$$\begin{aligned} \xi_l(Y) &\equiv V_l^L(Y) - V_l^F(Y) < 0 \text{ for } \forall Y \in \left[Y(0),\ Y_h^{F*}\right], \\ \text{and } Y_h^{F*} &< Y_l^{F*}. \end{aligned} \quad (16)$$

The second condition of (16) already holds in our model.

For the first condition of (16) to be held, we need to find $\left\{Y^*,\ \kappa^{d*}\right\}$ that satisfies the following two requirements.[4]

$$\xi_l\left(Y^*;\kappa^{d*}\right) = 0 \tag{17}$$

$$\left.\frac{\partial \xi_l\left(Y;\kappa^{d*}\right)}{\partial Y}\right|_{Y=Y^*} = 0 \tag{18}$$

After κ^{d*} is found, we ensure that the non-preemptive equilibrium (the preemptive equilibrium, respectively) occurs at $\kappa^d < (>$, respectively$)\,\kappa^{d*}$. That is, κ^{d*} represents the threshold of κ^d between the preemptive equilibrium and the non-preemptive equilibrium.

4.3.1 Derivation of the threshold κ^{d*} between the preemptive and the non-preemptive equilibria

Let us derive κ^{d*}. The following lemma shows the characterization of κ^{d*}.

Lemma 1 *The threshold of κ^d between the preemptive equilibrium and the non-preemptive equilibrium, κ^{d*}, is characterized by*

$$\begin{aligned}\left(\kappa^{d*}\right)^{\beta} - \beta\kappa^{d*} + \beta\chi(\theta) - (\chi(\theta))^{\beta} &= 0, \\ \textit{where } \chi(\theta) &\equiv \frac{\Pi_l^{m*}}{\Pi_h^{d*}(\theta)}.\end{aligned} \tag{19}$$

In addition, we obtain

$$\frac{d\kappa^{d*}}{d\theta} > 0. \tag{20}$$

Proof. See Appendix A. ■

We mention the meaning of κ^{d*}. κ^{d*} indicates the threshold of κ^d that distinguishes the non-preemptive equilibrium from the preemptive equilibrium. From (19), we ensure that κ^{d*} is influenced not only by firm l's monopoly profit Π_l^{m*} but also by β that depends on several stochastic parameters such as α and σ. In addition, κ^{d*} gets larger as the

[4] See Appendix B of Pawlina and Kort (2006) for the procedure of the analysis here.

proportion of savvy consumers, θ, increases. We should also note that for any given $\theta \in [0,\ 1]$, κ^d is uniquely defined and κ^{d*} uniquely exists, and they do not coincide in general.

4.3.2 Conditions for equilibria

Let us define $\Psi\left(\kappa^d\right)$ such that

$$\Psi\left(\kappa^d\right) \equiv \left(\kappa^d\right)^\beta - \beta\kappa^d + \beta\chi\left(\theta\right) - \left(\chi\left(\theta\right)\right)^\beta. \tag{21}$$

Because

$$\Psi'\left(\kappa^d\right) = \beta\left(\left(\kappa^d\right)^{\beta-1} - 1\right) < 0,$$

we verify that given θ, $\Psi\left(\kappa^d\left(\theta\right)\right) > (<)\, 0$ when $\kappa^d\left(\theta\right) < (>)\, \kappa^{d*}\left(\theta\right)$.

(Insert Figure 4 around here.)

Using these results, we find three different cases of equilibrium configuration, depending on the magnitudes of firms' profit flows. For example, we obtain a case in which there exists a unique interior value of $\widehat{\theta} \in (0,\ 1)$ such that the preemptive equilibrium (resp. the non-preemptive equilibrium) occurs for $\theta < \widehat{\theta}$ $\left(\text{resp. } \theta > \widehat{\theta}\right)$, as depicted in Figure 4. The next proposition indicates the conditions under which each of the two equilibria occurs, including the case of Figure 4.

Proposition 2 *There exists two types of entry equilibrium, i.e., the preemptive equilibrium and the non-preemptive equilibrium, in which firm h is a leader and firm l is a follower. Then, the following three cases, (i) to (iii), occur, depending on the magnitudes of profit flows.*

(i) When the following (22) and (23) hold, the preemptive equilibrium (resp. the non-preemptive equilibrium) occurs for $\theta < \widehat{\theta}$ (resp. $\theta > \widehat{\theta}$).

$$\beta \left(\Pi_h^{d*}(1)\right)^{\beta-1} \left(\Pi_l^{m*} - \Pi_l^{d*}(1)\right) > G(1), \text{ and} \tag{22}$$

$$\beta \left(\Pi_h^{d*}(0)\right)^{\beta-1} \left(\Pi_l^{m*} - \Pi_l^{d*}(0)\right) < G(0), \tag{23}$$

$$\text{where } G(\delta) \equiv \left(\Pi_l^{m*}\right)^{\beta} - \left(\Pi_l^{d*}(\delta)\right)^{\beta} \quad \text{for } \delta = 0,\ 1.$$

(ii) When the following (24) holds, the preemptive equilibrium occurs for any $\theta \in [0,\ 1]$.

$$\beta \left(\Pi_h^{d*}(1)\right)^{\beta-1} \left(\Pi_l^{m*} - \Pi_l^{d*}(1)\right) < G(1). \tag{24}$$

(iii) When the following (25) holds, the non-preemptive equilibrium occurs for any $\theta \in [0,\ 1]$.

$$\beta \left(\Pi_h^{d*}(0)\right)^{\beta-1} \left(\Pi_l^{m*} - \Pi_l^{d*}(0)\right) > G(0). \tag{25}$$

Proof. See Appendix B. ■

An interesting point of Proposition 2 is that a firm's entry timing is affected by the proportion of savvy consumers in the population, θ. In addition, in the case of (i) of the proposition, we have a jump of a leader's (firm h's) entry timing as θ reaches $\widehat{\theta}$; a leader's entry becomes suddenly later, even though the quality of its add-on services does not change.

Lastly, we mention some remarks on the different environments from the one analyzed in our model. First, when the two firms provide the same quality for add-on services, we definitely have the preemptive equilibrium for any $\theta \in [0,\ 1]$. Furthermore, the proportion of savvy consumers does not affect the entry timing of either a leader or a follower. This is because the duopoly profit flow under the same quality between the two firms is $\Pi_l^{d*} = \Pi_h^{d*} = 1/2$, which does not depend on θ. Also, because $\Pi_l^{d*} = \Pi_h^{d*} = 1/2 > \Pi_l^{d*}(\theta)$ for any $\theta \in [0,\ 1]$, we ensure that the entry timing of the follower in this case is always earlier than that in our model. On the other hand, the comparison of the leader's entry timing between the case of same quality and our model is very subtle and complicated. This additional property, however, seems to depend on the competition mode for the core goods.

Second, when core goods provided by the two firms exhibit a quality difference, each firm's entry timing in the equilibrium is not affected by the proportion of savvy consumers. Specifically, if the quality difference of core goods between the two firms is small, the preemptive equilibrium occurs for any $\theta \in [0,\ 1]$, and each firm's entry timing does not change despite the change in θ. On the contrary, if the quality difference of core goods between the two firms is large, the non-preemptive equilibrium occurs for any $\theta \in [0,\ 1]$, and each firm's entry timing does not change either, despite the change in θ.

5 Conclusion

This paper has examined the impact of the presence of non-savvy consumers on firms' entry incentives in durable goods markets when the add-on services are vertically differentiated.

As a preliminary result, we have firstly described the prices of a durable good and its add-on services set by firms. Specifically, we have shown that the price of a durable good with high quality add-on services is lower (resp. higher) than that with low quality add-on services when the proportion of non-savvy consumers in the population is high (resp. low). In addition, we have also verified that the price of add-on services is an increasing function of the proportion of non-savvy consumers in the population, which is a well-known fact called "exploitation by firms".

We then have characterized the entry equilibria in the durable goods market. After ensuring that a firm that supplies a durable good with high quality add-on services enters earlier than that with low quality add-on services despite the proportion of non-savvy consumers, we have distinguished two type of equilibrium; a preemptive equilibrium and a non-preemptive equilibrium. When the proportion of non-savvy consumers is high, there exists a preemptive equilibrium in which both firms have a preemptive incentive such that the entry timing of a leader (i.e., a firm with high quality add-on services) into the market becomes earlier than that when savvy consumers are prevalent. On the contrary, when the proportion of non-savvy consumers is low, a non-preemptive equilibrium occurs in which one of the firms does not have an preemptive incentive into the market. A remarkable feature of this result is that the occurrence of a type of entry equilibrium is affected by the proportion of non-savvy consumers in the population.

Appendix

Appendix A: the proof of Lemma 1

From (17) in the text, we obtain

$$\begin{aligned} \xi_l\left(Y^*;\kappa^{d*}\right) &= \frac{Y^*\Pi_l^{m*}}{r-\alpha} - I - \frac{Y_h^{F*}\left(\Pi_l^{m*}-\Pi_l^{d*}(\theta)\right)}{r-\alpha}\left(\frac{Y^*}{Y_h^{F*}}\right)^{\beta} \\ -\left[\frac{Y_l^{F*}\Pi_l^{d*}(\theta)}{r-\alpha} - I\right]\left(\frac{Y^*}{Y_l^{F*}}\right)^{\beta} &= 0. \end{aligned} \tag{26}$$

From (18), we obtain

$$\begin{aligned} \left.\frac{\partial\xi_l\left(Y;\kappa^{d*}\right)}{\partial Y}\right|_{Y=Y^*} &= \frac{\Pi_l^{m*}}{r-\alpha} - \beta\frac{\left(\Pi_l^{m*}-\Pi_l^{d*}(\theta)\right)}{r-\alpha}\left(\frac{Y^*}{Y_h^{F*}}\right)^{\beta-1} \\ -\beta\left[\frac{Y_l^{F*}\Pi_l^{d*}(\theta)}{r-\alpha} - I\right]\left(\frac{Y^*}{Y_l^{F*}}\right)^{\beta-1}\frac{1}{Y_l^{F*}} &= 0. \end{aligned} \tag{27}$$

By (27)$\times$ (Y^*/β),

$$\begin{aligned} & \frac{Y^*}{\beta}\frac{\Pi_l^{m*}}{r-\alpha} - I - \frac{Y_h^{F*}\left(\Pi_l^{m*}-\Pi_l^{d*}(\theta)\right)}{r-\alpha}\left(\frac{Y^*}{Y_h^{F*}}\right)^{\beta} \\ -\left[\frac{Y_l^{F*}\Pi_l^{d*}(\theta)}{r-\alpha} - I\right]\left(\frac{Y^*}{Y_l^{F*}}\right)^{\beta} &= 0. \qquad (28) \end{aligned}$$

Then, subtracting (28) from (26), we derive

$$Y^* = \frac{\beta}{\beta-1}\frac{r-\alpha}{\Pi_l^{m*}}I. \qquad (29)$$

Substituting (29) into (26), we obtain the following equation for κ^{d*}.

$$\begin{aligned} \left(\kappa^{d*}\right)^{\beta} - \beta\kappa^{d*} + \beta\chi(\theta) - (\chi(\theta))^{\beta} &= 0, \qquad (30) \\ \text{where } \chi(\theta) &\equiv \frac{\Pi_l^{m*}}{\Pi_h^{d*}(\theta)}. \end{aligned}$$

Totally differentiating (30), we obtain

$$\begin{aligned} \frac{d\kappa^{d*}}{d\theta} &= \frac{\chi'(\theta)\left[(\chi(\theta))^{\beta-1}-1\right]}{\left(\kappa^{d*}\right)^{\beta-1}-1}, \\ \text{where } \chi'(\theta) &= -\Pi_l^{m*}\left(\Pi_h^{d*}(\theta)\right)^{-2}\Pi_h^{d*\prime}(\theta) < 0. \end{aligned}$$

Furthermore, because $\beta > 1$, $\kappa^{d*} < 1$, and $\chi(\theta) > 1$, we conclude that

$$\frac{d\kappa^{d*}}{d\theta} > 0. \qquad (31)$$

This completes the proof. ■

Appendix B: the proof of Proposition 2

Because $\kappa^{d}(\theta)$ is decreasing in θ and $\kappa^{d*}(\theta)$ is increasing in θ, we have the following four cases. See Figure 4 as a reference.

Case 1: $\kappa^{d}(1) < \kappa^{d*}(0) < \kappa^{d}(0)$,

Case 2: $\kappa^{d*}(0) < \kappa^{d}(1)$ and $\kappa^{d}(1) < \kappa^{d*}(1)$,

Case 3: $\kappa^{d*}(0) < \kappa^{d}(1)$ and $\kappa^{d*}(1) < \kappa^{d}(1)$,

Case 4: $\kappa^{d}(0) < \kappa^{d*}(0)$.

We check each of these cases.

Case 1: $\kappa^{d}(1) < \kappa^{d*}(0) < \kappa^{d}(0)$.

This case corresponds to (i) in the proposition. That is, there exists a unique threshold $\widehat{\theta}$ such that the preemptive equilibrium (resp. the sequential equilibrium) occurs for $\theta < \widehat{\theta}$ (resp. $\theta > \widehat{\theta}$).

When $\kappa^{d}(1) < \kappa^{d*}(0)$, $\Psi\left(\kappa^{d}(1)\right) > 0$ because $\Psi'\left(\kappa^{d}\right) < 0$ and $\Psi\left(\kappa^{d*}(0)\right) = 0$. Substituting $\kappa^{d}(1) = \left(\Pi_{l}^{d*}(1)/\Pi_{h}^{d*}(1)\right)$ into $\Psi\left(\kappa^{d}(1)\right) > 0$ and rearranging it, we obtain (22).

Similarly, when $\kappa^{d*}(0) < \kappa^{d}(0)$, $\Psi\left(\kappa^{d}(0)\right) < 0$ because $\Psi'\left(\kappa^{d}\right) < 0$ and $\Psi\left(\kappa^{d*}(0)\right) = 0$. Substituting $\kappa^{d}(0) = \left(\Pi_{l}^{d*}(0)/\Pi_{h}^{d*}(0)\right)$ into $\Psi\left(\kappa^{d}(0)\right) < 0$ and rearranging it, we obtain (23).

Case 2: $\kappa^{d*}(0) < \kappa^{d}(1)$ and $\kappa^{d}(1) < \kappa^{d*}(1)$.

When $\kappa^{d*}(0) < \kappa^{d}(1)$, $\Psi\left(\kappa^{d}(1)\right) < 0$ because $\Psi'\left(\kappa^{d}\right) < 0$ and $\Psi\left(\kappa^{d*}(0)\right) = 0$. This is the opposite case of (22).

Similarly, when $\kappa^{d}(1) < \kappa^{d*}(1)$, $\Psi\left(\kappa^{d}(1)\right) > 0$ because $\Psi'\left(\kappa^{d}\right) < 0$ and $\Psi\left(\kappa^{d*}(1)\right) = 0$. However, this contradicts with the result derived for the case where $\kappa^{d*}(0) < \kappa^{d}(1)$; $\Psi\left(\kappa^{d}(1)\right) < 0$. Hence, case 2 cannot exist.

Case 3: $\kappa^{d*}(0) < \kappa^{d}(1)$ and $\kappa^{d*}(1) < \kappa^{d}(1)$.

This case corresponds to (ii) in the proposition. That is, the preemptive equilibrium occurs for any $\theta \in [0,\ 1]$ in this case. When $\kappa^{d*}(0) < \kappa^{d}(1)$, $\Psi\left(\kappa^{d}(1)\right) < 0$ because $\Psi'\left(\kappa^{d}\right) < 0$ and $\Psi\left(\kappa^{d*}(0)\right) = 0$. Substituting $\kappa^{d}(1) = \left(\Pi_{l}^{d*}(1)/\Pi_{h}^{d*}(1)\right)$ into $\Psi\left(\kappa^{d}(1)\right) < 0$ and rearranging it, we obtain the opposite case of (22), i.e., (24).

Similarly, when $\kappa^{d*}(1) < \kappa^{d}(1)$, $\Psi\left(\kappa^{d}(1)\right) < 0$ because $\Psi'\left(\kappa^{d}\right) < 0$ and $\Psi\left(\kappa^{d*}(1)\right) =$

0., which is the same as (24).

Case 4: $\kappa^{d}(0) < \kappa^{d*}(0)$.

This case corresponds to (iii) in the proposition. That is, the non-preemptive equilibrium occurs for any $\theta \in [0,\ 1]$ in this case. When $\kappa^{d}(0) < \kappa^{d*}(0)$, $\Psi\left(\kappa^{d}(0)\right) > 0$ because $\Psi'\left(\kappa^{d}\right) < 0$ and $\Psi\left(\kappa^{d*}(0)\right) = 0$. Substituting $\kappa^{d}(0) = \left(\Pi_{l}^{d*}(0) / \Pi_{h}^{d*}(0)\right)$ into $\Psi\left(\kappa^{d}(0)\right) > 0$ and rearranging it, we obtain the opposite case of (23), i.e., (25).

Summarizing the above results gives the proposition. ■

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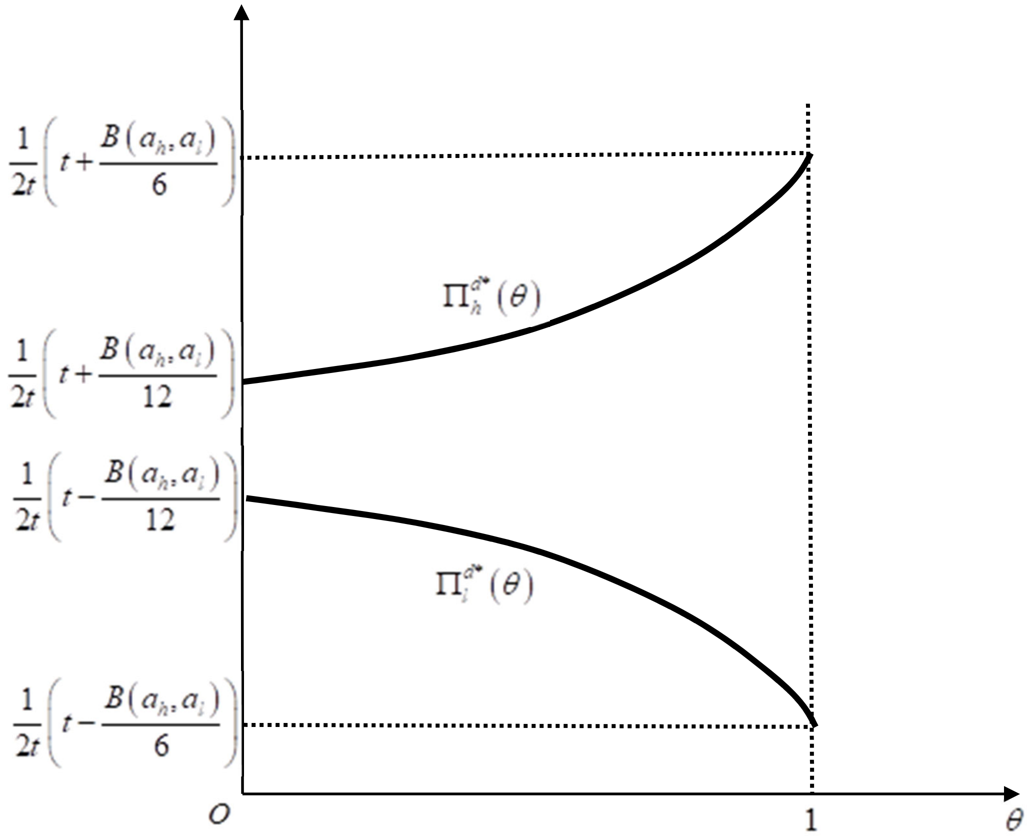

Figure 1 Vertical Product Differentiation in Add-On Services

Note: $B(a_h, a_l) \equiv (a_h - c)^2 - (a_l - c)^2$.

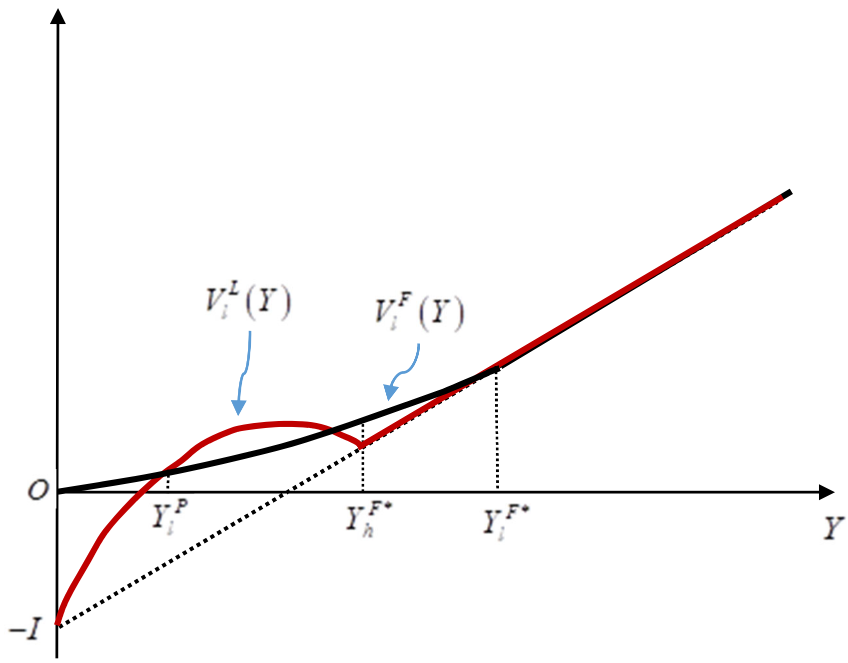

Figure 2-1 Firm *l*'s Value Functions in Preemptive Equilibrium

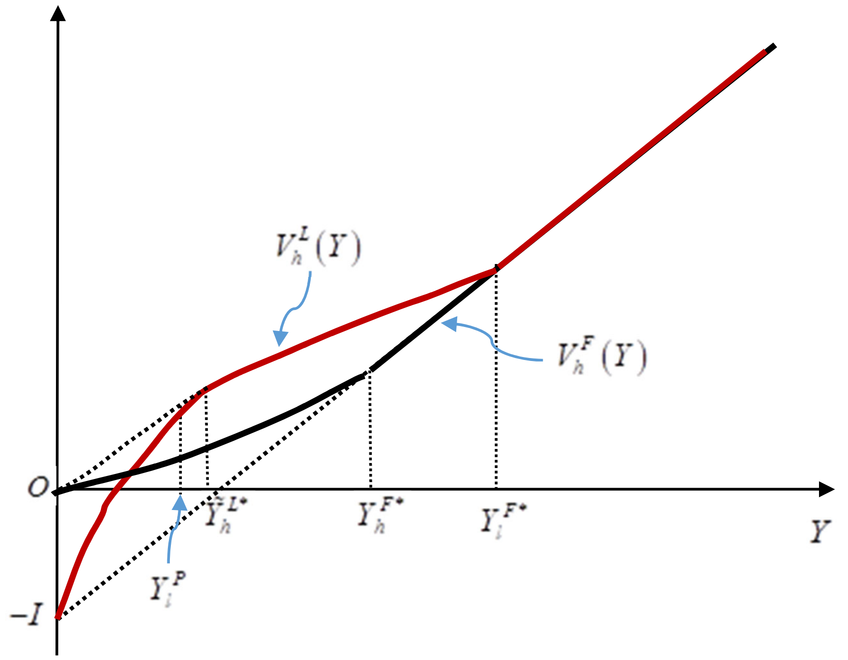

Figure 2-2 Firm *h*'s Value Functions in Preemptive Equilibrium

Figure 2 Preemptive Equilibrium

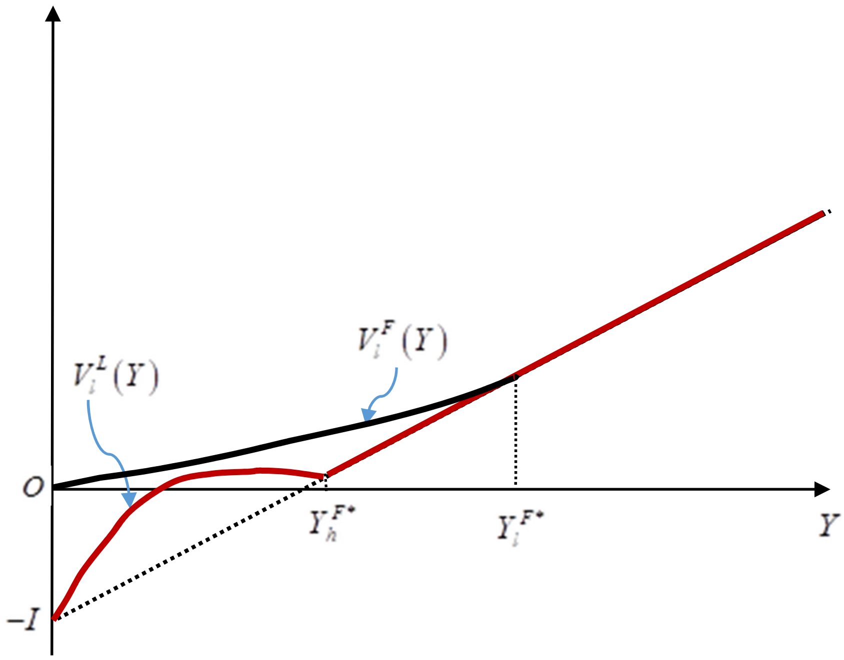

Figure 3-1 Firm *l*'s Value Functions in Non-Preemptive Equilibrium

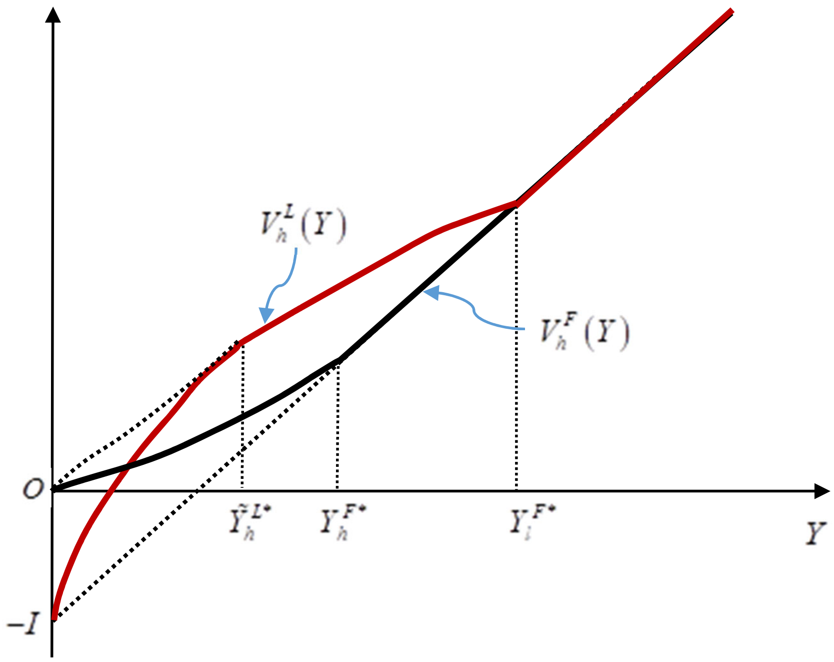

Figure 3-2 Firm *h*'s Value Functions in Non-Preemptive Equilibrium

Figure 3 Non-Preemptive Equilibrium

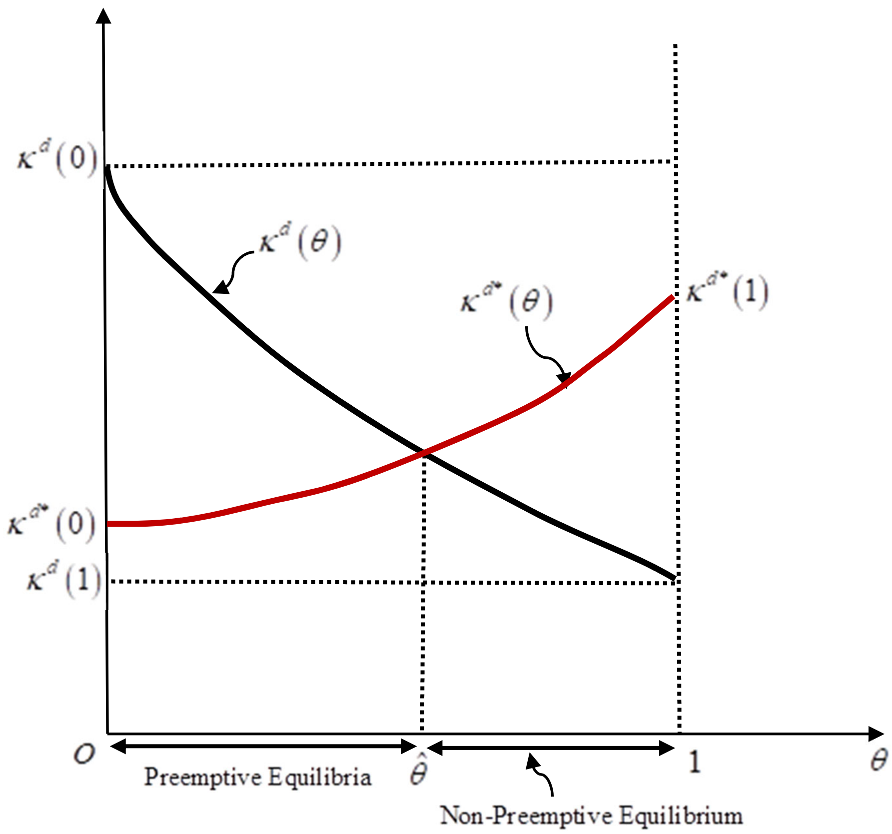

Figure 4 The Equilibrium Configuration

Note:

(i) $\kappa^{d}(\theta) \equiv \dfrac{\Pi_l^{d*}(\theta)}{\Pi_h^{d*}(\theta)}$.

(ii) $\kappa^{d*}(\theta)$ is defined as the solution of $\left(\kappa^{d*}\right)^{\beta} - \beta\kappa^{d*} + \beta\chi(\theta) - \left(\chi(\theta)\right)^{\beta} = 0$,

where $\chi(\theta) \equiv \dfrac{\Pi_l^{m*}}{\Pi_h^{d*}(\theta)}$.