Subsidies and sustainable tourism under uncertainty

Luciana Barbosa[a,b], José Carlos Dias[a,b,*], Margarida Marques[a]

[a] *Iscte - Instituto Universitário de Lisboa, Lisbon, Portugal.*
[b] *Business Research Unit (BRU-IUL), Lisbon, Portugal.*

Abstract

This paper considers the problem of taking a managerial decision associated to an investment project in a trail designed for tourist activities. Our real options models study the existence of alternative temporary minimum demand guarantee policies aiming to encourage new private investments, but without neglecting the potential environmental damages when the number of tourists is too excessive. We provide an elegant analytical characterization for both the value function of the active project and for the infinite-horizon optimal stopping problem.

Keywords: Tourist trails; Demand floors; Sustainable caps; Real options

JEL classification: G13; Q28; Z32; Z38.

*Corresponding author. Iscte - Instituto Universitário de Lisboa, Edifício II, Av. Prof. Aníbal Bettencourt, 1600-189 Lisboa, Portugal. Tel: +351 21 7650531. Declarations of interest: none.

Email addresses: luciana.barbosa@iscte.pt (Luciana Barbosa), jose.carlos.dias@iscte.pt (José Carlos Dias), margarida_silva_marques@iscte.pt (Margarida Marques)

1. Introduction

2. The investment problem and building blocks

Our model builds upon the work of Barbosa et al. (2020), in which optimal investment thresholds of two feed-in tariffs under market and regulatory uncertainty are analyzed. These tariffs include a minimum price guarantee and a sliding premium with a cap and a floor for renewable energy projects. We extend this prior research by incorporating uncertainty related to the number of users of a pedestrian path, as this is the primary source of uncertainty in the projects we are studying. Barbosa et al. (2020) assume the production is fixed. Additionally, our model incorporates a marginal cost associated with maintenance and security expenses. This is a departure from Barbosa et al. (2020), where the marginal cost can be considered to be zero. Nevertheless, the projects under analysis involve significant marginal costs that need to be taken into account when modeling the investment problem.

2.1. The investment problem

In this paper, we assume that the number of users per pedestrian path $\{Q_t, t \geq 0\}$ is governed by the risk-neutral dynamics

$$dQ_t = (r - q)\, Q_t dt + \sigma Q_t dW_t^{\mathbb{Q}}, \tag{1}$$

where r, q and σ are the (positive and constant) risk free interest rate, dividend yield (or rate of return shortfall) and volatility, respectively, while $\{W_t^{\mathbb{Q}} \in \mathbb{R} : t \geq 0\}$ is a standard Brownian motion under the risk-neutral measure $\mathbb{Q}$, initialized at zero and generating the augmented, right continuous and complete filtration $\mathbb{F} = \{\mathcal{F}_t : t \geq 0\}$.

The goal is to analyze the optimal decision of a firm (or private investor) to invest in a project that may include a given clause aiming to offer incentives for early adoption of investment projects. In particular, two finite-lived subsidy policies will be considered: a minimum revenue guarantee (or quantity floor) and a collar contract. For comparative purposes, the case with no subsidy will be also considered.

Let P be the (fixed) price (or tariff) that is paid by each user of the pedestrian path and K_2 be the marginal costs associated with maintenance and security expenses. Moreover, let the level L be understood as a quantity floor and H $(> L)$ be interpreted as a quantity cap that is imposed due to environmental restrictions. Then, the firm's profit functions for each case are the following:[1]

- Case without any quantity clause: $\Pi_w(Q_t) = PQ_t - K_2$. In this case there is no controlling clause and, hence, the stochastic quantity Q moves freely.
- Quantity floor scheme: $\Pi_f(Q_t) = P \times \max(Q_t, L) - K_2$. This corresponds to a scenario where the government subsidizes the firm ensuring a minimum guarantee quantity of tourists L and there is no environmental cap to limit the number of tourists.
- Quantity cap scheme: $\Pi_{cp}(Q_t) = P \times \min(Q_t, H) - K_2$. In this case there is no subsidy, though an environmental cap H is imposed to limit the number of tourists. As expected, this scenario should not be attractive to the investor.
- Quantity collar scheme: $\Pi_{co}(Q_t) = P \times \min(\max(Q, L), H) - K_2$. In this case the project is subsidized ensuring the firm a minimum guarantee quantity of tourists L, but there is an environmental cap H limiting the number of tourists.

Similarly to Barbosa et al. (2020), we start assuming that when the (fixed) duration T of the contract with a subsidy floor and/or an environmental cap ends, the firm's profit function becomes equal to a scenario without any clause. Hence, for $s \in \{w, f, cp, co\}$, the optimization problem that is proposed to be solved in this paper is represented by

$$F_s(Q) = \sup_{\tau \in \mathcal{T}} \mathbb{E}_{\mathbb{Q}} \left[\int_{\tau}^{\tau+T} \Pi_s(Q_t) e^{-rt} dt + \int_{\tau+T}^{\infty} \Pi_w(Q_t) e^{-rt} dt - K_1 e^{-r\tau} \middle| \mathcal{F}_0 \right], \qquad (2)$$

where K_1 is the investment cost to be made at the optimal investment (or stopping) time τ, whereas $\mathcal{T}$ is the set of all stopping times taking values in $[0, \infty[$ for the filtration $\mathbb{F}$.

[1]Hereafter, we use the subscript w, f, cp and co for the cases without subsidy, with a quantity floor regime, with a quantity cap regime and with a quantity collar regime, respectively.

We notice that at the optimal investment time τ the firm receives a project whose full value is represented by

$$V_s^F(Q) = \mathbb{E}_{\mathbb{Q}}\left[\int_{\tau}^{\tau+T} \Pi_s(Q_t)e^{-rt}dt + \int_{\tau+T}^{\infty} \Pi_w(Q_t)e^{-rt}dt \,\middle|\, \mathcal{F}_0\right]. \tag{3}$$

The floor, cap and collar schemes stated above consider that after the end of the contract the quantity of tourists is not controlled, i.e., the number of tourists in the path can be very low (i.e., much lower than L)—which influences negatively the firm without harming significantly the environment—or very high (i.e., much higher than H)—which is good news for the firm but might introduce several environmental damages that have the potential to jeopardize the future attractiveness of the path.

To overcome this problem, we will consider another scheme that consists of a collar until the end of the contract—that ensures both a subsidy via the floor and the environmental cap—and a perpetual (environmental) cap afterwards to warrant the fulfilment required by the policy maker to prevent serious environmental damages in the future. This originates a profit function of the form

$$\Pi_{co+cp\equiv cc}(Q_t) = \Pi_{co}(Q_t)\mathbb{1}_{\{t\leq T\}} + \Pi_{cp}(Q_t)\mathbb{1}_{\{t>T\}}, \tag{4}$$

implying that equations (2) and (3) need to be replaced by

$$F_{cc}(Q) = \sup_{\tau\in\mathcal{T}} \mathbb{E}_{\mathbb{Q}}\left[\int_{\tau}^{\tau+T} \Pi_{co}(Q_t)e^{-rt}dt + \int_{\tau+T}^{\infty} \Pi_{cp}(Q_t)e^{-rt}dt - K_1 e^{-r\tau} \,\middle|\, \mathcal{F}_0\right] \tag{5}$$

and

$$V_{cc}^F(Q) = \mathbb{E}_{\mathbb{Q}}\left[\int_{\tau}^{\tau+T} \Pi_{co}(Q_t)e^{-rt}dt + \int_{\tau+T}^{\infty} \Pi_{cp}(Q_t)e^{-rt}dt \,\middle|\, \mathcal{F}_0\right], \tag{6}$$

respectively.

The aim now is to solve the aforementioned optimization problems using the real options theory. To accomplish this purpose, the building blocks described next will be pivotal since they will allow us to simplify significantly the usual approach that is used in Barbosa et al.

(2020).

2.2. Building blocks

Let us now recall two relevant ingredients for our investment decision problem. Assuming instantaneous flow payoffs of the form $(Q_t - X)\mathbb{1}_{\{Q_t \geq X\}}$, Shackleton and Wojakowski (2007, equations 21 and 28) show that a finite-lived profit cap, $V(Q_0, X, T)$, and its delta, $\Delta(Q_0, X, T) := \partial V(Q_0, X, T)/\partial Q_0$, can be defined in compact form as

$$\begin{aligned} & V(Q_0, X, T) \\ = \; & \frac{Q_0}{q}\left[\mathbb{1}_{\{Q_0 \geq X\}} - e^{-qT} N(d_1(Q_0, X))\right] - \frac{X}{r}\left[\mathbb{1}_{\{Q_0 \geq X\}} - e^{-rT} N(d_0(Q_0, X))\right] \\ & + B(X) Q_0^{\beta_2}\left[\mathbb{1}_{\{Q_0 \geq X\}} - N(d_{\beta_2}(Q_0, X))\right] - A(X) Q_0^{\beta_1}\left[\mathbb{1}_{\{Q_0 \geq X\}} - N(d_{\beta_1}(Q_0, X))\right] \end{aligned} \tag{7}$$

and

$$\begin{aligned} & \Delta(Q_0, X, T) \\ = \; & \frac{1}{q}\left[\mathbb{1}_{\{Q_0 \geq X\}} - e^{-qT} N(d_1(Q_0, X))\right] + B(X)\beta_2 Q_0^{\beta_2 - 1}\left[\mathbb{1}_{\{Q_0 \geq X\}} - N(d_{\beta_2}(Q_0, X))\right] \\ & - A(X)\beta_1 Q_0^{\beta_1 - 1}\left[\mathbb{1}_{\{Q_0 \geq X\}} - N(d_{\beta_1}(Q_0, X))\right], \end{aligned} \tag{8}$$

respectively, with

$$A(X) = \frac{X^{1-\beta_1}}{\beta_1 - \beta_2}\left(\frac{\beta_2}{r} - \frac{\beta_2 - 1}{q}\right), \tag{9}$$

$$B(X) = \frac{X^{1-\beta_2}}{\beta_1 - \beta_2}\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q}\right), \tag{10}$$

$$\beta_{1,2} = \frac{1}{2} - \frac{r-q}{\sigma^2} \pm \sqrt{\left(\frac{r-q}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{11}$$

$$d_\beta(x, y) = \frac{\ln(x/y) + (r - q + (\beta - 0.5)\sigma^2)T}{\sigma\sqrt{T}}, \tag{12}$$

and where $N(d_\beta(.))$ represents the cumulative density function of the univariate standard normal distribution for $\beta \in \{0, 1, \beta_1, \beta_2\}$.

3. Investment decisions for the case without any quantity clause

For the sake of completeness and to offer a benchmark for the remaining cases, let us first consider the case without any quantity clause. In this situation, the total (or full) value of the project is given by

$$V_w^F(Q_0, \infty) = P\frac{Q_0}{q} - \frac{K_2}{r}. \tag{13}$$

The value of the option to invest in this project and the corresponding optimal trigger to invest can be easily computed as shown in the next proposition, which solves the optimization problem defined in equation (2) for the case without any quantity clause (i.e., with $s = w$).

Proposition 1. *The value of the (perpetual) option to invest in a project without any quantity clause is calculated as*

$$F_w(Q_0, \overline{Q}, \infty, K_1) = \begin{cases} \left(V_w^F(\overline{Q}, \infty) - K_1\right)\left(\frac{Q_0}{\overline{Q}}\right)^{\beta_1} & \Leftarrow \quad Q_0 < \overline{Q} \\ V_w^F(Q_0, \infty) - K_1 & \Leftarrow \quad Q_0 \geq \overline{Q} \end{cases}, \tag{14}$$

where the optimal threshold of investment $\overline{Q}$ is obtained in closed-form as

$$\overline{Q} = \frac{q}{P}\frac{\beta_1}{\beta_1 - 1}\left(K_1 + \frac{K_2}{r}\right). \tag{15}$$

Proof. Using the usual value-matching and smooth-pasting conditions it is possible to show that

$$\beta_1 V_w^F(\overline{Q}, \infty) - \beta_1 K_1 = \overline{Q}\, \Delta_w^F(\overline{Q}, \infty),$$

with $\Delta_w^F(Q, \infty) := \partial V_w^F(Q, \infty)/\partial Q$, which finally yields equation (15) and the option solution (14) after applying some straightforward calculus. ■

4. Investment decisions with a quantity floor regime

The instantaneous flow payoff for this investment problem is represented by the form $P \times \max(Q_t, L) - K_2 = P \times (Q_t - L)\mathbb{1}_{\{Q_t \geq L\}} + PL - K_2$. Therefore, using the arbitrage-free relation enunciated in Dias et al. (2024, equation 21), it follows that the value of a finite maturity project containing a minimum quantity guarantee L and requiring a marginal cost K_2 can be expressed as

$$V_f(Q_0, L, T) = P \times V(Q_0, L, T) + \frac{PL - K_2}{r}\left(1 - e^{-rT}\right). \tag{16}$$

Following, for example, Barbosa et al. (2018), Barbosa et al. (2020) and Dias et al. (2024), we consider that after the expiry date of the finite maturity guarantee the entrepreneur can still explore the pedestrian path with a present value equal to $P\frac{Q_0}{q}e^{-qT} - \frac{K_2}{r}e^{-rT}$, that is understood as the value of exploring the path after the maturity date of the FIT contract. Therefore, the total (or full) value of the project that includes the period of the FIT contract and the (perpetual) period thereafter is given by

$$\begin{aligned} V_f^F(Q_0, L, T) &= V_f(Q_0, L, T) + P\frac{Q_0}{q}e^{-qT} - \frac{K_2}{r}e^{-rT} \\ &= P \times V(Q_0, L, T) + P\frac{L}{r}\left(1 - e^{-rT}\right) + P\frac{Q_0}{q}e^{-qT} - \frac{K_2}{r}. \end{aligned} \tag{17}$$

Next proposition shows how to determine the value of the option to invest in a project in the presence of a quantity floor regime and the corresponding optimal trigger to invest.

Proposition 2. *The value of the (perpetual) option to invest in a project containing a quantity floor regime with a finite maturity is calculated as*

$$F_f(Q_0, \overline{Q}, L, T, K_1) = \begin{cases} \left(V_f^F(\overline{Q}, L, T) - K_1\right)\left(\frac{Q_0}{\overline{Q}}\right)^{\beta_1} & \Leftarrow \quad Q_0 < \overline{Q} \\ V_f^F(Q_0, L, T) - K_1 & \Leftarrow \quad Q_0 \geq \overline{Q} \end{cases}, \tag{18}$$

where the optimal threshold of investment $\overline{Q}$ is obtained as the numerical solution of the *nonlinear equation*

$$(\beta_1 - 1)\, P\frac{\overline{Q}}{q}\left[\mathbb{1}_{\{\overline{Q}\geq L\}} + e^{-qT}N\left(-d_1(\overline{Q},L)\right)\right]$$
$$+PL\left(\frac{\overline{Q}}{L}\right)^{\beta_2}\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q}\right)\left[\mathbb{1}_{\{\overline{Q}\geq L\}} - N\left(d_{\beta_2}(\overline{Q},L)\right)\right]$$
$$+\beta_1 P\frac{L}{r}\left[\mathbb{1}_{\{\overline{Q}<L\}} - e^{-rT}N\left(-d_0(\overline{Q},L)\right)\right] - \beta_1\left(K_1 + \frac{K_2}{r}\right) = 0. \tag{19}$$

Proof. Using the usual value-matching and smooth-pasting conditions it is possible to show that

$$\beta_1 V_f^F(\overline{Q},L,T) - \beta_1 K_1 = \overline{Q}\,\Delta_f^F(\overline{Q},L,T),$$

with $\Delta_f^F(Q,L,T) := \partial V_f^F(Q,L,T)/\partial Q$, which finally yields the nonlinear equation (19) and the option solution (18) after combining equations (7), (8) and (17), and applying some straightforward calculus. Additional details are available upon request. ■

The results offered in Proposition 2 solve the optimization problem defined in equation (2) for the case of a subsidy in the form of a quantity floor regime (i.e., with $s = f$) and allow us to derive analytically several interesting points from the point of view of both the government and the promoter of the project, as shown in the remainder of this section.

4.1. Optimal trigger level making the project to start immediately

For instance, taking the tariff price P, the investment cost K_1 and the marginal cost K_2 as given, it is possible to determine analytically the L^* level that would be required to be offered by the government so that the entrepreneur starts the project immediately. This is accomplished by replacing $\overline{Q}$ by L in equation (19) and solving it with respect to $L \equiv L^*$, yielding

$$L^* = \beta_1\frac{K_1 + \frac{K_2}{r}}{PY_f}, \tag{20}$$

with

$$Y_f = \frac{\beta_1 - 1}{q}\left[N\left(d_{\beta_2}\right) + e^{-qT}N\left(-d_1\right)\right] + \frac{\beta_1}{r}\left[N\left(-d_{\beta_2}\right) - e^{-rT}N\left(-d_0\right)\right] \tag{21}$$

and

$$d_\beta := d_\beta(x,x) = \frac{(r - q + (\beta - 0.5)\sigma^2)T}{\sigma\sqrt{T}}. \tag{22}$$

4.2. Tariff price making the current floor level the optimal trigger

We notice that we can immediately determine the tariff price P_L^* that makes the current floor level L the trigger by rearranging equation (20) to

$$P_L^* = \beta_1 \frac{K_1 + \frac{K_2}{r}}{LY_f}. \tag{23}$$

The following economic rational can be taken: (i) if $P_L^* > P$, the current floor level L is not enough for the firm to undertake the project; and (ii) if $P_L^* \leq P$, the current floor level L is sufficient and the firm should optimally undertake the project immediately.

4.3. Simple comparative statics between tariff prices and quantity floor levels

An interesting comparative static between the tariff price and the floor level can now be easily analyzed. In particular, defining dL^* as the derivative of the expression for L^* shown in equation (20) with respect to the tariff price P yields

$$dL^* := \frac{\partial}{\partial P} L^*(.) = -\beta_1 \frac{K_1 + \frac{K_2}{r}}{P^2 Y_f} = -\frac{L^*}{P} < 0. \tag{24}$$

Similarly, defining dP_L^* as the derivative of the expression for P_L^* shown in equation (23) with respect to the quantity floor level L yields

$$dP_L^* := \frac{\partial}{\partial L} P_L^*(.) = -\beta_1 \frac{K_1 + \frac{K_2}{r}}{L^2 Y_f} = -\frac{P_L^*}{L} < 0. \tag{25}$$

In summary, a combined policy between the fixed tariff price P that is settled by the firm and the quantity floor level L that is defined by the government may be used to accelerate the investment commitment. For example, if a government is reluctant (or is unable) in rising the floor level from L to L^*, the firm needs to increase the tariff price from P to P_L^* if the goal is to undertake the project immediately. By contrast, in the case of a rigid policy in terms of tariff prices, it is the government that needs to make an extra effort by supporting a higher quantity floor level L^* so that the investment is undertaken immediately. Hence, the flexibility for changing quantity floor levels (in the perspective of the government) or price tariffs (in the perspective of the firm) will dictate the optimal policy of investment.

4.4. Investment and marginal costs making the current floor level the optimal trigger

It is also possible to find the investment cost $K^*_{1,L}$ and the marginal cost $K^*_{2,L}$ that makes the current floor level L the trigger by rearranging equation (20) to

$$K^*_{1,L} = \frac{PLY_f}{\beta_1} - \frac{K_2}{r} \tag{26}$$

and

$$K^*_{2,L} = \left(\frac{PLY_f}{\beta_1} - K_1 \right) r, \tag{27}$$

respectively.

The following economic insights can be made: (i) if $K^*_{1,L} \leq K_1$ (or, similarly, if $K^*_{2,L} \leq K_2$), then the optimal trigger is in the region $\overline{Q} \geq L$. In particular, $\overline{Q} \in \left[L^*, \overline{Q}_{max}\right]$. Notice that the level $\overline{Q}_{max}$ corresponds to the special case where there is no guarantee—i.e., when $L = 0$ or, equivalently, if $T \to 0$—, in which the present value of the project is simply equal to $\text{PV}_{\overline{Q}_{max}} := P\frac{Q_0}{q} - \frac{K_2}{r}$; and (ii) if $K^*_{1,L} > K_1$ (or, similarly, if $K^*_{2,L} > K_2$), then the optimal trigger is in the region $\overline{Q} < L$. In particular, $\overline{Q} \in \left]0, L^*\right[$.

4.5. Investment and marginal costs making the current Q_0 level the optimal trigger

Assuming a given floor level L that is defined by the government and a fixed tariff price P settled by the firm, it is also possible to find the investment cost K^*_{1,Q_0} and the marginal cost K^*_{2,Q_0} that make the current quantity Q_0 the trigger by rearranging equation (19) to

$$K^*_{1,Q_0} = \frac{Z\left(Q_0, L\right)}{\beta_1} - \frac{K_2}{r} \tag{28}$$

and

$$K^*_{2,Q_0} = \left(\frac{Z\left(Q_0, L\right)}{\beta_1} - K_1 \right) r, \tag{29}$$

respectively, with

$$
\begin{aligned}
Z(Q_0, L) \quad = \quad & (\beta_1 - 1) P \frac{Q_0}{q} \left[\mathbb{1}_{\{Q_0 \geq L\}} + e^{-qT} N\left(-d_1(Q_0, L)\right) \right] \\
& + PL \left(\frac{Q_0}{L} \right)^{\beta_2} \left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q} \right) \left[\mathbb{1}_{\{Q_0 \geq L\}} - N\left(d_{\beta_2}(Q_0, L)\right) \right] \\
& + \beta_1 P \frac{L}{r} \left[\mathbb{1}_{\{Q_0 < L\}} - e^{-rT} N\left(-d_0(Q_0, L)\right) \right]. \qquad (30)
\end{aligned}
$$

As expected, if $K_1 \leq K^*_{1,Q_0}$ it would be better for the firm to invest immediately because $Q_0 = \overline{Q}$ at the K^*_{1,Q_0} level. Hence, when $K_1 > K^*_{1,Q_0}$, if the firm is able to reduce the investment cost from K_1 to K^*_{1,Q_0}, the current quantity Q_0 would be enough for the investor to exercise the option to invest immediately. Alternatively, for $K_1 > K^*_{1,Q_0}$, the amount $K_1 - K^*_{1,Q_0}$ can be understood as a subsidy value on investment that might be supported by the government if the goal is to deploy the investment immediately without the need of changing the policy with respect to the minimum quantity guarantee L. That is, the government supports the firm with a one-time subsidy amount that is paid upfront, while maintaining the fixed quantity floor L until the end of the finite-lived FIT scheme. A similar reasoning van be employed when comparing the marginal costs K_2 and K^*_{2,Q_0}.

4.6. Quantity floor level making the net present value equal to zero

A further interesting point from the policymaker perspective is the quantity floor level L_0 that turns the net present value (henceforth, NPV) of the project equal to zero, because any value of L above this point generates a positive NPV independently of the quantity Q. Notice that if $L \geq L_0$ the investment will be deployed immediately generating a risk-free profit and, hence, there is no waiting option. This point can be determined analytically by solving equation NPV $:= \lim_{Q_0 \to 0^+} V^F_f(Q_0, L_0, T) - K_1 = 0$ with respect to L_0 and with $V^F_f(Q_0, L_0, T)$ being defined as the project value given in equation (17), which yields

$$
L_0 = \frac{rK_1 + K_2}{P\left(1 - e^{-rT}\right)}, \qquad (31)
$$

after applying straightforward calculus.

4.7. Investment and marginal costs making the net present value equal to zero

Finally, assuming a given floor level L that is defined by the government, it is also possible to find the investment cost K^*_{1,L_0} and the marginal cost K^*_{2,L_0} that make the current L to coincide with the L_0 level, that is

$$K^*_{1,L_0} = \frac{PL\left(1 - e^{-rT}\right) - K_2}{r} \tag{32}$$

and

$$K^*_{2,L_0} = PL\left(1 - e^{-rT}\right) - rK_1, \tag{33}$$

respectively.

In summary, if $K_1 \leq K^*_{1,L_0}$ or $K_2 \leq K^*_{2,L_0}$, the investment will be deployed immediately generating a risk-free profit and, therefore, there is no waiting option.

5. Investment decisions with a quantity cap regime

The instantaneous flow payoff for this investment problem is represented by the form $P \times \min(Q_t, H) - K_2 = PQ_t - P \times (Q_t - H)\mathbb{1}_{\{Q_t \geq H\}} - K_2$. Therefore, using the arbitrage-free relation enunciated in Dias et al. (2024, equation 18), it follows that the value of a finite maturity project containing a maximum quantity H and requiring a marginal cost K_2 can be expressed as

$$V_{cp}(Q_0, H, T) = P\frac{Q_0}{q}\left(1 - e^{-qT}\right) - P \times V(Q_0, H, T) - \frac{K_2}{r}\left(1 - e^{-rT}\right). \tag{34}$$

Similarly to the quantity floor regime case, we consider that after the expiry date of the finite maturity guarantee the entrepreneur can still explore the pedestrian path with a present value equal to $P\frac{Q_0}{q}e^{-qT} - \frac{K_2}{r}e^{-rT}$. Therefore, the total (or full) value of the project that includes the period of the quantity cap contract and the (perpetual) period thereafter is given by

$$V_{cp}^{F}(Q_0,H,T) = V_{cp}(Q_0,H,T) + P\frac{Q_0}{q}e^{-qT} - \frac{K_2}{r}e^{-rT}$$
$$= P\frac{Q_0}{q} - P\times V(Q_0,H,T) - \frac{K_2}{r}. \qquad (35)$$

Next proposition shows how to determine the value of the option to invest in a project in the presence of a quantity cap regime and the corresponding optimal trigger to invest.

Proposition 3. *The value of the (perpetual) option to invest in a project containing a quantity cap regime with a finite maturity is calculated as*

$$F_{cp}(Q_0,\overline{Q},H,T,K_1) = \begin{cases} \left(V_{cp}^{F}(\overline{Q},H,T) - K_1\right)\left(\frac{Q_0}{\overline{Q}}\right)^{\beta_1} & \Leftarrow \quad Q_0 < \overline{Q} \\ V_{cp}^{F}(Q_0,H,T) - K_1 & \Leftarrow \quad Q_0 \geq \overline{Q} \end{cases}, \qquad (36)$$

where the optimal threshold of investment $\overline{Q}$ is obtained as the numerical solution of the nonlinear equation

$$(\beta_1 - 1)\,P\frac{\overline{Q}}{q}\left[\mathbb{1}_{\{\overline{Q}<H\}} + e^{-qT}N\left(d_1(\overline{Q},H)\right)\right]$$
$$-PH\left(\frac{\overline{Q}}{H}\right)^{\beta_2}\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q}\right)\left[\mathbb{1}_{\{\overline{Q}\geq H\}} - N\left(d_{\beta_2}(\overline{Q},H)\right)\right]$$
$$+\beta_1 P\frac{H}{r}\left[\mathbb{1}_{\{\overline{Q}\geq H\}} - e^{-rT}N\left(d_0(\overline{Q},H)\right)\right] - \beta_1\left(K_1 + \frac{K_2}{r}\right) = 0. \qquad (37)$$

Proof. Using the usual value-matching and smooth-pasting conditions it is possible to show that

$$\beta_1 V_{cp}^{F}(\overline{Q},H,T) - \beta_1 K_1 = \overline{Q}\,\Delta_{cp}^{F}(\overline{Q},H,T),$$

with $\Delta_{cp}^{F}(Q,H,T) := \partial V_{cp}^{F}(Q,H,T)/\partial Q$, which finally yields the nonlinear equation (37) and the option solution (36) after combining equations (7), (8) and (35), and applying some straightforward calculus. Additional details are available upon request. ■

The results offered in Proposition 3 solve the optimization problem defined in equation (2) for the case of an environmental cap in the form of a quantity cap regime (i.e., with

$s = cp$) and allow us to derive analytically several interesting points from the point of view of both the government and the promoter of the project, as highlighted below in this section.

5.1. Optimal trigger level making the project to start immediately

Taking the tariff price P, the investment cost K_1 and the marginal cost K_2 as given, it is possible to determine analytically the H^* level that would be required to be imposed by the regulator so that the entrepreneur admits the possibility of starting the project immediately. This is accomplished by replacing $\overline{Q}$ by H in equation (37) and solving it with respect to $H \equiv H^*$, yielding

$$H^* = \beta_1 \frac{K_1 + \frac{K_2}{r}}{PY_{cp}}, \tag{38}$$

with

$$Y_{cp} = \frac{\beta_1 - 1}{q} \left[N\left(-d_{\beta_2}\right) + e^{-qT} N\left(d_1\right) \right] + \frac{\beta_1}{r} \left[N\left(d_{\beta_2}\right) - e^{-rT} N\left(d_0\right) \right]. \tag{39}$$

Interestingly, it is possible to relate the optimal trigger levels L^* and H^* by combining equations (20) and (38), that is

$$H^* = \frac{Y_f}{Y_{cp}} L^*, \tag{40}$$

where the ratio $\frac{Y_f}{Y_{cp}}$ indicates how much higher is the H^* threshold with respect to the L^* trigger (for $H \geq L$ and assuming that the remaining parameters are the same for both contracts). Even though the quantity floor and the quantity cap under analysis are independent (i.e., L and H are inputs of two different contracts), this novel result might be important for managers and regulators when designing optimal floor and cap levels in the case of a collar contract (and, hence, with L and H being inputs of the same contract).

5.2. Tariff price making the current cap level the optimal trigger

We notice that we can immediately determine the tariff price P_H^* that makes the current cap level H the trigger by rearranging equation (38) to

$$P_H^* = \beta_1 \frac{K_1 + \frac{K_2}{r}}{HY_{cp}}. \tag{41}$$

In this case, it is possible to relate the prices P_L^* and P_H^* by combining equations (23) and (41), that is

$$P_H^* = \frac{L}{H}\frac{Y_f}{Y_{cp}}P_L^*. \tag{42}$$

Moreover, combining equations (40) and (42) yields the following simply relation

$$\frac{H^*}{L^*} = \frac{P_H^*}{P_L^*}\frac{H}{L}. \tag{43}$$

6. Investment decisions with a quantity collar regime

The instantaneous flow payoff for this investment problem is represented by the form $P \times \min\left(\max\left(L, Q_t\right), H\right) - K_2 = PL + P \times (Q_t - L)\mathbb{1}_{\{Q_t \geq L\}} - P \times (Q_t - H)\mathbb{1}_{\{Q_t \geq H\}} - K_2$. Therefore, using the arbitrage-free relation enunciated in Dias et al. (2024, equation 30), it follows that the value of a finite maturity project containing a minimum quantity guarantee L, a maximum cap level H ($> L$) and requiring a marginal cost K_2 can be expressed as

$$V_{co}(Q_0, L, H, T) = P \times V(Q_0, L, T) - P \times V(Q_0, H, T) + \frac{PL - K_2}{r}\left(1 - e^{-rT}\right). \tag{44}$$

Following, for instance, Adkins et al. (2019), Barbosa et al. (2020) and Dias et al. (2024), we consider that after the expiry date of the finite maturity guarantee the entrepreneur can still explore the pedestrian path with a present value equal to $P\frac{Q_0}{q}e^{-qT} - \frac{K_2}{r}e^{-rT}$, that is understood as the value of exploring the path after the maturity date of the collar arrangement. Therefore, the total (or full) value of the project that includes the period of the collar contract and the (perpetual) period thereafter is given by

$$\begin{aligned} & V_{co}^F(Q_0, L, H, T) \\ = \; & V_{co}(Q_0, L, H, T) + P\frac{Q_0}{q}e^{-qT} - \frac{K_2}{r}e^{-rT} \\ = \; & P \times V(Q_0, L, T) - P \times V(Q_0, H, T) + P\frac{L}{r}\left(1 - e^{-rT}\right) + P\frac{Q_0}{q}e^{-qT} - \frac{K_2}{r}. \end{aligned} \tag{45}$$

Next proposition shows how to determine the value of the option to invest in a project in the presence of a quantity collar regime and the corresponding optimal trigger to invest.

Proposition 4. *The value of the (perpetual) option to invest in a project containing a quantity collar regime with a finite maturity is calculated as*

$$F_{co}(Q_0, \overline{Q}, L, H, T, K_1) = \begin{cases} \left(V_{co}^F(\overline{Q}, L, H, T) - K_1\right)\left(\frac{Q_0}{\overline{Q}}\right)^{\beta_1} & \Leftarrow \quad Q_0 < \overline{Q} \\ V_{co}^F(Q_0, L, H, T) - K_1 & \Leftarrow \quad Q_0 \geq \overline{Q} \end{cases}, \tag{46}$$

where the optimal threshold of investment $\overline{Q}$ is obtained as the numerical solution of the nonlinear equation

$$\begin{aligned}
&(\beta_1 - 1)\, P\frac{\overline{Q}}{q}\left[\mathbb{1}_{\{L \leq \overline{Q} < H\}} + e^{-qT}\left(N\left(-d_1(\overline{Q}, L)\right) + N\left(d_1(\overline{Q}, H)\right)\right)\right] + P\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q}\right) \\
&\times \left[L\left(\frac{\overline{Q}}{L}\right)^{\beta_2}\left(\mathbb{1}_{\{\overline{Q} \geq L\}} - N\left(d_{\beta_2}(\overline{Q}, L)\right)\right) - H\left(\frac{\overline{Q}}{H}\right)^{\beta_2}\left(\mathbb{1}_{\{\overline{Q} \geq H\}} - N\left(d_{\beta_2}(\overline{Q}, H)\right)\right)\right] \\
&+\beta_1 P\left[\frac{L}{r}\left(\mathbb{1}_{\{\overline{Q} < L\}} - e^{-rT}N\left(-d_0(\overline{Q}, L)\right)\right) + \frac{H}{r}\left(\mathbb{1}_{\{\overline{Q} \geq H\}} - e^{-rT}N\left(d_0(\overline{Q}, H)\right)\right)\right] \\
&-\beta_1\left(K_1 + \frac{K_2}{r}\right) = 0.
\end{aligned} \tag{47}$$

Proof. Using the usual value-matching and smooth-pasting conditions it is possible to show that

$$\beta_1 V_{co}^F(\overline{Q}, L, H, T) - \beta_1 K_1 = \overline{Q}\, \Delta_{co}^F(\overline{Q}, L, H, T),$$

with $\Delta_{co}^F(Q, L, H, T) := \partial V_{co}^F(Q, L, H, T)/\partial Q$, which finally yields the nonlinear equation (47) and the option solution (46) after combining equations (7), (8) and (45), and applying some straightforward calculus. Additional details are available upon request. ■

The results offered in Proposition 4 solve the optimization problem defined in equation (2) for the case of a subsidy in the form of a quantity collar regime (i.e., with $s = co$) and allow us to derive analytically several interesting points from the point of view of both the government and the promoter of the project, as highlighted below in this section.

6.1. Optimal trigger levels making the project to start immediately

Taking the tariff price P, the cap level H, the investment cost K_1 and the marginal cost K_2 as given, it is possible to determine numerically the L^* ($< H$) level that would be required to be offered by the government so that the entrepreneur starts the project immediately. This is accomplished by replacing $\overline{Q}$ by L in equation (47) and solving it (numerically) with respect to $L \equiv L^*$, yielding

$$\begin{aligned}
&(\beta_1 - 1) P \frac{L^*}{q} \left[1 + e^{-qT} \left(N(-d_1) + N(d_1(L^*, H))\right)\right] \\
&+P\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q}\right)\left[L^* N(-d_{\beta_2}) + H\left(\frac{L^*}{H}\right)^{\beta_2} N(d_{\beta_2}(L^*, H))\right] \\
&-\beta_1 P\left[\frac{L^*}{r} e^{-rT} N(-d_0) + \frac{H}{r} e^{-rT} N(d_0(L^*, H))\right] - \beta_1\left(K_1 + \frac{K_2}{r}\right) = 0. \qquad (48)
\end{aligned}$$

Similarly, taking now the tariff price P, the floor level L, the investment cost K_1 and the marginal cost K_2 as given, it is possible to determine numerically the H^* ($> L$) level that would be required to be offered by the government so that the entrepreneur starts the project immediately. This is accomplished by replacing $\overline{Q}$ by H in equation (47) and solving it (numerically) with respect to $H \equiv H^*$, yielding

$$\begin{aligned}
&(\beta_1 - 1) P \frac{H^*}{q} e^{-qT} \left(N(-d_1(H^*, L)) + N(d_1)\right) \\
&+P\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q}\right)\left[L\left(\frac{H^*}{L}\right)^{\beta_2} N(-d_{\beta_2}(H^*, L)) - H^* N(-d_{\beta_2})\right] \\
&-\beta_1 P\left[\frac{L}{r} e^{-rT} N(-d_0(H^*, L)) - \frac{H^*}{r}\left(1 - e^{-rT} N(d_0)\right)\right] - \beta_1\left(K_1 + \frac{K_2}{r}\right) = 0. \quad (49)
\end{aligned}$$

6.2. Tariff price making the current floor level the optimal trigger

Taking the cap level H, the investment cost K_1 and the marginal cost K_2 as given, we can immediately determine the tariff price P_L^* that makes the current floor level L the trigger by rearranging equation (48) to

$$P_L^* = \beta_1 \frac{K_1 + \frac{K_2}{r}}{Y_L}, \qquad (50)$$

with

$$
\begin{aligned}
Y_L &= (\beta_1 - 1)\frac{L}{q}\left[1 + e^{-qT}\left(N(-d_1) + N(d_1(L,H))\right)\right] \\
&+ \left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q}\right)\left[LN(-d_{\beta_2}) + H\left(\frac{L}{H}\right)^{\beta_2} N(d_{\beta_2}(L,H))\right] \\
&- \beta_1\left[\frac{L}{r}e^{-rT}N(-d_0) + \frac{H}{r}e^{-rT}N(d_0(L,H))\right]. \qquad (51)
\end{aligned}
$$

The following economic rational can be taken: (i) if $P_L^* > P$, the current floor level L is not enough for the firm to undertake the project; and (ii) if $P_L^* \leq P$, the current floor level L is sufficient and the firm should optimally undertake the project immediately.

6.3. Tariff price making the current cap level the optimal trigger

Taking the floor level L, the investment cost K_1 and the marginal cost K_2 as given, we can immediately determine the tariff price P_H^* that makes the current cap level H the trigger by rearranging equation (49) to

$$
P_H^* = \beta_1 \frac{K_1 + \frac{K_2}{r}}{Y_H}, \qquad (52)
$$

with

$$
\begin{aligned}
Y_H &= (\beta_1 - 1)\frac{H}{q}e^{-qT}\left(N(-d_1(H,L)) + N(d_1)\right) \\
&+ \left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q}\right)\left[L\left(\frac{H}{L}\right)^{\beta_2} N(-d_{\beta_2}(H,L)) - HN(-d_{\beta_2})\right] \\
&- \beta_1\left[\frac{L}{r}e^{-rT}N(-d_0(H,L)) - \frac{H}{r}\left(1 - e^{-rT}N(d_0)\right)\right]. \qquad (53)
\end{aligned}
$$

The following economic rational can be taken: (i) if $P_H^* > P$, the current cap level H is not enough for the firm to undertake the project; and (ii) if $P_H^* \leq P$, the current cap level H is sufficient and the firm should optimally undertake the project immediately.

Finally, we notice that a simple relation between the tariff prices making the current floor level or the current cap level the optimal triggers arises by combining equations (50) and (52), that is

$$P_H^* = \frac{Y_L}{Y_H} P_L^*. \tag{54}$$

6.4. Investment and marginal costs making the current floor or cap level the optimal trigger

Taking the tariff price P, the floor level L, the cap level H and the marginal cost K_2 as given, it is possible to find the investment cost level $K_{1,L}^*$ that makes the current floor level L the trigger by rearranging equation (50) to

$$K_{1,L}^* = \frac{PY_L}{\beta_1} - \frac{K_2}{r}. \tag{55}$$

Taking now the tariff price P, the floor level L, the cap level H and the investment cost K_1 as given, we can determine the marginal cost level $K_{2,L}^*$ that makes the current floor level L the trigger by rearranging equation (50) to

$$K_{2,L}^* = \left(\frac{PY_L}{\beta_1} - K_1 \right) r. \tag{56}$$

Similarly, assuming the tariff price P, the floor level L, the cap level H and the marginal cost K_2 as given, it is possible to find the investment cost level $K_{1,H}^*$ that makes the current cap level H the trigger by rearranging equation (52) to

$$K_{1,H}^* = \frac{PY_H}{\beta_1} - \frac{K_2}{r}. \tag{57}$$

Taking now the tariff price P, the floor level L, the cap level H and the investment cost K_1 as given, we can determine the marginal cost level $K_{2,H}^*$ that makes the current cap level H the trigger by rearranging equation (52) to

$$K_{2,H}^* = \left(\frac{PY_H}{\beta_1} - K_1 \right) r. \tag{58}$$

6.5. Investment and marginal costs making the current Q_0 level the optimal trigger

Assuming a given floor level L and a given cap level H that are both defined by the government and a fixed tariff price P settled by the firm, it is also possible to find the investment cost K^*_{1,Q_0} and the marginal cost K^*_{2,Q_0} that make the current quantity Q_0 the trigger by rearranging equation (47) to

$$K^*_{1,Q_0} = \frac{Z\left(Q_0, L, H\right)}{\beta_1} - \frac{K_2}{r} \tag{59}$$

and

$$K^*_{2,Q_0} = \left(\frac{Z\left(Q_0, L, H\right)}{\beta_1} - K_1\right) r, \tag{60}$$

respectively, with

$$\begin{aligned}
& Z\left(Q_0, L, H\right) \qquad\qquad (61)\\
= \ & \left(\beta_1 - 1\right) P \frac{Q_0}{q} \left[\mathbb{1}_{\{L \leq Q_0 < H\}} + e^{-qT}\left(N\left(-d_1(Q_0, L)\right) + N\left(d_1(Q_0, H)\right)\right)\right] \\
& + P\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q}\right)\left[L\left(\frac{Q_0}{L}\right)^{\beta_2}\left(\mathbb{1}_{\{Q_0 \geq L\}} - N\left(d_{\beta_2}(Q_0, L)\right)\right)\right. \\
& \left. - H\left(\frac{Q_0}{H}\right)^{\beta_2}\left(\mathbb{1}_{\{Q_0 \geq H\}} - N\left(d_{\beta_2}(Q_0, H)\right)\right)\right] \\
& + \beta_1 P\left[\frac{L}{r}\left(\mathbb{1}_{\{Q_0 < L\}} - e^{-rT} N\left(-d_0(Q_0, L)\right)\right) + \frac{H}{r}\left(\mathbb{1}_{\{Q_0 \geq H\}} - e^{-rT} N\left(d_0(Q_0, H)\right)\right)\right].
\end{aligned}$$

As expected, similar conclusions can be taken such as those already highlighted for the quantity floor regime case.

6.6. Investment and marginal costs making the net present value equal to zero

We notice that the point L_0 can be determined analytically by solving equation NPV := $\lim_{Q_0 \to 0^+} V_c^F(Q_0, L_0, H, T) - K_1 = 0$ with respect to L_0 and with $V_c^F(Q_0, L_0, H, T)$ being defined as the project value given in equation (45), which also yields the analytic representation (31). Moreover, in the quantity collar regime that is now under consideration, the investment and marginal costs making the NPV equal to zero are still given by equations (32) and (33), respectively. Hence, the following economic insights can be made for the investment cost case: (i) if $K_1 \in]K^*_{1,L_0}, K^*_{1,L}[$, the investment trigger $\overline{Q} \in]0, L[$; (ii) if $K_1 \in [K^*_{1,L}, K^*_{1,H}[$, the investment trigger $\overline{Q} \in [L, H[$; and (iii) if $K_1 \in [K^*_{1,H}, \infty[$, the investment trigger $\overline{Q} \in [H, \infty[$. A similar rational can be used for the case of the marginal cost, that is: (i) if $K_2 \in]K^*_{2,L_0}, K^*_{2,L}[$, the investment trigger $\overline{Q} \in]0, L[$; (ii) if $K_2 \in [K^*_{2,L}, K^*_{2,H}[$, the investment trigger $\overline{Q} \in [L, H[$; and (iii) if $K_2 \in [K^*_{2,H}, \infty[$, the investment trigger $\overline{Q} \in [H, \infty[$.

7. Investment decisions with a combined quantity collar-cap regime

The profit function (4) implies that the total (or full) value of the project is this combined quantity collar-cap regime is given by

$$V^F_{cc}(Q_0, L, H, T) = V_{co}(Q_0, L, H, T) + e^{-rT}\mathbb{E}_{\mathbb{Q}}\left[V_{cp}(Q_T, H, \infty)|\mathcal{F}_o\right], \tag{62}$$

with the forward start perpetual value $V_{cp}(Q_T, H, \infty)$ being expressed as

$$V_{cp}(Q_T, H, \infty) = P\frac{Q_T}{q} - P \times V(Q_T, H, \infty) - \frac{K_2}{r}, \tag{63}$$

after computing the limit of equation (34).

Therefore, substituting equations (44) and (63) in expression (62) and rearranging yields

$$\begin{aligned}
& V^F_{cc}(Q_0, L, H, T) \\
= \; & P \times V(Q_0, L, T) - P \times V(Q_0, H, T) + P\frac{L}{r}\left(1 - e^{-rT}\right) - \frac{K_2}{r}\left(1 - e^{-rT}\right) \\
& + P\frac{Q_0}{q}e^{-qT} - Pe^{-rT}\mathbb{E}_{\mathbb{Q}}\left[V(Q_T, H, \infty)|\mathcal{F}_o\right] - \frac{K_2}{r}e^{-rT} \\
= \; & P \times V(Q_0, L, T) - P \times V(Q_0, H, \infty) + Pe^{-rT}\mathbb{E}_{\mathbb{Q}}\left[V(Q_T, H, \infty)|\mathcal{F}_o\right] + P\frac{L}{r}\left(1 - e^{-rT}\right) \\
& + P\frac{Q_0}{q}e^{-qT} - Pe^{-rT}\mathbb{E}_{\mathbb{Q}}\left[V(Q_T, H, \infty)|\mathcal{F}_o\right] - \frac{K_2}{r} \\
= \; & P \times V(Q_0, L, T) - P \times V(Q_0, H, \infty) + P\frac{L}{r}\left(1 - e^{-rT}\right) + P\frac{Q_0}{q}e^{-qT} - \frac{K_2}{r} \\
= \; & V^F_f(Q_0, L, T) - P \times V(Q_0, H, \infty),
\end{aligned} \tag{64}$$

after applying the time decomposition technique of Shackleton and Wojakowski (2007) to the finite-lived cap $V(Q_0, H, T)$ appearing in the first equality of equation (64) and with the perpetual cap $V(Q_0, H, \infty)$ being given by

$$V(Q_0, H, \infty) = A(H)Q_0^{\beta_1}\mathbb{1}_{\{Q_0<H\}} + \left(B(H)Q_0^{\beta_2} + \frac{Q_0}{q} - \frac{H}{r}\right)\mathbb{1}_{\{Q_0\geq H\}}. \quad (65)$$

In summary, and as expected, equation (64) reveals that the combined quantity collar-cap regime under analysis can be also understood as a long position in a finite maturity quantity floor and a short position in a perpetual (environmental) quantity cap.

Next proposition shows how to determine the value of the option to invest in a project in the presence of a combined quantity collar-cap regime and the corresponding optimal trigger to invest.

Proposition 5. *The value of the (perpetual) option to invest in a project containing a combined quantity collar-cap regime is calculated as*

$$F_{cc}(Q_0, \overline{Q}, L, T, K_1) = \begin{cases} \left(V_{cc}^F(\overline{Q}, L, H, T) - K_1\right)\left(\frac{Q_0}{\overline{Q}}\right)^{\beta_1} & \Leftarrow \quad Q_0 < \overline{Q} \\ V_{cc}^F(Q_0, L, H, T) - K_1 & \Leftarrow \quad Q_0 \geq \overline{Q} \end{cases}, \quad (66)$$

where the optimal threshold of investment $\overline{Q}$ is obtained as the numerical solution of the nonlinear equation

$$\begin{aligned} &(\beta_1 - 1)\,P\frac{\overline{Q}}{q}\left[\mathbb{1}_{\{L\leq\overline{Q}<H\}} + e^{-qT}N\left(-d_1(\overline{Q}, L)\right)\right] \\ &+P\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{q}\right)\left[L\left(\frac{\overline{Q}}{L}\right)^{\beta_2}\left(\mathbb{1}_{\{\overline{Q}\geq L\}} - N\left(d_{\beta_2}(\overline{Q}, L)\right)\right) - H\left(\frac{\overline{Q}}{H}\right)^{\beta_2}\mathbb{1}_{\{\overline{Q}\geq H\}}\right] \\ &+\beta_1 P\left[\frac{L}{r}\left(\mathbb{1}_{\{\overline{Q}<L\}} - e^{-rT}N\left(-d_0(\overline{Q}, L)\right)\right) + \frac{H}{r}\mathbb{1}_{\{\overline{Q}\geq H\}}\right] - \beta_1\left(K_1 + \frac{K_2}{r}\right) = 0. \quad (67) \end{aligned}$$

Proof. Using the usual value-matching and smooth-pasting conditions it is possible to show that

$$\beta_1 V_{cc}^F(\overline{Q}, L, H, T) - \beta_1 K_1 = \overline{Q}\,\Delta_{cc}^F(\overline{Q}, L, H, T),$$

with $\Delta_{cc}^F(Q, L, H, T) := \partial V_{cc}^F(Q, L, H, T)/\partial Q$, which finally yields the nonlinear equation (67) and the option solution (66) after combining equations (7), (8), (64) and (65), and applying some straightforward calculus. Additional details are available upon request. ■

The results offered in Proposition 5 solve the optimization problem defined in equation (5) and allow us to derive analytically several interesting points from the point of view of both the government and the promoter of the project, as shown in the remainder of this section.

8. Comparative statics

9. Conclusions

Acknowledgements

This work was supported by the Fundação para a Ciência e a Tecnologia [grant number UIDB/00315/2020].

References

Adkins, R., Paxson, D., Pereira, P.J., Rodrigues, A., 2019. Investment decisions with finite-lived collars. Journal of Economic Dynamics and Control 103, 185–204.

Barbosa, L., Ferrão, P., Rodrigues, A., Sardinha, A., 2018. Feed-in-tariffs with minimum price guarantees and regulatory uncertainty. Energy Economics 72, 517–541.

Barbosa, L., Nunes, C., Rodrigues, A., Sardinha, A., 2020. Feed-in tariff contract schemes and regulatory uncertainty. European Journal of Operational Research 287, 331–347.

Dias, J.C., Nunes, J.P., Silva, F.C., 2024. Novel analytic representations for caps, floors and collars on continuous flows, arbitrage-free relations and optimal investments Working Paper, Iscte Business School.

Shackleton, M.B., Wojakowski, R., 2007. Finite maturity caps and floors on continuous flows. Journal of Economic Dynamics and Control 31, 3843–3859.