A MODEL FOR A SUBWAY FLEET SIZING

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ABSTRACT

Capital investment in acquiring the fleet required for a subway line has relevant impacts on the system's total cost. Nonetheless, the traditional methods used to determine the optimal fleet typically involve static models that do not consider the uncertainties of shifting passenger demand or the technical uncertainties of the operation. The fundamental problem is knowing the specific and necessary number of trains for each circuit before planning the CAPEX of rolling stock. This article presents a dynamic model for the fleet sizing of a subway system. We propose establishing the number of trains needed for an efficient operation of the subway system to match the fluctuation in the number of passengers throughout the day. The dynamic model is devised based on two types of analysis: the time trains run through the carousel and the number of trains per day conditioned on the existing passengers' demand. The results indicate that the dynamic model offers information closer to reality considering the existing uncertainties. On the contrary, the static model only considers parameters based on means and averages, allowing inconsistencies. The presented approach can help the government or other authorities better plan subway systems in determining more accurate CAPEX values planning and the cost/benefit of rolling stock acquisition, considering the passengers' comfort and convenience.

KEYWORDS: subway, fleet sizing, passenger demand.

1. INTRODUCTION

Subway systems are a fundamental link in the public transport system in major urban centres. It is an efficient mechanism that optimises people's time as it has the fundamental function of directing individuals to their main destinations quickly, safely, and efficiently. Each vehicle has an unexceedable fixed capacity, and each customer has a known demand that the service provider must fully meet. Therefore, the concessionaire must serve each customer by exactly one visit from a single train, and each train must leave the yard and return after completing several cycles on the respective carousel (Rahimi-Vahed, Crainic, Gendreau, & Rei, 2015). One of the components of a subway system is the carousel, a circular circuit that sequentially goes past through all the stations and then returns in the opposite direction until reaching the starting point, thus completing a cycle. The correct functioning of a subway system considers some

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essential variables that must be adjusted to obtain an optimal and efficient operation for the public, such as departure and arrival times, number of passengers, line length, number of stations, number and capacity of cars, period of continuous operation and stopping time at each station.

Usually, according to L. Liu, Sun, Chen, and Ma (2019), the traffic planning process consists of five steps: latent demand study, route design, network design, schedule development and trains schedules. The first of these steps is the premise and foundation of the other steps that, in turn, affect the travel choice behaviour of passengers. At the same time Schöbel (2017), considers that, in the planning process of public transportation, one can generally divide it into three stages: line planning, timetable scheduling and vehicle scheduling. In any transport system, fleet sizing is a relevant decision due to the significant capital investment required and therefore, the objective is always to determine the minimum fleet necessary to meet passenger demand. Traditionally, managers perform this analysis through static models that consider only the fixed values presented in the project, which has many limitations. This paper presents a dynamic model for the sizing of urban subway fleet where we explicitly model uncertainties and show how this model contributes to an adequate solution. In addition, it works according to the number of minutes it takes a train to complete the cycle, thus returning to the beginning of the carousel, the number of train trips needed per day, the maximum number of trains that must be on the carousel at peak times and the maximum waiting time a passenger takes to board during these peak demand periods.

There are several difficulties in obtaining an adequate and balanced subway operation. According to L. Kroon, Maróti, Helmrich, Vromans, and Dekker (2008), Shafia, Aghaee, Sadjadi, and Jamili (2012), (Jamili, Shafia, Sadjadi, and Tavakkoli-Moghaddam (2012)) and Pascual, Martínez, and Giesen (2013), the service providers have not yet fully resolved the problem of modelling both the number of passengers and the surplus at peak times. The greater the capacity of trains on each trip implies fewer passengers waiting at each station, as long as the departure time between one train and another is short and continuous. This way, excess passengers at each station are reduced during peak hours. However, the time it takes for each train's travel between stations limits the problem. Also, the time it takes cycling the carousel and the number of cycles per day. Overall, the size of the fleet influences them all.

This article analyses a simulated case study of a subway system. Modelling defines the size of the train fleet considering the random delays and uncertainties. Some aspects considered are platform delays, signalling problems, unexpected stops between stations and passenger flow. Therefore, we defined this problem as the research question: What is the train fleet required to operate a subway system, given an increase in passenger demand or a possible delay between stations? We also intend to establish the number of trains needed in CAPEX planning. A schedule should be as compact and flexible as possible to meet travel demand and passenger satisfaction. Unfortunately, many metro systems need government subsidies to cover operating expenses. As a result, planners sometimes focus more on operating costs than passenger factors when designing a schedule or timetable (Robenek, Maknoon, Azadeh, Chen, & Bierlaire, 2016).

The analysis is of great relevance today, as subway systems are a fundamental transport and one of the most utilised by the urban population as it optimises their time in going back and forth from one place to another. Therefore, its efficient and effective functioning is a basic premise for both the government and the citizens. For the government, the correct specification of variables that influence the functioning of subway systems allows it to examine the general costs. At the same time, it helps the balance that the government must consider through financial analysis, which implies the possibility of a public-private contract, which is beneficial for all economic entities.

This work is organised as follows. After this introduction, a literature review is performed, followed by a description of the research problem. In sections 2 and 3, the models and the results obtained are presented, and then we conclude.

1.1 Literature review

1.1.1 OPERATION AND OPTIMISATION OF THE SUBWAY SYSTEM

Numerous authors in the literature address the problem of operation and optimisation of the subway system. In turn, other studies explained below aim to obtain the best operational capacity of the subway system through the precise combination of all the variables involved, adjusting more to the analysis of the case study addressed. For example, the Periodic Event Scheduling Problem (PESP) model solves the cyclical railway scheduling problem introduced by Serafini and Ukovich (1989). In turn, L. G. Kroon and Peeters (2003) streamlined and developed the PESP model to obtain a longer response time through some deviations in the period of an endless cycle. The PESP considers the problem as a set of periodically recurring events under time constraints. Also, Voorhoeve (1993) uses the same model to solve the problem of periodic railway schedules to reduce and control operational costs. Other authors, such as Nachtigall (1994) and Odijk (1996), use the same PESP analysis model in the perspectives of their studies. In the particular case of Nachtigall (1994), it seeks to minimise the waiting time of passengers in the face of an increase in demand. It is based on a cycle regularity model and utilises the limitations of the PESP model to create a systematic procedure of the schedules for each train.

Khmelnitsky (2000) and R. R. Liu and Golovitcher (2003) developed analyses and simulations using variables such as the speed limits and the gradients of each one of them based on the Pontryagin Principle, used in the theory of optimised monitoring to find the best possible control for changing dynamic systems from one state to another, especially in the presence of restrictions on the state or access controls. Albrecht and Oettich (2002), use the same principle mentioned above to solve the specific control problem of trains on a carousel. In addition, they provide a proposal to resolve the recommended travel schedule for each train. The design of robust cyclical timetables is an achievement in this area to face stochastic delays, which can worsen the flow of passengers due to increased demand at stations. Some of the studies that approach these dynamics from different perspectives are L. Kroon et al. (2008), Shafia et al. (2012) and Jamili et al. (2012). Pascual et al. (2013), studied asset management to calculate the size of a rolling stock fleet and the maintenance capacity that adjusts to a subway system. The proposed analytical model uses the overall cost rate, availability and performance as profit indicators. Within the overall cost components, these involve the opportunity cost associated with not operating at the correct time, the cost of vehicle downtime (can be adjusted for the train fleet) and the cost of downtime of maintenance resources — all to create a balance of command to be able to identify the main compensations of the system.

Gomes, e Aguiar, and Vils (2018), agree that the speed of trains significantly interferes with energy consumption and optimisation of the subway system. Several authors study this dynamic in different ways and consider that speed optimisation is beneficial for the better functioning of the subway system through train fleet optimisation (Xu, Li, & Li, 2016; Yin, Yang, Tang, Gao, & Ran, 2017), precise control of the train fleet (Douglas, Roberts, & Hillmansen, 2016; Su, Tang, & Wang, 2016), synchronisation between the train fleet (L. Li, Wang, Liu, & Chen, 2017; Tian, Weston, Hillmansen, Roberts, & Zhao, 2016) and the differentiation of rail line distributions (Popescu, Bitoleanu, Deaconu, & Dobriceanu, 2016). A fundamental premise of the metro system, also known as CBTC (Communication Based Train Control), is that ATP (Automatic Train Protection), as the whole security system of all metro lines on the train, recognises its location and communicates it directly. Through data communication (Wi-Fi or radio) to the ATP, which controls the traffic and route, demonstrated by Yu (2015). As the control system interprets the train's location, the track map, and its destination, it can manage its acceleration and braking curves and stops and breaks at stations. Zafar, Khan, and Araki (2012), suggest that a good synchronisation can be obtained in the general system from the proper analysis of these variables.

Efficient use of rolling stock (vehicle scheduling) is an important objective to be pursued by a railway company due to the investment required being capital intensive (Robenek et al., 2016). With this objective, Lai, Fan, and Huang (2015) develop an optimisation model to improve the efficiency of rolling stock use considering the necessary regulations and practical limitations. In addition, they designed a hybrid heuristic process to improve the quality and efficiency of the solution. Haahr, Wagenaar, Veelenturf, and Kroon (2016), use CPLEX (IBM optimisation software) and a column and row approach to assign rolling stock units to schedule services on passenger rail lines, prepare daily schedules, and verify their applicability in real-time testing different outage scenarios. CPLEX is used to construct a Pareto Boundary of contradictory objectives. The Pareto principle states that, for many events, approximately 80% of the effects come from 20% of the causes. Thus, with the use of the optimisation software it is intended to find the points of sensitivity of the train system for better functioning. S. Li, Dessouky, Yang, and Gao (2017), combine the dynamic rules of trains and passenger controls to minimise times and deviations in the service provision of subway lines, thus reducing the loss of operator revenues and delays for the passengers.

K. Li, Huang, and Schonfeld (2018), propose a continuous optimisation algorithm to obtain non-cyclical schedules considering passenger demand that varies over time and the effects of congestion at stations. It integrates goals into line planning (frequency), scheduling (conflicting goals, including passenger wait/time and vehicle power) and vehicle scheduling (train cost). They precisely model the dynamic evolution of the number of passengers on trains at each station, considering passenger arrival rates, limited train capacity, and actual passenger boarding/disembarkation rates associated with agglomeration. They also show that passengers boarding and arriving and the respective congestion ratios at stations point to the length of stay. Therefore, they should be considered parameters that depend on other decision variables, i.e., first station exit times and segment travel times.

1.1.2 USER SATISFACTION WITH THE SUBWAY SYSTEM

At the same time, many research models address user satisfaction based on the analysis of different variables that influence the effective functioning of the subway system. Börjesson and Rubensson (2019), for example, analyse the interaction between satisfaction and importance for new quality attributes in Stockholm data and explore the interaction between satisfaction and performance. The main focus is on agglomeration and reliability. One of the findings is that agglomeration is among the attributes with the lowest satisfaction and the only attribute in which satisfaction decreases over time. Still, crowding is less important than the cognitive attributes: reliability and frequency. The higher the reliability and less crowding, the more satisfied passengers are with these attributes. An intriguing result of this study is that, although the importance of agglomeration is negligible unless it reaches very high levels, it is still the attribute with the lowest levels of satisfaction.

Yap and Cats (2021), estimate a discrete choice model based on observed route alternatives to infer how passengers value waiting time after being denied boarding on crowded public transport networks. It provides a quantitative indication of how the passengers perceive the wait time compared to the initial wait time for the first arriving train and the time inside the vehicle. The analysis allows for a better understanding of the impact of overcrowding in public transport on passengers' travel experiences and route choice decisions. Furthermore, the results confirm that the extra waiting time is perceived more negatively after being denied boarding than the initial waiting time.

Björklund and Swärdh (2017), estimate values for comfort, defined by obtaining a seat and reducing the capacity on board local public transport in Sweden. They use stated preference data and present '*crowding'* as a neutral crowding among standing travellers portrayed in images presented to respondents. They analyse whether there are differences in the willingness to pay to have more comfort and reduction of agglomeration. In general, they argue that preferences vary according to the mode of travel, income, and purpose of the trip. Soza-Parra, Raveau, Muñoz, and Cats (2019), found that the primary source of insecurity is the progress of service reliability, which also affects waiting times and distributes passengers unevenly between vehicles. They investigate the existence of non-linearity in user satisfaction caused by both levels of agglomeration and the number of denied boardings using a survey of bus and subway users that assesses post-service satisfaction. They indicate that wait time reliability and concurrency levels have a powerful impact on user satisfaction assessment. In addition, the nonlinear relationship between satisfaction and level of crowding further exacerbates the impact of unreliability and overcrowding on the passenger experience.

In the particular event of this case study, the focus is on the user's waiting time and other variables to obtain the number of trains needed at peak times and the best possible functioning of the subway system.

1.2 Case Study

A public transport network is necessary to obtain a balance and an adequate quality in urban mobility. For this, an adequate distribution between the bus network and the collective transport formed by trains and subway is necessary. Trains and subways have an essential benefit in urban mobility. Some advantages are that the passenger goes through the ticket roulette before reaching the boarding platform, which is on the same level as the vehicles' deck. People also save time because they are quicker to get in and out nearly immediately when the train arrives. For these reasons, some cities with integrated urban subway and bus systems use this solution to improve traffic in a specific region. In this way, citizens can move with agility in large urban centres.

The case study focuses on analysing a line in a subway system. The main agents involved are the government and the concessionaire. The former is interested in obtaining an optimal service at the lowest possible cost for the end-user, meeting the required quality, reliability, and safety parameters. The latter, meanwhile, tries to make as much profit as possible by providing this service. The biggest challenge in the operation of the subway is when the system has an irregular operation at peak times when the number of passengers increases, there is often not enough rolling stock to meet the demand; agglomerations occur in this way due to the waiting time on the platforms by individuals and the lack of continuous circulation of trains. We carried out an operational analysis based on two models: a traditional static one and a dynamic one to analyse the existing problems. Considering that, the difference between one and the other is the flexibilities of the variables where the most significant uncertainties are concentrated.

2. TRADICIONAL STATIC MODEL

We analysed the dimensioning and operations of a subway system through the traditional model, which considers the static values of the operation parameters. In this way, possible variations that can modify the system's proper functioning are not considered. The model parameters are based on typical industry data and expert input. The basic specifications and assumptions adopted in the subway line case study are these:

Table 1 - Parameters and assumptions for a model of a subway line

Path length	18 Km
Number of stations	16
Average number of daily users	600.000 passengers
Average speed between stations	36 Km/h
Business hours	06:00 to 23:00 h
Average stop at stations	30 seconds
Maximum train capacity	1.650 passengers
Average commuters per train	1175 passengers
Average interval between trains	240 seconds

Therefore, we calculate the number of trains necessary for the operation in this model:

1) **Carousel:** a train travelling at 36 km/h takes an hour to travel, not counting stops at stations, along the carousel – the round trip system on the subway line route – and arrives again at the starting point and is ready to go and restart the route.

   Stopping an average of 30 seconds at each of the 16 stations, both on the way out and back, we have 30s × (2 × 16) = 960s = 16 minutes. Added to the travel time above, this train completes the cycle in 76 minutes.

2) **Number of Trains:** with the calculation of time on the carousel made above and with an interval of 240 seconds (4 minutes) between one train and another, we have that simultaneously 76/4 = 19 trains travel on the carousel and throughout the day they will complete 255 trips or whole cycles on that line.

3) **Waiting time:** with this static model, one of the leading quality measures of service, the maximum time a passenger needs to wait to embark is already given, the fixed interval of 240 seconds between trains (4 minutes).

   With all these values taken by the average, we concluded that 19 trains are enough to support the demand of this subway system, and thus the total rolling stock necessary for the business (CAPEX) is defined too simply.

3. DYNAMIC MODEL

Dynamic modelling allows incorporating some uncertainties of the daily activities of a subway transport system. To reach the same objective of the model discussed above, defining the minimum number of trains necessary for the operation of the subway, we added sources of uncertainty that influenced the model and divided the analysis also into three parts: (1) uncertainties regarding the time to complete a cycle in the carousel, (2) the number of trains operating inside the carousel at the same time and (3) the quality of service provided reflected in the waiting time that a passenger spends waiting to board at a station.

To execute these models, we used Microsoft Excel software with the complement (add-on) @Risk version 8.1.1 developed by the company Palisade LLC, and through it, we conducted Monte Carlo simulations with 100.000 iterations for each model.

3.1 Time on the carousel

It is how long a composition remains on the carousel. The sources of uncertainties determining this time are (i) stopping time at stations for boarding and disembarking passengers, (ii) the number of seconds accelerating the composition to reach cruising speed from the moment of departure and (iii) from the same forms the braking time when the train leaves cruising speed until the train comes to a complete stop at the next station.

For the time at the stations and the acceleration and braking processes, we considered minimum, average and maximum values in a triangular distribution according to Table 2. The cruising speed is a Normal distribution with an average of 10m/s (36Km/h) and a standard deviation σ of 1.5m/s (5.4Km/h). As the Normal distribution has infinite ends, we forced limitation boundaries at 2m/s (7.0Km/h) at the lowermost and a maximum at 22.22m/s (80Km/h). In Table 2, we describe these values.

Table 2 – Uncertainties cycling through the carousel and respective distributions.

Time	at the station	on the move		
	At a standstill	**Accelerating**	**Cruising**	**Braking**
Minimum	00:20 s	0,50m/s^2		0,50m/s^2
Average	00:30 s	0,85m/s^2	10m/s	0,85m/s^2
Maximum	00:40 s	1,00m/s^2		1,00m/s^2
Std deviation σ			1,5m/s	
Distribution	Triangular	Triangular	Normal	Triangular

Considering variations on these parameters, we obtained the result shown in Figure 1.

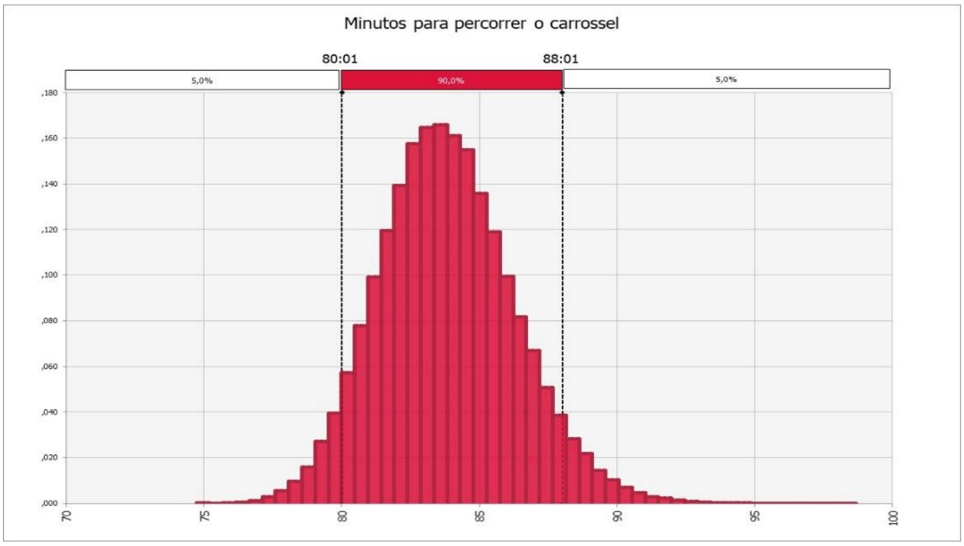

Figure 1 – How many minutes a train requires to go through a complete cycle on the carousel, repeated 100,000 times.

The simulation shows the number of minutes a train takes to complete a carousel cycle. For example, 95% of the possibilities are that a train will take up to 88:01 minutes and be ready to restart the next cycle on the carousel. The average is 83:49 minutes, which is 7:49 minutes higher than the traditional calculation. Furthermore, the minimum value of all these measurements, 74:43 minutes, is only 1:17 lower than the 76-minute average of the static model. With 100,000 simulations, in less than 1% of the simulations, the trains cruised the carousel in up to 76 minutes.

3.2 Number of trains per day

The number of trains required depends on the number of passengers throughout the day. Therefore, before calculating the number of trains, we need to calculate the demand caused by transporting people. The superposition of three distinct curves represents the public commuting on the subway line, each one with its uncertainties regarding the total number of passengers and the moment in which they reach the peak of demand: (1) “base” demand that encompasses the whole day, (2) the peak hour of people forming a peak in the morning and (3) the rush hour in the afternoon.

These curves are similar to a Normal distribution, and these passengers are quantified over time using the density function $N(\mu,\sigma^2)$. To count how many passengers (PAX) arrive at the stations in a given time interval, say between h_0 and h_1, the calculation for each of these curves is done with this integral, which in Excel is solved by numerical approximation:

$$PAX(h_0 < X < h_1) = \int_{h_0}^{h_1} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \tag{1}$$

The value obtained in each curve (base, morning peak, afternoon peak) is summed and represents the total passenger demand in this interval [h_0, h_1].

The “base” passengers are distributed through a Normal curve $N\left(\mu_b, \sigma_b^2\right)$ along the subway-operating interval, having their standard deviation σ_b fixed in 2h50min = 170 minutes and peak μ_b as shown in Table 3. The “base” curve follows a Normal distribution, but its mean, the point of the most significant influx of passengers, varies each day in a triangular distribution. As the Normal distribution admits that its extremes are infinite, the values before the opening and after the line operation closing time are allocated to both the first and the last train, respectively.

Table 3 – Triangular distribution of peak time for “base” passenger demand μ_b

Standard deviation σ_b	Minimum	Most probable	Maximum
2h50min	11:40	14:30	17:20

To define the morning and afternoon peak times, the model makes two triangular distributions as seen in Table 4: one to find the mean μ and the other to find the standard deviation σ of the respective periods.

Table 4 – Morning and afternoon peak times in triangular distribution.

	Minimum	Most probable	Maximum
Morning peak			
Time μ_m	8:15	8:30	8:45
Std deviation σ_m	0:20	0:30	0:40
Afternoon peak			
Time μ_t	18:00	18:30	19:30
Std deviation σ_t	0:20	0:30	0:40

The sum of these three inflows of passengers, base, morning and afternoon, can be represented mathematically by

$$PAX(h_0,h_1)=\int_{h_0}^{h_1} N\left(\mu_b,\sigma_b^2\right)+\int_{h_0}^{h_1} N\left(\mu_m,\sigma_m^2\right)+\int_{h_0}^{h_1} N\left(\mu_t,\sigma_t^2\right) \qquad (2)$$

totalled in Table 6 generates the daily demand proportion curve from Table 5, which is distributed throughout the day, as shown in Figure 2.

Table 5 - The subway line daily passenger (PAX) demand breakout.

PAX daily “base”	75%
PAX morning	15%
PAX afternoon	10%

Table 6 - Passenger demand, Normal distribution.

Daily PAX average	600.000
(Standard deviation σ)	18.000 (3%)

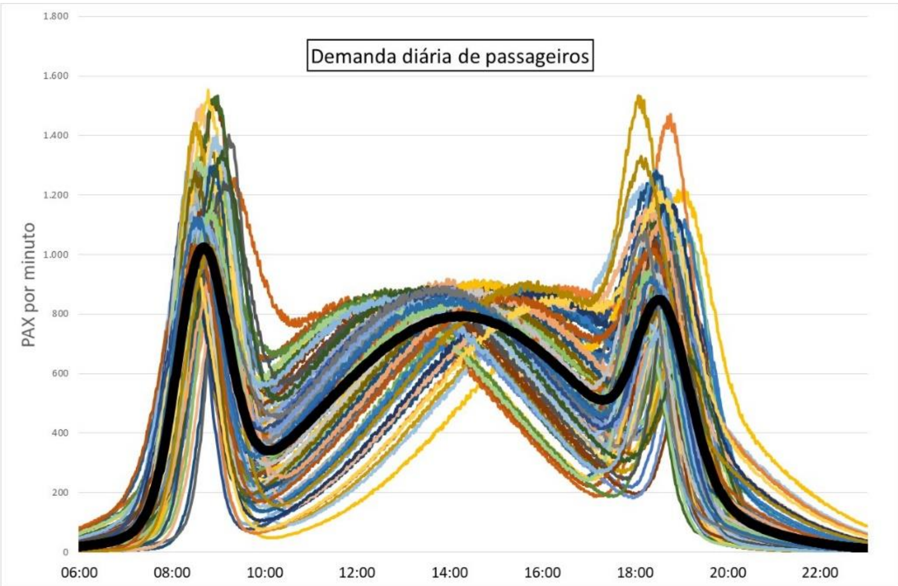

Figure 2 – Daily passenger demand (PAX).

We also defined a turnover rate of 1.25 passengers per individual seat on the train per trip. *Turnover* is when a passenger arrives at their destination station, gets off the train completing their journey, and another passenger takes their place, and the train continues the journey to further stations. In this case, the same seat is used more than once on the same trip, and this rate indicates that for every five passengers, only one gets off the train, giving way to another passenger on the same trip.

We have added a condition for the intervals (Table 7) between trains with two thresholds related to passenger capacity. The greater the number of passengers, the shorter the interval between trainsets with a consequent increase in train frequency to meet the system's demand more quickly. In addition, when that number of passengers decreases, the interval between trains increases.

Table 7- Intervals between trains

Minimum	120 seconds
Average	240 seconds
Maximum	360 seconds

When passengers are below 40% of the maximum capacity, the interval is 6 minutes between trains (360 seconds). When passing 70%, the interval is 2 minutes (120 seconds). Intermediate capacity ratios between 40% and 70% cause the respective decrease of 8 seconds for each 1% increase in capacity, as shown in Table 8. To calculate the interval between trains, we used the capacity average of the last two trains, where *l* is the train's capacity.

The range where capacity has to be calculated = $40\% \leq \left(\frac{l_{-2}+l_{-1}}{2}\right) \leq 70\%$ (3)

Then the interval between trains is calculated in seconds as:

$$\text{interval} = 360s - 8s\left(\frac{l_{-2}+l_{-1}}{2}\right) \quad (4)$$

Table 8 – Train capacity and thresholds

Minimum	1,650 PAX
Low demand threshold	40% of train capacity
High demand threshold	70% of train capacity
Range reduction factor	-8 seconds each +1%

At peak times, when the train capacity is not enough to meet the number of passengers at the stations, this number of passengers remaining accumulates waiting for the next train. During these periods, the interval between trains is always the minimum (120 seconds) as long as trains are available to enter the carousel. Moreover, the interval remains minimum until the average occupancy of two subsequent trains is less than 70%, as shown in Equations (3) and (4).

If there are no more trains available in the yard, t_x will be the time required for the first train on the carousel to complete the cycle and become available for another trip; however, if there are trains in the yard, $t_x = 0h00min$. In the model, the first interval is always 360 seconds, and with that l_{-2} and l_{-1}, occupancy rates of the two last trains are defined for the formula calculating the number of train trips for a day, the sum of intervals:

$$\text{trips per day } = 1 + \sum_{06:00:00}^{23:00:00} \left[t_x + 360s - 8s\left(\frac{l_{-2} + l_{-1}}{2} \right) \right] \tag{5}$$

With the variations of all these uncertainties presented in this section, we obtained the results of Figures 3, 4 and 5:

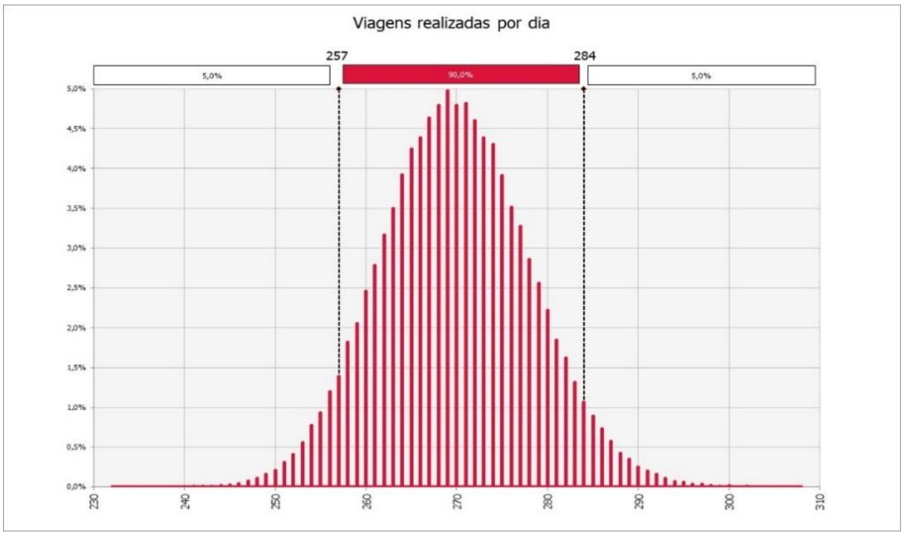

Figure 3 – Trips per day

The simulation shows the number of train trips required per day. The range of up to 284 trips meets 95% of the demand for carousel runs per day. The average is 269,9 train runs per day.

This simulation (Figure 4) reveals the maximum number of trains travelling in the carousel at the same time, and with a fleet of 40 trains, it is possible to transport up to 95% of the number of passengers at peak hours. The mean is 34.5 trains, and the mode is 33 trains. This number is high above the one obtained when calculating the “averages” that determined a fleet of 19 trains. Here we can see the importance of modelling uncertainties since CAPEX would be undersized in that case.

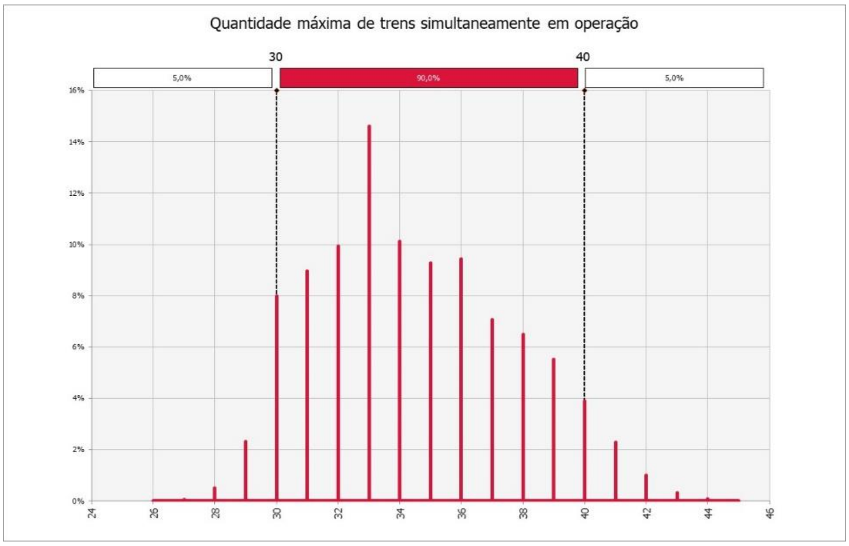

Figure 4 - Maximum number of trains simultaneously on the carousel

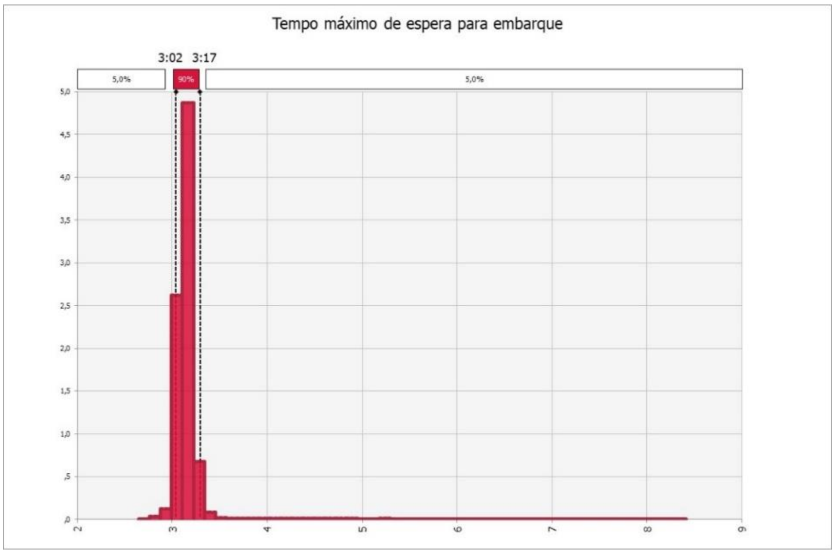

Figure 5 - Maximum waiting time to board

Figure 5 reproduces the maximum waiting time that a passenger spends to board at peak demand times. In 95% of the simulations, passengers have to wait a maximum of 3:17 minutes to board, with an average of 3:11 minutes.

This dynamic modelling aims to bring maximum comfort to the passenger with the shortest waiting time to board. However, just like the static modelling seen above, it can also lead to easy and wrong conclusions since it assumes there is no limitation on the number of trains in the yard as if the money needed for the CAPEX of rolling stock were infinite. That is why we carried out a sensitivity analysis in a complementary way to understand the relationship between the number of trains and the maximum time a passenger needs to wait before boarding. Starting from these contradictory ideas and with these mutually exclusive objectives between the (usually) government concessionary power and the company that carries out the subway operation, we show that a balance can be reached between the amount of rolling stock and the end user's satisfaction with the waiting time to board the train, thus satisfying both parties.

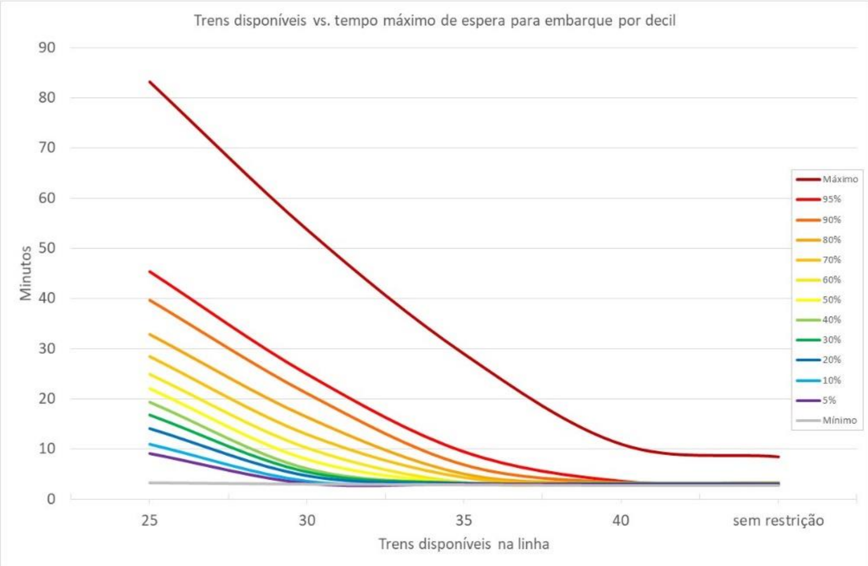

Figure 6 – Available trains and waiting time at the boarding platform

As shown in Figure 6, the waiting time changes little when there is a limitation of 40 compositions on the subway line. From the deciles analysis, around 35 compositions is a reasonable CAPEX because, in 95% of the simulations, the wait will be up to 9:27 minutes. With 30 trains, the same 95% curve will consume up to 24:58 minutes of waiting time for boarding, while the median will be 7:56 minutes. For 25 trains, the waiting time is 45:39 minutes for the same percentage curve. Thus, as the limit of available trains on the subway line decreases, the boarding waiting time increases, demonstrating an inversely proportional relationship that affects user satisfaction in the subway system. The possible conflict between user satisfactions through an on scheduled time subway train is evidenced concerning the amount of rolling stock.

In addition to all the analysis carried out previously, the real options method is executed based on the risk, uncertainties and flexibilities present in the case study.

4. CONCLUSION

The article presents an analysis of how to adjust better the number of rolling stock in a subway system. Different parameters are taken into account to design various simulations considering uncertainties from the first static situation. The main objective of the case study is to establish the number of trains necessary for an efficient fleet of the subway system and the consequent quality of service, measured by the time the passenger waits to board a train, given a variable demand of individuals over the hours of one day of operation, to satisfy the user at the same time.

The Static Analysis incorrectly shows that a fleet of 19 trains is the number necessary for a subway system operation, without considering the possibility of variations in the parameters. Dynamic analysis, on the contrary, shows that a more extensive fleet is needed, around 35 trains to transport up to 95% of the number of passengers at peak hours for the proper functioning of the subway system, almost twice the average calculated by the static analysis. In this way, the dynamic model is more relevant because it considers the different variations of the parameters, the existing uncertainties and the amount of rolling stock needed to attend to fulfil the user's satisfaction.

User satisfaction is an essential element that defines the efficient operation of a metro system. The case study shows that it is necessary to have around 35 trains available to achieve a reasonable waiting time for passengers to come across this satisfaction aspect. In addition, however, it is advisable, without exceeding the concessionaire's costs, to have some rolling stock above the lower limit in case of any breakdown or unforeseen event. All this, to meet passenger levels of trust and satisfaction.

The article contributes when using two analysis methods to obtain the rolling stock needed for optimal functioning metro systems at peak times. It also shows the imprecision and erroneous conclusion of the static analysis compared to the dynamic analysis that considers the existing uncertainties and shows more similarity that is remarkable with reality. In addition, it showed that the public power has a decision aid tool in defining the number of trains that the concessionaire needs to supply because the dynamic model predicts bottlenecks in the stations. We recommend improving the dynamic model in future studies by expanding the analysis to each subway station and allowing for energy optimisation as another base parameter in the two models developed. Furthermore, one can also be concerned about analysing other parameters, such as the satisfaction analysis of people sitting instead of standing during subway trips.

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