War of Attrition in oil exploration under uncertainty:

An asymmetrical geological case.

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Abstract

This article models asymmetry in the war of attrition in oil exploration under uncertainty. The optimum moment for the players to invest depends on price uncertainty, the length of the exploration concession contract, the geological assets parameters, and the other company's competitive action. Geometric Brownian Motion is the assumed long-term oil price model, and the two players (oil companies) War of Attrition is the game for the competitive conflict. This paper expands Dias and Teixeira (2009), adding a more realistic scenario with an asymmetric case.

The impact of strategic interactions is relevant, especially when the two firms have similar exploratory triggers. For the asymmetric games, the ratio between the exploratory and quit triggers defines the degeneration to drilling action of one of the operators. In order to perpetuate the conflict, the exploratory triggers need to be similar between the companies, even with different prospects and investment evaluations. This situation happens when the ratio between the premium and investment of the two companies' projects are similar. The upper price of the interval for perpetuating the conflict also needs to coincide in price, in effect a rare situation, since the quit trigger depends on the probability of finding hydrocarbons in the neighboring block.

Introduction

The exploratory portfolio of an oil company is composed of blocks acquired in auctions or farm-in from other oil companies. These blocks have prospects, becoming oil fields after successful exploration and appraisal campaign. There are many technical, market, and competitive uncertainties at this early stage. This article focuses on the competitive uncertainty after the auction and during the exploration contract. The competition occurs when the assets are geographically close and with geological similarities. In this situation, the well drilling may reveal information for the neighboring correlated assets, altering the optimal time for investment in the exploratory block.

In the oil exploration phase, the war of attrition occurs in the search for geological information at the lowest cost to filter out the most unlikely scenarios and readjust the planned investment to develop the production. Most petroleum agencies oblige companies to report the results of wildcat wells, and often the company itself disclosures for the news in its own interest. Depending on the companies' assets, none of the agents may invest, waiting for free information from the competitor drilling. Without cooperation between the companies, one of the agents can obtain information at no cost, known as a free rider. We will show that one player would rather drill (losing the game) than wait if the ratio of the cost and benefit of waiting is greater than the individual benefit of obtaining the information.

The positive externality is the way to analyze conflict in exploratory blocks. First, the geology dynamic creates a genetic similarity among opportunities, causing asset dependence. Therefore, there is an optimal chain of a priori investments in the entire geological basin, which depends on the probability of successful occurrence of hydrocarbons (PoS), the size, the net present value of each opportunity, and the correlation between opportunities (Smith and Thompson, 2008). It is strategic to have opportunities in the same geological play, which promptly abandons a sequence of untested opportunities in case of failure in wildcat wells. On the other hand, the success in the exploratory well generates a frontier opening for new projects. Nevertheless, there is competition between agents, as each asset is in an exploratory block with different operators. This optimal sequence may be broken or never performed due to the conflict.

The framework involves options games and the value of information theories. The case study considers two exploratory blocks with three-year contracts operated by different companies without exploratory's well obligation. Each block has a prospect with different geological features (figure 1) but at the same geological play.

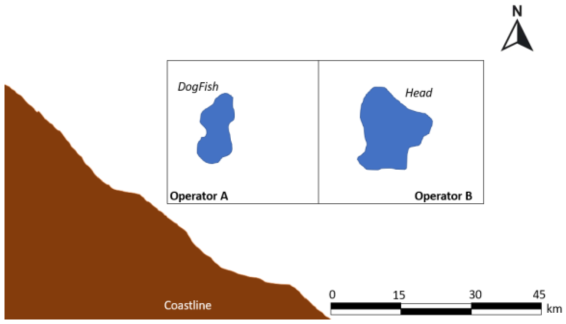

Figure 1 – Case Study

The assets can be different, but symmetry must exist to perpetuate the conflict, as we will see later. If the net present value, geological risks, and investments are similarly balanced, there are conditions for conflict and a free riders' agents. It depends on the interval of two triggers: the trigger for investing in the exploratory asset (P^{**}) and the trigger for abandoning (P^{Q}) the war of attrition.

The asymmetry case contributes to the research because it approximates the reality of the oil exploration industry. The perpetuation of a conflict for information can generate an investment gap in an exploratory oil basin. This situation alerts the regulatory bodies to develop a mechanism design to escape that trap. The government is interested in the end of the conflict since the investments increase tax revenue and boost other regional investments. However, a sequence of dry holes can devaluate the country's exploratory future activities, with the opposite effect on exploratory investment.

The first section presents the introduction of this study; section 2 shows the theoretical framework; Section 3 presents the methodology and model. The fourth part presents the results and sensitivity; Section 5 has the discussion and conclusion of the article.

2 -Bibliography Review

Smith and Price (1973) were the first to introduce the war of attrition game. It is a non-cooperative game, with no coalition between agents. Nash (1950, 1951) showed that every finite (number of players and number of strategies) game has at least one strategic equilibrium if we allow mixed strategies. Hawk and Dove game is a similar version of this game (Smith, 1976) and evolutionary game theory is the right tool for this model, a concept also defined in Smith and Price (1973). Evolutionarily stable strategy (ESS) is the equilibrium in this game, which a population of individuals is resistant to invasions or mutations. The ESS concept benefits the dynamic games with multiple equilibria, supporting the choice between subgame perfect Nash equilibrium (SPNE) more stable as a function of time, as are the case applied here.

In Smith (1974), fighting causes damage to different players just as waiting causes a cost of waiting. In this way, the player with aggressive type can fight even if the accumulated damage is more significant than the resource, and the retreat's types can also define a dispute due to a waiting cost (Smith, 1976). So, in the war of attrition game, the strategies are the delta time of resistance that the players endure the conflict. Then, the ESS occurs in mixed strategies between the fight and retreat types in this game.

The symmetry between players is the classical format to present the game; however, the asymmetry between the payoffs puts these games in a more realistic domain. According to Hammerstein and Selten (1994, p.965), in the hawk-dove game with n players and incomplete information, the ESS is pure in terms of Nash equilibrium, reflecting a situation without conflict between types. Furthermore, Hammerstein and Parker (1982), in their analysis of asymmetrical war of attrition, point out that the ratio between the benefit and cost of perpetuating the conflict is decisive to define the winning company in the dispute.

Hendricks, Weiss and Wilson (1988) formalize the continuous-time war of attrition game with complete information applied to oil exploration. They define the conflict as a function of two players' exercise time, with the leader being the first company to trigger the exploratory investment and the follower depending on the leader. In this model, the follower has an advantage in the subsequent action in exploring the field; that is, this operator prefers to wait for information from the competitor to decide to drill. Dias (1997) presents this conflict in oil exploration with the uncertainty of oil price in discrete time, using the retro-inductive binomial solution. Dias and Teixeira (2009) analyze this conflict in continuous time for a symmetrical payoff and price uncertainty, considering the migration to cooperative bargaining and presenting reflections on the asymmetrical games.

The positive externalities are the best way to analyze the exploration competition after the auction. This effect also occurs in declining market duopolies, a model presented by Ghemawat and Nalebuff (1985). Those who leave last have advantages arising from the market share left by the leading company, which is the first to exit the business.

Due to price uncertainty, the theoretical field of these studies is present in options games (Chevalier-Roignant et al., 2011). Perfect Markov Equilibrium (PME) considers games with a stochastic variable and the past events are irrelevant to the current state (Kapur, 1995; Maskin and Tirole, 1988). The PME depends only on the current state of the variable, and it is a refinement of the SPNE concept (PME is always SPNE, but not vice versa).

The game's payoffs consider the values of the defer options to explore and invest in the oil field, the concept in Dias (2004), the parameterization of the cash flow, and the value of the information. The positive externality is the information of drilling the well and the probability of geological success (PoS) dependency between opportunities (Dias and Teixeira, 2009), the same principle of portfolio effect presents in Dias and Calvette (2017). This boosted effect in valuation is due to the information from genetic connection of hydrocarbon occurrence, represented by correlation of exploratory opportunity (Dias and Calvette, 2017) and the best investment decision (Bickel, Smith, 2006; Smith and Thompson, 2008), without considering risk aversion (Bratvold et al., 2009).

3 - Model – War of Attrition in Portfolio

Stand-Alone Valuation – No conflict

An exploratory asset (equation 1) is measured by the field's Net Present Value (NPV) weighted by the probability of geological success (PoS) minus the cost of this wildcat well (I_w), a measure known as Expected Monetary Value (EMV).

$$EMV(P) = PoS\ NPV(P) - I_w \qquad (1)$$

NPV (equation 2) can be simplified by the developed oilfield minus the investment in the oil and gas production (I_d). The developed oilfield value is related to the recoverable volume (B), economic quality (q), and the long-term oil price P (Dias, 2004). The most relevant variable for investments in production (I_d) is the recoverable volume. Dias and Teixeira (2009) propose a linear function, by a gradient of line K_v and an intercept K_f.

$$NPV(P) = qBP - I_d(B) \qquad (2)$$

The long-term price is uncertain and follows a Geometric Brownian Motion (GBM – equation 3), with drift (α) and volatility (σ), where $dz \sim N(0, dt)$.

$$dP = \alpha P dt + \sigma P dz \qquad (3)$$

As the oil price is uncertain and there is an exploratory deadline for drilling the wildcat well I_w, the best way to value the exploratory opportunity E (equation 4) is with an American option to explore the field, defined by the last two boundary conditions of the partial differential equation (4).

$$\frac{\partial E}{\partial t} + (r - \delta)\frac{\partial E}{\partial P}P + \frac{1}{2}\frac{\partial^2 E}{\partial P^2}\sigma^2 P^2 = rE \qquad (4)$$

With the following boundary conditions:

- $P = 0, E(0,t) = 0$
- $t = T, E(P,T) = max(PoS \cdot NPV - I_w, 0)$
- $P = P^{**}, E(P^{**},T) = PoS \cdot NPV(P^{**}) - I_w$
- $P = P^{**}, \frac{\partial E(P^{**},t)}{\partial P} = PoS \cdot \frac{\partial NPV}{\partial P}$

Where σ and δ are the volatility and convenience yield of a barrel of oil, r is the risk-free rate and P** is the trigger to invest in the wildcat.

If we consider the development trigger (P*) to invest in the wildcat drilling, another differential equation R (equation 5), where R is the option to develop the already discovered oilfield, needs to be solved.

$$\frac{\partial R}{\partial t} + (r - \delta)\frac{\partial R}{\partial P}P + \frac{1}{2}\frac{\partial^2 R}{\partial P^2}\sigma^2 P^2 = rR \qquad (5)$$

With the following boundary conditions:

- $P = 0, R(0,t) = 0$
- $t = T, R(P,T) = max(NPV, 0)$
- $P = P^{*}, R(P^{*},T) = NPV(P^{*})$
- $P = P^{*}, \frac{\partial R(P^{*},t)}{\partial P} = \frac{\partial NPV}{\partial P}$

Similar to equation (1), Dias and Teixeira (2009) present a measure as a function of the price uncertainty and the exploratory contract, as show in equation (6). This EMV is always smaller or equal to E(P,t):

$$EMV(P,t) = PoS\ R(P,t) - I_w \qquad (6)$$

The decision to drill affects the premium received from exploring the field and the cost of obtaining that premium. As in Dias and Teixeira (2009), after an oil reserve discovery, the appraisal phase and the development study take about 2 years. Consequently, we consider a discount in the development parameters in order to consider present values. Thus, there is a discount of two year for the field premium by the risk-free rate in the risk neutral measure approach (see Trigeorgis, 1996, for risk-neutral approach details).

Portfolio Effect - Dependency on prospects

The probability of geological success (PoS) alters due to a drilling information disclosure and the correlation of the prospects (Dias, 2004). A positive result from a well drilling increases the PoS of other opportunities and raises the EMV. On the other hand, negative well drilling information shuts down this probability, and the EMV decreases (figure 2). EMVs that were negative can turn to positive with the new information, just as positive EMV can deflect the investment after negative information. The information that changes his/her own decision is what the free rider wants to get, which defines competition in a war of attrition. The maximum function is applied to conditional EMVs, as the value of information is estimated neutrally risk aversion (Bratvold et al., 2009).

Figure 2 - Conditional Probabilities and EMV

Dias and Calvette (2017) present these conditional probabilities (equations 7 and 8), where X is the informative prospect, X^+ is the success and X^- is the failure of this well drilling, Y is the affected prospect, and ρ_{XY} the correlation between the prospects PoS.

$$PoS_{Y|X^+} = PoS_Y + \frac{\sqrt{PoS_X(1-PoS_X)PoS_Y(1-PoS_Y)}}{PoS_X}\rho_{XY} \tag{7}$$

$$PoS_{Y|X^-} = PoS_Y - \frac{\sqrt{PoS_X(1-PoS_X)PoS_Y(1-PoS_Y)}}{1-PoS_X}\rho_{XY} \tag{8}$$

Model - War of Attrition

The leading firm will drill before the competitor's information, obtaining a payoff equal to the VME(P,t) (equation 6). It is also presented in Table 1 (drill, wait). This EMV also occurs in the simultaneous drilling payoff (drill, drill) of table (1). The (wait,wait) payoffs are equal to the option to explore E(P,t) when the price is below the trigger (P^{**}). When the price is above the trigger, awaiting monthly cost is equal to $c_i = VME_i(P,t)\left(1 - e^{\frac{-r}{12}}\right)$, as proposed in Dias and Teixeira (2009). This cost is a monthly penalty of the premium not acquired by the company. As we work with the risk-neutral measure, the risk-free rate r is used as a discount, and it penalizes the postponement of the deep in the money exploratory option. The follower will have the information of the occurrence of hydrocarbon provided by leader drilling result. The follower's value is a weighting by the unconditional PoS of the competitor j applied to the option to explore E conditioned by the PoS given the competitor's information, like the equation (9).

$$E_{i|j(P,t)} = PoS_j E_i\left(P,t,PoS_{i|j^+}\right) + \left(1 - PoS_j\right) E_i\left(P,t,PoS_{i|j^-}\right) \qquad (9)$$

The normal form is presented by the following matrix:

		Player 2	
		Drill (q)	Wait (1-q)
Player 1	Drill (p)	$EMV_1; EMV_2$	$EMV_1; \mathrm{E}_{2\|1}$
	Wait (1-p)	$\mathrm{E}_{1\|2}; EMV_2$	$E_1 - c_1; E_2 - c_2$

Table 1 - Normal Form - War of Attrition

Table 1 also shows the probabilities p and q for the mixed strategy equilibrium. Mixed strategy NE is the unique ESS equilibrium in the original (see Smith, 1974) and in this war of attrition game. Equation 10 shows the optimal probability p* of the player 1 drill the well maximizing the expected value of player 2:

$$p^* = \frac{EMV_2 - (E_2 - c_2)}{\mathrm{E}_{2|1} - (\mathrm{E}_2 - \mathrm{c}_2)} \qquad (10)$$

As q is the probability of player 2 investing, the optimal q* (equation 11) for maximizing the player 1 expected value is:

$$q^* = \frac{EMV_1 - (E_1 - c_1)}{\mathrm{E}_{1|2} - (\mathrm{E}_1 - \mathrm{c}_1)} \quad (11)$$

In the war of attrition, there is a price above the exploration trigger (P^{**}) that superiorly delimits the conflict over information. This oil price is called the quit trigger (P^Q) (Dias and Teixeira, 2009) and is defined for each player, and at this value, the operator is indifferent between waiting for information or drilling immediately, whereas above P^Q is strictly better to drill the prospect (even with negative neighboring information). So, by the following equality (12), we have:

$$E_{i|j}(P^Q, t) = EMV_i(P^Q, t) \quad (12)$$

With the barrel of oil at or above that price, all metrics presented so far are equal to the EMV (equation 1).

4 - Study and Sensibility

Inputs

Table 2 presents some numerical numbers for the case study. As can be seen in figure (1), the blocks have only two prospects: DogFish and Head which the recoverable volume B is in millions of barrels (MM bbl), and investments are in millions of dollars (MM$) (table 2). The DogFish is in the block operated by Company A and Head prospect is the opportunity in the block operated by Company B, both with 3 years for the exploration campaign. Operator A often will be designated as Player 1 and Operator B as Player 2.

	Operator	B (MM bbl)	q(%)	PoS(%)	I_w(MM$)	I_d (MM$)
DogFish	A	620	20	30	80	3967
Header	B	950	18	20	80	5732.50

Table 2 - Parameters of the opportunities

In this case study we adopt the parameters k_f=650 MMbbl and k_v=5.35. The correlations of the prospects are in the matrix below (table 3):

ρ	DogFish	Head
DogFish	1	0.5
Head	0.5	1

Table 3 - Dependency - Correlation

The risk-free rate is 3% p.a., the initial oil price is $50, the convenience yield is 3.8% p.a., and long-term volatility of a barrel of oil is 30% p.a.

Valuation – Stand Alone and Portfolio Effect

Figure (3) presents the option to explore the DogFish opportunity E(P,t), the EMV metric that considers the option to invest in development, and the static EMV metric, which is equivalent to the NPV of oil exploration. All are equal at considerably high prices, as all options are deep in the money at these prices. Note that the exploration option is always greater than or equal to zero for any oil price.

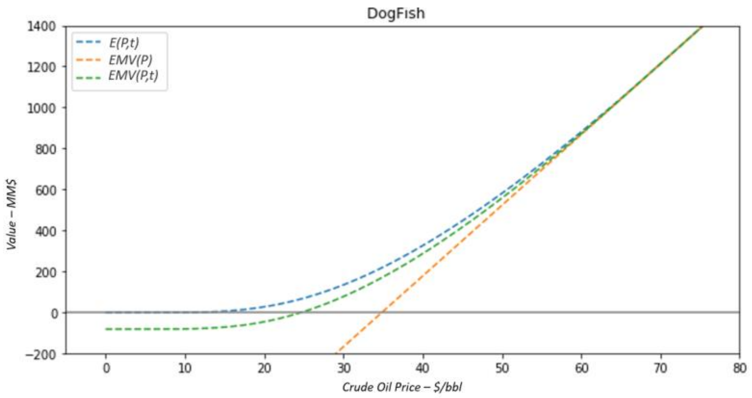

Figure 3- Option to explore (E), EMV(P,t) and EMV(P)

Table (4) presents the asset value by each metric at a current price of $50 a barrel of oil and the triggers of the option to explore. These triggers are relevant to the war of attrition, as when the current price hits the trigger, investors tend to drill the wild cat well. At the initial moment of the block concession, no opportunity will be invested, as the current price is below the trigger. However, considering the trigger price or value of wait, the first investment should be in DogFish opportunity.

	DogFish	Head
EMV (P) – (MM$)	523.08	425.12
EMV (P,t) - (MM$)	558.23	471.13
E(P,t) - (MM$)	581.28	497.71
Value of Wait (%) $\frac{E(P,t)}{EMV(P)}$	11.13	17.08
P** ($/bbl)	69.62	73.12

Table 4 - Values of the Opportunities for P = 50 $/bbl

The portfolio analysis assumes that both opportunities belong to the same operator. Thus, there is an optimal sequence exploration campaign, as one opportunity affects the value of the other. Figure (4) shows that the value of information of drilling first the DogFish opportunity is greater than the Head opportunity. At high prices, the operator is indifferent to drilling one another or both simultaneously, as the information cannot change the drilling decision. Indeed, at the current price, there is no value of information. Indeed, there is no value of the information at the current price if we do not consider it linked with the option of exploring. At 50 $\bbl, the portfolio with uncertainty considers a price downgrade chance, with the likelihood of hitting the interval with the highest value of the information of the figure (4).

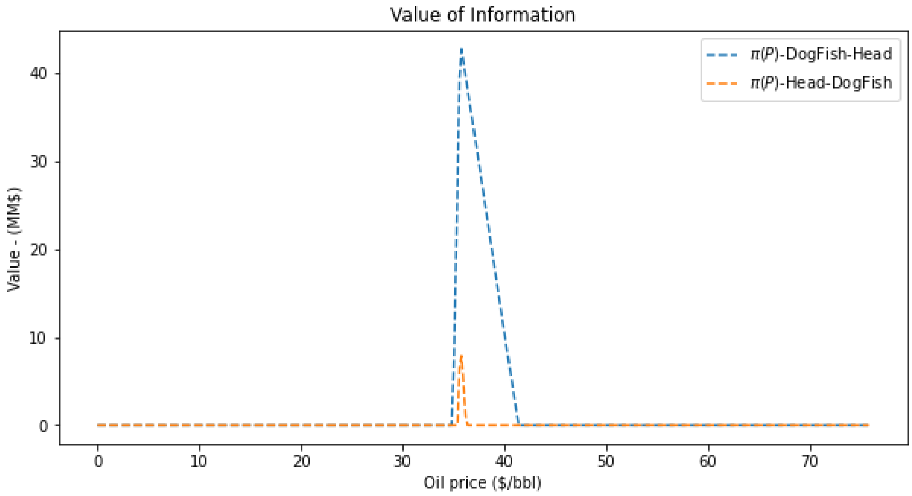

Figure 4 - Value of Information - DogFish and Head

Table (5) shows the combined value of option and portfolio effect. The sum of the wait options is always higher than the sum of the immediate decision - EMV(P,t). The difference between the portfolio with price uncertainty and the sum of the option of exploring is the VOI (P,t) parameter. This value indicates an association between waiting for the best time to drill and the portfolio effect.

Opportunities DogFish and Header	MM$
$\Sigma\, EMV\ (P)$	948.20
$\Sigma E(P,t)$	1078.99
$\Pi(P,t)$	1092.98
$VOI\ (P,t)$	13.99

Table 5 - Value - Dependency - Portfolio Effect for P=50 $/bbl

War of Attrition

The model is a Markovian game because the oil price is uncertain, with probabilistic transitions for both players in each repeated stage. The PME is the concept of equilibrium, which depends on the current price. Due to this, the game is analyzed at the beginning of the contract. The war of attrition matrix (table 1) can be mapped as a function of price using the matrix cells of the table (6). It captured the perfect Nash equilibrium in the subgames and consequently the PME. The war of attrition only occurs when the optimal action of waiting and investing for a player co-occurs with the waiting and investing for the other player, generating an impasse.

	Drill	Wait
Drill	1	2
Wait	3	4

Table 6 - Normal Form - Quadrant

Dias and Teixeira (2009) present a numerical example of symmetric game with indications of how the asymmetrical would be. To study symmetry, we can design a replica situation of the prospect in the two blocks with the same geological and investment parameters. The DogFish prospect was chosen for this exercise.

Pure holding strategies (figure 5 - quadrant 4) are PME (and also SPNE) at low prices until the exploratory trigger at P** = $69.62 bbl.

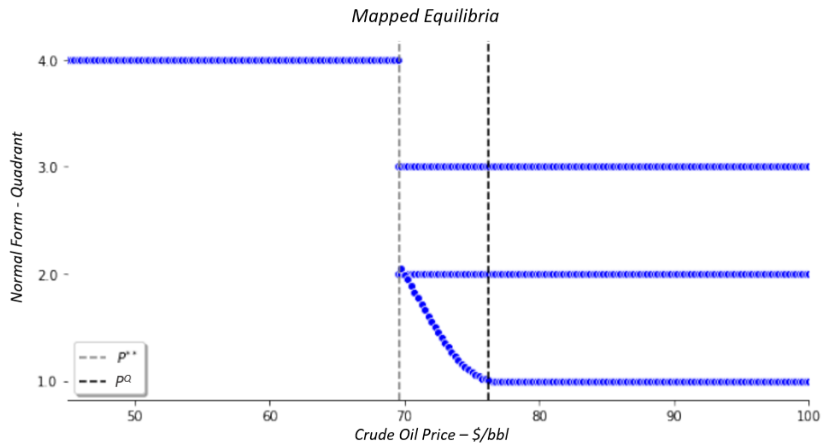

Figure 5 – Symmetric :Equilibria x Oil Price

After the trigger (P**), there are two pure equilibria (drill,wait) and (wait,drill) and one equilibrium in mixed strategies (figure 5). Figure 6 presents the phase plan for the mixed strategies probabilities when the oil price is at 70 $/bbl. The mixed strategies equilibrium is the only ESS in this interval, supporting small population oscillations (ϵ) compared to the other two degenerate equilibria (table 6- quadrants 2 and 3), as show in figure (6). Inside the conflict price interval [69.62; 76], the rising price increase the probability of one players drill the well, as the mixed strategy Nash equilibrium (between quadrants 1 and 2) get closer to the quadrant 1, which is the (drill,drill) strategy.

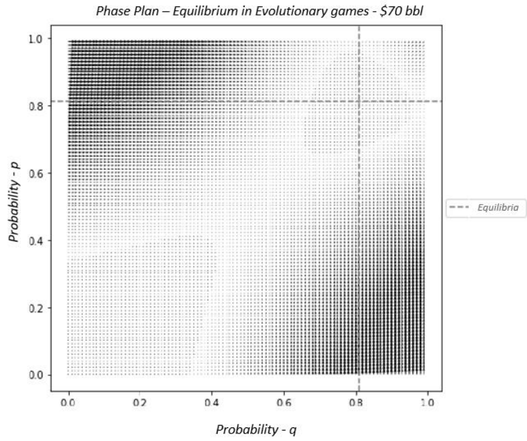

Figure 6 – Symmetric - Phase Plan -ESS: p and q probabilities for $ 70/bbl

When passing the $76 bbl quit trigger ($P^Q$), there are three equilibria (figure 5). However, the weakly dominant equilibrium is both investing (table 6- quadrant 1) and the only evolutionary stable equilibrium, as seen in phase plan of figure (7).

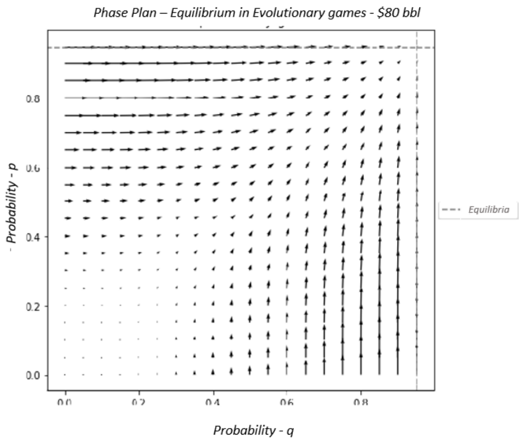

Figure 7 Symmetric - Phase Plan -ESS: p and q probabilities for $ 80/bbl

Thus, a conflict is configured in this price range, defined by the time of the first agent decides to drill (Dias e Teixeira, 2009). However, the conflict price interval changes over time till the end of the contract. As the lower bound is the exploration trigger (P^{**}), this optimum oil price of the project decreases over time. The exploration trigger is a concave function, with the maximum at the initial moment of the contract and the minimum at the end. Likewise, the upper bound (P^Q) decreases over time and approaches the oil price that VME(P) is zero. In other words, in figure (5), the price's interval of the conflict shifts to the left and decreases the window of occurrence as time as progress in the contract, collapsing at the EMV(P)=0 in a price equal to $34.83 the barrel of oil at expiration.

Nevertheless, the symmetric game is not realistic. Figure (1) presents a real situation, with two distinct opportunities. Figure (8) shows there is no interval of oil price with conflict, without mixed strategy equilibrium. At $69, player 1 drill the DogFish project, and player 2 collects the free information. Depending on DogFish result, operator B invests in Head opportunity. If the information is positive, Head's exploratory trigger jumps down from 73.12 to $ 67.37/bbl, according to equations (7 and 4) now deep in the money, as operator A invested in DogFish at 69.62 dollars per barrel.

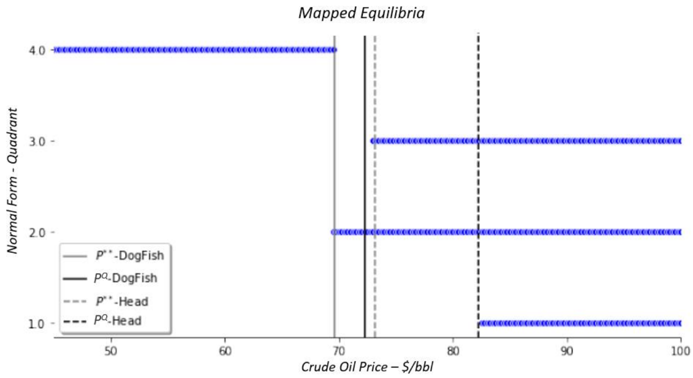

Figure 8 Asymmetric :Equilibria x Oil Price

In a remapping the Head prospect, company B's geologists increased the expected reserve to 1.31 billion of oil recover, raising the premium, investment I_d and decreasing the trigger to exploit this opportunity. This new prospect view changes the dynamics of the game (figure 9). The DogFish and Head triggers become coincident. However, the indifferent price between investing and waiting (P^Q) is still lower for DogFish's opportunity. At $76 and above, company A would prefer to drill than wait.

This overlapping of exploring triggers occurs when Head's recoverable volume changes properly. The ratio between value and investment of the DogFish is equal to the Head ratio with this new volume, as the equality (13).

$$\frac{V^D}{I^D} = \frac{V^H(B^*)}{I^H(B^*)} \tag{13}$$

Where the subscript D and H correspond to opportunities DogFish and Head, B^* is the recoverable volume of opportunity Head for this equality to be respected, $V^i = PoS_i q_i B_i P$ and $I^i = PoS_i I_{d_i} + I_{w_i}$, and i is the indicated D or H opportunity.

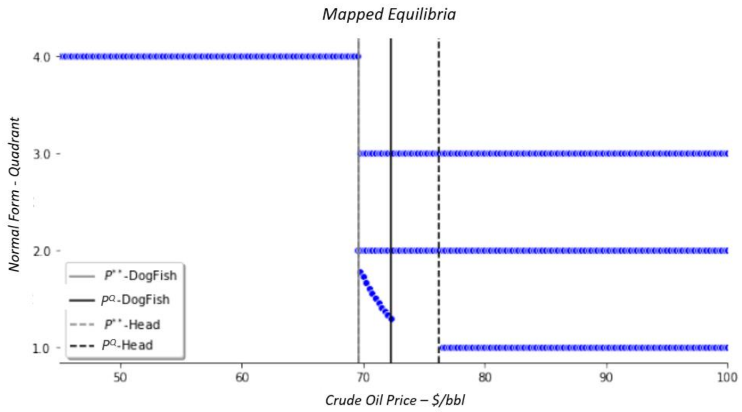

Figure 9 Asymmetric :Equilibria x Oil Price - Increase of Head Volume

In the conflict price conditions [69, 76], the payoffs are not equal for the leader and follower for each player. So, a minor disturbance of this balance can shift the probabilities in and lead to degenerate actions. In this way, the war friction may exist, but the equilibrium in mixed strategies in asymmetrical game is not ESS. The map below (figure 10) shows this instability at $70 per barrel. The curves around the equilibrium are not self-closing, with the possibility of small disturbing (ϵ) driving any player's probability to 100% for drilling.

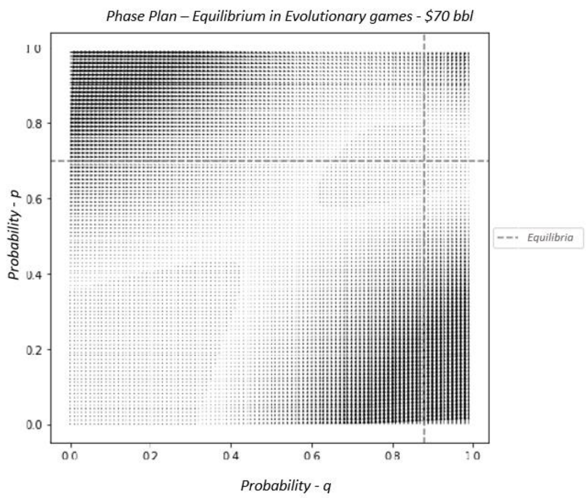

Figure 10 Asymmetric - Phase Plan -ESS: p and q probabilities for $ 70/bbl

6 – Conclusion

Even if considered the least realistic model, the symmetrical game is essential to delimit conflict conditions. There is no war of attrition at oil prices lower than the exploratory trigger as companies have no economic interest to drill. As soon as the oil price rises above the exploratory trigger, the tension between the two companies begins. As the neighboring situation is a mirror of their asset, the drilling triggers occur at the same price in both blocks and, therefore, the interest in free information begins at the same time for both agents. The end of the conflict depends not only on the oil price but also on the correlation between the assets, summarize in trigger P^{Q}. The deferment happens until the quit trigger price, common to both in this symmetrical case. With prices higher than the quit trigger, companies ignore the presence of the other operator, and invest without looking the neighbor yard.

In asymmetric models, the relationship between the waiting premium or the exploration trigger is crucial for start the game. The company with a low trigger price will not wait for the high trigger company to drill since it has an immediate economic interest in its project. On balance, it would be useless to wait, since the great moment has passed and the neighbor's trigger has not yet occurred, so there is no chance of getting free information. When the triggers of the company's prospects are disparate, the likelihood of attrition is low. In this way, the company that owns the lower trigger asset drills and shares with the other company the information.

In the case of asymmetric values, but with the same or close triggers, perfect balance occurs in subgames in mixed strategies in the interval between the exploration trigger and the lowest price of the quit trigger of one of the operators. However, this balance is not stable in evolutionary games. Given the practical effect of business, players may get "stuck" in this war of attrition and get confused about optimal projects. The phase plan helps to elucidate this issue as there is not a maximum point and therefore incentives players to deviate.

Several lines of research can extend the paper. First by the price model, considering stochastic processes with long-term mean reversion and jumps. This change has consequences for the mathematical solution of the problem. Furthermore, non-parametric cash flow models can bring nuances and different responses for each project. Future studies may detail the best contract designs for countries' energy development agencies to avoid this type of conflict and extract maximum social utility.

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