Auctions and Real Options: Bidding for Feed-in tariff

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Abstract

This paper presents novel models for firms' valuation functions of fixed-price and fixed-premium, under market uncertainty. Our models allow the identification of the equilibrium bidding, the optimal time to deploy a renewable energy project, and the optimal time of the auction. We present several findings that are aimed at policy-making decisions.

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1. Introduction

Investing in renewable energy projects is a key strategy for reducing carbon emissions and thus curbing climate change. Governments have used a myriad of policy instruments to accelerate renewable energy investments, such as feed-in tariffs (Barbosa, Nunes, Rodrigues & Sardinha 2020) and renewable energy certificate trading (Boomsma, Meade & Fleten 2012). Renewable auctions are becoming a popular mechanism for awarding projects to investors due to the potential for price discovery.

The main difference between the popular feed-in tariff and the renewable auction is the price discovery mechanism. While feed-in tariff has a price that is determined by policymakers, the price in renewable auctions is set by the participants through competitive bidding. In particular, renewable auctions are procurement auctions, in which many sellers compete to sell goods or services to a buyer, and the winning bids are the ones with the lowest prices. In renewable auctions, the buyer is the policymaker and the sellers are the renewable energy investors. In addition, most renewable auctions are multi-unit auctions which means the auction volume consists of multiple units of energy. Hence, the units are a subset of the total good. For example, the policymaker may want to auction a total power of 100 MW which is then split into blocks of 5 or 10 MW. Hence, investors submit bids with energy prices and units of energy.

In the scientific literature, some scholars have analyzed renewable auctions and investment decisions under uncertainty. For instance, Welisch & Poudineh (2020) develop an agent-based simulation to analyze UK's Contracts for Difference scheme where contracts are awarded through renewable auctions. Through a literature review, the authors state that UK's Contracts for Difference have several options, such as the option to invest all the capacity at once or in phases, and the option to default. However, the agent-based simulation does not implement these options and analyze them. Matthäus, Schwenen & Wozabal (2021) analyze the option to invest (or default) on the awarded projects. The results show that bidders with a high option value are more aggressive, regardless of the auction format. However, this work does not consider the optimal auction timing, which can lead to higher welfare.

This work proposes a novel model for renewable auctions that derives the optimal auction timing, optimal bidding, and optimal investment timing. We include in our model two different contracts, namely a fixed-price and premium contract. Our numerical analysis also includes findings of the investment threshold and bidding strategy when we change the parameters.

This paper is organized as follows. Section 2. presents the assumptions of our model. Section 3. derives our model with the optimal auction timing, optimal bidding strategy, and optimal investment timing. Section 4. discusses the results for the numerical analysis. Finally, Section 5. presents our concluding remarks.

2. Assumptions

In this section, we present the assumptions of our models. We assume that the energy market price P follows a Geometric Brownian Motion (GBM) in Equation (1), where $P = \{P_t, t \geqslant 0\}$.

$$dP_t = \mu P_t dt + \sigma_i P_t dW_t \tag{1}$$

where $\mu < r$ is a deterministic drift, r is the discount rate, $\sigma_i > 0$ is the volatility, and W_t is the standard Brownian motion process.

Equation (2) is the profit flow of the winning bidder (i.e., producer with awarded contract) within the two different types of auctions. The first auction awards a fixed-price contract to a winning bid b_F (i.e., the producer receives b_F for each unit of energy). The second auction awards a contract to a winning bidder that receives a premium b_P over the market price for each unit of energy. In addition, we assume that the production cost is zero, which is a reasonable assumption for renewable energy projects.

$$\begin{cases} \Pi_F(P_t, b_F) = b_F Q \\ \Pi_P(P_t, b_P) = (P_t + b_P)Q \end{cases} \tag{2}$$

where Q is the energy produced.

Equation (3) is the government's instantaneous utility function for both types of auctions where the first term is the public expenditure and the second term is related to the environmental benefit in producing renewable energy[1].

$$\begin{cases} W_F(P, b_F) = (P - b_F)Q + kQ^2 \\ W_P(P, b_P) = -b_P Q + kQ^2 \end{cases} \tag{3}$$

where the constant k represents an increment in the marginal environment. So the intuition behind the second term is that for one unit of renewable energy produced, one less unit of carbon energy sources is not produce, and consequently, this effect increases the welfare.

In addition, we consider that the value of public expenditure, the first term of Equation (3), can assume two different values. First, the public expenditure of the fixed price is equal to the difference between the market price and the winning bid multiplied by the energy produced. The intuition behind this formula is that the government pays b_F (the winning bid) for every unit of

[1]Note that Equation (3) does not contain the producer surplus and consumer surplus, because in our model, for simplicity, we assume that policymakers optimize only the public expenditure and environmental damage. For future work, we will include consumer surplus and producers surplus in the analysis. Consequently, we will optimize the social welfare

energy and sells it for market price. In addition, the public expenditure of the premium is equal to the winning bid multiplied by the energy produced.

We follow Matthäus et al. (2021) and also assume truthful bidding, hence all renewable producers bid their true valuation. In addition, the valuations are assumed to be private information and are modeled as identically distributed random variables with uniform distribution. Another important assumption is that the firm sells energy for the market price when the awarded contract terminates.

3. Model

Our model has two sequential stages. In the first stage, the government/policymaker rationally chooses the best moment to hold the auction, and investors/producers submit a bid to maximize their profit. In the second stage, the winning bidder rationally chooses the best moment to invest in the project. We solve the model using backward induction. Hence, we first solve the optimal investment strategy for the winning bidder. Then, we derive the winning bid by finding the Bayesian-Nash equilibrium. Lastly, we derive the optimal auction timing.

3.1. Optimal Investment Timing for the Winning Bidder

If the winning bidder invests at time τ then he receives the amount in Equation (4), where the first term is the firm's profit until the duration of the contract T. The second term is the amount received when the contract's duration is over, whereby the producers sell energy for the market price. We use the subscript $S = \{F, P\}$ to denote the particular contract that we are considering, which can be either fixed-price F or premium P.

$$V_S(P, b_S) = E\left[\int_{\tau}^{\tau+T} \Pi_S(P_t, b_S)e^{-rt}dt + \int_{\tau+T}^{+\infty} P_t Q e^{-rt}dt | P_0 = P\right] \tag{4}$$

Consequently, the investor's optimization problem that we propose to solve is the following:

$$F_S(P, b_S) = \sup_{\tau} E\left[(V_S(P, b_S) - Ie^{-r\tau})\right] \tag{5}$$

Equation (5) is a standard investment problem, where I is the investment cost of the winning bidder. Equation (6) is the auctioneer's optimization problem, where he chooses the optimal auction timing that maximizes the utility function in Equation (3).

$$FW_S(P) = \sup_{t_a} E[W_S(P_{t_a}, b_S)e^{-rt_a}|P_0 = P] \tag{6}$$

Next, we present the value of the project and the value of the option for the winning bidder within the two auctions, namely the fixed-price and premium contract auctions. Equation (7) is the value of the project for the winning bidder with a fixed-price contract:

$$V_F(P, b_F) = \frac{b_F Q}{r}\left(1 - e^{-rT}\right) + \frac{PQ}{r-\mu}e^{-(r-\mu)T} \tag{7}$$

Proposition 1: *The value of the investment option is given by:*

$$F_F(P, b_F) = \begin{cases} (V_F(P_F^*, b_F) - I)\left(\dfrac{P}{P_F^*}\right)^{\beta_1} & \textit{for } P < P_F^* \\ \\ V_F(P, b_F) - I & \textit{for } P \geqslant P_F^* \end{cases} \tag{8}$$

where the investment threshold P_F^* is equal to:

$$P_F^*(b_F) = \frac{\beta_1}{(\beta_1 - 1)} \frac{r-\mu}{Qe^{-(r-\mu)T}} \left(I_i - \frac{b_F Q}{r} \left(1 - e^{-rT}\right) \right) \tag{9}$$

Equation 10 is the value of the project for the winning bidder with a premium contract:

$$V_P(P, b_P) = \frac{PQ}{r-\mu} + \frac{b_{Pi}Q}{r} \left(1 - e^{-rT}\right) \tag{10}$$

Proposition 2: *The value of the investment option is given by:*

$$F_P(P, b_P) = \begin{cases} \left(V_P(P_P^*, b_P) - I\right) \left(\dfrac{P}{P_P^*}\right)^{\beta_1} & \text{for } P < P_P^* \\ \\ V_P(P, b_P) - I & \text{for } P \geqslant P_P^* \end{cases} \tag{11}$$

where the investment threshold P_P^* is given by:

$$P_P^*(b_P) = \frac{\beta_1}{\beta_1 - 1} \frac{r-\mu}{Q} \left(I - \frac{b_P Q}{r} \left(1 - e^{-rT}\right) \right) \tag{12}$$

3.2. Finding the Winning Bidder

Next, we derive the strategic equilibrium in order to find the optimal bid. Recall that we assume truthful bidding, hence investors bid their true valuation. In addition, support is given to the producer who bids the lowest bid. If lower bids are more likely to win, then the probability that the producer that bids b_i wins is the probability that all the other producers bid more than b_i. Hence, the probability to renewable energy producer (i) wins the Auction is given by:

$$P(win|b_{Si}) = P(b_{Si} \leq \min_{j \neq i} b_{Sj}) = (1 - CDF(b_{Si}))^{N-1} \tag{13}$$

where CDF is the cumulative distribution function of the valuation of the producers for the subsidy. Therefore, producers choose the bid that maximizes the following Equation (i.e.: the expected value of the option of the winning bidder):

$$\mathbb{E}[F_{Si}] = P(win|b_{Si}) F_i(P, b_{Si}) \tag{14}$$

3.3. Optimal Auction Timing

Next, we derive the optimal auction timing and the policymaker's utility function. Recall that the utility function includes the public expenditure and environmental benefit in producing renewable energy.

Proposition 3: *The policymaker's utility function for a fixed-price Auction at the time of the investment is given by:*

$$SW_F(P, b_F^*(P_F^*)) = -\frac{b_F^*(P_F^*)Q}{r} \left(1 - e^{-rT}\right) + \frac{PQ}{r-\mu}(1 - e^{-(r-\mu)T}) + \frac{kQ^2}{r} \tag{15}$$

In addition, the policymaker's utility function by considering the value of the flexibilities is:

$$FW_F(P, b_F^*(P_F^*)) = \begin{cases} W_F(P_A^*, b_F^*(P_F^*)) \left(\dfrac{P}{P_A^*}\right)^{\beta_1} & \text{for } P < P_A^* \\ \\ W_F(P, b_F^*(P_F^*)) & \text{for } P \geqslant P_A^* \end{cases} \tag{16}$$

where P_A^* is the optimal auction timing

Proposition 4: *At the time of the investment, the welfare for a fixed-premium Auction is:*

$$SW_P(P, b_P^*(P_P^*)) = -\frac{b_P^*(P_P^*)Q}{r}\left(1 - e^{-rT}\right) + \frac{kQ^2}{r} \tag{17}$$

Note that the policymaker's utility function for a fixed-premium Auction does not depend on the energy market price. Hence, we do not find the optimal auction timing.

4. Analytical and numerical study

Now, we present some results from our models. The blue curve is the investment trigger as a function of the duration of the contract when bids can be viewed as fixed-price. In addition, the red curve is the investment trigger when bids can be viewed as premium-price. We can see that the investment triggers decrease as the duration of the contract increases. However, before the point that both curves meet, the investment trigger for a premium price is lower than the fixed price, which suggests that before this point a premium price is a better option for the firm. In contrast, after this point, the investment trigger of a fixed price is lower, which suggests that after this point a fixed price is a better option for the firm.

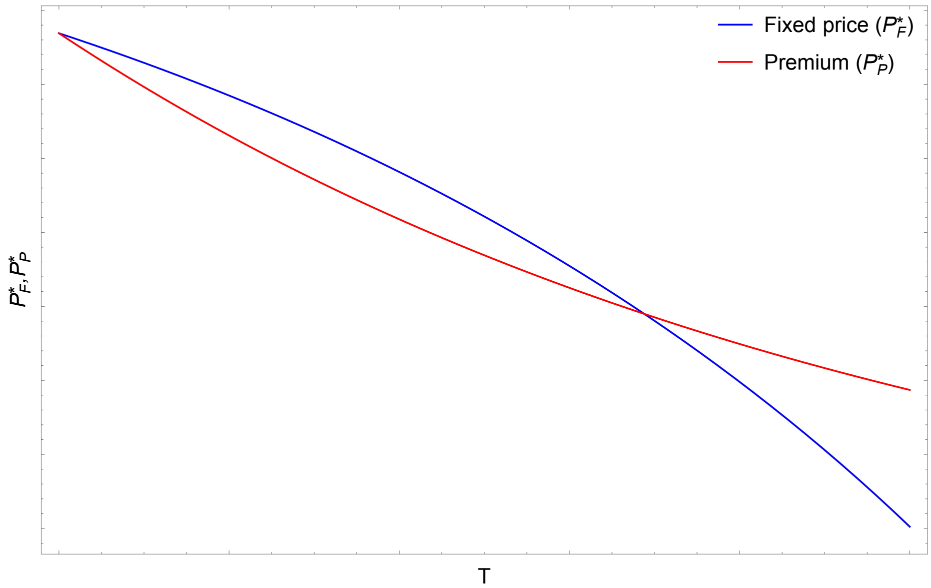


Figure 1: Triggers P_F^* and P_P^* as a function of T

Figure 2 represents the investment thresholds P_F^* and P_A^* as a function of T. The green curve is the auction trigger and the blue curve is the investment trigger of a fixed price scheme. We can see that firms do not expect that the auctioneer offers a duration of the contract greater than the point that both curves meet because it increases the public expenditure and does not accelerate investment.

Figure 3 presents the equilibrium biddings as a function of the duration of the contract. The dashed blue curve is the optimal bidding strategy of the fixed price. As we can see, the optimal bid increases as the duration of the contract increases. This result might suggest that the optimal bid increases because the investment trigger decreases as we have seen in Figure 1. Consequently, firms bid higher value because they will invest with a lower market price. In addition, the dashed red curve is the optimal bidding strategy for a premium price. In this case, the duration of the contract does not influence the optimal bidding strategy.

Figure 4 presents the equilibrium biddings and the investment thresholds for different values of the volatility. The dashed blue curve is the optimal bidding strategy of the fixed price, the dashed red curve is the optimal bidding strategy of the premium price, the blue curve is the investment

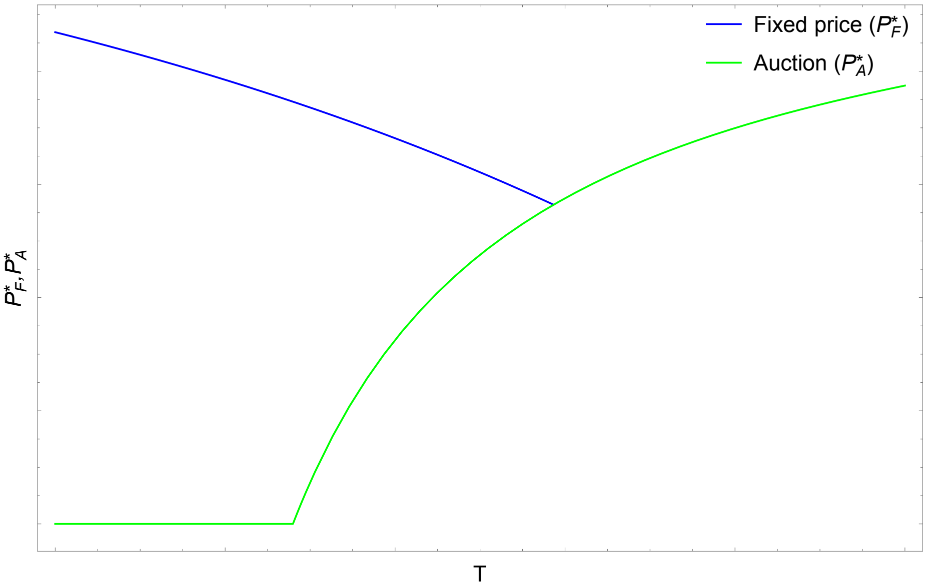
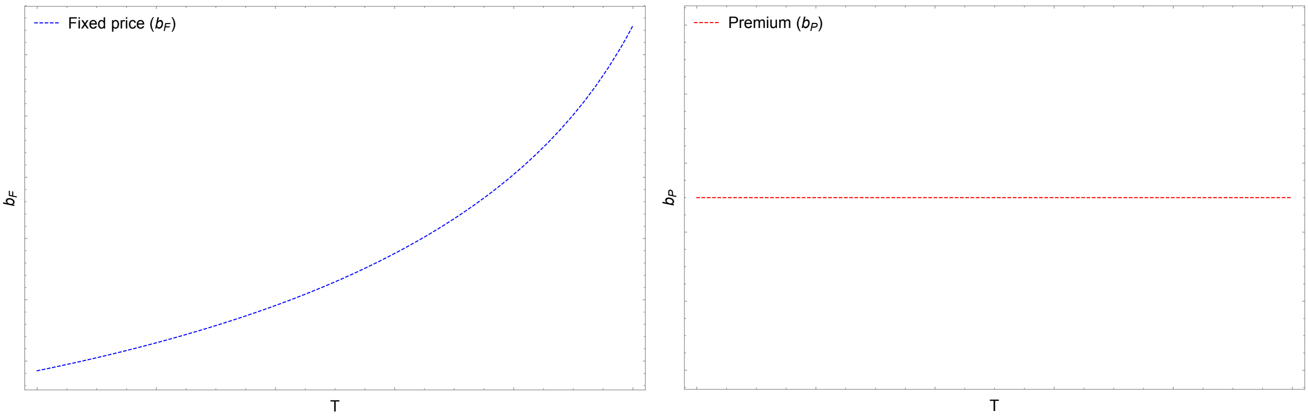


Figure 2: Triggers P_F^* and P_A^* as a function of T



(a) (b)

Figure 3: Optimal Biddings b_F^* and b_P^* as a function of T

trigger of a fixed price and the red curve is the investment trigger of the premium price. We can see that for both schemes the investment trigger increases as the volatility increases, which is consistent with the real options theory where higher volatilities increase the thresholds and consequently postpone the investment decision.

In addition, we can see that as the volatility increases, the equilibrium bidding decreases for a fixed price scheme and does not affect the equilibrium bidding for the premium price. For a fixed-price scheme, the result might suggest that the optimal bid decreases because the investment trigger increases. So, the firm bids lower value because it will invest with a higher market price.

Figure 5 present the optimal bid and the investment trigger as a function of the duration of the contract for different values of the volatility. As we already saw, the optimal bid decreases when the volatility increases, and the investment trigger increases as the volatility increases.

Figure 6 present the optimal bidding strategy and the investment trigger as a function of the contract for different values of the drift. We can see that the optimal bidding strategy and the investment trigger decrease as the drift increases because higher drifts lead to higher expected prices.

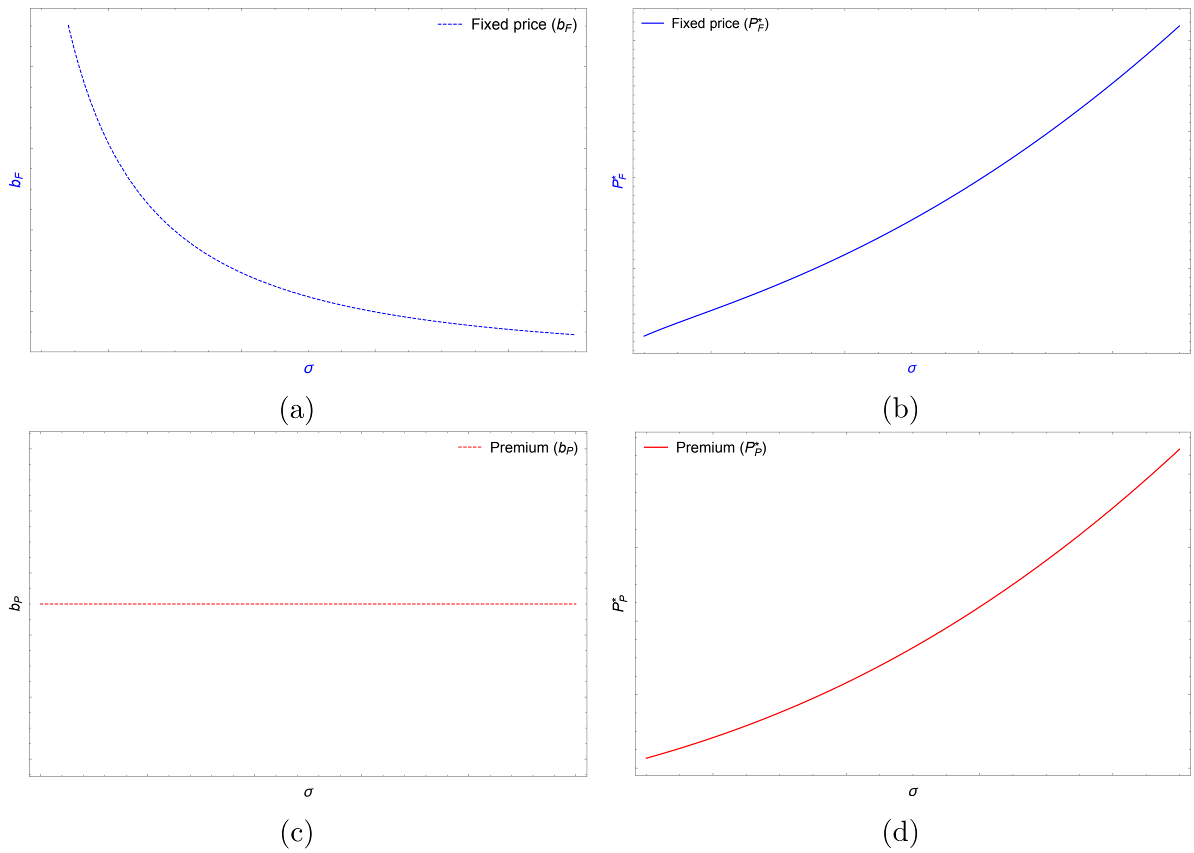


Figure 4: Optimal Biddings b_F^* b_P^* and P_F^* P_P^* as a function of σ

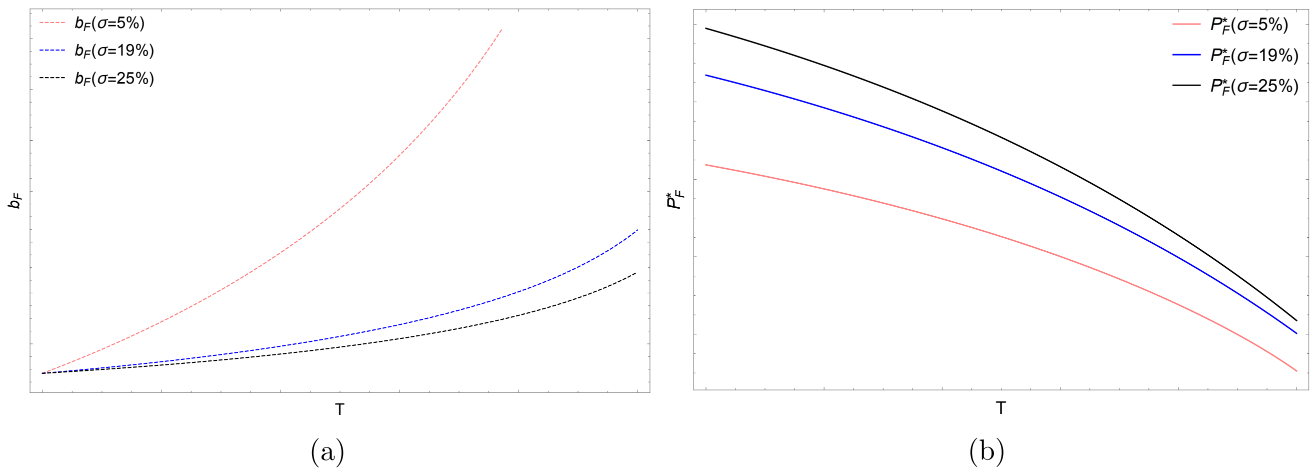
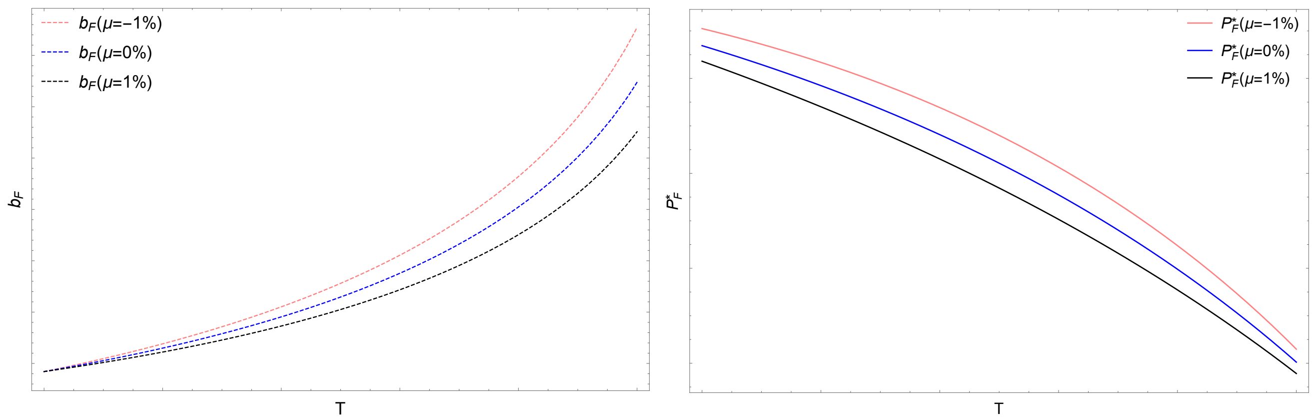


Figure 5: b_F^* and P_F^* as a function of T for different values of σ

5. Concluding Remarks

We present a novel model for a firm's valuation for fixed-price and premium contracts, which are allocated to an investor with renewable auctions. For each scheme, we use a real options framework in order to calculate the value of the project, the option value, the optimal investment threshold, and the auction optimal timing.

We find very interesting results. For example, for a fixed price, as the duration of the contract increases, the investment trigger decreases, and the equilibrium bidding increases. The model also allows for the analysis of the auctioneer's decisions.



(a) (b)

Figure 6: b_F^* and P_F^* as a function of T for different values of μ

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