Investment timing and capacity choice with overlapping ownership

(Preliminary and incomplete)

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Abstract

We study how common ownership affects the timing and size of capacity investments by duopolists leading to possible anticompetitive outcomes. Alongside accommodation and delay strategies, internalization also allows the leader to block follower entry permanently. We find that internalization magnifies the effect of leader capacity on the follower's capacity and timing. The leader generally prefers to delay the follower at low demand states. We show through a numerical example though that some internalization can be procompetitive by prompting the leader to switch to a strategy resulting in significantly larger capacities for both firms.

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1 Introduction

Horizontal ownership concentration, where a small number of investors hold significant minority stakes in otherwise rival firms, is an increasingly pervasive phenomenon which has raised regulatory concerns regarding potentially anticompetitive behavior ([3], [16]).[1] This trend is complemented by an increase in horizontal minority shareholdings since the beginning of the millennium where firms take non-controlling stakes in product market rivals. Although the empirical evidence of pricing distortions which sparked current interest in common ownership has been the subject of discussion ([2], [12]), new forms of evidence have continued to emerge such as natural and lab experiments ([7], [6]), which together lend credence to the thesis that managers respond to common owners by internalizing effects on rival firms in at least some of their decisions.

The causal mechanism linking owners to the managerial decisions that drive product market outcomes is a key theme in common ownership ([8], [1]). There is good reason to think that owners regularly engage with the management of their portfolio firms ([19]), and among the key decision that top management makes is the exercise of a firm's real options ([20]). These strategic decisions can take form of capacity commitments, which have the power to constrain the firm's short-run product market choices. In this paper we outline a novel channel for the effect of common ownership on product markets by studying how increasing internalization influences the timing and size of irreversible capacity investments under uncertainty. To do this, we extend a framework originally developed in [10] by allowing firms to internalize the effect of their investments on rivals. We identify several anticompetitive effects of common ownership, such as higher prices or blockaded follower entry, but also show that common ownership can shift the strategy

[1] These calls have not gone unheeded. For example, the Australian parliamanent recently launched an inquiry into this issue (https://www.aph.gov.au/commonownership).

of a leader firm favorably, by promoting capacity investment strongly enough to result in procompetitive long run outcomes.

We model two firms holding competing projects in a market which evolves over time according to a geometric Brownian motion. These firms have either common owners or cross-holdings, which compel them to factor rival value into their investments decisions. They determine when and how much capacity to install so as to operate in the market. In equilibrium, one of the firms acts as a leader and invests decision first, whereas the second firm is a follower which reacts to the leader's decision.

First of all, we find first that common ownership amplifies the effect of leader capacity for the follower firm, which enters later and at a smaller capacity (Proposition 1). In addition, the weight that the follower places on the leader's value creates a novel possibility for the leader to block the follower's entry permanently. The leader's strategic investment decision accordingly involves a choice between immediate duopoly, delayed duopoly, or a permanent monopoly position. We derive the leader's payoff at the capacity choice stage (Proposition 2), and find that the leader's optimal investment resembles that described by [10] at low levels of internalization? That is, the leader prefers to strategically delay the follower's entry at low demand states on and makes a more accommodating capacity choice at high demand states (Proposition 3). We show that this behavior of the leader at low demand states generalizes to arbitrary levels of internalization (Proposition 4), and that a preemption equilibrium exists if firm roles are endogenous in which the leader delays the follower's entry if the degree of internalization is not too high (Proposition 5).

We complement our analytical results by studying a numerical example. Internalization drives the leader to choose higher capacities, and makes delay relatively more advantageous for the leader. The leader therefore opts for strategic delay over a greater range of demand states, choosing a significantly higher capacity than if accommodation occurs, in the absence of internalization, so that total industry capacity ultimately increases. Finally, we calculate consumer welfare and find that the static procompetitive effects of internalization during the industry's monopoly and duopoly phases lead to higher overall consumer surplus.

Our study contributes to the growing literature on the effect of overlapping ownership on strategic behavior. A key aspect of strategy, which we do not pursue here, is innovation and it has been shown that overlapping ownership can have a positive effect on welfare in the presence of R&D spillovers, or facilitate welfare-enhancing technology transfers ([22], [17]). Yet another research stream addresses how internalization affects Stackelberg leadership, finding that it facilitates entry deterrence and may raise efficiency ([14], [15]). To our knowledge, few authors have otherwise studied how the fundamental insights regarding overlapping ownership and product market outcomes in a static setting [18] extend to a stochastic, dynamic market.

This paper also contributes to the literature on strategic investment in a dynamic setting ([10]), complementing other extensions of the baseline timing and capacity choice model which have allowed for pre-existing capacities or time to build for example ([9], [11]). The dimension of internalization which we add extends the space of strategies available to the leader firm so as to encompass three possibilities, accommodation, strategic delay, and blockade, which mirror the early work on this subject of [4] where the first mover either accommodates, deters, or blocks entry through its capacity choice. Arguably, the possibility of blockade which we identify is one which would arise in the baseline model if a suitable fixed cost were introduced, and internalization also alters the balance between accommodation and delay significantly.

Section 2 below gives the main assumptions of our model. Section 3 studies the follower problem. Section 4 studies the leader's capacity choice and derives its reduced form payoff.

Section 5 describes equilibrium investment. In section 6 we provide a numerical illustration showing a procompetitive common ownership effect.

2 Model

An industry consists of two firms which are initially inactive. Their ownership structures are symmetric and overlap, so each firm maximizes a perceived value

$$\Omega_i = V_i + \lambda V_{-i},\ \lambda \in [0, 1] \tag{1}$$

where V_i denotes the value of a firm's own assets and V_{-i} denotes the value of its rival's assets. The parameter λ is identical for both firms and represents the weight each gives to rival value. It is referred to as the *degree of internalization*, with $\lambda = 0$ representing purely self-interested behavior and $\lambda = 1$ representing joint value maximization.[2]

At any time $t \geq 0$, inverse demand is

$$X(t)\,(1 - \eta Q(t)) \tag{2}$$

where $\eta > 0$, $Q(t)$ is industry capacity, and $X(t)$ is an exogenous shock which evolves according to

$$dX(t) = \mu X(t)dt + \sigma X(t)d\omega(t) \tag{3}$$

where μ is the drift, $\sigma \geq 0$ the volatility, and $d\omega(t)$ is the increment of a standard Wiener process, uncorrelated across time and satisfying $E\,(d\omega_t) = 0$ and $E\,(d\omega_t)^2 = dt$. The discount rate r is constant with $r > \mu$ to focus on the case where the expected revenue

[2] [22] discusses common and cross-ownership structures that yield Eq. (1). Estimates of λ vary across both countries and industries, with [3] finding .7 for U.S. firms whereas [13] report .1 for Autstralian firms for example.

stream is bounded.

The firms both choose when and at what scale to enter an evolving market. Market entry involves a single capacity investment. Capacity has a constant unit cost $\delta > 0$ and can be neither altered nor resold once it is installed. There are no production costs and firms are assumed to operate at capacity.[3]

3 Follower investment

Suppose that when the current value of the demand state is X one of the firms, the *leader*, invests a capacity Q_L. The remaining firm, or *follower*, chooses when to invest and fixes a capacity level Q_F when it does. Letting T denote the stopping time at which the follower subsequently enters and $Q_F^*(T)$ its optimal capacity upon investment, the follower's perceived value is

$$\begin{aligned}\Omega_F(X) = \sup_{T\geq 0} E_X \Bigg[\lambda \left(\int_0^T X(s)\left(1-\eta Q_L\right) Q_L e^{-rs} ds - \delta Q_L \right) \\ + \int_T^\infty X(s)\left(1-\eta\left(Q_L+Q_F^*(T)\right)\right)\left(\lambda Q_L + Q_F^*(T)\right) e^{-rs} ds - \delta Q_F^*(T) e^{-rT} \Bigg) \Bigg] \quad (4)\end{aligned}$$

Inside the conditional expectation in Eq. (4), the first set of terms is the weighted discounted profit from the leader's monopoly phase, net of investment cost, which the follower perceives up until time T where it enters. The second set of terms is the follower's perceived discounted profit from the duopoly phase which consists of its own profit and a perceived share of the leader's, net of investment cost at the moment of follower entry.

The first step in determining the timing of the follower's entry is to characterize its capacity choice upon investment. If when the follower enters the demand state is $X(t) =$

[3] [5] argue that operating below capacity is technically inefficient in many real-world industries.

X', its perceived duopoly profit at that moment net of forgone internalized monopoly profit and investment cost is

$$
\begin{aligned}
& \left(\left(1-\eta\left(Q_L+Q_F\right)\right)\left(\lambda Q_L+Q_F\right)-\lambda\left(1-\eta Q_L\right) Q_L\right) E_{X'}\left[\int_0^{\infty} X(s) e^{-rs} ds\right]-\delta Q_F \\
= & \left(1-\eta\left(\left(1+\lambda\right) Q_L+Q_F\right)\right) Q_F \frac{X'}{r-\mu}-\delta Q_F . \qquad (5)
\end{aligned}
$$

Eq. (5) shows that if the degree of intenalization is positive the leader can block the follower's entry permanently while obtaining a positive price. It does this by choosing capacity $Q_L \in \left[\frac{1}{\eta(1+\lambda)}, \frac{1}{\eta}\right)$ for which the follower's net present value is negative for all positive Q_F and X'. Eq. (5) is strictly concave in Q_F, so optimizing over R_+ gives an optimal follower capacity upon investment which is a piecewise function of the state,

$$
Q_F^*\left(X'\right)=\max \left\{0, \frac{1}{2\eta}\left(1-\eta\left(1+\lambda\right) Q_L-\frac{\delta\left(r-\mu\right)}{X'}\right)\right\}. \qquad (6)
$$

In Eq. (6), the effect of internalization is to soften the follower's reaction which involves weakly lower capacity, all else equal. By choosing such an optimal capacity, the follower perceives an expected net present value

$$
\eta Q_F^{*2}\left(X'\right) \frac{X'}{r-\mu}, \qquad (7)
$$

which is also weakly decreasing with internalization.

Provided $Q_L < \frac{1}{\eta(1+\lambda)}$ so the leader does not block entry, the follower holds a valuable real option. The value of this option consists of a perceived flow dividend corresponding to the leader's monopoly position, and a terminal payoff corresponding to Eq. (7). A dynamic programming argument (see Appendix A.1) establishes that the follower's optimal policy is an investment threshold. That is, the follower invests once the demand state reaches

$$X_F^* = \begin{cases} \frac{\beta+1}{\beta-1}\frac{\delta(r-\mu)}{1-\eta(1+\lambda)Q_L}, & \text{if } Q_L < \frac{1}{\eta(1+\lambda)} \\ \infty, & \text{if } Q_L \geq \frac{1}{\eta(1+\lambda)} \end{cases} \tag{8}$$

where

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{9}$$

is a constant that reflects discounting in a stochastic environment. At this threshold, the follower's optimal capacity (Eq. 6) takes the value

$$Q_F^*\left(X_F^*\right) = \begin{cases} \frac{1-\eta(1+\lambda)Q_L}{(\beta+1)\eta}, & \text{if } Q_L < \frac{1}{\eta(1+\lambda)} \\ 0, & \text{if } Q_L \geq \frac{1}{\eta(1+\lambda)}. \end{cases} \tag{10}$$

Eqs. (8) and (10) indicate that (provided $Q_L < \frac{1}{\eta(1+\lambda)}$ so the follower invests after a finite delay) internalization has a monotonic effect on both the timing and size of the follower's investment. More precisely an increase in λ effectively scales up the leader's capacity by 100λ percent. This has a twofold effect, first by reducing the follower's capacity upon entry all else equal, and second by driving the follower to raise its investment threshold.

The next proposition sets out the main results concerning follower investment.

Proposition 1. *The follower's investment threshold is given by X_F^* (Eq. 8) and its perceived value $\Omega_F(X, Q_L)$ is*

$$\begin{cases} \lambda\left(\frac{(1-\eta Q_L)Q_L X}{r-\mu} - \delta Q_L\right) + \frac{\delta(1-\eta(1+\lambda)Q_L)}{(\beta-1)(\beta+1)\eta}\left(\frac{X}{X_F^*}\right)^\beta, & \textit{if } X < X_F^* \\ \frac{\left(1-\eta(1-\lambda)Q_L - \frac{\delta(r-\mu)}{X}\right)^2 X}{4\eta(r-\mu)}, & \textit{if } X \geq X_F^*, \end{cases} \tag{11}$$

if $Q_L < \frac{1}{\eta(1+\lambda)}$, and

$$\lambda\left(\frac{(1-\eta Q_L)\, Q_L X}{r-\mu} - \delta Q_L\right) \tag{12}$$

if $Q_L \geq \frac{1}{\eta(1+\lambda)}$.

In Proposition 1, the first case (Eq. 11) applies if the leader does not block follower entry so the follower holds a valuable real option, which consists of two pieces. The first piece is the follower value if the demand state is low so that it chooses to delay investment, so it obtains the sum of internalized leader value and its perceived option value. The second piece is the follower's perceived value if the demand state is high, so that it invests immediately. The second case (Eq. 12) is the follower value if its real option is not valuable, in which case it perceives only the internalized leader value.

4 Leader capacity choice

The leader's investment threshold cannot be lower than the net present value threshold for monopoly investment. We can therefore restrict attention to demand states $X > \delta(r-\mu)$.[4] The leader's perceived value from investing at given X has the general form

$$\begin{aligned}\Omega_L(X) &= E_X\left[\max_{Q_L\geq 0}\left(\int_0^T X(s)\left(1-\eta Q_L\right)Q_L e^{-rs}ds - \delta Q_L\right.\right. \\ &\left.\left. + \int_T^\infty X(s)\left(1-\eta\left(Q_L+Q_F^*\left(X(T)\right)\right)\right)\left(Q_L+\lambda Q_F^*\left(X(T)\right)\right)e^{-rs}ds - \lambda\delta Q_F^*\left(X(T)\right)e^{-rT}\right)\right] \end{aligned} \quad (13)$$

where $T = \inf\{t \geq 0 \,|\, X(t) \geq X_F^*\}$ is the follower's stopping time and $Q_F^*(X(T))$ is its capacity choice. Investment is assumed to be definitive, so the leader cannot reinvest at a later date if it chooses $Q_L = 0$.

[4] The value of monopoly investment in demand state X is

$$\begin{aligned} & E_X\left[\max_{Q\geq 0}\left(\int_0^\infty X(s)\left(1-\eta Q\right)Qe^{-rs}ds - \delta Q\right)\right] \\ = & \max_{Q\geq 0}\left[\frac{X}{r-\mu}\left(1-\frac{\delta(r-\mu)}{X}-\eta Q\right)Q\right] \end{aligned}$$

so monopoly capacity investment is only positive if $X > \delta(r-\mu)$.

In this section we focus on the leader's capacity choice upon investment. Because it affects the follower's entry timing and capacity, the leader's capacity choice is strategic. If its capacity is small enough, follower entry can be immediate if the demand state is large enough ($T = 0$). On the other hand if the leader's capacity is large enough, the follower may never enter ($T = \infty$). In addition, intermediate capacity levels can induce the follower to delay entry until a finite threshold $X_F^* > X$ is reached. The conditional expectation in Eq. (13) must therefore be evaluated over the set of admissible capacities and demand states $\aleph = \left[0, \frac{1}{\eta}\right] \times (\delta(r-\mu), \infty)$ with these three alternatives in mind. Below we partition $\aleph$ according to the three kinds of follower entry behavior to obtain explicit expressions that allow us to study the leader's capacity choice problem.

First, for all $X \geq \widehat{X}_L$ where

$$\widehat{X}_L = \min_{Q_L \in \left[0, \frac{1}{\eta}\right]} X_F^* = \frac{\beta+1}{\beta-1} \delta(r-\mu) \tag{14}$$

bounds the follower entry threshold from below, let

$$\widehat{Q}_L(X) = \frac{1}{\eta(1+\lambda)} \left(1 - \frac{\beta+1}{\beta-1} \frac{\delta(r-\mu)}{X}\right) \tag{15}$$

denote the leader capacity at which the follower's threshold takes the value $X_F^* = X$. In (Q_L, X) space, the locus $\widehat{Q}_L(X)$ discriminates between capacity levels at which follower entry is immediate and those at which it is delayed. Then let

$$\begin{aligned}
\aleph^{\mathrm{a}} &= \left\{ (Q_L, X) \in \aleph \middle| \, X \geq \widehat{X}_L \text{ and } Q_L \in \left[0, \widehat{Q}_L(X)\right] \right\}, \\
\aleph^{\mathrm{b}} &= \left\{ (Q_L, X) \in \aleph \middle| \, Q_L \geq \frac{1}{\eta(1+\lambda)} \right\}, \text{ and} \\
\aleph^{\mathrm{d}} &= \aleph \setminus \left(\aleph^{\mathrm{a}} \cup \aleph^{\mathrm{b}}\right).
\end{aligned} \tag{16}$$

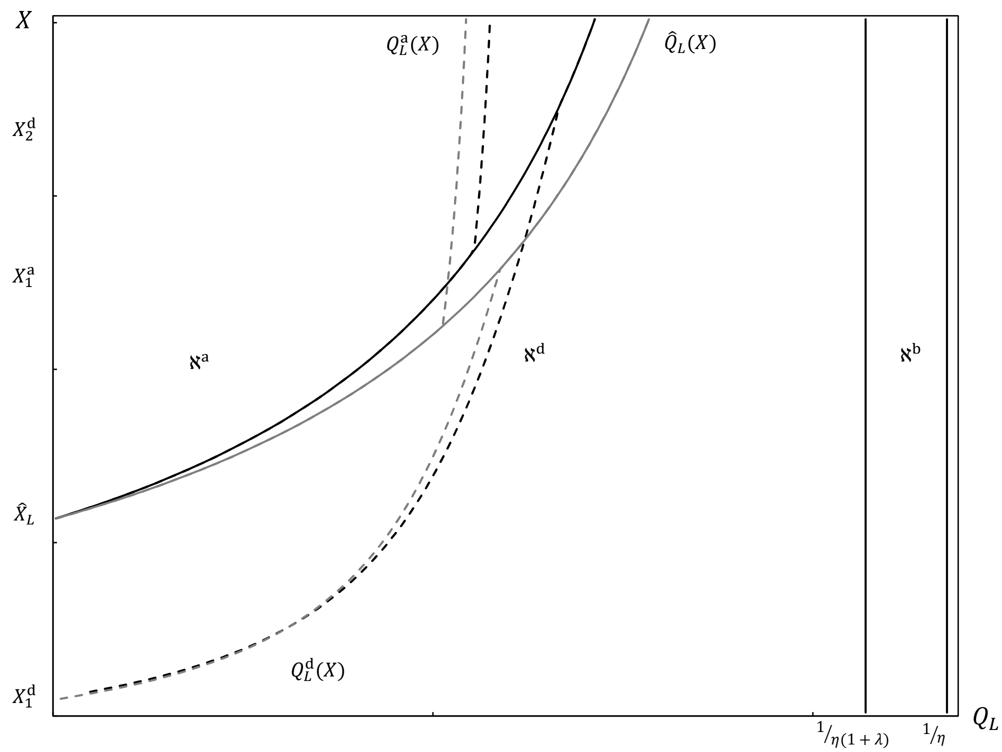

Figure 1: Leader capacity choice regions in (Q, X) space for $r = 0.1$, $\mu = 0.06$, $\sigma = 0.1$, $\delta = 0.1$, $\eta = 0.05$ and $\lambda = 0$ (gray) or 0.1 (black)

If $(Q_L, X) \in \aleph^{\mathrm{a}}$ there is immediate duopoly, which may be construed as a dynamic form of accommodation by the leader ($T = 0$). If $(Q_L, X) \in \aleph^{\mathrm{b}}$ the leader has a permanent monopoly and follower entry is blocked ($T = \infty$). Finally if $(Q_L, X) \in \aleph^{\mathrm{d}}$ there is delayed duopoly ($X_F^* > X$ with X_F^* finite). The forms that the conditional expectation term in Eq. (13) takes over $\aleph^{\mathrm{a}}$, $\aleph^{\mathrm{b}}$, and $\aleph^{\mathrm{d}}$ and the behavior with respect to Q_L over these regions are as follows.

$\aleph^{\mathrm{a}}$ **(immediate duopoly)**:

In this region, $T = 0$ and the conditional expectation term has the form

$$E_X \left[\int_0^\infty X(s) \left(1 - \eta \left(Q_L + Q_F^* \left(X\right)\right)\right) \left(Q_L + \lambda Q_F^* \left(X\right)\right) e^{-rs} ds - \delta \left(Q_L + \lambda Q_F^* \left(X\right)\right) \right]. \tag{17}$$

The expectation is over the leader's perceived perpetual duopoly profit net of investment cost, given that the leader internalizes a share of the follower's payoff. Although both firms effectively enter at the same moment, the follower's entry decision occurs "immediately after" the leader's. Because it observes the leader's investment and this investment is irreversible, the follower reacts to the leader's capacity through $Q_F^* \left(X\right)$. Evaluating the expectation gives

$$\frac{X \left(1 - \eta \left(1 - \lambda\right) Q_L - \frac{\delta(r-\mu)}{X}\right) \left(\lambda + \eta \left(2 + \lambda\right) \left(1 - \lambda\right) Q_L - \lambda \frac{\delta(r-\mu)}{X}\right)}{4\eta \left(r - \mu\right)}. \tag{18}$$

Viewed as a function of Q_L, Eq. (18) is concave with interior maximum

$$Q_L^{\mathrm{a}}(X) = \frac{1 - \frac{\delta(r-\mu)}{X}}{\eta \left(2 + \lambda\right) \left(1 - \lambda\right)}. \tag{19}$$

For $\lambda < \sqrt{2} - 1$, or for $\lambda > \sqrt{2} - 1$ and $\beta > \frac{3-\lambda^2}{\left(\lambda+1-\sqrt{2}\right)\left(\lambda+1+\sqrt{2}\right)}$, there exists a unique demand state $X_1^{\mathrm{a}} > \widehat{X}_L$ at which $Q_L^{\mathrm{a}}(X)$ intersects $\widehat{Q}_L \left(X\right)$ from below in (Q, X) space. For demand states above X_1^{a}, the maximum is interior, at $Q_L^{\mathrm{a}}(X)$. Otherwise (for $\lambda = \sqrt{2} - 1$ or $\lambda > \sqrt{2} - 1$ and $\beta \leq \frac{3-\lambda^2}{\left(\lambda+1-\sqrt{2}\right)\left(\lambda+1+\sqrt{2}\right)}$), Eq. (18) is increasing and reaches its maximum on the boundary, at $\widehat{Q}_L \left(X\right)$. Solving the condition $Q_L^{\mathrm{a}}(X) = \widehat{Q}_L \left(X\right)$ gives

$$X_1^{\mathrm{a}} = \frac{\beta \left(2 - (1 + \lambda)^2\right) + 3 - \lambda^2}{\left(\beta - 1\right) \left(2 - (1 + \lambda)^2\right)} \delta \left(r - \mu\right). \tag{20}$$

As λ increases, the boundary $\widehat{Q}_L \left(X\right)$ moves to left. Greater internalization therefore shrinks the accommodation region, whereas the interior maximum $Q_L^{\mathrm{a}}(X)$ shifts rightward.

$\aleph^{\mathrm{d}}$ **(delayed duopoly):**

In this region, $T = \inf\{t \geq 0 \,|\, X(t) \geq X_F^*\}$ with a finite follower threshold $X_F^* > X$ and the conditional expectation term has the form

$$E_X\left[\int_0^T X(s)(1-\eta Q_L)Q_L e^{-rs}ds - \delta Q_L \right. \tag{21}$$
$$\left. + \int_T^\infty X(s)\left(1-\eta\left(Q_L + Q_F^*\left(X_F^*\right)\right)\right)\left(Q_L + \lambda Q_F^*\left(X_F^*\right)\right)e^{-rs}ds - \lambda\delta Q_F^*\left(X_F^*\right)e^{-rT}\right]$$

Inside this expression, the first two terms are the discounted monopoly profit net of investment cost, and the last two terms are the perceived discounted duopoly profit and internalized follower investment cost. Evaluating the expectation yields

$$\frac{\left(1-\eta Q_L\right)Q_L X}{r-\mu} - \delta Q_L + \left(\frac{X}{X_F^*}\right)^\beta \frac{\delta\left(\lambda - \eta\left(1+\lambda\right)\left(\beta+1-\lambda\beta\right)Q_L\right)}{\eta\left(\beta^2-1\right)}, \tag{22}$$

where X_F^* is the follower's threshold reaction function (Eq. 8 above).

To characterize the behavior of Eq. (22) as leader capacity varies, differentiate with respect to Q_L to get the first-order condition at an interior optimum, which we denote by Q_L^{d}:

$$\frac{\left(1-2\eta Q_L^{\mathrm{d}}\right)X}{r-\mu} - \delta - \left(\frac{X}{X_F^*}\right)^\beta \frac{\delta\left(1+\lambda\right)\left(1-\eta\left(1+\lambda\right)\left(\beta+1-\lambda\beta\right)Q_L^{\mathrm{d}}\right)}{\left(\beta-1\right)\left(1-\eta\left(1+\lambda\right)Q_L^{\mathrm{d}}\right)} = 0. \tag{23}$$

The second-order condition is satisfied for β or λ small enough.[5] The range of demand states for which an interior optimum exists is determined as follows (see also Appendix A.2). First, setting $Q_L^{\mathrm{d}} = 0$ in Eq. (23) gives the lower bound of the demand states at

[5] A sufficient condition is $\beta < \frac{2}{\lambda(3-\lambda^2)}$.

which an interior optimum exists, X_1^{d}, as the lower root of

$$\frac{(1+\lambda)(\beta-1)^{\beta-1}}{(\beta+1)^{\beta}}\left(\frac{X_1^{\mathrm{d}}}{\delta(r-\mu)}\right)^{\beta}-\frac{X_1^{\mathrm{d}}}{\delta(r-\mu)}+1=0. \tag{24}$$

The left-hand side of Eq. (24) is convex in the demand state and negative at $\widehat{X}_L$, so the lower root satisfies $X_1^{\mathrm{d}} < \widehat{X}_L$, and X_1^{d} increases unambiguously as λ increases. If $\lambda \notin \left[\sqrt{\frac{\beta+1}{\beta}}-1, \frac{1}{\beta}\right]$, there exists a finite demand state at which $\widehat{Q}_L(X)$ solves the first-order condition (Eq. 23),

$$X_2^{\mathrm{d}}=\frac{(\beta+1)(2+\lambda)(1-\beta\lambda)}{(\beta-1)\left(1-2\beta\lambda-\beta\lambda^2\right)}\delta(r-\mu) \tag{25}$$

which is the upper bound of the demand states at which an interior optimum exists. Otherwise if $\lambda \in \left[\sqrt{\frac{\beta+1}{\beta}}-1, \frac{1}{\beta}\right]$ there is no such upper bound.

$\aleph^{\mathrm{b}}$ **(permanent monopoly):**

In this region $T=\infty$ and the conditional expectation term has the form

$$E_X\left[\int_0^{\infty} X(s)(1-\eta Q_L)Q_L e^{-rs}ds-\delta Q_L\right]. \tag{26}$$

Inside this expression is the perpetual monopoly profit net of investment cost. Evaluating the expectation gives

$$\left(\frac{X}{r-\mu}(1-\eta Q_L)-\delta\right)Q_L. \tag{27}$$

Because $\lambda \leq 1$, capacities in the $\aleph^{\mathrm{b}}$ region lie at or above the monopoly level. The leader's payoff is therefore decreasing in capacity, and hence maximized on the left boundary by setting $Q_L = \frac{1}{\eta(1+\lambda)}$, for which the leader obtains

$$\Omega_L\left(\frac{1}{\eta\left(1+\lambda\right)}, X\right) = \frac{\lambda X}{\eta\left(r-\mu\right)\left(1+\lambda\right)^2} - \frac{\delta}{\eta\left(1+\lambda\right)}. \tag{28}$$

To summarize, the following proposition gives the payoff $\Omega_L(Q_L, X)$ the leader obtains from choosing capacity Q_L upon investment.

Proposition 2. *At the capacity choice stage,*

$\Omega_L(Q_L, X) =$

$$\begin{cases} \frac{(1-\eta)Q_L X}{r-\mu} - \delta Q_L + \left(\frac{X}{X_F^*}\right)^\beta \frac{\delta(\lambda-\eta(1+\lambda)(\beta+1-\lambda\beta)Q_L)}{\eta\left(\beta^2-1\right)}, & \text{if } 0 \le Q_L < \frac{1}{\eta(1+\lambda)} \\ \left(\frac{X}{r-\mu}(1-\eta Q_L) - \delta\right) Q_L, & \text{if } \frac{1}{\eta(1+\lambda)} \le Q_L \le \frac{1}{\eta} \end{cases} \tag{29}$$

if $X < \widehat{X}_L$,

and

$$\begin{cases} \frac{X\left(1-\eta(1-\lambda)Q_L - \frac{\delta(r-\mu)}{X}\right)\left(\lambda+\eta(2+\lambda)(1-\lambda)Q_L - \lambda\frac{\delta(r-\mu)}{X}\right)}{4\eta(r-\mu)}, & \text{if } 0 \le Q_L \le \widehat{Q}_L\left(X\right) \\ \frac{(1-\eta Q_L)Q_L X}{r-\mu} - \delta Q_L + \left(\frac{X}{X_F^*}\right)^\beta \frac{\delta(\lambda-\eta(1+\lambda)(\beta+1-\lambda\beta)Q_L)}{\eta\left(\beta^2-1\right)}, & \text{if } \widehat{Q}_L\left(X\right) < Q_L < \frac{1}{\eta(1+\lambda)} \\ \left(\frac{X}{r-\mu}(1-\eta Q_L) - \delta\right) Q_L, & \text{if } \frac{1}{\eta(1+\lambda)} \le Q_L \le \frac{1}{\eta} \end{cases} \tag{30}$$

if $X \ge \widehat{X}_L$.

In Proposition 2, the first part (Eq. 29) indicates that the leader's capacity choice problem involves two pieces at low demand states, which correspond to the delay and blockade regions. The second part (Eq. 30) indicates that the payoff at high demand states includes an additional low capacity range over which accommodation occurs, and therefore consists of three pieces. The conditions under which the leader's payoff admits

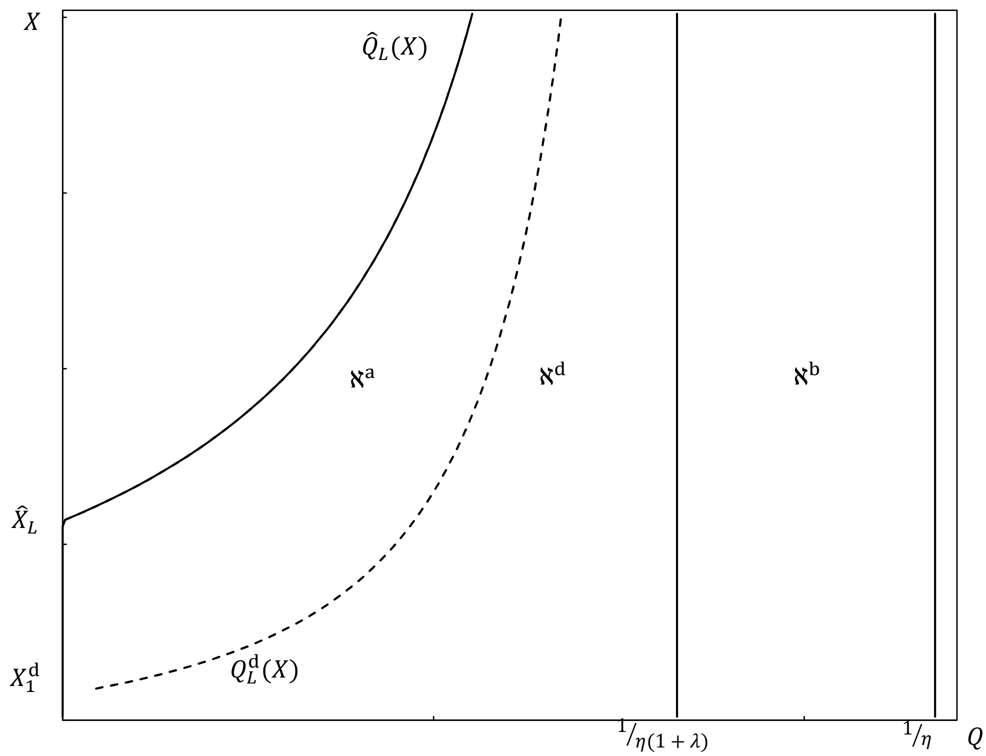

Figure 2: Leader capacity choice regions in (Q, X) space for $r = 0.1$, $\mu = 0.06$, $\sigma = 0.1$, $\delta = 0.1$, $\eta = 0.05$ and $\lambda = 0.42$

a local maximum and under which such a maximum is interior over each of these pieces depend on the level of the demand state, as described in the paragraphs above. Moreover because the optimal capacity under delay (Q_L^d) is defined only implicitly, there is no general analytic solution to the capacity choice problem $\max_{Q_L \in [0,1/\eta]} \Omega_L(Q_L, X)$. The set of candidate solutions can be significantly narrowed however in certain cases.

Figure 1 depicts $\aleph$ for a low degree of internalization ($\lambda = .1$, in black) and without internalization ($\lambda = 0$, in grey). The configuration is similar for all levels of internalization $\lambda < \sqrt{\frac{\beta+1}{\beta}} - 1$ at which X_1^a and X_2^d is finite. The regions on the left of the figure, $\aleph^a$ and $\aleph^d$, are similar to those in Figure 2 of [10]. Dotted curves plot the local maxima Q_L^a and

Q_L^{d}, suggesting that small increases in internalization have a gradual effect on the leader's payoff for capacities below $\frac{1}{\eta(1+\lambda)}$. The qualitative difference with internalization is the additional region $\aleph^{\mathrm{b}}$ over which the leader's capacity choice blocks the follower's entry, resulting in a jump in the payoff.

The intuition that the leader's capacity choice problem at low levels of internalization resembles the case without internalization (so the consequences of internalization are quantitative rather than qualitative) is developed in the following proposition.

Proposition 3. *There exists $\overline{\lambda}(\beta)$, positive and continuous, such that the leader's payoff is*

$$\Omega_L(X) = \begin{cases} \frac{\lambda(\beta-1)^{\beta-1}X^{\beta}}{(\beta+1)^{\beta+1}\eta\delta^{\beta-1}(r-\mu)^{\beta}}, & \text{if } \delta(r-\mu) \leq X \leq X_1^{\mathrm{d}} \\ \frac{\left(1-\eta Q_L^{\mathrm{d}}\right)Q_L^{\mathrm{d}}X}{r-\mu} - \delta Q_L^{\mathrm{d}} + \left(\frac{X}{X_F^*}\right)^{\beta} \frac{\delta\left(\lambda-\eta(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{\eta\left(\beta^2-1\right)}, & \text{if } X_1^{\mathrm{d}} < X < X_1^{\mathrm{a}} \\ \max\left\{ \frac{(1+\lambda)^2 X}{4\eta(2+\lambda)(r-\mu)}\left(1-\frac{\delta(r-\mu)}{X}\right)^2, \frac{\left(1-\eta Q_L^{\mathrm{d}}\right)Q_L^{\mathrm{d}}X}{r-\mu} - \delta Q_L^{\mathrm{d}} \right. & \\ \left. \quad + \left(\frac{X}{X_F^*}\right)^{\beta} \frac{\delta\left(\lambda-\eta(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{\eta\left(\beta^2-1\right)} \right\}, & \text{if } X_1^{\mathrm{a}} \leq X < X_2^{\mathrm{d}} \\ \frac{(1+\lambda)^2 X}{4\eta(2+\lambda)(r-\mu)}\left(1-\frac{\delta(r-\mu)}{X}\right)^2, & \text{if } X \geq X_2^{\mathrm{d}} \end{cases} \tag{31}$$

for all $\lambda \leq \overline{\lambda}(\beta)$.

Proof. See Appendix A.3.

The primary consequence of Proposition 3 is that with lower degrees of internalization the solution of the leader's capacity choice problem resembles the situation without internalization, despite the possibility of a blockade strategy. This result is all the more striking at high demand states ($X \geq X_2^{\mathrm{d}}$) where the outcomes of the leader's capacity choice might be expected to be either accommodation or blockade as in the standard entry deterrence model of [4]. In the absence of internalization, [10] observe that the accommodation and delay payoff cross only once in $\left(X_1^{\mathrm{a}}, X_2^{\mathrm{d}}\right)$, so that the leader chooses delayed duopoly at lower demand states and accommodation at higher demand states. In Section 6 we study a numerical example which bears this out, and allows us to assess the quantitative effect of internalization and find an unexpected consequence of internalization for the competitiveness of product market outcomes.

With higher levels of internalization, the leader's strategic behavior differs significantly from the situation without internalization. If $\lambda > \sqrt{2}-1$ and $\beta \leq \frac{1}{\lambda}$ for example, both X_1^{a} and X_2^{d} are undefined which implies that the relevant strategies for the leader induce either delayed duopoly or permanent monopoly, but never accommodation. Figure 2 illustrates such a possibility where the blockade region is significantly expanded, strategic delay can occur for any demand state above X_1^{d}, and accommodation is possible in principle for any demand state above $\widehat{X}_L$ but takes the form of the corner solution at $\widehat{Q}_L$ (and is never optimal).

One feature of Proposition 3 which holds more generally concerns the leader's capacity choice over a range of lower demand states, as the next proposition shows.

Proposition 4. *There exists* $\overline{X}_L \in \left(X_1^{\mathrm{d}}, \widehat{X}_L\right)$ *such that the leader's payoff is*

$$\Omega_L(X) = \begin{cases} \frac{\lambda(\beta-1)^{\beta-1} X^{\beta}}{(\beta+1)^{\beta+1}\eta\delta^{\beta-1}(r-\mu)^{\beta}}, & \text{if } \delta\,(r-\mu) \leq X \leq X_1^{\mathrm{d}} \\ \frac{\left(1-\eta Q_L^{\mathrm{d}}\right)Q_L^{\mathrm{d}} X}{r-\mu} - \delta Q_L^{\mathrm{d}} + \left(\frac{X}{X_F^*}\right)^{\beta} \frac{\delta\left(\lambda-\eta(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{\eta\left(\beta^2-1\right)}, & \text{if } X_1^{\mathrm{d}} < X < \overline{X}_L \end{cases} \tag{32}$$

Proof. See Appendix A.4.

According to Proposition 4, the leader's capacity choice invariably involves strategic delay at low demand states for all levels of internalization $\lambda < 1$, which allows us to describe the competitive equilibrium investment in the next section.

5 Equilibrium investment

To characterize industry investment, suppose that $\lambda < 1$ and the roles of each firm as leader or follower are determined noncooperatively. At any demand state X at which no investment has yet occurred, firms have the choice to either invest or to wait. We suppose that the degree of internalization is low enough, so that the payoff to a firm from investing immediately as a leader is given by $\Omega_L(X)$ in Eq. (31). If on the other hand it is the rival that invests immediately, a firm obtains the follower payoff $\Omega_F(X, Q_L)$ in Eq. 11 where $Q_L \in \left\{Q_L^{\mathrm{a}}, Q_L^{\mathrm{d}}\right\}$ is the leader's optimal capacity.

Let $f(X) = \Omega_L(X) - \Omega_F(X, Q_L)$ denote the payoff difference, which represents each firm's incentive to enter preemptively ahead of its rival. For initial states $X \in \left(X_1^{\mathrm{d}}, X_1^{\mathrm{a}}\right)$, by Proposition 3 the leader's optimum capacity is Q_L^{d}, which induces delayed investment by the follower. In this range, the preemption incentive has the form

$$f(X) = (1-\lambda)\left(\frac{\left(1-\eta Q_L^{\mathrm{d}}\right) Q_L^{\mathrm{d}} X}{r-\mu} - \delta Q_L^{\mathrm{d}} - \left(\frac{X}{X_F^*}\right)^{\beta}\left(\frac{\left(1-\eta Q_L^{\mathrm{d}}\right) Q_L^{\mathrm{d}} X_F^*}{r-\mu} - \delta Q_L^{\mathrm{d}}\right)\right. \tag{33}$$
$$\left. + \left(\frac{X}{X_F^*}\right)^{\beta}\left(\frac{\left(1-\eta\left(Q_L^{\mathrm{d}} + Q_F^*\left(X_F^*\right)\right)\right) X_F^*}{r-\mu} - \delta\right)\left(Q_L^{\mathrm{d}} - Q_F^*\left(X_F^*\right)\right)\right).$$

The first line of Eq. (33) is the rent that the leader obtains from the monopoly phase by entering ahead of the follower with capacity Q_L^{d}. The second line is the leader's relative profit during the industry's duopoly phase, which is positive if the leader has a larger capacity than the follower and negative if the reverse is true.

To determine the equilibrium pattern of investment, suppose that $X < X_1^{\mathrm{a}}$. For $X \leq X_1^{\mathrm{d}}$, a firm which invests as a leader sets $Q_L^{\mathrm{d}} = 0$ so $f(X) < 0$. At X_1^{d} for example the leader's capacity choice is $Q_L^{\mathrm{d}} = 0$, so the preemption incentive in Eq. (33) takes the value $f(X_1^{\mathrm{d}}) = -(1-\lambda)\,\Omega_F(X, Q_L) < 0$. At X_1^{a} on the other hand, the leader's payoff is at least as large as its accommodation payoff $\Omega_L(Q_L^{\mathrm{a}}, X_1^{\mathrm{a}})$. Moreover at this threshold both firms invest with the rival acting as a Stackelberg follower, so $\Omega_L(Q_L^{\mathrm{a}}, X_1^{\mathrm{a}}) > \Omega_F(X_1^{\mathrm{a}}, Q_L^{\mathrm{a}})$ and therefore, $f(X_1^{\mathrm{a}}) > 0$. We conclude that the preemption range (the range of demand states over which firms prefer to lead rather than follow) is nonempty and bounded from below by a lower bound $X_P \in \left(X_1^{\mathrm{d}}, X_1^{\mathrm{a}}\right)$. The lower bound of the preemption range is the first demand state at which firms prefer to lead rather than follow, and if one firm were to set an investment threshold above this lower bound $X' > X_P$, the rival would have an incentive to enter before it at a threshold in (X_P, X'). If the initial state is low enough therefore (if $X < X_P$), in equilibrium one of the firms (either with equal probability) invests as a leader at X_P. In the interior of the preemption range finally, both firms seek to invest simultaneously.[6]

Proposition 5. *For initial states $X < X_1^{\mathrm{a}}$ and with endogenous firm roles, in a preemption equilibrium the leader's investment occurs at $X_P = \inf\{X > 0, \text{ s.t. } f(X) > 0\}$ with capacity $Q_1^{\mathrm{d}}(X_P)$, and the follower invests at X_F^*.*

6 Numerical illustration

The two preceding sections establish that at low values of λ the leader's decision problem and equilibrium investment both resemble the case without internalization. To examine the effect of in internalization, we take parameter values similar to those in [10] and set $r = 0.1$, $\mu = 0.06$, $\sigma = 0.1$, $\delta = 0.1$ and $\eta = 0.05$. Figure 3 plots the value of the local maxima of $\Omega_L(Q_L, X)$ in each of the three regions, $\aleph^{\mathrm{a}}$, $\aleph^{\mathrm{b}}$, and $\aleph^{\mathrm{d}}$ for $\lambda = 0.1$ and $\lambda = 0$. As indicated Proposition 3 indicates, blockade is not a relevant possibility throughout

[6] This is a simplified description of preemption. See Thijssen, Huisman and Kort [21] for a formalization of this game, which requires specifying the outcomes in case both firms invest at the same threshold and an appropriate strategy space.

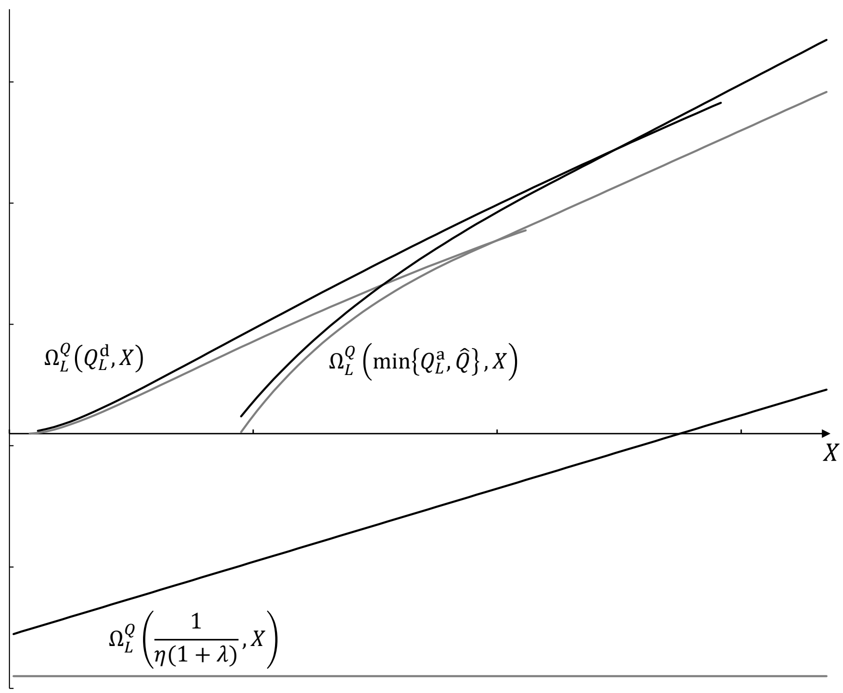

Figure 3: Leader capacity choice, optimal values over each region ($\aleph^{\mathrm{a}}$, $\aleph^{\mathrm{b}}$, and $\aleph^{\mathrm{d}}$) for $r = 0.1$, $\mu = 0.06$, $\sigma = 0.1$, $\delta = 0.1$, $\eta = 0.05$ and $\lambda = 0$ (gray) or 0.1 (black)

so we can focus on accommodation and deterrence. Figure 3 shows that delaying is preferable for the leader at low demand states, and that there is a single crossing point beyond which the leader prefers to accommodate. Factoring in the leader's capacity (or strategy) choice, the leader value $\Omega_L(X)$ is the upper envelope of $\Omega_L(Q_L^{\mathrm{d}}(X), X)$ and $\Omega_L(\min\left\{Q_L^{\mathrm{a}}(X), \widehat{Q}(X)\right\}, X)$ and presents an upward kink at the crossing point.

To obtain more insight into how overlapping ownership affects investment and market outcomes, we plot optimal capacities for the leader and follower against the demand state at which the leader invests in Figure 4. Higher demand states invariably result in higher leader capacity regardless of the leader's strategy (delay or accommodation), but the leader's capacity jumps down when it switches from delaying the follower to accommodation. The follower's capacity decreases with the current demand state if it invests at a higher demand state, but increases if the leader accommodates and it invests immediately. Internalization softens the follower's capacity reaction, increasing the leader's incentive to invest in capacity. In a dynamic setting, this strategic effect is reinforced if the leader opts to delay the follower, as internalization also softens the follower's threshold reaction. As the demand state increases, strategic delay becomes increasingly costly for the leader and its capacity eventually jumps down to an accommodating level. Because internalization has a stronger effect if the leader opts for delay as compared with accommodation, the shift to accommodation occurs at a higher demand state with internalization.

From a policy perspective it is overall competitiveness which matters, as measured by the total capacity ($Q(X)$, top curve). It is here that the comparison of market outcomes with and without internalization is most instructive. Internalization is anticompetitive, just as in the standard static setting, for demand states which are either sufficiently low or sufficiently high. However, Figure 4 also shows that there is an intermediate range of demand states where total capacity increases with internalization. Over this range, internalization drives the leader to opt for strategic delay, resulting in higher overall capacity under duopoly than if the leader and follower invested simultaneously as they would in the absence of internalization.

The procompetitive effect described above arises in the industry's duopoly phase, once both firms are active. Such an increase in static efficiency might not be welfare enhancing though, if the leader's strategic shifts increases the follower's investment threshold so as to delay the onset of duopoly excessively. To verify whether the procompetitive effect of internalization prevails, we adopt the stricter consumer surplus standard. Assuming that

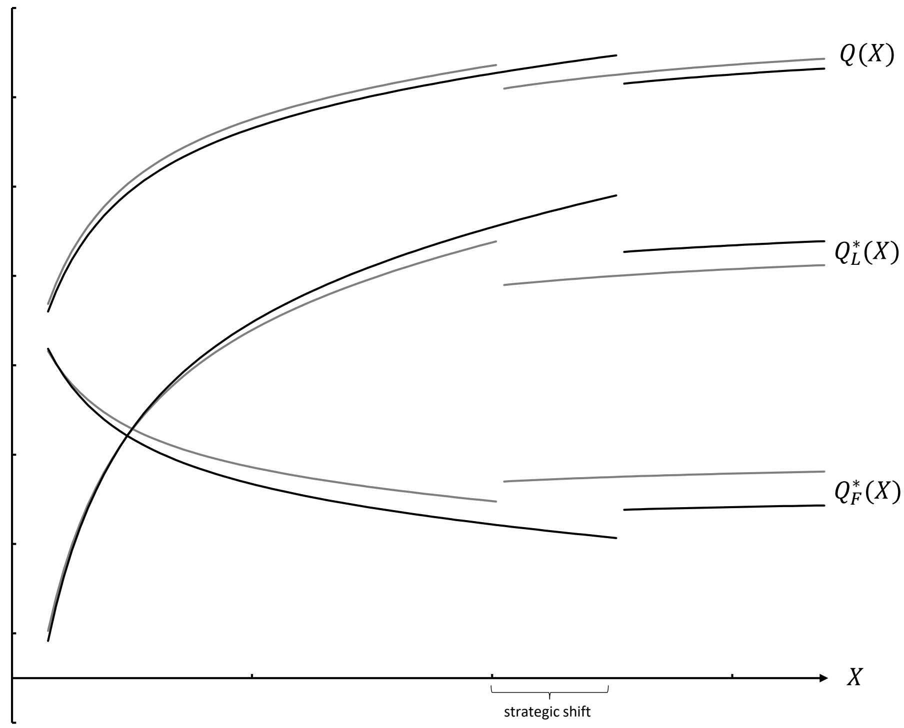

Figure 4: Optimal leader and follower capacities for $r = 0.1$, $\mu = 0.06$, $\sigma = 0.1$, $\delta = 0.1$, $\eta = 0.05$ and $\lambda = 0$ (gray) or 0.1 (black)

consumers have the same discount rate as firms, the consumer surplus at the moment that the leader invests is

$$S(X) = E_X \left[\int_0^T \frac{1}{2} X(s) \eta Q_L^2 e^{-rs} ds + \int_T^\infty \frac{1}{2} X(s) \eta \left(Q_L + Q_F^* \left(X(T) \right) \right)^2 e^{-rs} ds \right]. \quad (34)$$

Inside the conditional expectation in Eq. (34), the first term is the discounted consumer surplus during the industry's monopoly phase and the second term is the discounted consumer surplus during the industry's duopoly phase, evaluates at an optimal capacity choice Q_L for the leader. If the demand state is sufficiently high for the leader to choose positive capacity and low enough that the leader opts for delay (as occurs in a preemption equilibrium for instance), taking the expectation gives

$$S^{\mathrm{d}}(X) = \frac{\left(Q_L^{\mathrm{d}}\right)^2 X}{r-\mu} + \left(\frac{X}{X_F^*}\right)^\beta \frac{\left(\left(Q_L^{\mathrm{d}} + Q_F^*\left(X_F^*\right)\right)^2 - \left(Q_L^{\mathrm{d}}\right)^2\right) X_F^*}{r-\mu}, \tag{35}$$

whereas at demand states which are high enough that the follower enters immediate, using Eqs (6) and (19) to get values for $Q_F^*\left(X\right)$ and Q_L gives a consumer surplus of

$$S^{\mathrm{a}}(X) = \frac{(3+\lambda)^2}{4\eta^2\left(2+\lambda\right)^2}\left(1-\frac{\delta\left(r-\mu\right)}{X}\right)^2 \frac{X}{r-\mu}. \tag{36}$$

We evaluate consumer surplus at representative three demand, states, one of which ($X = .35$) lies inside the region where internalization shifts the leader's strategy and two of which ($X = .25$,.45) lie on either side, obtaining the values in Table 1. The level of internalization that we use ($\lambda = .1$) is on the lower end of the estimates found in the literature, and the effects we find are thus not extremely large. The key qualitative result concerns the intermediate demand state, where internalization shifts the leader's strategy. In this state, the loss of surplus consumers experience because of the delayed arrival of the second firm and the lower capacity at which it ultimately enters are offset by the additional surplus due to the significant increase in the leader's initial capacity investment and the high duopoly capacity which results. We conclude therefore that in a dynamic setting internalization can be procompetitive by shifting the leader's strategy to one with significantly more investment without excessively perturbing industry dynamics.

Table 1: Consumer surplus

X	.25	.35	.45
$S(X)$, $\lambda = 0$	2.53	3.89	5.25
$S(X)$, $\lambda = 0.1$	2.46	3.97	5.08

The welfare discussion of the preceding paragraph assumed that the leader's investment occurs at a given demand state, X. This situation can occur as an equilibrium outcome if the demand state is sufficiently high initially, so both firms seek to invest immediately. If the demand state is lower however, the equilibrium investment threshold may be the preemption threshold (see Proposition 5), which lies below the region over which internalization shifts the leader's strategy to produce a procompetitive outcome with the parameter values (and low internalization level) chosen here. Even in such cases though, the favorable effects of internalization for the leader firm may accelerate the initial investment in the industry, which we show in a related paper [23] may also lead to positive welfare effects

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A Appendix

A.1 Follower value

The follower's value is the sum of a (perpetual) internalized leader value and a growth option,

$$\Omega_F(X) = \frac{\lambda\left(1-\eta Q_L\right) Q_L X}{r-\mu} - \lambda\delta Q_L + O_F\left(X\right). \tag{37}$$

Over an inaction region $(0, X_F^*)$ the option component of follower value $O_F\left(X\right)$ satisfies the asset equilibrium condition

$$rO_F(X) = \frac{E_X dO_F(X)}{dt}. \tag{38}$$

Applying Itô's lemma and taking the expectation gives

$$rO_F(X) = \mu X O_F'(X) + \frac{1}{2}\sigma^2 X^2 O_F''(X) \tag{39}$$

as $dt \to 0$. The boundary conditions associated with Eq. (39) are

$$O_F(0) = 0 \text{ and } O_F\left(X_F^*\right) = \frac{\eta Q_F^{*2}\left(X_F^*\right) X_F^*}{r-\mu}, \tag{40}$$

along with the smooth pasting condition

$$O_F'\left(X_F^*\right) = \frac{\eta}{r-\mu}\frac{d\left(Q_F^{*2} X\right)}{dX}\left(X_F^*\right). \tag{41}$$

Conjecture a solution of the form $O_F(X) = A_F X^\beta$, where $\beta > 1$ (Eq. (9) in the text) is the upper root of the fundamental quadratic, $\frac{1}{2}\sigma^2 b(b-1) + \mu b - r = 0$. As $O_F'(X) = \frac{\beta}{X} O_F(X)$, substituting the upper boundary condition gives $O_F'(X_F^*) = \frac{\beta\eta\left(Q_F^*\left(X_F^*\right)\right)^2}{r-\mu}$. Moreover

$$\frac{d\left(Q_F^{*2}X\right)}{dX}\left(X_F^*\right) = 2Q_F^*\left(X_F^*\right)X_F^*\frac{dQ_F^*}{dX}\left(X_F^*\right) + Q_F^{*2}\left(X_F^*\right) \tag{42}$$

where $\frac{dQ_F^*}{dX}\left(X_F^*\right) = \frac{\delta(r-\mu)}{2\eta X_F^{*2}}$. The smooth pasting condition can therefore be expressed as

$$(\beta - 1)\,Q_F^*\left(X_F^*\right) = \frac{\delta\left(r-\mu\right)}{\eta X_F^*}. \tag{43}$$

Substituting for $Q_F^*(X)$ (Eq. (6)) gives the exercise threshold X_F^* (Eq. (8)) and $A_F = \frac{\eta Q_F^{*2}\left(X_F^*\right)X_F^{*1-\beta}}{r-\mu}$ by the second boundary condition, which gives the expression in the text. Finally to verify the optimal policy is a threshold, we check that the terminal payoff satisfies $X\left(\frac{\eta Q_F^{*2}(X)X}{r-\mu}\right)'' \leq (\beta-1)\left(\frac{\eta Q_F^{*2}(X)X}{r-\mu}\right)'$ for $X \geq X_F^*$. □

A.2 Leader payoff with strategic delay

For the first-order condition (Eq. 23), differentiating Eq. (22) gives

$$\begin{aligned}&\frac{\left(1-2\eta Q_L\right)X}{r-\mu} - \delta - \frac{\beta}{X_F^*}\left(\frac{X}{X_F^*}\right)^{\beta}\frac{\delta\left(\lambda - \eta\left(1+\lambda\right)\left(\beta+1-\lambda\beta\right)Q_L\right)}{\eta\left(\beta^2-1\right)}\frac{\partial X_F^*}{\partial Q_L} \\ &-\left(\frac{X}{X_F^*}\right)^{\beta}\frac{\delta\left(1+\lambda\right)\left(\beta+1-\lambda\beta\right)}{\left(\beta^2-1\right)}\end{aligned} \tag{44}$$

and substituting for $\frac{\partial X_F^*}{\partial Q_L}\frac{1}{X_F^*} = \frac{\eta(1+\lambda)}{1-\eta(1+\lambda)Q_L}$ gives the condition in the text. The second-order condition is

$$\begin{aligned}&-\frac{2\eta X}{r-\mu} + \frac{X^{\beta}\left(\beta-1\right)^{\beta-1}\beta\eta\left(1+\lambda\right)^2\left(1-\eta\left(1+\lambda\right)Q_L^{\mathrm{d}}\right)^{\beta-2}\left(2-\lambda-\eta\left(1+\lambda\right)\left(\beta+1-\lambda\beta\right)Q_L^{\mathrm{d}}\right)}{\left(\beta+1\right)^{\beta}\delta^{\beta-1}\left(r-\mu\right)^{\beta}} \\ =\;&\frac{\eta X}{r-\mu}\left(\left(\frac{X}{X_F^*}\right)^{\beta-1}\frac{\beta\left(1+\lambda\right)^2\left(2-\lambda-\eta\left(1+\lambda\right)\left(\beta+1-\lambda\beta\right)Q_L^{\mathrm{d}}\right)}{\left(\beta+1\right)\left(1-\eta\left(1+\lambda\right)Q_L^{\mathrm{d}}\right)} - 2\right) < 0.\end{aligned} \tag{45}$$

Observe that $\frac{2-\lambda-\eta(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}}{1-\eta(1+\lambda)Q_L^{\mathrm{d}}} = 1+(1-\lambda)\frac{1-\beta\eta(1+\lambda)Q_L^{\mathrm{d}}}{1-\eta(1+\lambda)Q_L^{\mathrm{d}}} < 2-\lambda$. As $X < X_F^*$, the second order condition therefore holds if

$$\frac{\beta+1}{\beta} > \frac{(1+\lambda)^2(2-\lambda)}{2} \tag{46}$$

which upon rearrangement gives the condition in the footnote.

To establish that X_1^{d} is unique and constitutes a lower bound, we verify that $\frac{dQ_L^{\mathrm{d}}}{dX}(X_1^{\mathrm{d}}) > 0$ (uniqueness of Q_L^{d} by the second-order condition then implies that $Q_L^{\mathrm{d}}(X)$ separates $\aleph^{\mathrm{d}}$ into two distinct subregions). Differentiating Eq. (23) gives

$$\frac{1-2\eta Q_L^{\mathrm{d}}}{r-\mu} - \left(\frac{X}{X_F^*}\right)^{\beta-1}\frac{\beta(1+\lambda)\left(1-\eta(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{(\beta+1)(r-\mu)}. \tag{47}$$

The value of this derivative at X_1^{d} is

$$\frac{1}{r-\mu} - \left(\frac{X_1^{\mathrm{d}}}{\delta(r-\mu)}\right)^{\beta-1}\frac{\beta(\beta-1)^{\beta-1}(1+\lambda)}{(\beta+1)^{\beta}(r-\mu)} = \frac{1}{r-\mu}\left(\beta\frac{\delta(r-\mu)}{X_1^{\mathrm{d}}} - (\beta-1)\right) \tag{48}$$

(using Eq. (24) to substitute for $\left(\frac{X_1^{\mathrm{d}}}{\delta(r-\mu)}\right)^{\beta-1}\frac{(\beta-1)^{\beta-1}(1+\lambda)}{(\beta+1)^{\beta}}$). Evaluating Eq. (24) at $X = \frac{\beta}{\beta-1}\delta(r-\mu)$ gives $\frac{1}{\beta-1}\left(\left(\frac{\beta}{\beta+1}\right)^{\beta}(1+\lambda)-1\right) < 0$, so $X_1^{\mathrm{d}} > \frac{\beta}{\beta-1}\delta(r-\mu)$ and therefore $\frac{dQ_L^{\mathrm{d}}}{dX}(X_1^{\mathrm{d}}) > 0$.

To establish that X_2^{d} (if finite) is an upper bound, we verify that $\frac{dQ_L^{\mathrm{d}}}{dX}(X_2^{\mathrm{d}}) < \frac{d\widehat{Q}}{dX}(X_2^{\mathrm{d}})$ From Eq. (23),

$$\frac{dQ_L^{\mathrm{d}}}{dX}\left(X_2^{\mathrm{d}}\right) = -\frac{1-2\eta Q_L^{\mathrm{d}} - \frac{\beta(1+\lambda)}{\beta+1}\left(1-\eta(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{X_2^{\mathrm{d}}\eta\left(\frac{\beta(1+\lambda)^2}{\beta+1}\frac{\left(2-\lambda-\eta(1+\lambda)(\beta+1-\lambda\beta)Q_L^{\mathrm{d}}\right)}{1-\eta(1+\lambda)Q_L^{\mathrm{d}}}-2\right)} \tag{49}$$

and from Eq. (15),

$$\widehat{Q}'_L\left(X_2^{\mathrm{d}}\right) = \frac{\beta+1}{\beta-1}\frac{\delta\left(r-\mu\right)}{\eta X_2^{\mathrm{d}2}\left(1+\lambda\right)}. \tag{50}$$

Comparing the two, we find that $\widehat{Q}'_L\left(X_2^{\mathrm{d}}\right) > \frac{dQ_L^{\mathrm{d}}}{dX}\left(X_2^{\mathrm{d}}\right)$ if $\beta\lambda\left(2-\lambda(1+\lambda)\right) < 1-\lambda$, the condition for the existence of X_2^{d}. $\square$

A.3 Proof of Proposition 3

From Eq. (28), blocking the follower is profitable for the leader only if the demand state is greater than

$$\underline{X}^{\mathrm{b}} = \frac{1+\lambda}{\lambda}\delta\left(r-\mu\right). \tag{51}$$

Comparing Eqs. (20) and (51), $X_1^{\mathrm{a}} \leq \underline{X}^{\mathrm{b}}$ if and only if $\beta \geq 1 + \frac{2\lambda(2-\lambda(1+\lambda))}{2-(1+\lambda)^2}$. The right hand side is increasing in lambda over $\left[0, \sqrt{2}-1\right)$. It is therefore invertible over this range and there exists an increasing function $\widetilde{\lambda}\left(\beta\right)$ with $\widetilde{\lambda}\left(1\right) = 0$ and $\lim_{\beta\to\infty}\widetilde{\lambda}\left(\beta\right) = \sqrt{2}-1$ such that $X_1^{\mathrm{a}} \leq \underline{X}^{\mathrm{b}}$ for $\lambda \leq \widetilde{\lambda}\left(\beta\right)$. Define $\widehat{\lambda}\left(\beta\right)$ decreasing with $\widehat{\lambda}\left(1\right) = 1$ and $\lim_{\beta\to\infty}\widehat{\lambda}\left(\beta\right) = 0$ by $\beta = \frac{2}{\lambda\left(3-\lambda^2\right)}$ so that the second-order condition Eq. (45) holds and set $\overline{\lambda}\left(\beta\right) = \min\left\{\widetilde{\lambda}\left(\beta\right), \widehat{\lambda}\left(\beta\right)\right\}$. $\lambda \leq \overline{\lambda}\left(\beta\right)$ therefore rules out the choice of any capacity $Q_L \geq \frac{1}{\eta(1+\lambda)}$ for demand states $X \leq X_1^{\mathrm{a}}$ and ensures Q_L^{d} is well-defined. Capacities above $\frac{1}{\eta(1+\lambda)}$ are not optimal for $X > X_1^{\mathrm{a}}$ if $\Omega_L\left(\frac{1}{\eta(1+\lambda)}, X\right)$ is lower than the payoff from accommodation (Eq. 18) evaluated at $Q_L^{\mathrm{a}}(X)$, that is if

$$\frac{\lambda X}{\eta\left(r-\mu\right)\left(1+\lambda\right)^2} - \frac{\delta}{\eta\left(1+\lambda\right)} < \frac{\left(1+\lambda\right)^2 X}{4\eta\left(2+\lambda\right)\left(r-\mu\right)}\left(1-\frac{\delta\left(r-\mu\right)}{X}\right)^2 \tag{52}$$

or after rearrangement,

$$\left(\frac{2-\left(1+\lambda\right)^2}{\left(1+\lambda\right)^2}\right)^2\left(\frac{X}{\delta\left(r-\mu\right)}\right)^2 - 2\frac{\lambda^3+3\lambda^2+\lambda-3}{\left(1+\lambda\right)^3}\frac{X}{\delta\left(r-\mu\right)} + 1 \tag{53}$$

which is positive by virtue of $\lambda < \sqrt{2} - 1$ if X_1^{a} is finite. Capacities larger than $\frac{1}{\eta(1+\lambda)}$ can therefore be globally ruled out for $\lambda \leq \widetilde{\lambda}(\beta)$.

For the first piece of Eq. (31), as $X_1^{\mathrm{d}} < \widehat{X}_L$, Proposition 2 implies that only the region $\aleph^{\mathrm{d}}$ is relevant. Moreover, the leader's payoff is decreasing in capacity over this range so the global maximum is at $Q_L = 0$, and the leader obtains the internalized follower payoff $\lambda A_F X^\beta$, which works out to

$$\Omega_L(0, X) = \frac{\lambda (\beta - 1)^{\beta - 1} X^\beta}{(\beta + 1)^{\beta + 1} \eta \delta^{\beta - 1} (r - \mu)^\beta} \tag{54}$$

For the second piece, there are two subcases corresponding to demand states in $\left(X_1^{\mathrm{d}}, \widehat{X}_L\right)$ and $\left[\widehat{X}_L, X_1^{\mathrm{a}}\right)$. For $X < \widehat{X}_L$, the reasoning is as for the first piece except that the leader's payoff has a global maximum at Q_L^{d}. For $X \geq \widehat{X}_L$, by Proposition 2 the leader's payoff lies initially in $\aleph^{\mathrm{a}}$ but $\Omega_L(Q_L, X)$ is strictly increasing in this range because $X < X_1^{\mathrm{a}}$ and the global maximum is therefore also Q_L^{d}.

For the third piece, because $X \geq X_1^{\mathrm{a}}$, $\Omega_L(Q_L, X)$ has two interior local maxima, Q_L^{a} and Q_L^{d}. Substituting $Q_L^{\mathrm{a}}(X)$ into Eq. (18) gives an explicit form for the payoff at the first local maximum,

$$\Omega_L(Q_L^{\mathrm{a}}, X) = \frac{(1 + \lambda)^2 X}{4\eta (2 + \lambda)(r - \mu)} \left(1 - \frac{\delta (r - \mu)}{X}\right)^2. \tag{55}$$

Finally for the fourth piece, because $X \geq X_2^{\mathrm{d}}$ there is no interior maximum in the range $Q_L > \widehat{Q}_L(X)$. □

A.4 Proof of Proposition 4

Proof. Because $\widehat{X}_L < X_1^{\mathrm{a}}$, accommodation ($Q_L = \widehat{Q}_L(X)$ if $X < X_1^{\mathrm{a}}$ or $Q_L^{\mathrm{a}}(X)$ if $X \geq X_1^{\mathrm{a}}$) is ruled out in the range described by the proposition. We show by contradiction that the leader does not choose blockade either. From Eqs. (54) and (28), setting $Q_L = \frac{1}{\eta(1+\lambda)}$ is more profitable than setting zero capacity if

$$\frac{(1+\lambda)(\beta-1)^{\beta-1}}{(\beta+1)^{\beta}}\left(\frac{X}{\delta(r-\mu)}\right)^{\beta} - \frac{\beta+1}{1+\lambda}\frac{X}{\delta(r-\mu)} + \frac{\beta+1}{\lambda} < 0 \tag{56}$$

whereas X_1^{d} satisfies

$$\frac{(1+\lambda)(\beta-1)^{\beta-1}}{(\beta+1)^{\beta}}\left(\frac{X_1^{\mathrm{d}}}{\delta(r-\mu)}\right)^{\beta} - \frac{X_1^{\mathrm{d}}}{\delta(r-\mu)} + 1 = 0. \tag{57}$$

The left hand sides of both of the above conditions are convex in X and X_1^{d} respectively, and decreasing at $\frac{X_1^{\mathrm{d}}}{\delta(r-\mu)} = 1$. There is a unique intersection at $X^* = \frac{1+\lambda}{\lambda}\frac{\beta+1-\lambda}{\beta-\lambda}\delta(r-\mu)$ where the first cuts the second from above. In order for Eq. (56) to have a lower root equal to or smaller than X_1^{d}, $i)$ X^* must lie to the left of the minimum $X_0 = \frac{(\beta+1)^{\frac{\beta}{\beta-1}}}{(\beta-1)\beta^{\frac{1}{\beta-1}}(1+\lambda)^{\frac{1}{\beta-1}}}\delta(r-\mu)$ Eq. (57) and $ii)$ the left hand side of Eq. (57) must be positive at X^*. Setting $X^{*\beta-1} < X_0^{\beta-1}$ to satisfy (i) and rearranging gives

$$\Theta := \frac{\beta(\beta-1)^{\beta-1}(1+\lambda)^{\beta}(\beta+1-\lambda)^{\beta-1}}{(\beta+1)^{\beta}\lambda^{\beta-1}(\beta-\lambda)^{\beta-1}} < 1, \tag{58}$$

whereas (ii) implies

$$1 < \frac{(\beta-1)^{\beta-1}(1+\lambda)^{\beta+1}(\beta+1-\lambda)^{\beta}}{(\beta+1)^{\beta+1}\lambda^{\beta-1}(\beta-\lambda)^{\beta-1}} = \frac{(1+\lambda)(\beta+1-\lambda)}{\beta(\beta+1)}\Theta \tag{59}$$

contradicting $\Theta < 1$. Therefore, $X^* > X_1^{\mathrm{d}}$ which implies that $Q_L = \frac{1}{\eta(1+\lambda)}$ is strictly suboptimal up to X_1^{d}. □

A.5 Static Stackelberg with internalization

See also [15] for a model of Stackelberg competition with cross-holding. Supposing $P = 1 - \eta Q$ and constant unit cost c, follower profit is

$$\pi_F = \left(1 - \eta\left(q_L + q_F\right) - c\right)\left(\lambda q_L + q_F\right) \tag{60}$$

so

$$q_F^* = \max\left\{0, \frac{1 - \eta\left(1+\lambda\right) q_L - c}{2\eta}\right\} \tag{61}$$

Provided $q_L \leq \frac{1-c}{\eta(1+\lambda)}$ therefore,

$$\begin{aligned} \pi_L &= \left(1 - \eta\left(q_L + q_F^*\right) - c\right)\left(q_L + \lambda q_F^*\right) \qquad (62) \\ &= \frac{1}{4\eta}\left(1 - \eta\left(1-\lambda\right) q_L - c\right)\left(\lambda + \eta\left(1-\lambda\right)\left(2+\lambda\right) q_L - \lambda c\right), \end{aligned}$$

so

$$q_L^{\mathrm{acc}} = \frac{1-c}{\eta\left(1-\lambda\right)\left(2+\lambda\right)}. \tag{63}$$

Observe that $q_L^{\mathrm{det}} = \frac{1-c}{\eta(1+\lambda)} < \frac{1-c}{2\eta}$, so if the leader deters total output is below the standard Stackelberg level. Otherwise, total output is

$$q_L^{\mathrm{acc}} + \frac{1 - \eta\left(1+\lambda\right) q_L^{\mathrm{acc}} - c}{2\eta} = \frac{3+\lambda}{2\left(2+\lambda\right)}\frac{1-c}{\eta} \leq \frac{3}{4}\frac{1-c}{\eta} \tag{64}$$

so total output is lower with internalization than in the standard Stackelberg equilibrium.