Optimal R&D investment from a private and social perspective

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Abstract

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Extended Abstract

In this paper, we study the research and development (R&D) investment decisions of a monopolist from a private and social perspective. The firm has a finite selection of R&D projects of different sizes to his disposal. A larger R&D project is more costly, but at the same time increases the probability of a breakthrough. The technological progress is modeled by a Poisson process with one jump. We assume that the R&D investment size has a positive effect on the arrival rate, where this relationship exhibits decreasing returns to scale. As such this model feature extends Weeds (2002) in which the arrival rate is a constant.

Upon a breakthrough the firm can launch the newly-developed product and receive a revenue stream. Here, we distinguish between immediate launch and the option to defer immediate launch. While the firm wants to maximize his producer's surplus, a government wants to maximize the total surplus. A government can provide subsidies to influence the timing and size of the firm's R&D project as well as the firm's production upon an innovation breakthrough. We analyze whether a government is able to influence the firm's investment decisions by providing subsidies, and if so, to what extent. Moreover, we study the effects of market conditions, e.g., the product price uncertainty, the product price growth and the interest rate, on the decisions of the firm and a government.

We conclude the paper with a case study to show how real options can contribute to the understanding of the interactions between a firm and a government with respect to innovation.

Reference: Weeds, H. (2002). Strategic Delay in a Real Options Model of R&D Competition. *The Review of Economic Studies, 69*(3), 729-747. Available here: `http://www.jstor.org/stable/1556717`

1 Introduction

2 Literature review

3 R&D from a private perspective

The R&D model in this section puts the monopolistic model of Huisman and Kort (2015) in an R&D framework. In the model of Huisman and Kort (2015), the monopolist is considering entering a market. The firm determines the optimal investment timing as well as the optimal quantity or capacity level. Here, on the other hand, the firm has yet to develop the product and thus has to go through a research and development phase before it can enter the market.

A typical timeline for the firm is given in Figure 3.1. Currently, the firm has not yet started its R&D project. In this waiting region, the firm has the option to commence its R&D project. If the firm exercises the option, it undertakes an irreversible R&D investment. The duration of the R&D phase is random, so the firm faces technological uncertainty. As soon as there is a breakthrough the firm launches its product and receives a revenue stream.

Figure 3.1: A typical timeline for the firm.

The technological progress is modeled as a Poisson process with one jump, given by

$$dq(t) = \begin{cases} u & \text{with probability } \lambda dt, \\ 0 & \text{with probability } 1 - \lambda dt, \end{cases}$$

where $q(0) = 0$. A jump in the Poisson process indicates that there has been a breakthrough so that we have $q(t) = u$ forever. The arrival rate is given by λ and the probability of innovation in time interval dt is equal to λdt.

The firm faces price uncertainty in addition to technological uncertainty. The product price is governed by a geometric Brownian motion with drift. A geometric Brownian motion is a continuous-time stochastic process $X(t)$ in which the rate of change in $X(t)$ is given by

$$dX(t) = \mu X(t)dt + \sigma X(t)dz(t), \tag{3.1}$$

where μ is the drift rate, $\sigma > 0$ is a variance parameter, and $dz(t)$ is the increment of a Wiener process, i.e., $dz(t) = \varepsilon_t \sqrt{dt}$, with $\varepsilon_t \sim N(0, 1)$ and $\mathbb{E}[\varepsilon_t \varepsilon_s] = 0$ for $t \neq s$. The starting value, $X(0)$, of the geometric Brownian motion is henceforth denoted by X and is assumed to be strictly positive.

The following market model is analogous to Huisman and Kort (2015). The market of the new product is assumed to be homogeneous with linear demand. The price at time t is given by

$$P(t) = (\theta - \alpha K)X(t),$$

where K is the quantity and θ and α are positive constants. In contrast to Huisman and Kort (2015), the parameter θ is added which can be a proxy for consumer's anticipation of the new product — after all, a larger θ implies a higher price. The price of the product is subject to stochastic shocks, $X(t)$, that follow a geometric Brownian motion with drift as given in (3.1). The instantaneous profit is given by

$$\pi(t) = KP(t) = K(\theta - \alpha K)X(t).$$

The firm is risk neutral and discounts against rate r (> 0). It is assumed that $\mu < r$, otherwise the problem does not make sense since the firm will wait indefinitely and thus will never undertake an R&D investment.

Let T denote the stochastic innovation time. The random variable $T(\geq 0)$ is exponentially distributed with mean $\frac{1}{\lambda}$ and its probability density function, for all $t \geq 0$, is given by $f_T(t) = \lambda e^{-\lambda t}$.

We assume that the firm immediately launches the product upon innovation at time T and incurs an investment cost equal to δK, where δ represents the cost of one unit of capacity. The corresponding *termination value* of the firm when it launches the product at time T follows from Proposition 1 of Huisman and Kort (2015) and is given by

$$\Omega(X(T)) = \frac{(\theta X(T) - \delta(r-\mu))^2}{4\alpha X(T)(r-\mu)}. \tag{3.2}$$

However, from Huisman and Kort (2015) it follows that the firm immediately launches the product if only if $\delta = 0$. Hence, we set $\delta = 0$ so that the termination value (3.2) simplifies to

$$\Omega(X(T)) = \frac{\theta^2}{4\alpha(r-\mu)} X(T), \tag{3.3}$$

which is a linear function of $X(T)$. The assumption that $\delta = 0$ is a reasonable assumption if we assume that the firm incurs a fixed sunk R&D cost R to start its R&D project and the firm is investing in the innovation of a digital product.

If the firm invests R with corresponding arrival rate λ, then the investment problem the firm is facing boils down to an optimal stopping problem:

$$V(X) = \max_{\tau \geq 0} \mathbb{E}_X \left[\int_{t=\tau}^{\infty} \lambda e^{-\lambda t} e^{-rt} \Omega(X(t)) dt - e^{-r\tau} R \middle| X(0) = X \right], \tag{3.4}$$

where the expectation is conditional on $X = X(0)$, which is the the current level of the geometric Brownian motion, and where τ is the time at which the firm starts R&D.

We let X^* denote the optimal R&D investment trigger. This is the value of the geometric Brownian motion whereupon the firm starts its R&D project. It is the value where the firm is indifferent between investing and not investing. In other words, the optimal investment timing τ is the moment in time where the level of the geometric Brownian motion hits X^*. On the basis of X^*, just as in Figure 3.1, two regions can be considered. If $X < X^*$, it is optimal for the firm to wait, hence the firm is in the waiting region. If $X \geq X^*$, it is optimal for the firm to undertake the investment immediately, hence the firm is in the R&D investment region.

In this paper, we assume that the firm can either invest in a small or large R&D project. If the firm invests in a small (large) R&D project, it incurs a fixed sunk cost of R_S (R_L) with corresponding arrival rate λ_S (λ_L). These fixed sunk costs include, but are not limited to, the cost of a new laboratory or research facility, equipment costs as well as possible licensing costs. We assume a one-to-one correspondence between the investment sizes and the arrival rates. In particular we assume $R_S < R_L$, $\lambda_S = c\sqrt{R_S}$ and $\lambda_L = c\sqrt{R_L}$ with $c > 0$, so that $\lambda_S < \lambda_L$ and so that the R&D investment exhibits decreasing returns to scale.

For each of the two cases we solve the optimal stopping problem given in (3.4).

Proposition 3.1. *The value of the firm if it invests in a small R&D project is equal to*

$$V_S(X) = \begin{cases} A_S X^{\beta} & \text{if } X < X_S^*, \\ \dfrac{\theta^2 X}{4\alpha(r-\mu)} \dfrac{\lambda_S}{\lambda_S + r - \mu} - R_S & \text{if } X \geq X_S^*; \end{cases} \tag{3.5}$$

the value of the firm if it invests in a large R&D project is equal to

$$V_L(X) = \begin{cases} A_L X^{\beta} & \text{if } X < X_L^*, \\ \frac{\theta^2 X}{4\alpha(r-\mu)} \frac{\lambda_L}{\lambda_L + r - \mu} - R_L & \text{if } X \geq X_L^*; \end{cases} \tag{3.6}$$

where

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1.$$

The optimal investment triggers are given by

$$X_S^* = \left(\frac{\beta}{\beta - 1}\right) \frac{4\alpha(r-\mu)R_S(\lambda_S + r - \mu)}{\theta^2 \lambda_S},$$
$$X_L^* = \left(\frac{\beta}{\beta - 1}\right) \frac{4\alpha(r-\mu)R_L(\lambda_L + r - \mu)}{\theta^2 \lambda_L};$$

and the constants are given by

$$A_S = \frac{(X_S^*)^{-\beta+1}}{\beta} \frac{\theta^2}{4\alpha(r-\mu)} \frac{\lambda_S}{\lambda_S - r - \mu},$$
$$A_L = \frac{(X_L^*)^{-\beta+1}}{\beta} \frac{\theta^2}{4\alpha(r-\mu)} \frac{\lambda_L}{\lambda_L - r - \mu}.$$

The optimal investment timing of the small R&D project always precedes the optimal investment timing of the large R&D project because $R_S < R_L$ and $\lambda_S < \lambda_L$, i.e., it holds that $X_S^* < X_L^*$. Therefore, we let the current value of the Brownian motion be such that it is not optimal to immediately undertake an R&D project, i.e., we have $X \equiv X(0) < X_S^*$. The firm invest in a small R&D project if and only if $V_S(X_S^*) > V_L(X_S^*)$. This means that at X_S^* the termination value of a small R&D project is strictly larger than the option value of investing in a large R&D project. The above condition is equivalent to

$$\beta > \frac{\log\left(\frac{R_L}{R_S}\right)}{\log\left(\frac{R_L}{R_S}\right) - \log\left(\frac{\lambda_L(\lambda_S + r - \mu)}{\lambda_S(\lambda_L + r - \mu)}\right)} (> 1). \tag{3.7}$$

Conversely, the firm invests in a large R&D project if and only if $V_S(X_S^*) \leq V_L(X_S^*)$. Hence, market conditions determine the investment decision of the firm. In particular, from $\frac{\partial \beta}{\partial \sigma} < 0$ and (3.7) it follows that the firm prefers a large R&D project if the price uncertainty increases (ceteris paribus). However, the effect of a change in the drift rate or discount rate on the investment decision of the firm is not so clear. If the drift rate increases, then both the left-hand side and right-hand side of (3.7) decrease. Moreover, if the discount rate increases, then both the left-hand side and the right-hand side of (3.7) increase.

4 R&D from a social perspective

A government can provide subsidies to influence the timing and size of a firm's R&D project as well as the firm's production upon a breakthrough.

4.1 Timing and size

We first assume that a government will only provide subsidies to large R&D projects to encourage innovation — after all, a firm expects to innovate sooner if it invests more in R&D.

Let S_L denote the subsidy of the government that the firm receives if it undertakes a large R&D project. We assume $R_L - S_L > R_S$. The value of the firm if it invests in a large R&D project consequently becomes

$$V_L(X, S_L) = \begin{cases} A_L X^\beta & \text{if } X < X_L^*, \\ \frac{\theta^2 X}{4\alpha(r-\mu)} \frac{\lambda_L}{\lambda_L + r - \mu} - R_L + S_L & \text{if } X \geq X_L^*, \end{cases}$$

with investment trigger

$$X_L^*(S_L) = \left(\frac{\beta}{\beta-1}\right) \frac{4\alpha(r-\mu)(R_L - S_L)(\lambda_L + r - \mu)}{\theta^2 \lambda_L}.$$

Hence, any subsidy by the government encourages a firm to start its R&D sooner.

Proposition 4.1. *It holds that* $\frac{\partial}{\partial\mu} X_L^*(S_L) > 0$.

Proof. We need to show that

$$\frac{\partial}{\partial\mu}\left(\frac{\beta}{\beta-1}\right) \frac{4\alpha(r-\mu)(R_L - S_L)(\lambda_L + r - \mu)}{\theta^2 \lambda_L} > 0,$$

which is equivalent to

$$\frac{\partial}{\partial\mu}\left(\frac{\beta}{\beta-1}\right)(r-\mu)(\lambda_L + r - \mu) > 0. \tag{4.1}$$

Recall that

$$\frac{\partial\beta}{\partial\mu} = \frac{-\beta}{(\sigma^2\beta + (\mu - \frac{1}{2}\sigma^2))}.$$

Using the above, condition (4.1) becomes

$$\lambda_L > (r-\mu)\left[\frac{(\beta-1)(\sigma^2\beta + (\mu - \frac{1}{2}\sigma^2))}{(r-\mu) - (\beta-1)(\sigma^2\beta + (\mu - \frac{1}{2}\sigma^2))} - 1\right].$$

To verify the above condition, it suffices to show that

$$r - \mu - (\beta-1)(\sigma^2\beta + (\mu - \frac{1}{2}\sigma^2)) < 0.$$

Using the fact that β is a root of

$$Q(\beta) = \frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0,$$

we find that

$$
\begin{aligned}
r-\mu-(\beta-1)(\sigma^2\beta+(\mu-\frac{1}{2}\sigma^2)) &= r-\mu-(r+\frac{1}{2}\sigma^2\beta^2-\sigma^2\beta-(\mu-\frac{1}{2}\sigma^2)) \\
&= \frac{1}{2}\sigma^2(\beta-1)^2>0.
\end{aligned}
$$

□

The firm prefers a large R&D project over a small one if and only if

$$
\log\left(\frac{R_L-S_L}{R_S}\right)\leq\frac{\beta}{\beta-1}\log\left(\frac{\lambda_L(\lambda_S+r-\mu)}{\lambda_S(\lambda_L+r-\mu)}\right),
$$

which is equivalent to

$$
S_L\geq R_L-R_S\left(\frac{\lambda_L(\lambda_S+r-\mu)}{\lambda_S(\lambda_L+r-\mu)}\right)^{\frac{\beta}{\beta-1}}. \tag{4.2}
$$

It holds that

$$
\frac{\lambda_L(\lambda_S+r-\mu)}{\lambda_S(\lambda_L+r-\mu)}>1 \text{ and } \frac{\beta}{\beta-1}>1
$$

We have

$$
\frac{\partial}{\partial\sigma}\frac{\beta}{\beta-1}=-\frac{1}{(\beta-1)^2}\frac{\partial\beta}{\partial\sigma}>0
$$

Therefore, the right-hand side of (4.2) decreases if the the price uncertainty increases. In other words, if there is more price uncertainty, then the government spends less on subsidies.

4.2 Social welfare

Goal: maximize expected total surplus.

5 Generalization to k scenarios

In this section, we generalize the results of previous sections to $k>2$ scenarios.

6 Positive product investment costs

In this section, we relax the assumption that $\delta=0$. If $\delta>0$, the firm will also have an option to start production upon a breakthrough, which complicates the analysis. Hence, we expect that this section comprises a numerical analysis.

7 Case Study

8 Conclusion

References

Huisman, K. J., & Kort, P. M. (2015). Strategic capacity investment under uncertainty. *The RAND Journal of Economics*, *46*(2), 376-408.