Production flexibility and trade credit under demand uncertainty

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Abstract

This paper analyzes optimal capacity investment decisions of a buyer firm receiving trade credit from a supplier under demand uncertainty. Within a real option framework, we analyze two scenarios: a flexible and an inflexible (rigid) buyer firm. The flexible firm can temporarily suspend operations when market conditions deteriorate, while the inflexible firm always produces at full capacity. We find that the flexible firm orders higher quantities from the supplier and trade credit value is higher for the flexible firm, however trade credit as a proportion of buyer firm value is higher for the inflexible firm. Moreover, we find that the supplier extends higher trade credit durations to the flexible firm. Our framework provides predictions regarding the effects of extending credit duration on trade credit values and default policies of the flexible compared to inflexible firm. We also analyze the effects of supplier's pricing, uncertainty in downstream markets, recovery value of trade credit for the supplier at default, operating profit margins of buyer firms and buyer capacity constraints on the differences in trade credit values, ordered quantities and trade credit maturity between the flexible and rigid firm. Finally, we extend the setting to study the effect of switching costs on the flexible firm's policies in the presence of trade credit.

Keywords: production flexibility; real options; trade credit; supply chain

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1. Introduction

In highly uncertain economic environments, the ability to adjust operating variables such as the level of production to cope with unpredictable changes in market conditions is essential. This ability is referred to as “operational/volume/production flexibility” in the literature and has been shown to influence a firm’s investment timing, capacity and capital structure decisions (Hagspiel et al., 2016; Rithcken and Wu, 2020).

However, production flexibility’s impact on firm value is not limited to a firm’s investment and debt financing decisions. A firm’s ability to adjust production level may affect its relationships with suppliers and the trade credit provided. Firms rely greatly on trade credit, especially in developing countries with poorly developed financial markets and weak financial institutions (Fisman and Love, 2003). Moreover, empirical evidence shows that trade credit is the largest single source of short-term financing (Petersen and Rajan, 1997) even in a well-developed market as the US. Additionally, it is a key financing source not only for small and medium enterprises, but also for large ones. Yang and Birge (2018) report accounts payable 3.3 times larger than bank loans for US nonfinancial firms and document that for large public retailers in North America, accounts payable represent one third of their liabilities.

In this paper, we analyze the effect of production flexibility on trade credit values and maturity. We develop a continuous time real options framework in which the demand shock follows a geometric Brownian motion. We consider a buyer firm with capacity constraints that orders input goods from a supplier. These goods are provided on credit. Given the short-term nature and roll-over character of trade credit, we build on Leland (1994,1998) models of finite debt used in the capital structure literature to model short-term trade credit. This framework assumes a stationary debt structure where at every instant of time a constant fraction of the credit matures and must be refinanced to keep the total amount of credit outstanding constant. The payment rate then determines an average credit duration. To analyze the impact of production flexibility on trade credit we consider two cases for the buyer firm. On the one hand, we have a flexible firm that has the possibility to temporarily suspend operations when the price drops below the marginal cost (idle mode). If market conditions improve, the firm retakes full-scale production (active mode). However, if market conditions further deteriorate the firm defaults at an endogenous threshold. On the other hand, we model a rigid firm that is restricted to always produce at full capacity and has the option to default when the demand shock drops to an optimally chosen threshold. The buyer firm selects optimally its capacity and quantities to be ordered from the supplier, as well as default timing. The supplier optimally chooses the trade credit maturity to extend to the buyer, internalizing how his choice of maturity will influence the buyer's optimal quantity and default threshold.[1]

We show that operational flexibility affects trade credit value and maturity through various channels. First, a flexible firm can avoid operational losses by temporarily suspending production. This leads to the flexible firm investing in larger capacity (in line with the findings of Hagspiel et al., 2016). In terms of trade credit, this implies that a flexible firm orders larger quantities from the supplier. This is broadly in line with Fisman (2001) who provides empirical evidence of a positive association between trade credit and capacity utilization. Second, a flexible firm defaults later than a rigid one, i.e., shareholders are more willing to keep the firm alive. This effect is consistent with the results of Ritchken and Wu (2020) and Charupat and Sarkar (2020) who analyze the impact of operational flexibility on capital structure. However, unlike Ritchken and Wu (2020) who find that the flexible firm uses less corporate debt, we show that the flexible firm has higher trade credit values. Indeed, since trade credit is short-term (finite) debt, the fact that shareholders postpone default means that the supplier can maintain a longer duration of business relationship with the buyer which increases trade credit values. Moreover, a lower default risk for the flexible firm also leads to higher ordered quantities. Both effects, larger ordered quantities and later default lead to a higher trade credit value for the flexible firm compared to the rigid one. The above-mentioned results are broadly consistent with Petersen and Rajan (1997) who show that higher credit quality buyers obtain larger amounts of credit.[2] Nevertheless, despite the larger trade credit value for the flexible firm, in line with Ritchken and Wu (2020) and Charupat and Sarkar (2020), we find a higher leverage ratio for the rigid firm.[3] Data comparing flexible with rigid firms' trade credit "leverage" ratios is not readily available since much of the focus of the literature is on debt (capital structure decisions). MacKay (2003) finds that firms that can easily adjust their production use less financial leverage, and less public debt. Our analysis shows that indeed flexible firms have lower trade leverage, however ,we generally find larger absolute levels of trade credit and order quantities provided to flexible firms. Reinartz and Schmid (2016) focus on energy utilities and show that more flexible firms have higher leverage ratios attributing this to lower costs of financial distress and higher tax benefits of debt.

[1] In line with previous literature, we assume an exogenous price for the input good, since price discrimination is often forbidden or can lead to a price war since it is observable by competitors. Discriminating through trade credit maturity is a less aggressive and more flexible instrument (Fabbri and Klapper, 2016).

[2] Note that the flexible firm always defaults later than the rigid firm in our analysis which indeed indicates higher credit worthiness.

[3] The value of the flexible firm increases more compared to trade credit values, leading to a lower leverage ratio.

Regarding trade credit maturity, the supplier faces a trade-off when extending credit.[4] On the one hand, extending a larger trade credit maturity gives incentives to the buyer firm to order larger quantities and to reduce default risk. This implies a larger and more durable business with the buyer. On the other hand, a larger trade credit maturity results in delayed payments, thus reducing the present value of the cash flows received by the supplier. From this trade-off an optimal maturity is determined. Since for a flexible firm the positive effects of extending trade credit are higher (a flexible firm orders higher quantities and postpones default), the supplier will extend larger maturities to a flexible firm compared to the rigid one. Our results are broadly in line with evidence from Klapper et al. (2012) who find that most credit worthy buyers obtain longer trade credit maturity. MacKay (2003) finds that flexible firms have shorter maturities, however he analyzes debt maturity, not trade credit durations.

Our overall findings are that greater production flexibility results in higher trade credit values and larger maturities of credit. The impact is larger in uncertain environments, in industries with less intense competition between suppliers and downstream markets with higher capacity constraints, lower recovery rates, lower gross profit margins or lower capacity holding costs.

Our analysis highlights the “real option” effects under uncertainty arising due to the presence of limited liability (option to default) and operating flexibility that create convexities in the underlying profits. Our setting should thus be contrasted with frameworks such as Chod et al. (2010) who focus on concave firm values as a function of revenues creating incentives for risk reduction and hedging. Our analysis also differs from Iancu et al. (2015) in that they focus on risk-shifting incentives of firms with flexibility to adjust inventory levels. Such flexibility has adverse effects on creditors. In particular, they focus on debt and how covenants can be used to alleviate these risk-shifting incentives. Our focus, on the other hand, is on how operating flexibility to adjust production level of the buyer firm enables suppliers to enjoy larger order quantities.

Our paper is related to two strands of the literature. On the one hand, we contribute to the growing literature on production flexibility. Hagspiel et al. (2016) analyze the impact of production flexibility on capacity investment size and timing. They find that a flexible firm invests in higher capacity compared to the inflexible firm, and this difference increases with uncertainty. Moreover, in highly uncertain economic environments the flexible firm invests later compared to the inflexible one. Ritchken and Wu (2020) and Charupat and Sarkar (2020) introduce corporate debt in this framework and analyze the impact of production flexibility on leverage and capital structure. Both models identify two opposing effects of flexibility on debt usage. On the one hand, there is a positive effect since flexibility increases firm value. On the other hand, greater flexibility

[4] A similar trade-off is highlighted in Koussis and Silaghi (2020).

increases the agency conflict between the shareholders and the creditors and leads to a negative effect on debt usage. Overall, both papers find that higher flexibility results in lower leverage ratios, although while in Ritchken and Wu (2020) flexibility leads to a lower debt level, in Charupat and Sarkar (2020) it leads to a higher debt level. In a related study, Charalambides and Koussis (2018) analyze the possibility of debt rescheduling when the firm is in the idle region. Unlike these papers that focus either on the investment option or on corporate debt, we analyze trade credit. Building on models of finite maturity debt, we contribute to the literature by modeling trade credit and deriving optimal trade credit durations. In our model, the supplier optimally chooses the trade credit horizon to extend to the buyer, anticipating the effect that this choice will have on the optimal capacity choice of the buyer. Our framework therefore differs in several aspects from previous models of corporate debt in order to reflect the characteristics of trade credit. First, while a main determinant of the usage of corporate debt is its tax advantage, trade credit implies no tax benefits. Second, unlike corporate debt, trade credit value depends on the order quantities, and thus on the capacity choice of the firm. Third, while previous papers study perpetual debt, we analyze finite debt, given the short-term and roll-over nature of trade credit. This allows us to explore the effect of operational flexibility on debt maturity.

On the other hand, we also extend the trade credit literature by including operational flexibility which has not been considered in existing models. The closest related papers are Koussis and Silaghi (2020), and Silaghi and Moraux (2020). Our rigid firm corresponds to the buyer firm analyzed in Koussis and Silaghi (2020).

The rest of the paper is organized as follows. Section 2 presents the model setup and the valuation for trade credit and buyer firm values for the two cases of a flexible and rigid firm. Section 3 presents numerical results and sensitivity analysis. In section 4 we consider an extension to our framework that includes switching costs. Finally, section 5 concludes.

2. The model

2.1. Model setup

A buyer firm produces a final good using as an input an intermediate good that it buys from the supplier firm. The price of the final good in the downstream market is given by the following inverse demand function: $X = x\, Q^{\varepsilon_B}$, where ε_B is the price sensitivity of demand ($-1 \leq \varepsilon_B \leq 0$).[5] For analytical convenience, we assume that $\varepsilon_B = 0$, i.e., demand is perfectly (infinitely)

[5] This demand function has been widely used in the literature (see among others Aguerrevere, 2009; Dixit and Pindyck, 1994; Dobbs, 2004; Silaghi and Sarkar, 2020). The model can be extended to a linear demand function as in Hagspiel et al. (2016) or Sarkar (2018). We also note that one can define the elasticity of demand as $\left|\frac{1}{\varepsilon_B}\right|$. A parameter value $\varepsilon_B = 0$ implies perfect competition and no market power, and the larger ε_B, the larger the market power of the firm.

elastic, so that the quantity produced has no impact on price and the demand shock process coincides with the price process. That is, the firm participates in a competitive market and cannot influence the commodity price. The demand shock x introduces uncertainty in the model and is driven by variables such as population size, prices of substitutes, the level of industrial production, etc. Following previous literature (Aguerrevere, 2009; Dixit and Pindyck, 1994; Dobbs, 2004; Silaghi and Sarkar, 2020), we assume that x follows a geometric Brownian motion:

$$\frac{dx}{x} = \mu dt + \sigma dZ \tag{1}$$

where μ is the expected rate of change, σ is the volatility and dZ is a standard Weiner process.

Both firms (buyer and supplier) are risk neutral, thus they maximize expected profits. We denote by r the risk-free interest rate, and have that $r > \mu$ such that there is a rate of return shortfall similar to a convenience yield $\delta = r - \mu$. A higher δ (while keeping r constant) captures a lower rate of growth of the good's demand in the downstream market. The buyer optimally chooses the quantity of goods to produce and order from the supplier by solving an optimal capacity problem (see for example, Hagspiel et al, 2016; Nishihara et al., 2019).[6] In particular, the buyer firm needs to incur a one-time investment cost κQ^{η} at time $t=0$ to install capacity, where Q represents the capacity, Q^{η} is the amount of capital required to produce at that capacity (with $\eta > 1$), and κ is the cost of capital per unit (in dollars).

The buyer firm faces both fixed costs of production C_b, as well as variable costs w. Fixed costs include two components: $C_b = c_f + c_h Q$, where c_h is the cost of holding one unit of capacity. We will initially assume $c_h = 0$, so that there are no capacity holding costs. We then elaborate on the effect of capacity holding costs in a later section. Additionally, the buyer procures from the supplier the input goods obtained on trade credit, thus he has to incur the debt payments. The supplier continuously provides a quantity of goods Q to the buyer.[7] The initial debt principal is $P_S Q$, where P_S defines the price per unit of goods charged. We follow Leland's (1994, 1998) framework for finite debt to model trade credit. That is, we assume that at any instant of time a constant fraction m of the debt matures and the buyer reimburses a fraction payment m of the value of goods, i.e., $mP_S Q$ of the goods are repaid. Therefore, the outstanding balance of trade credit at any instant of time $t>0$ gets reduced by e^{-mt}. As Leland (1994, 1998) shows, the average maturity of repayment is given by $1/m$. Moreover, the credit is continuously renewed, i.e., each time a fraction of the credit is reimbursed by the buyer, the supplier provides a new credit identical and of equal size to the one repaid, so that trade credit is fully rolled over. This is consistent with the roll-over character of short-term financing in general and of trade credit in particular (see

[6] For simplicity, we assume a one to one relation between the input and the output good.
[7] Note that the capacity size fixes the production level and thus the order quantity. We do not explore other roles of flexibility such as the possibility of producing below capacity, as in Hagspiel et al. (2016).

Amberg et al., 2020; Auboin and Engemann, 2014; Ferrando and Wolski, 2018; Garcia-Appendini, 2011). This also implies that the total outstanding amount of trade credit remains constant, i.e., we have a stationary trade credit structure. Similarly, Dangl and Zechner (2016), Diamond and He (2014), and Leland (1994, 1998) have a stationary structure for corporate debt.

Finally, the buyer firm has the option to default. This occurs when buyer value falls to zero and the buyer has no incentives to inject funds into the firm.[8] Formally, default takes place when the demand shock drops to an endogenously determined trigger, x_B. At this point the buyer stops payments to the supplier. We assume that in the event of bankruptcy, the supplier receives the recovery value $V_B = bP_SQ$, i.e., the supplier could repossess inventory that has not been converted into finished goods. In our base-case analysis we assume $b = 0$ (no recovery in bankruptcy) in line with suppliers' low priority in case of default. We study the impact of b in a later section.

We now analyze two different scenarios. First, we consider a rigid buyer firm with no operational flexibility which always produces at full capacity. Second, we analyze a flexible firm that can temporarily shut down when market conditions deteriorate, limiting losses. A rigid firm has the option to switch between a full-scale operating mode and an absorbing state where the firm defaults at the default threshold x_B. A flexible firm on the other hand has the options to switch between a full-scale operating mode (active mode), an idle mode and an absorbing state where the firm declares bankruptcy.[9] We assume that switching between the active and idle mode is costless. Since we want to compare trade credit policies for a flexible versus a rigid firm, we consider the two extreme cases, a flexible firm that can temporarily shut down without any switching costs, and a rigid firm that always produces at full capacity.[10] The firm switches from the active mode to the idle mode when the demand shock reaches the switching threshold $\bar{x} = w$. While being in this operating mode, the firm has the option to resume full-scale operations when the demand shock increases and reaches $\bar{x}$. On the contrary, if the demand shock falls further, the firm may default, thus $x_B < \bar{x}$. Theoretically, it is also possible that $x_B > \bar{x}$ if P_S is sufficiently high. In this case, the firm never shuts down temporarily and operational flexibility is not used. Therefore, the appropriate firm value is given by the value of the rigid firm. In our analysis below, we focus on the case where $x_B < \bar{x}$, that is, the flexible firm defaults at a value where the firm is operating in the idle mode.

2.2. Inflexible case

[8] Similarly, in the models of Diamond and He (2014) and Leland (1994, 1998), the equityholders are assumed to have access to funds to cover the investment costs and losses at refinancing.

[9] The possibility to temporarily shut down is similar to mothballing discussed in Dixit and Pindyck (1994), Chapter 7 with the added feature of trade credit. In a related paper, Charalambides and Koussis (2018) considered debt rescheduling when temporarily shutting down.

[10] We relax this assumption and incorporate switching costs in section 4.

2.2.1. Trade credit value under inflexible case

Following Dangl and Zechner (2016), Diamond and He (2014) and Leland (1994, 1998), the value of trade credit $D(x)$ follows the following second order ordinary differential equation:

$$rD_r(x) = \frac{\sigma^2}{2}x^2 D_r''(x) + (r-\delta)xD_r'(x) + m(P_S Q - D_r(x)) \quad (2)$$

At each instant of time a fraction $m\ dt$ of credit matures. The buyer firm thus makes the principal repayment of $mP_S Q\ dt$. At the same time, an identical credit is issued of a similar amount, so the supplier provides a new credit of $mD_r\ dt$. Combining the two amounts we obtain the change in trade credit value due to debt retirement given by the last term on the right-hand side of the previous equation.

The solution is given by:

$$D_r(x) = \left(\frac{mP_S Q}{r+m}\right) + A_1^D x^{\gamma_1} + A_2^D x^{\gamma_2} \quad (3)$$

The interpretation is intuitive. The first term represents the risk-free value of credit, while the additional terms capture the adjustments needed due to buyer's default option. γ_1 and γ_2 are the solutions of the following fundamental quadratic equation: $q = \frac{1}{2}\sigma^2\gamma(\gamma-1) + (r-\delta)\gamma - (r+m) = 0$, and are given by the following equations:

$$\gamma_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+m)}{\sigma^2}} > 1 \quad (4a)$$

$$\gamma_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+m)}{\sigma^2}} < 0 \quad (4b)$$

Parameters A_1^D and A_2^D are constants to be determined by the following boundary conditions:

$$\lim_{x\to\infty} D_r(x) = \frac{mP_S Q}{r+m} \quad (5)$$

$$D_r(x_{Br}) = V_B \quad (6)$$

Extremely profitable buyers never default, and the default-free trade credit value is $\frac{mP_S Q}{r+m}$, as in equation (5). On the other hand, equation (6) indicates that the buyer defaults when $x = x_{Br}$ and the supplier receives the recovery value $V_B = bP_S$Q.

Solving equation (2) with boundary conditions (5) and (6) we obtain that:

$$A_1^D = 0 \quad (7a)$$

$$A_2^D = \left(V_B - \frac{mP_S Q}{r+m}\right) x_{Br}^{-\gamma_2} \quad (7b)$$

This leads to the following solution for the value of trade credit:

$$D_r(x) = \frac{mP_SQ}{r+m} - \left(V_B - \frac{mP_SQ}{r+m}\right)\left(\frac{x}{x_{Br}}\right)^{\gamma_2} \tag{8}$$

2.2.2. Buyer value under inflexible case

Following a similar reasoning, the rigid buyer value $B_r(x)$ satisfies the following second order ordinary differential equation:

$$rB_r(x) = \frac{\sigma^2}{2}x^2 B_r''^{(x)} + (r-\delta)xB_r'(x) + (xQ - wQ - C_b) - m(P_sQ \ - D_r(x)) \tag{9}$$

The last two terms of equation (9) represent the cash flows of the inflexible buyer firm. The first of these two terms captures the profits of the buyer comprising of revenues xQ net of variable costs wQ and fixed costs C_b, while the second one represents the rollover gains/losses of paying the principal, i.e., the price of goods, mP_SQ, and receiving the trade credit proceeds, mD.

The solution of the above equation is given by:

$$B_r(x) = \left(\frac{xQ}{\delta} - \frac{C_b}{r}\right) - D_r(x) + A_1\, x^{\beta_1} + A_2\, x^{\beta_2} \tag{10}$$

where $D_r(x)$ is given in equation (8) and the exponents β_1 and β_2 are given by:

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{11a}$$

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{11b}$$

The particular solution (first two terms in equation 10) captures the perpetuity value of the buyer accounting for revenues, operating costs and repayment of credit, while the additional terms capture the buyer's default option. We provide details on the derivation of the particular solution in Appendix. The constant terms A_1 and A_2 are determined by applying the following boundary conditions:

$$\lim_{x\to\infty} B_r(x) = \left(\frac{xQ}{\delta} - \frac{wQ+C_b}{r}\right) - \frac{mP_SQ}{r+m} \tag{12a}$$

$$B_r(x_{Br}) = 0 \tag{12b}$$

The first condition (12a) implies that the buyer's value is the present value of net cash flows (first term) minus the (risk-free) value of credit in the absence of default risk. Condition (12b) implies that the buyer's value in the event of bankruptcy becomes zero. We thus obtain the following solution for rigid buyer value:

$$B_r(x) = \left(\frac{xQ}{\delta} - \frac{wQ+C_b}{r}\right) - D(x) - \left(V_B - \frac{x_{Br}Q}{\delta} + \frac{wQ+C_b}{r}\right)\left(\frac{x}{x_{Br}}\right)^{\beta_2} - \kappa Q^{\eta} \quad (13)$$

Note that we have also accounted for the one-time investment cost incurred by the buyer at time *t=0*, κQ^{η} (last term of equation 13). The optimal default threshold is obtained by applying the following smooth pasting condition:

$$\frac{\partial B_r}{\partial x}\Big|_{x=x_{Br}(Q)} = 0 \quad (14)$$

Applying this smooth pasting condition, we obtain the following default threshold:

$$x_{Br}(Q) = \frac{\delta}{Q(1-\beta_2)}\left[\gamma_2\left(V_B - \frac{mP_SQ}{r+m}\right) - \beta_2\left(V_B + \frac{wQ+C_b}{r}\right)\right] \quad (15)$$

The buyer does not only choose optimally the default trigger, but also the optimal quantity. The optimal Q maximizes $B_r(x)$ and therefore satisfies:

$$\frac{\partial B_r}{\partial Q} = 0 \quad (16)$$

This condition results in a non-linear implicit equation (see Appendix A) that has no closed-form solution. Given the non-linearity of this equation, this maximization problem is solved numerically. We run a dense grid search of Q choices for the buyer. Among the grid of Q choices created we then select the choice that maximizes the rigid buyer firm value.

2.3. Flexible case

2.3.1. Trade credit value under flexible case

Although the buyer firm has operational flexibility and can temporarily shut down, we follow Ritchken and Wu (2020) and Charupat and Sarkar (2020) who model corporate debt and assume that the firm continues to make debt/trade credit payments in idle mode. This would be the case for example if the supplier is a rigid firm that cannot adjust production or if the supplier imposes minimum order quantities. Thus, trade credit value will be derived in a similar fashion as in the inflexible case:

$$D_f(x) = \frac{mP_SQ}{r+m} - \left(V_B - \frac{mP_SQ}{r+m}\right)\left(\frac{x}{x_{Bf}}\right)^{\gamma_2} \quad (17)$$

However, the optimal quantity chosen by the buyer might be different, as well as the optimal maturity selected by the supplier.

2.3.2. Buyer value under flexible case

When the buyer firm has operational flexibility and can temporarily shut down, buyer value will have two operating regions:

$$B_1(x) = \frac{xQ}{\delta} - \frac{wQ}{r} - \frac{C_b}{r} - D_f(x) + A_3 x^{\beta_2}, x > \bar{x} \quad (18a)$$

$$B_0(x) = -\frac{C_b}{r} - D_f(x) + A_4 x^{\beta_1} + A_5 x^{\beta_2}, x \leq \bar{x} \quad (18b)$$

Equation (18a) applies to the active mode, while equation (18b) corresponds to the idle mode. The first four terms in equation (18a) capture the discounted value of the cash flows net of variable costs, fixed costs and trade credit payments. The last term in this equation captures the buyer's option to switch to idle mode. Similarly, the first four terms in equation (18b) capture the net cash flows of the buyer, while the fifth term adjusts for the option to switch to full-scale production, and the last term adjusts for the option to default. The constants A_3 , A_4 , and A_5 and the optimal threshold x_{Bf} are determined from the following value matching and smooth pasting conditions:

$$B_0(\bar{x}) = B_1(\bar{x}), \quad (19a)$$

$$B_0(x_{Bf}) = 0, \quad (19b)$$

$$B_0^{'}(\bar{x}) = B_1^{'}(\bar{x}), \quad (20a)$$

$$B_0^{'}(x_{Bf}) = 0. \quad (20b)$$

The buyer switches from the active to the idle mode (and vice-versa) when the demand shock drops (raises) to $\bar{x}$. The buyer function value needs to be continuously differentiable at this point (equations 19a and 20a). When the demand shock drops to x_{Bf}, the buyer firm defaults and its value becomes zero (equation 19b). Since the default threshold is optimally chosen, the smooth pasting condition applies (equation 20b).

The expressions of the constants A_3 , A_4 , and A_5 are relegated to Appendix A. The optimal threshold is the implicit solution of the following equation:

$$\beta_1 A_4 x_B^{\beta_1 - 1} + \beta_2 A_5 x_B^{\beta_1 - 1} - \frac{\gamma_2}{x_B}\left(V_B - \frac{mP_S Q}{r+m}\right) = 0 \quad (21)$$

Finally, the buyer optimally chooses the quantity to produce:

$$\frac{\partial B_1}{\partial Q} = 0 \quad (22)$$

This condition results in a non-linear implicit equation that has no closed-form solution. We thus solve it numerically by performing a dense grid search for Q choices, among which we select the one that maximizes the flexible buyer firm value net of the capacity cost. Additionally, we always ensure that the flexible buyer starts operations at full capacity, not in idle mode.

2.4 Interactions buyer-supplier

In this section we describe the interactions between the buyer and the supplier. Following Koussis and Silaghi (2020), we assume that for a given trade credit maturity, the buyer optimally selects capacity Q (the quantity of goods ordered from the supplier), and the default threshold that maximize buyer value. The supplier internalizes how his choice of trade credit maturity influences the buyer's optimal choices of Q and x_B, and optimally selects the trade credit maturity that maximizes trade credit value. These interactions are formally described in the following maximization problem:

$$\underset{(\frac{1}{m})}{MAX}\ D(x) \tag{23}$$

$$s.t.\ \underset{Q\ \ ,x_B}{MAX}\ B(x)$$

Given the non-linearity of the equations involved, this maximization problem is solved numerically. We run a dense grid search of $(1/m)$ choices for the supplier subject to the optimal solutions Q and x_B that maximize the buyer's value. Among the grid of $(1/m)$ choices created we then select the choice that maximizes the trade credit value. We do this for both the rigid and flexible case and compare the results to understand the effect of flexibility on trade credit values and maturity.

3. Numerical results

We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0$, $w = 7$, $P_S = 5$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2$, $b = 0$. Our base parameters used for r, δ and σ are in line with other real options models (e.g., Mauer and Sarkar, 2005 and Hackbarth and Mauer, 2011).[11] η is the same as in Nishihara et al. (2019), while our base case parameters for x and C_b were chosen alongside κ and η which define optimal order quantity levels to allow the buyer to operate with positive value. c_h is assumed initially to be zero to see more clearly the impact of initial capacity choices (we perform sensitivity subsequently on c_h). We assume initially no recovery at default (b = 0) which is a reasonable assumption since suppliers typically have low priority when buyer firm defaults compared to other claimants. The price per unit sold x is chosen to allow for a reasonable, albeit not significantly high gross profit margin (x-w) which is commensurate with assumptions used in other related studies (e.g., Hagspiel et al., 2016). We

[11] Mauer and Sarkar (2005) use r = 0.05, δ = 0.02 and $\sigma = 0.25$, while Hackbarth and Mauer (2011) use r = 0.06, δ = 0.05 and $\sigma = 0.25$. Our use a lower volatility level reflects product markets with a moderate effect of variations in the target market size, disposable income, tastes, and prices of substitute products (see, for instance, Aguerrevere, 2003 and 2009). We consider higher volatility which is more reflective of industries with a larger variation such as luxury or technology products in our sensitivity results.

have no empirical benchmark for P_S and we provide sensitivity results that reveals the pricing impact of supplier on trade credit duration levels, trade credit level quantities and buyer and trade credit values.

In all figures we vary *m* with increments of 0.1 and minimum (1/m) maturity of 0.1 and maximum maturity (1/m) of 1 year. Unless otherwise stated *Q* is optimal chosen by the buyer firm where we use increments for *Q* search of 0.01. Figure 1 shows our base case sensitivity results.

[Insert Figure 1 here]

In the first panel we observe that extensions of trade credit duration improve the value for both the rigid and flexible firm, albeit the impact of longer trade credit horizons on the value of the flexible firm is more significant. This is reflected in the quantity choices of the two firms where the flexible firm chooses higher quantities for all credit duration levels compared to the rigid firm, however the differences between the two firms are increasing at higher durations. This result is in line with the findings of Hagspiel et al. (2016) who show that optimal capacity is larger for a flexible firm. We also observe that in terms of default thresholds the flexible firm delays default more compared to the rigid firm and the differences become more significant for longer horizons. This result is in line with the results of Ritchken and Wu (2020) and Charupat and Sarkar (2020) who focus on optimal capital structure decisions of flexible compared to inflexible firms. However, while Ritchken and Wu (2020) find that the flexible firm uses less debt (a lower optimal coupon), we find that the flexible firm uses more trade credit. The reason is twofold. First, since we model finite short-term credit (not perpetual debt), a later default implies that the supplier trades with the buyer for a longer period, leading to higher trade credit values. Second, the lower default risk also results in higher order quantities, which further increases trade credit values. Thus, when comparing trade credit values, we observe that the flexible firm has a higher trade credit value compared to the rigid one.

Since we focus on finite debt, our model allows us to derive novel results regarding the impact of flexibility on debt maturity. Interestingly, our analysis reveals that trade credit value has a hump shape for both the flexible and rigid firm which results in an optimal trade credit horizon. This is driven by the trade-offs involved in the supplier's choice of increasing the duration of credit: on the negative side the supplier receives the payment with a delay, whereas on the positive side, increasing credit duration allows the buyer to order more (*Q* increases) and results in delayed default and thus an extended period where the supplier trades with the buyer. The interaction of these two opposite effects determines the optimal credit duration. Since the impact of extending credit duration on order quantities and default is more significant for the flexible firm, the optimal trade duration provided by the supplier is longer for the flexible firm.

When excluding the positive impact on order quantities for the supplier the supplier has less incentives (only arising due to delayed default) and may not extend credit duration. To see this, in Figure 2 we investigate a fixed (instead of optimal) Q. Indeed, we observe that in the absence of the positive effect of increasing quantities at longer trade horizons, the optimal credit policy of the supplier is to request immediate repayment, i.e., the supplier value, in the absence of gains of extending credit, is strictly decreasing in the trade credit horizon.

[Insert Figure 2 here]

It is important to emphasize that extensions of trade credit duration may not be achieved if the buyer firm free rides on extensions by ordering more from other suppliers (e.g. see Chod et al., 2019). This effect is not considered here, however our analysis is consistent since as shown in Figure 2 when the supplier does not anticipate changes in order quantities then the supplier does not extend credit.

Previous literature on the effect of operational flexibility on capital structure (Ritchken and Wu, 2020; Charupat and Sarkar, 2020) also shows the effect of operational flexibility on leverage ratios, not only on debt levels. In particular, they find that leverage ratios are higher for inflexible compared to flexible firms. In Figure 3 we examine trade credit leverage.

[Insert Figure 3 here]

Consistently with this work, and despite the fact that trade credit is larger for flexible firms, we find that the trade credit leverage ratio is higher for the inflexible firm. This is due to the fact that flexibility increases the buyer firm value even more than it increases its trade credit value. The figure also shows that the trade credit leverage ratio differences widen for longer trade credit maturities. The trade credit leverage ratios obtained are in line with empirical evidence. According to Rajan and Zingales (1995), accounts payable constituted around 15% of the total book value for the average non-financial firm in 1991.

The previous findings are summarized in the following result:

Result 1: Flexible vs rigid firm and the optimal choice of trade credit duration

The optimal order quantities and trade credit value are higher for the flexible firm compared to the rigid firm. However, the trade credit leverage ratio is higher for the rigid firm and the difference widens for longer trade credit durations. The flexible firm defaults later compared to the rigid firm. The supplier generally has more incentive to provide a longer trade credit duration for the flexible firm.

Unfortunately, to our knowledge, there is no empirical evidence on the relationship between operational flexibility and trade credit. Nevertheless, Harris (2015) finds that increases in financial flexibility are associated with higher levels of trade credit. Our model predicts that *operational* flexibility is also positively related to trade credit levels. Similarly, we are not aware of empirical studies on the relationship between operational flexibility and trade credit duration. Our results are broadly in line with the findings of Klapper et al. (2012) who document longer trade credit maturities for more credit worthy buyers. MacKay (2003) finds that flexible firms have lower leverage and shorter maturities for corporate debt. However, he does not analyze trade credit.

Figure 4 provides sensitivity with respect to lower prices P_S per unit of credit charged by the supplier for providing credit. In this case we observe that the order quantities increase relative to the base case (see Figure 1), however, the impact of extending trade credit duration becomes of less importance on optimal selected Q_S. Indeed, optimal Q_S become flat with respect to the trade credit duration. The same appears for the default thresholds which appear relatively flatter with respect to the trade credit duration. Thus, with lower prices charged P_S, the negative effect of delayed payments for the supplier becomes more significant relative to the positive effects of increased quantities and delayed default. This results in a shortening of the credit horizon for both the flexible and rigid firm. This is consistent with evidence provided by Molina and Preve (2009) showing that suppliers facing cash flow problems reduce trade credit. We observe that the differences in value between the flexible and rigid firm do not widen as much with respect to trade credit duration when suppliers prices are lower and thus the optimal trade durations differences between the flexible and rigid firm are of less importance (and actually converge at low trade credit horizons). We summarize the following result.

[Insert Figure 3 here]

Result 2: The impact of prices Ps charged for trade credit on trade credit duration

Lower prices per unit of trade credit result in lower trade credit values and a shortening of the trade credit horizon for both the flexible and rigid firm. The optimal trade credit duration is reduced for both flexible and inflexible firm and there is a lower difference in trade credit durations provided to the flexible vs rigid firm. The order quantities are higher for both the flexible and inflexible firm and less sensitive to the duration of credit.

Result 2 highlights that in industries with less intense competition between suppliers which may result in higher prices charged for provided credit (e.g., specialized inputs in technology products), trade credit duration may be a more effective mechanism for suppliers to affect order quantities of the buyer firms. On contrary, when supplier firms face intense competition resulting in lower prices or face cash flow concerns such as in Molina and Preve (2009), the optimal policy is to reduce trade credit and trade credit duration.

Figure 5 shows that an extension of trade credit duration becomes more important for both the rigid and flexible in retaining higher quantities when downstream firms operate in more volatile downstream markets.

[Insert Figure 5 here]

Thus, optimal credit duration increases for both firms. However, we observe a more noticeable impact of extending credit on the flexible firm's optimal quantities and on its timing of default. Hence the analysis supports the following result[12]:

Result 3: The impact of downstream volatility on trade credit duration

A higher volatility in the downstream market results in an extension of trade credit horizon for both a rigid and flexible firm. Due to a more significant positive impact of extension in credit duration on optimal quantities and default timing on the flexible firm, the optimal credit duration is extended more for the flexible firm.

Figure 6 explores the impact of more significant capacity constraints (η). For brevity we do not show the sensitivity results with respect to per unit cost of capital, k which has similar implications. We observe that a longer credit duration by the supplier can have a less significant positive impact on order quantities for the rigid buyer when it faces more severe capacity constraints (a higher η) compared to when η was low (see base case of Figure 1). The optimal duration of credit is thus lower for a higher η for the rigid firm. While higher capacity constraints also have a negative impact for the flexible firm, we observe an extension of trade credit duration for the flexible firm since this helps the supplier retain quantities at the largest possible level. Thus, with more significant capacity constraints we observe a somewhat more significant difference between extending duration for the flexible vs the inflexible firm.

[Insert Figure 6 here]

Result 4: The impact of higher capacity constraints (η) on trade credit duration

An increase in capacity constraints (η) results in a larger difference in trade credit horizon for the flexible compared to the rigid firm. Order quantities are adversely affected for both the rigid and flexible firm, but credit credit duration has a somewhat stronger positive effect on quantities and trade credit for the flexible firm.

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[12] Our results relating the duration of credit are consistent with Koussis and Silaghi (2020). Compared to Koussis and Silaghi (2020), however here higher volatility results in higher value for buyer firm. This is due to the fact that in Koussis and Silaghi (2020) there are no variable costs of production for the buyer relating to quantity. Thus, buyer firms were more "in-the-money" and increases in volatility resulted in a reduction in buyer value. Here, since the buyer also faces variable costs it becomes more "out-of-money" and thus higher volatility helps improve its value. Interestingly, in this case, higher volatility results in the buyer employing larger quantities and this also has a positive impact on supplier trade credit value.

Result 4 extends the insights on the capacity choice of firms provided in the real options literature focusing on firms' capacity choices (e.g., see Huberts et al., 2015 for a review). Compared to this literature we provide insights on the effect of capacity constraints on trade credit and order quantities and the different behavior of flexible versus rigid firms across these dimensions. Recent developments in Hagspiel et al. (2016) have analyzed flexible and inflexible firms but they focus on the timing and optimal capacity level while we focus on trade credit and the interactions between supplier and buyer firm in the choice of trade credit duration and order quantities. Richten and Wu (2020) focus on flexible and rigid firms capacity choices and optimal capital structure by focusing on debt financing and not on trade credit.

Figure 7 explores the impact of a higher recovery rate of supplier value in the event of default (*b)*. With higher recovery value in the event of default the supplier becomes less concerned in extending credit (since in the event of default a more significant part of the value can be recovered). Thus, the trade credit value is maximized at shorter credit durations. We still observe that the flexible firm obtains longer credit, but as recovery becomes more significant the supplier will optimally select very short (or immediate) payments irrespective of flexibility levels. We summarize the following result.

Result 5: The impact of the recovery rate in default (b) on trade credit duration

A higher recovery rate in default (b) results in higher order quantities and trade credit and a shortening of trade credit horizon for both the rigid and the flexible firm. The difference in trade credit duration between the flexible and rigid firm is expected to be of less importance the more significant the recovery value in default.

The positive effect of recovery values on trade credit is consistent with evidence in Costello (2019) who shows that an improvement in suppliers' rights to the liquidation value of collateral results in an increase in the amount of credit. This is broadly in line also with Petersen and Rajan (1997) results who show higher levels of trade credit value when buyers have higher credit ratings (since it is reasonable to assume that higher recovery values would improve ratings). In comparison with Koussis and Silaghi (2020) our above results higlight the differences in trade credit, durations and quantities one would observe between the flexible firm and rigid firm under alternative recovery rate scenarios.

Figure 8 shows the impact of lowering the variable production costs *w*. We still observe that extensions in trade credit duration create a significant impact on firm's values, however the marginal impact appears more important for short maturities. Hence the optimal credit duration is shorter for both the flexible and rigid firm and their differences become less important. A similar effect is observed when we analyze a higher *x* which has a similar effect on gross profit (operating) margins (see Figure A1). We thus summarize the following result in terms of gross profit margins.

Result 6: The impact of gross profit margins for downstream (buyer) firms

Higher gross profit margins of downstream firms resulting from either lower variable costs of production (w) or higher prices per unit sold (x) result in a more significant positive impact on order quantities and trade credit for shorter credit horizons. We thus observe a shortening of trade credit horizon for both a rigid and flexible firm and the difference in trade credit duration between the flexible and rigid firm is expected to be of less importance the higher the gross profit margin levels of downstream firms.

We note that the above result on the impact of gross profit margin on order quantities and trade credit is consistent with Petersen and Rajan (1997) who show that more profitable buyers obtain larger amounts of credit.

Figure 9 shows the impact of higher capacity holding cost *(h).* A higher holding cost of capacity reduces installed capacity and order quantities for both the flexible and rigid buyer firms. Extending trade credit duration mitigates the impact on order quantities and we thus observe that supplier firms will somewhat extend credit duration (see relative to base case of Figure 1).

[Insert Figure 9 here]

The capacity holding cost is a fixed cost that will be incurred by both the flexible and rigid firm, thus despite the flexible firm's ability to truncate variable costs in the idle mode this cost will still need to be incurred by the flexible firm. We thus observe that maturity differences in the trade credit horizon become less significant between the rigid and flexible firm. We summarize the following result:

Result 7: The impact of capacity holding cost (h) for downstream (buyer) firms

A higher capacity holding cost (h) results in lower order quantities and credit and extensions of trade credit horizon for both a rigid and flexible firm. The difference in trade credit duration between the flexible and rigid firm is expected to be of less importance the higher the capacity holding cost.

4. Extension: Switching costs

So far we have assumed that the flexible firm can costlessly switch between the active and the idle mode. We now introduce switching costs into the framework. The presence of switching costs creates hysteresis that causes a delay of the flexible firm switching to idle region or vice versa when switching from idle to active mode. We have the following adjustments to the previous framework. The flexible firm starts production at $x > x_H$ and then may temporarily stop production when the demand shock reaches the threshold x_L. When switching to this lower region, the buyer firm needs to incur a switching cost ϵ_{10}. While being in this operating mode, the firm has the option to resume full-scale operations when the demand shock increases and reaches x_H (from below). In this case, the firm starts producing at full capacity again, and incurs a switching cost ϵ_{01}. On the contrary, if the demand shock falls further, the firm may default, thus $x_B < x_L < x_H$. Theoretically, it is also possible that $x_B > x_L$ if P_S is sufficiently high. In this case, the firm never reduces scale temporarily and operational flexibility is not used. Therefore, the appropriate firm value is given by the value of the rigid firm. In our analysis below, we focus on the case where $x_B < x_L$, that is, the flexible firm defaults at a value where the firm is operating at a low production scale. The zone between x_H and x_L is called in the literature a hysteresis zone since within that region the firm remains active if previously in the active mode or idle if previously in the idle mode (e.g. Dixit and Pindyck, 1997, ch.7). This is intuitive since the firm postpones entering a different mode to avoid incurring the switching costs involved. Compared to the framework without switching costs, the switching thresholds satisfy $x_L < w < x_H$. The mathematical details of the model including boundary conditions and buyer value are provided in Appendix C.

In Figure 10 we investigate a case with positive switching costs $\epsilon_{10} = \epsilon_{01} = 20$. The results show that when switching costs are positive there is a hysteresis zone created due to the difference between x_H and x_L. This implies that when the firm's revenues cross x_H from above but $x > x_L$ the firm remains in the active mode despite possibly incurring some losses. The firm enters the idle mode once x first crosses x_L from above. If x crosses x_L again from below, the firm remains in the idle mode until x_H is now reached from below. Interestingly, the operating policies of the firm regarding the utilization of operating flexibility remain unchanged irrespective of the trade horizon. The flexible firm in the presence of switching costs only adjusts the default trigger as the trade credit horizon is altered similarly to the changes taking place for the fully flexible firm.

[Insert Figure 10 here]

Compared to the fully flexible firm (no switching costs), the flexible firm with positive switching costs has slightly lower order quantities and trade credit. The differences between the flexible firm with positive switching costs and the rigid firm thus remain (albeit become smaller). Of course, the differences between the flexible firm and rigid firm become zero when $x_B > x_L$ in which case the flexible firm faces such significant costs which makes it impossible to utilize operational flexibility options.

Figure 11 shows sensitivity results of the model with switching costs in the special case where the switching costs are zero **(**$\epsilon_{01} = \epsilon_{10} = 0$). The results confirm that in the case that switching costs are zero we obtain an identical solution to the flexible model since optimal switching is $x_H = x_L = w$. We thus obtain the solutions of the flexible firm of the previous section.

[Insert Figure 11 here]

5. Conclusions

This paper investigates the effect of operational flexibility on trade credit values, order quantities and trade credit maturity. We find that firms that have the ability to temporarily shut down will order higher quantities and have larger trade credit values. Moreover, suppliers will extend larger trade credit maturities to flexible firms. Our analysis shows however that trade credit as a proportion of buyer firm value is higher for the inflexible firm because flexible firm's value increases more rapidly than the associated increase in trade credit. This result may be useful for empirical researchers usually focusing on ratios rather than levels.

We provide a number of predictions such as that the effect of flexibility on trade credit and credit durations is more significant in more uncertain environments, in industries with less intense competition between suppliers and in downstream markets with higher capacity constraints, lower recovery rates, lower gross profit margins or lower capacity holding costs.

There are some factors that we have overlooked in our analysis and provide the basis for future extensions. Firstly, although our analysis provides for some adjustments in production, it would be interesting to investigate how the framework can be adjusted to incorporate capacity utilization, so that productioncan be upscaled and downscaled in relation to demand. Secondly, we have assumed a single buyer and supplier and it would thus be interesting to investigate competitive interactions in the supplier or/and the buyer markets. Finally, we analyze a single product sold by the buyer firm while in many cases firms operate and manage multiple products.

Appendix A: Details on the derivation of buyer's value

In this appendix we provide details on our derivation of the buyer value for both the inflexible and the flexible cases.

Inflexible buyer value

In order to derive the particular solution in equation (10) of the buyer we proceed by applying the solution:

$$B(x) = B + C_0 x + C_1 x^{\gamma_2} \tag{A1}$$

that satisfies differential equation (9) obtaining the following solutions for C_0 and C_1 and B:

$$B = \frac{m}{r}\frac{mP_SQ}{r+m} - \frac{mP_SQ+C_b}{r}, C_0 = -A_2^D\ ,\ C_1 = \frac{Q}{\delta} \tag{A2}$$

We note that to derive C_0 we have used the fact that $m = -(r-(r-\delta)\gamma_2 - \frac{1}{2}\gamma_2(\gamma_2-1)\sigma^2)$ which simplifies the presentation of the solution. Note also that unlike the standard particular solution, the term $C_1x^{\gamma_2}$ in (A1) is used to capture trade credit value in the differential equation (9) (which as seen in equation (8) depends on γ_2 term).

Replacing solutions of constants C_0, C_1 and B from (A2) back in (A1) and noting that

$$\frac{mP_SQ}{r} - \frac{m}{r}\frac{mP_SQ}{r+m} = \frac{mP_SQ}{r+m} \tag{A3}$$

one can verify first three terms in equation (10).

Next, we note that since $\beta_1 > 0$ applying (12a) to equation (10) implies that:

$$A_1 = 0 \tag{A4}$$

Applying the boundary condition (12b) to equation (10) we obtain that:

$$A_2 = -\left[\left(\frac{x_{Br}}{\delta} - \frac{(mP_SQ+C_b)}{r}\right)(1-\tau) + \frac{m}{r}\frac{mP_SQ}{r+m} - \left(V_B - \frac{mP_SQ}{r+m}\right)\right](x_{Br})^{-\beta_2}$$

Using (A3) we can further simplify A_2 and we obtain that:

$$A_2 = -\left[\left(\frac{x_{Br}}{\delta} - \frac{C_b}{r}\right) - V_B\right](x_{Br})^{-\beta_2} \tag{A5}$$

Replacing (A4) and (A5) into equation (10) together with the particular derived above we thus derive the final solution for the buyer value in equation (13).

The optimal quantity ordered by the buyer maximizes buyer value. From equation (16) we obtain the implicit equation for Q:

$$\begin{aligned}
&\frac{x}{\delta} - \frac{w}{r} - \frac{mP_S}{r+m} - \left(bP_S - \frac{mP_S}{r+m}\right)\left(\frac{x}{x_{Br}(Q)}\right)^{\gamma_2} \\
&\qquad + \frac{\gamma_2}{x_{Br}(Q)}\left(bP_SQ - \frac{mP_SQ}{r+m}\right)\left(\frac{x}{x_{Br}(Q)}\right)^{\gamma_2}\frac{\partial x_{Br}(Q)}{\partial Q} \\
&\qquad + \left[bP_S - \frac{x_{Br}(Q)}{\delta} - \frac{Q}{\delta}\frac{\partial x_{Br}(Q)}{\partial Q} + \frac{w}{r}\right]\left(\frac{x}{x_{Br}(Q)}\right)^{\beta_2} \\
&\qquad - \frac{\beta_2}{x_{Br}(Q)}\left[bP_SQ - \frac{x_{Br}(Q)Q}{\delta} + \frac{wQ + C_b}{r}\right]\left(\frac{x}{x_{Br}(Q)}\right)^{\beta_2}\frac{\partial x_{Br}(Q)}{\partial Q} - \kappa\eta Q^{\eta-1} \\
&\qquad = 0,
\end{aligned}$$

where we have used the fact that recovery value is given by $V_B = bP_SQ$.

Flexible buyer value

The constants A_3, A_4, A_5 determined from the boundary conditions are given by the following expressions:

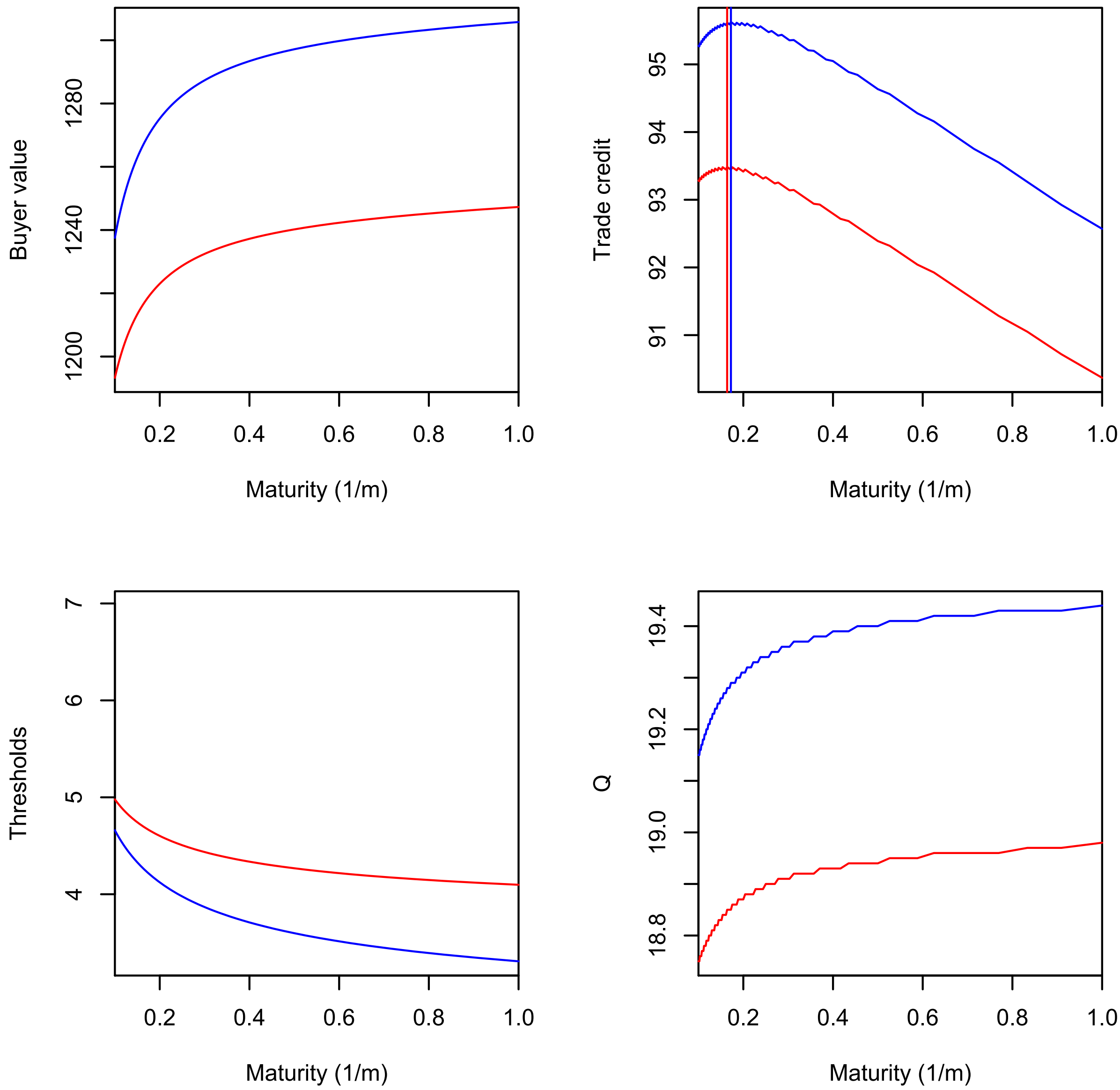
$$A_4 = \frac{1}{\beta_2 - \beta_1} \frac{Q}{\bar{x}^{\beta_1}} \left(\frac{(\beta_2 - 1)\bar{x}}{\delta} - \frac{\beta_2 w}{r} \right)$$

$$A_5 = x_B^{-\beta_2} \left(\frac{C_b}{r} + V_B - A_4 x_B^{\beta_1} \right)$$

$$A_3 = A_5 + \frac{Q \left[\frac{(\beta_1 - 1)\bar{x}}{\delta} - \frac{\beta_1 w}{r} \right]}{(\beta_2 - \beta_1)\bar{x}^{\beta_2}}$$

Appendix B. Additional results

Fig. A1. Sensitivity with respect to revenue level *x*: higher revenue level for optimal Q



Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0$, $w = 7$, $P_S = 5$, $\sigma = 0.15$, $x = 10$, $k = 5$, $\eta = 2$, $b = 0$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01. “Thresholds” in the above figures refer to the bankruptcy thresholds x_{Br} and x_{Bf} for the rigid and flexible firm respectively.

Appendix C. Flexible buyer value with switching costs

The buyer value has two operating regions:

$$B_1(x) = \frac{xQ_1}{\delta} - \frac{wQ_1}{r} - \frac{C_b}{r} - D(x) + A_3 x^{\beta_2}, x > x_H \tag{18a}$$

$$B_0(x) = -\frac{C_b}{r} - D(x) + A_4 x^{\beta_1} + A_5 x^{\beta_2}, x \leq x_L \tag{18b}$$

The constants A_3 , A_4 , and A_5 and the optimal thresholds x_H, x_L, and x_B are determined from the following value matching and smooth pasting conditions:

$$B_0(x_H) = B_1(x_H) - \epsilon_{01}, \tag{19a}$$

$$B_1(x_L) = B_0(x_L) - \epsilon_{10}, \tag{19b}$$

$$B_0(x_B) = 0, \tag{19c}$$

$$B_0'(x_H) = B_1'(x_H), \tag{20a}$$

$$B_1'(x_L) = B_0'(x_L), \tag{20b}$$

$$B_0'(x_B) = 0. \tag{20c}$$

The expressions of the constants A_3 , A_4 , and A_5 are given by:

$$A_4 = \frac{x_H^{1-\beta_1}}{\beta_1}\frac{Q}{\delta}\left(1 + \frac{x_L^{1-\beta_1} - x_H^{1-\beta_1}}{x_H^{\beta_2-\beta_1} - x_L^{\beta_2-\beta_1}} x_H^{\beta_2-1}\right)$$

$$A_5 = -\frac{\beta_1}{\beta_2} A_4 x_B^{\beta_1-\beta_2} - \left(V_B - \frac{mP_S Q}{r+m}\right)\frac{\gamma_2}{\beta_2} x_B^{-\beta_2}$$

$$A_3 = A_5 + \frac{Q}{\delta\beta_2}\frac{x_L^{1-\beta_1} - x_H^{1-\beta_1}}{x_H^{\beta_2-\beta_1} - x_L^{\beta_2-\beta_1}}$$

The optimal thresholds are the implicit solutions of the following equations:

$$A_4 x_H^{\beta_1} + (A_5 - A_3) x_H^{\beta_2} + \epsilon_{01} = \left(\frac{x_H}{\delta} - \frac{w}{r}\right) Q$$

$$A_4 x_L^{\beta_1} + (A_5 - A_3) x_L^{\beta_2} - \epsilon_{01} = \left(\frac{x_L}{\delta} - \frac{w}{r}\right) Q$$

$$A_4 x_B^{\beta_1} + A_5 x_B^{\beta_2} = \frac{C_b}{r} + V_B$$

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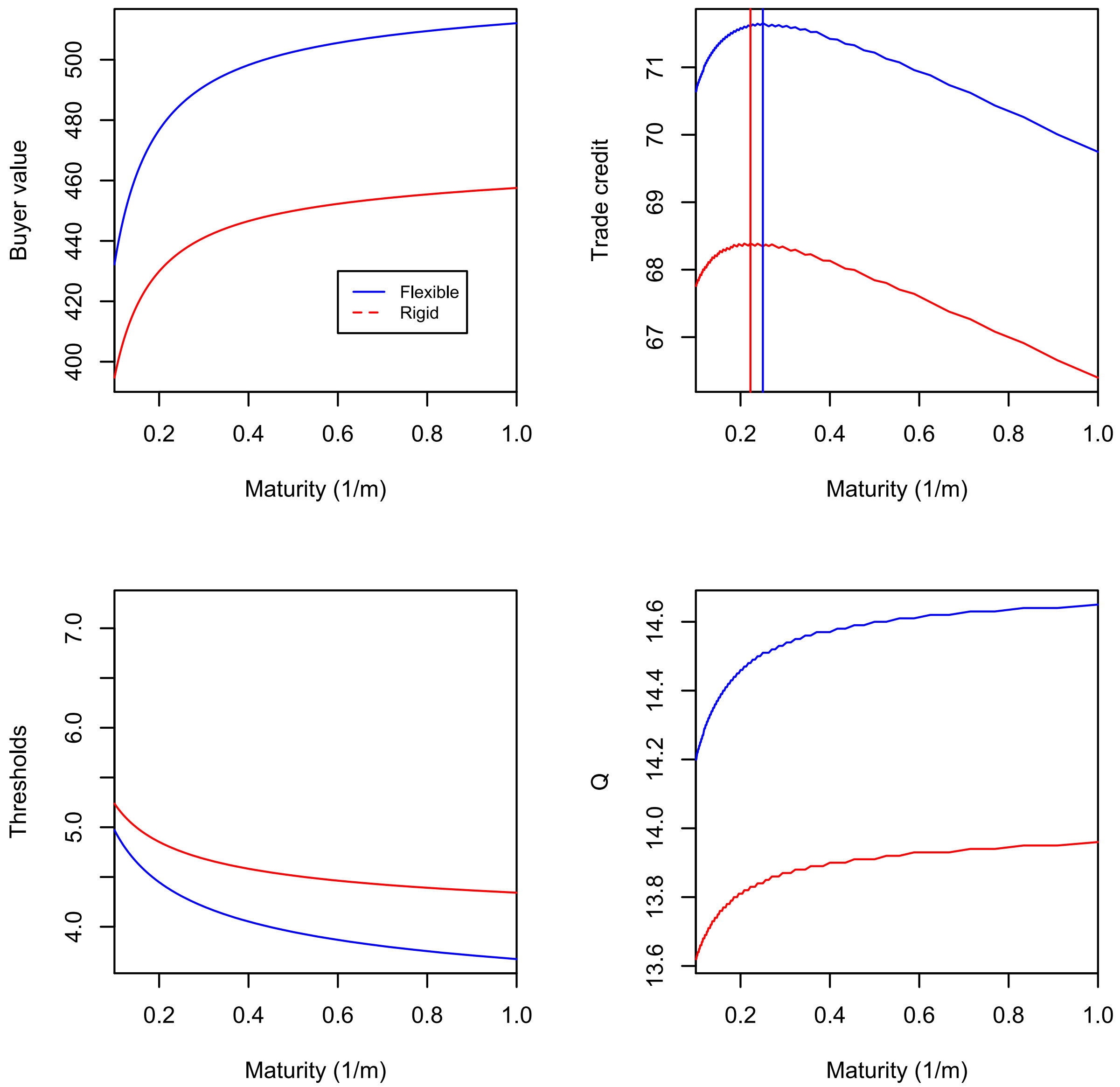
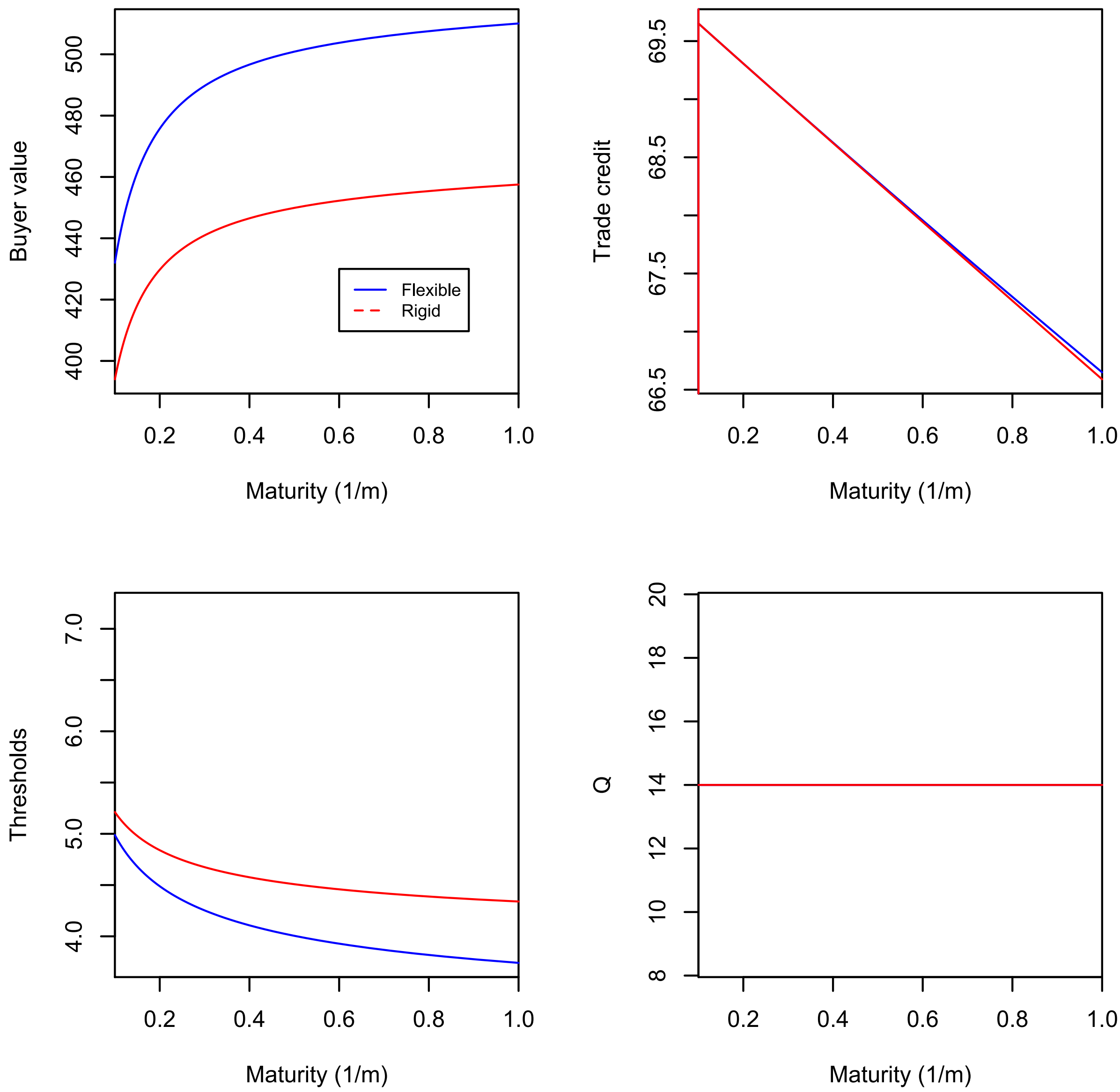
Figures

Fig.1 Baseline results: sensitivity with respect to credit duration for optimal Q



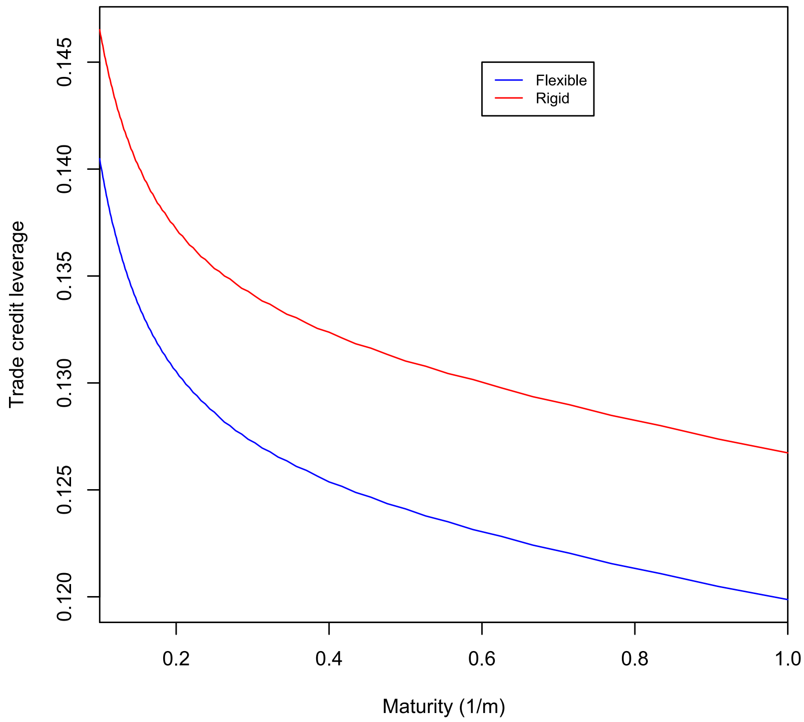
Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0$, $w = 7$, $P_S = 5$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2$, $b = 0$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01. "Thresholds" in the above figures refer to the bankruptcy thresholds $x_{Br}$and x_{Bf} for the rigid and flexible firm respectively.

Fig.2 Sensitivity with respect to credit duration with fixed Q



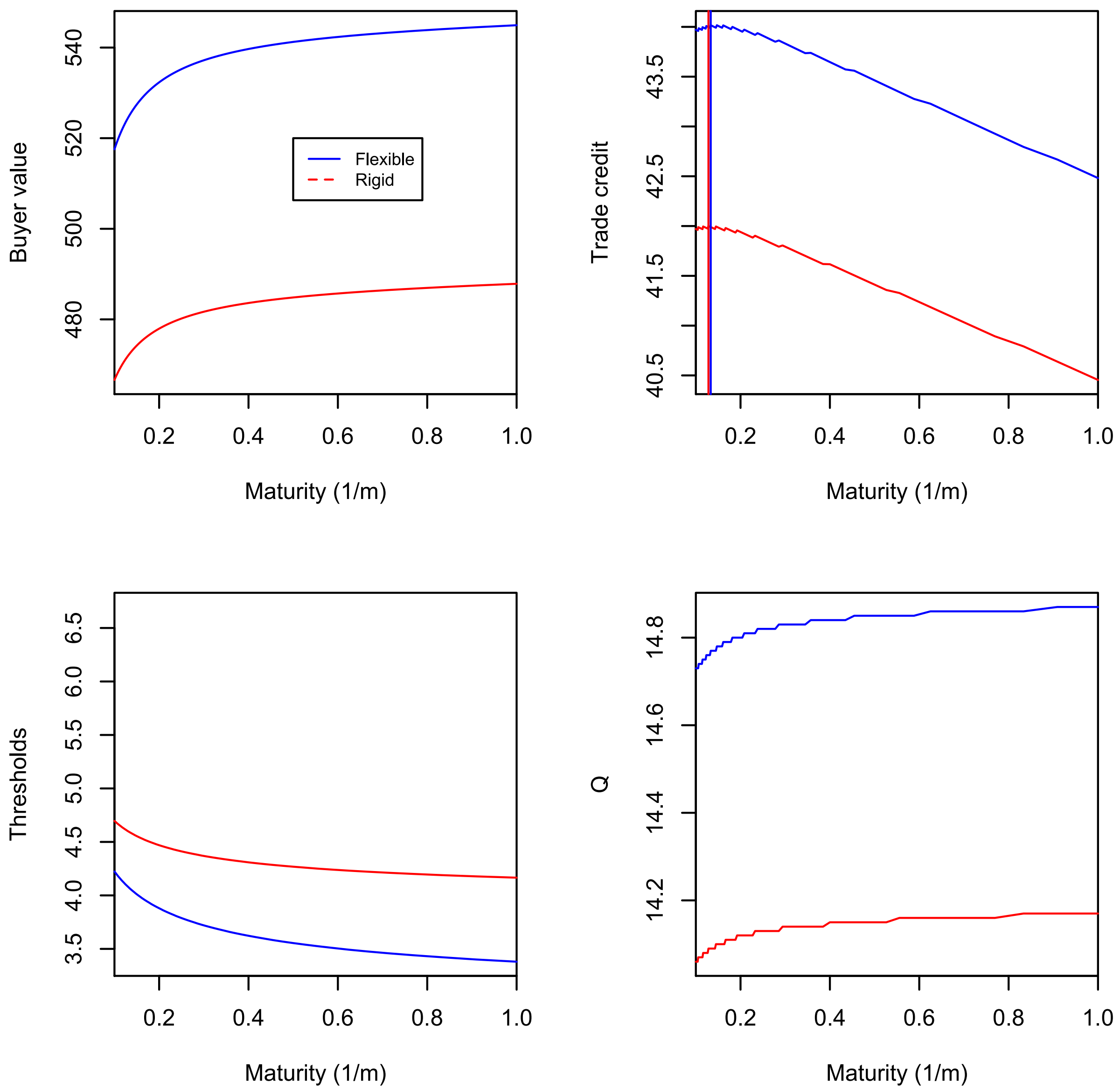
Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0$, $w = 7$, $P_S = 5$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2$, $b = 0$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q assumed fixed at Q =14. “Thresholds” in the above figures refer to the bankruptcy thresholds x_{Br} and x_{Bf} for the rigid and flexible firm respectively.

Fig. 3 Trade credit leverage



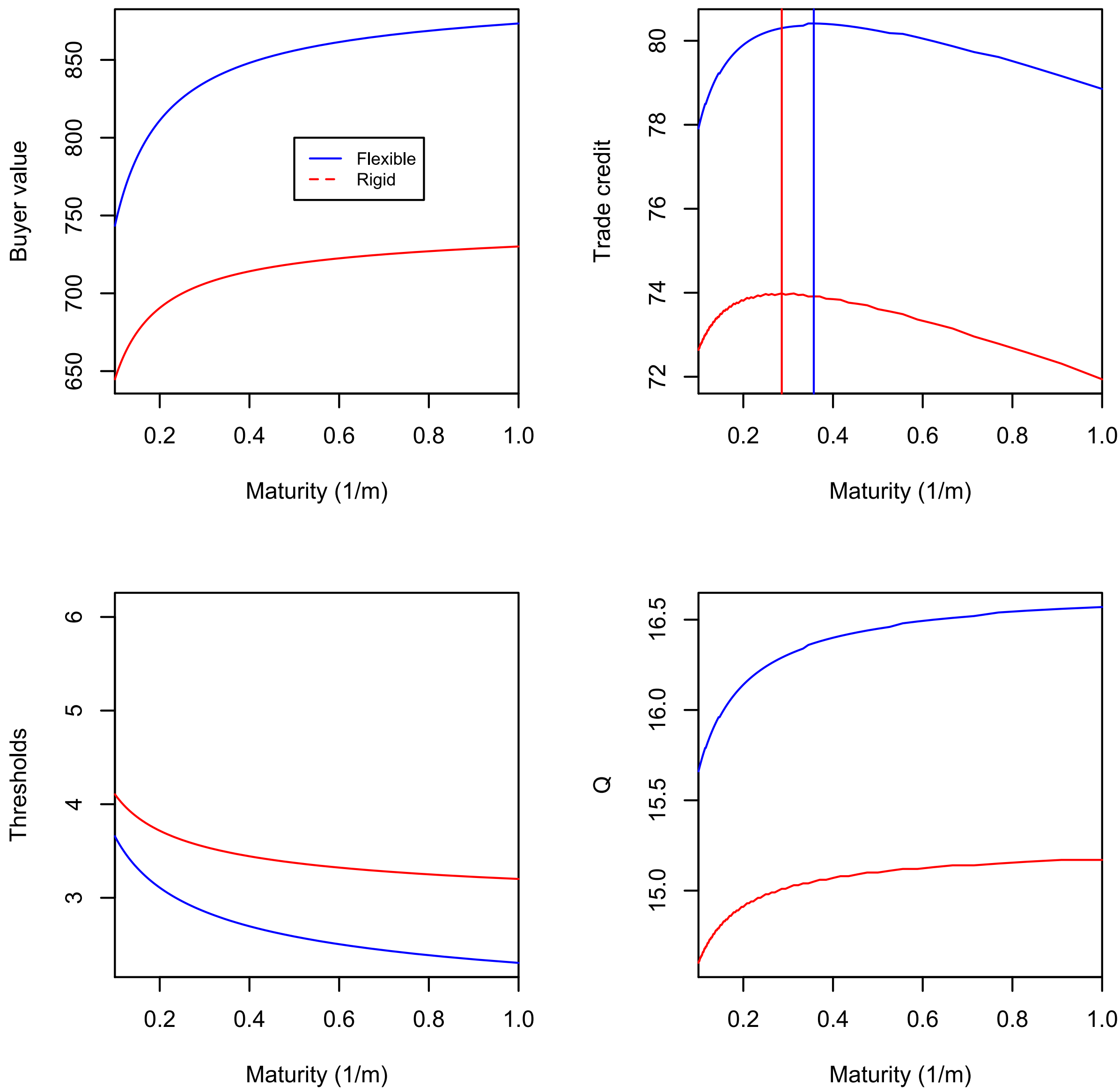
Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0$, $w = 7$, $P_S = 5$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2$, $b = 0$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01. The figure shows trade credit relative to the sum of net of capacity value plus trade credit for the rigid and flexible firms respectively.

Fig. 4 Sensitivity with respect to prices charged by the supplier Ps: Low pricing of the supplier



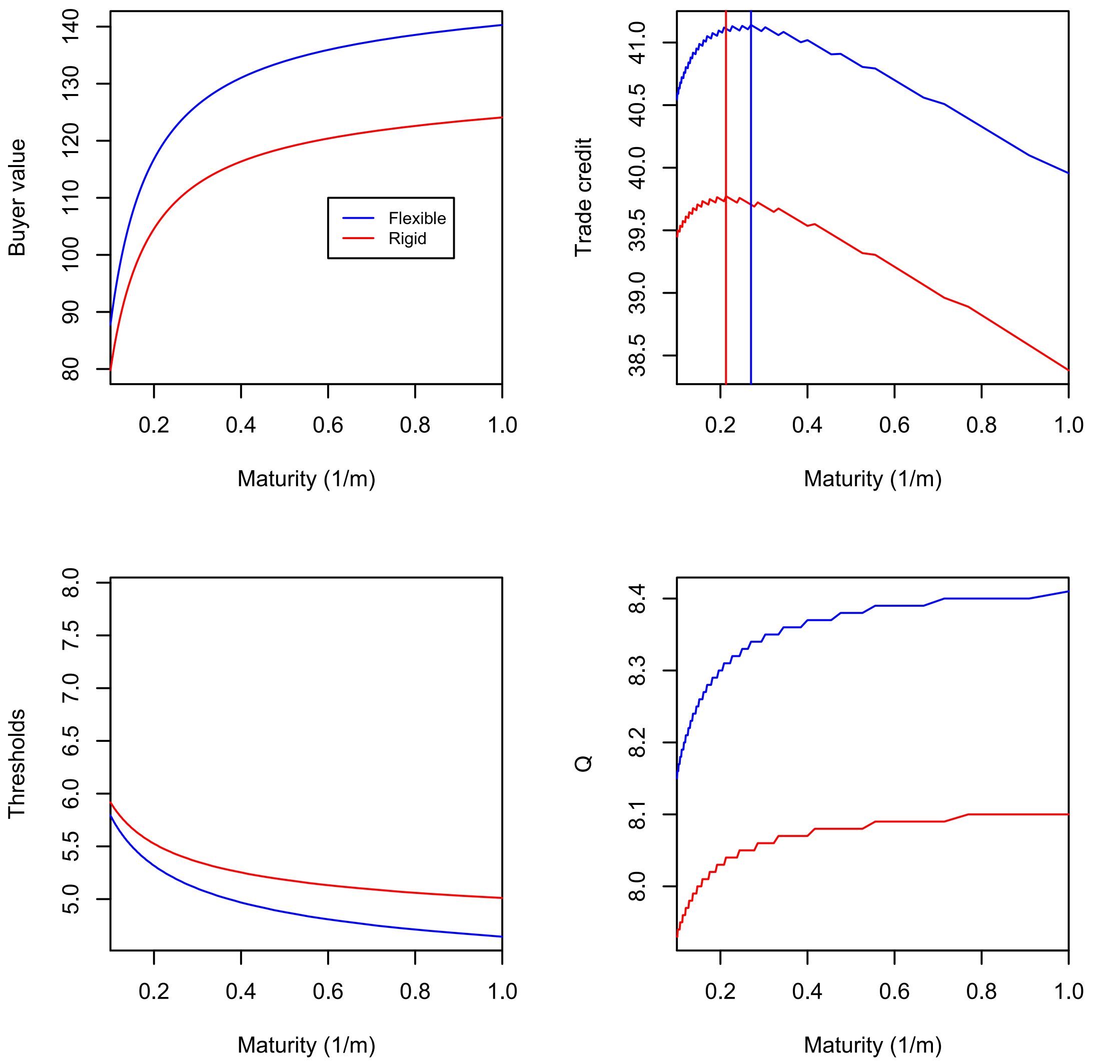
Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0$, $w = 7$, $P_S = 3$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2$, $b = 0$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01. "Thresholds" in the above figures refer to the bankruptcy thresholds x_{Br} and x_{Bf} for the rigid and flexible firm respectively.

Fig.5 Sensitivity with respect to volatility of price in downstream market: higher volatility



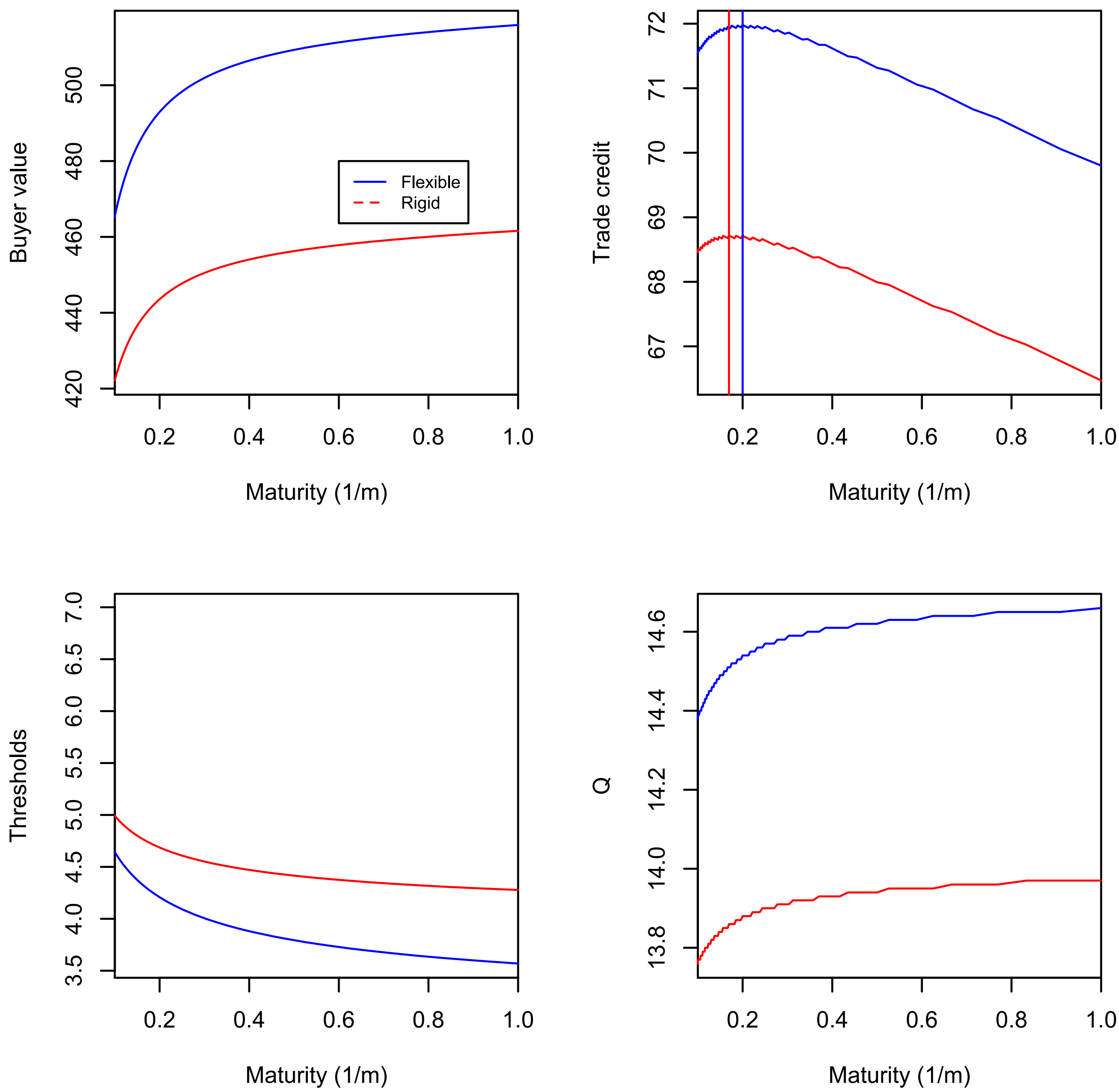
Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0$, $w = 7$, $P_S = 5$, $\sigma = 0.25$, $x = 8.5$, $k = 5$, $\eta = 2$, $b = 0$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01. "Thresholds" in the above figures refer to the bankruptcy thresholds x_{Br} and x_{Bf} for the rigid and flexible firm respectively.

Fig.6 Sensitivity with respect to capacity constraints (η)



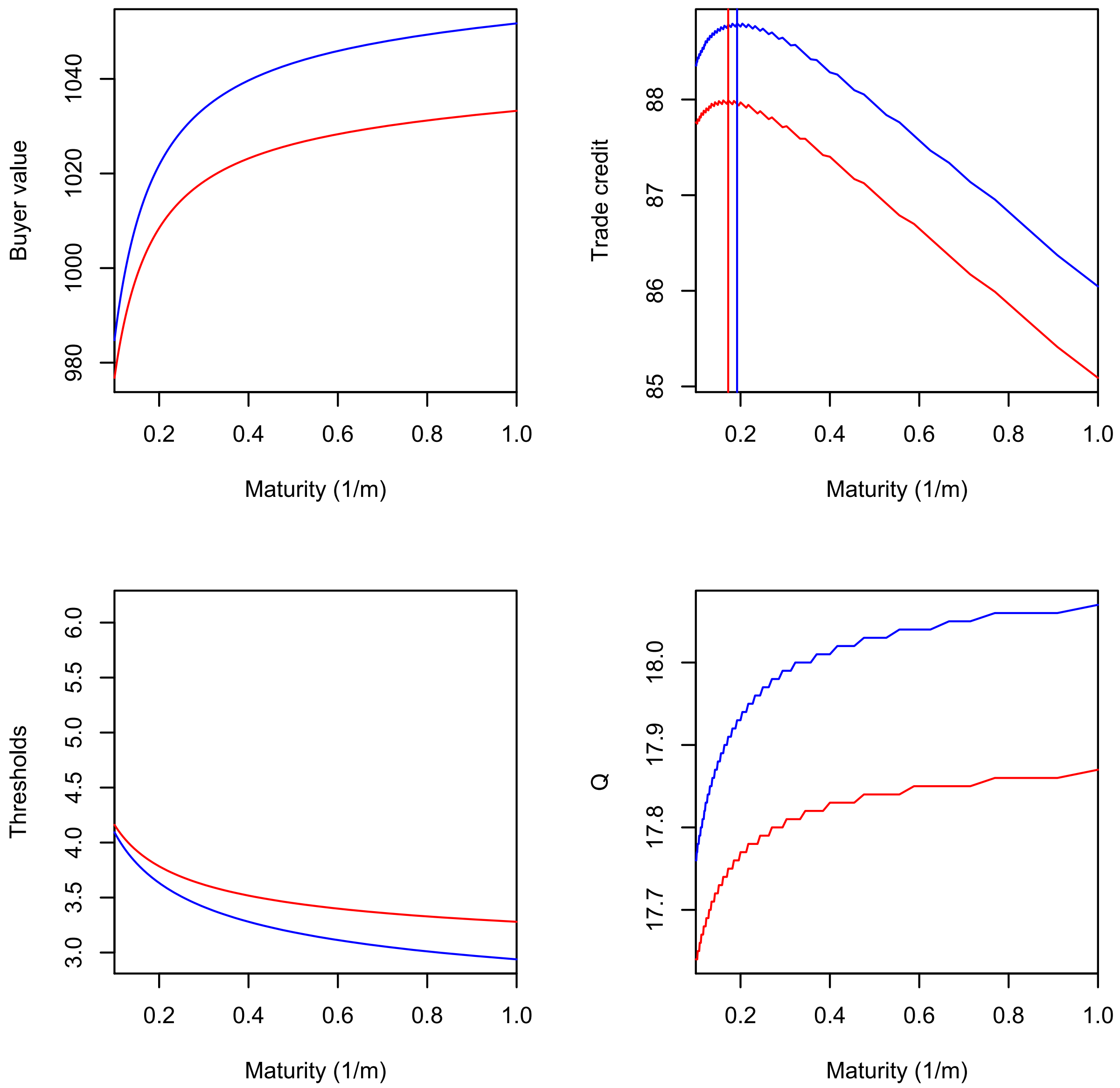
Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0$, $w = 7$, $P_S = 5$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2.2$, $b = 0$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01. "Thresholds" in the above figures refer to the bankruptcy thresholds x_{Br} and x_{Bf} for the rigid and flexible firm respectively.

Fig. 7. Sensitivity with respect to recovery rate in default (b): higher recovery value



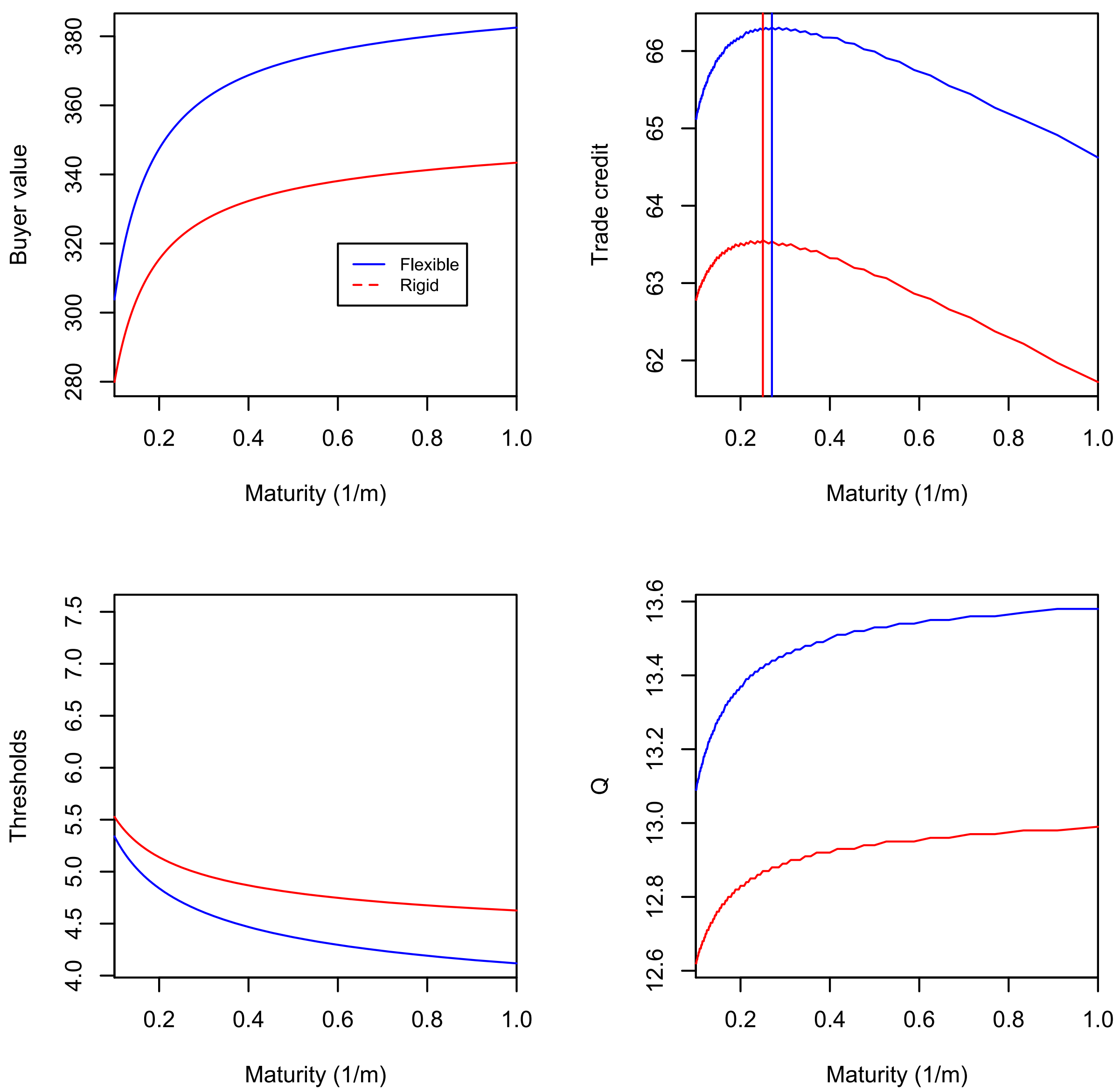
Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0$, $w = 7$, $P_S = 5$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2$, $b = 0.2$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01. “Thresholds” in the above figures refer to the bankruptcy thresholds x_{Br} and x_{Bf} for the rigid and flexible firm respectively.

Fig. 8. Sensitivity with respect to variable production costs *w*: lower costs *w*.



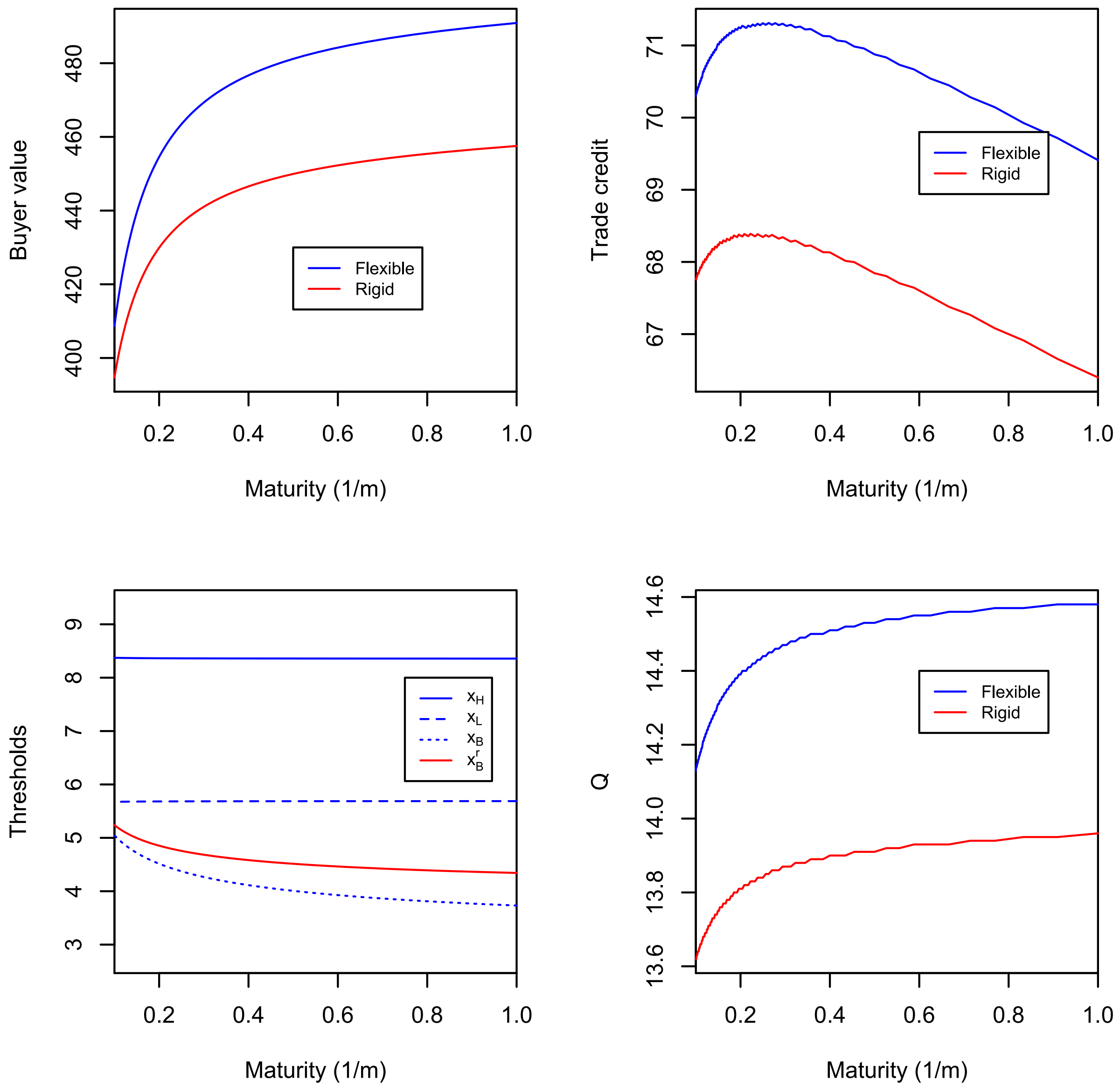
Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0$, $w = 5$, $P_S = 5$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2$, $b = 0$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01. "Thresholds" in the above figures refer to the bankruptcy thresholds x_{Br} and x_{Bf} for the rigid and flexible firm respectively.

Fig. 9. Sensitivity with respect to fixed holding costs *h*: higher holding costs



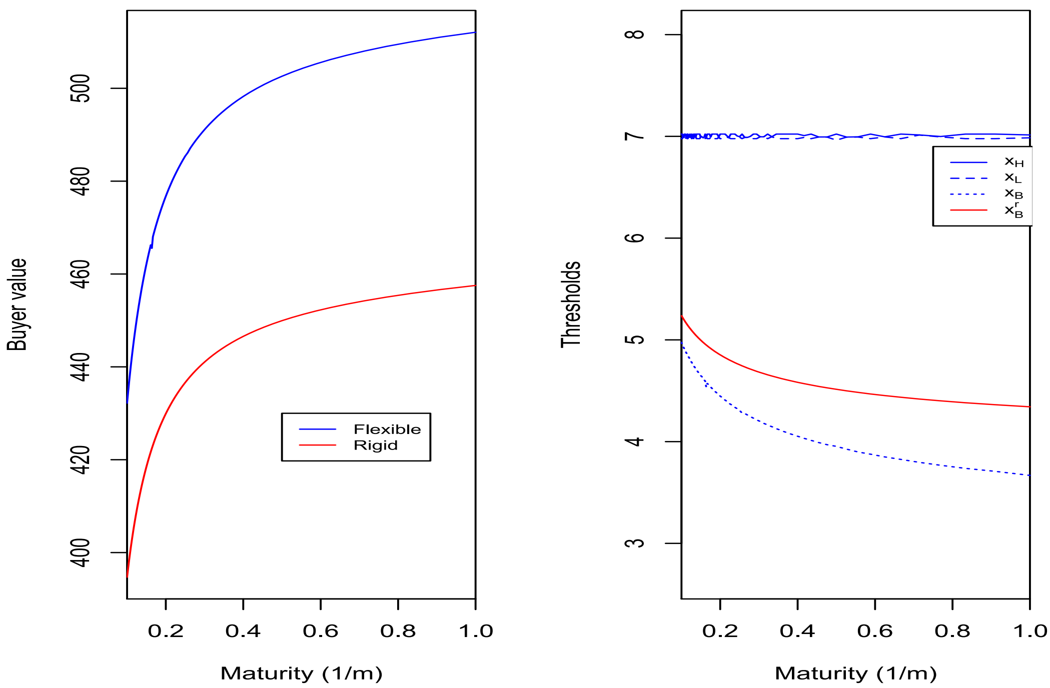
Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0.5$, $w = 7$, $P_S = 5$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2$, $b = 0$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01. "Thresholds" in the above figures refer to the bankruptcy thresholds x_{Br} and x_{Bf} for the rigid and flexible firm respectively.

Fig. 10. Flexible vs rigid firm with positive switching costs



Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0.5$, $w = 7$, $P_S = 5$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2$, $b = 0$. The flexible firm corresponds to the firm with switching costs ϵ_{01} and ϵ_{10}. In this figure we set $\epsilon_{01} = \epsilon_{10} = 20$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01 with solutions satisfying $x_H > x_L$ and $x_L > x_B$.

Fig. 11. Flexible vs rigid firm with zero switching costs



Notes: We consider the following base case parameters: $r = 0.05$, $\delta = 0.03$, $c_f = 30$, $c_h = 0.5$, $w = 7$, $P_S = 5$, $\sigma = 0.15$, $x = 8.5$, $k = 5$, $\eta = 2$, b = 0. The flexible firm corresponds to the firm with switching costs ϵ_{01} and ϵ_{10}. In this figure we set $\epsilon_{01} = \epsilon_{10} = 0$. In this figure we vary m with increments of 0.1 and minimum (1/m = 0.1) and maximum maturity (1/m) of 1 year. Q is optimally chosen by the buyer firm. The increment for Q search is 0.01.