Investing in flexible combined heat and power generation

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Abstract

We find the optimal investment timing and capacity of flexible combined heat and power (CHP) units. We show that flexibility guarantees earlier investment but has an ambiguous effect in terms of optimal capacity with respect to investments in standard CHP units. A numerical exercise using data from the pulp and paper industry concludes the paper.

1 Introduction

Among the policies that are required to achieve a low-carbon economy in Europe, there are the decarbonisation of the electricity sector and the efficiency improvement of the industrial processes (see European Commission, 2011). The pulp and paper industry is one of the most energy-intensive sectors in Europe since the production of 1 tonne of paper requires 5-17 GJ of process heat, depending on the paper type and on the technology used (see Szabó et al., 2009). This is also evident from the cost structure of this sector where the average energy costs represent around 16% of production costs, and in some cases up to 30%. This makes the energy content of the pulp and paper process comparable to that of other energy-intensive sectors, such as cement or steel (see Schlomann et al., 2015).

The two most important steps in paper making are pulping and paper finishing where both heat and power are needed. First, pulp is made by blending wood and water or by using recycled fibres. Then, it is supplied to the paper machine to form a sheet of paper. At this stage, water is removed from the paper sheet through a press section and then evaporated in a dryer section. As a final step, the paper is smoothed by passing it through high pressure rollers. The consumption of energy and raw materials depends on the technologies adopted in these phases. For instance, the two main methods of pulping are mechanical and chemical processing. Chemical pulping uses twice as much wood per tonne compared to that of mechanical pulping, which, in turn, results to be electrical energy-intensive and yields paper with less strength compared to that produced by the chemical pulping process. Mechanical pulping produces heat as a by-product and it is used as

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drying steam in paper processing (see Das and Houtman, 2004; Szabó et al., 2009). Around the 55% of the energy used by the industry comes from biomass and the remaining 38% is obtained by burning natural gas. This derives from the fact that modern pulp mills are self-sufficient in their energy consumption as half of the wood is dissolved and used as fuel in the chemical recovery phase.

To improve their energy efficiency, large and complex paper mills typically take control of their energy needs by building combined heat and power (CHP) plants that also guarantee a cost and an emission reduction. For instance, thanks to the investments in CHP in the last decades, the European paper industry now produces almost half the electrical energy it consumes, also implying an increase in pulp and paper production (see European Commission et al., 2011). Although paper-making technology has developed significantly, by improving its efficiency using CHP plants, it is estimated that only 40% of cogeneration potential capacity has been installed in this industry, meaning that some paper mills have not done this investment or have done it in a small size. Moreover, traditionally CHP systems have been sized to match the own energy request of pulp and paper mills. A critical factor that influences this expansion is represented by the spread between the price of the fuel used to run the CHP and the price of the electricity generated. A prioritized access to the grid giving the possibility to the pulp and paper industry to dispatch the excess electricity produced could lead to a quicker and wider implementation of CHP investments in this sector (see European Commission et al., 2011). As highlighted by the U.S. Department of Energy (2018a,b) and by Jones and Kelly (2017), a flexible CHP technology that connects to the grid and it is able to provide grid services, such as frequency regulation, could be win-win solution for both manufacturers and grid operators. Moreover, since CHP units used to support industrial activities typically operate continuously, they would respond rapidly whenever grid services or added power are needed. This could be applicable only if CHP units become more "flexible". Technological improvements in data analytics and forecasting tools to enable seamless and automated interaction with the grid and manufacturing site operations are needed to make CHP units flexible in this direction.

Considering this framework, this work investigates CHP investments operated by a pulp and paper mill through a real options approach. We concentrate our analysis on the pulp and paper sector because CHP is a technology already adopted by this industry. In the analysis we take the point of view of a pulp and paper plant manager who is contemplating the opportunity of investing in a CHP plant. In particular, s/he needs to decide i) when to exercise the investment option and ii) what is the capacity of the installed unit. The potential investor is also accounting for the fact that the unit will be connected to the power grid and hence the plant manager will benefit from a profit flow associated with the provision of balancing services. We first solve this problem for the case where the CHP technology is standard in the sense that the unit cannot provide ancillary services in a flexible way. This is the status quo. Then, we resolve the problem assuming a flexible CHP technology which allows for such units to be an integrated part of the power grid. Considerable R&D effort is in fact currently devoted to the development of these critically needed technologies.

In the following, we denote these two cases as "operationally rigid CHP" and "operationally flexible CHP", respectively.

Our findings suggest that while flexibility favors earlier (in expected terms) investments, the effect in terms of capacity is ambiguous and depends on the value of flexibility. The potential investor opts for a larger CHP unit when operational flexibility is very valuable but s/he is investing more conservatively otherwise. We complement the theoretical part of the paper with a numerical analysis using data from the paper and pulp industry giving an approximation of the computed investment thresholds, capacity measures and option values.

2 The basic set-up

An industrial plant (e.g. a pulp and paper production unit) produces a fixed amount of output the market price and production cost of which are assumed to be certain and given. Power and heat are among the inputs needed for the production of the output. The plant manager is contemplating the opportunity of investing in a combined heat and power (CHP) production system to cover these needs in-house. The problem of the plant manager involves the choice of both the timing of the investment and the level of capacity. The potential investor is also accounting for the fact that upon installment:

1. the plant can provide balancing services to the grid by selling to the grid's system operator a quantity $\theta\alpha$ of the power produced in-house whenever this is profitable where $\alpha \in [0, 1]$ corresponds to the overcapacity installed and $\theta \in [0, 1)$ is a capacity-to-power coefficient and
2. the periodic operating cost (e.g. the fuel cost) of the CHP unit is $c > 0$.

If the plant manager chooses $\alpha = 0$, then the installed CHP unit covers exactly the needs of the plant and no ancillary services can be provided whereas, if $\alpha > 0$, then the plant manager invests in overcapacity in an attempt to benefit from its access to the grid. The maximum α is normalized without loss of generality to unity and captures the technical and budget constraints that the investor faces.

The cost of investing in α is assumed to be $I(\alpha) = j + i\frac{\alpha^\gamma}{\gamma}$ where i, $j > 0$ and $\gamma > 1$. $I(\alpha)$ has two components. The first component j captures costs independent of the chosen overcapacity, as for instance the administrative and technical costs of connecting the unit to the grid. The second component $i\frac{\alpha^\gamma}{\gamma}$ accounts for costs that depend on the level of α such as the cost of investing in a large rather than a small CHP unit. As it is standard in the literature, we assume that $I(\alpha)$ is a convex function of the chosen overcapacity ($\gamma > 1$).

Since the revenue and cost related to the production of the output are fixed, the plant manager's objective to maximize its periodic profits reduces to the minimization of the following periodic cost function:

$$c_t = \begin{cases} c & \text{, when balancing services are not provided to the grid} \\ -\alpha\theta\,(p_t - c) + c & \text{, when balancing services are provided to the grid} \end{cases}$$

The first piece of c_t refers to periods of time during which ancillary services are not provided to the grid. During these periods the plant is producing power and heat to cover exclusively its own needs. The second piece of c_t refers instead to periods of time during which ancillary services are provided to the grid. During these periods the manager accounts also for the profit from the provision of ancillary services, $\alpha\theta\left(p_t - c\right)$, where p_t is the price of power provided to the grid.

p_t is assumed to fluctuate over time according to the following geometric Brownian motion

$$\frac{dp_t}{p_t} = \mu dt + \sigma dW_t \text{ with } p_0 = p$$

where μ is the drift, σ is the instantaneous volatility and W_t is a standard Wiener process.[1]

Notably, the provision of balancing services is the cost-minimizing alternative when $p_t > c$ which means that the cost function can be written as:

$$c_t = \begin{cases} c & \text{, when } p_t \leq c \\ -\alpha\theta\left(p_t - c\right) + c & \text{, when } p_t > c \end{cases}$$

Hence, the net periodic benefit for an industrial plant that has invested in a flexible CHP unit is equal to b_t:[2]

$$b_t = \begin{cases} 0 & \text{when } p_t \leq c \\ \alpha\theta\left(p_t - c\right) & \text{when } p_t > c \end{cases}$$

Summing up, the investment problem that the plant manager needs to solve involves the choice of both the timing of the investment and the level of α given that the cost associated with this investment is $I(\alpha)$. For the sake of simplicity, we assume that the plant manager contemplates the investment over an infinite time horizon starting at the current time point $t = 0$. Further, we assume that, once the investment takes place at a time point $\tau \geq 0$: i) the project runs forever,[3]

[1] We deviate from models that incorporate seasonality, mean-reversion and price spikes, for three reasons. The first one is that industrial plants that employ CHP systems have long lifecycles covering spans of several years. Thus, the presence of daily or weekly spikes and seasonalities can be neglected in the long run. This can be justified if we interpret the price p_t as, e.g., a weekly average price (see Bosco et al., 2010 and Gianfreda et al., 2016). The second reason is mathematical tractability. A mean-reverting process would lead to similar results but at the cost of a higher mathematical sophistication and of the lack of explicit solutions. Finally, the mean-reversion effect in electricity prices is not unanimously recognized. For example, de Vany and Walls (1999) find unit roots (i.e., random-walk-like dynamics) in U.S. markets, and Bosco et al. (2010) reach the same result for several European markets. In fact, Bosco et al. (2010) even claim that previous findings suggesting mean-reversion could be due to the use of non-robust statistical techniques.

[2] Upon investment the power plant manager saves also the periodic cost that it had to pay for the provision of heat and power prior to the investment in the CHP plant. However, since this is a benefit for the plant no matter if $p_t \leq c$ or $p_t > c$, it is factored out of the net periodic benefit function b_t. Note also that instead of $b_t = \max\left\{\alpha\theta\left(p_t - c\right), 0\right\}$ we can have $b_t = \max\left\{\alpha\theta\left(p_t - c - \omega\right), 0\right\}$ where $\omega > 0$ corresponds to any environmental cost/penalty that the plant is facing for heat that cannot be consumed in-house or sold in the market and consequently is emitted to the environment. The addition of such a term would leave our findings qualitatively unaffected.

[3] The relevance of this assumption becomes clear in the following sections where we treat terms capturing the present values of flows. If for instance the plant has a long but finite life cycle ending at a time point $T > 0$, the present value of the flow of c at any time point $t \in (0, T)$ is $\int_t^T ce^{-r(s-t)}ds = \frac{c}{r}\left(1 - e^{-r(T-t)}\right)$. If T is sufficiently large this can be approximated by $\frac{c}{r}$. This approximation is obviously allowing for a more compact presentation of the mathematical formulas and is also reasonable considering that costs paid at a very far future date have little

ii) the potential investor is risk neutral and discounts future payoffs using the interest rate $r > \mu$,[4] and iii) the CHP system installed does not "rust", i.e., no maintenance is required.[5]

3 Investing in an operationally rigid CHP

Before we discuss the problem of a potential investor who contemplates investing in an operationally flexible CHP unit, we present here the opposite case. This is in fact the status-quo. The analysis of this section serves as our standard of comparison and allows us to isolate the effect of operational flexibility in the following sections. The presentation evolves as follows. First, we derive the operating value of a CHP plant for an arbitrary level of p_t and α, $v(p_t;\alpha)$. Then we compute the optimal level of α, that is, the level of overcapacity that maximizes the net present value $v(p_t;\alpha) - I(\alpha)$ for any p_t and last, we derive the price level that corresponds to the optimal investment timing.

Operational flexibility facilitates the provision of ancillary services to the power producers. A supplier observes the realizations of p_t in real time providing ancillary services when this is profitable ($p_t > c$) and abstaining from this practice otherwise ($p_t \leq c$) with (almost) immediate response. Nevertheless, real time pricing and response are not prerequisites for the provision of balancing services. Typical CHP allows for the provision of ancillary services as well. However, in this case the power supplier is operationally rigid since it might miss out on the opportunity to profitably provide balancing services when $p_t > c$ or might be producing power when the price is too low ($p_t \leq c$) because the relevant piece of information does not reach it on time. In the following, we model the operational rigidity that is associated with the provision of ancillary services from a typical CHP assuming that the net periodic benefit has the form: $\tilde{b}_t = \alpha\theta\left(p_t - c\right)$. A power supplier benefits from a positive $\tilde{b}_t$ whenever $p_t > c$ but makes losses whenever $p_t \leq c$. The potential negativity of $\tilde{b}_t$ is exactly what captures the operational rigidity for the power supplier. Notably, $\tilde{b}_t$ differs from b_t since the latter is always non-negative.

Let $v(p_t;\alpha)$ represent the operating value of a typical CHP unit upon investment. In the Appendix we show that:

$$v(p_t;\alpha) = \alpha\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right)$$

Given $v(p_t;\alpha)$, the plant manager chooses the optimal level of α taking into account the potential future evolution of cash flows from energy production and the operational rigidity of the plant. Since the capacity of the plant to provide ancillary services does not come for free, the corresponding benefits must be traded off with the investment cost. In the following, we determine the optimal α for the scenario where the investment in the CHP plant occurs at a time point t where the operating value of the plant is positive, i.e., $\frac{p_t}{r-\mu} > \frac{c}{r}$. The reason is that the investment will never take place

weight in current terms.

[4]This condition ensures that the problem that we study is economically meaningful (see p. 138 in Dixit and Pindyck, 1994).

[5]A complete analysis of the case where capital maintenance costs are considered can be found in Dixit and Pindyck (1994, Ch. 7).

for $\frac{p_t}{r-\mu} \leq \frac{c}{r}$ since there is no reason for the investor to spend $I(\alpha)$ in order to gain access to a plant with negative operating value.

The optimal level of overcapacity, $\underline{\alpha}$, must be such that the expected net present value associated with the current and future operations is maximized, that is:

$$\underline{\alpha} = \arg\max_{\alpha} NPV(p_t;\alpha), \text{ s.t. } 0 \leq \alpha \leq 1$$

where,

$$\begin{aligned} NPV(p_t;\alpha) &= v(p_t;\alpha) - I(\alpha) \\ &= \alpha\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right) - \left(j + i\frac{\alpha^\gamma}{\gamma}\right) \end{aligned}$$

$\underline{\alpha}$ is set on the basis of the expected net present value taken at a time point t which is obtained by subtracting the investment cost $(I(\alpha))$ from the expected present value of the flow of profits accruing from that time t onward $(v(p_t;\alpha))$. The evolution of this flow over time depends on the fluctuations of p_t and the ability of the plant to provide ancillary services. Solving $\max_{\alpha} NPV(p_t;\alpha)$ we obtain:

Proposition 1 *The optimal level of overcapacity for a typical CHP unit is*

$$\underline{\alpha}(p_t) = \begin{cases} \left(\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right)/i\right)^{\frac{1}{\gamma-1}} & \text{for} \quad c\left(1-\frac{\mu}{r}\right) < p_t < \underline{p} \\ 1 & \text{for} \quad \underline{p} \leq p_t \end{cases}$$

where $\underline{p} = \left(\frac{i}{\theta} + \frac{c}{r}\right)(r-\mu)$.

Proof. The first-order condition yields,[6]

$$\underline{\alpha}(p_t) = \left(\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right)/i\right)^{\frac{1}{\gamma-1}}$$

Since $c\left(1-\frac{\mu}{r}\right) < p_t$ we have $\underline{\alpha}(p_t) > 0$. Also for $p_t \geq \underline{p} = \left(\frac{i}{\theta} + \frac{c}{r}\right)(r-\mu)$ we obtain $\underline{\alpha}(p_t) = 1$. ■

The optimal level of overcapacity depends on the level of p_t. For $c\left(1-\frac{\mu}{r}\right) \leq p_t$ the price of power is large enough to allow for $\underline{\alpha}(p_t) > 0$. In fact, if $p_t > \underline{p}$, it is reasonable for the plant manager to choose $\underline{\alpha}(p_t) = 1$ where $\underline{p}$ is the minimum price level that allows for $\underline{\alpha}(p_t) = 1$. Unsurprisingly, $\underline{\alpha}(p_t)$ is locally increasing in p_t. This is thanks to the term $\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right)$, that is, the marginal (with respect to overcapacity) operating value of the plant. Consistently, the larger the marginal operating value, the higher the optimal level of overcapacity.

[6] The second-order condition is always satisfied thanks to the convexity of $I(\alpha)$.

Given $\underline{\alpha}(p_t)$ we have:

$$NPV(p_t;\underline{\alpha}(p_t)) = \begin{cases} \left(\theta\left(\frac{p_t}{r-\mu}-\frac{c}{r}\right)/i\right)^{\frac{\gamma}{\gamma-1}}\left(1-\frac{1}{\gamma}\right)i-j & \text{for} \quad c\left(1-\frac{\mu}{r}\right)<p_t<\underline{p} \\ \theta\left(\frac{p_t}{r-\mu}-\frac{c}{r}\right)-\left(j+\frac{i}{\gamma}\right) & \text{for} \quad \underline{p}\leq p_t \end{cases}$$

This is the expected net present value associated with the operation of the plant when the optimal level of overcapacity $\underline{\alpha}(p_t)$ has been adopted.

Given $\underline{\alpha}(p_t)$ and $NPV(p_t;\underline{\alpha}(p_t))$, we can determine the value of the option to invest and the optimal investment timing. Since in our set-up all of the information about the future evolution of the price of power is embodied in p_t, there exists an optimal investment rule of the form: "Invest immediately if p_t is at, or above, a critical threshold and wait otherwise".[7] Denoting by $\widetilde{p}$ the price level triggering investment and assuming that the current market price p is below this threshold, i.e. $p<\widetilde{p}$, the value of the option to invest is equal to

$$F(p;\widetilde{p}) = \max_{\tau>0}[E_0\left(e^{-r\tau}\right)NPV(\widetilde{p};\underline{\alpha}(\widetilde{p}))],$$

where $E_0\left(e^{-r\tau}\right)$ is the expected value of the stochastic discount factor $e^{-r\tau}$ associated with the investment time $\tau=\inf\{t>0 \mid p_t=\widetilde{p}\}$. In the Appendix we show that $E_0\left(e^{-r\tau}\right)=(p/\widetilde{p})^{\beta_1}$ where $\beta_2<0$ and $\beta_1>1$ are the roots of the quadratic equation $\Lambda(\beta)=\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-r$. This implies

$$F(p;\widetilde{p}) = \max_{\widetilde{p}>p}[(p/\widetilde{p})^{\beta_1}NPV(\widetilde{p};\underline{\alpha}(\widetilde{p}))].$$

Optimality requires that the following first-order condition holds:

$$\widetilde{p} = \beta_1\frac{NPV(\widetilde{p};\underline{\alpha}(\widetilde{p}))}{\frac{\partial NPV(\widetilde{p};\underline{\alpha}(\widetilde{p}))}{\partial\widetilde{p}}}$$

Following Dixit et al. (1999), for this problem to be well-posed, the following condition must also hold:

$$\left.\frac{\partial^2 F(p_t;\widetilde{p})}{\partial p_t^2}\right|_{p_t=\widetilde{p}} > \left.\frac{\partial^2 NPV(p_t;\underline{\alpha}(p_t))}{\partial p_t^2}\right|_{p_t=\widetilde{p}}$$

This condition guarantees that for $p_t<\widetilde{p}$ the potential investor keeps the investment option alive ($F(p_t;\widetilde{p})>NPV(p_t;\underline{\alpha}(p_t))$).

From the first-order condition and given that $NPV(p_t;\underline{\alpha}(p_t))=\underline{\alpha}(p_t)\theta\left(\frac{p_t}{r-\mu}-\frac{c}{r}\right)-\left(j+i\frac{\underline{\alpha}(p_t)^{\gamma}}{\gamma}\right)$ we have:

Proposition 2 *The optimal investment threshold for a typical CHP unit is*

$$\widetilde{p} = \frac{\beta_1}{\beta_1-1}(r-\mu)\left(\frac{c}{r}+\frac{j+i\frac{(\underline{\alpha}(\widetilde{p}))^{\gamma}}{\gamma}}{\theta\underline{\alpha}(\widetilde{p})}\right)$$

[7] See Dixit et al. (1999) for more details.

When $p_t \geq \underline{p}$ *this is equal to* $\widetilde{p}^* = \frac{\beta_1}{\beta_1 - 1}(r-\mu)\left(\frac{c}{r} + \frac{j+\frac{i}{\gamma}}{\theta}\right)$ *whereas when* $p_t \in \left(c\left(1-\frac{\mu}{r}\right), \underline{p}\right)$ *we have* $\widetilde{p}^{**} = \frac{\beta_1}{\beta_1 - 1}(r-\mu)\left(\frac{c}{r} + \frac{j+i\frac{\underline{\alpha}(\widetilde{p}^{**})^\gamma}{\gamma}}{\theta\underline{\alpha}(\widetilde{p}^{**})}\right)$.

Last, plugging $\widetilde{p}$ and $\underline{\alpha}(\widetilde{p})$ in $F(p;\widetilde{p})$ we obtain $F(p;\widetilde{p}) = \left(\underline{\alpha}(\widetilde{p})\theta\left(\frac{\widetilde{p}}{r-\mu} - \frac{c}{r}\right) - \left(j + i\frac{\underline{\alpha}(\widetilde{p})^\gamma}{\gamma}\right)\right)\left(\frac{p}{\widetilde{p}}\right)^{\beta_1}$ where $\widetilde{p} = \widetilde{p}^*$ and $\underline{\alpha}(\widetilde{p}) = 1$ when $p_t > \underline{p}$ whereas $\widetilde{p} = \widetilde{p}^{**}$ and $\underline{\alpha}(\widetilde{p}) < 1$ when $p_t \in \left(c\left(1-\frac{\mu}{r}\right), \underline{p}\right)$.

Summing up, since the net present value of providing balancing services is non-negative $\left(\frac{p_t}{r-\mu} - \frac{c}{r} \geq 0\right)$, a potential investor will opt for investing in overcapacity ($\underline{\alpha} > 0$). This will take place as soon as the stochastic price p_t reaches the investment threshold $\widetilde{p}$ and the value of the option to make such an investment is $F(p;\widetilde{p})$. Again, it is important to stress that the provision of ancillary services is not necessarily profitable all the time. However, as long as the operating value of the project is non-negative, the potential investor has an incentive to realize the investment along the lines described above.

4 Investing in an operationally flexible CHP

In the following we discuss the problem of a potential investor who contemplates investing in cogeneration overcapacity provided that it has access to an operationally flexible CHP unit. As in the previous section, we first derive the operating value of the plant and then we compute the optimal level of overcapacity, the optimal investment threshold and the value of the option to invest.

4.1 The operating value of the plant

Let $V(p_t;\alpha)$ represent the operating value of the plant upon investment. In the Appendix we show that:

$$V(p_t;\alpha) = \begin{cases} \widetilde{B}p_t^{\beta_1} & \text{when } p_t \leq c \\ \widetilde{A}p_t^{\beta_2} + \alpha\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right) & \text{when } p_t > c \end{cases}$$

where:

$$\begin{aligned} \widetilde{A} &= \alpha\theta A = \alpha\theta\frac{r-\mu\beta_1}{(\beta_1-\beta_2)\,r\,(r-\mu)}c^{1-\beta_2} > 0 \\ \widetilde{B} &= \alpha\theta B = \alpha\theta\frac{r-\mu\beta_2}{(\beta_1-\beta_2)\,r\,(r-\mu)}c^{1-\beta_1} > 0 \end{aligned}$$

The term $\widetilde{A}p_t^{\beta_2}$ represents the value of the option to suspend providing balancing services when this becomes unprofitable. Instead, $\widetilde{B}p_t^{\beta_1}$ is the option to restart providing these services when this becomes profitable again. The option constants $\widetilde{A}$ and $\widetilde{B}$ are both positive and linearly increasing in the level of installed overcapacity α. $\widetilde{A}p_t^{\beta_2}$ decreases in the price level p_t and increases in the production cost c. This makes sense considering that the option to suspend becomes more valuable when profits from the provision of ancillary services fall. In contrast, $\widetilde{B}p_t^{\beta_1}$ increases in the price level p_t and decreases in the production cost c. This is because the option to restart becomes more valuable when profits associated with power supply rise.

The term $\alpha\theta\left(\frac{p_t}{r-\mu}-\frac{c}{r}\right)$ is known from the previous section and corresponds to the operating value of an operationally rigid plant ($v(p_t;\alpha)$). The operating value of an operationally flexible CHP is clearly larger than the operating value of a typical plant ($V(p_t;\alpha) > v(p_t;\alpha)$). This is attributed to the fact that when $p_t > c$, the plant manager benefits from selling power to the grid but has the option to suspend operations in case p_t drops below c. Similarly when $p_t < c$, the plant manager does not produce electricity avoiding making losses but keeps the option to restart producing as soon as p_t becomes larger than c.

4.2 The optimal level of overcapacity

The plant manager chooses the optimal α taking into account the potential future evolution of profits from power production and the operational flexibility of the plant. Consistently, in the following we determine the optimal α for the scenario where the investment occurs at a time point t where power production is profitable, i.e., for $p_t > c$. The reason is that the investment will never take place for $p_t \leq c$ since there is no reason for the investor to spend $I(\alpha)$ only to keep the CHP plant idle for some time (p. 190, Dixit and Pindyck, 1994). It is worth noting that this condition is stricter than the condition $\frac{p_t}{r-\mu} > \frac{c}{r} \rightarrow p_t > c\left(1-\frac{\mu}{r}\right)$ of the previous section.

The optimal level of overcapacity, $\overline{\alpha}$, is the argument maximizing the net present value of the plant:

$$\overline{\alpha} = \arg\max_{\alpha} NPV(p_t;\alpha), \text{ s.t. } 0 \leq \alpha \leq 1$$

where,

$$\begin{aligned}
NPV(p_t;\alpha) &= V(p_t;\alpha) - I(\alpha) \\
&= \widetilde{A}p_t^{\beta_2} + \alpha\theta\left(\frac{p_t}{r-\mu}-\frac{c}{r}\right) - \left(j + i\frac{\alpha^\gamma}{\gamma}\right) \\
&= \alpha\theta\Omega\left(p_t\right) - \left(j + i\frac{\alpha^\gamma}{\gamma}\right)
\end{aligned}$$

and $\Omega\left(p_t\right) = Ap_t^{\beta_2} + \frac{p_t}{r-\mu} - \frac{c}{r}$.

$\overline{\alpha}$ is set on the basis of the expected net present value taken at a time point t which is obtained by subtracting the investment cost ($I(\alpha)$) from the expected present value of the flow of profits accruing from that time t onward ($V(p_t;\alpha)$). The evolution of this flow over time depends on the fluctuations of p_t and the ability of the plant to provide ancillary services when $p_t > c$ and abstain from this practice otherwise.

Solving $\max\limits_{\alpha} NPV(p_t;\alpha)$, we find that:

Proposition 3 *Provided that $\Psi = \theta\Omega(c) - i < 0$, the optimal level of overcapacity is*

$$\overline{\alpha}(p_t) = \begin{cases} (\theta\Omega(p_t)/i)^{\frac{1}{\gamma-1}} & \textit{for} \quad c < p_t < \overline{p} \\ 1 & \textit{for} \quad \overline{p} \leq p_t \end{cases}$$

where $\overline{p}$ is such that $\theta\Omega(\overline{p}) = i \rightarrow \overline{p} = \underline{p} - A\overline{p}^{\beta_2}(r-\mu)$. If instead $\Psi \geq 0$, the optimal level of overcapacity is $\overline{\alpha}(p_t) = 1$.

Proof. Available in the Appendix. ■

$\overline{\alpha}(p_t)$ is locally increasing in p_t. This results from two opposing forces within $\Omega(p_t) = Ap_t^{\beta_2} + \frac{p_t}{r-\mu} - \frac{c}{r}$. On one hand, the term $\frac{p_t}{r-\mu} - \frac{c}{r}$ is increasing in p_t. This term represents the marginal (with respect to power) expected net present value of a plant that is always providing balancing services to the grid. Consistently, the higher the $\frac{p_t}{r-\mu} - \frac{c}{r}$, the higher the optimal level of overcapacity. In contrast, the term $Ap_t^{\beta_2}$, that is, the value of the option to stop providing power to the grid when this is not profitable, decreases in p_t. This is because the higher the p_t, the less likely is the plant's interruption of provision of this service. As shown in the proof of the Proposition in the Appendix, the first force prevails ($\Omega(p_t)$ is increasing in p_t and $\overline{\alpha}(p_t)$ is increasing in $\Omega(p_t)$ for any $p_t \in (c, \overline{p})$).

As for the condition $\Psi < 0$, the term $\Psi = \theta\Omega(c) - i$ represents the difference between the marginal operating value of the plant evaluated at the minimum p_t in the interval of interest ($\theta\Omega(c)$) and the marginal cost evaluated at the maximum p_t in this interval (i). If this difference is negative, there is a region of p_t in which investments in partial overcapacity are optimal. If instead, this difference is non-negative then this region is vacuous and $\overline{\alpha}(p_t) = 1$ for any $p_t > c$.

Comparing $\underline{\alpha}(p_t)$ and $\overline{\alpha}(p_t)$ we first observe that in both cases the potential investor opts for positive overcapacity. The main difference between the two is that with a flexible CHP unit it is possible to have $\overline{\alpha}(p_t) = 1$ for any $p_t > c$. This is the case when $\Psi \geq 0$. On the contrary, the choice of $\underline{\alpha}(p_t) = 1$ is always conditional on the price level at the time of the investment. The intuition behind this result is as follows. As long as operational flexibility guarantees non-negative profits, the potential investor will opt for $\overline{\alpha}(p_t) = 1$ if the variable investment cost is low enough ($\Psi \geq 0 \rightarrow \theta\Omega(c) \geq i$) no matter the level of p_t. If instead a potential investor does not benefit from operational flexibility it will always condition the choice of $\underline{\alpha}(p_t) = 1$ on the level of p_t accounting for the fact that it is costlier to have a large ($\underline{\alpha}(p_t) = 1$) rather than a small ($\underline{\alpha}(p_t) < 1$) CHP unit operating when $p_t \leq c$.

A second observation has to do with the price interval delimiting investment in partial overcapacity. We require $p_t \in \left(c\left(1 - \frac{\mu}{r}\right), \underline{p}\right)$ for $\underline{\alpha}(p_t) < 1$ and instead $p_t \in (c, \overline{p})$ for $\overline{\alpha}(p_t) < 1$ where $\underline{p} > \overline{p}$. The interval $(c, \overline{p})$ is contained in the interval $\left(c\left(1 - \frac{\mu}{r}\right), \underline{p}\right)$ which means that, on one hand, operational flexibility discourages investments when the price is too low setting a lower bar at c, and, on the other, it lowers the price level that makes investment in full overcapacity optimal $\left(\underline{p} > \overline{p}\right)$.

Last, thanks to $Ap_t^{\beta_2} > 0$ we have $(\theta\Omega(p_t)/i)^{\frac{1}{\gamma-1}} > \left(\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right)/i\right)^{\frac{1}{\gamma-1}}$. This means that when investing in partial overcapacity is optimal, operational flexibility favors investment in a larger plant. We summarize all this in the following proposition.

Proposition 4 *Operational flexibility:*

1. *allows for investments in full overcapacity for any* $p_t > c$ *when* $\Psi \geq 0$,
2. *contracts the price interval associated with investments in partial overcapacity* $\left((c, \overline{p}) \subset \left(c\left(1-\frac{\mu}{r}\right), \underline{p}\right)\right)$,
3. *favors, ceteris paribus, larger partial overcapacity investments* $\left(\left(\theta\Omega\left(p_t\right)/i\right)^{\frac{1}{\gamma-1}} > \left(\theta\left(\frac{p_t}{r-\mu}-\frac{c}{r}\right)/i\right)^{\frac{1}{\gamma-1}}\right)$.

Last, provided $\Psi < 0$ and given $\overline{\alpha}\left(p_t\right)$ we have:

$$NPV(p_t; \overline{\alpha}(p_t)) = \begin{cases} \left(\theta\Omega\left(p_t\right)/i\right)^{\frac{\gamma}{\gamma-1}}\left(1-\frac{1}{\gamma}\right)i - j & \text{for} \quad c < p_t < \overline{p}, \\ \theta\Omega\left(p_t\right) - \left(j + \frac{i}{\gamma}\right) & \text{for} \quad \overline{p} \leq p_t. \end{cases}$$

When instead $\Psi \geq 0$, we have $NPV(p_t; \overline{\alpha}(p_t)) = \theta\Omega\left(p_t\right) - \left(j + \frac{i}{\gamma}\right)$ for any $p_t > c$.

$NPV(p_t; \overline{\alpha}(p_t))$ is the expected net present value associated with the operations of the plant where the optimal level of overcapacity $\overline{\alpha}(p_t)$ has been adopted. Thanks to $V(p_t; \alpha) > v(p_t; \alpha)$ we have $NPV(p_t; \overline{\alpha}(p_t)) > NPV(p_t; \underline{\alpha}(p_t))$, that is, operational flexibility increases the net present value of the project.

Up to now we have seen that the operational flexibility is changing the fundamentals of the investment option held by the plant manager. This is reflected in the operating value of the plant ($V(p_t; \alpha) > v(p_t; \alpha)$), the magnitude of the investment (Proposition 4) and the corresponding expected net present value ($NPV(p_t; \overline{\alpha}(p_t)) > NPV(p_t; \underline{\alpha}(p_t))$). In the following, we discuss the effect on the timing and value of the investment option.

4.3 The optimal investment threshold and the investment option value

Let us now determine the value of the option to invest and the optimal investment threshold. Denoting by $\widehat{p}$ the price threshold triggering investment and assuming that the current market price p is below this threshold, i.e. $p < \widehat{p}$, the value of the option to invest is equal to

$$F(p; \widehat{p}) = \max_{\widehat{p} > p}[(p/\widehat{p})^{\beta_1} NPV(\widehat{p}; \overline{\alpha}(\widehat{p}))].$$

As in the previous section, optimality requires that the following conditions hold

$$\begin{aligned} \widehat{p} &= \beta_1 \frac{NPV(\widehat{p}; \overline{\alpha}(\widehat{p}))}{\frac{\partial NPV(\widehat{p}; \overline{\alpha}(\widehat{p}))}{\partial \widehat{p}}} \\ \left.\frac{\partial^2 F(p_t; \widehat{p})}{\partial p_t^2}\right|_{p_t = \widehat{p}} &> \left.\frac{\partial^2 NPV(p_t; \overline{\alpha}(p_t))}{\partial p_t^2}\right|_{p_t = \widehat{p}} \end{aligned}$$

Let us now consider the two investment scenarios proposed above, namely, the scenario where $\overline{\alpha}\left(p_t\right) = 1$ and the scenario where $\overline{\alpha}\left(p_t\right) < 1$.

When the plant manager opts for $\overline{\alpha}(p_t) = 1$, that is, when $\overline{p} \leq p_t$, we have the following:

Proposition 5 *Provided that $\frac{\overline{p}}{r-\mu} - \frac{c}{r} \geq \Delta$, the optimal investment threshold p^* for a project with capacity $\overline{\alpha}(p^*) = 1$ is the solution of the following equation:*

$$p^* + \frac{\beta_1 - \beta_2}{\beta_1 - 1} A p^{*\beta_2} (r - \mu) - \widetilde{p}^* = 0$$

where $\Delta = \left(\frac{c}{r} + \frac{i\beta_2 - \beta_1 \left(i\frac{\gamma-1}{\gamma} - j \right)}{\theta} \right) / (\beta_2 - 1)$.

Proof. Available in the Appendix. ■

Notably, investment in $\overline{\alpha}(p^*) = 1$ is conditional on having at $\overline{p}$, i.e. at the minimum price in the interval of interest, an expected profitability associated with the provision of balancing services, $\frac{\overline{p}}{r-\mu} - \frac{c}{r}$, higher than the level Δ. Otherwise, the project is not worth investing in. The reason is that while operational flexibility guarantees a non-negative flow of profits associated with power production, for $\overline{\alpha}(p_t) = 1$ to make sense the project must have a minimum net present value. Otherwise, as we shall see below, the potential investor should opt for $\overline{\alpha}(p_t) < 1$ rather than $\overline{\alpha}(p_t) = 1$.

Rearranging the formula for p^* we obtain $\widetilde{p}^* - p^* = \frac{\beta_1-\beta_2}{\beta_1-1} A p^{*\beta_2} (r - \mu)$. Thanks to the positivity of the option to temporally suspend operations when necessary ($A p^{*\beta_2} > 0$), we have $\widetilde{p}^* > p^*$:

Proposition 6 *Operational flexibility hastens the investment in a CHP unit characterized by maximum overcapacity ($\widetilde{p}^* > p^*$).*

The importance of operational flexibility is obvious. The positive difference $\widetilde{p}^* - p^*$ is linearly increasing in the term associated with the option to temporally suspend power provision (A). This means that operational flexibility is an investment stimulus and the magnitude of the stimulating effect depends directly on the value of operating flexibility. If the option to suspend is very valuable, because e.g. of generally low power prices or high operating costs, the distance between $p^*$and $\widetilde{p}^*$ is large. If instead operational flexibility is less valuable, as for example in the case where power prices are high or operating costs are low, then the distance between $p^*$and $\widetilde{p}^*$ is small.

Summarizing, as soon as p_t reaches p^* the potential investor chooses a CHP unit of size $\overline{\alpha}(p^*) = 1$ which has operating value $V(p^*; 1)$, expected net present value $NPV(p^*; 1)$ and investment option value $F(p; p^*) = NPV(p^*; 1)(p/p^*)^{\beta_1}$. From $NPV(p^*; 1) - NPV(\widetilde{p}^*; 1) = V(p^*; 1) - v(\widetilde{p}^*; 1) = \theta A p^{*\beta_2} \frac{\beta_2-1}{\beta_1-1} < 0$ we see that operational flexibility reduces the operating value and consequently the expected net present value of the plant. Nevertheless, it has an ambiguous effect on the investment option value.[8] The intuition is as follows. Since operational flexibility serves as a hedge against losses, the potential investor opts for investing sooner than later. However, a lower investment threshold ($p^* < \widetilde{p}^*$) is reflected in a lower operating value ($V(p^*; 1) < v(\widetilde{p}^*; 1)$) and consequently

[8] One can easily show that $F(p; p^*) \gtrless F(p; \widetilde{p}^*)$ if $A p^{*\beta_2} \gtrless \frac{1}{\theta} \frac{\beta_1-1}{\beta_2-1} NPV(\widetilde{p}^*; 1) \left(\left(\frac{p^*}{\widetilde{p}^*} \right)^{\beta_1} - 1 \right)$.

a lower expected net present value for the plant ($NPV(p^*; 1) < NPV(\widetilde{p}^*; 1)$)). The effect on the investment option value is nevertheless ambiguous since, what the potential investor is losing in terms of NPV, is in some cases, remunerated by the earlier investment. Thus, while operational flexibility stimulates investment it might be detrimental in terms of investment option value since it reduces the value of the option of the potential investor to defer a risky and irreversible investment.

When instead the plant manager opts for $\overline{\alpha}(p_t) < 1$, that is, when $p_t \in (c, \overline{p})$, we have the following:

Proposition 7 *Provided that $\frac{\overline{p}}{r-\mu} - \frac{c}{r} < \Delta$, the optimal investment threshold p^{**} for a project with capacity $\overline{\alpha}(p_t) < 1$ is the solution of the following equation:*

$$p^{**}\frac{\partial\overline{\alpha}(p^{**})}{\partial p^{**}} - \beta_1\frac{\overline{\alpha}(p^{**})}{\gamma} + \beta_1\frac{j}{\theta\Omega(p^{**})(\gamma-1)} = 0$$

where $\Delta = \left(\frac{c}{r} + \frac{i\beta_2 - \beta_1\left(i\frac{\gamma-1}{\gamma} - j\right)}{\theta}\right) / (\beta_2 - 1)$.

Proof. Available in the Appendix. ■

Here the project's realization is conditional on having at $\overline{p}$ an expected profitability, $\frac{\overline{p}}{r-\mu} - \frac{c}{r}$, lower than the level Δ. Otherwise, investing in a project with $\overline{\alpha}(p_t) < 1$ would not make sense since the potential investor should rather consider investing in a project with $\overline{\alpha}(p_t) = 1$.

Comparing p^{**} and $\widetilde{p}^{**}$ we have:

Proposition 8 *Operational flexibility hastens the investment in a CHP unit characterized by partial overcapacity ($\widetilde{p}^{**} > p^{**}$).*

Proof. Available in the Appendix. ■

As we show in the Appendix, the operational flexibility is once again the factor that makes the difference. From $\widetilde{p}^* > p^*$ and $\widetilde{p}^{**} > p^{**}$ it is clear that, irrespective of the overcapacity choice made by the potential investor, partial or full, the investment takes place earlier under operational flexibility.

It is worth noting that while the inequality $\widetilde{p}^* > p^*$ refers to investments of the same size ($\underline{\alpha}(\widetilde{p}^*) = \overline{\alpha}(p^*) = 1$) the inequality $\widetilde{p}^{**} > p^{**}$ refers to $\underline{\alpha}(\widetilde{p}^{**})$ and $\overline{\alpha}(p^{**})$ that are sensitive to the chosen investment threshold. In fact, comparing $\underline{\alpha}(\widetilde{p}^{**})$ and $\overline{\alpha}(p^{**})$ we find:

Proposition 9 *The optimal level of partial overcapacity at the time of the investment in a flexible CHP plant is smaller (larger) when the value of operational flexibility $\left(Ap^{**\beta_2}\right)$ is lower (higher) than $\frac{\widetilde{p}^{**} - p^{**}}{r-\mu}$.*

Proof. *$\underline{\alpha}(\widetilde{p}^{**})$ is equal to $\left(\theta\left(\frac{\widetilde{p}^{**}}{r-\mu} - \frac{c}{r}\right)/i\right)^{\frac{1}{\gamma-1}}$ and $\overline{\alpha}(p^{**})$ is equal to $(\theta\Omega(p^{**})/i)^{\frac{1}{\gamma-1}}$. A straightforward comparison shows that $\underline{\alpha}(\widetilde{p}^{**}) \gtrless \overline{\alpha}(p^{**})$ when $\frac{\widetilde{p}^{**} - p^{**}}{r-\mu} \gtrless Ap^{**\beta_2}$.* ■

While, as we have seen in Proposition 4, for any given p_t operational flexibility favors investments in higher partial overcapacity, this is not necessarily the case when we compare the optimal levels $\underline{\alpha}(\widetilde{p}^{**})$ and $\overline{\alpha}(p^{**})$. In fact, as the previous proposition suggests, operational flexibility is associated with larger investments ($\overline{\alpha}(p^{**}) > \underline{\alpha}(\widetilde{p}^{**})$) only when the value of the option to suspend the provision of ancillary operations in the future, $Ap^{**\beta_2}$, is large enough, namely, larger than $\frac{\widetilde{p}^{**}-p^{**}}{r-\mu}$. Otherwise, the potential investor opts for investing in a lower level of overcapacity ($\overline{\alpha}(p^{**}) < \underline{\alpha}(\widetilde{p}^{**})$). The intuition behind this result is as follows. Suppose that being operationally flexible is important ($Ap^{**\beta_2} > \frac{\widetilde{p}^{**}-p^{**}}{r-\mu}$) because, for instance, of generally high operating costs or low power prices. If this is the case, the plant manager opts for investing in a larger α in an attempt to benefit as much as possible, by producing more, in the periods during which the provision of ancillary services is a profitable practice. If instead, being operationally flexible is not as important ($Ap^{**\beta_2} < \frac{\widetilde{p}^{**}-p^{**}}{r-\mu}$), because, for instance, of generally low operating costs or high electricity prices, the plant manager will opt for investing in a smaller plant which will be both, more frequently operational and less costly to install.

Last, given that $\widetilde{p}^{**} > p^{**}$, a sufficient but not necessary condition for $F(p;p^{**}) > F(p;\widetilde{p}^{**})$ is $NPV(p^{**};\overline{\alpha}(p^{**})) > NPV(\widetilde{p}^{**};\underline{\alpha}(\widetilde{p}^{**}))$ which holds when $Ap^{**\beta_2} > \frac{\widetilde{p}^{**}-p^{**}}{r-\mu}$, that is, when $\overline{\alpha}(p^{**}) > \underline{\alpha}(\widetilde{p}^{**})$. Thus, when the value of operational flexibility is high enough so that the potential investor chooses a larger level of overcapacity, the investor opts for investing earlier in a plant that guarantees a larger NPV at the time of the investment and has a larger investment option value. If instead the value of operational flexibility is not as high, and consequently $\overline{\alpha}(p^{**}) < \underline{\alpha}(\widetilde{p}^{**})$ and $NPV(p^{**};\overline{\alpha}(p^{**})) < NPV(\widetilde{p}^{**};\underline{\alpha}(\widetilde{p}^{**}))$, the inequality $F(p;p^{**}) > F(p;\widetilde{p}^{**})$ still holds as long as $NPV(p^{**};\overline{\alpha}(p^{**}))$ is larger than $NPV(\widetilde{p}^{**};\underline{\alpha}(\widetilde{p}^{**}))\left(p^{**}/\widetilde{p}^{**}\right)^{\beta_1}$.

5 Discussion and Conclusions

As demonstrated in the previous section, operational flexibility is affecting both the timing and the capacity choice of a potential investor. Starting with the timing effect, Propositions 6 and 8 show that the potential investor is always setting a lower investment threshold which is instead implying an, in expected terms, earlier investment. This is attributed to the fact that operational flexibility guarantees a non-negative cash flow. In an attempt to gain access to this cash flow as soon as possible, the plant manager chooses a lower investment threshold.

The capacity choice effect presented in Proposition 9 is less straightforward and has important implications for a policy maker interested in the ability of CHP plants to provide balancing services but is also concerned about the problems associated with the installment of CHP overcapacity. While operational flexibility constitutes an investment stimulus in terms of timing, it is not necessarily implying investments in projects with larger capacity. The reason is that a project with lower investment threshold ($\widetilde{p}^{**} > p^{**}$) has a lower per-unit-of-power expected net revenue $\left(\frac{\widetilde{p}^{**}}{r-\mu} - \frac{c}{r} > \frac{p^{**}}{r-\mu} - \frac{c}{r}\right)$ but also a valuable option to suspend the provision of ancillary services in the future $\left(Ap^{**\beta_2} > 0\right)$. While operational flexibility guarantees non-negative cash flows perpetually, the plant manager will still face periods of anemic profit flows in the future, particularly if the investment option is not sufficiently "in the money" when it takes place because of the lower investment threshold. Hence, if the option of being operationally flexible is valuable enough to cover the loss in expected net revenue $\left(Ap^{**\beta_2} > \frac{\widetilde{p}^{**}-p^{**}}{r-\mu}\right)$ the potential investor opts for both earlier and larger in terms of capacity investment. Otherwise, if being operationally flexible is not as valuable and the loss in expected net revenue dominates $\left(Ap^{**\beta_2} < \frac{\widetilde{p}^{**}-p^{**}}{r-\mu}\right)$, the potential investor will still invest earlier but also more conservatively. This is because under $Ap^{**\beta_2} < \frac{\widetilde{p}^{**}-p^{**}}{r-\mu}$ it is not the promising character of the project in terms of value that is driving the investment but rather the hedge against making losses. Since the ability of the plant to be operationally flexible does not depend on its magnitude, the efficient level of overcapacity is lower.

These findings demonstrate the importance of the operational flexibility, not only for the potential investor, but for the power system as a whole. Operational flexibility (i) stimulates new investments in power-production contributing this way to the solution of the security-of-supply-problem while (ii) hedging power suppliers from making losses in the energy market decoupling this way the choice of capacity for the provision of balancing services from the plant's profitability throughout its lifetime. In other words, operational flexibility qualifies as a de-facto investment stimulus that is at the same time alleviating capacity choice distortions attributed to the operational rigidity of typical plants.

6 CHP investments in the pulp and paper industry: a numerical analysis

-work in progress-

A Appendix

A.1 Proof of $v(p_t;\alpha) = \alpha\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right)$

$v(p_t;\alpha)$ can be expressed as the sum of the income accruing over the current production season and a continuation value regarding future uses. $v(p_t;\alpha)$ can be determined by solving the following Bellman equation:

$$v(p_t;\alpha) = \alpha\theta\left(p_t - c\right) + e^{-rdt}E_t\left\{v(p_t + dp_t;\alpha)\right\}$$

Expanding the right-hand side of this equation using Ito's lemma and rearranging yields the following differential equation:

$$\frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 v(p_t;\alpha)}{\partial p_t^2} + \mu p_t \frac{\partial v(p_t;\alpha)}{\partial p_t} - rv(p_t;\alpha) = -\alpha\theta\left(p_t - c\right)$$

This equation must be solved subject to the following boundary conditions:

$$\begin{aligned} \lim_{p_t \to 0} v(p_t;\alpha) &= 0 \\ \lim_{p_t \to \infty} v(p_t;\alpha) &= \alpha\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right) \end{aligned}$$

The two conditions guarantee that the operating value of the plant does not explode either for low or high values of p_t.

Thus, the general solution takes the form:

$$v(p_t;\alpha) = \alpha\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right)$$

A.2 Proof of $E_0\left(e^{-r\tau}\right) = (p/\widetilde{p})^{\beta_1}$

Keeping in mind that $\tau = \inf\{t > 0 \mid p_t = \widetilde{p}\}$, we define

$$D(p;\widetilde{p}) = E_0\left\{e^{-r\tau}\right\}.$$

In the continuation region $(p < \widetilde{p})$, $D(p;\widetilde{p})$ is the solution of the following Bellman equation:[9]

$$D(p;\widetilde{p}) = e^{-rdt}E_0\left(D(p + dp;\widetilde{p})\right).$$

Expanding the right-hand side of this equation using Ito's lemma and noting that $e^{-rdt} = 1 - rdt$ for sufficiently small dt yields the following differential equation

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 D(p;\widetilde{p})}{\partial p^2} + \mu p \frac{\partial D(p;\widetilde{p})}{\partial p} - rD(p;\widetilde{p}) = 0.$$

[9] See pp. 315-316 in Dixit and Pindyck (1994).

The general solution is

$$D(p;\widetilde{p}) = H_1 p^{\beta_1} + H_2 p^{\beta_2}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the quadratic equation $\Lambda(\beta) = (1/2)\sigma^2\beta(\beta-1) + \mu\beta - r$.

$D(p;\widetilde{p}) = H_1 p^{\beta_1} + H_2 p^{\beta_2}$ must be solved subject to the following boundary conditions:

$$\begin{aligned} \lim_{p \to 0} D(p;\widetilde{p}) &= 0, \\ \lim_{p \to \widetilde{p}} D(p;\widetilde{p}) &= 1. \end{aligned}$$

Using these conditions, we get $H_2 = 0$ and $H_1 = \widetilde{p}^{-\beta_1}$. Hence:

$$D(p;\widetilde{p}) = (p/\widetilde{p})^{\beta_1}$$

A.3 Derivation of $V(p_t;\alpha)$

$V(p_t;\alpha)$ can be expressed as the sum of the income accruing over the current production season and a continuation value regarding future uses. Denoting by $V^H(p_t;\alpha)$ the value of the plant when $p_t > c$ and by $V^L(p_t;\alpha)$ the value of the plant when $p_t \leq c$, $V(p_t;\alpha)$, can be determined by solving the following system of Bellman equations:

$$\begin{array}{ll} V^L(p_t;\alpha) = e^{-rdt} E_t\left\{V^L(p_t + dp_t;\alpha)\right\} & \text{for } p_t \leq c \\ V^H(p_t;\alpha) = \alpha\theta\,(p_t - c) + e^{-rdt} E_t\left\{V^H(p_t + dp_t;\alpha)\right\} & \text{for } p_t > c \end{array}$$

Expanding the right-hand side of these equations using Ito's lemma and rearranging yields the following differential equations:

$$\begin{array}{ll} \frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 V^L(p_t;\alpha)}{\partial p_t^2} + \mu p_t \frac{\partial V^L(p_t;\alpha)}{\partial p_t} - rV^L(p_t;\alpha) = 0 & \text{for } p_t \leq c \\ \frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2 V^H(p_t;\alpha)}{\partial p_t^2} + \mu p_t \frac{\partial V^H(p_t;\alpha)}{\partial p_t} - rV^H(p_t;\alpha) = -\alpha\theta\,(p_t - c) & \text{for } p_t > c \end{array}$$

These equations must be solved subject to the following boundary conditions:

$$\begin{array}{ll} \lim_{p_t \to 0} V^L(p_t;\alpha) = 0 & \text{for } p_t \leq c \\ \lim_{p_t \to \infty} V^H(p_t;\alpha) = \alpha\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right) & \text{for } p_t > c \end{array}$$

The two conditions guarantee that the operating value of the plant does not explode either for low or high values of p_t.

Thus, the general solution takes the form

$$V(p_t;\alpha) = \begin{cases} \widetilde{B} p_t^{\beta_1} & \text{when } p_t \leq c \\ \widetilde{A} p_t^{\beta_2} + \alpha\theta\left(\frac{p_t}{r-\mu} - \frac{c}{r}\right) & \text{when } p_t > c \end{cases}$$

where $\widetilde{B}$ and $\widetilde{A}$ are constants to be determined. At $p_t = c$, standard optimality conditions, i.e. the value matching and smooth pasting conditions, require

$$\begin{aligned}\widetilde{A}c^{\beta_2} + \alpha\theta\left(\frac{c}{r-\mu} - \frac{c}{r}\right) &= \widetilde{B}c^{\beta_1}, \\ \widetilde{A}\beta_2 c^{\beta_2 - 1} + \frac{\alpha\theta}{r-\mu} &= \widetilde{B}\beta_1 c^{\beta_1 - 1}.\end{aligned}$$

Solving the system we obtain:

$$\begin{aligned}\widetilde{A} &= \alpha\theta A = \alpha\theta\frac{r-\mu\beta_1}{(\beta_1-\beta_2)\,r\,(r-\mu)}c^{1-\beta_2} > 0 \\ \widetilde{B} &= \alpha\theta B = \alpha\theta\frac{r-\mu\beta_2}{(\beta_1-\beta_2)\,r\,(r-\mu)}c^{1-\beta_1} > 0\end{aligned}$$

A.4 Derivation of $\overline{\alpha}$

The first-order condition yields,[10]

$$\overline{\alpha} = \left(\theta\Omega\left(p_t\right)/i\right)^{\frac{1}{\gamma-1}}$$

For $\overline{\alpha} \in [0,1]$ we require:

$$0 \leq \Omega\left(p_t\right) \leq \frac{i}{\theta}$$

Note that

$$\begin{aligned}\Omega\left(c\right) &= Bc^{\beta_1} > 0 \\ \Omega'\left(c\right) &= \beta_1\frac{r-\mu\beta_2}{(\beta_1-\beta_2)\,r\,(r-\mu)} > 0 \\ \Omega''\left(p_t\right) &= A\beta_2\left(\beta_2-1\right)p_t^{\beta_2-2} > 0\end{aligned}$$

Hence, by the convexity of $\Omega\left(p_t\right)$, it follows that $\Omega\left(p_t\right) > 0$ and $\Omega'\left(p_t\right) > 0$ for any $p_t > c$.

Let us now check the conditions under which $\theta\Omega\left(p_t\right) \leq i$, i.e. $\overline{\alpha} \leq 1$. Define the function:

$$\Psi = \theta\Omega\left(c\right) - i$$

Then if $\Psi < 0 \rightarrow \overline{p} > c$

$$\overline{\alpha}\left(p_t\right) = \begin{cases} \left(\theta\Omega\left(p_t\right)/i\right)^{\frac{1}{\gamma-1}} & \text{for} \quad c < p_t < \overline{p}, \\ 1 & \text{for} \quad c < \overline{p} \leq p_t, \end{cases}$$

where $\overline{p}$ is such that $\theta\Omega(\overline{p}) = i \rightarrow \overline{p} = \underline{p} - A\overline{p}^{\beta_2}\left(r-\mu\right)$.

If instead $\Psi \geq 0 \rightarrow \overline{p} \leq c$,

$$\overline{\alpha}\left(p_t\right) = 1,\, \overline{p} \leq c < p_t$$

[10] The second-order condition is always satisfied.

A.5 The investment threshold under $\overline{\alpha}=1$

Substituting $NPV(p_t;\overline{\alpha}(p_t)) = \theta\Omega\left(p_t\right) - \left(j+\frac{i}{\gamma}\right)$ in the optimality condition $\widehat{p} = \beta_1 \frac{NPV(\widehat{p};\overline{\alpha}(\widehat{p}))}{\frac{\partial NPV(\widehat{p})}{\partial \widehat{p}}}$ and denoting $\widehat{p}=p^*$ for $\overline{\alpha}=1$ we obtain:

$$p^* + \frac{\beta_1-\beta_2}{\beta_1-1}Ap^{*\beta_2}\left(r-\mu\right) - \frac{\beta_1}{\beta_1-1}\left(r-\mu\right)\left(\frac{c}{r}+\frac{j+\frac{i}{\gamma}}{\theta}\right)=0$$

Thanks to $\widetilde{p}^* = \frac{\beta_1}{\beta_1-1}\left(r-\mu\right)\left(\frac{c}{r}+\frac{j+\frac{i}{\gamma}}{\theta}\right)$, this inequality reduces to $p^*+\frac{\beta_1-\beta_2}{\beta_1-1}Ap^{*\beta_2}\left(r-\mu\right)-\widetilde{p}^*=0$.

Regarding existence and uniqueness of p^* we have the following. We define the function

$$\Phi(x) = x + \frac{\beta_1-\beta_2}{\beta_1-1}Ax^{\beta_2}\left(r-\mu\right) - \frac{\beta_1}{\beta_1-1}\left(r-\mu\right)\left(\frac{c}{r}+\frac{j+\frac{i}{\gamma}}{\theta}\right)$$

$\Phi(x)$ is convex in x. Hence, the equation $\Phi(p^*)=0$ may admit up to two roots. However, we also have:

$$\frac{\partial NPV(p^*;\overline{\alpha}(p^*))}{\partial p^*} > \frac{p^*}{\beta_1-1}\left.\frac{\partial^2 NPV(p_t;\overline{\alpha}(p_t))}{\partial p_t^2}\right|_{p_t=p^*}$$

This can be written as

$$\Omega'\left(p^*\right) > \frac{p^*}{\beta_1-1}\Omega''\left(p^*\right)$$

or alternatively

$$\Phi'(p^*)>0.$$

This implies that if a solution p^* exists, then it is unique since $\Phi'(p^*)>0$.

By the convexity of $\Phi(x)$, two necessary conditions for the existence of a solution p^* are:

$$\Phi(c)<0,\ \Phi(\overline{p})\leq 0$$

$\Phi(c)<0$ holds always thanks to $i,j>0$. As for $\Phi(\overline{p})\leq 0$ we have the following necessary condition:

$$\frac{\overline{p}}{r-\mu}-\frac{c}{r}\geq \Delta = \frac{\frac{c}{r}+\frac{i\beta_2-\beta_1\left(i\frac{\gamma-1}{\gamma}-j\right)}{\theta}}{\beta_2-1}$$

A.6 The investment threshold under $\overline{\alpha}<1$

We denote by p^{**} the $\widehat{p}$ under $\overline{\alpha}<1$. This needs to satisfy $p^{**}=\beta_1\frac{NPV(p^{**};\overline{\alpha}(p^{**}))}{\frac{\partial NPV(p^{**};\overline{\alpha}(p^{**}))}{\partial p^{**}}}$ and $\frac{\partial NPV(p^{**};\overline{\alpha}(p^{**}))}{\partial p^{**}} > \frac{p^{**}}{\beta_1-1}\left.\frac{\partial^2 NPV(p_t;\overline{\alpha}(p_t))}{\partial p_t^2}\right|_{p_t=p^{**}}$. From the former we obtain

$$p^{**}\frac{\partial\overline{\alpha}(p^{**})}{\partial p^{**}} - \beta_1\frac{\overline{\alpha}\left(p^{**}\right)}{\gamma} + \beta_1\frac{j}{\theta\Omega\left(p^{**}\right)\left(\gamma-1\right)}=0$$

Regarding the existence and uniqueness of p^{**} we have the following. We define $\Theta(x) = x\frac{\partial NPV(x;\overline{\alpha}(x))}{\partial x} - \beta_1 NPV(x)$. The first derivative of $\Theta(x)$ with respect to x is

$$\Theta'(x) = x\frac{\partial^2 NPV(x;\overline{\alpha}(x))}{\partial x^2} - (\beta_1 - 1)\frac{\partial NPV(x;\overline{\alpha}(x))}{\partial x}.$$

Thus, the inequality $\frac{\partial NPV(p^{**};\overline{\alpha}(p^{**}))}{\partial p^{**}} > \frac{p^{**}}{\beta_1 - 1}\frac{\partial^2 NPV(p_t;\overline{\alpha}(p_t))}{\partial p_t^2}\Big|_{p_t = p^{**}}$ implies that at p^{**} we have $\Theta'(p^{**}) < 0$. The second derivative of $\Theta(x)$ with respect to x is

$$\Theta''(x) = (2 - \beta_1)\frac{\partial^2 NPV(x;\overline{\alpha}(x))}{\partial x^2} + x\frac{\partial^3 NPV(x;\overline{\alpha}(x))}{\partial x^3}.$$

Given that in the interval of interest we have

$$NPV(x;\overline{\alpha}(x)) = \left(\frac{\theta\Omega(x)}{i}\right)^{\frac{\gamma}{\gamma-1}}\frac{\gamma-1}{\gamma}i - j,$$

the effect of x on NPV passes exclusively through $\delta(\Omega(x)) = \left(\frac{\theta\Omega(x)}{i}\right)^{\frac{\gamma}{\gamma-1}}$. Since $\frac{\partial NPV(\delta)}{\partial\delta} > 0$, $\frac{\partial\delta(\Omega)}{\partial\Omega} > 0$ and, in the considered interval, $\Omega(x)$ is increasing and convex in x with $\Omega'''(x) < 0$ we have:[11]

$$\begin{aligned}
\frac{\partial NPV(x;\overline{\alpha}(x))}{\partial x} &= \frac{\partial NPV(\delta)}{\partial\delta}\frac{\partial\delta(\Omega)}{\partial\Omega}\Omega' > 0,\\
\frac{\partial^2 NPV(x;\overline{\alpha}(x))}{\partial x^2} &= \frac{\partial NPV(\delta)}{\partial\delta}\frac{\partial\delta(\Omega)}{\partial\Omega}\Omega'' > 0,\\
\frac{\partial^3 NPV(x;\overline{\alpha}(x))}{\partial x^3} &= \frac{\partial NPV(\delta)}{\partial\delta}\frac{\partial\delta(\Omega)}{\partial\Omega}\Omega''' < 0.
\end{aligned}$$

From these we obtain

$$\Theta''(x) = \frac{\partial NPV(\delta)}{\partial\delta}\frac{\partial\delta(\Omega)}{\partial\Omega}A\beta_2(\beta_2 - 1)x^{\beta_2 - 2}[\beta_2 - \beta_1] < 0$$

This means that $\Theta(x)$ is concave and consequently there is at most one p^{**} satisfying $\Theta'(p^{**}) < 0$.

Two conditions that guarantee the existence of p^{**} are $\lim_{x\to\overline{p}}\Theta(x) < 0$ and $\lim_{x\to c}\Theta(x) > 0$. Let us start with the first. In the interval of interest we have $NPV(x;\overline{\alpha}(x)) = \overline{\alpha}(x)\theta\Omega(x)\frac{\gamma-1}{\gamma} - j$ and $\frac{\partial NPV(x;\overline{\alpha}(x))}{\partial x} = \theta\frac{\gamma-1}{\gamma}\left(\overline{\alpha}'(x)\Omega(x) + \overline{\alpha}(x)\Omega'(x)\right)$. Since $\overline{\alpha}'(p_t) = \frac{\overline{\alpha}(p_t)}{\gamma-1}\frac{\Omega'(p_t)}{\Omega(p_t)}$ we have $\frac{\partial NPV(x;\overline{\alpha}(x))}{\partial x} = \theta\overline{\alpha}(x)\Omega'(x)$. Then, $\Theta(x)$ can be written as:

$$\Theta(x) = \theta\overline{\alpha}(x)\left(x\Omega'(x) - \beta_1\Omega(x)\frac{\gamma-1}{\gamma}\right) + \beta_1 j$$

[11] We have $\Omega'(p_t) = A\beta_2 p_t^{\beta_2-1} + \frac{1}{r-\mu}$, $\Omega''(p_t) = A\beta_2(\beta_2-1)p_t^{\beta_2-2} > 0$, $\Omega'''(p_t) = A\beta_2(\beta_2-1)(\beta_2-2)p_t^{\beta_2-3} < 0$.

Given that $\Omega\left(\overline{p}\right) = \frac{i}{\theta}$ we have:

$$\lim_{x \to \overline{p}} \Theta(x) = \theta \left(1 - \beta_2\right) \left[\frac{\overline{p}}{r - \mu} - \frac{c}{r} - \frac{\frac{c}{r} + \frac{\beta_2 i - \beta_1 \left(i \frac{\gamma - 1}{\gamma} - j\right)}{\theta}}{\beta_2 - 1} \right]$$

For $\lim_{x \to \overline{p}} \Theta(x) < 0$ we require $\frac{\overline{p}}{r-\mu} - \frac{c}{r} < \Delta = \frac{\frac{c}{r} + \frac{i\beta_2 - \beta_1 \left(i \frac{\gamma-1}{\gamma} - j\right)}{\theta}}{\beta_2 - 1}$.

Last, regarding $\lim_{x \to c} \Theta(x) > 0$ we have

$$\lim_{x \to c} \Theta(x) = \overline{\alpha}(c)\theta \left(c\Omega'\left(c\right) - \beta_1 \Omega\left(c\right) \frac{\gamma - 1}{\gamma} \right) + \beta_1 j$$

One can easily show that $c\Omega'\left(c\right) - \beta_1 \Omega\left(c\right) \frac{\gamma - 1}{\gamma} = \frac{r - \mu\beta_2}{(\beta_1 - \beta_2)\gamma r (r - \mu)} c\beta_1 > 0$, consequently, $\lim_{x \to c} \Theta(x) > 0$.

A.7 Proof of $\widetilde{p}^{**} > p^{**}$

We need to compare

$$p^{**} = \beta_1 \frac{NPV(p^{**}; \overline{\alpha}(p^{**}))}{\frac{\partial NPV(p^{**}; \overline{\alpha}(p^{**}))}{\partial p^{**}}}$$

and

$$\widetilde{p}^{**} = \beta_1 \frac{NPV(\widetilde{p}^{**}; \underline{\alpha}(\widetilde{p}^{**}))}{\frac{\partial NPV(\widetilde{p}^{**}; \underline{\alpha}(\widetilde{p}^{**}))}{\partial \widetilde{p}^{**}}}.$$

Given that the chosen overcapacity is NPV-maximizing from the envelope theorem we have

$$\frac{\partial NPV(p_t; \overline{\alpha}\left(p_t\right))}{\partial p_t} = \overline{\alpha}\left(p_t\right) \theta \frac{\partial \Omega\left(p_t\right)}{\partial p_t}$$

and

$$\frac{\partial NPV(p_t; \underline{\alpha}\left(p_t\right))}{\partial p_t} = \underline{\alpha}\left(p_t\right) \theta \frac{1}{r - \mu}.$$

From these, the formula for p^{**} can be written as

$$p^{**} \frac{\partial \Omega\left(p^{**}\right)}{\partial p^{**}} = \beta_1 \left(\Omega\left(p^{**}\right) \left(1 - \frac{1}{\gamma}\right) - \frac{j}{\overline{\alpha}\left(p^{**}\right)\theta} \right),$$

or, substituting for $\frac{\partial \Omega(p^{**})}{\partial p^{**}}$ and $\Omega\left(p^{**}\right)$ we have

$$Ap^{**\beta_2} \left(\beta_2 - \beta_1 \frac{\gamma - 1}{\gamma}\right) + \frac{p^{**}}{r - \mu} \left(1 - \beta_1 \frac{\gamma - 1}{\gamma}\right) + \beta_1 \left(\frac{\gamma - 1}{\gamma} \frac{c}{r} + \frac{j}{\overline{\alpha}\left(p^{**}\right)\theta} \right) = 0.$$

Similarly, the formula for $\widetilde{p}^{**}$ can be written as

$$\frac{\widetilde{p}^{**}}{r-\mu}\left(1-\beta_1\frac{\gamma-1}{\gamma}\right)+\beta_1\left(\frac{\gamma-1}{\gamma}\frac{c}{r}+\frac{j}{\underline{\alpha}\left(\widetilde{p}^{**}\right)\theta}\right)=0.$$

The two formulas are obviously symmetric. The only difference between them is the term capturing operational flexibility (A). We can see how this term affects the investment threshold in the following way.

p^{**} is the root of

$$\begin{aligned}\Theta(x) &= x\frac{\partial NPV(x;\overline{\alpha}(x))}{\partial x}-\beta_1 NPV(x;\overline{\alpha}(x)) \\ &= Ax^{\beta_2}\left(\beta_2-\beta_1\frac{\gamma-1}{\gamma}\right)+\frac{x}{r-\mu}\left(1-\beta_1\frac{\gamma-1}{\gamma}\right)+\beta_1\frac{\gamma-1}{\gamma}\frac{c}{r}+\beta_1\frac{j}{\theta\left(\frac{\theta\Omega(x)}{i}\right)^{\frac{1}{\gamma-1}}}.\end{aligned}$$

If $A=0$, $\Theta(x)$ reduces to $\frac{x}{r-\mu}\left(1-\beta_1\frac{\gamma-1}{\gamma}\right)+\beta_1\frac{\gamma-1}{\gamma}\frac{c}{r}+\beta_1\frac{j}{\theta\left(\frac{\theta\left(\frac{x}{r-\mu}-\frac{c}{r}\right)}{i}\right)^{\frac{1}{\gamma-1}}}$ the root of which is $\widetilde{p}^{**}$. Applying the implicit function theorem we have:

$$\frac{dp^{**}}{dA}=-\frac{\left.\frac{\partial\Theta}{\partial A}\right|_{x=p^{**}}}{\left.\frac{\partial\Theta}{\partial n}\right|_{x=p^{**}}}$$

As we have seen above, $\Theta'(p^{**})<0$ at the point of interest. This means that the sign of $\frac{dp^{**}}{dA}$ is the sign of $\left.\frac{\partial\Theta}{\partial A}\right|_{x=p^{**}}$. One can easily show that

$$\frac{\partial\Theta}{\partial A}=x^{\beta_2}\left(\beta_2-\beta_1\frac{\gamma-1}{\gamma}-\frac{\frac{\beta_1}{\gamma-1}j}{\theta\Omega\left(x\right)\overline{\alpha}\left(x\right)}\right)<0$$

Hence, $\frac{dp^{**}}{dA}<0$ which means $p^{**}<\widetilde{p}^{**}$.

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