External Financing and Double Marginalization: Capacity Investment under Uncertainty⋆

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Abstract

This paper considers a firm's investment decision in a market environment with stochastic evolution of the (inverse) demand, where the investment is financed by borrowing. The lender has market power, generating a capital market inefficiency. The investment decision of the firm involves to determine the timing and the capacity level given a coupon rate schedule offered by the lender. It is shown that a double marginalization effect arises in the sense that the lender's market power results in a considerably smaller investment compared to internal financing, while the timing of the investment stays the same. Introducing the bankruptcy option mitigates the double marginalization effect. In particular the firm's investment size is increasing in the costs the lender faces when taking over a bankrupt firm's capital, albeit at the expense of an investment delay. For initial conditions in the stopping region welfare increases with increasing bankruptcy costs, whereas for initial conditions in the continuation region an inverse U-shaped dependence might arise.

Keywords: Double Marginalization; Uncertainty; Debt; Bankruptcy; Capacity Investment

Preliminary draft: Please do not cite or circulate

1 Introduction

Firm's innovation investment needs financing. When a firm is already operating in the market, investment can be financed internally (Chandy and Tellis, 2000). However, for a startup firm, investment has to be financed externally via bank loans or the capital market. Startups usually lack stable cash flows or collaterals, but rely greatly on intangible resources (Hall, 2002). Thus, firm's innovation activities are sensitive to the availability of capital (Cerqueiro et al., 2017). In some countries there exist loan guarantee programs to ease the access to financial resources (Minniti, 2008), e.g., Italian Startup Act (Giraudo et al., 2019). Venture

⋆Herbert Dawid and Xingang Wen gratefully acknowledge support from the German Research Foundation (DFG) via CRC 1283.

capital is another important financing resource for startups (see e.g., Kortum and Lerner (2000)). The difference between these two financing sources is that venture capital appreciates the high-risk projects with high returns, whereas the bank lender appreciates the startups with a steady and foreseeable growth path (Giraudo et al., 2019). Financing the innovation investment requires not only capital but also a willingness to fail (Nanda and Rhodes-Kropf, 2013). Hall and Woodward (2010) report in their sample about 50% of startups had zero-value exits. Many factors can lead to the startup failure, e.g., less capital, less brand presence, fewer strategic alliances and so on (Freeman and Engel, 2007). Apart from these factors, the market uncertainty has direct impact on the profitability of the firm and thus influences the firm's survival.

Incorporating the corporate finance aspects into an industrial organization model, this paper studies in a dynamic economic setting how the potential bankruptcy of a startup firm influences the strategic interactions between the startup firm and its debt holder in terms of coupon rate, the investment timing and size, and also the welfare effect. More specifically, this paper considers that a startup firm approaches a creditor/lender for capital to carry out innovation investment. After the investment, for the event of bankruptcy, the firm decides the optimal timing of default and the corresponding scrap value transfers to the debt holder.

This paper builds on the vast literature that uses real options framework to study investment decisions (Dixit and Pindyck, 1994). Some outstanding examples include Pindyck (1988), who develops a model with irreversible investment and capacity choice, Huisman and Kort (2015) extend the monopoly model to a duopoly setting and investigate the deterrence and accommodation interactions between firms. The traditional real options literature based on all equity financing has been extended to settings with debt financing. The extension have relied on numerical procedures to draw out the relationship between optimal investment and financing decisions (see e.g., Mello and Parsons (1992); Hennessy and Whited (2005); Sundaresan and Wang (2007); Pawlina (2010)).

For a given size of investment, several literature shows that risky debts accelerates the investment timing. The basic intuition is that a higher debt level increases the probability of future default, so the risky debt reduces the value of the option to wait and thus accelerate the investment (Boyle and Guthrie, 2003). This intuition is supported by Mauer and Sarkar (2005), who study the impact of stockholder-debtholder conflict on the timing to exercise the investment. They assume the equity holders issue debt to finance investment and they have an incentive to exercise early, i.e., to issue debts at a time when it is riskier and the market price is lower. Lyandres and Zhdanov (2010) also find that in the absence of wealth expropriation by a levered firm's debt holders, its shareholders exercise their investment options earlier. By incorporating the size of investment, Sarkar (2011) finds that the effect of debt financing on investment depends on the amount of debt used: The optimal amount of debt financing results in delayed but larger investment. Similarly, Lukas and Thiergart (2019) also find that levered firms invest more than unlevered firms, and their optimal investment threshold can also be higher than that of their unlevered counterpart. Some other literature considers the impact of the capital structure on dynamic investment. For instance, under the debt constraint with an upper limit of the debt issuance, Shibata and Nishihara (2015) show that firms are more likely to issue market debts than bank debts when the debt constraint increases; Shibata and Nishihara (2018) find the debt constraint does not always delay investment or affect the investment quantity, but may change the capital structure during financial distress. The difference between our research and the existing literature is that we consider the outside lender has its own preference on the investment, i.e., timing and size, and can influence the firm's investment decision. In particular, the lender can charge a significantly large coupon rate to stop the firm's access to capital. Then the firm's investment decision in our model depends on the perfection of the capital market, i.e., the market power of the lender.

The market power of the lender, especially for firms that rely on bank debts, has been well recognized in literature. Rajan (1992) finds that bank debt has more incentive to monitor the borrower, and the private information that the bank gains through monitoring allows it to "hold up" the borrower, i.e., if a borrower seeks to switch banks, it may be deemed as a "lemon" regardless of its true financial condition. So the bank can hold up borrowers for higher interest rates. Hale and Santos (2009) and Santos and Winton (2008) provide empirical evidence that the bank lends at lower interest rates when firms have access to the public bond market. Schenone (2009) supports that the information asymmetry grants the lending banks an information monopoly compared with prospective lenders. Schwert (2020) finds also empirically that banks earn a large premium relative to the bond-implied credit spread, and questions about the nature of competition in the loan market. Petersen and Rajan (1995) find that, although banks charge higher when they have monopoly power, they also extend loans to riskier young firms because their future rents on the survivors make up for the additional failures. Our model in the frame of real options offers more insight about the impact of the monopoly power on the charged loans. When lending is an option for the lender, the ratio of "debt price" to "risk free interest rate", i.e., Tobin's q, exceeds unity [1].

Moreover, we find by comparing to the without financial constraint scenario that, not only the investment becomes more costly, but the investment is also less [2]. This indicates a double marginalization effect. The well-known double marginalization effect arises when two firms that are different levels of supply chain, have market power and apply a mark-up to their prices. There is abundant literature on double marginalization, see e.g., Rochet and Tirole (2003, 2004) and Weyl (2010) for the double marginalization in two-sided markets, and Liu et al. (2007) and Li et al. (2014) for the double marginalization in supply chains. In our analogous financing model, the bank as supplier "Upstream" provides capital at the cost of risk-free interest rate and sets a price as coupon rate/lending rate to a monopolistic product producer "Downstream". Downstream uses capital and charges consumer at a monopolistic price for the final product. Thus, we can characterize capital as the intermediate input. To the best of our knowledge, this paper is the first to look at double marginalization from a financial perspective. Previous research work, such as Roy et al. (2019) and Desai et al. (2010), considers a two-stage setting and in each stage the upstream firm produces and sells to retailer and then retailer sells to customers. Anand et al. (2008) argues that as the number of periods increases, the qualitative results from two-period model still hold. Our dynamic setting allows to insight about the influence on timing by both players' market power. Double marginalization in our financial constraint model does not influence the timing of investment, but halves the investment capacity, i.e., the final product's market price doubles after the investment. Without financial constraint as in the traditional real options literature, the upstream and downstream can be considered vertically integrated and there is no double marginalization influence.

This paper first considers the firm and the lender's optimal decisions in imperfect capital market, i.e., the lender exerts market power. More specifically, the lender's optimal coupon rate (price to lend) scheme, and the firm's investment timing and capacity for without bankruptcy and with bankruptcy are derived. It comes up in our analysis that the lender has its own preferences about the investment decisions. If the firm approaches the lender prior to its preferred timing, the lender can set sufficiently high coupon rates such that the investment is temporarily delayed. If the firm approaches the lender within the range of lender's preferred time interval, then investment happens immediately. This has resemblance to the Stackelberg leader's accommodation strategy as by Huisman and Kort (2015). The interaction also works the other way around. When the lender prefers early investment and charges less for lending, it is possible that the firm

[1] Tobin's q is defined as the ratio of "the value of existing capital goods, or of titles to them" to "their current reproduction cost". When $q > 1$, the firm can increase its market value by increasing its capital stock, so a firm should invest. Otherwise, the firm does not invest. In a real options framework, q is larger than 1, reflecting the market value of existing asset (the numerator in q) should be the difference between the project value and the option value, see e.g., Dixit and Pindyck (1994).

[2] Without financial constraint implies that the firm can either finance through its own capital as in the traditional real options framework, or there is no market power of the lender, i.e., no "hold up" by the bank.

still waits to invest according to its own optimal decision.

Our analysis on the bankruptcy reveals that the influence of bankruptcy on investment is non-monotonic, and it depends on the interaction between the firm and the lender. When the bankruptcy costs are small, the firm's preference dominates and both the investment timing and size decrease with the bankruptcy costs. When the bankruptcy costs are large, the lender's preference dominates and both the investment timing and size increases with bankruptcy costs. Our result differs from previous research such as Sarkar (2011) and Lukas and Thiergart (2019) that the levered firm invests later but more. This is because they focus on the capital structure, i.e., only part of the investment is financed by debt, and they also assume a constant coupon rate as Leland (1994). Furthermore, the welfare analysis indicates that financial constraint decreases the total welfare.

The structure of this paper is organized as follows. Section 2 builds up the model and formulates the problem for the firm and the lender. Section 3 derives the optimal decisions for both the firm and the lender, and carries out numerical analysis on the influence of bankruptcy and market uncertainty on the optimal decisions. Section 4 conducts a robustness analysis and compares with that the lender has no market power. Section 5 concludes.

2 Model

Consider the situation of a risk-neutral value-maximizing monopolist that has the option to enter a new market through undertaking investment and a (private) lender that has the opportunity to provide external financing to the firm. Assuming that the firm has no equity, it fully relies on debt to finance the investment. The debt structure considered in this paper takes the form of coupon payments by the firm to the lender in exchange for a lump-sum amount upon investment that covers the cost of investment. Coupon payments are incurred after investment and ex-ante the firm is assumed to hold a perpetual American-style option to issue debt and undertake investment. Moreover, we assume that the lender has the opportunity to provide funds, but is not obliged.

When making the investment decision, the firm has to decide when and how much to invest, where the latter relates to the production capacity. The lender decides on the coupon rate to be charged. For a stipulatory coupon rate, the firm has to repay the coupon to the lender, until the firm defaults. In case the firm defaults the lender receives a scrap value corresponding to a certain proportion of the firm value at the time of default.

Market Environment Denote by I the scale of investment by the firm. The market is characterized by the inverse demand function that reads,

$$p(t) = x(t)(1 - \eta I), \quad \text{with} \;\; \mathrm{d}x(t) = \mu x(t)\mathrm{d}t + \sigma x(t)\mathrm{d}\omega(t).$$

Here, $p(t)$ denotes the market-clearing price, $\eta > 0$ denotes the price sensitivity parameter, and $x(t)$ is an exogenous shock process. The process $(x(t))_{t\geq 0}$ follows a geometric Brownian motion with trend μ and volatility parameter σ. The term $\mathrm{d}\omega(t)$ represents the increment of a Wiener process with expected value 0, standard deviation $\sqrt{t}$, and has the property that $(\mathrm{d}\omega(t))^2 = \mathrm{d}t$. Denote the corresponding probability measure by $\mathbb{P}$ and let $\mathbb{E}_t$ be the associated conditional expectation operator $\mathbb{E}[\cdot|\mathcal{F}_t^x]$, $t \geq 0$, where $(\mathcal{F}_t^x)_{t\geq 0}$ is the natural filtration of state process. Further, denote by X the initial value of the state process, i.e., $X = x(0)$.

The opportunity cost for the lender of the funds provided to the firm is linear in the scale of investment where δ is the unit investment cost, i.e. the total cost of investment equals δI. They encompass the total investment cost. Denote by ρ the coupon rate so that instantaneous profits, after investment has been undertaken, are given by

$$\pi(x(t), I; \rho) = Ix(t)(1 - \eta I) - \rho\delta I,$$

for $t \geq 0$. Discounting is done under fixed rate r, where we make the usual assumption that $r > \mu$ to ensure that investment is undertaken in finite time.

Equilibrium Concept This paper considers a Markov Perfect Equilibrium for a Stackelberg-like framework. At $t = 0$ the lender offers a scheme $\rho(x)$ for $x \geq 0$ determining the coupon rate if the firm invests at a value x of the state. The lender stays committed to this scheme throughout the game, making it the Stackelberg leader. The firm takes this scheme into account when subsequently deciding on the timing and scale of investment.

Problem of the Firm Once investment is undertaken, the firm is assumed to operate until it defaults. We assume during this period an immediate-investment-inducing coupon scheme. In line with the literature (see, e.g., Leland (1994)), bankruptcy is modeled as an stopping timing problem given by

$$\tau_B(X, I; \tilde{\rho}) = \arg\ \sup_{\tau \geq 0} \mathbb{E}_0 \int_0^{\tau} \pi(x(t), I; \tilde{\rho}) e^{-rt} \mathrm{d}t,$$

Here τ_B denotes the (stochastic) bankruptcy time, at an initial state $x(0) = X$, fixed investment I and the coupon rate $\tilde{\rho}$ fixed at the time of investment. Following, e.g., Dixit and Pindyck (1994), the optimal stopping problem will be written in terms of the state process. Then, the state space can be divided into two regions, for $X > X_B(I, \rho)$, for some $X_B(I, \rho)$, the firm remains active in the market and for $X \leq X_B$ the firm defaults. Then, τ_B is given by the first hitting time $\tau_B(X, I; \tilde{\rho}) = \inf\{t \mid x(t) \leq X_B(I, \tilde{\rho}), x(0) = X\}$.

Consequently, if investment is undertaken at some time τ_F, then the firm's net present value is given by

$$J_F(x(0), \rho(\cdot), \tau_F, I) = \mathbb{E}_0 \int_{\tau_F}^{\tau_F + \tau_B(x(\tau_F), I; \rho(x(\tau_F)))} e^{-rt} \pi(x(t), I; \rho(x(\tau_F))) \mathrm{d}t. \tag{1}$$

Similarly, the optimal stopping problem will be written in terms of the state process, distinguishing a region where investment is optimal, the *stopping region*, denoted by $S \subseteq [0, \infty)$, and a region where it is optimal to delay investment, the *continuation region*. Without making any additional assumptions on the shape of the function $\rho(\cdot)$, strictly speaking it is not evident that the stopping region of the problem is given by an interval. [3] We proceed in our analysis by assuming that $S = [X_F^*(\rho(\cdot)), \infty)$ for some threshold $X_F^*(\rho(\cdot)) > 0$ and will later verify that this assumption is true for the optimal coupon scheme $\rho^*(\cdot)$ of the lender. Hence, for $X < X_F^*(\rho(\cdot))$ the firm optimally delays investment and for $X \geq X_F^*(\rho(\cdot))$ it is optimal to immediately undertake investment.

Given a scheme $\rho(\cdot)$, the firm's investment problem is given by

$$V_F(x(0); \rho(\cdot)) = \sup_{\tau_F \geq 0, I \geq 0} J_F(x(0), \rho(x(\tau_F)), \tau_F, I),$$

so that τ_F is the first time of X_F^*. The scale of investment that follows from the solution to the optimization problem is denoted by $I^*(x(\tau_F), \rho(x(\tau_F)))$, where $x(\tau_F) = X$ if $\tau_F = 0$ and $X_F^*(\rho(\cdot))$ if $\tau_F > 0$, or $x(\tau_F) = \max\{X, X_F^*(\rho(\cdot))\}$.

[3] Dixit and Pindyck (1994), e.g., show that the state space can be split up into two consecutive regions for standard real options problems giving the stopping region and continuation region in this fashion. For models where capacity choice is explicitly modeled, their result is extended by Huberts et al. (2019). Using a verification theorem based on, e.g., Gozzi and Russo (2006), optimality can be shown. This result however does not cover the case of a state-dependent coupon rate.

Problem of the lender In our base model it is assumed that the capital market is imperfect, that is, the lender has market power. As such the lender sets a coupon scheme $\rho(\cdot)$ as to maximize its net present value.

We consider a strategy of the following form, the optimality of which we will later verify. Let $\rho^{imm}(X)$ denote the value maximizing coupon rate of the lender assuming immediate investment by the firm. Then, the coupon rate offered by the lender is given by

$$\rho^*(X) = \begin{cases} \rho^{imm}(X) & \text{for all } X \in D, \\ \infty & \text{for all } X \in \mathbb{R}_+ \backslash D, \end{cases}$$

with $D = [X_D^*, \infty)$ for some $X_D^* \geq 0$. For all $X \in D$ and $X \geq X_F^*(\rho^{imm}(X))$, the value $\tilde{\rho} = \rho^{imm}(X)$ follows from the optimization problem

$$\sup_{\tilde{\rho}} \tilde{J}_D(X, \tilde{\rho}, I^*(X, \tilde{\rho})), \tag{2}$$

with

$$\tilde{J}_D(X, \tilde{\rho}, I) = \mathbb{E}_0 \left\{ \int_0^{\tau_B(X,I;\tilde{\rho})} \tilde{\rho} \delta I e^{-rt} \mathrm{d}t + (1-\alpha) \int_{\tau_B(X,I;\tilde{\rho})}^{\infty} \pi(x(t), I; 0) e^{-rt} \mathrm{d}t - \delta I \right\}.$$

The first integral term represents the coupon payment from the firm. The second integral term captures the scrap value taken over by the lender after the firm defaults. Upon bankruptcy, following the similar formulation as Miao (2005) and Nishihara and Shibata (2021), the lender receives a proportion $1-\alpha$ of the firm value, i.e., the project is supposed to lose a proportion $\alpha \in (0,1)$ of its value and the scrap value is transferred to the lender. In what follows we refer to α as the bankruptcy cost parameter.

The equilibrium coupon schedule has to satisfy (2) for $X \in D, X \geq X_F^*(\rho^{imm}(X))$ because in a Markov Perfect Equilibrium the coupon rate $\rho^*(X)$ must be value maximizing for the lender in any subgame with $x(0) = X$ where the firm invests immediately. In light of the discussion above the strategy $\rho^*(X)$ is fully characterized by the choice of the threshold X_D and we can write the investment threshold of the firm as a function $X_F^*(X_D)$ Using this notation the threshold X_D^* can be found using the optimal stopping problem

$$\sup_{X_D : X_F^*(X_D) \leq X_D} \mathbb{E}_0 \left[e^{-r\tau_D(X_D)} \tilde{J}_D(X_D, \rho^{imm}(X_D), I^*(X_D, \rho^{imm}(X_D))) \right],$$

with $\tau_D(X) = \min[t \geq 0 : x(t) \geq X]$. It should be noted that we do not need to consider the case $X_F^*(X_D^*) < X_D^*$ since $\rho = \infty$ is never optimal for the firm.

As a result, the value of the lender is given by

$$V_D(x(0)) = \mathbb{E}_0 \left[e^{-r\tau_D(X_D^*)} \tilde{J}_D(X_D^*, \rho^{imm}(X_D^*), I^*(X_D^*, \rho^{imm}(X_D^*))) \right].$$

The timeline of our problem can be illustrated by the following figure.

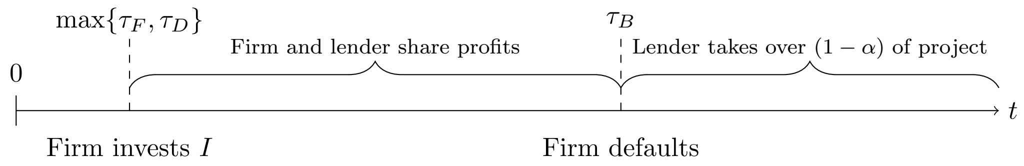


Figure 1: Illustration of the time line for the model.

3 Equilibrium Analysis and Economic Implications

In this section we characterize the optimal decisions of the firm and the lender in a Markov Perfect Equilibrium (MPE) of the game. In order to gain additional intuition for the key mechanisms at work, we first consider a simplified version of the model where the firm does not have the bankruptcy option, i.e. after investment the firm is committed to pay the coupon rate perpetually. From this we can analyze the equilibrium scale and timing in isolation, without the effect of bankruptcy playing a part.

3.1 No Bankruptcy Option

We proceed in several steps. First, we determine the firm's optimal choice of the investment scale for of given coupon rate $\tilde{\rho}$ under the assumption that $x(t)$ is in the stopping region. Based on this we derive the optimal investment threshold X_F^* of the firm. Different from a standard real option problem, different investment thresholds lead to different unit costs of investment because the coupon rate depends on the state $x(t)$ at the time of investment. The lender then takes into account the firm's optimal investment strategy and its dependence on the coupon scheme when determining the scheme $\rho^*(\cdot)$. Proceeding in this way results in the following proposition describing equilibrium behavior.

Proposition 1 *Assume that there is no bankruptcy option. Then, the lender's optimal strategy is given by*

$$\rho^*(X) = \begin{cases} \frac{r(X+\delta(r-\mu))}{2\delta(r-\mu)} & \text{for all } X \geq \frac{\beta_1+1}{\beta_1-1}\delta(r-\mu), \\ \infty & \text{otherwise.} \end{cases} \tag{3}$$

For $X < X_F^*$ the firm waits until the state process reaches X_F^* to install capacity $I^{opt} = I^*(X_F^*, \rho^*(X_F^*))$. Then the firm will pay a coupon rate $\rho^{opt} = \rho^*(X_F^*)$. The boundary of the stopping region for the firm, the associated investment size and the coupon rate are given by

$$\begin{aligned} X_F^* &= \frac{\beta_1+1}{\beta_1-1}\delta(r-\mu), \\ I^{opt} &= \frac{1}{2\eta(\beta_1+1)}, \\ \rho^{opt} &= r\frac{\beta_1}{\beta_1-1} > r \end{aligned} \tag{4}$$

with $\beta_1 > 1$ is the larger root of $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$. For $X \geq X_F^*$ the firm invests immediately and installs capacity $I^*(X, \rho^*(X))$ with a coupon rate $\rho^*(X)$. The optimal investment is then given by

$$I^*(X, \rho^*(X)) = \frac{1}{4\eta}\left(1 - \frac{\delta(r-\mu)}{X}\right).$$

In (4), as standard in real option models, the term $\frac{\beta_1}{\beta_1-1}$ can be interpreted as a mark-up of the price, sometimes referred to as 'wedge' (see, e.g., Dixit and Pindyck (1994)), where β_1 is fully related to the underlying state process.

The proposition provides several important insights about the effects of the interplay between an lender and a firm, where both have market power. To interpret these insights it is useful to compare the outcome of this strategic interaction with the scenario where the firm can finance investments internally and hence faces unit investment costs of r, where again r is the risk-free interest rate. This problem has been analyzed in Huisman and Kort (2015). Interestingly, the threshold X_F^* at which the firm invests is identical in both settings, however the size of the investment is only half in our framework with an endogenous coupon rate compared to that under internal financing. This reduction of the investment size has clear negative welfare implications since it was shown in Huisman and Kort (2015) that even under internal financing the socially optimal investment level is twice as high as that chosen by the firm. The reason why the firm is investing less under external than under internal financing is that the coupon rate requested by the lender is above the risk free rate. Hence our result can be interpreted as an instance of the phenomenon of double marginalization in the sense that the exploitation of market power on two subsequent stages of the value chain leads to distortions that are more pronounced than those resulting under an integrated monopoly. Although the double marginalization phenomenon occurs in numerous supply chain studies, to our knowledge so far double marginalization has not been identified as an important factor in the framework of optimal investment under external financing.

Compared to the case of internal financing, in which case only opportunity costs occur, the endogenous choice of the coupon rate in our model gives rise to two qualitative effects influencing the timing and size of the firm investment. First, the equilibrium coupon rate is larger than the risk free interest rate r and, second, by choosing the investment threshold X_F^* the firm can influence the size of the coupon rate, which is an increasing function of X (see (3)). Concerning the first of these effects it can easily be derived that the optimal investment threshold under a fixed coupon rate $\rho > r$ is increasing in ρ, whereas the optimal investment size is not affected. The second effect, driven by the market power of the lender, however gives the firm an incentive to accelerate the investment in order to keep the cost of investment low. Hence, contrary to standard double marginalization models, where the market power of the input supplier gives incentives for the final producer to reduce the quantity, here the market power of the credit supplier induces the firm to invest earlier and therefore to choose a smaller investment size. Overall, in our framework the two countervailing effects exactly cancel such that the timing of investment under external financing is identical to that under internal financing.

It follows from (3) that the coupon rate in the stopping region is increasing in X, i.e., a higher willingness-to-pay by consumers (a shift in the demand curve) allows the lender to extract more rents from the market by charging the firm a higher coupon rate. (Assuming the firm is willing to investment for a given coupon rate for a given X, an increase in X with the same coupon rate will not change this willingness. Since the firm's surplus increases, the lender is able to increase the coupon rate, i.e., the firm's marginal cost, in order to maximize its NPV.)

Considering the effect of market uncertainty on the coupon rate and the equilibrium investment pattern, we observe that for a given level of $x(t)$ at the time of investment the coupon rate does not depend on σ (see (3)). This is very intuitive since the income stream of the lender does not depend on the evolution of market demand once the firm has invested for the scenario without bankruptcy. Nevertheless, increased uncertainty induces a larger coupon rate in equilibrium. This is due to the fact that the coupon rate is an increasing function of the value of $x(t)$ at the time of investment, and a larger σ triggers a larger investment threshold, as is standard in real option models of this type, see Dixit and Pindyck (1994).

As shown in the proof of Proposition 1, the value of the lender is given by

$$V_D = \begin{cases} \frac{\rho^*(X)-r}{r}\delta I^*(X,\rho(X)) & \text{for all } X \geq X_F^*, \\ \left(\frac{X}{X_F^*}\right)^{\beta_1} \frac{\rho^*(X_F^*)-r}{r}\delta I^*(X_F^*,\rho(X_F^*)) & \text{for all } X < X_F^*, \end{cases}$$
$$= \begin{cases} \frac{X}{8\eta(r-\mu)}\left(1-\frac{\delta(r-\mu)}{X}\right)^2 & \text{for all } X \geq X_F^*, \\ \left(\frac{X}{X_F^*}\right)^{\beta_1} \frac{\delta}{2\eta(\beta_1^2-1)} & \text{for all } X < X_F^*. \end{cases}$$

Since both $\rho^*(X)$ and $I^*(X,\rho^*(X))$ are increasing in X, it is no surprise that V_D is increasing in X in the stopping region: since the lender and the firm are sharing profits from the downstream market, a higher willingness-to-pay by consumers upon investment results into a higher instantaneous cash-inflow for the lender. Without the bankruptcy option, the net result of X on V_D on the net present value is hence positive.

Welfare generated is given by

$$W = \begin{cases} \frac{X}{r-\mu}(I^*(X) - \frac{\eta}{2}(I^*(X))^2) - \delta I^*(X) & \text{for all } X \geq X^*, \\ \left(\frac{X}{X_F^*}\right)^{\beta_1} \left(\frac{X_F^*}{r-\mu}(I^*(X_F^*) - \frac{\eta}{2}(I^*(X_F^*))^2) - \delta I^{opt}\right) & \text{for all } X < X^*. \end{cases}$$
$$= \begin{cases} \frac{X}{r-\mu}\frac{7}{32\eta}\left(1 - \frac{\delta(r-\mu)}{X}\right)^2 & \text{for all } X \geq X^*, \\ \left(\frac{X}{X_F^*}\right)^{\beta_1} \frac{7\delta}{8\eta(\beta_1^2-1)} & \text{for all } X < X^*. \end{cases}$$

3.2 Bankruptcy Option

We now consider the full problem with a bankruptcy option for the firm, as described in Section 2. More precisely, we first treat the problem of the firm to choose a stopping region S, an investment schedule $I(\cdot)$ and a bankruptcy threshold X_B in order to maximize its expected payoff given in (1) for a given coupon scheme $\rho(\cdot)$. Then we determine the optimal coupon scheme to be offered by the lender.

Before solving the firm's investment problem, we consider the firm's exit option, which is only active once the firm has invested. After investment, the market demand evolves stochastically, and $x(t)$ reaches $X_B^*(I, \rho)$ for the first time at $\tau_B(X, I; \rho)$. The value of the firm at $X_B^*(I, \rho)$ is zero and it no longer pays coupons to the lender. The firm exercises the bankruptcy option at the threshold characterized as follows.

Lemma 1 *The default threshold for a given coupon rate $\tilde{\rho}$ and capacity size $\tilde{I}$ is equal to*

$$X_B^*(\tilde{I}, \tilde{\rho}) = \frac{\beta_2}{\beta_2 - 1}\frac{\tilde{\rho}\delta}{r}\frac{r-\mu}{1-\eta\tilde{I}}. \tag{5}$$

Here, $\beta_2 < 0$ is the smaller root of $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$.

The bankruptcy threshold in Lemma 1 implies that, for a given investment size $\tilde{I}$, $X_B^*(\tilde{I}, \tilde{\rho})$ increases with $\tilde{\rho}$. Because $x(t)$ reaches this trigger from above after investment, an increased $X_B^*(\tilde{I}, \tilde{\rho})$ is reached sooner. So a larger coupon rate makes it more likely for the firm to default up to a given point in time. Similarly, for a given coupon rate $\tilde{\rho}$, an increase in the capacity size also leads to a higher exit trigger. Intuitively, these results can be explained by noting that the future coupon payments increase in a linear way both with respect to $\tilde{\rho}$ and $\tilde{I}$, whereas market revenues are constant in $\tilde{\rho}$ and concave in $\tilde{I}$. Hence, a larger value of $\tilde{\rho}$ respectively $\tilde{I}$ implies that a larger value of $x(t)$ is needed to compensate the higher coupon payments.

3.2.1 Firm's investment decision

For a given $X \in S$ and $\tilde{\rho} = \rho(X)$, the firm's investment capacity follows from

$$\begin{aligned} &\sup_I \mathbb{E}_0 \int_0^{\tau_B(X,I;\tilde{\rho})} \exp\left(-rt\right)\Big(x(t)(1-\eta I)I - \tilde{\rho}\delta I\Big)\mathrm{d}t \\ &\quad = \sup_I \frac{I(1-\eta I)}{r-\mu}\left(X - \left(\frac{X}{X_B^*(I,\tilde{\rho})}\right)^{\beta_2} X_B^*(I,\tilde{\rho})\right) - \frac{\tilde{\rho}}{r}\delta I\left(1 - \left(\frac{X}{X_B^*(I,\tilde{\rho})}\right)^{\beta_2}\right), \end{aligned} \tag{6}$$

Taking the derivative of (6) with respect to I yields that $I^*(X, \tilde{\rho})$ satisfies

$$\left(\frac{-rX}{\beta_2(r-\mu)\tilde{\rho}\delta}\right)^{\beta_2} = \left(\frac{\tilde{\rho}\delta}{r} - \frac{X(1-2\eta I)}{r-\mu}\right)\frac{r}{\tilde{\rho}\delta}\frac{((1-\beta_2)(1-\eta I))^{1-\beta_2}}{1-(\beta_2+1)\eta I}. \tag{7}$$

Since $\frac{1}{2\eta}$ is the monopoly quantity on the market without taking into account any investment costs, the optimal investment level must satisfy $I \in \left[0, \frac{1}{2\eta}\right]$. In order to establish conditions under which equation (7) has a solution in this interval, we first observe that the left hand side of the equation is independent of I and positive. Furthermore, for $I = 0$ the right hand side is negative, and for $I = \frac{1}{2\eta}$ the right hand side is larger than or equal to the left hand side. This shows that a positive optimal investment level exists for sufficiently large values of X. The next step is to determine the stopping region. Relying on our analysis in the previous section we again assume that this region is of the form $S = [X_F^*, \infty)$ such that the firm invests immediately for $X \geq X_F^*$. Proposition 2 summarizes the firm's investment decision for a given coupon scheme $\rho(\cdot)$ with $[X_F^*, \infty) \subseteq D$.

Proposition 2 *Assume that $S = [X_F^*, \infty) \subseteq D$ and $\rho(\cdot)$ is differentiable on D. Then for $X < X_F^*$ the firm optimally delays investment till the threshold X_F^* is reached and then invests I^*, where $\{X_F^*, I^*\}$ satisfies*

$$\frac{\beta_1 I(1-\eta I)}{r-\mu}\left(X - \left(\frac{X}{X_B(I,\rho(X))}\right)^{\beta_2} X_B(I,\rho(X))\right) - \frac{\beta_1\rho(X)}{r}\delta I\left(1 - \left(\frac{X}{X_B(I,\rho(X))}\right)^{\beta_2}\right)$$
$$+\frac{\delta I}{r(\beta_2-1)}\left(\beta_2\rho(X) - X(\beta_2-1)\frac{\mathrm{d}\rho(X)}{\mathrm{d}X}\right)\left(\frac{rX(\beta_2-1)(1-\eta I)}{\beta_2\delta(r-\mu)\rho(X)}\right)^{\beta_2} - \frac{I(1-\eta I)X}{r-\mu} + \frac{\delta I X}{r}\frac{\mathrm{d}\rho(X)}{\mathrm{d}X} = 0 \tag{8}$$

and (7). *For $X \geq X_F^*$ the firm invests immediately with investment size determined by* (7).

3.2.2 Lender's coupon scheme

After the firm's investment, the lender starts receiving coupon payment until τ_B, and (reduced) profits afterwards. Since we consider MPE strategies, for any $X \in S \cap D$, where the firm invests immediately, the coupon rate $\tilde{\rho} = \rho^*(X)$ has to maximize the lender's expected payoff and therefore solves the optimization problem (2), where $I^*(X, \rho)$ satisfies the equation (7). Based on this we can provide the following characterization of the optimal coupon scheme.

Proposition 3 *For any $X \in S \cap D$ the coupon rate $\tilde{\rho} = \rho^*(X)$ under the equilibrium coupon scheme satisfies*

$$\begin{aligned}(1-\alpha\beta_2)\left(\frac{rX\big(1-\eta I^*(X,\tilde{\rho})\big)(\beta_2-1)}{\beta_2(r-\mu)\tilde{\rho}\delta}\right)^{\beta_2}&\left(I^*(X,\tilde{\rho}) + \frac{\tilde{\rho}\big(1-\eta I^*(X,\tilde{\rho})(1+\beta_2)\big)}{(1-\beta_2)\big(1-\eta I^*(X,\tilde{\rho})\big)}\frac{\partial I^*}{\partial \rho}\right)\\ &-I^*(X,\tilde{\rho}) - (\tilde{\rho}-r)\frac{\partial I^*}{\partial \rho} = 0.\end{aligned} \tag{9}$$

Although intuition might suggest that also in the case with bankruptcy the equilibrium coupon scheme as well as the lender's value function V_D are increasing with respect to X on $S \cap D$ this is less clear cut if the firm has the option to default. The reason is that an increase in X induces an increase of the firm's investment size (for a given value of the coupon rate). This has several implications for V_D. First, it increases the coupon payments the lender receives till τ_B. Second, it increases the bankruptcy threshold, reducing the expected time till bankruptcy, which has a negative implication for the lender. Third, the size of the lender's loss in case the firm defaults increases with the size of investment. The interplay of these effects makes it difficult to establish monotonicity of the lender's value function and of the coupon scheme. This ambiguity is reflected in the degree of complexity of expression (9), which prevents an analytical proof of the monotonicity of $\rho^*(\cdot)$. In light of this, it is also not possible to establish analytically that the stopping region of the lender or of the firm has the usual threshold structure as in the previous section. However, our numerical analysis below indicates that also in the presence of the bankruptcy option the optimal investment strategy and coupon scheme are characterized by (unique) thresholds. In such a scenario the optimal investment threshold arising in equilibrium can be described as follows.

Proposition 4 *Assume that $D = [X_D^*, \infty)$. Then there is an MPE such that $X_F^* = X_D^* = \max\{\tilde{X}_F, \tilde{X}_D\}$, where $\tilde{X}_F$ solves* (8) *with $\rho = \rho^*$ as given in* (9) *and $\tilde{X}_D$ solves*

$$(1-\alpha\beta_2)\rho^*(X)\left(\frac{rX(\beta_2-1)\,(1-\eta I^*(X,\rho^*(X)))}{\beta_2\delta(r-\mu)\rho^*(X)}\right)^{\beta_2}\left(\frac{\big(1-(\beta_2+1)\eta I^*(X,\rho^*(X))\big)X}{(\beta_2-1)\big(1-\eta I^*(X,\rho^*(X))\big)}\frac{\partial I^*}{\partial X}\right.$$
$$\left.+\frac{\beta_2-\beta_1}{\beta_2-1}I^*(X,\rho^*(X))\right)-(\rho^*(X)-r)\left(\beta_1 I^*(X,\rho^*(X))-X\frac{\partial I^*}{\partial X}\right)=0, \tag{10}$$

with $\rho^*(X)$ as given by (9).

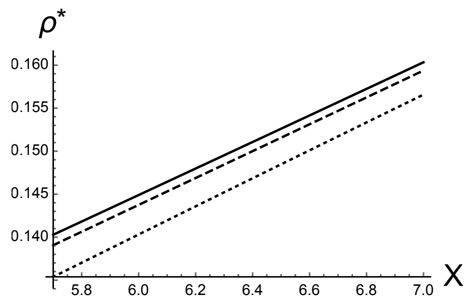
3.3 Numerical Analysis

The characterization of MPE provided in Propositions 2 - 4 unfortunately does not allow for a closed form representation of the firm's equilibrium investment strategy and the lender's coupon scheme. Therefore, in this section we resort to numerical analysis to gain insights about the effect of key parameters on investment. In particular, we will analyze the outcomes of the model with bankruptcy option, labeled as BO, and contrast them with the model without bankruptcy option, labeled as NBO.

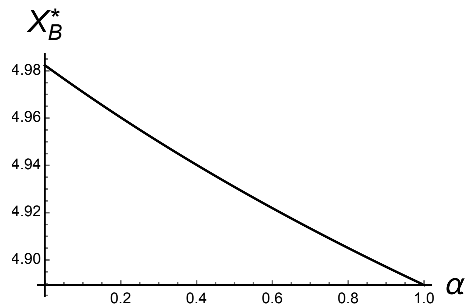
3.3.1 Effect of bankruptcy cost α

Let us first focus on the direct effect of α. This parameter determines the fraction of the value of firm that is lost when upon bankruptcy the lender takes over the firm. Hence, α determines the loss of project value for the lender in case the firm defaults. We start our analysis by considering the implications of a change in α for the optimal choice of the coupon rate in the stopping region. In Figure 2(a) we show $\rho^*(X)$ for an interval of X-values, $X \in \mathcal{S}$, in the stopping region and for different values of α as well as for the case without bankruptcy option (model NBO). The figure confirms that also in the model with bankruptcy option the optimal coupon scheme is an increasing function of X. In addition, it demonstrates, that for a given value of X the coupon rate under $\alpha = 0$ is higher than that in the NBO case. Furthermore the coupon goes down if α is increased and for high bankruptcy costs ($\alpha = 1$) the coupon rate in the BO model is lower than that under NBO.[4] To explain these observations *first* note that the existence of a bankruptcy option induces the firm to invest more, since it can avoid the losses in case of a negative development of demand (see also Figure 3b below). However, the lender now covers this risk and hence wants the firm to invest less compared to the NBO case. This generates an incentive for the lender to offer a higher coupon rate, thereby reducing the firm's investment size. *Second* in the BO scenario the choice of the coupon rate also affects the bankruptcy trigger for the firm, directly and indirectly through the firm's optimal investment size. As is shown in Figure 2(b) a larger coupon rate actually implies a larger bankruptcy trigger, meaning that the firm will default sooner. Due to this effect there emerges an incentive for the lender to lower the coupon rate and this incentive is larger the larger is the bankruptcy loss parameter α. This explains why for a sufficiently large value of α the lender's optimal coupon rate is not only lower than that for $\alpha = 0$ but also lower than the rate in the NBO scenario.

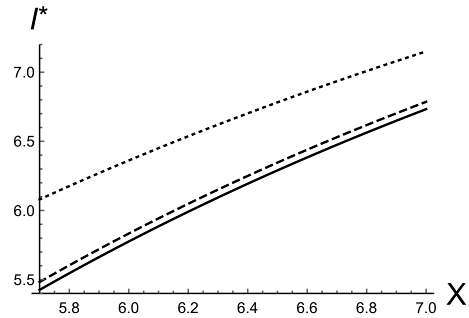
[4] Depicting $\rho^*(X)$ for a given value of X in the stopping region and continuous variation of $\alpha \in [0, 1]$ shows a decreasing shape with respect to α in the entire interval.



(a) Optimal coupon scheme for $\alpha = 0$ (solid), $\alpha = 1$ (dotted) and for the NBO scenario (dashed)

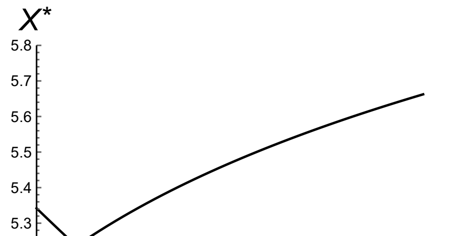


(b) Bankruptcy trigger $X_B^*(I^*(X), \rho^*(X))$ for $X = 6$.

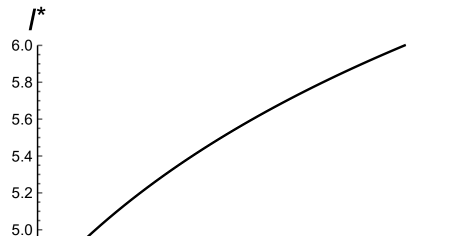


(c) Optimal investment size for $\alpha = 0$ (solid), $\alpha = 1$ (dotted) and for the NBO scenario (dashed)

Figure 2: Effect of α.
$\mu = 0.02,\ r = 0.1,\ \sigma = 0.05,\ \delta = 40$, and $\eta = 0.02$.



(a) Equilibrium investment threshold $X^* = \max\{\tilde{X}_D, \tilde{X}_F\}$ for the scenarios with (solid line) and without (dashed line) bankruptcy option.

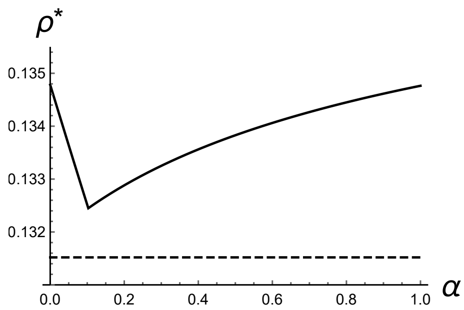


(b) Optimal scale of investment $I^*(X^*; \rho^*(X^*))$ at the investment threshold for the scenarios with (solid line) and without (dashed line) bankruptcy option.

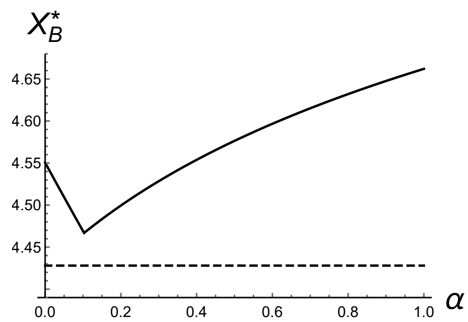
Figure 3: Effect of α.
$\mu = 0.02,\ r = 0.1,\ \sigma = 0.05,\ \delta = 40$, and $\eta = 0.02$.

Next we investigate how the investment threshold $X^* := X_D^* = X_F^*$ depends on the bankruptcy cost α. As shown in the previous section this threshold is given by $\max[\tilde{X}_D, \tilde{X}_F]$, where $\tilde{X}_D$ is the lender's threshold under the assumption that the firm invests immediately and $\tilde{X}_F$ is the firm's threshold under the assumption that the lender offers credit at any value of X. Concerning $\tilde{X}_D$, since an increase in bankruptcy cost lowers the lender's net present value of the project for each X "financing" is delayed, i.e. the threshold is increasing in α. We find the opposite for the firm: threshold $\tilde{X}_F$ is decreasing in α. The bankruptcy cost parameter α only has an indirect effect on the firm's investment problem in the sense that the coupon scheme $\rho^*(\cdot)$ is shifted downwards if α goes up. Lower investment costs imply earlier investment for the firm and therefore $\tilde{X}_F$ decreases with α.

The interplay between the opposite monotonicities of $\tilde{X}_D$ and $\tilde{X}_F$ imply that the equilibrium investment threshold X^* has a V-shape, as illustrated in Figure 3a. For low values of α we have $X^* = \tilde{X}_F$ since the low bankruptcy cost makes the project more attractive for the lender and therefore the investment timing depends on the willingness of the firm to carry out the investment. On the contrary, for large α the willingness of the lender to provide the credit is the bottleneck and we have $X^* = \tilde{X}_D$. As can be seen in Figure 3b the dependence of the size of equilibrium investment for $x(0) \leq X^*$ from α closely follows the shape of the investment threshold. In particular, also this relationship is characterized by a V-shape. This is driven by the standard reasoning that the marginal return from investment is higher the larger is $x(t)$ at the time of investment. Similarly, the size of the coupon rate realized in equilibrium for $x(0) \leq X^*$ is mainly driven by the positive dependence of the optimal coupon rate from the level of $x(t)$ at the time of investment (see Figure 4a). The fact that both the coupon rate and the investment size have a V-shaped dependence on α furthermore implies that also the dependence of the bankruptcy threshold X_B^* on α has this structure (see Figure 4b). The reason is that there is a positive relationship between the value of ρI and the bankruptcy trigger. In face of a commitment to a higher stream of coupon payment $\rho\delta I$ the firm has higher incentives to declare bankruptcy and therefore chooses a higher bankruptcy trigger.



(a) Coupon rate $\rho^*(X^*)$ in equilibrium for the scenarios with (solid line) and without (dashed line) bankruptcy option.



(b) Bankruptcy trigger $X_B^*(I^*(X^*), \rho^*(X^*))$ in equilibrium.

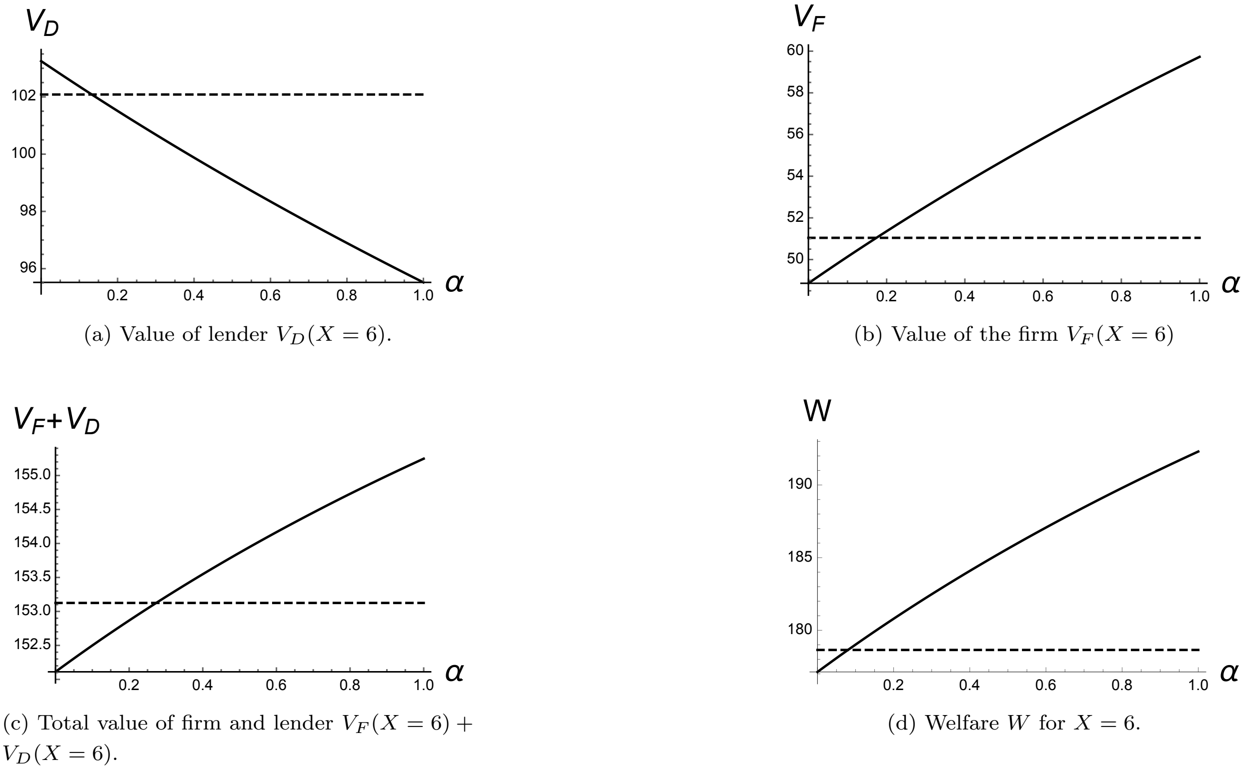
Figure 4: Effect of α.
$\mu = 0.02,\ r = 0.1,\ \sigma = 0.05,\ \delta = 40$, and $\eta = 0.02$.

Analysis welfare (TO BE ADDED AFTER FIGURES ARE COMPLETE)

Welfare generated is given by

$$W = \begin{cases} \frac{X}{r-\mu}(I^*(X) - \frac{\eta}{2}(I^*(X))^2) - \delta I^*(X) & \text{for all } X \geq X^*, \\ \left(\frac{X}{X^*}\right)^{\beta_1} \left(\frac{X^*}{r-\mu}(I^*(X^*) - \frac{\eta}{2}(I^*(X^*))^2) - \delta I^*(X^*)\right) & \text{for all } X < X^*. \end{cases}$$

Next we will study the effect of an increase in bankruptcy cost α for the value of both parties, the lender and the firm, as well as the sum of the values for both. Figure 5 shows how these value change with α



(a) Value of lender $V_D(X = 6)$.

(b) Value of the firm $V_F(X = 6)$

(c) Total value of firm and lender $V_F(X = 6) + V_D(X = 6)$.

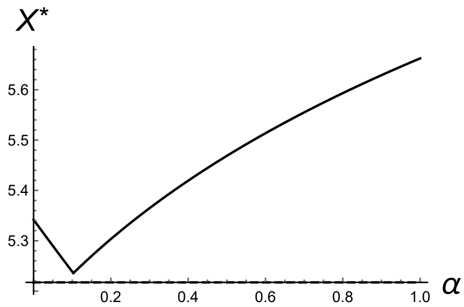
(d) Welfare W for $X = 6$.

Figure 5: Effect of α for the scenarios with (solid line) and without (dashed line) bankruptcy option and X in the stopping region.

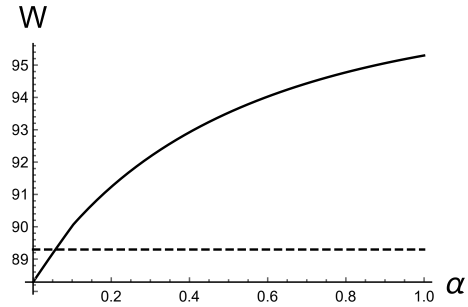
$$\mu = 0.02,\ r = 0.1,\ \sigma = 0.05,\ \delta = 40, \text{ and } \eta = 0.02$$

in a scenario with $X \geq X^*$ where the firm invests immediately. Not surprisinlgy, the value of the lender decreases with α (see panel (a)), whereas the value of the firm increases with α (panel (b)). The first of these observations is directly driven by the increased bankruptcy costs the lender faces, and the second effect is due to the reduction in the coupon rate, which is induced by a larger α. Interestingly, the indirect effect on the value of the firm dominates, such that the sum of the value of the firm and lenders increases as bankruptcy costs become larger (panel (c)). Hence, an increase in costs leads to an increase in the total expected value of the investment option for both agents. Intuitively, since the larger costs associated with a bankruptcy of the firm induces the lender to choose a lower coupon rate the inefficiency associated with a too large coupon rate (due to the double marginalisation problem discussed above) is reduced. The bankruptcy threat therefore diminishes the negative implications of sequential market power on different stages of the vertical chain. Comparing the total value with and without the bankruptcy option (i.e. comparing the solid and dashed lines in Figure 5(c) shows that for large bankruptcy costs the existence of the bankruptcy option indeed increases the total value. In this respect it should be noted that the direct effect of bankruptcy on the total generated value is always negative, since even for values of X below the bankruptcy threshold X_B^* the project, after investments have been sunk, generates a non-negative payoff stream. The positive effect of the bankruptcy option on the total value for firm and lender is therefore entirely driven by the effect of the option on the coupon rate and the investment size. Since it is not clear how to evaluate consumer surplus after the firm has declared bankruptcy[5] we abstain from incorporating consumer surplus into our

[5] In particular, if the lender sells the invested capital upon firm bankruptcy, thereby facing a loss of a fraction α of the



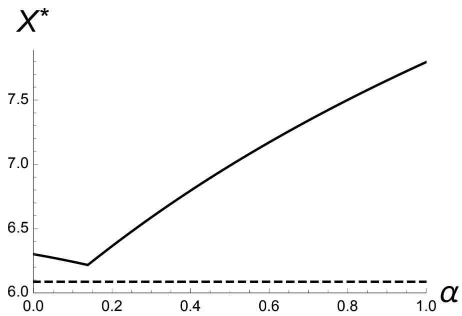
(a) Optimal investment threshold



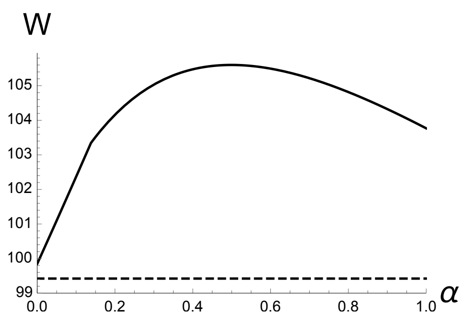
(b) Social welfare $W(X = 5, \alpha)$

Figure 6: Effect of α on social welfare for $\sigma = 0.05$ in scenarios with (solid line) and without (dashed line) bankruptcy option.

$\mu = 0.02,\ r = 0.1,\ \sigma = 0.05,\ \delta = 40$, and $\eta = 0.02$.



(a) Optimal investment threshold

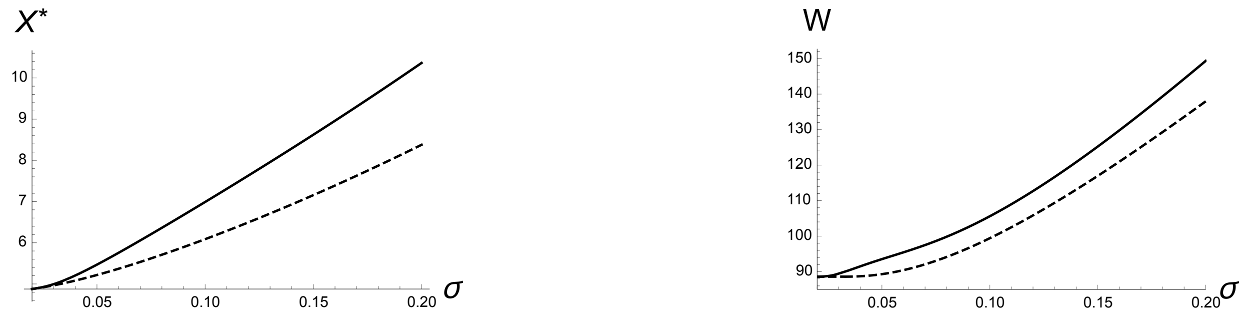


(b) Social welfare $W(X = 5, \alpha)$

Figure 7: Effect of α on social welfare for $\sigma = 0.1$ in scenarios with (solid line) and without (dashed line) bankruptcy option.

$\mu = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \delta = 40$, and $\eta = 0.02$.

consideration and therefore cannot provide a full welfare analysis. However, it is quite obvious that, at least if the invested capital is still used to sell the product on the market after firm bankruptcy, under large values of α the reduction in the coupon scheme and the increase in investment imply that the existence of the bankruptcy option would also increase consumer surplus. The fact that for small values of the bankruptcy cost parameter α the value of the lender is larger than without the bankruptcy option whereas that for the firm is smaller, is again driven by the effect of α on the coupon scheme. With the bankruptcy option the firm has stronger incentives to invest and the lenders exploits this by setting a higher coupon rate without facing substantial direct costs in the case of firm bankruptcy.



(a) Optimal investment threshold

(b) Social welfare $W(X, \alpha)$ for $X = 5$.

Figure 8: Effect of σ on social welfare for $\alpha = 0.5$ in scenarios with (solid line) and without (dashed line) bankruptcy option.

$\mu = 0.02,\ r = 0.1,\ \alpha = 0.5,\ \delta = 40$, and $\eta = 0.02$.

4 Robustness/Extensions/Sensitivity

4.1 Robustness

Figure 3 shows that the optimal investment decisions X^* and I^* with bankruptcy option are larger than that without bankruptcy option (NBO). In Figure 4 it is shown that the optimal coupon rate ρ^* is also larger compared with that in NBO. The following table shows results of the robustness check for the illustration in these two figures.

Parameter	Description	Baseline	Tested Interval	Robustness
σ	Volatility parameter	0.05	$[0.01, 0.3]$	✓
r	Discount rate	0.1	$[0.021, 0.2]$	✓
μ	Trend parameter	0.02	$[-0.01, 0.09]$	✓
η	Elasticity parameter	0.02	$[0.0005, 0.3]$	✓
δ	Unit investment cost	40	$[0.02, 80]$	✓

Table 1: Range of parameter values for which the firm's investment decisions X^* and I^*, and the lender's coupon rate ρ^* are larger than those in the scenario without bankruptcy. A checkmark indicates this result is robust.

current firm value, it is hard to determine consumer surplus generated by that capital after it is sold.

5 Concluding remarks

To be completed...

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Appendix

Proof of Proposition 1 As explained in the text we first determine the firm's optimal investment size, followed by the derivation of standard value matching and smooth pasting conditions providing necessary optimality conditions for the investment threshold for the firm. As the next step we determine the coupon rate for any $X \in D \cap S$. Using this coupon scheme we then first assume that S is included in the interior of D and show that there is a unique threshold satisfying the necessary optimality conditions for the firm. Hence, $S = [X_F, \infty)$, where we will be able to provide X_F in closed form. Finally, we will consider the problem of the lender. Clearly there is an optimal coupon scheme with $D \subseteq [X_F, \infty)$ and we will concentrate on optimal coupon schemes of this form.[6] We show that when taking this into account there exists a unique threshold X_D^* such that $D = [X_D^*, \infty)$ solves the lender's optimization problem. and that $X_D^* = X_F$. From this it follows that $X_F^* = X_F = X_D^*$.

In order to determine the optimal investment size we start out by calculating the firm's net present value in the stopping region. It is given by

$$J_F(X, \rho(X), 0, I) = \mathbb{E}_0 \int_0^\infty e^{-rt}\pi(t, I; \rho)\mathrm{d}t = \frac{X}{r-\mu} I(1-\eta I) - \frac{\rho(X)}{r}\delta I.$$

To find the optimal scale of investment, the first order condition gives

$$I^*(X, \rho(X)) = \frac{1}{2\eta}\left(1 - \frac{\delta(r-\mu)}{X}\frac{\rho(X)}{r}\right). \tag{11}$$

The second order condition confirms that (11) yields a (global) maximum. We will show later that values of X such that $I^* < 0$ are not considered, so that (11) gives a solution to the optimization problem. Inserting the optimal investment gives the value function for $X \in S$:

$$W_F(X) = \tag{12}$$

Consider now the firm's optimal stopping problem. In particular we first treat the auxiliary problem where the lender offers a differentiable coupon scheme with finite values for all $X \in (0, \infty)$. Denote by $\mathcal{L}$ the infinitesimal generator, i.e.

$$\mathcal{L} = \mu X \frac{\partial}{\partial X} + \tfrac{1}{2}\sigma^2 X^2 \frac{\partial}{\partial X}.$$

Let C denote the continuation region, and let ∂C denote a (potential) boundary. As standard for these problems (see, e.g., Peskir and Shiryaev (2006)), the firm's value function is given by some function ϕ that solves a free boundary problem, so that then $V_F = \phi$. That is, in the stopping region it holds that $\phi = J_F$ (i.e., $S = \mathbb{R}_+ \backslash C$) and in the continuation region ϕ solves $\mathcal{L}\phi = r\phi$ with conditions $\frac{\partial}{\partial X}\phi(\tilde{X}) = \frac{\partial}{\partial X} W_F(\tilde{X})$ ("*smooth pasting*"), and $\phi(\tilde{X}) = W_F(\tilde{X})$ for all $\tilde{X} \in \partial C$ ("*value matching*").

The solution to $\mathcal{L}\phi = r\phi$ (see, e.g., Dixit and Pindyck (1994)) is given by $\phi(X) = AX^{\beta_1}$ where A follows from the free boundary conditions and where β_1 is the positive root of the quadratic polynomial of

$$\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0.$$

Inserting $\phi(X) = AX^{\beta_1}$ into the value matching and smooth pasting conditions gives after some transformation the following equation to be satisfied for any X at the boundary ∂C,

$$\beta_1\left(\frac{X}{r-\mu} I(1-\eta I) - \frac{\rho(X)}{r}\delta I\right) = \frac{X}{r-\mu} I(1-\eta I) - \frac{X}{r}\delta I \frac{\partial}{\partial X}\rho(X). \tag{13}$$

[6] If the lender has a coupon scheme with $[X_F, \infty) \subset D$ then, since the firm never invests at $x(t) \in D \backslash [X_F, \infty)$, an alternative scheme in which $\rho(X)$ is unchanged for $X \in [X_F, \infty)$ and $\rho(X) = \infty$ for $X \notin [X_F, \infty)$ gives the same expected payoff for the lender.

Then turning to the lender's problem, let us first determine $\rho^{imm}(X)$ for all $X \in D$. The lenders net present value, for all $X \in D \cup S$, is given by

$$\begin{aligned} J_D(X,\rho,0,I^*(X,\rho)) &= \mathbb{E}_0 \int_0^\infty e^{-rt}\rho\delta I^*(X,\rho)\mathrm{d}t - \delta I^*(X,\rho) \\ &= \frac{\rho-r}{r}\delta I^*(X,\rho) \\ &= \frac{\rho-r}{2\eta r}\delta\left(1-\frac{\delta(r-\mu)}{X}\frac{\rho}{r}\right). \end{aligned} \tag{14}$$

Taking the derivative with respect to ρ yields

$$\frac{\partial}{\partial\rho}J_D = \frac{1}{2\eta r}\delta\left(1-\frac{\delta(r-\mu)}{X}\frac{\rho}{r}\right) - \frac{\rho-r}{2\eta r}\delta\frac{\delta(r-\mu)}{X}\frac{1}{r}.$$

From $\frac{\partial}{\partial\rho}J_D = 0$ we obtain

$$\rho^{imm}(X) = \frac{r(X+\delta(r-\mu))}{2\delta(r-\mu)}. \tag{15}$$

Hence,

$$I^*(X,\rho^{imm}(X)) = \frac{X-\delta(r-\mu)}{4\eta X}. \tag{16}$$

Solving (13) and (11) simultaneously, using (15), gives the unique solution

$$\begin{aligned} X_F &= \frac{\beta_1+1}{\beta_1-1}\delta(r-\mu), \\ I^*(X_F,\rho^{imm}(X_F)) &= \frac{1}{2\eta(\beta_1+1)}. \end{aligned}$$

Hence, under the coupon scheme, where $\rho(X) = \rho^{imm}(X)$ for all $X > 0$ the stopping region under the firm's optimal investment strategy is given by $[X_F, \infty)$.

Now we consider how the region D should be optimally determined by the lender. As shown in the beginning of the proof we can restrict attention to coupon schemes with $D \subseteq [X_F,\infty)$. For any such scheme we have that the firm immediately invests for any $X \in D$. Hence, the value function for the lender for $X \in D$ is given by inserting (15) into (14). Value matching and smooth pasting conditions using this value function imply that for any X on the boundary of D we must have

$$\beta_1\left(\frac{X}{\delta(r-\mu)}-1\right)^2 = \left(\frac{X}{\delta(r-\mu)}-1\right)\left(\frac{X}{\delta(r-\mu)}+1\right),$$

which has two solutions: $\tilde{X} = \delta(r-\mu)$ and

$$X_D^* = \frac{\beta_1+1}{\beta_1-1}\delta(r-\mu).$$

Under the first of these solutions $I^*(\tilde{X},\rho^{imm}(\tilde{X})) = 0$, which implies that the only candidate for the boundary of D is X_D^*. Since $X_D^* = X_F$ under this solution we indeed have that $D \subseteq [X_F,\infty)$. This establishes that $X_F^* = X_F = X_D^*$. At the threshold, it then holds that

$$\rho^{imm}(X_F^*) = r\frac{\beta_1}{\beta_1-1}.$$

Finally, notice that $I^*(X_F^*,\rho(X_F^*)) > 0$ so that $I^* > 0$ for all $X \in \mathbb{R}_+ \backslash C$. Furthermore,

$$J_D(X_D^*,\rho^*(X_D^*),I^*(X_D^*,\rho^*(X_D^*))) = \frac{\delta}{2\eta(\beta_1^2-1)} > 0,$$

which shows that it is optimal for the lender to provide a coupon scheme with a non-empty set D. □

Proof of Lemma 1: Because the default option is conditional on the firm being active in the market, we reset the present time to a point t' after investment ($t' \geq T$) and denote the corresponding geometric Brownian motion as $x(t') = x$. Then it holds that the firm's value equals

$$V_B(x; I, \tilde{\rho}) = \begin{cases} \frac{x(1-\eta I)I}{r-\mu} - \frac{\tilde{\rho}\delta I}{r} + A_B x^{\beta_2} & \text{for } \ x > X_B^*, \\ 0 & \text{for } \ x \leq X_B^*. \end{cases}$$

The value matching and smooth pasting conditions at the default threshold yield that

$$X_B(I, \rho) = \frac{\beta_2}{\beta_2 - 1} \frac{\rho\delta(r-\mu)}{r(1-\eta I)}.$$

□

Proof of Proposition 2: The firm's investment problem for the given coupon scheme $\rho(\cdot)$ and optimal investment size I can be written as

$$V_F(X) = \begin{cases} W_F(X; \rho), & \text{if } X \geq X_F^* \\ AX^{\beta_1} & \text{if } X < X_F^*. \end{cases}$$

where $W_F(X;\rho) = \frac{I(1-\eta I)}{r-\mu}\left(X - \left(\frac{X}{X_B(I,\rho(X))}\right)^{\beta_2} X_B(I, \rho(X))\right) - \frac{\rho(X)}{r}\delta I \left(1 - \left(\frac{X}{X_B(I,\rho(X))}\right)^{\beta_2}\right)$, X_F^* is the investment threshold and I satisfies (7). The firm's investment threshold X_F^* according to the value matching and smooth pasting condition at X_F^* satisfies that

$$\begin{aligned} \beta_1 V_F(X_F^*) &= X_F^* \left(\frac{\partial W_F(X_F^*)}{\partial \rho}\frac{\mathrm{d}\rho(X_F^*)}{\mathrm{d}X} + \frac{\partial W_F(X_F^*)}{\partial X}\right) \\ &= -\frac{\delta I}{r(\beta_2-1)}\left(\beta_2\rho(X_F^*) - X_F^*(\beta_2-1)\frac{\mathrm{d}\rho(X_F^*)}{\mathrm{d}X}\right)\left(\frac{rX_F^*(\beta_2-1)(1-\eta I)}{\beta_2\delta(r-\mu)\rho(X_F^*)}\right)^{\beta_2} \\ &\quad + \frac{I(1-\eta I)X_F^*}{r-\mu} - \frac{\delta I X_F^*}{r}\frac{\mathrm{d}\rho(X_F^*)}{\mathrm{d}X}, \end{aligned}$$

Rearranging the terms yields expression (8). □

Proof of Proposition 3: From the moment of the firm's investment, the coupon rate is fixed, and the lender's value as a function of the coupon rate $\tilde{\rho}$ is given by

$$J_D(X, \tilde{\rho}, 0, I^*(X; \tilde{\rho})) = \frac{\rho - r}{r}\delta I^*(X, \rho) - \frac{\rho\delta I^*(X,\rho)}{r}\frac{(1-\alpha\beta_2)}{1-\beta_2}\left(\frac{X}{X_B(I^*(X,\rho),\rho)}\right)^{\beta_2}$$

where $I^*(X, \tilde{\rho})$ satisfies equation (7). Taking the first order condition yields equation (9). □

Proof of Proposition 4: Recall that $D = [X_D^*, \infty)$ and we restrict our attention to $D \subseteq [X_F^*, \infty)$, then the firm invests immediately for $X \in D$ and the corresponding coupon scheme $\rho^*(X)$ is as specified by (9).

The lender's value reads

$$V_D(X) = \begin{cases} A_D X^{\beta_1} & \text{if } X < X_D^*, \\ W_D(X) & \text{if } X \geq X_D^*. \end{cases}$$

with $W_D(X) = \frac{\delta I^*(X,\ \rho^*(X))}{r} \left(-\frac{\rho^*(X)(\alpha\beta_2 - 1)}{\beta_2 - 1} \left(\frac{X}{X_B(I^*(X,\ \rho^*(X)),\ \rho^*(X))} \right)^{\beta_2} + \rho^*(X) - r \right)$ and $I^*(X;\rho^*(X))$ satisfies (7). According to the value matching and smooth pasting consitions at X_D^* it holds that

$$\begin{aligned} \beta_1 W_D(X_D^*) &= X_D^* \left(\frac{\partial W_D(X_D^*)}{\partial \rho} \frac{\mathrm{d}\rho^*(X_D^*)}{\mathrm{d}X} + \frac{\partial W_D(X_D^*)}{\partial I} \frac{\partial I^*(X_D^*;\rho^*(X_D^*))}{\partial \rho} \frac{\mathrm{d}\rho^*(X_D^*)}{\mathrm{d}X} \right. \\ &\quad \left. + \frac{\partial W_D(X_D^*)}{\partial I} \frac{\partial I(X_D^*;\rho^*(X_D^*))}{\partial X} + \frac{\partial W_D(X_D^*)}{\partial X} \right) \\ &= X_D^* \frac{\partial W_D(X_D^*)}{\partial I} \frac{\partial I(X_D^*;\rho^*(X_D^*))}{\partial X} + X_D^* \frac{\partial W_D(X_D^*)}{\partial X}. \end{aligned}$$

Rearranging the terms yields (10). □