Investment timing, capacity choice and social welfare under price floors and ceilings regimes*

Dean Paxson[a], Paulo J. Pereira[b], and Artur Rodrigues[c]

[a] *Alliance Manchester Business School, University of Manchester, UK.*
[b] *CEF.UP and Faculdade de Economia, Universidade do Porto, Portugal.*
[c] *NIPE and School of Economics and Management, University of Minho, Portugal.*

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Abstract

We develop a model for determining the optimal timing and capacity choice of investment under price floors and ceilings (collars). We study how a welfare maximizing finite-lived collar can be optimally designed. Our findings show that for a linear demand function, multiple collar arrangements are optimal, including pure floors and fixed price regimes. Departing from previous literature, we show that pure ceiling regimes are never optimal. When the demand function is iso-elastic, optimal schemes are not attainable.

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1 Introduction

In addition to subsidies and tax reductions, government policies such as collar schemes, combining price floors and ceilings, may be useful tools in controlling the size and timing of infrastructure and other social investments (De Miera et al., 2008; Brandao and Saraiva, 2008; Couture and Gagnon, 2010; Shaoul et al., 2012). A collar scheme allows the government to modify the cash flow structure of the project (offering the firm a downside protection and limiting the upside potential) without spending any initial funds for supporting its implementation (Adkins et al., 2019). When properly arranged, collars can be a viable instrument for fostering large investments in infrastructures (e.g.: energy, transports, and toll roads).

The study of the effects of regimes with price controls following a real options approach goes back to Dixit (1991), where the impact of price-ceilings on the dynamics of irreversible investments is analyzed. The study is extended in Dixit and Pindyck (1994) for regimes with both price floors and ceilings (collar regimes). Notably, the effects of price controls are mostly dynamic and include option-like characteristics, influencing the way firms and agents take their decisions, and, therefore, real options theory allows a richer understanding of price controls on investment dynamics (Dixit and Pindyck, 1994). Recent contributions for the valuation of collars arrangements following a real options approach appear in Adkins and Paxson (2017, 2019), for the case of perpetual collars, and Adkins et al. (2019), where the solution for the valuation of finite-lived collars is presented.

In this previous literature, the main focus is the impact of price control schemes in value and timing of irreversible investments, provided the scale of investment is fixed and exogenously determined. In other words, the impact of price floors and ceilings on the investment size is ignored. The are several models where investment size and timing are jointly determined. Dangl (1999) considers choosing both the timing and upper boundary for capacity with uncertain demand. Huisman and Kort (2015) extend Dangl's approach using an inverse demand function for a duopoly in a real options setting. Huberts et al. (2015) summarize some capacity model developments, allowing for production suspension, and also for bounded capacity. Hagspiel et al. (2016) consider holding costs per unit of capacity, and also a linear demand structure. Chronopoulos et al. (2017) finds step-wise investment leads to greater optimal capacity. De Giovanni and Massabò (2018) focus on volume flexibility, but follow the Dangl's approach with utilization considerations.

Few models in the literature address price control schemes and investment size. The effect of price cap regulation imposed to a monopolist is studied by Dobbs (2004). The author finds that, even when the price cap is optimally determined, the firm tends to under-invest and to impose quantity restrictions to consumers. Evans and Guthrie (2012) find that regulated firms by price caps tend to invest more frequently, but in smaller increments, than a social planner. The effects of price ceilings in the context of competitive markets appear in Roques and Savva (2009) and Lemus and Moreno (2017), whereas Sarkar (2015) studies its impact on consumers surplus. None of these articles consider the case of collar schemes, that combine price caps with price floors.

Our article integrates past literature, namely, by studying the joint effect of price controls in the scale and timing of irreversible investments, but innovates in three main aspects. First, we do not pre-assign a particular scheme to be used by a regulator or a government granting a concession. In our model, the price regime is kept open (generically designated as a collar) allowing us to find endogenously the optimal regime. Our results show that the optimal scheme, and depending on the parameters, can either be a pure price floor regime (an infinite price ceiling), or a pure fixed price (the price ceiling equals the price floor), or, in-between, a collar scheme (a price floor and a finite price ceiling). Secondly, the scheme is assumed to last for a finite period of time, not perpetually. This is an important and realistic assumption with a relevant impact on the investment decision. Finally, by considering simultaneously price floors and ceilings, we show how price floors are important instruments of risk-sharing and investment promotion. Departing from previous literature, we show that pure price ceilings regimes are never optimal, suggesting that price floors also play a central role in promoting social welfare.

We study how the elements that characterize the collar (i.e. the price floor, the price ceiling, as well as its duration), impact the decision of the firm with respect to the investment timing and the corresponding capacity choice. For a linear demand function, the collar impacts only the investment timing. Our findings show that for a finite-lived collar, the duration and the price ceiling have a non-monotonic effect on investment timing, and there is an optimal duration or price ceiling that induce the earliest investment, whereas a higher price floor always hastens investment.

We also study how to design optimal finite-lived collars, such that the social welfare is maximized. Our findings show that it is possible to maximize the total surplus by optimally defining the collar components. There are multiple optimal collar schemes, with different floors and ceilings, including pure floor and fixed price regimes, all producing the same wealth increment. The duration of the scheme has no impact on the incremental surplus attained, and it only determines the range of the optimal price floors and ceilings: for longer durations, collars, pure price floors and fixed price regimes may all be optimal, whereas for short durations only pure price floors and fixed price regimes become optimal. The choice of the parameters by the regulator or government affects only the cost/benefit of the intervention or the cost/benefit for the firm.

We extend the analysis to the iso-elastic demand function, showing that the optimal scheme would be an infinitesimally short and infinite price floor, which is not feasible. Therefore, the optimal collar arrangement that maximizes social welfare can only be implemented for the case a linear demand function.

The article unfolds as follows. In Section 2 we develop the models for active and idle projects subject to collar arrangements, both finite-lived as well as perpetual for the case of a linear demand function. In Section 3 we perform a comparative statics analysis and show the main results. In Section 4 we study the impact of finite-lived collars on social welfare and we show how to optimally design a collar scheme offered by a government. Section 5 extends the analysis to the case of an iso-elastic demand function. Section 6 concludes and suggests some extensions.

2 The model

Let us assume a monopolistic firm facing the opportunity to invest and operate in a market where the demand function is linear:

$$P(t) = X(t)(1 - \eta Q(t)) \tag{1}$$

where $Q(t)$ is the total market output, $\eta > 0$ is the demand slope, and $X(t)$ is an exogenous shock which affects the output price and follows a geometric Brownian motion (gBm) given by:

$$dX(t) = \alpha X(t)dt + \sigma X(t)dw(t) \tag{2}$$

where $X(0) > 0$, α (with $\alpha < r$) is the risk-neutral drift, r is the risk-free interest rate, σ is the instantaneous volatility, $dw(t)$ is the increment of a Wiener process.

Following Huisman and Kort (2015), let us also assume that the firm enters the market with a capacity Q and the investment cost is δQ. We also assume that after investing the firm operates at full capacity ($Q(t) = Q$).

Under this setting, the firm's objective function is given by:

$$\max_{\tau \geqslant 0, Q \geqslant 0} E\left[\int_{t=\tau}^{\infty} X(t)(1 - \eta Q)Qe^{-rt}dt - \delta Q e^{-r\tau} | X(0) = X\right] \tag{3}$$

where τ is the optimal time to invest, Q is the optimal entry capacity level.

The solution for Equation (3) is attained in two steps (see Huisman and Kort, 2015). In a first step, we select the optimal capacity ($Q^*(X)$) for a given $X(t)$, through:

$$\max_{Q \geqslant 0} E\left[\int_{t=0}^{\infty} X(t)(1 - \eta Q)Qe^{-rt}dt - \delta Q | X(0) = X\right] \tag{4}$$

which yields:

$$Q^*(X) = \max\left[\frac{1}{2\eta}\left(1 - \frac{(r-\alpha)\delta}{X}\right), 0\right] \tag{5}$$

In a second step, we replace Q in Equation (3) by equation (5) and obtain the optimal investment threshold (X^*), given by:

$$\max_X \left[\left(\frac{X(1-\eta Q^*(X))Q^*(X)}{r-\alpha} - \delta Q^*(X) \right) \left(\frac{X(0)}{X} \right)^{\beta_1} \right] \tag{6}$$

where β_1 is the positive root of the fundamental quadratic equation $\mathcal{Q}(\beta) = 0$[1], i.e. :

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{7}$$

The firm invests at the following optimal threshold:

$$X^* = \frac{\beta_1 + 1}{\beta_1 - 1}(r-\alpha)\delta \tag{8}$$

with capacity:

$$Q^* \equiv Q^*(X^*) = \frac{1}{\eta(\beta_1 + 1)} \tag{9}$$

Assume now that this investment opportunity consists of a concession assigned by the government.[2] A collar − consisting of a price floor and a price ceiling (or price cap) − can be offered to the firm, with the objective, for example, of hastening investment. Under this arrangement, the revenues of the firm have a downside protection at the floor price P_L (if P drops below P_L the firm receives from the government a subsidy equal to $(P_L - P)Q^*$) but, at the same time, the revenues are capped at P_H (if P exceed P_H the firm has to transfer the excess revenues $(P_H - P)Q^*$ to the government). Accordingly, the firm receives the instantaneous revenue $R(P, Q^*, P_L, P_H) = \min\{\max\{P_L, P\}, P_H\}Q^*$, where Q^* is the optimal installed capacity. We study how the choice of P_L and P_H affects the timing and scale of investment and how they be used by the government to achieve a desired outcome (e.g.: to prompt the investment or to maximize welfare).

Moreover, the collar arrangement offered to the firm can either have a perpetual or a finite duration. We start with the perpetual collar and then we present the more realistic case of a finite-lived collar, where the floor and the ceiling only prevail for $T < \infty$.

2.1 Perpetual collars

The solutions for an investment opportunity with a perpetual collar for a price-taker firm can be found in Adkins and Paxson (2017) and Adkins et al. (2019). Following similar steps for the current case of a firm facing a linear demand function, we derive first the

[1]Please refer to Dixit and Pindyck (1994) for details.

[2]We could be referring to the investment in an infrastructure, in facilities for producing renewal energy, or in any other activity where the government concedes operating licenses and may intervene to stimulate private investment.

value of the active firm and then the value of the investment opportunity. To facilitate notation we represent the state variable $X(t)$ simply by X.

The value of the active firm subject to a perpetual collar

Let $V_p(X,Q)$ denote the value of an active project whose output price P is bounded by a price floor $P_L = X_L(1-\eta Q)$ and a price ceiling $P_H = X_H(1-\eta Q)$. The solution for $V_p(X,Q)$ satisfies the following non-homogeneous differential equation:

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_p(X,Q)}{\partial X^2} + \alpha X \frac{\partial V_p(X,Q)}{\partial X} - rV_p(X,Q) + R(X,Q) = 0, \tag{10}$$

where $R(X,Q) = R(X,Q,P_L,P_H) = \min\{\max\{P_L,P\},P_H\}Q$ for convenience, i.e. $R(X,Q) = QP_L$ for $X < X_L = P_L/(1-\eta Q)$, $R(X,Q) = QP_H$ for $X \geqslant X_H = P_H/(1-\eta Q)$, and $R(X,Q) = QP(X) = QX(1-\eta Q)$ for $X_L \leqslant X < X_H$.

The general solution of (10) is:

$$V_p(X,Q) = A_a X^{\beta_1} + A_b X^{\beta_2} \tag{11}$$

where β_1 is as in Equation (7), and

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{12}$$

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The solutions for the non-homogeneous part of (10), the particular solutions, depend on where $P(X)$ stands in relation to $P_L(X_L)$ and $P_H(X_H)$. Accordingly, the particular solution for $X < X_L$ is QP_L/r, for $X \in [X_L, X_H)$ is $QX(1-\eta Q)/(r-\alpha)$, and for $X > X_H$ becomes QP_H/r. Considering that $V_p(0,Q) = 0$, then $A_b = 0$ for $X < X_L$. Additionally, given that $V_p(X,Q)$ has an upside limit of QP_H/r whenever $X \geqslant X_H$, then A_a must be set equal to 0 in this region. Putting together the solutions for all the regions we get:

$$V_p(X,Q) = \begin{cases} A_{11}X^{\beta_1} + \dfrac{QP_L}{r} & \text{for } X < X_L \\ A_{21}X^{\beta_1} + A_{22}X^{\beta_2} + \dfrac{Q(1-\eta Q)X}{r-\alpha} & \text{for } X_L \leqslant X < X_H \\ A_{32}X^{\beta_2} + \dfrac{QP_H}{r} & \text{for } X \geqslant X_H \end{cases} \tag{13}$$

The constants $A_{11}, A_{21}, A_{22}, A_{32}$ are found by ensuring that $V_p(X,Q)$ is continuous and continuously differentiable along X. The solutions for the constants are as follows:[3]

$$A_{11} = \frac{\left(P_H^{1-\beta_1} - P_L^{1-\beta_1}\right)}{\beta_1 - \beta_2}\left(\frac{\beta_2 - 1}{r-\alpha} - \frac{\beta_2}{r}\right) Q(1-\eta Q)^{\beta_1} \equiv a_{11} Q(1-\eta Q)^{\beta_1} \tag{14}$$

$$A_{21} = \frac{P_H^{1-\beta_1}}{\beta_1 - \beta_2}\left(\frac{\beta_2 - 1}{r-\alpha} - \frac{\beta_2}{r}\right) Q(1-\eta Q)^{\beta_1} \equiv a_{21} Q(1-\eta Q)^{\beta_1} \tag{15}$$

$$A_{22} = -\frac{P_L^{1-\beta_2}}{\beta_1 - \beta_2}\left(\frac{\beta_1 - 1}{r-\alpha} - \frac{\beta_1}{r}\right) Q(1-\eta Q)^{\beta_2} \equiv a_{22} Q(1-\eta Q)^{\beta_2} \tag{16}$$

$$A_{32} = \frac{\left(P_H^{1-\beta_2} - P_L^{1-\beta_2}\right)}{\beta_1 - \beta_2}\left(\frac{\beta_1 - 1}{r-\alpha} - \frac{\beta_1}{r}\right) Q(1-\eta Q)^{\beta_2} \equiv a_{32} Q(1-\eta Q)^{\beta_2} \tag{17}$$

The value of the idle firm and its investment policy

Moving backwards, let us consider an idle firm having a perpetual option to invest in a project subject to a perpetual collar. The firm needs to decide on the investment policy, i.e. the investment timing along with the scale of the project. Being a contingent claim, the value of the investment opportunity, $F_p(X)$, must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 F_p(X)}{\partial X^2} + \alpha X \frac{\partial F_p(X)}{\partial X} - rF_p(X) = 0. \tag{18}$$

The general solution has the form $F_p(X) = B_a X^{\beta_1} + B_b X^{\beta_2}$. Considering that the option becomes worthless at the absorbing barrier $X = 0$ (i.e. $F_p(0, Q) = 0$) we set $B_b = 0$. The optimal capacity that the firm chooses when investing at $X(t) = X$ is obtained with the following maximization condition:

$$\left.\frac{\partial\left(V_p(X,Q) - \delta Q\right)}{\partial Q}\right|_{Q=Q_p^*(X)} = 0 \tag{19}$$

The investment trigger, X_p^*, is obtained by solving the smooth-pasting condition:

$$\beta_1\left(V_p\left(X_p^*, Q_p^*(X_p^*)\right) - \delta Q_p^*(X_p^*)\right) = \left(\left.\frac{\partial\left(V_p(X, Q_p^*(X)) - \delta Q_p^*(X)\right)}{\partial X}\right|_{X=X_p^*}\right) X_p^* \tag{20}$$

Solving simultaneously Equations (19) and (20) allows finding the optimal capacity $Q_p^* = Q_p^*(X_p^*)$ and the investment trigger X_p^*. For the region $[0, X_L)$ the smooth-pasting condition has no solution, i.e. the trigger X_p^* can only be larger than X_L, either below or above X_H, i.e. $X_p^* \in [X_L, \infty)$. For these two regions the solution for the optimal scale is

[3]These solutions are obtained equalizing the value and the first derivatives of the upper and middle branches of Equation (13) at X_L, and of the middle and lower branches at X_H.

closed-form and coincides with the plain investment case (Equation (9)):[4]

$$Q_p^* = \frac{1}{\eta(\beta_1 + 1)} = Q^* \tag{21}$$

The optimal scale is, therefore, independent of price floors or ceilings.

For the region $[X_H, \infty)$, Equation (20) has the following closed-form solution:

$$X_p^* = \frac{\beta_1 + 1}{\beta_1} \left(\frac{\beta_1}{a_{32}(\beta_1 - \beta_2)} \left(\frac{P_H}{r} - \delta \right) \right)^{\frac{1}{\beta_2}} \tag{22}$$

where the condition $\delta < P_H/r$ is needed for obtaining positive solutions.[5] For the region $[X_L, X_H)$ it reduces to the following equation:

$$(\beta_1 - \beta_2) a_{22} Q_p^* \left(1 - \eta Q_p^*\right)^{\beta_2} X_p^{*\beta_2} + (\beta_1 - 1) \frac{Q_p^*(1 - \eta Q_p^*) X_p^*}{r - \alpha} - \beta_1 \delta Q_p^* = 0 \tag{23}$$

which needs to be solved numerically to find the investment trigger.

The condition $P_H/r > \delta > P_H/r + ((\beta_1 - \beta_2)/\beta_1) a_{32} {P_H}^{\beta_2}$ places the solution in the region $[X_H, \infty)$.[6] If δ is smaller but greater than P_L/r the solution is in the region $[X_L, X_H)$.[7] Setting a price floor above δr creates a risk-free project value, that would not be negative, even for $X = 0$.

The value of the investment opportunity subject to a perpetual collar arrangement is:

$$F_p(X) = \left(V_p\left(X_p^*, Q_p^*\right) - \delta Q_p^*\right) \left(\frac{X}{X_p^*}\right)^{\beta_1} \qquad \text{for } X < X_p^* \tag{24}$$

2.2 Finite-lived collars

Let us now consider the more realistic case of a collar with a finite duration of $T < \infty$ years. The collar arrangement remains in force during the period $[t^*,\ t^* + T]$, where t^* is the investment timing and $t^* + T$ is the expiring date, after which both the floor and the ceiling are withdrawn and the firm's revenues depend only on price P. We extend the model of Adkins et al. (2019) by allowing the firm to optimize the investment capacity. As before, we start with the active firm and then move backwards to the idle stage.

[4] See Appendix A.

[5] For $\delta = P_H/r$ the net project value is null, i.e. $V_p(X, Q_p^*(X)) - \delta Q_p^*(X) = 0$.

[6] Obtained by solving $X_p^* = X_H(Q_p^*) = P_H/(1 - \eta Q_p^*)$, where X_p^* is given by Equation (22) and Q_p^* by Equation (21).

[7] P_L/r is obtained by setting $X_p^* = X_L(Q_p^*) = P_L/(1 - \eta Q_p^*)$ in Equation (23), with Q_p^* given by Equation (21), and solving it for δ. Alternatively, P_L/r can be found as the value of δ that makes the net project value null for $X = 0$, i.e. $V_p(0, Q) - \delta Q = 0$ (Equation (13)).

The value of the active firm subject to a finite-lived collar

Immediately after being implemented, the value of a project subject to a finite-lived collar arrangement corresponds to a finite integral of European floorlets and caplets. The individual floorlets and caplets can be continuously exercised during the life of the collar (i.e. during T years) just by being in the money. From Shackleton and Wojakowski (2007) we know that this finite integral can be replicated by a portfolio that includes: (i) a long position in a perpetual collar, (ii) a short position in a forward-start perpetual collar that begins after T years.[8]

Additionally, considering a project that lasts perpetually, beyond the finite period of the collar, it is necessary to add (iii) a long position in the profits that are expected to be generated after moment T. In sum, combining (i) and (ii) we replicate the finite-collar, whereas with (iii) we capture the value of the project operating perpetually without any constrains in P, after the end of the collar.

Accordingly, the value of an active project with a finite-lived collar is given by:

$$V_f(X,Q) = V_p(X,Q) - S(X,Q,T) + \frac{Q(1-\eta Q)X}{r-\alpha}e^{-(r-\alpha)T}. \tag{25}$$

The first term, $V_p(X,Q)$, is as presented in equation (13). The second term, $S(X,Q,T)$, represents the present value of a forward-start perpetual collar (a collar that starts at the moment T), which is given by:[9]

$$\begin{aligned} S(X,Q,T) = {} & a_{11}Q(1-\eta Q)^{\beta_1}X^{\beta_1}N(-d_{\beta_1}(X,Q,P_L)) + \frac{QP_L}{r}e^{-rT}N(-d_0(X,Q,P_L)) \\ & + a_{21}Q(1-\eta Q)^{\beta_1}X^{\beta_1}\left(N(d_{\beta_1}(X,Q,P_L)) - N(d_{\beta_1}(X,Q,P_H))\right) \\ & + a_{22}Q(1-\eta Q)^{\beta_2}X^{\beta_2}\left(N(d_{\beta_2}(X,Q,P_L)) - N(d_{\beta_2}(X,Q,P_H))\right) \\ & + \frac{Q(1-\eta Q)X}{r-\alpha}e^{-(r-\alpha)T}\left(N(d_1(X,Q,P_L)) - N(d_1(X,Q,P_H))\right) \\ & + a_{32}Q(1-\eta Q)^{\beta_2}X^{\beta_2}N(d_{\beta_2}(X,Q,P_H)) + \frac{QP_H}{r}e^{-rT}N(d_0(X,Q,P_H)), \end{aligned} \tag{26}$$

where $N(.)$ is the standard normal cumulative distribution, and

$$d_\beta(X,Q,P) = \frac{\ln X - \ln\left(\frac{P}{1-\eta Q}\right) + \left(\alpha + (\beta - 0.5)\sigma^2\right)T}{\sigma\sqrt{T}}, \qquad \beta \in \{0, 1, \beta_1, \beta_2\}. \tag{27}$$

Finally, the last term of Equation (25) captures the present value of the expected

[8] According to Shackleton and Wojakowski (2007), this replicating portfolio is feasible due to the fact that the individual floorlets and caplets contained within the integral are independent.

[9] Naturally, the negative sign represents the short position in the forward-start perpetual collar. Please refer to Adkins et al. (2019) for details.

profits that will start after T.[10]

The value of the idle firm and its investment policy

Let as now move backwards to the idle stage. The value of the opportunity to invest in a project subject to a finite-lived collar, $F_f(X)$, must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 F_f(X)}{\partial X^2} + \alpha X \frac{\partial F_f(X)}{\partial X} - rF_f(X) = 0, \tag{28}$$

whose general solution is $F_f(X) = C_a X^{\beta_1} + C_b X^{\beta_2}$. Considering that the option to invest must be worthless at $X = 0$ (i.e. $F_f(0, Q) = 0$) we set $C_b = 0$. The optimal capacity that the firm chooses when investing at $X(t) = X$ is obtained with the following maximization condition:

$$\frac{\partial \left(V_f(X, Q) - \delta Q\right)}{\partial Q} \Bigg|_{Q=Q_f^*(X)} = 0 \tag{29}$$

The investment trigger, X_f^*, is obtained by solving the smooth-pasting condition:

$$\beta_1 \left(V_f\left(X_f^*, Q_f^*(X_f^*)\right) - \delta Q_f^*(X_f^*)\right) = \left(\frac{\partial \left(V_f(X, Q_f^*(X)) - \delta Q_f^*(X)\right)}{\partial X} \Bigg|_{X=X_f^*} \right) X_f^* \tag{30}$$

For the finite collar, the transition between the idle and the active stages can occur for any X $\left(X_L \lesseqqgtr X_f^* \lesseqqgtr X_H\right)$. However, the price floor must be lower than $\delta r/(1 - e^{-rT})$, otherwise it produces a risk-free profit.[11].

Appendix B provides the equations that need to be solved to obtain X_f^* and Q_f^*. It also shows that, as for the perpetual case, the optimal scale is closed-form and coincides with the plain investment case:

$$Q_f^* = \frac{1}{\eta(\beta_1 + 1)} \tag{31}$$

The value of the investment opportunity subject to a finite collar arrangement is:

$$F_f(X) = \left(V_f\left(X_f^*, Q_f^*\right) - \delta Q_f^*\right) \left(\frac{X}{X_f^*}\right)^{\beta_1} \qquad \text{for } X < X_f^* \tag{32}$$

[10]A generic expression for component (iii) is $\frac{Q(1-\eta Q)X}{r-\alpha}\left(e^{-(r-\alpha)T} - e^{-(r-\alpha)(T_d+T)}\right)$, where $T_d \geqslant 0$ is the life of the project after moment T (the end of the collar). In case of $T_d = 0$ the end of the project happens together with that of the collar, and so component (iii) collapses; when $0 < T_d < \infty$ the project continues, but has a finite duration after the end of the collar; finally, if $T_d = \infty$ the project lasts perpetually, corresponding to the last term of Equation (25).

[11]For $P_L = \delta r/(1 - e^{-rT})$ the net project value is null, i.e. $V_f(0, Q) - \delta Q = 0$ (Equation (25)).

3 Comparative statics

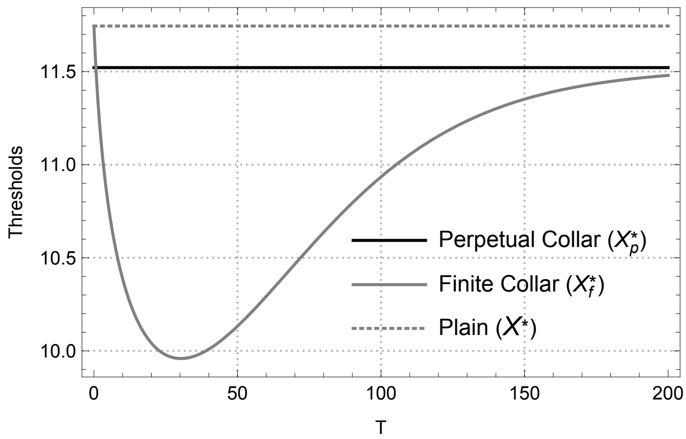
In this section we perform a numerical comparative statics analysis for the main parameters of the models. For that purpose, Table 1 shows the base case parameter values for a firm operating under a price floor and ceiling.

Parameter	Description	Value
P_L	Price floor	\$2
P_H	Price cap	\$6
η	Slope of demand function	2
σ	Volatility	0.2
r	Risk-free rate	0.04
α	Risk-neutral drift rate	0.01
δ	Variable investment cost	\$100
T	Duration of the collar (years)	15

Table 1: The base case parameter values.

We first study the effect of the collar parameters (T, P_H, P_L) on investment timing, because, as shown in the previous section, the investment scale is unaffected by price floors and ceilings, for a firm facing a linear demand function. Figures 1-3 illustrate the sensitivity of the investment triggers (X_p^*, X_f^*, and X^*) to changes in the collar parameter values.

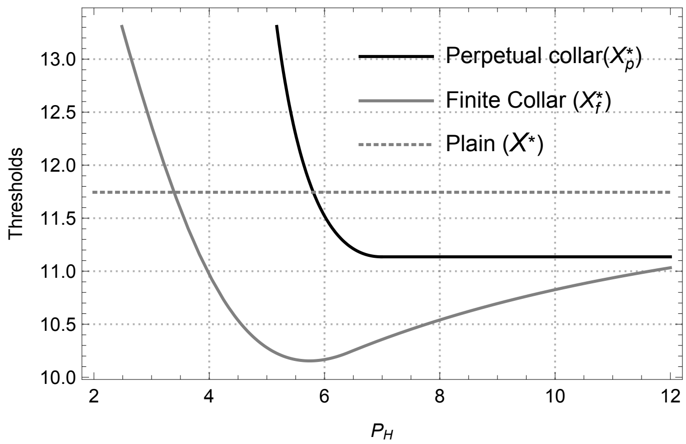
Figure 1 depicts the effects of the collar duration, T, on the investment dynamics. As expected, we see that the threshold of a finite collar arrangement ranges between the plain (no collar) and the perpetual case. This means that, when compared to the plain case, a collar induces the firm to invest sooner, and this effect is less pronounced for extreme (short or long) durations of the collar. Therefore, the duration of the collar has a non-monotonic effect on the investment timing. There is a duration of the finite collar that induces the earliest investment. As expected, as T increases beyond the duration inducing the earliest investment, the finite collar trigger converges to the perpetual trigger.



$P_L = \$2; P_H = \$6; \sigma = 0.2; r = 0.04; \alpha = 0.01; \eta = 2; \delta = \$100.$
$Q_f^* = Q_p^* = Q^* = 0.186.$

Figure 1: The effect of the collar duration

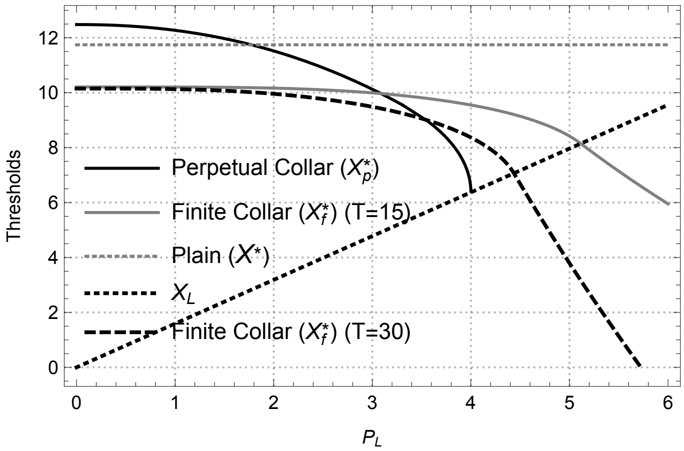
The effect of the price ceiling is analyzed in Figure 2. As for the duration, the effect of the price ceiling on the finite collar trigger is non-monotonic. Initially, as P_H increases it induces earlier investments but after a certain level of P_H the opposite effect occurs. There seems to exist a limit to the capacity of inducing earlier investment by reducing the price ceiling, i.e. there is a level of P_H, given the other parameter values, that induces the earlist invetment. In the case of a perpetual collar, the effect of the price ceiling on the investment trigger is monotonically decreasing until it reaches a limit where it ceases to have an effect.



$P_L = \$2; T = 15; \sigma = 0.2; r = 0.04; \alpha = 0.01; \eta = 2; \delta = \$100.$
$Q_f^* = Q_p^* = Q^* = 0.186.$

Figure 2: The effect of the price ceiling

Figure 3 show the effect of the price floor. By increasing P_L it is possible to hasten the implementation of the project. A finite-lived collar induces the firm to invest sooner, when compared to the plain (no collar) alternative, and, contrary to the perpetual case, if the price floor is sufficiently large, for a price level below the price floor ($X_f^* < X_L$ or $P^* < P_L$)[12]. Different durations produce an important effect on the effect of the hastening investment. For the base-case duration ($T = 15$), the maximum price floor is the price ceiling, for which the collar become a fixed price regime. When $P_L = P_H = 6$, the price trigger is 5.953 and it is impossible to decrease it further, unless a longer duration is offered. For $T = 30$, in which case the maximum price floor is 5.724,[13] the investment trigger can be reduced to zero, making immediate investment optimal for any level of X.

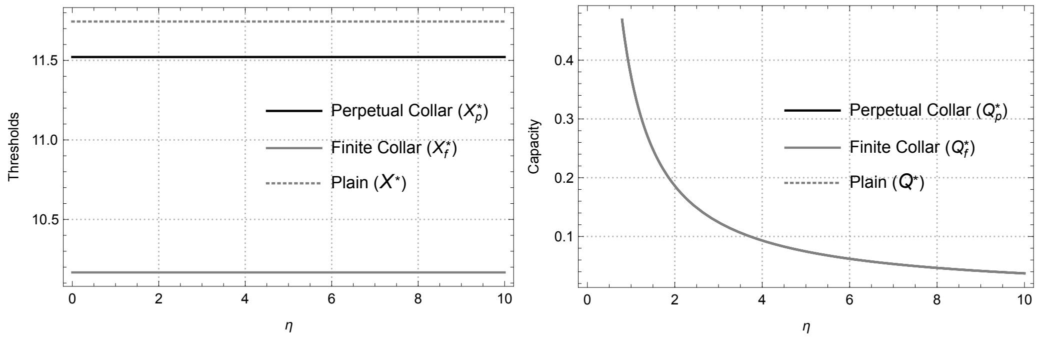


$P_H = \$6; \sigma = 0.2; r = 0.04; \alpha = 0.01; \eta = 2; \delta = \$100.$
$Q_f^* = Q_p^* = Q^* = 0.186.$

Figure 3: The effect of the price floor

The optimal capacity is only affected by the $\beta_1(r, \alpha, \sigma)$ and η. Figure 4 depicts that the demand slope η has no effect in the investment triggers (Figure 4(a)) and reduces the investment scale (Figure 4(b)). In fact, as the optimal capacity solution is closed form, the effect on capacity choice can be generalized to any set of parameters, given that $\partial Q^*/\partial\eta < 0$.

[12] P^* is the price level when $X = X^*$.
[13] The maximum price floor is $\delta r/(1 - e^{-rT})$.

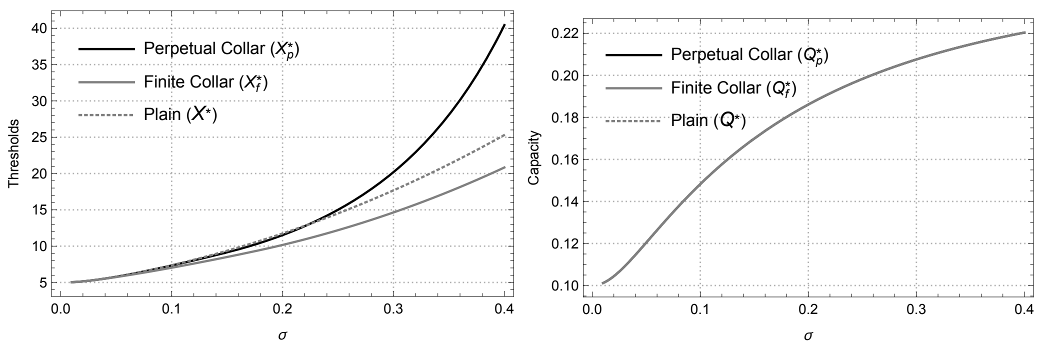


(a) Threshold (b) Capacity

$P_L = \$2; P_H = \$6; T = 15; \sigma = 0.2; r = 0.04; \alpha = 0.01; \delta = \$100.$

Figure 4: The effect of η

Finally, Figure 5(a) shows the well known effect of uncertainty: the higher the volatility the larger the thresholds. Figure 5(b) shows that a higher uncertainty induces the firm to choose a larger scale of investment. This effect is again general, given that $\partial Q^*/\partial\sigma > 0$.[14]



(a) Threshold (b) Capacity

$P_L = \$2; P_H = \$6; T = 15; r = 0.04; \alpha = 0.01; \eta = 2; \delta = \$100.$

Figure 5: The effect of σ

4 Welfare analysis for finite-lived collars

Assuming that the main motivation of the government is to prompt the investment (for instance, in the context of a concession contract), the introduction of a collar can be used as an effective mechanism for achieving this goal. In fact, it would be in general possible to design a collar, by manipulating parameters $\{T, P_H, P_L\}$, in such a way that the investment threshold equals the current realization of X.

[14]Notice that $\partial Q^*/\partial\sigma = \partial Q^*/\partial\beta_1 \times \partial\beta_1/\partial\sigma$, $\partial Q^*/\partial\beta_1 < 0$, and $\partial\beta_1/\partial\sigma < 0$.

However, the policy to stimulate immediate investment can be questionable as it may not necessarily maximize social welfare. To address this issue, we study the design of a finite-lived collar from the perspective of a social planner that aims to maximize the total surplus that arises from the arrangement.

Therefore, we analyze the conditions under which a collar offered by a government can improve social welfare. In particular, we study how to optimally design a finite-lived collar, by manipulating the components that characterize the collar $\{T, P_H, P_L\}$, with the purpose of maximizing the total surplus, i.e. the sum of the producer and consumer surpluses net of the cost of the collar. Noticing that the government's expenses/revenues with the collar are transfers to/from the firm (i.e. they have no effect in aggregate terms), a social welfare optimizer should simply maximize the total surplus (the sum of producer surplus and consumer surplus) in the absence of the collar. However, the investment timing is determined by the firm, knowing the collar arrangement offered by the government, i.e. X_f^* is dependent on the collar parameters. Accordingly, the collar arrangement offered by the government affects social welfare by modifying firm's behavior with respect to the timing of investment.

Before investment becomes optimal ($X < X^*$), the value of the producer surplus without the collar is

$$PS(X) = \left(\frac{X^* Q^* (1 - \eta Q^*)}{r - \alpha} - \delta Q^* \right) \left(\frac{X}{X^*} \right)^{\beta_1}, \tag{33}$$

and the value of the consumer surplus is

$$CS(X) = \frac{X^* {Q^*}^2 \eta}{2(r - \alpha)} \left(\frac{X}{X^*} \right)^{\beta_1}, \tag{34}$$

where X^* and Q^* are, respectively, the trigger and the optimal capacity choice.[15]

Computing the total surplus, $TS(X) = PS(X) + CS(X)$, leads to:

$$TS(X) = \left(\frac{X^* Q^* (2 - \eta Q^*)}{2(r - \alpha)} - \delta Q^* \right) \left(\frac{X}{X^*} \right)^{\beta_1} \tag{36}$$

A social planner, pursuing the goal of maximizing the total surplus, has the following

[15] For the linear demand function (1), the instantaneous consumer surplus when investment becomes optimal is:

$$CS = \int_{X^*(1-\eta Q^*)}^{X^*} \frac{1}{\eta} \left(1 - \frac{P}{X^*} \right) dP = \frac{1}{2} X^* {Q^*}^2 \eta \tag{35}$$

investment trigger and capacity choice (Huisman and Kort, 2015):

$$X_w^* = \frac{\beta_1 + 1}{\beta_1 - 1}(r - \alpha)\delta = X^* \tag{37}$$

$$Q_w^* = \frac{2}{(\beta_1 + 1)\eta} = 2Q^* \tag{38}$$

The monopolistic firm invests optimally but with half of capacity, producing to a welfare loss. Is it possible to eliminate or, at least, mitigate the welfare loss by offering a collar? Knowing that the collar parameters do not change the firm's capacity choice, we know that eliminating the welfare loss is impossible. However, because the collar changes the behavior of the firm regarding investment timing, a welfare loss mitigation may be possible.

When a collar is offered, the total surplus, $TS_f(X)$, changes because X^* becomes X_f^*:[16]

$$TS_f(X) = \left(\frac{X_f^* Q^* (2 - \eta Q^*)}{2(r - \alpha)} - \delta Q^* \right) \left(\frac{X}{X_f^*} \right)^{\beta_1} \tag{39}$$

The consumer surplus is affected by the change in the investment trigger:

$$CS_f(X) = \frac{X_f^* Q^{*2} \eta}{2(r - \alpha)} \left(\frac{X}{X_f^*} \right)^{\beta_1}, \tag{40}$$

and the producer benefits from the transfers and bears the payments related with the collar, with its surplus being the value of the investment opportunity:

$$PS_f(X) = F_f(X) = \left(V_f\left(X_f^*, Q^* \right) - \delta Q^* \right) \left(\frac{X}{X_f^*} \right)^{\beta_1} \tag{41}$$

The cost of the collar for the government corresponds to the difference between the total surplus and the consumer and producer surpluses:

$$G_f(X) = TS_f(X) - CS_f(X) - PS_f(X) \tag{42}$$

In order to maximize social welfare, the government chooses the optimal level of a collar parameter (T^w, P_H^w, P_L^w), that can be found by solving numerically the following optimization problem:

$$\max_{y \in \{T, P_H, P_L\}} TS_f(X) = \max_{y \in \{T, P_H, P_L\}} \left[\left(\frac{X_f^*(y) Q^* (2 - \eta Q^*)}{2(r - \alpha)} - \delta Q^* \right) \left(\frac{1}{X_f^*(y)} \right)^{\beta_1} \right] \tag{43}$$

[16] Notice that $Q_f^* = Q^*$.

This optimization problem is equivalent to:

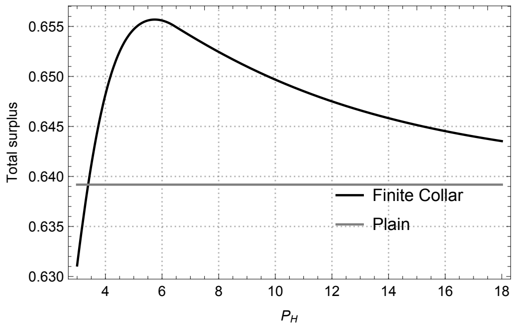
$$\max_{X_s^*}\left[\left(\frac{X_s^* Q^*(2-\eta Q^*)}{2(r-\alpha)}-\delta Q^*\right)\left(\frac{1}{X_s^*}\right)^{\beta_1}\right] \tag{44}$$

where X_s^* is the optimal trigger for a social planner that optimizes investment timing for a given capacity ($Q = Q^*$). Therefore, the optimal levels of the collar parameters (T^w, P_H^w, P_L^w) are such that $X_f^*(y) = X_s^*$. The solution to the optimization problem (44) is:

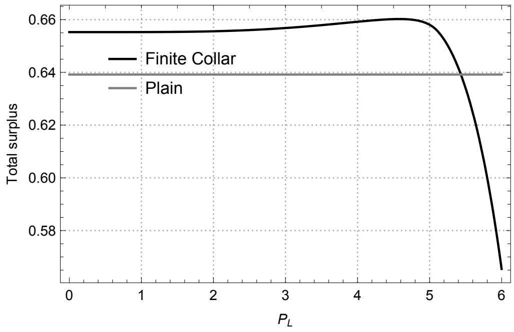
$$X_s^* = \frac{\beta_1+1}{\beta_1-1}\frac{2}{2-\eta Q^*}(r-\alpha)\delta = \frac{2}{2-\eta Q^*}X^* < X^*(= X_w^*) \tag{45}$$

The social planner that optimizes investment timing with price floors and ceilings, invests sooner than the monopolistic firm ($X_s^* < X^*$). However, the total surplus attained is, of course, smaller than that of a social planner that also optimizes capacity (Q_w^*), because it has one degree of freedom less.

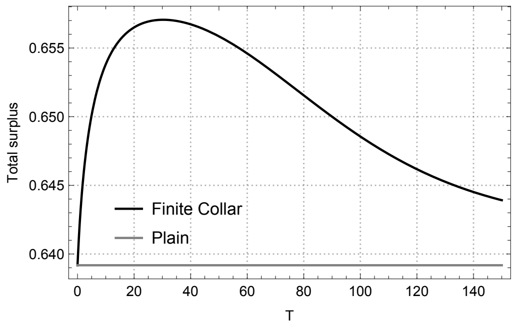
Figure 6 shows how the value of the total surplus changes with the three collar parameters: the price ceiling (Figure 6(a)), the price floor (Figure 6(b)), and the collar duration (Figure 6(c)). The figures also show the total surplus for the case of a plain project without the collar (Equation (36)). The social welfare of a project subject to a finite collar can be higher or lower than that of a plain investment, depending on the collar parameters set by the government. More importantly, the figures also show that a maximum social welfare can be attained, which means that, for our base-case parameters, the price ceiling, the price floor or the collar duration can be effectively used as instruments to achieve a social welfare maximum.



(a) P_H



(b) P_L



(c) T

$X = 1; P_L = \$2; P_H = \$6; T = 15; \sigma = 0.2; r = 0.04; \alpha = 0.01; \eta = 2; \delta = \$100.$

Figure 6: Total surplus as a function of the collar parameters

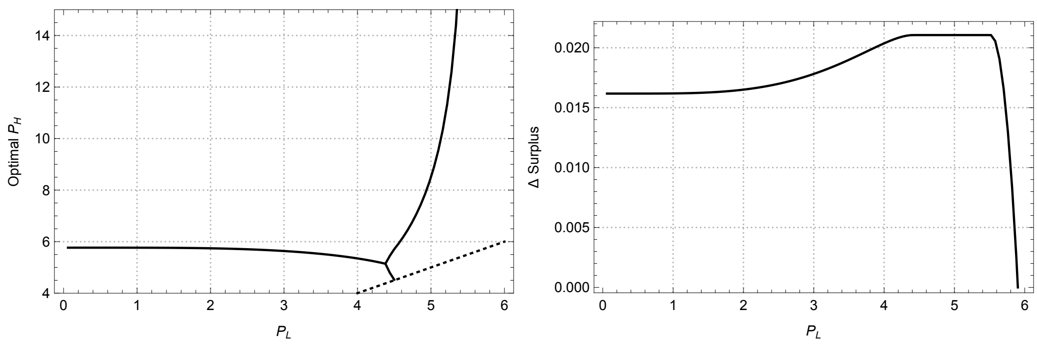
Let us now focus on the price ceiling as the instrument used by a social optimizer to maximize the total surplus. The optimal P_H^w is found solving numerically the optimization problem (43).[17]

A price ceiling changes the firm's behavior (see Figure 2): a higher price ceiling has a non-monotonic effect on investment timing. Specifically, starting from a price ceiling close to the price floor, increasing P_H induces earlier investments, but, after a certain level, this effect is reversed: additional increments in P_H start increasing the investment trigger.

Figure 7(a) depicts the optimal price ceiling for different levels of the price floor. We see that P_H^w initially decreases as P_L increases. For price floors from 4.413 to 4.505 there are two optimal price ceilings. At 4.505, one of the optimal price ceilings becomes the floor (a fixed price scheme), and above that level, the optimal ceiling increases rapidly to infinity. Figure 7(b) shows that the total surplus reaches a maximum between $P_L = 4.413$ and $P_L = 5.513$, suggesting that there are multiple optimal collar arrangements, including a fixed price scheme (at 4.505), and a floor only scheme (at 5.513). Offering a collar with a price floor below 4.413, or a price floor without ceiling above 5.513 is detrimental for

[17]The problem is highly nonlinear and involves several equations that need to be solved numerically. In order to ensure that the solution is a global maximum, a careful choice of the seeds is needed.

the social planner.[18]



(a) P_H^w (b) Additional surplus

$X = 1; T = 15; \sigma = 0.2; r = 0.04; \alpha = 0.01; \eta = 2; \delta = \$100.$

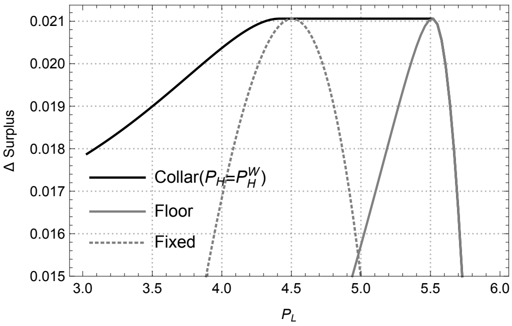
Figure 7: Optimal price ceilings for different price floors.

In summary, for the base-case parameter values, there are multiple optimal collar schemes, including a floor only and fixed price regimes, all producing the same wealth increment. It may be possible that, for other parameter values, the multitude of alternative optimal schemes differs.

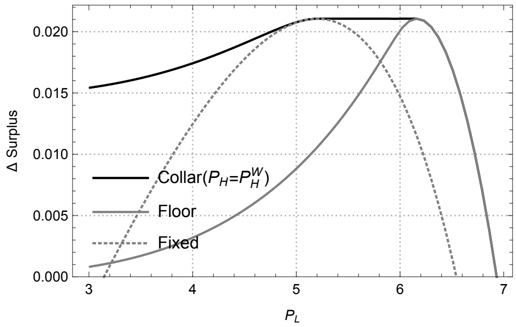
4.1 The effect of collar duration

Figure 8 shows the impact of the price floor on the incremental surplus, for the collar, with an optimal price ceiling, the floor, and the fixed price schemes, for three alternative durations. As the optimal scheme is such that it induces the firm to invest always at the same trigger (X_s^* in Equation (45)), the duration of the price scheme is irrelevant for the incremental surplus attained. However, it has important implications for the optimal schemes. In fact, for the base-case duration ($T = 15$), all schemes are optimal (Figure 8(a)). When the duration is reduced to $T = 10$, the three schemes are still optimal, but the minimum price floor is the optimal fixed price level, i.e. the optimal ceiling for the minimum price foor, is the price floor (Figure 8(b)). If the duration is further reduced to $T = 2.5$, imposing a price ceiling is no longer an optimal scheme: the optimal fixed price coincides with the optimal price floor (Figure 8(c)).

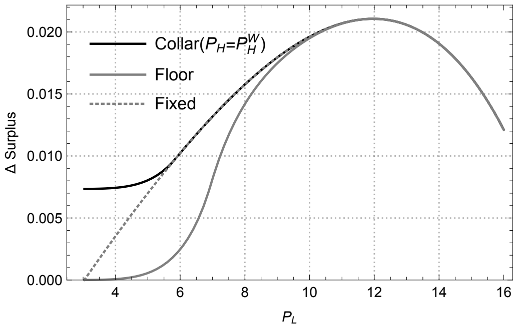
[18]For the price floor scheme, the value functions and the equations to obtain the thresholds and capacity can be obtained by letting $P_H \to \infty$, or derived following similar steps to those of the collar case (see Appendix C). Similarly, for the fixed price scheme the solutions are obtained letting $F = P_L = P_H$ or derived following similar steps (see Appendix D).



(a) $T = 15$



(b) $T = 10$



(c) $T = 2.5$

$X = 1; \sigma = 0.2; r = 0.04; \alpha = 0.01; \eta = 2; \delta = \$100.$

Figure 8: The effect of the price floor (P_L) on social welfare for collar and floor schemes.

These effects can also be found in Table 2, which shows the optimal scheme with the minimum price floor.[19] For the base case duration ($T = 15$), the optimal price floor and ceiling are $P_L^w = 6.484$, and $P_H^w = 21.974$, respectively. Furthermore, the effect of the duration in the optimal scheme, suggested in Figure 8 can be confirmed: For $T = 5$ or $T = 10$, the optimal minimum price floor is the optimal fixed price, and for $T = 1$ or $T = 2.5$, the optimal ceiling is infinite. Notice that for the smaller durations, an infinite ceiling (price floor scheme, shown in the table) and a ceiling $P_H^w = P_L^w$ (fixed price scheme) are equivalent in terms of total surplus.

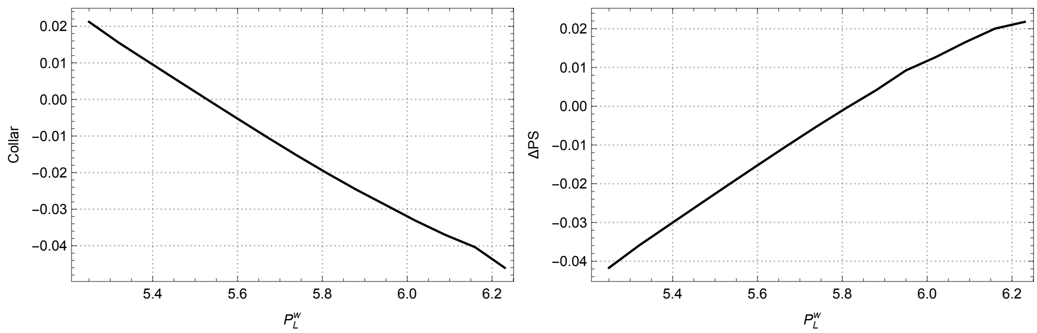
[19]When multiple collar schemes are optimal, we show the one with the minimum price floor.

Table 2: The effect of the duration (T) on optimal price floors and ceilings

T	P_L^w	P_H^w	Scheme	P_f^*	X_f^*	Q_f^*	X^*	Q^*	X_w^*	Q_w^*	ΔCS	ΔPS	Collar	ΔTS
1	25.657	∞	Floor	5.688	9.058	0.186	11.745	0.186	11.745	0.372	0.042	0.068	-0.089	0.021
2.5	11.966	∞	Floor	5.688	9.058	0.186	11.745	0.186	11.745	0.372	0.042	0.046	-0.067	0.021
5	7.418	7.418	Fixed	5.688	9.058	0.186	11.745	0.186	11.745	0.372	0.042	0.012	-0.033	0.021
10	5.200	5.200	Fixed	5.688	9.058	0.186	11.745	0.186	11.745	0.372	0.042	-0.049	0.028	0.021
15	4.413	5.117	Collar	5.688	9.058	0.186	11.745	0.186	11.745	0.372	0.042	-0.084	0.063	0.021
20	3.921	5.212	Collar	5.688	9.058	0.186	11.745	0.186	11.745	0.372	0.042	-0.113	0.092	0.021

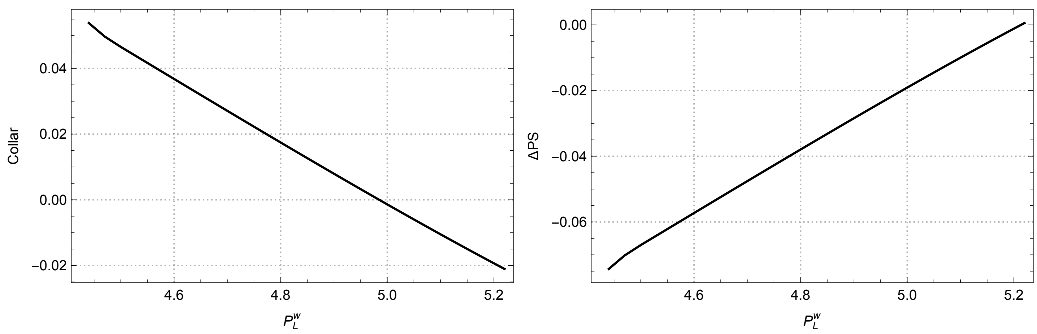
$X = 1; \sigma = 0.2; r = 0.04; \alpha = 0.01; \eta = 2; \delta = \$100$. P_L^w and P_H^w are the optimal price floor and ceiling, obtained from Equation (43). $P_f^* = X_f^*(1 - \eta Q_f^*)$ is the price at the investment trigger for the finite collar. X_f^* and Q_f^*, obtained from Equations (30) and (29), are optimal capacity and investment trigger for the finite collar. The equivalent values for the plain project and the social planner are X^* and Q^* (Equations (8) and (9)), and Q_w^* and X_w^* (Equations (37) and (38)), respectively. The last 4 columns show the incremental surpluses of the optimal finite collar, for the consumer (ΔCS, from Equations (40) and (34)), the producer (ΔPS, from Equations (41) and (33)), the cost of the collar (Equation (42)), and the total surplus (ΔTS, from Equations (39) and (36)).

Additional insights can be found in Table 2. Because the capacity is unaffected by price floors and ceilings, and the investment trigger for the optimal collar is always X_s^*, the price level when investment becomes optimal is constant ($P_f^* = X_s^*(1-\eta Q^*) = 5.688$). Therefore, the incremental total surplus (ΔTS) and the incremental consumer surplus (ΔCS) are independent of the (optimal) collar parameters. Different collars change only the cost for the government and the producer surplus, as expected. For the collars presented in the table, a higher duration benefits the government at the cost of the firm. Is is possible for the government to obtain a net gain by choosing longer durations, as the benefit of the price ceiling outweighs the cost of the price floor. Notice, however, that the table shows only one of the possible collars for durations higher than 2.5. Figure 9 shows the effect of the alternative optimal collars for $T = 10$ and $T = 15$. Holding the duration constant, the government can choose the price floor and price ceiling to achieve a desired cost (benefit), including a zero-cost collar for a sufficient long duration. In summary, a government can manage the collar parameters (price floor, price ceiling and duration), without changing the total surplus, and choose, within limits, the cost/benefit of the intervention or the cost/benefit for the firm.



(a) Collar ($T = 10$)

(b) Producer surplus ($T = 10$)



(c) Collar ($T = 15$)

(d) Producer surplus ($T = 15$)

$X = 1; \sigma = 0.2; r = 0.04; \alpha = 0.01; \eta = 2; \delta = \$100.$

Figure 9: The effect of the optimal floor (in an optimal collar) on the collar cost and the producer surplus.

4.2 The effect of uncertainty

In Table 3 we can see the well known effect of volatility on the timing and scale of investment: a larger σ induces the firm (both with and without a collar scheme), as well the central planner, to invest later at a larger scale. However, some additional insights can be found. Volatility has a non-monotonic effect on the optimal minimum price floor, and a monotonic positive effect on the corresponding price ceiling. When volatility is low, the optimal minimum price floor scheme corresponds to a fixed price scheme (the price ceiling equals the price floor). For higher levels of volatility, the optimal scheme with a minimum price floor is a collar (a price ceiling higher than the price floor). The consumer and society benefit from the optimal collar scheme, independently from the level of σ. Interestingly, we see that the impact on the consumer and total surpluses increase with volatility. For $T = 15$, the minimum price floor induces always a cost for the firm and a benefit for government, because of the corresponding price ceiling. As before, other combinations of price floors and ceilings are possible entailing different costs/benefits for the government and the firm.

Table 3: The effect of uncertainty (σ) on optimal price floors and ceilings

σ	P_L^w	P_H^w	Scheme	P_f^*	X_f^*	Q_f^*	X^*	Q^*	X_w^*	Q_w^*	ΔCS	ΔPS	Collar	ΔTS
0.05	3.987	3.987	Fixed	3.797	4.996	0.120	5.788	0.120	5.788	0.241	0.004	-0.003	0.001	0.002
0.10	4.197	4.197	Fixed	4.287	6.089	0.148	7.372	0.148	7.372	0.297	0.014	-0.016	0.009	0.007
0.15	4.365	4.493	Collar	4.918	7.451	0.170	9.369	0.170	9.369	0.340	0.028	-0.046	0.032	0.014
0.20	4.413	5.117	Collar	5.688	9.058	0.186	11.745	0.186	11.745	0.372	0.042	-0.084	0.063	0.021
0.25	4.310	5.968	Collar	6.596	10.920	0.198	14.508	0.198	14.508	0.397	0.052	-0.128	0.102	0.027
0.30	4.063	6.773	Collar	7.620	13.048	0.208	17.679	0.208	17.679	0.415	0.059	-0.184	0.155	0.030

$X = 1; T = 15; r = 0.04; \alpha = 0.01; \eta = 2; \delta = \$100$. P_L^w and P_H^w are the optimal price floor and ceiling, obtained from Equation (43). $P_f^* = X_f^*(1 - \eta Q_f^*)$ is the price at the investment trigger for the finite collar. X_f^* and Q_f^*, obtained from Equations (30) and (29), are optimal capacity and investment trigger for the finite collar. The equivalent values for the plain project and the social planner are X^* and Q^* (Equations (8) and (9)), and Q_w^* and X_w^* (Equations (37) and (38)), respectively. The last 4 columns show the incremental surpluses of the optimal finite collar, for the consumer (ΔCS, from Equations (40) and (34)), the producer (ΔPS, from Equations (41) and (33)), the cost of the collar (Equation (42)), and the total surplus (ΔTS, from Equations (39) and (36)).

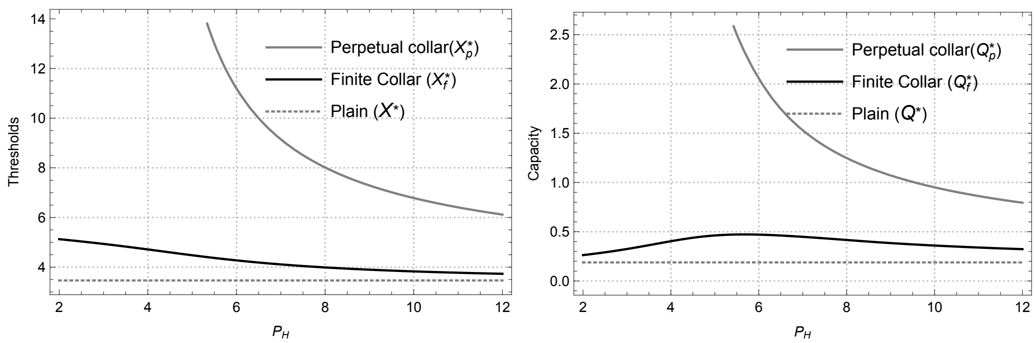
5 The paradox of the iso-elastic demand case

The previous results are conditional on a linear demand function. As in Huisman and Kort (2015) we study the case of an iso-elastic demand function:

$$P(t) = X(t)Q(t)^{-\gamma} \tag{46}$$

Ir order to obtain a meaningful solution, we need to assume that the investment cost comprises two components: a fixed cost (δ_0) and a cost per output unit (δ_1). The total investment is therefore given by: $\delta_0 + \delta_1 Q$.

Following similar steps as for the linear demand function, Appendix E presents the case of an iso-elastic demand function. When deriving the investment trigger and optimal capacity choice, a crucial difference is that the optimal capacity is no longer independent of the collar parameters, i.e. Equations (85) and (86) have solutions for X_f^* and Q_f^* that vary with the collar parameters. For example, Figure 10 show the effect of the price ceiling. A larger P_H decreases the investment trigger. Regarding the optimal scale, we observe a non-monotonic effect when a finite collar is in place.



(a) Threshold (b) Capacity

$P_L = \$2; T = 15; \sigma = 0.2; r = 0.04; \alpha = 0.01; \gamma = 0.825; \delta_0 = \$20; \delta_1 = \$80.$

Figure 10: The effect of the price ceiling for an iso-elastic demand function

When optimizing the collar parameters to maximize the social welfare the objective function of the government becomes:

$$\max_{y \in \{T, P_H, P_L\}} \left[\left(\frac{1}{1-\gamma} \frac{X_f^*(y)^{1-\beta_1} Q_f^*(y)^{1-\gamma}}{r-\alpha} - (\delta_0 + \delta_1 Q_f^*(y)) \right) \left(\frac{1}{X_f^*(y)} \right)^{\beta_1} \right] \tag{47}$$

Because now the optimal capacity is affected by the collar parameters, this optimization problem is not equivalent to that of a central planner optimizing investment timing for a constant capacity choice.

This has important implications on the optimal collar schemes. Table 4 shows the effect of the collar duration on the optimal scheme. For short durations ($T < 15$), the optimal decision is to offer a scheme with a price floor but no price ceiling. As the duration increases, the optimal price floor decreases and the optimal price ceiling becomes finite, also decreasing with T. In other words, only for a sufficiently long duration, the collar becomes an optimal scheme. Contrary to the linear demand case, there is a single combination of a price floor and a price ceiling that maximizes the social welfare.

For shorter durations, the best decision for the government is to offer a simple floor contract. Furthermore, a larger duration of the scheme induces the firm to invest later but in larger capacity. When compared to a plain investment (X^* and Q^*) we see that the collar scheme, depending on its duration, may accelerate or deter the investment, but always promote investment in larger scale. Comparing the decision with that of a central planner (X_w^* and Q_w^*) the collar scheme leads to a later investment but in a much larger scale.

The last four columns show the incremental surpluses (ΔCS, ΔPS, and ΔTS), when compared to a plain investment, as well as the cost of the collar for the government (collar). We see that the consumer is the entity that benefits more from the collar scheme, due to the impact on the scale of the project, capturing the major portion of both the total surplus and the cost of the collar. However, this effect decreases as the duration increases, mainly because of the impact on the timing of the investment. The producer's incremental surplus can either be positive or negative, depending on the duration of the scheme; with a small T the producer captures part of the incremental surplus, whereas with a large T the producer transfers surplus to the consumer. Finally, and more importantly, we see that the incremental total surplus of an optimal collar increases as the duration decreases. Paradoxically, the optimal scheme is an infinitesimally short infinite price floor. Faced with an iso-elastic demand function the optimal collar that a government should offer is not feasible.

Table 4: The effect of the duration (T) on optimal price floors and ceilings for an iso-elastic demand

T	P_L^w	P_H^w	Scheme	P_f^*	X_f^*	Q_f^*	X^*	Q^*	X_w^*	Q_w^*	ΔCS	ΔPS	Collar	ΔTS
1	70.529	∞	Floor	1.919	2.517	1.389	3.464	0.189	0.606	0.189	38.318	4.492	-19.704	23.106
2.5	29.269	∞	Floor	1.915	2.611	1.456	3.464	0.189	0.606	0.189	36.823	3.843	-18.763	21.903
5	15.537	∞	Floor	1.909	2.776	1.574	3.464	0.189	0.606	0.189	34.383	2.846	-17.309	19.920
10	8.708	∞	Floor	1.916	3.139	1.819	3.464	0.189	0.606	0.189	29.555	1.136	-14.602	16.089
15	6.484	21.974	Collar	2.050	3.512	1.920	3.464	0.189	0.606	0.189	24.356	-0.145	-11.248	12.963
20	5.436	10.174	Collar	2.132	3.835	2.037	3.464	0.189	0.606	0.189	20.722	-0.993	-9.257	10.472
25	4.818	7.827	Collar	2.150	4.182	2.240	3.464	0.189	0.606	0.189	17.764	-1.713	-8.048	8.004
30	4.401	6.631	Collar	2.152	4.579	2.498	3.464	0.189	0.606	0.189	14.902	-2.362	-7.063	5.476

$X = 1; \sigma = 0.2; r = 0.04; \alpha = 0.01; \gamma = 0.825; \delta_0 = \$20; \delta_1 = \$80$. P_L^w and P_H^w are the optimal price floor and ceiling, obtained from Equation (47). $P_f^* = X_f^* Q_f^{*-\gamma}$ is the price at the investment trigger for the finite collar. X_f^* and Q_f^*, obtained from Equations (86) and (85) in Appendix E, are optimal capacity and investment trigger for the finite collar. The equivalent values for the plain project and the social planner are X^* and Q^* (Equations (74) and (75) in Appendix E), and Q_w^* and X_w^* (Equations (80) and (81) in Appendix E), respectively. The last 4 columns show the incremental surpluses of the optimal finite collar, for the consumer (ΔCS, from Equations (88) and (78) in Appendix E), the producer (ΔPS, from Equations (89) and (76) in Appendix E), the cost of the collar (Equation (90) in Appendix E), and the total surplus (ΔTS, from Equations (87) and (79) in Appendix E).

6 Conclusion

We develop a model that aims to jointly determine the timing and the optimal capacity of investment opportunities that are subject to a collar scheme, both perpetual and finite. For a linear demand function, we show that the choice of the elements that characterize the collar (i.e. the price floor, the price ceiling, as well as its duration), do shape the decision of the firm with respect to the investment timing, but not to the capacity choice. Our main results show that, (i) the duration of a finite-lived collar has a non-monotonic effect on the investment trigger, and there is an optimal duration of the finite collar that induces the earliest investment; (ii) a larger price ceiling reduces the investment trigger for a perpetual collar, whereas its impact is non-monotonic for finite-lived collars (independently from its duration); (iii) finally, it is possible to induce the firm to invest sooner by increasing the price floor. Additionally, regarding the demand slope and volatility, we show that, independently from the arrangements (perpetual, finite and plain), (iv) a higher demand slope has no impact in the investment timing but reduces capacity, and (v) larger volatilities induce the firm to invest later in a larger scale.

We extend our analysis to study the conditions under which a collar arrangement can improve social welfare. The aim is to optimally design the finite-lived collar such that the total surplus is maximized. Our findings show that, depending on the parameters of the collar, the social welfare attained by a project subject to a finite collar can be higher or lower than that of a plain investment. More importantly, we show that it is possible to maximize the total surplus by optimally setting the components that characterize the collar. As capacity remains constant, regardless of changes in the collar parameters values, the incremental total surplus and the incremental consumer surplus are independent of the collar, but they can be improved by setting optimal price floors and ceilings. There are multiple optimal collar schemes, with different floors an ceilings, including a floor only and fixed price regimes, all producing the same wealth increment. The duration of the price scheme is irrelevant for the incremental surplus attained. However, whereas for longer durations, collars, floors only, and fixed price regimes may be optimal, for short durations only price floors (without a ceiling) and fixed price regimes become optimal. A government can choose the collar parameters (price floor, price ceiling and duration), within limits, and affect only the cost/benefit of the intervention or the cost/benefit for the firm. We show that price floors are important instruments of risk-sharing, investment promotion, and social welfare, as pure price ceilings regimes are never optimal.

Extending the analysis to the iso-elastic demand function, these results change considerably. The optimal capacity choice is affected by changes in the collar parameter values, but the optimal scheme creates a paradox: an infinitesimally short and infinite price floor, which would not be feasible. Therefore, for an iso-elastic demand function, it is possible to improve welfare, but never to optimize it.

Plausible extensions are to consider the welfare implications of layered collars, retractable schemes, other stochastic processes, and competitive markets.

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A Perpetual collar

For the region $[X_H, \infty)$, Equations (19) and (20), for $X = X_p^*$ and $Q_p^* = Q_p^*(X_p^*)$, reduce to:

$$\left(1 - \beta_2 \frac{\eta Q_p^*}{1 - \eta Q_p^*}\right) a_{32}(1 - \eta Q_p^*)^{\beta_2} X_p^{*\beta_2} + \frac{PH}{r} - \delta = 0 \tag{48}$$

$$(\beta_1 - \beta_2) a_{32} Q_p^* \left(1 - \eta Q_p^*\right)^{\beta_2} X_p^{*\beta_2} + \beta_1 \left(\frac{P_H}{r} - \delta\right) Q_p^* = 0, \tag{49}$$

which yield solutions (21) and (22).

For the region $[X_L, X_H)$, Equations (19) and (20), for $X = X_p^*$ and $Q_p^* = Q_p^*(X_p^*)$, reduce to:

$$\begin{aligned}&\left(1 - \beta_1 \frac{\eta Q_p^*}{1 - \eta Q_p^*}\right) a_{21}(1 - \eta Q_p^*)^{\beta_1} X^{\beta_1} \\ &\qquad + \left(1 - \beta_2 \frac{\eta Q_p^*}{1 - \eta Q_p^*}\right) a_{22}(1 - \eta Q_p^*)^{\beta_2} X^{\beta_2} + \frac{(1 - 2\eta Q_p^*) X_p^*}{r - \alpha} - \delta = 0\end{aligned} \tag{50}$$

and Equation (23). The solution for Q_p^* is (21).

B Finite-lived collar

Considering the three regions of V_p (Equation (13)) and depending on the value of δ, Equations (29) and (30), for $X = X_f^*$ and $Q_f^* = Q_f^*(X_f^*)$, reduce to:

$$\begin{aligned}0 = &- Z_Q(X_f^*, Q_f^*) + \frac{(1 - 2\eta Q_f^*) X_f^*}{r - \alpha} e^{-(r-\alpha)T} - \delta \\ &+ \begin{cases} \dfrac{P_L}{r} + \left(1 - \beta_1 \dfrac{\eta Q_f^*}{1 - \eta Q_f^*}\right) a_{11}(1 - \eta Q_f^*)^{\beta_1} X_f^{*\beta_1} & \text{for } \delta_f^0 < \delta < \delta_f^L \\ \left(1 - \beta_1 \dfrac{\eta Q_f^*}{1 - \eta Q_f^*}\right) a_{21}(1 - \eta Q_f^*)^{\beta_1} X_f^{*\beta_1} & \\ \quad + \left(1 - \beta_2 \dfrac{\eta Q_f^*}{1 - \eta Q_f^*}\right) a_{22}(1 - \eta Q_f^*)^{\beta_2} X_f^{*\beta_2} & \\ \quad + \dfrac{(1 - 2\eta Q_f^*) X_f^*}{r - \alpha} & \text{for } \delta_f^L \leqslant \delta < \delta_f^H \\ \dfrac{P_H}{r} + \left(1 - \beta_2 \dfrac{\eta Q_f^*}{1 - \eta Q_f^*}\right) a_{32}(1 - \eta Q_f^*)^{\beta_2} X_f^{*\beta_2} & \text{for } \delta_f^H \leqslant \delta < \delta_f^1 \end{cases}\end{aligned} \tag{51}$$

$$
\begin{aligned}
0 =& Z_X(X_f^*, Q_f^*) + (\beta_1 - 1)\frac{Q_f^*(1-\eta Q_f^*)X_f^*}{r-\alpha}e^{-(r-\alpha)T} - \beta_1 \delta Q_f^* \\
&+ \begin{cases}
\beta_1 \dfrac{Q_f^* P_L}{r} & \text{for } \delta_f^0 < \delta < \delta_f^L \\[2ex]
(\beta_1-\beta_2)a_{22}Q_f^*(1-\eta Q_f^*)^{\beta_2}X_f^{*\beta_2} + (\beta_1 - 1)\dfrac{Q_f^*(1-\eta Q_f^*)X_f^*}{r-\alpha} & \text{for } \delta_f^L \leqslant \delta < \delta_f^H \\[2ex]
(\beta_1-\beta_2)a_{32}Q_f^*(1-\eta Q_f^*)^{\beta_2}X_f^{*\beta_2} + \beta_1 \dfrac{Q_f^* P_H}{r} & \text{for } \delta_f^H \leqslant \delta < \delta_f^1
\end{cases}
\end{aligned} \tag{52}
$$

where

$$
\begin{aligned}
Z_X(X,Q) =& -\beta_1 S(X,Q,T) + \frac{\partial S(X,Q,T)}{\partial X}X \\
=& -(\beta_1-\beta_2)\Big[a_{22}Q(1-\eta Q)^{\beta_2}X^{\beta_2}\left(N(d_{\beta_2}(X,Q,P_L)) - N(d_{\beta_2}(X,Q,P_H))\right) \\
&\qquad + a_{32}Q(1-\eta Q)^{\beta_2}X^{\beta_2}N(d_{\beta_2}(X,Q,P_H))\Big] \\
&- (\beta_1 - 1)\,\frac{Q(1-\eta Q)X}{r-\alpha}e^{-(r-\alpha)T}\left(N(d_1(X,Q,P_L)) - N(d_1(X,Q,P_H))\right) \\
&- \beta_1\left[\frac{QP_L}{r}e^{-rT}N(-d_0(X,Q,P_L)) + \frac{QP_H}{r}e^{-rT}N(d_0(X,Q,P_H))\right]
\end{aligned} \tag{53}
$$

$$
\begin{aligned}
Z_Q(X,Q) =& \frac{\partial S(X,Q,T)}{\partial Q} \\
=& \left(1 - \beta_1\frac{\eta Q}{1-\eta Q}\right)\Big[a_{11}(1-\eta Q)^{\beta_1}X^{\beta_1}N(-d_{\beta_1}(X,Q,P_L)) \\
&\qquad + a_{21}(1-\eta Q)^{\beta_1}X^{\beta_1}\left(N(d_{\beta_1}(X,Q,P_L)) - N(d_{\beta_1}(X,Q,P_H))\right)\Big] \\
&+ \left(1 - \beta_2\frac{\eta Q}{1-\eta Q}\right)\Big[a_{22}(1-\eta Q)^{\beta_2}X^{\beta_2}\left(N(d_{\beta_2}(X,Q,P_L)) - N(d_{\beta_2}(X,Q,P_H))\right) \\
&\qquad + a_{32}(1-\eta Q)^{\beta_2}X^{\beta_2}N(d_{\beta_2}(X,Q,P_H))\Big] \\
&+ \frac{(1-2\eta Q)X}{r-\alpha}e^{-(r-\alpha)T}\left(N(d_1(X,Q,P_L)) - N(d_1(X,Q,P_H))\right) \\
&+ \frac{P_L}{r}e^{-rT}N(-d_0(X,Q,P_L)) + \frac{P_H}{r}e^{-rT}N(d_0(X,Q,P_H))
\end{aligned} \tag{54}
$$

$$
\delta_f^0 = \frac{P_L}{r}\left(1 - e^{-rT}\right) \tag{55}
$$

$$
\delta_f^1 = \frac{P_H}{r}\left(1 - e^{-rT}\right), \tag{56}
$$

δ_f^L is found solving the first branches of equations (51) and (52) substituting X_L for X_f^*, and δ_f^H is found solving the third branches of equations (51) and (52) substituting X_H for X_f^*.

The partial derivatives are obtained following the same steps of Appendix B.1 in Adkins et al. (2019).

C Finite-lived floor

Following similar steps as for the collar, the following equations compute the value and triggers of a finite-lived price floor.

The value of an active project with a finite-lived price floor is given by:

$$V_{fF}(X,Q) = V_{pF}(X,Q) - S_F(X,Q,T) + \frac{Q(1-\eta Q)X}{r-\alpha}e^{-(r-\alpha)T}. \tag{57}$$

where $V_{pF}(X,Q)$, is the value of a perpetual price floor:

$$V_{pF}(X,Q) = \begin{cases} B_{11}X^{\beta_1} + \dfrac{QP_L}{r} & \text{for } X < X_L \\ B_{22}X^{\beta_2} + \dfrac{Q(1-\eta Q)X}{r-\alpha} & \text{for } X_L \leqslant X < X_H \end{cases} \tag{58}$$

with

$$B_{11} = -\frac{P_L^{1-\beta_1}}{\beta_1-\beta_2}\left(\frac{\beta_2-1}{\delta} - \frac{\beta_2}{r}\right)Q(1-\eta Q)^{\beta_1} = b_{11}Q(1-\eta Q)^{\beta_1} \tag{59}$$

$$B_{22} = -\frac{P_L^{1-\beta_2}}{\beta_1-\beta_2}\left(\frac{\beta_1-1}{\delta} - \frac{\beta_1}{r}\right)Q(1-\eta Q)^{\beta_2} = b_{22}Q(1-\eta Q)^{\beta_2}, \tag{60}$$

$S_F(X,Q,T)$, represents the present value of a forward-start perpetual price floor:

$$\begin{aligned} S_F(X,Q,T) =& b_{11}Q(1-\eta Q)^{\beta_1}X^{\beta_1}N(-d_{\beta_1}(X,Q,P_L)) + \frac{QP_L}{r}e^{-rT}N(-d_0(X,Q,P_L)) \\ &+ b_{22}Q(1-\eta Q)^{\beta_2}X^{\beta_2}N(d_{\beta_2}(X,Q,P_L)) \\ &+ \frac{Q(1-\eta Q)X}{r-\alpha}e^{-(r-\alpha)T}N(d_1(X,Q,P_L)), \end{aligned} \tag{61}$$

where $N(.)$ is defined as before.

The optimal capacity that the firm chooses when invests at $X(t) = X$ is obtained with the following maximization condition:

$$\frac{\partial\left(V_{fF}(X,Q) - \delta Q\right)}{\partial Q}\Bigg|_{Q=Q^*_{fF}(X)} = 0 \tag{62}$$

The investment trigger, X^*_{fF}, is obtained by solving the smooth-pasting condition:

$$\beta_1\left(V_{fF}\left(X^*_{fF}, Q^*_{fF}(X^*_{fF})\right) - \delta Q^*_{fF}(X^*_{fF})\right) = \left(\frac{\partial V_{fF}(X, Q^*_{fF}(X))}{\partial X}\bigg|_{X=X^*_{fF}}\right) X^*_{fF} \quad (63)$$

Considering the two regions of V_{pF} Equation (58) and depending on the value of δ, Equations (62) and (63) reduce to nonlinear equations that need to be solved numerically:

$$\begin{aligned} 0 = & -Y_Q(X^*_{fF}, Q^*_{fF}) + \frac{(1-2\eta Q^*_{fF})X^*_{fF}}{r-\alpha} e^{-(r-\alpha)T} + \delta \\ & + \begin{cases} \dfrac{P_L}{r} + \left(1 - \beta_1 \dfrac{\eta Q^*_{fF}}{1-\eta Q^*_{fF}}\right) b_{11}(1-\eta Q^*_{fF})^{\beta_1} {X^*_{fF}}^{\beta_1} & \text{for } \delta^0_{fF} < \delta < \delta^L_{fF} \\ \left(1 - \beta_2 \dfrac{\eta Q^*_{fF}}{1-\eta Q^*_{fF}}\right) b_{22}(1-\eta Q^*_{fF})^{\beta_2} {X^*_{fF}}^{\beta_2} + \dfrac{(1-2\eta Q^*_{fF})X^*_{fF}}{r-\alpha} & \text{for } \delta \geqslant \delta^L_{fF} \end{cases} \end{aligned} \quad (64)$$

$$\begin{aligned} 0 = & \, Y_X(X^*_{fF}, Q^*_{fF}) + (\beta_1 - 1)\frac{Q^*_{fF}(1-\eta Q^*_{fF})X^*_{fF}}{r-\alpha} e^{-(r-\alpha)T} - \beta_1 \delta Q^*_{fF} \\ & + \begin{cases} \beta_1 \dfrac{Q^*_{fF} P_L}{r} & \text{for } \delta^0_{fF} < \delta < \delta^L_{fF} \\ (\beta_1 - \beta_2) b_{22} Q^*_{fF}(1-\eta Q^*_{fF})^{\beta_2} {X^*_{fF}}^{\beta_2} + (\beta_1 - 1)\dfrac{Q^*_{fF}(1-\eta Q^*_{fF})X^*_{fF}}{r-\alpha} & \text{for } \delta \geqslant \delta^L_{fF} \end{cases} \end{aligned} \quad (65)$$

where

$$
\begin{aligned}
Y_X(X,Q) =& -\beta_1 S_F(X,Q,T) + \frac{\partial S_F(X,Q,T)}{\partial X} X \\
=& -(\beta_1-\beta_2)\left[b_{22} Q(1-\eta Q)^{\beta_2} X^{\beta_2} N(d_{\beta_2}(X,Q,P_L))\right. \\
& -(\beta_1-1)\frac{Q(1-\eta Q)X}{r-\alpha} e^{-(r-\alpha)T} N(d_1(X,Q,P_L)) \\
& -\beta_1 \frac{QP_L}{r} e^{-rT} N(-d_0(X,Q,P_L))
\end{aligned} \tag{66}
$$

$$
\begin{aligned}
Y_Q(X,Q) =& \frac{\partial S_F(X,Q,T)}{\partial Q} \\
=& \left(1-\beta_1\frac{\eta Q}{1-\eta Q}\right)\left[b_{11}(1-\eta Q)^{\beta_1} X^{\beta_1} N(-d_{\beta_1}(X,Q,P_L))\right. \\
& + \left(1-\beta_2\frac{\eta Q}{1-\eta Q}\right)\left[b_{22}(1-\eta Q)^{\beta_2} X^{\beta_2} N(d_{\beta_2}(X,Q,P_L))\right. \\
& + \frac{(1-2\eta Q)X}{r-\alpha} e^{-(r-\alpha)T} N(d_1(X,Q,P_L)) \\
& + \frac{P_L}{r} e^{-rT} N(-d_0(X,Q,P_L))
\end{aligned} \tag{67}
$$

$$
\delta_{fF}^0 = \frac{P_L}{r}\left(1-e^{-rT}\right) \tag{68}
$$

δ_{fF}^L is found solving the first branches of equations (64) and (65) substituting X_L for X_{fF}^*.

D Finite-lived fixed price

The value of an active project with a finite-lived fixed price F is given by:

$$
V_{fC}(X,Q) = \frac{F}{r}\left(1-e^{-rT}\right) + \frac{Q(1-\eta Q)X}{r-\alpha} e^{-(r-\alpha)T}. \tag{69}
$$

The optimal capacity that the firm chooses when invests at $X(t)=X$ is obtained with the following maximization condition:

$$
\frac{\partial\left(V_{fC}(X,Q)-\delta Q\right)}{\partial Q}\Bigg|_{Q=Q_{fC}^*(X)} = 0 \tag{70}
$$

The investment trigger, X_{fC}^*, is obtained by solving the smooth-pasting condition:

$$
\beta_1\left(V_{fC}\left(X_{fC}^*, Q_{fC}^*(X_{fC}^*)\right) - \delta Q_{fC}^*(X_{fC}^*)\right) = \left(\frac{\partial V_{fF}(X, Q_{fC}^*(X))}{\partial X}\Bigg|_{X=X_{fC}^*}\right) X_{fC}^* \tag{71}
$$

Solving simultaneously Equations (70) and (71), for $X = X_{fC}^*$ and $Q_{fC}^* = Q_{fC}^*(X_{fC}^*)$, allows finding the optimal capacity Q^*_{fC} and the investment trigger X^*_{fC}:

$$Q^*_{fC} = \frac{1}{\eta(\beta_1+1)} = Q^* \tag{72}$$

$$X^*_{fC} = \frac{\beta_1+1}{\beta_1-1}(r-\alpha)e^{(r-\alpha)T}\left(\delta - \frac{F}{r}\left(1-e^{-rT}\right)\right) \tag{73}$$

E The iso-elastic demand function case

Huisman and Kort (2015) study the case of a plain investment project for the iso-elastic demand function. Without a collar, the firm invests at the following optimal threshold:

$$X^* = \left(\frac{\beta_1(1-\gamma)\delta_0}{(\beta_1\gamma-1)\delta_1}\right)^\gamma \frac{r-\alpha}{1-\gamma}\delta_1 \tag{74}$$

with capacity:

$$Q^* = \frac{\beta_1(1-\gamma)\delta_0}{(\beta_1\gamma-1)\delta_1} \tag{75}$$

with $\beta_1\gamma > 1$, otherwise the firm would postpone investment forever (see Dixit and Pindyck, 1994) and $\delta_0 > 0$, otherwise the firm would invest immediately (see Huisman and Kort, 2015).

The value of the producer surplus without the collar is

$$PS(X) = \left(\frac{X^*Q^{*1-\gamma}}{r-\alpha} - (\delta_0+\delta_1 Q^*)\right)\left(\frac{X}{X^*}\right)^{\beta_1}, \tag{76}$$

and the value of the consumer surplus is:[20]

$$CS(X) = \frac{\gamma}{1-\gamma}\frac{X^*Q^{*1-\gamma}}{r-\alpha}\left(\frac{X}{X^*}\right)^{\beta_1}, \tag{78}$$

The total surplus is:

$$TS(X) = \left(\frac{1}{1-\gamma}\frac{X^*Q^{*1-\gamma}}{r-\alpha} - (\delta_0+\delta_1 Q^*)\right)\left(\frac{X}{X^*}\right)^{\beta_1} \tag{79}$$

A social planner, pursuing the goal of maximizing the total surplus, has the following

[20]For the iso-elastic demand function, the instantaneous consumer surplus is:

$$\int_{XQ^{-\gamma}}^{\infty}\left(\frac{X}{P}\right)^{\frac{1}{\gamma}} dP = \frac{\gamma}{1-\gamma}XQ^{1-\gamma} \tag{77}$$

investment trigger and capacity choice:

$$X_w^* = \left(\frac{\beta_1(1-\gamma)\delta_0}{(\beta_1\gamma-1)\delta_1}\right)^\gamma (r-\alpha)\delta_1 = (1-\gamma)X^* \tag{80}$$

$$Q_w^* = \frac{\beta_1(1-\gamma)\delta_0}{(\beta_1\gamma-1)\delta_1} = Q^* \tag{81}$$

Following similar steps as for the linear demand function, the value of an active project with a finite-lived collar is given by:

$$V_f(X,Q) = V_p(X,Q) - S(X,Q,T) + \frac{Q^{1-\gamma}X}{r-\alpha}e^{-(r-\alpha)T}. \tag{82}$$

where

$$V_p(X,Q) = \begin{cases} A_{11}X^{\beta_1} + \dfrac{QP_L}{r} & \text{for } X < X_L \\ A_{21}X^{\beta_1} + A_{22}X^{\beta_2} + \dfrac{Q^{1-\gamma}X}{r-\alpha} & \text{for } X_L \leqslant X < X_H \\ A_{32}X^{\beta_2} + \dfrac{QP_H}{r} & \text{for } X \geqslant X_H \end{cases} \tag{83}$$

$X_L = P_L Q^\gamma$, $X_H = P_H Q^\gamma$, $A_{11} = a_{11}Q^{1-\beta_1\gamma}$, $A_{21} = a_{21}Q^{1-\beta_1\gamma}$, $A_{22} = a_{22}Q^{1-\beta_2\gamma}$, $A_{32} = a_{32}Q^{1-\beta_2\gamma}$, and

$$\begin{aligned} S(X,Q,T) = {} & a_{11}Q^{1-\beta_1\gamma}X^{\beta_1}N(-d_{\beta_1}(X,Q,P_L)) + \frac{QP_L}{r}e^{-rT}N(-d_0(X,Q,P_L)) \\ & + a_{21}Q^{1-\beta_1\gamma}X^{\beta_1}\left(N(d_{\beta_1}(X,Q,P_L)) - N(d_{\beta_1}(X,Q,P_H))\right) \\ & + a_{22}Q^{1-\beta_2\gamma}X^{\beta_2}\left(N(d_{\beta_2}(X,Q,P_L)) - N(d_{\beta_2}(X,Q,P_H))\right) \\ & + \frac{Q^{1-\gamma}X}{r-\alpha}e^{-(r-\alpha)T}\left(N(d_1(X,Q,P_L)) - N(d_1(X,Q,P_H))\right) \\ & + a_{32}Q^{1-\beta_2\gamma}X^{\beta_2}N(d_{\beta_2}(X,Q,P_H)) + \frac{QP_H}{r}e^{-rT}N(d_0(X,Q,P_H)) \end{aligned} \tag{84}$$

The optimal capacity that the firm chooses when investing at $X(t) = X$ is obtained with the following maximization condition:

$$\left.\frac{\partial\left(V_f(X,Q) - (\delta_0 + \delta_1 Q)\right)}{\partial Q}\right|_{Q=Q_f^*(X)} = 0 \tag{85}$$

The investment trigger, X_f^*, is obtained by solving the smooth-pasting condition:

$$\beta_1\left(V_f\left(X_f^*, Q_f^*(X_f^*)\right) - \left(\delta_0 + \delta_1 Q_f^*(X_f^*)\right)\right) = \left(\left.\frac{\partial\left(V_f(X, Q_f^*(X)) - \delta_1 Q_f^*(X)\right)}{\partial X}\right|_{X=X_f^*}\right) X_f^* \tag{86}$$

When a collar is offered, the total surplus is affected by both X_f^* and Q_f^*:

$$TS_f(X) = \left(\frac{1}{1-\gamma} \frac{X_f^* {Q_f^*}^{1-\gamma}}{r-\alpha} - (\delta_0 + \delta_1 Q_f^*) \right) \left(\frac{X}{X_f^*} \right)^{\beta_1}, \tag{87}$$

with the consumer surplus being:

$$CS_f(X) = \frac{\gamma}{1-\gamma} \frac{X_f^* {Q_f^*}^{1-\gamma}}{r-\alpha} \left(\frac{X}{X_f^*} \right)^{\beta_1}, \tag{88}$$

where the producer benefits becomes:

$$PS_f(X) = F_f(X) = \left(V_f \left(X_f^*, Q_f^* \right) - \left(\delta_0 + \delta_1 Q_f^* \right) \right) \left(\frac{X}{X_f^*} \right)^{\beta_1} \tag{89}$$

The cost of the collar for the government is:

$$G_f(X) = TS_f(X) - CS_f(X) - PS_f(X) \tag{90}$$