Real options approach for a staged field development with optional wells

Semyon Fedorov[1], Verena Hagspiel[1], and Thomas Lerdahl[2]

[1]Norwegian University of Science and Technology (NTNU)
[2]OKEA ASA

ABSTRACT

The decreasing average size of new discoveries in mature production areas makes the base of oil field investment decisions more uncertain than ever before. Fewer appraisal wells, which allow to decrease the amount of subsurface uncertainty, are typically drilled before the development of a small field compared to large fields. Therefore, new solutions are required to make small discoveries commercial given both technical and market uncertainties. In such conditions, accounting for managerial flexibilities that enable to change the course of the project due to new information, becomes even more important for investment valuation.

Combining the real options approach and decision analysis, we present a novel model that allows to identify additional value created by a sequential drilling strategy for field development under oil price and resource uncertainty. We capture the sequence of key investment and operating decisions of a marginal field development in cooperation with an oil industry partner, building a synthetic (yet realistic) project case. Addressing the flexibility to divide production wells drilling into two stages, we evaluate the option to wait to expand the production by drilling additional wells using the least-squares Monte Carlo algorithm.

We identify the conditions under which the staged (phased) development is preferred compared to standard development. Furthermore, we propose a decision rule determining the optimal expansion timing based on new information on the reservoir and the oil price uncertainty. Our results suggest that staged development carries large upside potential for marginal field development under extensive reservoir uncertainty. We also illustrate that partial hedging against the downside risks within a staged development can improve project's economy significant enough to justify investment.

Keywords

Real options   Decision analysis   Marginal fields   Staged field development

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1 INTRODUCTION

The decreasing average size of new oil discoveries (Norwegian Petroleum Directorate 2019) is one of the most crucial issues that oil and gas exploration and production (E&P) companies are currently facing in mature production areas such as the Norwegian Continental Shelf (NCS). Exploration and development of smaller reservoirs, located in more challenging formations, demands costly technology and advanced engineering solutions to access it (Lund 1999). Furthermore, fewer appraisal wells are typically drilled before the development of a small field than in case of larger discoveries. This makes the investment and development decision base relatively more uncertain. Together with the adverse price environment, it led industry's majors to explore less actively or even withdraw from such areas in recent years[1]. Nevertheless, smaller reservoirs could still represent substantial value and might be attractive investment opportunities if the decision making process addresses field development risks and upside potentials adequately. Better informed decisions that exploit the data generated during the course of a project become highly important to produce hydrocarbons in an efficient and cost effective way when prominent downside risks are present. In their Resource report 2019, the Norwegian Petroleum Directorate emphasizes "the importance of continuing to find good solutions in order to make small discoveries commercial". This puts particular focus on flexible instruments, that allow to react on the outcome of uncertain parameters by changing the course of the project.

In this study, we analyse one of the possible solutions - an opportunity to invest in field development sequentially within a staged drilling strategy. This allows to collect additional information between the stages and tune the development plan at the subsequent stages of the project. In order to address this flexibility within the economic valuation of a project, we propose a novel methodology built to provide a decision support tool for oil companies. We put a particular focus on the relevance of this methodology for small field development cases. The key contributions of this paper are: (1) propose the staged drilling as a strategy for the field development under prominent reservoir uncertainty (2) present a methodology to optimize the production expansion decision during the production phase based on new information on two types of uncertainty - the estimated production rate and the oil price (3) account for the whole range of probable outcomes of technical uncertainty within the optimization and valuation procedure.

In order to evaluate the investment under uncertainty with implied flexibility, we apply the real option approach (ROA) combined with decision analysis (DA). Combining these tools enables the decision maker to accommodate both technical and market uncertainties in the economic model representing a complex E&P project (Jafarizadeh et al. 2009). A static Discounted Cash Flow (DCF) approach, traditionally used in the industry, fails to capture multiple uncertainties and inherent flexibilities within the investment valuation. We show how the decision maker can optimize the drilling strategy based on the ROA by choosing for staged development amid substantial reservoir uncertainty. To the best of our knowledge, such models do not exist neither for small nor for large field development.

We then analyse the value of flexibility created by a staged oil field development strategy and compare it to what we refer to as standard development. In standard development a predefined set of production and injection wells are planned to be drilled and completed prior to production start of a field. All necessary facilities required to produce the field over the expected lifetime are put in place before production start. This implies that measures to increase hydrocarbon recovery during the operation of the field, such as drilling additional wells or improved oil recovery (IOR), may be identified, but are not included in the economic analysis at the time of the investment decision. Compared to that, the basis for staged development is a first

[1] ExxonMobil, for example, sold their Norwegian assets to Vår for $4.5bn in 2019 (see https://www.ft.com/content/f03fec96-e085-11e9-b112-9624ec9edc59)

development phase (Stage 1) which contains a number of predefined wells, that is lower than in case of the standard development, and facilities that shall be in place at production start-up. In addition, the operator has the opportunity to expand the development by drilling additional production wells after some time of operation during Stage 1. This time is needed to gather and process additional information on the reservoir, which might improve the field development decision making at the second stage of the project. Such a strategy is highly relevant when subsurface uncertainty is prominent. If the information generated during the initial stage indicates that the reservoir properties are poor, the decision maker is able to avoid drilling superfluous wells. This flexibility provides a partial hedge against reservoir risk and can be addressed within the valuation procedure already at the stage of the investment decision. The methodology we present in this paper allows the decision maker to exploit the benefits of the staged development approach and identify the potential to create additional value for both large and small fields. A special focus, however, is made on small discoveries due to their sensitivity to the downside risks, as in case of an unfavorable outcome of uncertain conditions, project's economy might be disrupted.

Our methodology is built upon several blocks. We first build the field production forecast, accounting for the uncertainty in initial production rates. This allows us to realistically reflect the decision maker's knowledge on the reservoir characteristics and account for a possibility of facing a "low reservoir performance" case that might lead to a negative overall project value. With the field design basis, drainage strategy and production forecast given, we estimate capital and operational expenses throughout the whole lifetime of the field. We then proceed with building future oil price curves, based on the two-factor stochastic price model (Schwartz & Smith 2000), calibrated using the Kalman filter and historical market data. We assume that the production at Stage 1 generates perfect information on the technical uncertainty. Based on this data and the conditions of the oil market, we consider that the operator can make a decision to expand the production by drilling additional wells within a predetermined period of time. Combining the simulations of the production rate, cost profiles and oil prices, we, furthermore, construct several sets of the expected yearly project cash flows associated with the respective expansion decisions. The waiting option to expand is then formulated using the least-squares Monte Carlo (LSM) framework (Longstaff & Schwartz (2001)). In order to optimize the expansion decision within the LSM algorithm for American options by comparing the immediate exercise value with the estimated value from continuation at each decision point, a regression function accommodating both the oil price and production rate parameters is used. We then identify the optimal expansion timing (drilling for Stage 2) for each simulated case and construct a threshold boundary representing combinations of the production rate and the oil price which are considered to trigger the expansion decision. Finally, we calculate the values of the project under standard and staged development strategies and identify under which conditions the staged development is the preferred choice.

In Fedorov et al. (2020) we analysed the potential of the staged development on a benchmark reservoir model, Olympus (Fonseca et al. 2018), where well control optimization based on several realizations of the reservoir model was used. With this paper we aim to propose a methodology to evaluate the staged development strategy, rather than study a specific case, where project parameters and detailed reservoir characterization might raise barriers in the understanding of how to benefit from the flexibility. The value of the staged development depends strongly on the problem at hand. In order to find out whether this strategy can create additional value, any decision maker can use their project-specific input such as development plan, expected production profiles, costs, etc. Overall, our results lead to recommendations that can both facilitate and improve the field development decision making process. The modelling approach, therefore, is expected to be of both academic and industry value.

1.1 Literature review

With this paper we aim to contribute to three different strands of literature. The first strand is represented by literature focusing on specifics of economic assessment of small oil projects and decision making under high amount of subsurface uncertainty. Among the few contributions in this field are Laine et al. (1997), who use an example of two Norwegian fields in order to model deferral, expansion and abandonment options and demonstrate that option valuation techniques can add substantial value to marginal discoveries. Lund (1999) analyzes an investment in a small oil field on the Norwegian continental shelf and emphasizes the importance of the operator's flexibility to change the course of the project during the operating phase, for overall project value, especially "when the uncertainty surrounding the reservoir is high". Galli et al. (2001) study a small satellite gas field in the North Sea using a real option framework to evaluate the impact of drilling decisions on the project value. Armstrong et al. (2004) use information from production logging and a copula-based Bayesian updating scheme for real options valuation of small oil projects. Dias (2004) briefly discusses the possibility of phasing the investment in several stages accounting for an option to expand the production by drilling additional wells. He proposes this strategy as a method to deal with high amount of subsurface uncertainty, which is a typical problem for small field development. Dias (2004) discusses a hypothetical method to analyze this flexibility as a sequence of actions that a decision maker should make to apply the ROA to this problem. Jafarizadeh & Bratvold (2015) demonstrate the differences in economic analysis of small and large discoveries by evaluating a waiting-to-invest option in two hypothetical exploration opportunities (a large and a small prospect). Jafarizadeh & Bratvold (2015) show that projects with smaller recoverable volumes, shorter lead times for development, and a steeper production decline are more sensitive to the variability of oil prices and discount rate.

However, none of the existing literature, including Dias (2004), has studied the effect of the staged approach on the value of a E&P project and showed how to optimize the decision to drill optional wells under technical and market uncertainties neither for large nor for small oil fields. We, in our turn, contribute to this strand literature by building a formal model to quantify the value of this strategy, which is based on real options analysis and a production expansion optimization algorithm, and provide numerical results for a case study. The discussion in Dias (2004), in fact, underpins the value of our work and its contribution both to industry and to academia.

Second, this work aims to contribute to the strand of literature that combines the real options approach and decision analysis within a single valuation procedure. The classic real options literature applies methods that are based on the possibility to find a market-traded portfolio that could replicate a real-world investment in order to perform a real option valuation. Introducing technical uncertainty to these methodologies is challenging and can lead to inaccurate valuation results (Jafarizadeh et al. 2009). In the decision analysis approach, in turn, the decision maker's beliefs about the project are captured by assessing subjective probabilities for the uncertainties. The valuation of a risky project is typically done using a decision tree or a dynamic program, neglecting market opportunities for hedging price risks. Smith & Nau (1995) is a pioneering contribution to propose an integrated approach combining option pricing methods and decision analysis to accommodate market uncertainty that can be hedged and technical uncertainty that can not be hedged. Copeland & Antikarov (2001) and Brandão et al. (2005) both use traditional decision analysis tools - binomial decision trees and binomial lattices - for solving real-options problems. They use a mix of discounted cash flow and risk-neutral methods, which is criticized by Smith (2005), who suggests to use a fully risk-neutral approach leading to a single coherent valuation model that can be used to value projects with and without options. Comparing the competing methodologies, Smith (2005) analyses an investment opportunity in an oil production project and concludes that ”there is much to be gained from integrating the real options and decision analysis approaches to project evaluation”.

By applying the simulation-based risk-neutral valuation approach, we adopt the method presented by Smith & Nau (1995) and Smith (2005) and contribute to the literature on the integrated ROA and DA by accounting for the whole range of possible outcomes of the technical uncertainty in accordance with the decision maker's arbitrary probabilities within the valuation procedure. Unlike Dias (2002) and Santos et al. (2017*b*), we therewith capture not only a discretized representation of the technical uncertainty, as typically used for a decision tree and lattice model approach. Using a simulation approach instead, allows us to construct a better representation of how the reservoir risk affects the decision making process within a field development case, which would give us more accurate valuation results.

Another important aim of our analysis is to study the effect of new information on decision making. The data generated during the initial stage of the project is used to update the decision maker's knowledge on the reservoir in order to optimize the further development strategy. This might create additional value, which can be identified already at the investment decision phase. Therefore, the third strand of literature that we aim to contribute to are publications focusing on the value of information for natural resource projects taking an ROA approach. Among contributions that optimize timely decisions using new information are Chorn et al. (1997), who study the application of option pricing techniques to value information on offshore gas field reserve volume and selection of production strategy. Gallant et al. (1999) use "learning models"[2] to capture changing expectations as new information is gained over an E&P project's life. Dias et al. (1997) and Dias (2002) study the effect of timing and "drilling games" with strategic interaction in E&P projects explicitly modelling the value of learning. Cunningham & Begg (2008) analyze various scenarios of a sequential drilling program using new information. Therewith, they provide an example of how the value of information can be used proactively in the construction of drilling strategies. This allows to avoid the overspending on costly tests, which cannot change initial beliefs on the project and contribute to better decision making. Santos et al. (2017*a*) introduce an uncertainty management method complementing techniques of acquiring new information and adding flexibility to the production system in order to reduce the downside risk within a robust production strategy. Kullawan et al. (2018) develop a discretized stochastic dynamic programming approach for sequential decisions in geosteering operations based on real-time information. Accounting for this information allows to optimize well trajectory and increase economic value. Hanea et al. (2019) assess the value of learning created by the data that is generated within a sequential drilling strategy. Using a synthetic reservoir case they demonstrate how history matching and updating the development strategy more frequently can improve the field development.

Although we assume that the decision maker is able to acquire perfect technical information, we contribute with this paper to above-mentioned selection of literature by demonstrating how the information generated at the initial production phase can be used to optimize the drilling strategy and therewith, to increase the economic value. This is achieved by including the reservoir uncertainty parameter into the LSM regression. By introducing a threshold boundary representing combinations of the production rate and the oil price which are considered to trigger the expansion decision, we directly show how the updated knowledge on the reservoir uncertainty influences the decision-making. We also demonstrate that the additional information that can be potentially acquired with the future production experience, should not be ignored when making the investment decision. For a small field development this information is, in fact, crucial when dealing with the

[2]A "learning model" is defined by Gallant et al. (1999) as a depiction of how new information allows a decision maker to revise their initial belief on an uncertain event. Gallant et al. (1999) argue that the ”learning model helps us take advantage of new information in the evaluation of a project's potential, not just in its execution”.

subsurface uncertainty. In our further research, we plan to extend the proposed methodology by incorporating a reservoir model, which will allow to realistically account for the impact of imperfect information using history-matching and Bayesian updating.

The remainder of this paper is organized as follows. Section 2 introduces key features of the staged development and explains the limitations of the classic approach to evaluation of an investment with flexibilities. In Section 3 we formulate and develop the modelling approach for the project valuation of a staged development with an option to expand. In Section 4, we present a case study to apply our modelling approach on a realistic problem. Section 5 presents results including a sensitivity analysis and robustness check. Section 6 concludes the paper.

2 BACKGROUND

In this section we provide the motivation behind adopting the staged drilling strategy (see Section 2.1) including both the risks and benefits associated with deferring of the second stage of an oil field development project. Table 1 summarizes the main features of the staged development strategy compared to the standard one. Furthermore, we explain the operator's decision making process for the optimization of the production expansion once the technical uncertainty is revealed. In Section 2.2 we discuss the essence of using a more advanced economic analysis compared to a static discounted cash flow approach in order to evaluate investment with embedded options. We motivate the need for using a combination of real option approach and decision analysis, for cases when managerial flexibility is of high importance for the project value.

2.1 Staged development with an option to expand

Decisions related to petroleum exploration and production are very complex because of the large number of aspects involved in the process (Suslick et al. 2009). During the design phase, the project team has – amongst many other things - to decide on the optimal number of development wells (both producers and injectors) and their placement. This decision is particularly challenging in the face of both technical and market uncertainties. Moreover, the investment in wells is considered to be irreversible as an operator cannot recover drilling expenditures once they are made. Consequently, the team has to make a fundamental decision that cannot be changed in the course of the project, by finding a trade-off between the expected value creation by an additional well being drilled and the associated costs within prevailing uncertainties.

Table 1: Key features of the staged development strategy compared to the standard one

Risk	Staged development
Low reservoir/oil price scenario	Ability to mitigate downside risks by not drilling uneconomic wells
Timing	Loss of value due to waiting (time value of money) and probable oil loss due to migration during Stage 1
Well placement	Improved well placement at the Stage 2 based on production experience and acquired data leading to increase of expected recovery
Capital expenditure (CAPEX)	Possibility to defer a significant amount of CAPEX until Stage 2

A standard way to decrease reservoir uncertainty in order to improve the quality of well placement and field design, is drilling appraisal wells. In some projects, however, this might not be advisable as the investment in the extensive appraisal program is considered to be inadequately costly compared to the potential of information revelation (Dias 2004). This is especially true in the case of small field development with marginal economy.

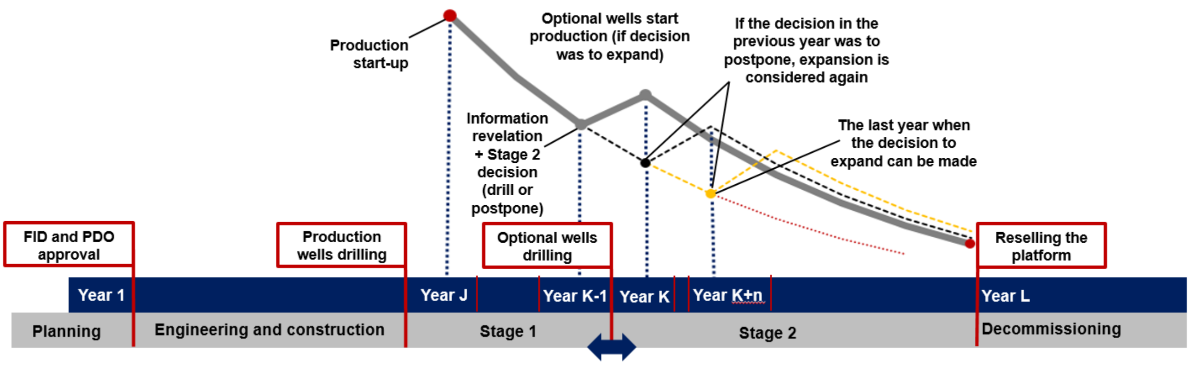

Figure 1: Decision gates of a general project under the staged development strategy

Under such conditions, a better strategy to cope with the reservoir uncertainty might be to start drilling production wells based on the available information without an additional appraisal program. However, instead of drilling all potential production wells, some of which are likely to fail in ensuring an economical production rate, the operator can prioritize wells and drill them sequentially, i.e. developing an oil field in several stages. Under a staged development strategy, the decision maker first drills several wells in those locations that are less exposed to reservoir risk. This is done to start the production at the initial stage of the field life, that may last from some months to several years. During the Stage 1 the operator gathers data and performs specific tests that they use to optimize the Stage 2 drilling decision. Figure 1 provides a sketch of the timeline and production profile for such a staged development strategy. After the final investment decision (FID) and authority approval of the Plan for development and operation (PDO), the project enters the engineering and construction phase. Drilling of production wells typically commences 1-2 years prior to production start. After a certain period, experiences from drilling and production of these wells can be evaluated and used to update the underlying reservoir models. Based on the updated knowledge on the reservoir, managers can make the decision to drill additional production wells to increase the field production potential. New information also allows to optimize the number, placement and choose the best time to drill optional wells. Alternatively, the decision maker might refrain from drilling optional wells and keep producing with the same number of wells in order to avoid investing in wells that could potentially prove to be uneconomic. In that case, the process of considering optional wells can be repeated after additional data has been collected. The staged approach allows to mitigate downside risk, which could reduce the economic value of the project if the company commits to drill all the wells before actually starting to produce the field (Willigers et al. 2017). This feature is particularly relevant for marginal fields, where drilling costs may represent a large part of capital expenditure and highly influence the overall value of the project.

Once the data is gathered and analysed, managers are able to take advantage of the flexibility to expand the production by drilling optional wells only in favorable scenarios. The management may also decide to wait with the Stage 2 drilling in case if market conditions are not good enough. For large field development, whose lifetime might be more than 30 years, holding this option for a long time could be reasonable. But in the case of small fields, holding the expansion option for more than a couple of years may not make much sense due to the relatively short lifetime of the project. Moreover, the longer the optional wells remain undrilled, the higher is the potential loss of value through hydrocarbon migration away from the location of these optional wells and towards the location of existing producers. The effect of this phenomenon is described by (Dias 2004) ”as a dividend lost by the option holder", giving them "a higher incentive to drill the optional well earlier".

In addition, by drilling not all the possible wells immediately, the operator might loose some portion of value due to the depreciation effect of delayed production. This is especially true if the reservoir performance is better than expected. In reality, an E&P company has also to account for additional investment costs that the expansion might require. The amount is mainly dictated by the state of the production system at the moment when the decision to expand is made. The least costly way is to fill spare capacity of the facilities as soon as the field enters the production decline. Other strategies imply that the field operator designs the field development process by leaving some extra capacity idle at Stage 1 to be able to increase the production rate while the end of plateau during Stage 1 is not reached yet.

Within a staged development strategy, field engineers and economists perform joint assessment to identify production wells (and their location) to be drilled for the first stage of the project. Typically, these will be wells that are expected to generate a large amount of production and/or are less exposed to the reservoir uncertainty and thus ensure more economical production not only at the initial stage, but throughout the whole lifetime of the project. Potential candidates for Stage 2 are, therefore, the locations with the highest risk of losing value due to technical uncertainty.

However, the field development is exposed not only to technical uncertainty, but to market conditions as well. The decision to expand the production during a drastic oil price downturn could disrupt the revenue, particularly of a small field, whose production lifetime might be limited to 5-8 years (Rasmussen 2015) compared to a 30+ years lifespan of large discoveries. Contrarily, the decision to expand could be still beneficial amid a price surge, despite the fact that the cumulative production during Stage 1 could indicate that reservoir performance was quite poor. Therefore, the optimal expansion policy must be based not only on the reservoir information revelation, but on market development as well. The moment when managers can exercise the option is not limited to the point in time right after the information revelation. Price uncertainty might motivate to postpone the decision to exercise the option. Assuming that the expansion decision can be made once a year, Figure 1 shows that the operator can shift the last year of Stage 1, i.e. Year K-1, when the optional wells are drilled, and postpone Stage 2.

2.2 Limitations of the DCF approach for investment evaluation

As we stated in the introduction, E&P companies typically apply a Discounted Cash Flow (DCF) approach to evaluate investment opportunities. In the classical DCF approach, investment decisions are based on the net present value (NPV) that is calculated by discounting the cash flows using a risk-adjusted discount rate as given by

$$NPV = \sum_{t=t_0}^{T} \frac{\mathbb{E}\left[CF_t)\right]}{(1+R_a)^t}, \tag{1}$$

where $\mathbb{E}(t)\left[CF_t\right]$ is the expected cash flow of the period t, T is the number of periods and R_a is the risk-adjusted discount rate. The risk-adjusted discount rate reflects both the time value of money and risk. It is a compensation demanded by investors for the risks that are associated with holding an asset (Brealey et al. 2012). Most companies use a rate equal to their weighted average cost of capital (WACC), arguing that the investment should cover both costs of debt and capital (Jafarizadeh & Bratvold 2019). In fact companies often apply a single discount rate to all their project when evaluating investment decisions or communicating with the authorities[3], ignoring specific features of each individual project. This might result in incorrect valuation

[3] Oil companies acting on the NCS have to submit a plan for development and operation of a petroleum deposit to the The Ministry of Petroleum and Energy of Norway before they start the oil field development. The official guidelines recommend using a standard rate

leading to poor decision making, especially in case when investment opportunities are exposed to various uncertainties.

The two main types of uncertainties that we aim to account for in the current study, reservoir and oil price uncertainty, are of completely different risk nature. Using a risk-neutral pricing approach that is an integral part of our method, proved to be a correct way to perform the valuation from a methodological point of view (Smith & Nau (1995), Smith & McCardle (1999)). It distinguishes between market risks, hedgeable by trading securities, and private risks, which are project-specific and cannot be hedged with any market instrument. In risk-neutral approach, instead of risk-adjusting the whole cash flow, the decision maker treats uncertainties separately. In our case this means that we use the risk adjusted stochastic process for market risks (oil price) and true probabilities for private risks (reservoir uncertainty in order to calculate the expected value of investment. The resulting cash flows are then discounted at the risk-free rate (Jafarizadeh et al. 2012), as given by

$$NPV = \sum_{t=t_0}^{T} \frac{\mathbb{E}[CF_t)]}{(1+R_f)^t} \tag{2}$$

where R_f is the risk-free discount rate.

We illustrate this technique by building the production profiles in Section 3.1 based on the decision maker's beliefs on the probabilities associated with the technical uncertainty, and construct the risk-neutral process for the future oil prices in Section 3.3. The risk-neutral valuation procedure is then implemented in Section 3.4.

However, the difference between the traditional DCF and the method that we use in this work is not only in how we treat risk in the project cash flow. The DCF approach does not allow to accurately capture the managerial ability to change the course of the project. It also ignores values of embedded options, as it is based on a "static" view, in which future decisions are assumed to depend only on information available now, while additional information that would be available at later stages is ignored. The real option approach, in turn, is a tool that allows to account for this flexibility and evaluate the additional value associated with it (Jafarizadeh et al. 2012). Different approaches can be used to apply the real options analysis for the project valuation. These might be decision tree models, binomial lattices or simulation-based approaches. In this paper, we apply a simulation-based approach - the least-squares Monte Carlo (LSM) algorithm, which allows us to represent the influence of project parameters, uncertainties and the flexibility on the project performance using the simulation. In Section 3.4 we demonstrate how the risk-neutral procedure can be implemented using the LSM algorithm to evaluate the project with an option to expand.

3 METHODOLOGY

We now introduce the key components of the proposed procedure allowing to evaluate staged investment in a small offshore field with an option to expand under the presence of technical and market uncertainty. Figure 2 represents the main building blocks and information flow of a modern development project, that we propose in this study. First, based on available data, a subsurface assessment is carried out in order to generate reservoir model(s) that allow to simulate the outcome of alternative drainage strategies in terms of production. At the same time, a design basis for the field development and a proposed drainage strategy are matured. Based on these, both a production forecast (see Section 3.1) and cost estimates (see Section 3.2) for the envisioned development are established, including uncertainty estimates. This data can be combined

of 7% to justify profitability of the project. - The Norwegian Petroleum Directorate PDO Guidelines https://www.npd.no/globalassets/1-npd/regelverk/forskrifter/en/pdo-and-pio.pdf with economic assumptions regarding oil prices (see Section 3.3), exchange rates and similar to model the cash flows of the project. Together, this serves as an input to a real option valuation of alternative drainage strategies. This is done using the LSM algorithm (see Section 3.4), which allows to optimize the decision to expand the production by accounting for both technical and price uncertainty. As our main goal is to compare the staged development strategy with the standard one, we perform a symmetric analysis of the two strategies based on the same price and production assumptions, for a fair comparison.

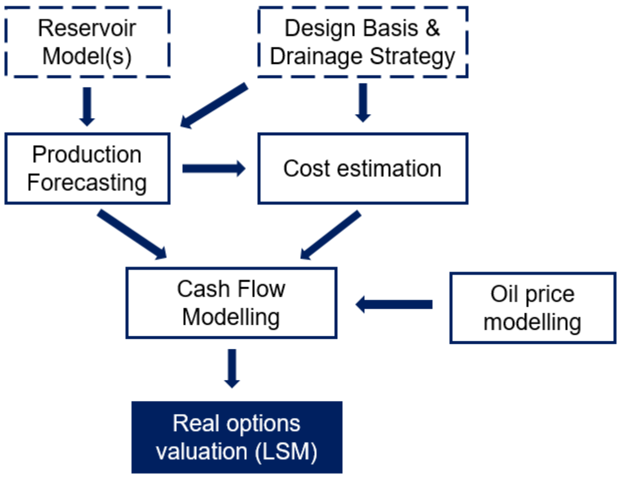

Figure 2: Valuation procedure

In this paper, we focus on the introduction and implementation of this methodology to evaluate the opportunity to phase a field development into two stages with an option to expand. The case study that is presented to illustrate the application of this method is a synthetic data set, described in detail in Section 4. We do not include any underlying reservoir model nor a specific design basis of the field development, allowing the methodology be easily adopted for other industry cases. In Fedorov et al. (2020) we used the well control optimization based on several realizations of the benchmark reservoir model, Olympus (Fonseca et al. 2018) to verify whether the staged development can create additional value. Using an analytical approach to model the reservoir uncertainty brings both benefits and limitations for the valuation procedure.

3.1 Production profiles

We start our modelling procedure with the estimation of the yearly production rate for each well for both standard (non-staged) and staged development strategies. It is vital to realistically assess the amount of underlying reservoir uncertainty that the operator is expected to deal with, before committing to develop the field. The field development decision making process must aim to capture all probable outcomes of the uncertain parameters and exploit the available flexibility to react on these outcomes.

We use the probability density functions of each well's initial production rate (in the first year of production) as input data to estimate the field production potential. Such probability density functions are typically generated by the reservoir and production engineers and are a representation of the technical uncertainty that affects the production rates. Figure 3 illustrates potential probability distribution functions of the initial production rates for wells within the standard development strategy. Let S denote the number of wells drilled for Stage 1 and N the total number of wells. Then $N - S$ is the number of wells considered for Stage 2. In case of the standard development strategy, all wells $i = 1...N$ are drilled before the production start-up, while in case of the staged development, wells $i = 1...S$ are drilled at Stage 1 and wells $i = S + 1..N$ are candidates (with possibly adjusted positions) for Stage 2.

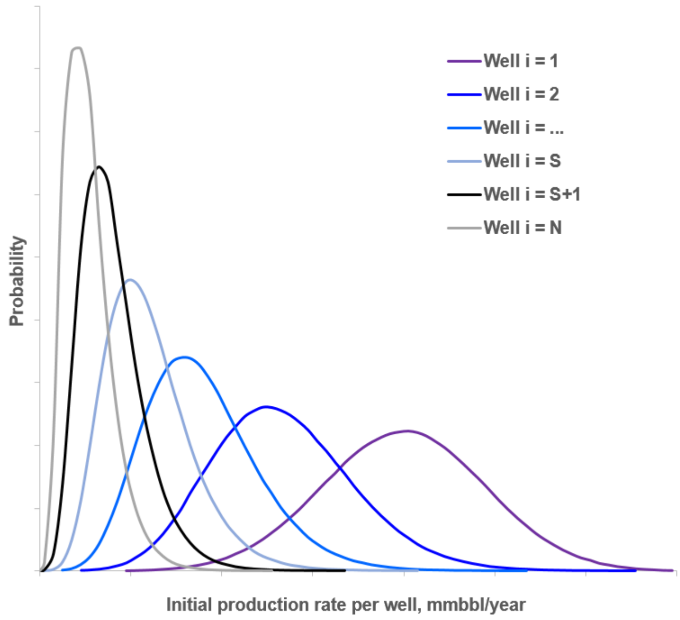

Figure 3: Illustration of potential probability density functions of the initial production rates per well (standard development case)

When studying an opportunity to phase the drilling strategy into two stages, the decision maker has to identify which wells should be drilled at the first stage and which should be kept for later, i.e. Stage 2. Following the approach by Dias (2004), we assume that the locations with the lowest risk and highest expected recovery are selected for Stage 1, while the ones which tend to be more exposed to risk and uncertainty are chosen as candidates for Stage 2. In our case, the well reservoir risk and prioritization are given by the distributions of the initial production rate.

We start the procedure with well $i = 1$. We use Monte Carlo simulation to generate its initial production rate samples in the whole range of the distribution that is provided by the field engineers as the input data. Accounting for geological dependencies between the wells, we then proceed with sampling simulation cases for each subsequent well. For the standard development, where all the wells $i = 1..N$ are drilled immediately, sampling of wells $i = 2..N$ is explicitly dependent on the generated initial rate of the well $i - 1$ as given by

$$q_{0_i} = q_{0_{i-1}} k_i, \tag{3}$$

where q_{0_i} is the initial production rate of well i = 2..N, $q_{0_{i-1}}$ is the initial production rate of well i-1 and k_i is the factor that defines the extent of geological dependency between wells i and $i - 1$, which is randomly generated from a given distribution, $0 < k_i < 1$. This describes the expected decrease/growth of the initial rate $q_{0_{i-1}}$ compared to q_{0_i}.[4]

For the purpose of comparison, we assume that the wells $i = 1..S$, drilled at Stage 1 within the staged development, have the same properties as the respective wells under the standard development and that the wells $i = 2..S$ are dependent on the generated initial rate of well $i - 1$ in the same manner as given by Eq.3. However, due to the data generated during Stage 1, the decision maker is able to update the reservoir model and therewith, the probability density functions for the initial rates, and reassess the initial positions of the optional wells. Therefore, the uncertainty about their initial production rates is expected to decrease, but is not fully eliminated.

[4]As illustrated in Figure 3, for our case the expected value of k_i for each specific well is below 1, resulting in the fact that the expected value of q_{0_i} is less than $q_{0_{i-1}}$.

Here we assume that there are two potential scenarios for the update of expected initial rates of the Stage 2 wells, referred as low- and high- case. In the low-case scenario, the data generated during Stage 1 reveals that the optional wells have lower production potential than expected. We denote the probability of the low case by x. In the high-case scenario, with assigned probability is $1-x$, the new information and re-positioning of the optional wells allows to increase the expected initial rate of the optional wells against the standard development case. These probabilities depend on the operator's beliefs on their ability to alter the drilling program by improving Stage 2 wells locations based on the data gathered during Stage 1. The higher is the probability of the high case, the more the operator can benefit from decreased subsurface uncertainty and increased expected initial production rate associated with the locations of Stage 2 wells.

The initial rate of well $i = S+1$ is calculated as follows

$$q_{0_{i=S+1}} = \frac{\sum_{i=1}^{S} q_{0_i}}{S} k_i^*, \tag{4}$$

where $q_{0_{i=S+1}}$ is the initial production rate of the well i = S+1, S is the number of wells drilled for Stage 1, k_i^* is the factor that describes the expected decrease of the initial rate $q_{0_{i=S+1}}$ compared to the average initial rate of the Stage 1 wells (i = 1..S) of a specific sample case within the Monte Carlo simulation. k_i^* is also sampled from a distribution and has a respective probability of x and $1-x$ to have a low or high mean, defining the low/high scenario for the update.

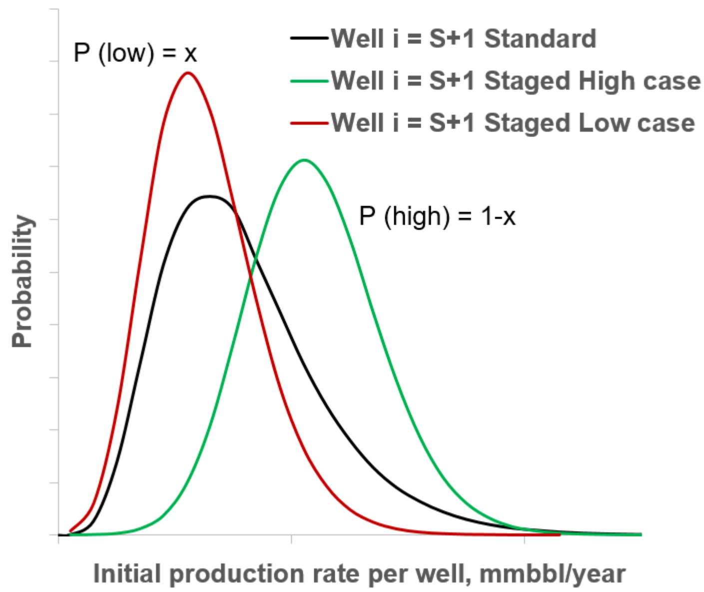

Figure 4: Example of probability density functions of the initial production rates for well S+1 (standard development vs. staged development)

Figure 4 illustrates the possible results of updating the distribution of the initial rates for well $i = S+1$.

The initial rates of the remaining Stage 2 wells $i = S+2..N$ are then calculated using the relationship presented in Eq.3, with initial rates dependent on $q_{0_{i=S+1}}$.

When estimating the production rates for the Stage 2 wells we also have to take into account that their production potential is decreasing while we are waiting to drill them due to the loss of some portion of oil that might have migrated towards the Stage 1 production areas. We account for this by assuming that $q_{0_{i=S+1}}$ is reduced by n% with each consecutive year of waiting for Stage 2.

Once we simulated all samples for the initial production rates per well under staged and standard development respectively, we proceed with the production estimation for the whole lifetime of the field. The production rate is assumed to follow the general exponential equation (Fetkovich 1980),

$$q_{i_t} = q_{i_{t-1}} e^{-a}, \tag{5}$$

where q_{i_t} is the production rate in year t and a denotes the nominal decline rate.

The exponential decline a special case of a hyperbolic decline curve introduced by (Arps et al. 1945), and is widely used both by academic literature and industry professianals owing to its simplicity and ease of use. It is based on the assumptions of constant bottom-hole pressure production and boundary-dominated flow (Fetkovich 1980) and is often used as a first-order approximation to a production forecast, especially in a situation where little or no observed production data is available. The decline rate in Eq.5 is individual for each field design and may vary significantly from one case to another depending mainly on the reservoir depletion. The exponential form of the decline is only one of several options of mathematical approximation of future production rate behaviour. Our method is flexible enough to handle any other production rate estimation approach. In Section 5.3.2 we test alternative ways to model the production decline, also adding the uncertainty to the decline rate.

The yearly total production of the field Q_t is then equal to the sum of yearly production rates per well as given by

$$Q_t = \sum_{i=1}^{N} q_{i_t}. \tag{6}$$

For simplicity, we assume that the production facilities are large enough to handle even the high reservoir case without putting any limit on the production rate, which is typical for small fields. This allows us to avoid the technical complications that stem from adjusting production profiles to a plateau rate.

The total recoverable reserves per well denoted by Res_i are equal to the sum of yearly production of the well as given by

$$Res_i = \sum_{t=1}^{t} q_{i_t}. \tag{7}$$

The total reserves of the field denoted by Res are then equal to the sum of the reserves of each well, i.e.

$$Res = \sum_{i=1}^{N} Res_i. \tag{8}$$

The fairly simple, but realistic enough representation of technical uncertainty used in this paper can be easily applied for other case studies and is considered to deliver a sufficient input for the main part of the methodology - the economic modelling. In order to parameterize the apriori distributions of the initial recovery (q_0 and k_i in Eq. 3) and sample production rates we only need input from engineers, which is typically provided before making the final investment decision. This analytical approach to modelling the production uncertainty is in line with Dias (2004), Armstrong et al. (2004) and Guedes & Santos (2016) who also use Monte Carlo simulation and binomial trees based an input distribution to represent the reservoir uncertainty. Despite presenting a simplified way of modelling production uncertainty, we account for the geological dependencies between wells by reproducing the declining marginal productivity of new wells being drilled in the same reservoir. k_i in Eq. 3 is assumed to be less than 1, which makes the expected initial rate q_0 of well $i+1$ less than q_0 of well i. It means that the more wells we drill in the same reservoir, the less is the initial rate of a new well.

We, however, disregard possible interference between wells that might reduce the production rate of already existing wells. The expected production rate of well i in our model is not affected by additional wells $i + ...$ being drilled. The way that the well interference works in practice would be very much dependent on the problem at hand and might affect the value of the staged development significantly. In this paper, we address this issue in a relatively limited fashion, while in Fedorov et al. (2020) we used a benchmark reservoir model simulation, where the interference is accounted for. Staged development in fact presents us with a strategy that can help to avoid drilling wells that would interfere with each other. The reason for that is that the data generated by Stage 1 production allows the operator to optimize positions of new wells, which is likely to further increase the value of the project under the staged development. This is an iterative approach, where the decision maker learns about the remaining uncertainty to update the drilling strategy. We have not accounted for this learning effect in Fedorov et al. (2020), while in this paper we assume to acquire perfect information that might either be a realization of "high or "low" case. In reality the reservoir properties and potentials for learning during the course of the project can be even more complex, but our economic valuation procedure allows to account for that.

The advantage of the proposed approach is that we are able to capture the whole range of probable production rates per well, and not only discretized values representing "high", "medium" and "low" cases, as typically done in decision analysis problems using decision trees. With that we facilitate the process of replacing the scenario based thinking with a probabilistic approach. This allows us to make better use of a seismic data and assess the influence of the reservoir uncertainty on the optimization of the decision to expand the production by drilling optional wells. Using the Monte Carlo simulation of the expected production rates based on the input distributions and running the whole valuation procedure described in Sections 3.1-3.4 takes a few seconds. The Monte Carlo simulation approach allows us to build several thousand production scenarios, while in Fedorov et al. (2020) using the reservoir simulation we were limited only to 5 realizations of the reservoir uncertainty because of high computational demand of the production optimization.

3.2 Costs

The next step in the procedure is the cost estimation based on the technical features of the production system and production rates. An appropriate estimate of cost parameters and underlying uncertainties is very important for investment valuation. Underestimating costs may lead to difficulties associated with cost overruns, and ultimately, lower profitability than stakeholders had expected. On the other hand, exaggerating cost estimates can lead the management to unnecessarily renounce a project.

For yearly capital investment ($CAPEX_t$) estimation we assume it consists of drilling costs ($DRILLEX_t$), cost of the platform(PL_t), which has to be constructed and installed, and other associated costs (FC_t) as given by

$$CAPEX_t = DRILLEX_t + PL_t + FC_t. \tag{9}$$

While overall drilling costs depend on the number of wells drilled, the platform cost component is determined by the chosen capacity size.

Yearly operating costs ($OPEX_t$) are related mainly to the maintenance of platforms and wells and the costs of day-to-day operation of the facilities (labour costs and maintenance). OPEX are assumed to consist of a fixed (FO) and a variable parameters, the latter of which depends on the yearly production rate of the field (Q_t) and a coefficient b, representing the relationship between the production rate and OPEX, and is given by

$$OPEX_t = FO_t + bQ_t. \tag{10}$$

We use basic cost estimates for our synthetic case with associated uncertainty estimation of individual cost elements based on both contractors' data and the operator's own assessment, which will be presented in Section 4.3.

Another important cost component that offshore operators incur are the decommissioning costs, which generate "unavoidable negative cash flow" (Parente et al. 2006). Companies have to cease their offshore operations, and depending on regulators' requirements have to ensure that offshore production will not generate environmental damage after that. The abandonment expenditure is expected to be fixed and consist of three cost elements: decommissioning planning, removal of facility and plug and abandonment of well.

However, as a new platform is built for our field case, the operator could benefit from reselling it (or using it on other fields) once the field is decommissioned. This could generate an overall positive cash flow at the abandonment. Our methodology is, however, flexible enough to account for additional cost components that might be important for other case studies.

3.3 Oil price modelling

We now proceed with parametrizing the underlying market uncertainty, which in our case is represented by oil price risk. Oil price is one of the main factors that drive uncertainty in economic value assessment of oil field development. As we apply the real options approach, we use a stochastic price model, that replicates the characteristics of real market uncertainty. There exists a rich literature on oil and gas price modeling. Much of it has been motivated by the desire to improve the quality of investment valuation under price uncertainty. Early literature mostly used the geometric Brownian motion (GBM) approach to stochastic oil-price modeling, which is based on an analogy in behavior of oil prices and stocks in the capital markets (see, e.g. Cox et al. (1985), Smith & McCardle (1999)). Later on, the literature has noted a mean-reverting characteristic inherent to oil prices due to ability of producers to react on the market conditions (see, e.g. Lund (1999)). Modelling the future oil price as a mean-reverting stochastic process allows short-term disequilibrium from a constant long-term equilibrium. Trying to better mimic the nature of oil markets, further research was extending this approach by adding more levels of uncertainty leading to two-, three- and more factors in the model.

In this study, we assume that future oil prices follow the two-factor stochastic price process proposed by Schwartz & Smith (2000). The two-factor price process allows to account for the mean reversion in short-term prices and uncertainty in the long-term equilibrium level to which prices revert. The equilibrium prices are modeled as a Brownian motion, reflecting expectations of the exhaustion of existing supply, improved exploration and production technology, inflation, and political and regulatory effects. The advantage of this two-factor process is that it is relatively easy to calibrate, while it is at the same time based on realistic assumptions. It has clear advantages over one-factor models due to uncertainty in both the short- and long-term factors, and other multi-factor models, which are harder to calibrate and are less intuitive to be communicated with industry representatives. The Schwartz & Smith (2000)'s process mimics not only features of the physical commodity market with a mean-reverting nature, but the derivative market, where the volatility of the near-maturity futures contracts is significantly higher than the volatility of far-maturity futures contracts.

We denote P_t as the commodity price at time t, where

$$ln(P_t) = \xi_t + \chi_t. \tag{11}$$

ξ_t denotes the long-term equilibrium price level and χ_t represents the short-term deviation from equilibrium prices. The long-term factor ξ_t is modeled as a Brownian motion with drift rate μ_ξ and volatility σ_ξ. Its dynamics are given by

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi. \tag{12}$$

The short-term deviations from the equilibrium prices reflect events in the market, that affect the price in the short-term, but are smoothed in the long-term by the ability of market participants to adjust the production and inventory levels in response to market conditions. These short-term disequilibriums are expected to fade away in time. χ_t is therefore, modeled as an Ornstein-Uhlenbeck process, given by

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi, \tag{13}$$

where κ is the mean-reversion coefficient, σ_χ is the volatility of the short-term factor and dz_ξ and dz_χ are correlated increments of standard Brownian motion processes with $dz_\xi dz_\chi = \rho_{\xi\chi}$dt.

As we mentioned in Section 2.2, we choose to use a risk-neutral valuation technique (Cox et al. 1985) to value an investment under multiple sources of uncertainty instead of using a risk-adjusted discount rate. We, therefore, risk adjust individual uncertainties in the model. The short-term and long-term factors in the risk-neutral version of the two-factor price process can be described by the following equations:

$$d\xi_t^* = (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dz_\xi^*, \tag{14}$$

$$d\chi_t^* = (-\kappa\chi_t - \lambda_\chi)dt + \sigma_\chi dz_\chi^*, \tag{15}$$

where, as before, dz_ξ^* and dz_χ^* are correlated increments of standard Brownian motions such that $dz_\xi^* dz_\chi^* = \rho_{\xi\chi}$dt and λ_χ and λ_ξ are risk premiums that are being subtracted from the drifts of each process.

This means that the risk-neutral short-term factor is now reverting to $-\lambda_\chi/\kappa$. The drift of the long-term factor in the risk-neutral model is equal to $\mu_\xi^* = (\mu_\xi - \lambda_\xi)$.

As we use Monte Carlo simulation to generate prices and cash flows, we next need to discretize the price process. The discretization for the long-term component in Eq.14 is given by

$$\xi_t^* = \xi_{t-1}^* + \mu_\xi^* \Delta t + \sigma_\xi \varepsilon_\xi \sqrt{\Delta t}. \tag{16}$$

The discretized version for the risk-neutral short-term component process is given by

$$\chi_t^* = \chi_{t-1}^* e^{-\kappa\Delta t} - (1 - e^{-\kappa\Delta t})\frac{\lambda_\chi}{\kappa} + \sigma_\chi \varepsilon_\chi \sqrt{\frac{(1 - e^{-2\kappa\Delta t})}{2\kappa}}, \tag{17}$$

where ε_ξ and ε_χ in Eqs.16 and 17 are standard normal random variables and are correlated in each time period with the correlation coefficient $\rho_{\xi\chi}$.

The oil price model has a total of seven parameters (κ, σ_ξ, σ_χ, μ_ξ, λ_χ, $\rho_{\xi\chi}$, and λ_ξ), along with two initial values ξ_0 and χ_0 that need to be estimated. As model parameters are not directly observable in the commodity markets, we need a tool to calibrate them. Here we adopt the approach by Goodwin (2013)[5], using Kalman filter and maximum likelihood estimation. The Kalman filter recursively computes estimates for unknown parameters in the form of a posteriori conditional distribution on a given dataset of spot and/or futures prices and measurement covariance matrix. The calibration that we performed using the Kalman filter for a market data set is presented in Section 4.2. In Section 5.3.3 we also perform a robustness check in order to test alternative price models and to see how they affect our results.

[5] Goodwin (2013) uses a MATLAB function to estimate the parameters of the two-factor process. It is computationally efficient and allows to calibrate the process using a large data set of historical prices, which is required as an input.

3.4 Option to expand at Stage 2 and LSM algorithm implementation

In the current study, we focus on the valuation of the flexibility to drill additional wells during the production phase. We assume that the field operator holds an option to expand the production from the moment of the reservoir information revelation, which happens as soon as necessary production data is collected and processed. This point in time serves as the option holder's lower time constraint. The production experience at Stage 1 is assumed to generate perfect information about the remaining reserves. Knowing the initial rates per each well, the decision maker is able to estimate the future field production profile using the exponential decline curve. After the reservoir information revelation the operator has a certain number of years to exercise the option. Drilling activities are assumed to be done by the end of the year when the decision to expand was made. If the optional wells are drilled, the field production potential increases from the following year on (see Figure 1 for an example of how the decision to drill optional wells can affect the production profile). Managers are assumed to able to reevaluate this decision once a year after the information revelation. This is considered to be a realistic assumption for oil field development in terms of time needed to process data for decision support. The upper time constraint of the option, i.e. the moment when drilling optional wells is considered to be no longer reasonable, is defined by the operator based on the development strategy. In the case of small field development, it might be limited to a few years after the decision to expand can be made for the first time due the reservoir depletion. This means that we evaluate an American call option, that can be exercised at the predetermined discrete points of time.[6]

Based on the information on the initial production rates per well and then-current state of the oil market, the operator optimizes the expansion decision by choosing whether to exercise the expansion option at a given point in time. The valuation procedure of problems as ours, with midway decisions that change the course of a project, is typically performed in a backward fashion (Jafarizadeh et al. 2009). This means that one first determines the optimal exercise strategy at the last decision point in time. Proceeding backwards in time, an optimization algorithm determines the optimal strategy for precedent choices. To do so, we choose to apply a LSM simulation approach, "a state-of-the-art approximate dynamic programming approach used in financial engineering and real options analysis to value and manage options with early or multiple exercise opportunities" (Nadarajah et al. 2017). It is considered to be well suited for investment valuation problems, when the investment decision depends on multiple sources of uncertainties and involves multiple decision points. This is due to the fact that LSM approach does not suffer from the curse of dimensionality (Longstaff & Schwartz (2001), Willigers et al. (2009)). Real option valuation methods based on the LSM approach have been compared and verified by (Nadarajah et al. 2017) and used in several oil and gas applications (Jafarizadeh et al. 2009), Willigers et al. (2009), Hong et al. (2018).

In our model, we first determine the expected yearly cash flows by combining simulated production and cost profiles as well as the trajectories for oil prices based on the risk-neutral process. We, therewith, generate several sets of cash flows, where each set represents the simulated cash flows associated with the respective time when the decision to expand is made (i.e. project cash flow given that the optional wells are drilled in Year K, Year K-1, etc.). These cash flows serve as the main input for the LSM algorithm. At each decision point t_n the algorithm compares the immediate exercise value denoted by $\Pi(t_n, P_{t_n}, Q_0))$, which is known, with the estimated discounted value from continuation denoted by $\Phi(t_n, P_{t_n}, Q_0)$. Φ is not known and is equal to the expected conditional value of future cash flows $\mathbb{E}^*_{t_n}\left[F(t_{n+1}, P_{t_{n+1}}, Q_0)\right]$. Both parameters are dependent on the the oil price P_{t_n} at time t_n and sum of initial production rates per well Q_0. Remember that the option can only be exercised at discrete time steps, in the interval between year $K-1$ - when the reservoir information is gathered

[6]This type of an option is also called Bermuda option.

and processed, and year $K+n$ - when the decision to expand can be last made (see Figure 1). Therefore, the optimal value function $F(t_n, P_{t_n}, Res_{t_n})$ at time step t_n can be obtained using the following Bellman equation (Rodrigues & Rocha Armada 2006):

$$F(t_n, P_{t_n}, Q_0) = max\{\Pi(t_n, P_{t_n}, Q_0)), \Phi(t_n, P_{t_n}, Q_0)\}, \tag{18}$$

where

$$\Phi(t_n, P_{t_n}, Q_0) = e^{(-r(t_{n+1}-t_n))}\mathbb{E}^*_{t_n}\left[F(t_{n+1}, P_{t_{n+1}}, Q_0)\right], \tag{19}$$

where r is the risk-free rate.

The algorithm starts with the last decision point and maximizes the expected project value along each simulated path. At the maturity date $K+n$, i.e. the last year when the decision to expand can be made, the continuation value equals zero. The algorithm, therefore, determines the optimal decision by comparing expected project values associated with the decision to drill optional wells now and the decision to keep the option unexercised by producing only with the wells drilled at Stage 1 until the field shut-down. We then consider these decision policies in the precedent years and find the optimal expansion time (if undertaking the expansion is optimal) for each path of the simulation. At time steps $K-1 < t_n < K+n$, the holder of the option must compare the payoff from immediate exercise, $\Pi(t_n, P_{t_n}, Q_0)$ (drill optional wells now), with the continuation value (wait until the next year to reevaluate the drilling decision).

The main challenge in this process is to determine the continuation value. Here we follow Longstaff & Schwartz (2001), who suggest to use linear regression that relates the continuation value to the underlying uncertainties. The LSM algorithm uses the regression as the estimator of the expected value of future cash flows conditional on the current information on the state variables (i.e. in our case, the simulated oil prices and the sum of initial production rates per well), as given by

$$\mathbb{E}^*_{t_n}\left[F(t_{n+1}, P_{t_{n+1}}, Q_0)\right] = \alpha_1 P_{tn} + \alpha_2 Q_0 + \alpha_3 P_{tn}^2 + \alpha_4 Q_0^2 + \alpha_5 P_{tn} Q_0, \tag{20}$$

where $\alpha_{1...5}$ are the regression coefficients.

As suggested by Longstaff & Schwartz (2001), we use only in-the-money paths for the estimation of regression parameters, "since it allows us to better estimate the conditional expectation function in the region where exercise is relevant". It means that in the year $K+n-1$ we consider only project values of those simulated samples, in which the optimal decision was to expand in year $K+n$, and regress them on the current oil price and sum of initial production rates per well.

In order to calculate the regression coefficients α_i in Eq.20 based on the in-the-money paths at each time step, we have used the MATLAB "backslash" solver[7], following the approach by Jafarizadeh & Bratvold (2015). The fitted values from this regression are used as an estimate for the expected continuation function. The continuation value is then discounted with the risk-free rate and compared with the value created by the decision to drill the optional wells now. If the immediate exercise value is higher than the estimated continuation value, the optimal decision is to exercise the option. Otherwise, holding the option is optimal.

Moving backwards in time, we calculate the maximum value at each time step. In our case the algorithm stops after evaluating the decision in Year $K-1$, in which the data on the reservoir is gathered and the decision whether to exercise the option can be made for the first time. The cash flows for the years when the decision maker does not have any flexibility, in our case before Year $K-1$, are evaluated based on a simple DCF

[7] The backslash solver gives m unknowns for n system of equations when $n = m$. If $n > m$ this function uses the linear least-squares regression to estimate m (Jafarizadeh & Bratvold 2015).

procedure. For the consistency of the results, the DCF evaluation is based on the same simulated oil price paths as the real options procedure. We then calculate the overall project value by summing the cash flows that the project generates before Year $K-1$ and the values resulting from the LSM algorithm from Year $K-1$, all discounted with the risk-free rate.

Using the traditional DCF approach to evaluate the project with flexibility would have not allowed to capture the value of waiting to expand. This additional value stems from the fact that the operator can optimize the decision to drill optional wells based on the new knowledge. To the best of our knowledge, models to evaluate the staged development with an option to drill additional wells do not exist in the literature. Previous contributions have been looking at the value of the option to expand production by (1) connecting a tie-in field (Fleten et al. 2011) (2) initiate IOR solution (Hong et al. 2018). The focus of these contributions is, however, on the upside potential that can be exploited by production increase, while the main focus of our paper is to show how the staged development strategy allows to mitigate the prominent downside risk, especially in case of a marginal project.

The LSM algorithm is considered to be computationally efficient, flexible and transparent due to the fact that it is based on a simple least squares regression. If there are additional risk factors that will affect the expected continuation values, they can be easily added to the regression model (Willigers et al. 2009).

In the next section we proceed with the implementation of the proposed methodology to a synthetic case study.

4 CASE STUDY

We now illustrate how the proposed method is applied to the evaluation of an investment in a small oil field based on a synthetic, yet realistic industry case. The relevant technical inputs for the model were chosen such as to build a realistic case taking into account the main features of an offshore field development project, while disregarding components that are considered to be of secondary importance and are likely to make the illustration of the procedure less intuitive and transparent. The goal of the case study is not to analyse a specific project investment, but rather to illustrate the valuation algorithm based on the synthetic case, and show that the workflow is flexible enough to be easily implemented for other project cases.

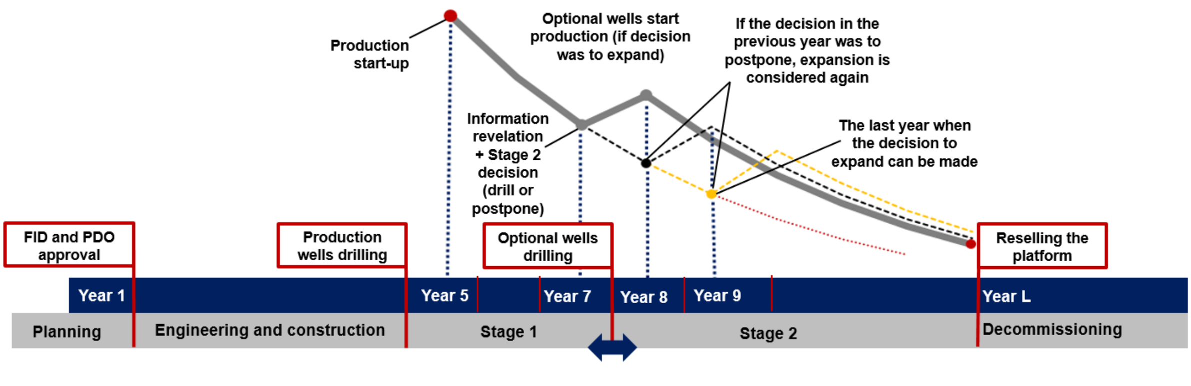

Figure 5: Decision gates of the project under the staged development strategy

Following the proposed modelling procedure, we first build 30,000 simulation samples[8] of production profiles per well for each of the expansion scenarios (in total 5 sets of 30,000 paths), which serves as an input for the

[8] 30,000 iterations proved to be computationally reasonable and produce a consistent and stable result, that deviates insignificantly throughout several simulations

cost profiles estimation. We then construct the project's cash flows based on the simulated production profiles and oil price paths to run the LMS algorithm in order to find an optimal strategy for the staged development.

Figure 5 illustrates the flexibility that an operator has in our project case. Year 7 is the year when the data on the reservoir is gathered and the decision to expand can be made for the first time. The operator is assumed to hold the expansion option for a timespan of 3 years, reevaluating the decision once a year within this period of time. Such a restriction for the number of decision points is consistent with the project's expected lifespan and other features such as loss of migrating oil and time value of money that were already discussed. This means that the LSM algorithm is applied from Year 9 (when the optional wells can be drilled last) working backwards to Year 7. Shortening the time needed for the initial stage of the project to make a decision upon Stage 2 drilling would clearly increase the value of the staged development. We, however, remain conservative, giving an operator more than enough time to gather and process the Stage 1 production data.

4.1 Production profiles

The case study mimics the production and cost profiles for a standalone development of an offshore oil discovery with relatively low reservoir properties, allowing only an ultimate recovery factor of order 20%. The main components of the field design consist of a single platform and subsea templates. The oil is offloaded to tanker and transported to the mainland. A part of the gas production is used for power generation on the platform, while the rest is re-injected into the reservoir.

In total, 6 oil production, 3 water injection and 1 water production wells are planned for the standard (non-staged) development, while for the staged development 4 oil production, 2 water injection and 1 water production wells are drilled at Stage 1, with an option to drill the remaining 2 oil production and 1 water injection wells at Stage 2.

Table 2: Input parameters for the Monte Carlo simulation

	Distribution	Mean	Unit	Std. dev.
Initial production rate, Well 1	Normal	4.0	million bbl/year	0.9
k_i	Normal	0.667	-	0.12
k_i^* high case	Normal	0.45	-	0.015
k_i^* low case	Normal	0.26	-	0.07

We estimate the initial production rates per well, accounting for geological dependencies using the Monte Carlo simulation as described in Section 3.1. The procedure starts with assigning the initial rate for Well 1 and then proceeds with the remaining wells applying Eq.3 for Wells $i = 2..6$ in the standard development case and for Wells $i = 2..4$ in the staged development case. The input parameters that were used for the Monte Carlo simulation based on the input from field engineers are presented in Table 2.

In general, the expected initial rate of Wells $i = 2..6$ is decreasing due to the factor k_i in Eq.3, which is less than 1. In our case, the expected value of the initial rate of Well 2 is 33.3% lower than the initial rate q_0 of Well 1. Given the simulation inputs, however, there is some probability that Well 2 will have a greater initial rate than Well 1. Figure 6 shows the probability distributions for initial rates of wells $i = 2..6$ with a given initial rate of Well 1 for two example cases within the standard development. For case A we assume that q_0 of Well 1 equals 5.2 mmbbl/year. For case B we assume a lower q_0 that equals 2.8 mmbbl/year. If the initial rate of the Well 1 is relatively high (as for case A in Figure 6), there is a higher chance that wells $i = 2..6$ will produce at a higher than average rate as well, due to geological dependencies between the wells. The opposite holds for

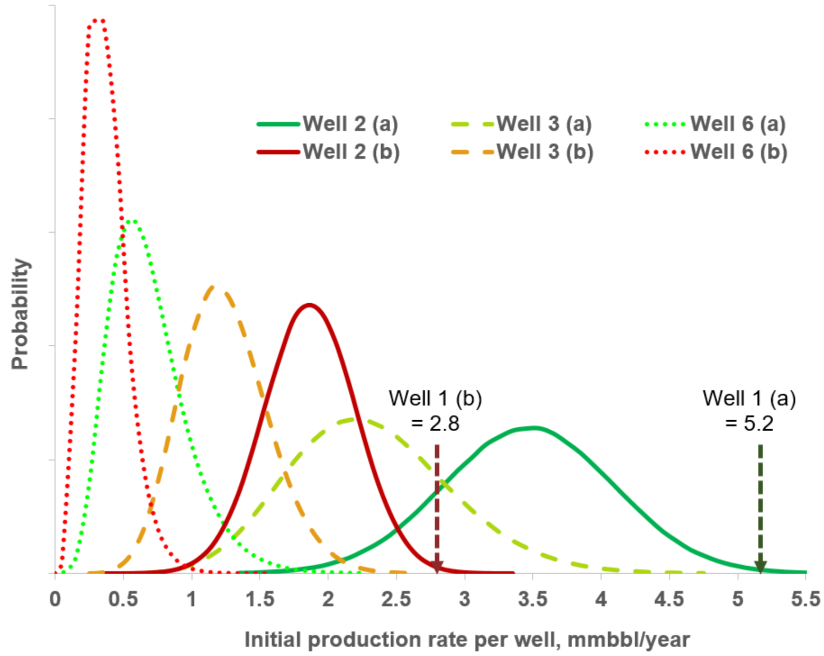

Figure 6: Probability density functions of initial production rates per well with a fixed initial rate of Well 1 (cases *A* and *B*) for the standard development case

case *B*.

In order to account for a learning effect during Stage 1 within staged development, we generate samples of the initial rates of the first optional well, i.e. Well 5, using the dependency presented in Eq.4. As discussed in Section 3.1, the information revealed during Stage 1, might lead to either increase or decrease of the expected initial production rate for optional wells compared to the respective apriori initial rates. We assign the probabilities for the high and low case to be equal to 60% and 40%, respectively. Given these probabilities, the factor k_i^* in Eq.4 is then sampled from a normal distribution with mean 0.45 and standard deviation 0.015 in 60% of simulated cases, while in 40% of cases the distribution with mean 0.26 and standard deviation 0.07 is used (see Table 2). The initial rate for Well 6 within the staged development is then sampled using Eq.3 based on the generated values for Well 5.

Although the low case has a relatively high assigned probability for our case, the expected overall recovery from Wells 5 and 6 under the staged development is expected to slightly increase. However, this does not necessarily mean that in all the cases the staged development (given that Wells 5 and 6 are always drilled) increases the recoverable volume compared to the standard development. Our simulation results show that in 34.2% of the cases, the total reserves under the standard development strategy are larger than under the staged development.

Additionally, we consider that the production potential of the Stage 2 wells decreases by 1% yearly from Year 7 (when the expansion decision is made for the first time) due to the loss of migrated oil. This assumption accounts for the fact that while Stage 2 wells remain undrilled, the oil from these locations migrates towards the areas which are already under development at Stage 1. Some portion of this oil is then trapped in the rock and cannot be recovered at Stage 2. The decision maker, therefore, additional to the time value of money and depreciation effects, has another incentive to drill Stage 2 wells earlier.

Based on the assumptions that the production rate follows an exponential decline function and that the rate is not limited by the capacity constraint, we then simulate the production profiles. Given the initial production rate distributions by each well, we estimate 30,000 possible production profiles realizations for each of the

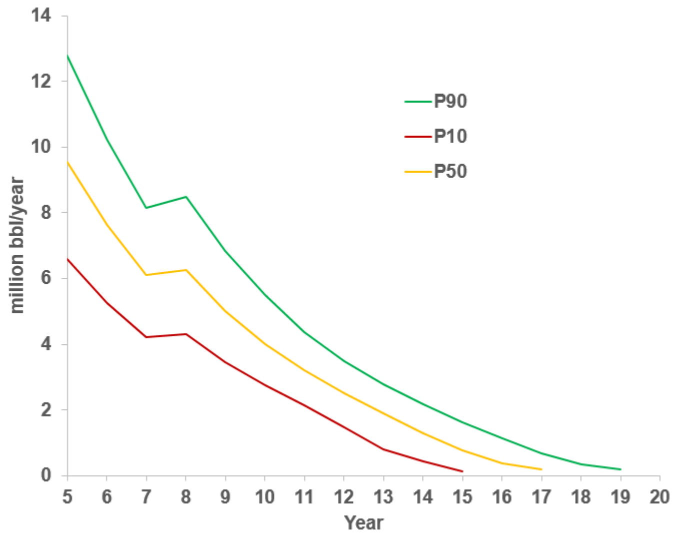

Figure 7: Confidence bands of the expected production profile under the staged development (expansion in Year 7)

expansion scenarios using Eq.5 with a nominal decline rate of 22.5%. We perform a sensitivity analysis in Section 5.2 in order to see how different decline rate assumptions affect our results. The minimal production rate per well is set to be equal to 0.05 million bbl per year. Once this threshold is reached, a well is assumed to stop production. Figure 7 illustrates the confidence bands of the field production profiles for the case of the staged development, given that in each case the expansion takes place in Year 7.

Summing the production profiles of all wells, we then obtain the total recoverable reserves for all simulation samples. Those are shown in Figure 8 for all production wells, with the confidence bands resulting from the Monte Carlo simulation. The results show that the additional expected recovery is decreasing with more wells being drilled, meaning that the P50 value is continuously decreasing from Well 1 to Well 6 in the standard development case. In case of the staged development strategy (the right side of Figure 8), we can note that the expected recoverable volume for the optional wells that are drilled at Stage 2 increases compared to the respective values under the standard development.

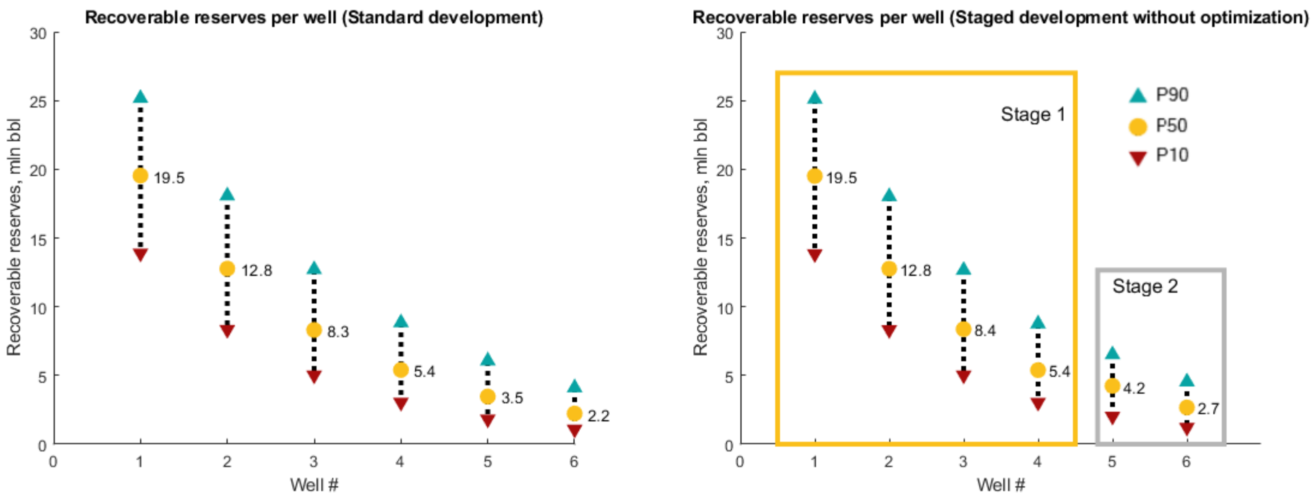

Figure 8: Total recoverable reserves per well under the standard development (left graph) and staged development (right graph) strategies

Table 3 states the total recoverable reserves of the field under the standard development and staged development strategies, respectively. For the staged development, we show the results for three different cases. The confidence bands stated in the first row result from the expansion optimization, discussed further in Section 5. The LSM algorithm indicates that in 24.7% of the simulated cases, the decision maker should not drill additional wells and should produce only with the Stage 1 wells. Therewith, the recoverable reserves under two strategies are at the same level (the staged development's mean is even slightly lower), which allows us to make a fair comparison of their economic value. Values in the last two rows are limited to extreme points when optional wells are drilled in all and none of the simulated cases, respectively. If optional wells are always drilled, the field production potential increases, but production of these reserves might be uneconomical.

Table 3: Confidence bands of the recoverable reserves, mmbbl

	P10	P50	Mean	P90
Optimized staged (optional wells are drilled when it is optimal)	**33.8**	**52.0**	**52.8**	**72.4**
Standard	**35.2**	**52.0**	**53.0**	**72.2**
Staged (optional wells are drilled in all the 30,000 cases)	36.4	53.2	53.9	72.5
Staged (optional wells are drilled in none of the 30,000 cases)	31.8	46.4	46.9	62.6

4.2 Oil price simulation

As mentioned in Section 3.3, we calibrate the oil price process parameters based on historical market data using the Kalman filter. We use Thomson Reuters weekly (averaged daily) data on ICE Brent historical futures contracts from March 2006 (the first available forward curve with 81 months maturity) to November 2019, along with Dated Brent FOB North Sea spot price for the same period of time. We average monthly maturity contracts with mid- and long- range maturities to construct forward curve vectors with respective maturities of 1,2,3...12,14...18,21,25,28,34,40,46,55,66,78 months to decrease the computational effort for the Kalman filter algorithm. The overall data set for the model calibration forms a 715x24 matrix. The resulting oil price process parameters are reported in Table 4. Figure 9 illustrates the confidence bands and the expected value for the risk-neutral price process based on 30,000 simulated paths. The thin colored lines represent examples of simulated price paths that were used for our valuation procedure. The resulting expected (mean) price increases from 56 USD/bbl in 2021 (Year 2 in our procedure) to 65 USD/bbl in 2040 (Year 21).

Table 4: Calibrated parameter values used for the Schwartz-Smith two-factor price process simulation

ξ_0	4.0	χ_0	0.1
σ_ξ	13%	σ_χ	58%
μ_ξ^*	-0.4%	$\rho_{\xi\chi}$	0.07
κ	0.52	λ_χ	9.82%

4.3 Costs and abandonment

As mentioned in Section 3.2, the main components of the CAPEX are the facility cost and the cost of production drilling. The former is assumed to consist of a fixed parameter and a variable component that depends on the chosen capacity of the platform. For our specific case, the facility cost is payable over several years as shown in Table 5 and Figure 10. The cost of drilling and including one oil production well into the production system is

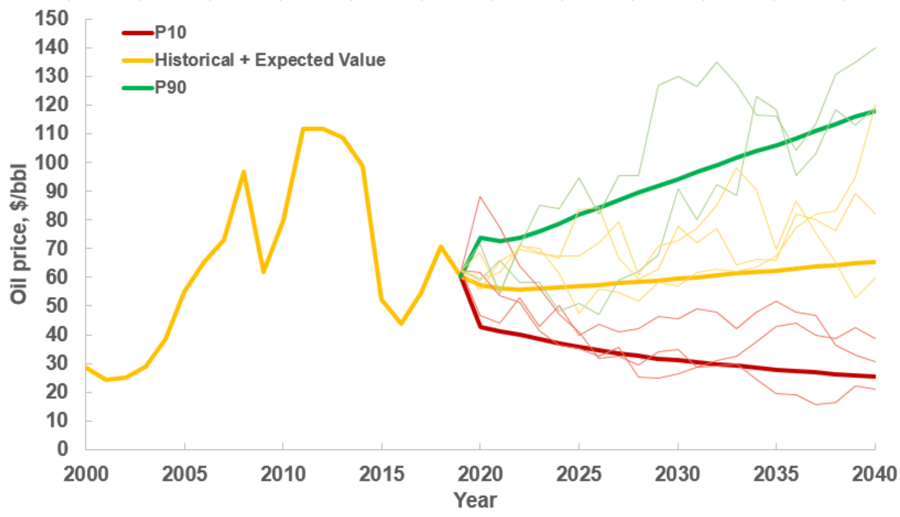

Figure 9: Historical Brent crude oil prices, confidence bands and simulation examples for the risk-neutral price process

equal to 48 MM USD. The CAPEX is assumed to have a random component that can increase/decrease the cost estimate withing a range of 15% for a specific simulated cost profile. The resulting confidence intervals for the CAPEX estimates are presented in Table 5. Staged development implies that Wells 5 and 6 remain undrilled in Year 4. The decision maker can consider whether to drill those wells in Year 7, 8 or 9 or not to drill them at all.

Table 5: P10-P50-P90 CAPEX estimation under the staged development vs. standard one, MM USD

	Year 2	Year 3	Year 4	Year 7, 8 or 9
Staged (optional wells are drilled)	85-96-108	32-36-40	602-683-765	127-145-162
Standard	85-96-108	32-36-40	729-828-927	0-0-0
Staged (optional wells are not drilled)	85-96-108	32-36-40	602-683-765	0-0-0

The main cost elements of OPEX are storage vessel leasing, facility manning and operations, and fuel costs. The fixed component in Eq.10 equals 84 MM USD, while the b-coefficient is set to 0.6.

We also need to consider the end of the field lifetime as it bears some important features for the field management strategy and project cash flow. We first introduce a decision rule to cease the production and abandon the field. Once the project cash flow reaches negative values due to the reservoir depletion, the only reason that can drag the revenue from the negative to the positive values is an oil price upturn. By performing simulations we empirically determine a threshold which is considered to be a ”point of no return” for the project's development. The production is assumed to be ceased as soon as the cash flow falls below 0.93 MM USD. For the calibrated parameters for the oil price process, the probability to reach a positive cash flow after reaching this threshold is reasonably low (less than 0.5%). One year after the production is halted, the company incurs some losses due to decommissioning. However, it has an opportunity to resell the production platform, which is expected to generate an overall revenue of 100 MM USD in the last year of the project life. Figure 10 illustrates confidence bands for project's expected yearly cash flows. Unfavourable price and production scenarios that represent the P10 case, force the oil company to shut-down the field and resell the platform early.

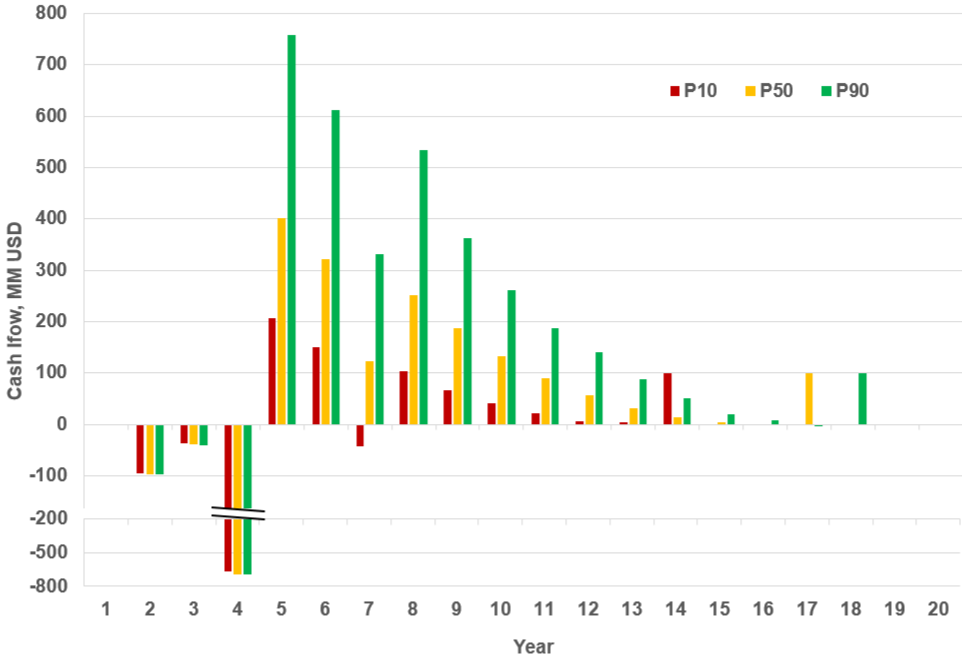

Figure 10: Project cash flows under the staged development strategy (optional wells are drilled in Year 7)

5 RESULTS

The main issue that the decision maker faces in our case is whether to commit to investment in drilling all the wells before the oil production start-up or to choose for the staged development strategy. Our goal is to evaluate whether the staged development with the option to drill additional wells creates value and to analyze under which conditions the staged development is the preferred strategy.

We first study the staged development case with the option to expand. Based on the input from the technical part of the workflow we use the LSM algorithm to optimize the expansion decision. The simulation with 30,000 trials for each of the development strategy (expand in Year 7/ expand in Year 8/ expand in Year 9/ do not expand/ choose standard development) generates 5 sets of 30,000 production and cost profiles as well as risk-adjusted paths for oil prices. The value of the investment at a respective decision node is calculated by determining the expected project value discounted at the risk-free rate of 2.5%.

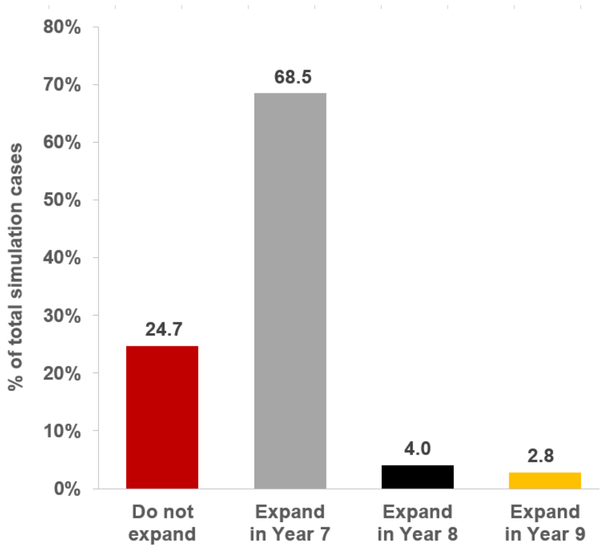

Figure 11: Optimal expansion timing as percentage of total number of simulated paths

As mentioned in Section 3.4, the LSM algorithm works in a backward manner starting from Year 9, and then moves to precedent time steps, i.e. Year 8 and Year 7 optimizing the expansion policy for each simulation case.

Figure 11 illustrates the results related to the optimal timing of exercising the option to expand that are produced by the LSM algorithm. It shows that in 68.5% of all simulated cases it is optimal to drill optional wells as soon as the reservoir information is gathered, i.e. in Year 7. The field production potential and oil prices in these cases favour the early expansion. However, the decision maker is recommended to defer drilling of optional production wells until Year 8 and Year 9 in 4.0% and 2.8% of cases, respectively. Finally, in 24.7% of cases the optimal decision is to leave the option to expand unexercised and avoid drilling optional wells. These cases represent realizations of unfavourable production and/or price scenarios.

Knowing the optimal expansion decision for each of the simulated case, we sum the value that project generates before the optional wells are drilled (if the expansion is optimal) and the value that project generates over Stage 2. As mentioned in Section 3.4 the former is evaluated using a simple DCF approach, while the latter is the result of risk-neutral real option valuation procedure.

We then derive the value of the project when using the standard development strategy, which will serve as a reference point. As the standard development considers an irreversible commitment at Year 4 to drill all production and injection wells, it can be evaluated using a simple DCF approach. We do so using the same data set (Monte Carlo simulation for production rates for wells 1..4, cost profiles and oil price paths).

Table 6 shows the results of the project valuation for the staged and standard development case respectively. Due to flexibility to optimize the expansion decision, the staged development allows to increase values of all the confidence bands and, most importantly, the expected value of the project. In Section 5.3.1 we discuss the potential to increase the value of flexible project even more by accounting for additional flexibility to choose the number of optional wells.

Table 6: Confidence bands of the pre-tax values of the flexible and inflexible projects, MM USD

	P10	P50	Expected value (mean)	P90
Optimized Staged	-164	675	885	2186
Standard	-188	655	851	2127

The choice between staged and standard development is affected by several factors. One important factor here is the amount of the additional oil production that is generated at Stage 2. As we stated above, the decision maker is able to improve the drainage strategy by adjusting locations and design of the Stage 2 wells, due to the information gathered during Stage 1. This increases the expected recovery for Well 5 and 6 for the high case in comparison with the standard development. However, for the low case, the optional wells could have lower recovery than expected. The assigned probabilities to the high and low cases prove to be a key element in the choice between the staged and standard development. In order to isolate the value of flexibility, we chose the parameters for updating the Stage 2 wells recoverable volumes (60% high case) so that the P50 recoverable reserves of the flexible project (under the optimized staged development strategy) are equal to the P50 recoverable reserves under the standard development. This allows us to make a fair comparison between the staged and standard development as we can identify how much value is created solely by the flexibility to optimize the expansion decision, not accounting for the ability to significantly increase the recoverable volume with the staged development.

Figure 12 shows how the values of the two strategies evolve as a function of the probability of the high case for updating the Stage 2 wells initial rate under the staged development. The probability of the high case must be at least 35.5%[9]. for the staged development to be the preferred option. This means that if the engineers expect that the information being gathered during Stage 1 would ensure an expected total recoverable volume of at least 51.4 mln bbl, then choosing the staged development is optimal. If not, the value lost due to waiting for the information revelation, is not compensated by the production increase at Stage 2 nor by the ability to partly hedge against the downside risk by not drilling optional wells.

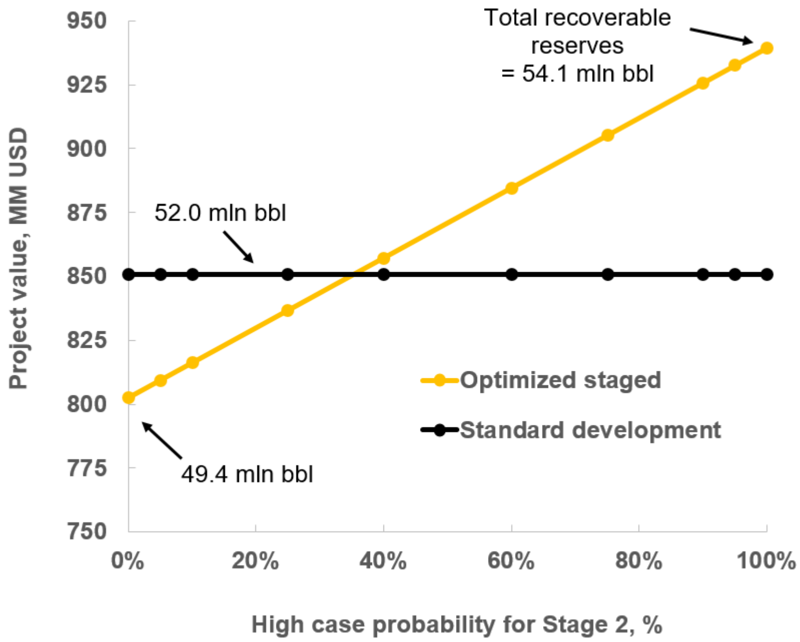

Figure 12: Project value under the staged and standard development depending on the probability of the high case for the Stage 2 wells

Furthermore, our methodology allows to derive the investment boundaries, at which it is optimal to exercise the expansion option. These investment boundaries are given as functions of the sum of initial production rates per well, which was indicated by the data collected during Stage 1, and the observed oil price in the market when the decision is made. The threshold boundaries result from hyperbolic fitting for the boundary combinations of the production rates/oil price (dots in Figure 13) that result in the decision to exercise the option in the LSM algorithm. Figure 13 illustrates the derived investment boundaries for each year when the expansion is possible. If the combination of the current oil price and the sum of initial production rates lay to the upper right of the respective investment boundary, it is optimal to exercise the expansion option in this year. If the combination of the two parameters lays to the lower left, it is optimal to wait and revisit the decision one year later. The continuation region becomes wider as the investment boundary moves each year to the upper right, while the decision maker defers the decision to expand. Higher oil prices are then needed to justify an expansion later in time.

5.1 Downside risk mitigation

One of the main advantages of the staged development strategy is the possibility to leave the expansion option unexercised if the reservoir information indicates too low potential production inflow due to drilling more wells and/or the oil prices are at a level, which does not trigger the investment. Therefore, this strategy allows to partly hedge against the downside risk in unfavourable reservoir/oil price scenario. We, however, should also note that for the case of real options, unlike financial options, a perfect hedge is almost never possible.

[9] total expected recoverable volume under optimized staged development in this case is 51.1 mln bbl

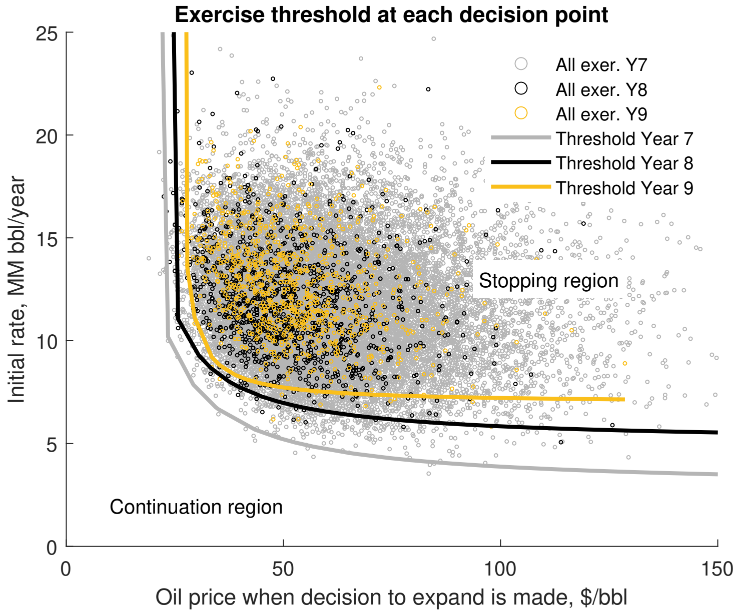

Figure 13: Results of the expansion timing optimization by the LSM algorithm. Threshold boundaries

This is due to multiple sources of uncertainty that have to be considered, and decisions that have long-lasting impacts. The optimization algorithm proposes an optimal decision given then-current knowledge, while the project remains exposed to some risks in the future that are not hedged. We, therefore, acknowledge that the staged development can provide only a partial hedge.

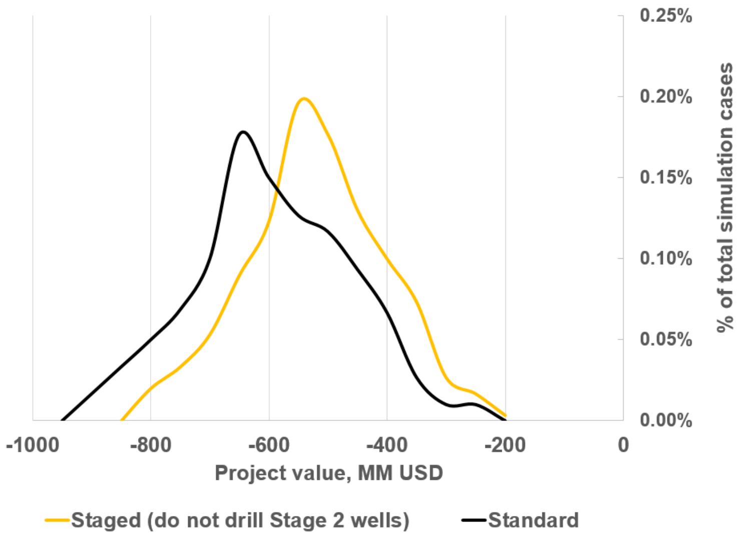

Figure 14: Project value distribution under staged and standard development in the low reservoir/price scenarios

In the standard development case, all the wells are drilled at the early stage of the project, ignoring the effect of new information received during the course of the project. The staged development, however, allows to save the cost of drilling optional wells and, therewith, decrease the downside risk. In a low reservoir/oil price scenario the project value is at risk, making the decision maker more likely to turn away from the investment, when there is no opportunity to react on this outcome of the uncertainty. Figure 14 shows the project value distribution in the low reservoir/price scenario under the staged and standard development, respectively. It represents only those simulated cases, where the sum of initial production rates per well for the staged development was less than 7 MM bbl and the oil price was less than 40 USD/bbl in Year 7 (the decision not to drill Stage 2 wells was optimal in all these cases). The staged development strategy in these cases tends to decrease the project value losses compared to the standard development case.

Performing a simulation with 30,000 samples allows us to capture a wide range of probable combinations of production and cost profiles, oil price paths and scenarios of information revelation after Stage 1. Figure 15 illustrates the probability that the project value under the standard development (the right axis) is below a certain level (see the horizontal axis). The yellow line in Figure 15 illustrates the average project value increase that would be added if the staged development is used instead of the standard development for the same simulation cases (given the same initial production rates for the Stage 1 wells, oil prices, etc.) (see the left axis).

The average increase of the project value due to the staged development if the project value under the standard development is below -500 MM USD is 82 MM USD. Overall, our results show that in 81% of the simulated cases where standard development results in a negative value, using the staged development increases the project value. In those simulated cases, where the expected project value under the standard development is high (due to a large reservoir or high oil prices), the effectiveness of the sequential drilling decreases. However, for the range of simulated project values below 3000 MM USD, the staged development still adds value in 59% of the cases.

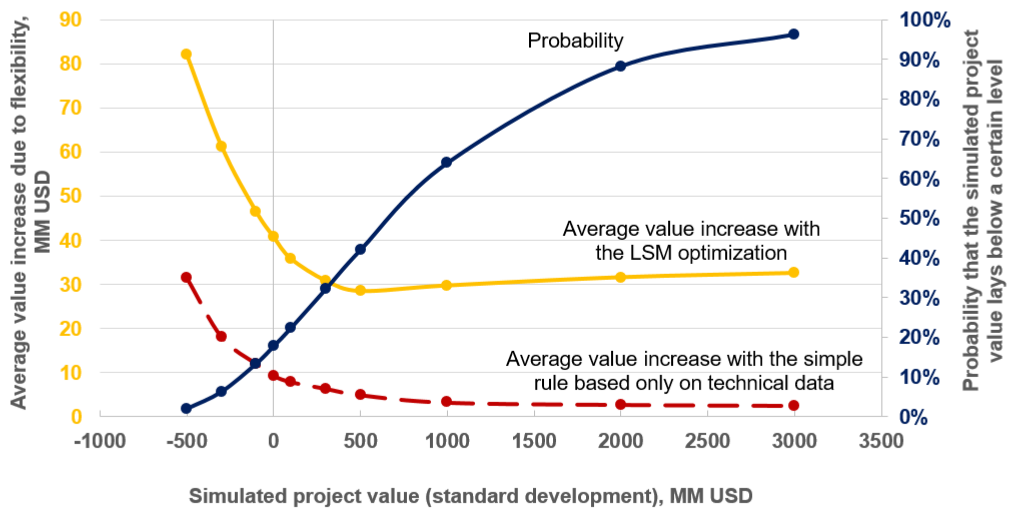

Figure 15: Project value increase due to the staged strategy standard development

It is also interesting to compare the results from the optimization using the LSM algorithm with a simple approach based on a now or never decision. Assume that managers decide to drill Wells 5 and 6 in Year 7 only if the information indicates that their compound initial rate would exceed 0.7 mln bbl/year. By following this rule, the decision maker disregards the market uncertainty and the value of waiting, focusing only on the technical part of the problem. The red dashed line in Figure 15 (the left axis) illustrates the average NPV added by such a strategy within staged development. Although such an optimization approach also allows to hedge some part of the downside reservoir risk, only 55% of the simulated cases with the negative project value under the standard development are improved due to this strategy (compared to 81% when using the LSM algorithm). In the worst cases (when project value is below -500 MM USD) this flexibility adds on average 31 MM USD. For the range of project simulated values below 3000 MM USD the average increase of the value is about 3 MM USD.¨

These findings are confirmed in Fedorov et al. (2020), where we used a benchmark reservoir model simulation to identify the value of the staged development in the presence of prominent reservoir uncertainty. However, as we mentioned in Introduction, whether the staged development creates additional value depends strongly on the problem at hand. With this paper we aim to introduce a methodology that could be easily used for other project cases in order to assess whether the staged approach would be beneficial.

5.2 Sensitivity analysis

In this section we study the sensitivity of the results to key factors. Specifically, we focus on the effect of changes in the production decline rate, oil price volatility and drilling costs.

In our case study we assumed that production profiles follow an exponential decline curve with a constant nominal decline rate of 22.5% based on the input from the field engineers. It is, however, important to see how our results would change if the reservoir depletes with another pace. Note that in order to keep recoverable reserves in line with the reference case (about 53 mmbbl) we also have to adjust the initial production rate of wells When reducing/increasing the nominal decline rate.

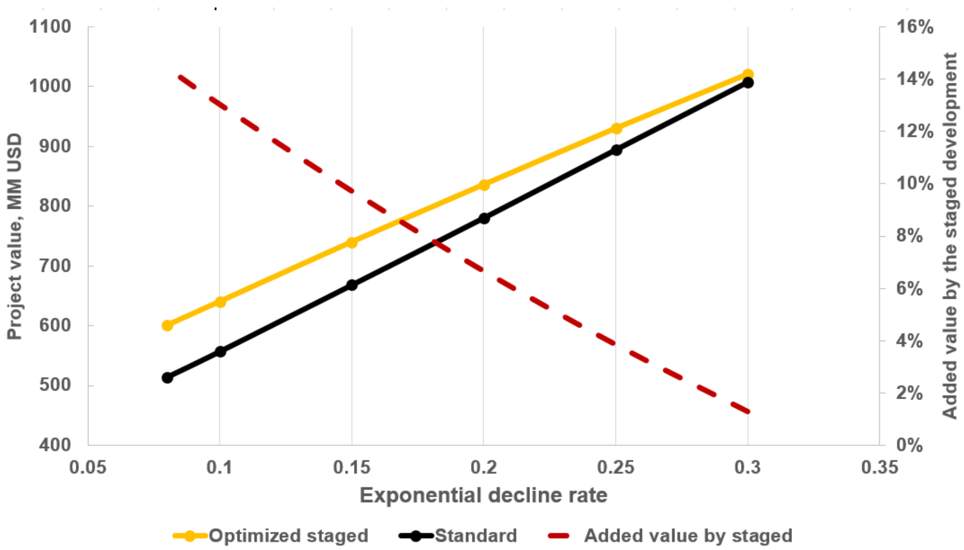

Figure 16: Sensitivity of project value to changes in the exponential decline rate (with adjusted initial production rates)

Figure 16 illustrates that the lower production decline is, the more beneficial is the staged development as mid- and late life decisions play a more significant role in the project value. We can argue that we were quite conservative in our analysis as the added value by the staged development in our reference case was 4%, when using 22.5% nominal decline rate, while when using 8% decline rate, the staged development can add 14% of the value compared to the standard development.

Although the calibration based on the historical market data provides good knowledge on how future oil prices might evolve based on the information we have so far, it is not a perfect estimate. In other words, historical data is based on past expectations in the market and, thus, is backward looking in nature. Because of that, it cannot fully reflect the possible changes in the future and parameters that are used to build a stochastic price process might change as time passes. A sensitivity analysis is then needed to illustrate how probable changes in underlying uncertain parameters might affect the investment decisions and project value.

The sensitivity analysis reveals that the project value depends on variations in both the short- σ_χ and long-term σ_ξ factors we use in the oil price modelling. However, the sensitivity to σ_χ is rather small. This is a result of the long build-up phase of the project (4 years), which decreases the effect of the short-term price variations on the project. The long-term factor, however, is more important due to the fact that the field production spans over a time horizon that starts in 5 years and ends in 12-18 years from the investment decision (depending

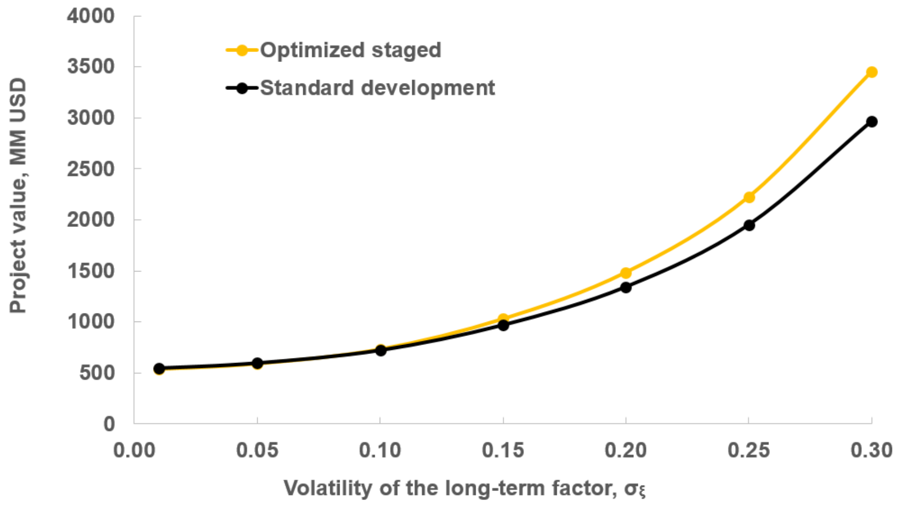

Figure 17: Sensitivity of project value to changes in the long-term oil price volatility factor, σ_ξ

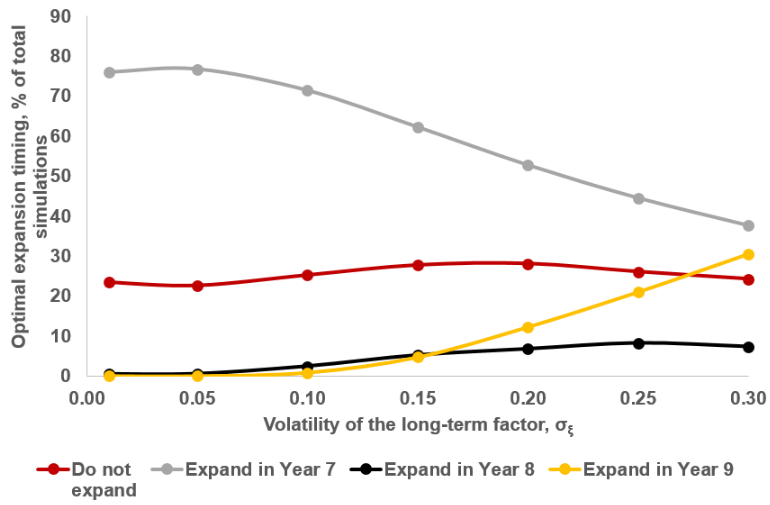

Figure 18: Shift in optimal expansion timing due to changes in the long-term oil price volatility factor, σ_ξ

on the optimal shut-down time). With these results about the sensitivity of the petroleum production investment (in a small oil field as well) to the long-term volatility, we support findings in Jafarizadeh & Bratvold (2015). Jafarizadeh & Bratvold (2015) argue that the uncertainty in the long-term price equilibrium makes the Schwartz-Smith oil price model particularly suitable for valuing long-term investments affected by commodity prices.

As illustrated in Figure 17, the project value increases with σ_ξ, as does the difference between the value of the flexible and non-flexible projects. This is a result of a shift in expansion timing optimization (see Figure 18). The yellow line in Figure 18 illustrates that the higher long-term volatility is, the more the decision maker is willing to postpone the production expansion decision awaiting for higher oil prices.

Another parameter that strongly affects both project values and the expansion policy, is the drilling cost of the production well. We used a fixed value of 48 MM USD for our case study. This gave the decision maker some incentive to wait for the information revelation before committing to drill all the production wells. Figure 19 shows that the values of the flexible and non-flexible projects diverge as the drilling cost of the production well grows. This is due to the increasing amount of simulation cases where the optimal decision is to stay with Stage 1 wells and do not expand in order to prevent extensive spending on drilling. The flexibility to leave the

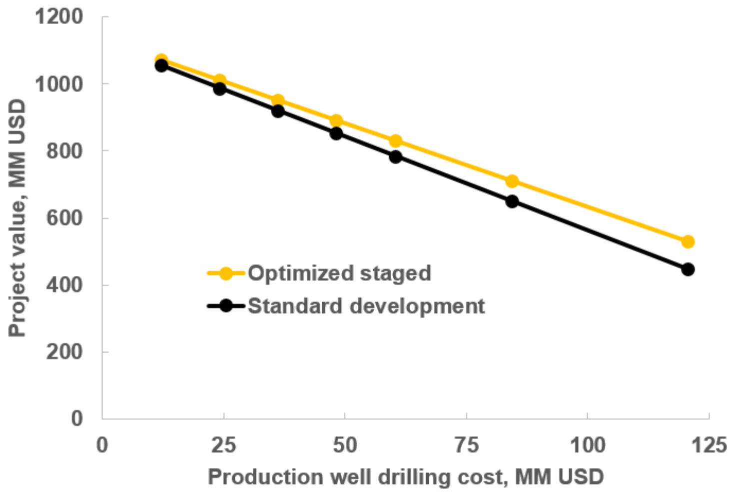

Figure 19: Sensitivity of project value to changes in the the production well drilling cost

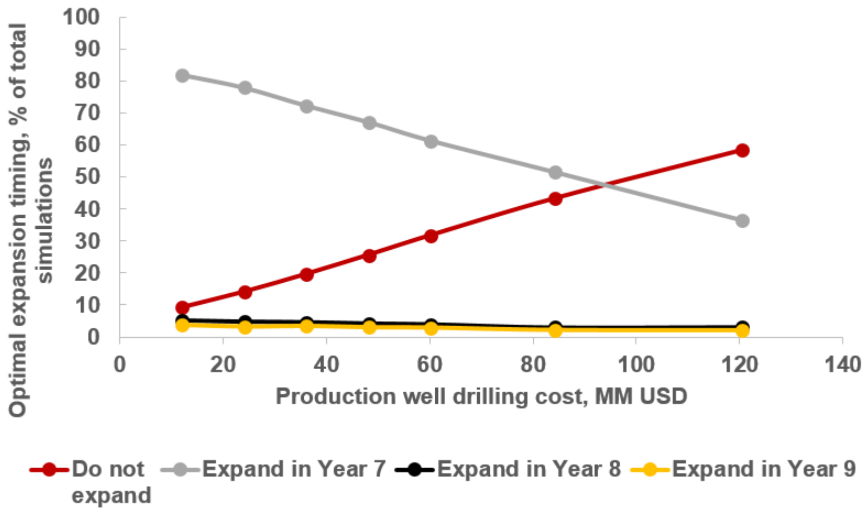

Figure 20: Shift in optimal expansion timing due to changes in the production well drilling cost

expansion option unexercised allows to hedge against this risk.

However, if these expenditures are rather low, the added value of the staged development strategy decreases, as the decision maker tends to drill optional wells even in the low-case scenario. However, increasing drilling expenditures make the decision maker more hesitant to invest in Stage 2 wells. As shown in Figure 20, the larger the well cost, the higher the likelihood that the expansion is not optimal.

5.3 Robustness check

In this section we present some additional analysis on our modelling approach by including more flexibility in the drilling strategy (see Section 5.3.1), accounting for additional uncertainty in production rates (see Section 5.3.2) and by testing different assumptions on how future oil price is modelled (see Section **??**).

5.3.1 Altering the number of optional wells at Stage 2

One of the assumptions we made in our modelling approach is the fact that the operator chooses whether to drill two optional wells at Stage 2 or to drill none of them. In reality, the operator might not be limited to a fixed number of wells that must be drilled after the initial stage of the project. The staged development allows for more flexibility due to the ability to react on the information generated at Stage 1. Therefore, it would be interesting to perform a robustness check of the model and investigate whether altering the number of wells for Stage 2 has a significant effect on the expected value.

Additional data generated during Stage 1 might indicate that some of the optional wells have little potential to create additional value if drilled. For the robustness check, we first introduce an opportunity to choose only one optional well of the two to be drilled at Stage 2. Accounting for this flexibility requires us to increase the number of simulation sets as now the operator can choose between three alternative policies each year instead of two: to drill only one optional well with the highest expected recovery now, to drill two wells now, or postpone the drilling decision until next year. The LSM algorithm, however, allows to straight forwardly incorporate this feature into the analysis. Our results show that in only 1.6% of the simulated cases the optimal decision is to drill only one of the optional wells. The data in Table 7 illustrates the results of the project valuation accounting for additional flexibility. It shows that incorporating the possibility to choose one best or drill both optional wells into the valuation procedure leads to only a minor increase of the expected value of the project by 0.02%. For our case, both optional wells have enough production potential to ensure positive additional cash flow if drilled, in most of the simulated cases.

Table 7: Impact on the project value due to adding more flexibilities

	Expected value (MM USD)	Change vs. reference case (MM USD)	Change vs. reference case (%)
Staged development, reference case (two optional wells without an opportunity to decrease/increase the number)	885.14	-	-
Possibility to choose one best or drill both optional wells	885.29	0.15	+0.02
Possibility to add a third well and choose optimal number of wells (one/two/three)	892.06	6.92	+0.78

The updated knowledge on the subsurface after Stage 1 might also indicate that the operator has potential to drill more than two optional wells due to a higher than expected reservoir performance. As mentioned before, we assume for our case study that the expected recovery under the staged and standard drilling strategies are equal in order to be able to make a fair comparison. In order to perform another robustness check we now, however, assume that in 20% of the simulated cases that resulted in a realization of the ”high case” for Stage 2, the operator is able to drill three additional wells instead of two. The initial production rate of the third optional well is modelled in the same manner as done for the second well by using Eq.3. Given that 60% of all simulated cases were a realization of a ”high case”, this means that the decision maker can decide to drill three wells in 12% of total number of cases. By including this flexibility in our model, we allow to optimize the number of optional wells drilled (i.e. only the best one, the two best or all three[10]). The results in Table 7 show that accounting for this flexibility increases the expected value of the project under the staged development by 0.8% (6.9 MM USD). This increase, however, is caused to a large extent by the additional expected recovery (+0.6%), that is provided by the third optional well, which is optimal to be drilled in 9.1% of all simulated cases.

Figure 21 illustrates the optimal expansion policy accounting for an opportunity to choose the number of optional wells as percentage of all simulated cases. Comparing Figure 11 for the reference case of the staged

[10] remember that drilling the third well is available only in 12% of the total simulated cases

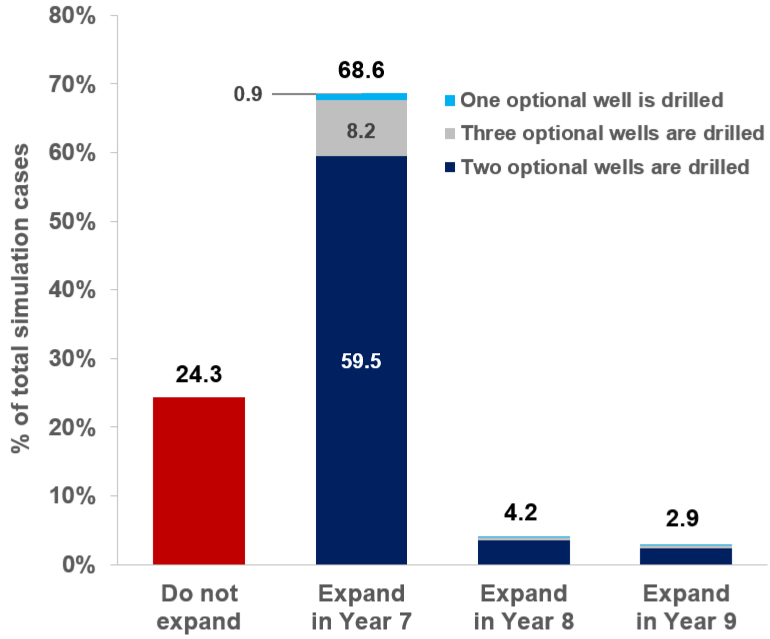

Figure 21: Optimal expansion timing accounting for the flexibility to optimize the number of optional wells

development with Figure 21, one can notice that accounting for the flexibility to change the number of optional wells leads to rather small changes. Overall, the percentage of simulated cases where optional wells remain undrilled decreases due to the fact that the decision maker now can choose to drill only one of optional wells that creates the highest value, rather than having to drill both. Our results also show that it is optimal to drill three wells in 78.1% of cases when the operator is able to add a third well, whereas in 6.3% of the cases it is optimal to drill the two best wells out of the three available. In 0.7% of the cases only one well is drilled, while in 15.0% of the cases the optimal decision is not to drill optional wells at all (most likely due to low oil prices in these simulated cases).

As we have shown, our proposed methodology can straight forwardly be used to account for the flexibility to choose an optimal amount of optional wells. For our case study none of the additional flexibilities in terms of the choice of optional wells, added a significant value directly. For other case studies, however, such flexibilities might create substantial additional value in case of higher uncertainty about the expected production rate of the optional wells and/or due to a higher number of wells.

5.3.2 Adding uncertainty to the production profiles

Another assumption made in our methodology was that production profiles follow an exponential decline curve with a constant nominal decline rate. Therewith, the operator could build a production forecast until the field shut-down once the initial rate of each well is sampled with the Monte Carlo simulation. In Section 4.1 we used an exponential decline equation to model the production rates of each well. This approximation suited the expected drainage from the reservoir case we model. We assumed the nominal decline rate to be fixed at 22.5% based on the input from the field engineers. However, there exist several alternative ways to represent mathematically the future production rate. A general discussion on decline curve analysis methods can be found in Höök et al. (2009). Duong et al. (2011) present a novel approach to estimate the performance of wells producing from tight or shale reservoirs in which fracture flow is dominant. Chang & Lin (1999) show that conventional decline curve analysis does not take into account the uncertainties. Probabilistic or stochastic analysis must then be used to reflect the uncertainty of the production. In this section, we will test alternative approaches to model the field production rate in order to see how our results might be affected by accounting for the uncertainty and how our conclusions depend on the choice of the type of the production decline curve.

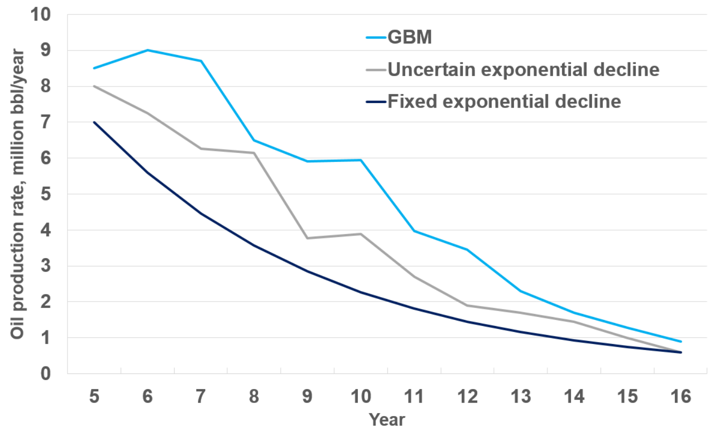

Figure 22: Production profile simulation examples (expected nominal decline rate = 22.5%)

There exist three variations of (Arps et al. 1945)'s decline curve: hyperbolic, harmonic and exponential. The latter two are a special case of hyperbolic decline equation. Höök et al. (2009) argue that "the disadvantage of the exponential decline curve is that it sometimes tends to underestimate production far out in the tail part of the production curve, as decline often flattens out towards a more harmonic and hyperbolic behaviour in that region". As seen in Figure 23 the difference between exponential and hyperbolic fitting for an example oil field historical production from Höök et al. (2009) appears after 15 years of production. Therefore, the difference between these approaches would be very marginal in our case of a marginal field, whose production lifetime is limited to 12 years on average.

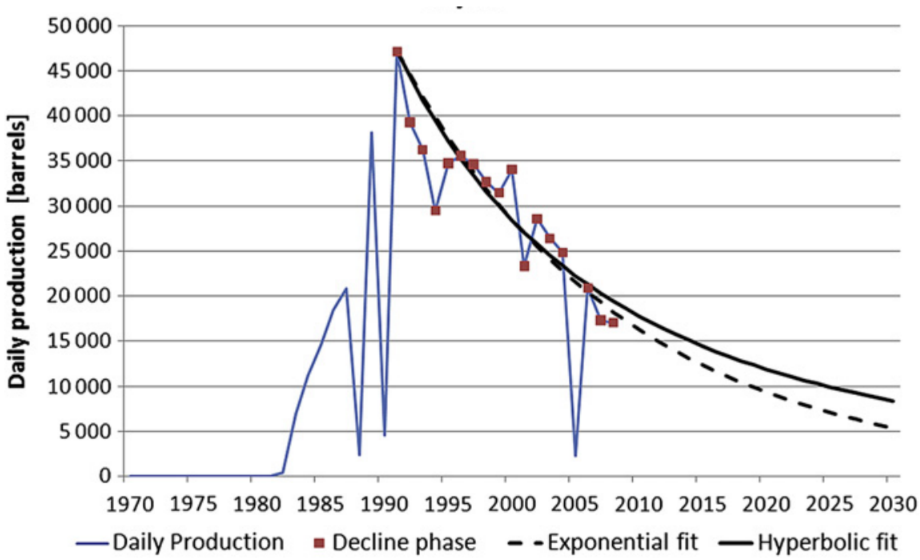

Figure 23: Hyperbolic and exponential decline curves applied an oil field historical production (from Höök et al. (2009))

It is, however, interesting to know how adding uncertainty in our production forecast can affect our results. A straight-forward way to add uncertainty to the exponential decline model, is to assume that the nominal decline rate in Eq. 5 might deviate from the expected mean. The decline rate might either be the same for all wells, or modelled for each individual well. For the purpose of this robustness check, we will use normal distribution with the same mean of 0.225 and the standard deviation of 0.1, to sample the nominal decline rate for each year, for each simulation case. It would generalize the whole field's production rate decline, making all the wells deplete proportionally. Figure 22 shows examples of simulated production profiles using (1) fixed exponential decline, (2) exponential decline with uncertainty and (3) GBM model, which is discussed below.

When including uncertainty in production decline, the project's expected value under the standard development remains the same (851 MM USD) as we kept the expected mean of the nominal decline unchanged. Under the staged development, however, the operator can proactively exploit the flexibility to optimize the expansion policy when additional uncertainty is accounted for. As variability of the technical uncertainty increases and the operator is not certain anymore about future production level once the initial production level is known, the project value under the staged development increases from 885 MM USD to 901 MM USD. The decision maker, therefore, can benefit more from the learning effect while waiting for the expansion, when uncertainty in production rates is accounted for. The additional value created by the staged development will further increase with the amount of the uncertainty in decline rate (if standard deviation is greater than 0.1).

A classic approach to address the uncertainty in production rates in real options literature is to assume that it follows a stochastic process such as geometric Brownian motion (GBM) (Smith & McCardle 1998), Jafarizadeh et al. (2009)). The GBM allows to account for probable deviations from the expected decline rate, adding uncertainty in the production profile modelling. With GBM process, the current production will depend on the previous year's rate and a probabilistic element representing variability, as given by

$$q_{t_n} = q_{t-1_n} e^{(-\mu - 0.5\sigma)\Delta t + \sigma N(0,1)\sqrt{\Delta t}}, \tag{21}$$

where μ is the expected decline rate and σ is the volatility.

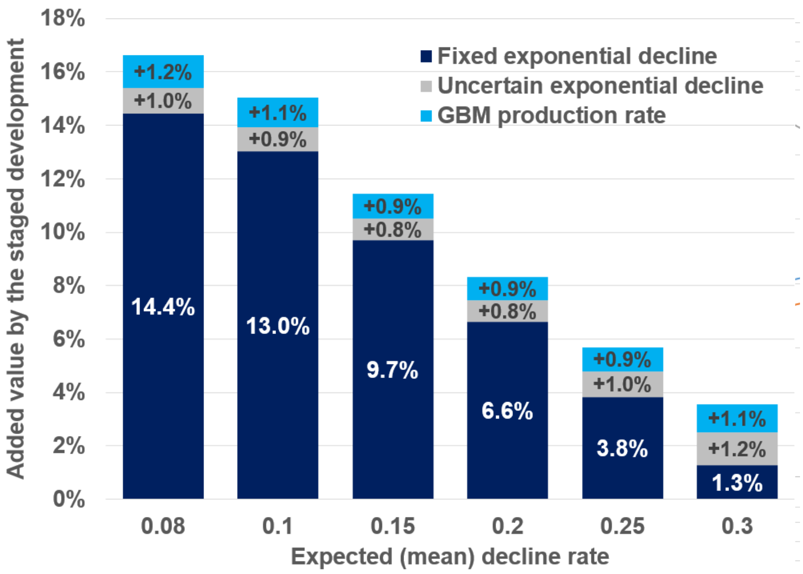

Figure 24: Added value by the staged development depending on the field's expected production decline rate

In GBM model we also assume an expected decline rate (μ) of 22.5%, while the volatility (σ) is set to 0.25 to ensure that confidence bands of recoverable reserves remain the same as in the base case in order to perform a fair comparison with the reference case results. The project value under the standard development again results in 851 MM USD, while the value of the staged development increases to 910 MM USD due to slightly higher variability of production rates, that further increases the real option value due to opportunity to react on the outcome of more uncertain conditions.

We also performed a sensitivity analysis using three models presented in order to illustrate how added value created by the staged development evolves when changing the nominal decline rate (the initial production rate of all wells is then also changed to keep the recoverable volume the same (about 53 mmbbl)). As Figure 24 shows, a steeper production decline, with higher recovery at early stages of the project, reduces the value added by the staged development. With lower expected decline rate, when late life decisions are crucial for the project economy, the staged development creates more value due to the field development strategy optimization. Accounting for the uncertainty in the decline curve adds on average 1% of the project value, with additional 1% when using the GBM process to model the production rate.

6 CONCLUSIONS

In this paper we present a novel methodology to evaluate investment in a small offshore oil field under technical and price uncertainty, addressing the managerial flexibility to phase the drilling strategy into two stages. By investing sequentially, the operator is able to gather additional information on the reservoir uncertainty during the initial stage of the project. We implement the least-squares Monte Carlo (LSM) algorithm for real options valuation, modelling oil price as a two-factor oil price process and accounting for production rate uncertainty, in order to optimize the production expansion decision. Applying the methodology on a synthetic project case, we show that accounting for such flexibility as staged development is crucial for the valuation of an investment in a marginal discovery.

Being flexible enough to be applied to another case studies, the presented methodology also produces results that can be easily communicated with oil E&P companies' managers. The main output of modelling are the recommendations for the companies' managers which could possibly facilitate the decision making. The operator's investment policy can be optimized based on the developed algorithm and threshold boundaries. We identify key features that may affect the choice between the standard and staged field development strategy and calculate associated project values. We perform a sensitivity analysis, which reveals the influence of drilling costs and oil price process parameters on the optimal decision and values of the project. By performing a robustness check of the modelling approach, we also prove that our assumptions on the oil price process are realistic and the methodology can potentially account for more flexibilities in terms of drilling strategy.

Some of the limitations that we face in this paper, i.e. the simplistic reservoir modelling, may be avoided by introducing a realistic reservoir simulation that allows to track geological dependencies between possible production wells locations and to assess the effect of Stage 1 production data on the reservoir model based on history-matching and Bayesian updating. We leave this for the further research in cooperation with our industry partner in order to account for even more flexibilities, e.g. possibility for optimize the number of wells drilled at Stage 2.

Among possible extensions to this work might be further analysis of various approaches to include managerial flexibility in the development strategy of a small offshore field. This flexibility may stem from using a floating production storage and offloading (FPSO) vessel (whose cost is typically higher than one of a platform) that can be re-positioned given the production experience during the initial stage of a project or used for another discovery in case of a failure with the existing field. Among other potential measures that can create additional value during the course of a project could be including possibility of IOR solution, additional capacity for a probable tie-in, etc. Incorporating the concept of imperfect information regarding the technical data generated throughout Stage 1, might also be regarded as a potential direction for our future research.

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