The dynamics of preemptive and follower investments with overlapping ownership

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Abstract

We study how overlapping ownership affects investments in a preemption race with market uncertainty. Internalization of rival payoffs delays follower entry if product market effects are moderate, implying longer incumbency which intensifies dynamic competition. Preemptive and follower investment thresholds increase with volatility as in standard real option models whereas firm value can decrease, and greater volatility makes internalization more profitable. From a welfare perspective there is a tradeoff between dynamic benefit and static costs of overlapping ownership. Whereas it is socially optimal not to have any overlapping ownership in some markets, at low volatility levels we find firms have an insufficient incentive to internalize.

Keywords: common ownership, dynamic competition, real options.

J.E.L. classification: C73, G32.

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1. Introduction

Ownership structures in many industries present important degrees of overlap nowadays, leading researchers to question whether firms maximize only their own value or also internalize rival values (Backus, Conlon and Sinkinson [6]), and prompting concern among regulators and academics about possible anticompetitive effects (Posner, Scott Morton and Weyl [19], Frazzani, Noti, Schinkel et al. [9]).[1] Theory suggests a clear link between overlapping ownership and weakened product market competition, which has been the object of significant empirical study (Reynolds and Snapp [20], Azar, Schmalz and Tecu [4]). The theoretical consequences of overlapping ownership for non-price competition are more involved, and empirical studies find the effect of overlapping ownership on investment measures to be either positive (He and Huang [13]), negative (Gutiérrez and Philippon [12], Newham, Seldeslachts and Banal-Estañol [17]) or insignificant (Koch, Panayides and Thomas [14]). R&D investment has been shown to increase with overlapping ownership if spillovers are large, providing a counterweight to anticompetitive product market effects (López and Vives [16]). But factors other than R&D spillovers are likely to be germane to investments in many industries, such as consumer goods where the timing of product

[1] Several empirical studies document an upward trend in overlapping ownership over past decades due to evolution of the asset management industry and the development of new instruments for investing in diversified portfolios. He and Huang [13] report that the fraction of U.S. public firms held by institutional investors that simultaneously hold at least 5% of the equity of other firms in their industry increased from less than 10% in 1980 to more than 60% in 2014. Backus, Conlon and Sinkinson [6] find that three institutional investors, BlackRock, Vanguard and State Street, owned approximately 21% percent of the average S&P 500 firm at the end of 2017, compared with 6% in 2000. Seldeslachts, Newham and Banal-Estañol [23] document a similar phenomenon in Germany.

introductions accelerates with overlapping ownership (Aslan [3]).

Among the decisions which overlapping ownership is most likely to weigh upon is the exercise of a firm's real options in the face of competition and uncertainty, which represents a key strategic choice for top management (Smit and Trigeorgis [25]). In this article we propose to account for the contrasting patterns of over- and under-investment in the literature therefore by studying a preemption race between firms with overlapping ownership. Irreversibility and uncertainty play a key role in this framework (Dixit and Pindyck [8]), which results in heterogeneous investment outcomes. As our model is prompted by common ownership (where third parties own shares in several firms in an industry) but applies equally if firms have symmetric cross holdings, we follow other authors by using the umbrella term overlapping ownership to refer to ownership structure.

We model two symmetric firms holding competing projects that await opportune market conditions to invest. Investment is discrete and allows firms to access a profit flow, either as leader by investing first or as follower by investing second. Overlapping ownership in the industry drives firms to factor rival value into the timing of their investments. We also allow the control of overlapping owners to extend to output or pricing decisions, in which case overlapping ownership is said to have positive product market effects.[2]

Once the leader is operating as an incumbent, the follower determines the timing of its entry based on its perceived profit flow, which consists of expected duopoly profit net of

[2]Some models of overlapping ownership assume positive product market effects whereas others do not. In López and Vives [16] for example, both R&D investment and price or quantity decisions internalize rival firm value, while in Antón, Ederer, Giné and Schmalz [2] managerial effort decisions factor in overlapping ownership but product market decisions do not.

the incumbent rent which the follower internalizes. High internalization levels can result in a negative perceived profit flow which deters entry (Proposition 1). We find increasing internalization has an ambiguous effect on the follower's perceived profit. If the product market effects of overlapping ownership are not too strong, greater internalization lowers the follower's perceived profit flow and delays its entry, in line with empirical evidence of generic entry in the pharmaceutical industry for example (Newham et al. [17]). We show product market effects are not too strong if firms compete in quantities (Proposition 2), and provide an example involving price competition to illustrate the theoretical possibility that strong product market effects lead to earlier follower investment. Greater volatility unequivocally delays the follower's investment (Proposition 3), but has an ambiguous effect on firm values in contrast with standard real option models because the follower's investment threshold is inefficient with respect to maximization of individual asset value (Proposition 4).

Firm roles are endogenously determined by preemption in our model. The preemptive threshold of the leader and the follower threshold are inversely related, so that if product market effects are not too strong, the delayed follower entry induced by overlapping ownership also lengthens the monopoly phase and raises the incentive for each firm to enter ahead of its rival (Proposition 5). Relaxing *ex-post* static competition thus intensifies dynamic competition *ex-ante*, implying over-investment by the industry leader. As dynamic competition leads to rent equalization, internalization paradoxically reduces firm values in the absence of product market effects. Turning to the effect of market uncertainty, we find that greater volatility causes firms to delay preemptive investment (Proposition

6) and attribute more weight to product market effects which leads them to prefer more internalization (Proposition 7).

Finally, we study welfare assuming that regulators can target a level of internalization by restricting overlapping ownership while firms are free to time their investments. If product market effects are not too strong, there is a welfare tradeoff between the dynamic benefit of overlapping ownership due to increased preemption and the static costs associated with delayed follower entry and relaxed product market competition. We show total welfare is quasiconcave if there are no product market effects and firms compete in quantities (Proposition 8) and study its behavior with product market effects numerically. We show that extending owner control to the product market can lead to higher welfare because it accelerates follower investment. Turning to the effect of uncertainty, at lower volatility levels the dynamic benefit of increasing overlapping ownership outweighs static costs leading regulators to prefer more internalization than firms (Proposition 9), though in markets with moderate uncertainty levels it is socially optimal not to have any overlapping ownership at all.

Our model relates to a theoretical literature dating back to Reynolds and Snapp [20] which studies how overlapping ownership affects product market competition, and to recent articles which have incorporated different forms of non-price competition. López and Vives [16] find that for firms competing in both innovation and product markets, overlapping ownership increases R&D and welfare if spillovers are large. Brito, Ribeiro and Vasconcelos [7] find increasing quality investment in a vertical product differentiation model with overlapping ownership. Li, Ma and Zeng [15] study entry and find that cross-shareholdings can induce deterrence in a sequential move game, whereas Sato and Matsumura [22], show that overlapping ownership mitigates excess entry in a circular-market model. Both of these findings are reminiscent of the follower under-investment arising in our model if product market effects are weak. In an overlapping generations model, Shy and Stenbacka [24] show that common ownership lowers real investment, distorting intertemporal consumption choices. These studies all assume as we do that firms behave competitively, and an alternative stream exemplified by Gilo, Moshe and Spiegel [11] studies coordinated rather than unilateral effects of overlapping ownership. Finally, by incorporating internalization of rival value into a preemption race our work contributes to the literature on preemption under uncertainty (see Azevedo and Paxson [5] for a survey).

The rest of the paper is organized as follows. Section 2 presents our model. Section 3 describes how overlapping ownership affects leader and follower investment. Section 4 relates welfare to overlapping ownership and uncertainty. Section 5 concludes.

2. The Model

An industry consists of two firms, A and B, whose ownership overlaps. Neither firm earns any profit initially but both compete over time for an evolving market. In order to lay the groundwork for our analysis this section specifies the objectives of the firms, the industry environment in which competition unfolds, the values firms obtain in different situations, and the strategic interaction governing market entry.

2.1. Overlapping ownership

The structure of ownership in the industry leads firms to maximize

$$\Omega_i = V_i + \lambda V_j,\ i,j \in \{A,B\},i \neq j,\ \lambda \in [0,1] \tag{2.1}$$

where V_A and V_B denote the value of firm A and firm B's real assets. The parameter λ, assumed to be identical for both firms, is the weight given to rival value. It is referred to as the *degree of internalization*, with $\lambda = 0$ representing purely self-interested behavior whereas $\lambda = 1$ represents joint value-maximizing behavior.

Eq. (2.1) accommodates several ownership structures, which López and Vives [16] refer to as cross ownership, proportional control and silent financial interests. With cross ownership between two firms, λ represents each firm's stake in the other. With common ownership, λ depends on shareholder portfolios through a specification of owner control. Let $m \geq 2$ shareholders hold shares in each firm and denote shareholder i's share of firm j by η_{ij}, $i \in \{1,...,m\}$ and $j \in \{A,B\}$, with $\sum_{i=1}^{m} \eta_{ij} = 1$. Each firm j maximizes a weighted average of the portfolio values of its shareholders, $\sum_{i=1}^{m} \nu_{ij}\left(\eta_{iA}V_A + \eta_{iB}V_B\right)$, where ν_{ij} measures the control exercised by shareholder i. Proportional control equates the weights ν_{ij} to the shares η_{ij}, in which case firm j's objective is $\left(\sum_{i=1}^{m} \eta_{ij}^2\right)V_j + \left(\sum_{i=1}^{m} \eta_{iA}\eta_{iB}\right)V_k$, $j,k \in \{A,B\},j \neq k$. Provided that $\sum_{i=1}^{m} \eta_{iA}^2 = \sum_{i=1}^{m} \eta_{iB}^2$, normalizing yields Eq. (2.1). An alternative is if firms (A,B) each have a single controlling shareholder $(1,2)$ and all other financial interests are silent, so $\nu_{1A} = \nu_{2B} = 1$ and $\nu_{ij} = 0$ otherwise. Firm j's objective is then $\eta_{iA}V_A + \eta_{iB}V_B$, where $i \in \{1, 2\}$ is firm j's controlling shareholder, and symmetric internalization requires $\eta_{1B}/\eta_{1A} = \eta_{2A}/\eta_{2B} = \lambda$.

2.2. Industry environment

Entry barriers shield both firms from further competition. The firms hold projects for which they await opportune market conditions. Each project involves an instantaneous and irreversible investment whose cost is a constant I.

A firm that invests first starts earning a baseline monopoly profit flow π^M. As soon as the second firm invests the market becomes a duopoly, with each firm earning a positive baseline duopoly profit flow $\pi^D(\lambda) \leq \pi^M$. Profit flows are assumed to be perpetual for simplicity. $\pi^D(\lambda)$ is a reduced-form representation of a product market outcome, which we allow to be either a constant π^D if product market effects are absent or a continuously differentiable and increasing function if overlapping ownership has product market effects.

The baseline profit flows are scaled by a measure of current market size Y_t, $t \geq 0$, which follows a geometric Brownian motion

$$dY_t = \alpha Y_t dt + \sigma Y_t dz_t \tag{2.2}$$

where z_t is a standard Wiener process, α is the drift, σ the volatility and $Y_0 = y$ the starting point.

Both firms are risk-neutral and have the same discount rate ρ, with $\rho > \alpha$ so that the problem we study is economically meaningful.

2.3. Firm values

The timing of each firm's investment results from a non-cooperative game which we describe in the next subsection. The equilibrium outcome of this game involves sequential investments, with either firm equally likely to invest first. The first firm is said to be the *leader* and the second the *follower*. Upon investing the leader is an incumbent monopolist and the follower holds a real option. Once the follower invests, both firms operate as duopolists. Denote the values of the leader and the follower's real assets by V^L and V^F. To obtain expressions for these values, let Y and Y^F denote arbitrary leader and follower investment thresholds.

The follower value at the moment of leader investment is

$$V^F(Y;Y^F) = E_Y\left[\int_{T^F}^{\infty} Y_s \pi^D(\lambda)\, e^{-\rho s} ds - I e^{-\rho T^F}\right]. \tag{2.3}$$

where $T^F = \inf\left\{t \geq 0 : Y_t \geq Y^F\right\}$ is the stochastic time at which Y^F is first hit. Inside the expectation, the first term measures discounted duopoly profit and the second is the discounted investment cost. As Y_t follows a geometric Brownian motion,

$$V^F(Y;Y^F) = \begin{cases} \left(\frac{Y}{Y^F}\right)^{\beta}\left(\frac{Y^F}{\rho-\alpha}\pi^D(\lambda) - I\right) & \text{if } Y < Y^F \\ \frac{Y}{\rho-\alpha}\pi^D(\lambda) - I & \text{if } Y \geq Y^F, \end{cases} \tag{2.4}$$

where $\beta > 1$ is the positive root of $0.5\sigma^2 b\,(b-1) + \alpha b - \rho = 0$ (see Appendix A.1). In Eq. (2.4), the first piece is the follower value if it delays investment, which is the product of an expected discount factor and the net present value of investment at Y^F. The second piece is the net present value of immediate duopoly investment.

The leader value at the moment of investment is

$$V^L(Y;Y^F) = E_Y\left[\int_0^{T^F} Y_s \pi^M e^{-\rho s} ds + \int_{T^F}^{\infty} Y_s \pi^D(\lambda)\, e^{-\rho s} ds\right] - I. \tag{2.5}$$

The first term in the expectation is the discounted monopoly profit the firm obtains until the follower's anticipated investment at threshold Y^F, the second term is discounted duopoly profit, and the last term is the investment cost. $V^L(Y;Y^F)$ has the specific form

$$V^L(Y;Y^F) = \begin{cases} \frac{Y}{\rho-\alpha}\pi^M - I + \left(\frac{Y}{Y^F}\right)^\beta \frac{Y^F}{\rho-\alpha}\left(\pi^D(\lambda) - \pi^M\right) & \text{if } Y < Y^F \\ \frac{Y}{\rho-\alpha}\pi^D(\lambda) - I & \text{if } Y \geq Y^F \end{cases} \tag{2.6}$$

(see Appendix A.1), where the first piece is the value if the follower delays investment, consisting of the net present value of perpetual monopoly profits and the product of an expected discount factor with a profit flow correction due to follower investment. The second piece is the net present value if the follower invests immediately.

2.4. Preemption

Whereas the firms solve a monopoly investment problem if $\lambda = 1$, for $\lambda < 1$ the timing of investments and the roles of each firm as leader or follower are determined non-cooperatively. The strategic interaction involved is a preemption race, whose equilibrium runs as follows.

Denote the value of the leader and follower objectives for a given current market size Y by $\Omega^L(Y,\lambda)$ and $\Omega^F(Y,\lambda)$, so $\Omega^L(Y,\lambda) = \Omega_i$ and $\Omega^F(Y,\lambda) = \Omega_j$ if firm i invests first and firm j invests second, $i, j \in \{A, B\}, i \neq j$. $\Omega^L(Y, 0)$ and $\Omega^F(Y, 0)$ correspond to a standard real option game payoffs as in Nielsen [18], whereas for $\lambda > 0$ the specifications are those we obtain in the next section. The difference $f(Y,\lambda) = \Omega^L(Y,\lambda) - \Omega^F(Y,\lambda)$ represents the incentive of each firm to invest ahead of the other. $f(Y,\lambda)$ is initially negative and attains positive values over an interval $\left(Y^P(\lambda), Y^F(\lambda)\right)$, referred to as the *preemption range*. Figure 2.1 illustrates a typical payoff configuration.

If the initial market size is $y \leq Y^P(\lambda)$, then in equilibrium the first investment occurs once the lower bound of the preemption range $Y^P(\lambda)$ is first reached. Intuitively, entry at a threshold $Z \in \left(Y^P(\lambda), Y^F(\lambda)\right)$ cannot be an equilibrium because if one firm enters at threshold Z, the other has a positive incentive to preempt by entering at a threshold in $\left(Y^P(\lambda), Z\right)$. Moreover, as $\Omega^L\left(Y^P(\lambda),\lambda\right) - \Omega^F\left(Y^P(\lambda),\lambda\right) = 0$, positional competition dissipates rents (Fudenberg and Tirole [10]). Compared to a standard real option game, overlapping ownership induces each firm to attribute a positive weight to the other's value. With moderate product market effects, this shifts $\Omega^L(Y,\lambda)$ and the rightmost part of $\Omega^F(Y,\lambda)$ upward as depicted in the figure.

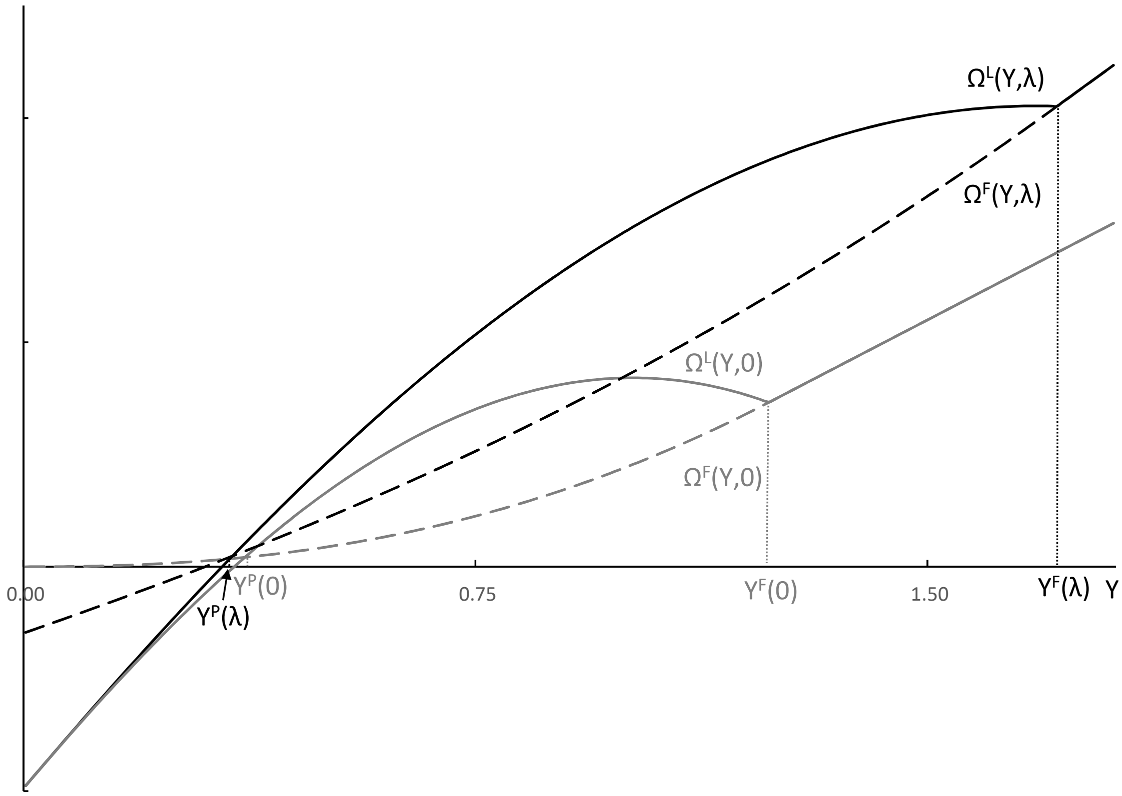


Figure 2.1: Leader and follower payoffs for standard preemption (in grey, $\Omega^L(Y,0)$ and $\Omega^F(Y,0)$) and overlapping ownership (in black, $\Omega^L(Y,\lambda)$ and $\Omega^F(Y,\lambda)$), assuming $\lambda = 0.3$, Cournot duopoly, and $\alpha = 0.01$, $\rho = 0.06$, and $\sigma = 0.15$.

Formally, the outcome described above obtains as the symmetric subgame perfect equilibrium in mixed strategies within an appropriate strategy space where firms choose continuously whether or not to enter (Riedel and Steg [21]). Both firms attempt investment once the threshold $Y^P(\lambda)$ is first reached, and only one randomly succeeds. In subgames starting at a market size in $\left(Y^P(\lambda), Y^F(\lambda)\right)$ on the other hand, both firms also seek to invest immediately even though joint investment is suboptimal. There is a positive probability of simultaneous investment in this case, which is calibrated so rent dissipation holds (Thijssen, Huisman and Kort [26]). Continuous time preemption games can present another symmetric equilibrium where players invest simultaneously at a jointly optimal threshold, but this possibility is ruled out in our model because firms do not earn positive initial profit flows. Finally, Fudenberg and Tirole [10] assume a closed-loop information structure but in an open-loop equilibrium rent dissipation need not hold.

3. Dynamics of industry investments with overlapping ownership

Having laid the foundations of the model in the preceding section, we can now describe the equilibrium timing of investments in the industry. Initially, first-mover advantage drives firms to race, and the outcome of this race attributes leader and follower roles. Once leader investment occurs, the follower chooses its investment timing but internalizes its effect on the leader. Proceeding by backward induction, we study the follower problem first and address the preemption equilibrium second.

3.1. Entrant (follower) investment

Once the leader has invested at a given market size Y and become an incumbent in the product market, the follower is left with an option on a duopoly profit stream. The main consequence of overlapping ownership is to translate the incumbent's positional rent into an opportunity cost for the follower, which perceives a net profit flow of

$$\Delta^F(\lambda) = \pi^D(\lambda) - \lambda\left(\pi^M - \pi^D(\lambda)\right) \tag{3.1}$$

from duopoly investment.

The perceived flow $\Delta^F(\lambda)$ may take negative values. For example if product market effects are absent ($\pi^D(\lambda)$ constant), the follower perceives investment to be unprofitable if $\lambda \geq \lambda^d := \pi^D/(\pi^M - \pi^D)$. If products are close substitutes ($\pi^D < \pi^M/2$, so $\lambda^d < 1$) and the degree of internalization is sufficiently large, its entry can therefore be deterred. Similarly to Li et al. [15]'s finding in a discrete time entry game, such deterrence occurs at levels of internalization that fall short of joint-value maximization. We thus have:

Proposition 1. *Without product market effects, entry is deterred if the degree of internalization exceeds λ^d.*

For sufficiently low internalization (provided $\pi^D(0) > 0$) or sufficiently high duopoly profit levels, the perceived flow $\Delta^F(\lambda)$ is positive. In this case the follower's entry decision is non-trivial and involves determining an investment threshold Y^F so as to maximize the perceived value

$$\begin{aligned}\Phi\left(Y;Y^F\right) &= V^F\left(Y;Y^F\right)+\lambda V^L\left(Y,Y^F\right)\\ &= \begin{cases}\lambda\left(\frac{Y}{\rho-\alpha}\pi^M-I\right)+\left(\frac{Y}{Y^F}\right)^\beta\left(\frac{Y^F}{\rho-\alpha}\Delta^F(\lambda)-I\right) & \text{if } Y<Y^F\\ (1+\lambda)\left(\frac{Y}{\rho-\alpha}\pi^D-I\right) & \text{if } Y\geq Y^F.\end{cases}\end{aligned} \tag{3.2}$$

In Eq. (3.2), the first piece is the perceived value if investment is delayed and the second piece is the perceived value of immediate duopoly investment. In the first piece, the first summand is the internalized incumbent value and the second is perceived benefit from delayed entry, which is the product of an expected discount factor and the perceived net present value at Y^F. The objective $\Phi\left(Y; Y^F\right)$ is strictly quasiconcave in Y^F, with global maximum

$$Y^F(\lambda) = \frac{\beta}{\beta - 1}\frac{\rho - \alpha}{\Delta^F(\lambda)} I \tag{3.3}$$

The follower invests once the threshold $\max\left\{Y, Y^F(\lambda)\right\}$ is reached, and obtains the payoff $\Omega^F(Y, \lambda) = \Phi\left(Y; Y^F(\lambda)\right)$.

By Eq. (3.3), the follower investment threshold is inversely related to the perceived profit flow $\Delta^F(\lambda)$, which captures the entire effect of overlapping ownership on the follower's investment decision. Changing λ has an ambiguous effect on the perceived profit flow. The direct effect of greater internalization is negative, because internalizing a greater share of the incumbency rent $\pi^M - \pi^D(\lambda)$ increases the follower's opportunity cost of investment. But there is also a positive indirect effect, because relaxing product market competition increases the attractiveness of duopoly entry by $(1 + \lambda)\left(\pi^D\right)'(\lambda)$. Because the sign of $\left(\Delta^F\right)'(\lambda)$ plays a key role throughout our analysis, we will say that overlapping ownership has:

- *weak product market effects* if the direct effect dominates, so $\left(\Delta^F\right)'(\lambda) < 0$, and

- *strong product market effects* if the indirect effect dominates, so $\left(\Delta^F\right)'(\lambda) > 0$.

The absence of product market effects ($\pi^D(\lambda)$ constant) is an extreme case of weak product market effects. Even if strong product market effects are theoretically possible, weak product market effects commonly arise in standard oligopoly models.

Proposition 2. *If product market competition is à la Cournot with constant marginal cost* *and decreasing marginal revenue, internalization delays entry* $\left(\left(Y^F\right)'(\lambda) \geq 0\right)$.

Proof. See Appendix A.2.

Proposition 2 provides a range of circumstances where internalization drives the follower to delay entry, whether or not product market effects are present. By specifying the demand function, it can be verified directly as the following example illustrates.

Example 1. *Let inverse demand be given by*

$$P(Q) = \begin{cases} a - bQ^{\eta} & \text{if } \eta \neq 0 \\ a - b\log Q & \text{if } \eta = 0. \end{cases} \tag{3.4}$$

where Q denotes total output, $a \geq 0$, $b \neq 0$ with $b\eta > 0$, and $\eta \geq -1$. This specification nests constant elasticity ($a = 0$, $b < 0$, $\eta \in [-1, 0)$) and linear ($a, b > 0$, and $\eta = 1$) demands. Both firms have constant unit costs c, with $c \in [0, a]$ if $a > 0$. With overlapping ownership and product market effects, firm i maximizes $\pi^i(q_i, q_j) + \lambda\pi^j(q_j, q_i)$, $i, j \in \{A, B\}, i \neq j$, resulting in equilibrium outputs

$$q_A^* = q_B^* = \left(\frac{2^{1-\eta}}{2 + \eta(1+\lambda)}\frac{a-c}{b}\right)^{\frac{1}{\eta}} \ (\eta \neq 0),\ \frac{1}{2}10^{\frac{a-c}{b} - \frac{1+\lambda}{2\ln 10}} \ (\eta = 0), \tag{3.5}$$

and profits

$$\pi^D(\lambda) = b\eta\frac{1+\lambda}{4}\left(\frac{2}{2+\eta(1+\lambda)}\frac{a-c}{b}\right)^{\frac{1+\eta}{\eta}} \ (\eta \neq 0),\ \frac{b(1+\lambda)}{4\ln 10}10^{\frac{a-c}{b} - \frac{1+\lambda}{2\ln 10}} \ (\eta = 0). \tag{3.6}$$

Noting that $\pi^M = 2\pi^D(1)$,

$$\Delta^F(\lambda) = \begin{cases} b\eta\left(\frac{(1+\lambda)^2}{4}\left(\frac{2}{2+\eta(1+\lambda)}\right)^{\frac{1+\eta}{\eta}} - \lambda\left(\frac{1}{1+\eta}\right)^{\frac{1+\eta}{\eta}}\right)\left(\frac{a-c}{b}\right)^{\frac{1+\eta}{\eta}} & \text{if } \eta \neq 0 \\ b10^{\frac{a-c}{b}}\left(\frac{(1+\lambda)^2}{4\ln 10}10^{-\frac{1+\lambda}{2\ln 10}} - \frac{\lambda}{10}\right) & \text{if } \eta = 0. \end{cases} \tag{3.7}$$

$\Delta^F(\lambda)$ *is positive and decreasing, implying that* $Y^F(\lambda)$ *is finite with* $\left(dY^F/d\lambda\right)(\lambda) > 0$.

The pattern of delayed follower entry in Example 1. is consistent with empirical evidence from the pharmaceutical industry (Newham et al. [17]), but there is also evidence that entrants overinvest if technological spillovers override business-stealing (Antón et al. [1]) which is consistent with strong product market effects. In Appendix A.3, we give an example using price competition and differentiated products where overlapping ownership locally accelerates follower entry for some parameter values, even without spillovers.

The sensitivity of the follower's investment threshold to volatility is, however, unambiguous.

Proposition 3. *Provided that* $\Delta^F(\lambda) > 0$, *the follower threshold satisfies*

$$\frac{\partial Y^F}{\partial \sigma}(\lambda) > 0. \tag{3.8}$$

Proof. See Appendix A.4.

Proposition 3 states that with overlapping ownership the follower exhibits a standard response to increasing volatility. However, because the follower exercises its investment option at a threshold which is suboptimal with respect to maximization of the value of its real assets, the value of the follower's assets $V^F(Y,\lambda)$ does not react to volatility in the usual way. Because greater volatility exacerbates the inefficiency of the follower's option exercise, there is both a standard positive effect due to the flexibility of investment timing and an additional negative effect attributable to the perceived profit flow. If the follower's option is deep out of the money the positive flexibility effect dominates and volatility increases firm value, but near the exercise threshold inefficiency dominates and additional volatility reduces firm value.

Proposition 4. *The effect of volatility on follower value is ambiguous, with*

$$\lim_{Y\to 0}\frac{\partial V^F}{\partial \sigma}(Y,\lambda) > 0 \text{ and } \lim_{Y\to Y^F(\lambda)}\frac{\partial V^F}{\partial \sigma}(Y,\lambda) < 0. \tag{3.9}$$

Proof. See Appendix A.5.

3.2. Preemptive (leader) investment

Anticipating that the follower subsequently invests at threshold $Y^F(\lambda)$, leading at market size Y results in the payoff

$$\Omega^L(Y,\lambda) = \begin{cases} \frac{Y}{\rho-\alpha}\pi^M - I + \left(\frac{Y}{Y^F(\lambda)}\right)^\beta \left(\frac{Y^F(\lambda)}{\rho-\alpha}\left(\Delta^F(\lambda) - (1-\lambda)\pi^M\right) - \lambda I\right) & \text{if } Y < Y^F(\lambda) \\ (1+\lambda)\left(\frac{Y}{\rho-\alpha}\pi^D(\lambda) - I\right) & \text{if } Y \geq Y^F(\lambda). \end{cases} \tag{3.10}$$

The first piece consists of three terms, the first two representing the net present value of monopoly profits and the last being the product of an expected discount factor and the perceived net value of follower entry. The second piece is perceived net present value of duopoly investment, as the follower is expected to enter immediately. The difference $\Omega^L(Y,\lambda) - \Omega^F(Y,\lambda)$ between leader and follower payoff, which measures the incentive to lead at Y, is therefore

$$f(Y,\lambda) = \begin{cases} (1-\lambda)\left(\frac{Y}{\rho-\alpha}\pi^M - I - \left(\frac{Y}{Y^F(\lambda)}\right)^\beta \left(\frac{Y^F(\lambda)}{\rho-\alpha}\pi^M - I\right)\right) & \text{if } Y < Y^F(\lambda) \\ 0 & \text{if } Y \geq Y^F(\lambda). \end{cases} \tag{3.11}$$

As described in Section 2.4, for $\lambda < 1$ there exists a range of market sizes over which $f(Y,\lambda)$ takes on positive values and preemption occurs. In a symmetric equilibrium, the first firm (A or B with equal probability) invests at $Y^P(\lambda) = \inf\{Y \geq y : f(Y,\lambda) > 0\}$, the lower bound of the preemption range. Letting $Y^m = (\rho - \alpha) I/\pi^M$ denote the competitive or NPV threshold, rearranging Eq. (3.11) yields the implicit definition

$$\frac{Y^P - Y^m}{Y^F(\lambda) - Y^m} = \left(\frac{Y^P}{Y^F(\lambda)}\right)^\beta. \tag{3.12}$$

The preemption threshold therefore depends on internalization only through the follower threshold $Y^F(\lambda)$, to which it is inversely related. If $Y^F(\lambda)$ is infinite, $Y^P(\lambda)$ attains the NPV threshold.

For $\lambda = 1$ on the other hand, the objectives of both firms are entirely aligned and the first firm (either A or B, assumed equally likely) invests at the monopoly threshold $Y^M = (\beta/(\beta-1))\left((\rho-\alpha)/\pi^M\right) I$.

Summing up,

Proposition 5. *The equilibrium threshold for leader investment is* $Y^P(\lambda) \in \left[Y^m, Y^M\right]$ *defined by Eq. (3.12), with*

$$\left(Y^P\right)'(\lambda) < 0 \;(\, > 0) \text{ if and only if } \left(\Delta^F\right)'(\lambda) < 0 \;(\, > 0). \tag{3.13}$$

Proof. See Appendix A.6.

In the case of weak product market effects, the intuition behind Proposition 5 is as follows. Greater internalization delays follower entry and thereby lengthens the monopoly phase. This exacerbates positional competition and accelerates preemptive investment. Conversely, strong product market effects relax positional competition and result in later preemptive entry.

The sensitivity of the preemptive investment threshold to volatility is, however, unambiguous:

Proposition 6. *Provided that* $\Delta^F(\lambda) > 0$, *the preemptive threshold satisfies*

$$\frac{\partial Y^P}{\partial \sigma}(\lambda) > 0. \tag{3.14}$$

Proof. See Appendix A.7.

Proposition 6 establishes that the response of preemptive investment to volatility remains standard with overlapping ownership.

Preemption dissipates positional rents in equilibrium so the equilibrium value of both objectives is $\Omega^F\left(Y^P(\lambda),\lambda\right)$ at the moment the leader invests regardless of subsequent firm roles. At an initial market size $y \leq Y^P(\lambda)$, the perceived payoffs are therefore $\Omega_i(y,\lambda) = \left(y/Y^P(\lambda)\right)^\beta \Omega^F\left(Y^P(\lambda),\lambda\right)$, $i \in \{A,B\}$. By Eq. (2.1), the values of real assets are also equalized, yielding

$$\hat{V}_i(y,\lambda) = \left(\frac{y}{Y^F(\lambda)}\right)^\beta \left(\frac{Y^F(\lambda)}{\rho-\alpha}\pi^D(\lambda) - I\right), i \in \{A,B\} \tag{3.15}$$

in equilibrium. The equilibrium value of real assets is therefore that of a follower's duopoly option exercised at the suboptimal threshold $Y^F(\lambda) = \left(\pi^D(0)/\Delta^F(\lambda)\right)Y^F(0)$.

Because $\Delta^F(\lambda) < \pi^D(0)$ if product market effects are weak, the follower's exercise threshold is inefficiently high with respect to maximization of follower asset value ($Y^F(\lambda) > Y^F(0)$ for $\lambda > 0$), and this inefficiency increases with internalization. If in addition product market effects are absent, greater internalization lowers equilibrium firm values. It is therefore not a foregone conclusion that firm values are positively affected by overlapping ownership, particularly if it is driven by quasi-indexing which drives institutional investors to acquire their stakes mechanically. By the same token, in industries where dynamic competition is important but product market effects are not, there is a strong incentive for owners to coordinate investments if they can rather than engage in preemption.

With product market effects, internalization also affects equilibrium value positively through $\pi^D(\lambda)$, and we can meaningfully speak of an efficient internalization level from the perspective of firms, $\lambda^V \in \arg\max_{\lambda\in[0,\overline{\lambda}]} \hat{V}_i(y,\lambda)$ where $\overline{\lambda} < 1$ is an upper bound that antitrust authorities may set (otherwise invariably $\lambda^V = 1$). This level of internalization provides a benchmark for our subsequent welfare analysis, and reflects the outcome that would arise if investors adjusted ownership so as to maximize expected value, provided that their firms engage in entry competition. Whether λ^V is interior or not depends on the strength of product market effects, and if it is interior, greater volatility complements overlapping ownership by raising the efficient internalization level.

Proposition 7. *Without product market effects, $\lambda^V = 0$. With weak product market effects, $\lambda^V > 0$, and*

$$\frac{\partial \lambda^V}{\partial \alpha} > 0 \text{ and } \frac{\partial \lambda^V}{\partial \sigma} > 0 \tag{3.16}$$

if $\lambda^V < \overline{\lambda}$. With strong product market effects, $\lambda^V = \overline{\lambda}$.

Proof. See Appendix A.9.

The logic behind Proposition 7 is that $\partial \hat{V}_i(y, \lambda) / \partial \lambda$ is globally negative if product market effects are absent and globally positive if there are strong product market effects. In the intermediate case of weak product market effects, positive internalization is profitable but more intense dynamic competition lowers firm values and λ^V can lie in the interior of $[0, \overline{\lambda}]$. In the Cournot case with linear demand for example, $\hat{V}_i(y, \lambda)$ is quasiconcave (see Appendix A.8) and the antitrust constraint need not bind. As for the comparative statics, raising drift or volatility increases the relative importance of product market effects and therefore the incentive for internalization.

4. Welfare analysis

We next analyze the effect of overlapping ownership on industry investments from a normative standpoint. We first describe the social welfare function and identify the tradeoffs associated with internalization. Next, assuming that industry investments take place as described in Section 3 and that policy-makers maximize a social welfare function by targeting the level of internalization, we identify conditions for positive internalization levels to be socially optimal and the sensitivity of optimum to uncertainty parameters.

Assume that society discounts at the same rate ρ as firms. Monopoly and duopoly respectively generate positive baseline consumer surplus flows s^M and $s^D(\lambda)$ with $s^M \leq s^D(\lambda)$, which are scaled by Y_t. If product market effects are present, consumer surplus under duopoly $s^D(\lambda)$ is assumed to be continuously differentiable and decreasing in λ.

Total welfare is therefore

$$W(y,\lambda) = E_y\left[\int_{T^P(\lambda)}^{T^F(\lambda)} Y_s\left(\pi^M + s^M\right)e^{-\rho s}ds - Ie^{-\rho T^P(\lambda)}\right.$$
$$\left. + \int_{T^F(\lambda)}^{\infty} Y_s\left(2\pi^D(\lambda) + s^D(\lambda)\right)e^{-\rho s}ds - Ie^{-\rho T^F(\lambda)}\right], \quad (4.1)$$

where $T^P(\lambda) = \inf\left\{t \geq 0 : Y_t \geq Y^P(\lambda)\right\}$ and $T^F(\lambda) = \inf\left\{t \geq 0 : Y_t \geq Y^F(\lambda)\right\}$ are the stochastic times at which $Y^P(\lambda)$ and $Y^F(\lambda)$ are first hit. In Eq. (4.1) the first and third terms measure welfare from the monopoly and duopoly phase respectively, whereas the second and fourth terms are the discounted costs of the first and second investment.

Provided that market size is initially small ($y \leq Y^P(\lambda)$), Eq. (4.1) has the form

$$W(y,\lambda) = 2\hat{V}_i\left(y,\lambda\right) + \left(\frac{y^\beta}{\left[Y^P(\lambda)\right]^{\beta-1}} - \frac{y^\beta}{\left[Y^F(\lambda)\right]^{\beta-1}}\right)\frac{s^M}{\rho-\alpha} + \frac{y^\beta}{\left[Y^F(\lambda)\right]^{\beta-1}}\frac{s^D\left(\lambda\right)}{\rho-\alpha}. \quad (4.2)$$

In Eq. (4.2), the first summand is equilibrium industry profit, the second is consumer surplus accruing during the monopoly phase, and the third is consumer surplus accruing during the duopoly phase.

To see how internalization affects welfare, suppose that $Y^F(\lambda)$ is finite and consider first the case of weak product market effects. The marginal effect of internalization can be broken down into

$$\begin{aligned}\frac{\partial W}{\partial\lambda}(y,\lambda) = 2\frac{\partial\hat{V}_i}{\partial\lambda}\left(y,\lambda\right) & \underbrace{-\left(\beta-1\right)\left(\frac{y}{Y^P(\lambda)}\right)^\beta\frac{s^M}{\rho-\alpha}\left(Y^P\right)'\left(\lambda\right)}_{\text{dynamic benefit of overlapping ownership}} \\ & \underbrace{-\left(\beta-1\right)\left(\frac{y}{Y^F(\lambda)}\right)^\beta\frac{s^D\left(\lambda\right)-s^M}{\rho-\alpha}\left(Y^F\right)'\left(\lambda\right) + \left(\frac{y}{Y^F(\lambda)}\right)^\beta\frac{Y^F(\lambda)}{\rho-\alpha}\left(s^D\right)'\left(\lambda\right)}_{\text{static costs of overlapping ownership}}. \quad (4.3)\end{aligned}$$

Eq. (4.3) highlights two offsetting effects of internalization on consumer surplus, which are related respectively to the $\left(Y^P\right)'$ term and to the $\left(Y^F\right)'$ and $\left(s^D\right)'$ terms. The *dynamic benefit of internalization* represents the flow of new surplus for consumers resulting from accelerated preemptive investment due to increased positional competition. The *static costs of internalization* are twofold. First, consumers experience a baseline increase in deadweight loss of $s^D(\lambda) - s^M$ during the period over which follower investment is delayed. Second, in the presence of product market effects, weakened product market competition due to internalization reduces the baseline consumer surplus flow during the industry's duopoly phase by $\left(s^D\right)'(\lambda)$. Finally, if there are strong product market effects, the sensitivity of investment thresholds is reversed ($\left(Y^P\right)'(\lambda) > 0$ and $\left(Y^F\right)'(\lambda) < 0$) and internalization generates a dynamic cost along with an ambiguous static effect.

As noted further above, our welfare analysis is premised on initial market size being low enough that both firms wait to invest. If the initial market size is greater than the preemption threshold, investment is immediately attempted and rent equalization implies a positive probability of simultaneous entry (see Section 2.4) which is increasing in λ. Because firms cannot enter any earlier in this case, the dynamic benefit of overlapping ownership takes the form of an increased probability of simultaneous investment, which raises consumer surplus by increasing the likelihood of immediate duopoly.[3]

To study total welfare further and identify the socially optimal level of internalization λ^W, we begin with the simplest case. If product market effects are absent, the total welfare function admits the following partial characterization:

Proposition 8. *If product market competition is à la Cournot with constant marginal cost and concave inverse demand and there are no product market effects, then $W(y, \lambda)$ is quasiconcave in λ over $[0, 1)$, and there exist σ_0 such that $\lambda^W > 0$ for $\sigma < \sigma_0$.*

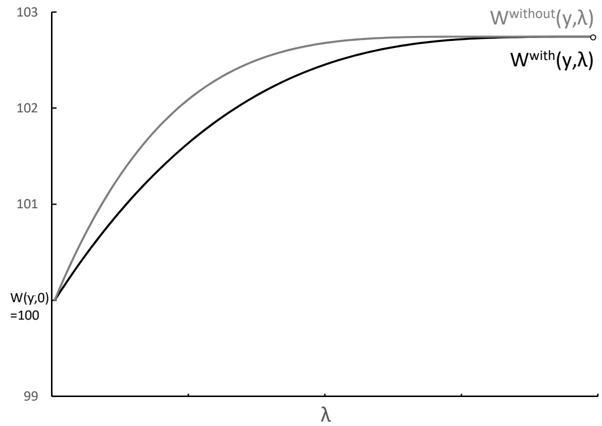
Proof. See Appendix A.10.

[3] We are grateful to a referee for suggesting this point.

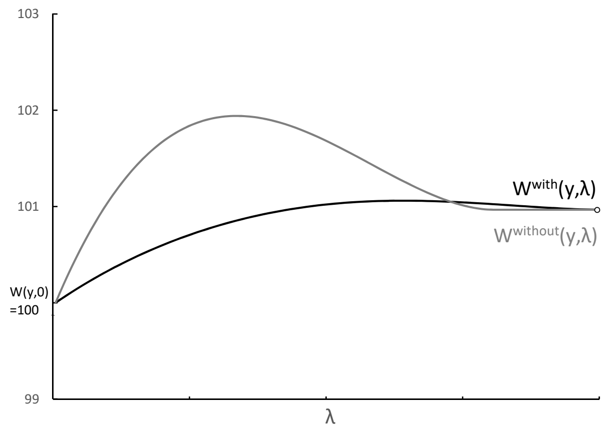
Proposition 8 asserts that there is a tradeoff between the dynamic benefit and static costs of internalization without product market effects which results in a positive socially optimal level of internalization if volatility is low enough. The logic behind this last statement is that the opportunity cost of inducing preemption in terms of foregone option value is important if volatility is high, whereas at low volatility strategic effects matter more making it socially preferable to exacerbate preemption by increasing internalization. Our characterization is limited because $W(y, \lambda)$ is discontinuous at $\lambda = 1$ where the first investment threshold in the industry jumps up to the monopoly threshold Y^M.

With product market effects, the behavior of total welfare is more involved so we focus on the case of Cournot competition with linear demand in the product market.[4] Figure 4.1 plots $W(y, \lambda)$ with and without product market effects. Without product market effects, $W(y, \lambda)$ is quasiconcave (Proposition 8) and there is a constant segment once λ reaches $\pi^D/(\pi^M - \pi^D)$, where $\Delta^F(\lambda) \leq 0$ so follower entry is deterred (Proposition 1). In panel a), $W(y, \lambda)$ is monotonic in λ and any level of internalization $\lambda^W \in [0.8, 1)$ which induces entry deterrence is socially optimal. In panels (b) and (c), $W(y, \lambda)$ has a unique interior optimum λ^W, which is lower in panel (c) where volatility is higher. With product market effects, total welfare is increasing over $[0, 1)$ in panel (a), single-peaked in panel (b), and decreasing in panel (c). Although we do not have an analytic characterization of this case, our numerical analysis suggests that positive internalization levels become socially desirable

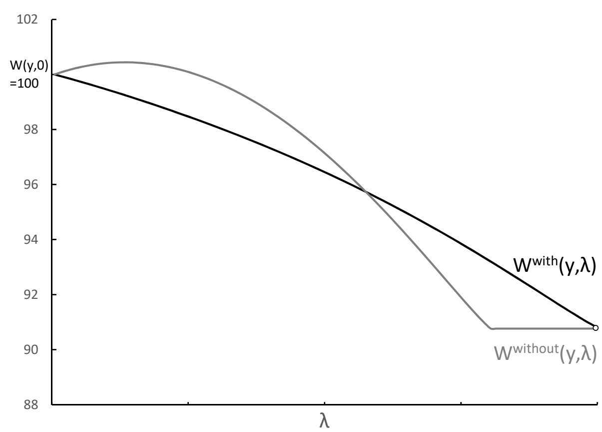
[4] Taking the specification of Eq. (3.4) with $\eta = 1$, profits are given by Eq. (3.6) and consumer surplus is $s^M = (a-c)^2/8b$ or $s^D(\lambda) = 2(a-c)^2/b(3+\lambda)^2$. For Figures 4.1 and 4.2 we set $(a-c)^2/b = y = I = 1$.



(a) $\sigma = 0.05$



(b) $\sigma = 0.125$



(c) $\sigma = 0.2$

Figure 4.1: Total welfare $W(y, \lambda)$ for Cournot payoffs with (black) and without (grey) product market effects, $\alpha = 0.01$, $\rho = 0.07$. Welfare levels at $\lambda = 1$ (not shown) are 60, 77, and 86 respectively.

at lower volatility.

By plotting the total welfare functions with and without product market effects together, Figure 4.1 is instructive as to the effect of extending the control of common owners to product market decisions. Because weakened product market competition implies higher profits and hence follower value, equilibrium firm values unambiguously increase. But, analogously with our analysis of increasing internalization above (see Eq. (4.3)), greater control by common owners has an ambiguous effect on welfare because it hastens follower investment (perceived profit is higher) while relaxing preemption. In panels (b) and (c), at high internalization levels the static benefit of earlier follower entry outweighs the dynamic cost of relaxed preemption, and total welfare increases with the reach of common owners.

Figure 4.1 shows a well-defined maximum of total welfare with product market effects for intermediate volatility values, which are situated in a range between 0.12 and 0.15 with our chosen parameter values. To study the interaction between internalization and volatility more closely, we first state a proposition concerning the social internalization incentive.

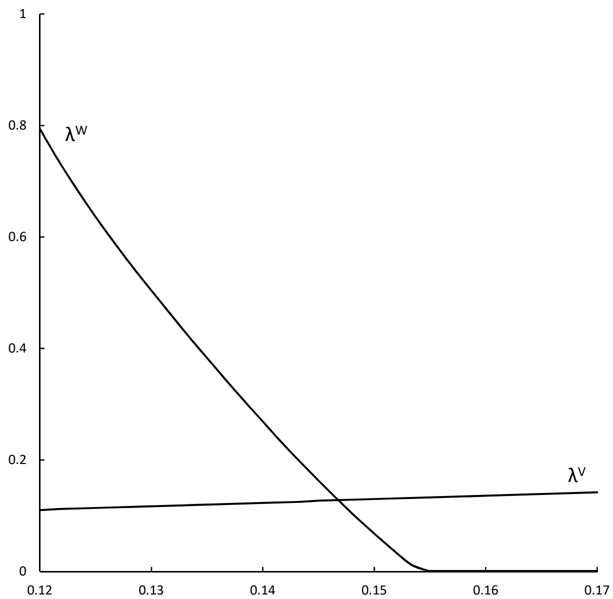
Proposition 9. *If λ^V is interior, there exists σ_1 such that*

$$\sigma < \sigma_1 \Rightarrow \frac{\partial W}{\partial \lambda}(y, \lambda^V) > 0. \tag{4.4}$$

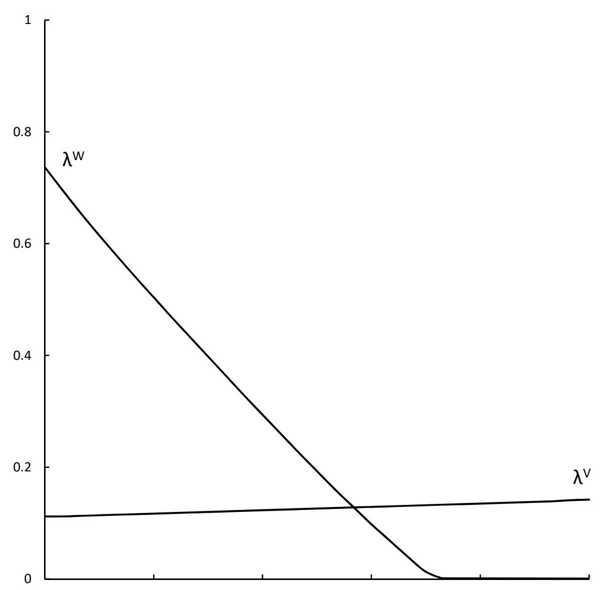
Proof. See Appendix A.11.

If firm value is quasiconcave as is the case for Cournot competition (Appendix A.8), the industry efficient internalization level λ^V is well-defined and Proposition 9 implies that regulators have a greater incentive to internalize at low volatility than firms do. If the social optimum is also interior (as in panel (b) of Figure 4.1, both with and without product market effects), then $\lambda^W > \lambda^V$. The intuition here is that the incremental welfare effect of internalization at λ^V depends only on the net consumer surplus effect, because $\left(\partial \hat{V}_i / \partial \lambda\right)\left(y, \lambda^V\right) = 0$. As this effect becomes positive if volatility decreases sufficiently, the efficient level of internalization for the industry is socially insufficient.

Figure 4.2 illustrates how firms and regulators react to varying uncertainty. In panel (a), λ^V increases with respect to volatility whereas λ^W decreases. With our parameters, λ^W is more responsive to volatility than λ^V. At higher volatility levels firms prefer more internalization than regulators, and regulators eventually prefer that there be no internalization at all. This policy implication is consistent with the prevalent view of overlapping ownership. However, if volatility is sufficiently low, the preferences of firms and regulators reverse, with regulators preferring higher levels of internalization than industry to achieve more intense preemption. Panel (b) shows an analogous pattern with respect to market drift. The effect of drift is similar to that of volatility but involves an additional increase in the value of discounted profit and surplus streams which renders the response of λ^W more elastic. In industries with low levels of market uncertainty we therefore reach the conclusion that to the extent that low uncertainty sufficiently exacerbates dynamic competition between firms, higher internalization is socially preferable.



(a) effect of σ, $\alpha = 0.01$



(b) effect of α, $\sigma = 0.13$

Figure 4.2: Industry efficient and socially optimal internalization levels λ^V and λ^W, for Cournot payoffs and $\rho = 0.07$.

5. Conclusion

This article incorporates overlapping ownership into a model of competitive investment in an evolving new market with both uncertainty and heterogeneous investment outcomes. In industries where overlapping ownership has weak product market effects, such as the Cournot specifications we focus on, increasing internalization lowers the entrant's perceived profit flow and delays its investment consistently with underinvestment patterns identified in the literature. In markets where incumbents already operate therefore, along with its possible anticompetitive product market effects, overlapping ownership is likely to have the additional perverse effect of weakening entry incentives.

But our model also highlights a potential economic benefit of overlapping ownership. In those markets in which no firm is yet active, internalization intensifies existing positional competition and accelerates preemptive investment. This happens because firms anticipate the entrant's accommodating behavior once they have invested, which raises their incentive to lead. From a welfare perspective, internalization therefore produces a dynamic benefit which can offset the static inefficiency of reduced product market competition. We find that it is in markets with low volatility, where option value is less important, that these welfare effects are more likely to arise. However, it is in higher volatility markets that firms themselves perceive greater benefits from internalization.

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A. Appendix

A.1. Value functions

The follower value $V^F\left(Y;Y^F\right)$ defined by Eq. (2.3) satisfies the dynamic programming condition $\rho V^F dt = E_Y dV^F$ over the inaction region $\left(0,Y^F\right)$. Applying Itô's lemma, taking the expectation and rearranging yields

$$\frac{1}{2}\sigma^2 Y^2 V^F_{YY} + \alpha Y V^F_Y - \rho V^F = 0 \tag{A.1}$$

along with boundary conditions

$$V^F\left(0;Y^F\right) = 0 \tag{A.2}$$

and

$$V^F\left(Y^F;Y^F\right) = E_{Y^F}\left[\int_0^\infty Y_s \pi^D\left(\lambda\right)e^{-\rho s}ds\right] - I = \frac{Y^F}{\rho-\alpha}\pi^D\left(\lambda\right) - I. \tag{A.3}$$

The first condition implies a solution of the form $V^F\left(Y;Y^F\right) = A^F\left(Y^F\right)Y^\beta$ and by the second $A^F\left(Y^F\right) = \left[Y^F\right]^{-\beta}V^F\left(Y^F;Y^F\right)$, which yields the first piece of Eq. (2.4).

The leader value satisfies $\rho V^L dt = \left(Y\pi^M - \rho I\right) dt + E_Y dV^L$ for $Y < Y^F$. Applying Itô's lemma, taking the expectation and rearranging yields

$$\frac{1}{2}\sigma^2 Y^2 V^L_{YY} + \alpha Y V^L_Y - \rho V^L + Y\pi^M - \rho I = 0 \tag{A.4}$$

along with boundary conditions

$$V^L\left(0; Y^F\right) = -I \tag{A.5}$$

and

$$V^L\left(Y^F; Y^F\right) = E_{Y^F}\left[\int_0^\infty Y_s \pi^D(\lambda)\, e^{-\rho s} ds\right] - I = \frac{Y^F}{\rho - \alpha}\pi^D(\lambda) - I. \tag{A.6}$$

The first condition implies a solution of the form $V^L\left(Y; Y^F\right) = A^L\left(Y^F\right) Y^\beta + BY - I$ where $B = \pi^M / (\rho - \alpha)$, and by the second $A^L\left(Y^F\right) = \left[Y^F\right]^{1-\beta}\left(\pi^D - \pi^M\right)/(\rho - \alpha)$, which yields the first piece of Eq. (2.6).

In several of our comparative statics results, we refer to the following two standard properties of β: $(\partial\beta/\partial\alpha) < 0, (\partial\beta/\partial\sigma) < 0$.

A.2. Proof of Proposition 2

Assume constant unit costs c and twice differentiable, decreasing inverse demand $P(Q)$ where Q denotes total output, with $P(0) > c$ and $P''(Q)Q/P'(Q) \geq -2$ (this last condition ensures existence and uniqueness of Cournot equilibrium). With overlapping ownership firm i's objective is

$$\omega_i\left(q_i, q_j\right) = \left(P\left(Q\right) - c\right)\left(q_i + \lambda q_j\right), \; i, j \in \left\{A, B\right\}, i \neq j. \tag{A.7}$$

The first-order condition is

$$\frac{\partial \omega_i}{\partial q_i}\left(q_i^*, q_j\right) = P\left(q_i^* + q_j\right) - c + P'\left(q_i^* + q_j\right)\left(q_i^* + \lambda q_j\right) = 0 \tag{A.8}$$

and the second-order condition is

$$\frac{\partial^2 \omega_i}{\partial q_i^2}\left(q_i^*, q_j\right) = 2P'\left(q_i^* + q_j\right) + P''\left(q_i^* + q_j\right)\left(q_i^* + \lambda q_j\right) \leq 0. \tag{A.9}$$

Existence and uniqueness of equilibrium follow from similar arguments to the Cournot ($\lambda = 0$) case: payoffs are quasi-concave and the sum of first-order conditions is decreasing in Q. The symmetric equilibrium condition is

$$P\left(2q^*\right) - c + P'\left(2q^*\right)\left(1 + \lambda\right) q^* = 0. \tag{A.10}$$

Differentiating gives

$$\frac{dq^*}{d\lambda}\left(\lambda\right) = \frac{-q^*}{3 + \lambda + \left(1 + \lambda\right)\frac{P''(2q^*)2q^*}{P'(2q^*)}} \tag{A.11}$$

which is negative for $\lambda < 1$ because the denominator is positive.

As $Y^F(\lambda)$ is inversely proportional to $\Delta^F(\lambda)$, its behavior depends on

$$\left(\Delta^F\right)'(\lambda) = \pi^D(\lambda) + (1+\lambda)\left(\pi^D\right)'(\lambda) - \pi^M. \tag{A.12}$$

To determine the sign of Eq. (A.12), first calculate

$$\left(\pi^D\right)'(\lambda) = \frac{d\left(\left(P\left(2q^*\right)-c\right)q^*\right)}{d\lambda}(\lambda) = \left(2P'\left(2q^*\right)q^* + P\left(2q^*\right)-c\right)\frac{dq^*}{d\lambda}(\lambda). \tag{A.13}$$

By the symmetric equilibrium condition, $P'\left(2q^*\right)q^* = -\left(\left(P\left(2q^*\right)-c\right)\right)/(1+\lambda)$. Substituting this and plugging $\left(dq^*/d\lambda\right)(\lambda)$ into Eq. (A.13) gives

$$\left(\pi^D\right)'(\lambda) = \left(\frac{1}{1+\lambda}\right)\frac{1-\lambda}{3+\lambda+(1+\lambda)\frac{P''(2q^*)2q^*}{P'(2q^*)}}\pi^D(\lambda). \tag{A.14}$$

Substituting back into Eq. (A.12) and regrouping terms gives

$$\pi^D(\lambda)\left(1+\frac{1-\lambda}{3+\lambda+(1+\lambda)\frac{P''(2q^*)2q^*}{P'(2q^*)}}\right) - \pi^M. \tag{A.15}$$

As products are perfect substitutes, $\pi^D(\lambda) < \pi^M/2$ for $\lambda < 1$ and a sufficient condition for Eq. (A.12) to be negative and hence for Y^F to be an increasing function of λ is

$$\frac{1-\lambda}{3+\lambda+(1+\lambda)\frac{P''(2q^*)2q^*}{P'(2q^*)}} \leq 1, \tag{A.16}$$

which follows from the inverse demand curvature assumption. Because $\Delta^F(\lambda)$ is decreasing in λ and $\Delta^F(1) = 0$ finally, $Y^F(\lambda)$ is finite for all $\lambda < 1$. $\square$

A.3. Bertrand with differentiated products

Suppose firm i's demand is

$$q_i(p_A, p_B) = \frac{a(1-\theta) - p_i + \theta p_j}{(1-\theta^2)\, b}, \tag{A.17}$$

$i \in \{A, B\}$, $a, b > 0$ where $\theta \in (0,1)$ is a differentiation parameter and both firms have constant unit costs $0 \leq c < a$. With overlapping ownership firm i maximizes $\pi^i(p_i, p_j) + \lambda \pi^j(p_j, p_i)$, $i, j \in \{A, B\}, i \neq j$, resulting in equilibrium prices $p_A^* = p_B^* = (a(1-\theta) + c(1-\lambda\theta)) / (2 - \theta(1+\lambda))$ and profits

$$\pi^D(\lambda) = \frac{(a-c)^2}{b} \frac{1-\theta}{1+\theta} \frac{1-\lambda\theta}{(2-\theta(1+\lambda))^2}; \tag{A.18}$$

so

$$\Delta^F(\lambda) = \left(\frac{1-\theta}{1+\theta} \frac{(1+\lambda)(1-\lambda\theta)}{(2-\theta(1+\lambda))^2} - \frac{\lambda}{4} \right) \frac{(a-c)^2}{b}. \tag{A.19}$$

It can be shown analytically that $\left(\Delta^F\right)'(0) < 0$, implying finite and increasing $Y^F(\lambda)$ at low levels of internalization. Numerical computation (available from the authors) establishes that $Y^F(\lambda)$ is globally increasing and finite for $\theta \leq 0.78$ but exhibits non-monotonic behavior thereafter. For $\theta \in (0.78, 0.86)$, $Y^F(\lambda)$ is finite for all λ but decreases over a non-empty interval. Finally for $\theta \geq 0.86$, $Y^F(\lambda)$ is successively increasing, infinite, then U-shaped.

A.4. Proof of Proposition 3

Provided $\Delta^F(\lambda) > 0$, evaluating the relevant derivatives gives

$$\frac{\partial Y^F}{\partial \sigma}(\lambda) = -\frac{1}{\beta(\beta-1)} Y^F(\lambda) \frac{\partial \beta}{\partial \sigma} > 0. \tag{A.20}$$

As $(\partial\beta/\partial\sigma) < 0$, the sign in the proposition follows. $\square$

A.5. Proof of Proposition 4

Provided $Y < Y^F(\lambda)$, evaluating the relevant derivative gives

$$\frac{\partial V^F}{\partial \sigma}(Y,\lambda) = \ln\left(\frac{Y}{Y^F(\lambda)}\right) V^F(Y,\lambda) \frac{\partial \beta}{\partial \sigma} - \lambda \left(\frac{Y}{Y^F(\lambda)}\right)^{\beta} \frac{(\beta-1)\left(\pi^M - \pi^D(\lambda)\right)}{\rho-\alpha} \frac{\partial Y^F}{\partial \sigma}(\lambda). \tag{A.21}$$

The first summand is positive whereas the second is negative by Proposition 3, and the proposition follows by taking limits as Y tends to 0 or $Y^F(\lambda)$. $\square$

A.6. Proof of Proposition 5

As the $\lambda = 1$ case is straightforward (firms maximize industry value by setting the first investment threshold at Y^M), we focus on the $\lambda < 1$ case. If $\Delta^F(\lambda) > 0$ so $Y^F(\lambda)$ is finite, $f\left(Y,\lambda\right)$ is strictly concave in Y over $\left(0,Y^{F}(\lambda)\right)$ with $f(0,\lambda)=-\left(1-\lambda\right)I<0$ and $f\left(Y^{F}(\lambda),\lambda\right)=0$. As

$$\frac{\partial f}{\partial Y}\left(Y^{F},\lambda\right)=-\frac{\left(\beta-1\right)\left(1-\lambda^{2}\right)\left(\pi^{M}-\pi^{D}\left(\lambda\right)\right)}{\rho-\alpha}<0, \tag{A.22}$$

$f(Y)$ crosses the horizontal axis from below once over $\left(0,Y^{F}(\lambda)\right)$, at the lower root of

$$\frac{Y}{\rho-\alpha}\pi^{M}-I-\left(\frac{Y}{Y^{F}(\lambda)}\right)^{\beta}\left(\frac{Y^{F}(\lambda)}{\rho-\alpha}\pi^{M}-I\right)=0. \tag{A.23}$$

Because f is concave in Y and Y^{P} is the lower root, $\left(\partial f/\partial Y\right)\left(Y^{P},\lambda\right)>0$ holds and the implicit function theorem can be applied yielding $\left(dY^{P}/d\lambda\right)\left(\lambda\right)=-\left(\partial f/\partial\lambda\right)\left(Y^{P},\lambda\right)$ $/\left(\partial f/\partial Y\right)\left(Y^{P},\lambda\right)$. Evaluating the numerator gives

$$\frac{\partial f}{\partial \lambda}\left(Y^{P},\lambda\right)=\frac{\left(\beta-1\right)\left(1-\lambda^{2}\right)\left(\pi^{M}-\pi^{D}\left(\lambda\right)\right)}{\rho-\alpha}\left(\frac{Y^{P}}{Y^{F}(\lambda)}\right)^{\beta}\left(Y^{F}\right)'\left(\lambda\right) \tag{A.24}$$

so $\left(Y^{P}\right)'\left(\lambda\right)\left(Y^{F}\right)'\left(\lambda\right)<0$.

If $\Delta^{F}\left(\lambda\right)\leq 0$ so $Y^{F}(\lambda)$ is infinite,

$$f\left(Y,\lambda\right)=\left(1-\lambda\right)\left(\frac{Y}{\rho-\alpha}\pi^{M}-I\right) \tag{A.25}$$

is linear and increasing, implying $Y^{P}\left(\lambda\right)=Y^{m}$. $\square$

A.7. Proof of Proposition 6

To sign $\left(\partial Y^{P}/\partial\sigma\right)(\lambda)$, use Eq. (3.12) to define

$$F_{\mathrm{A.7}}\left(Y,\beta\right)=\left(Y-Y^{m}\right)\left[Y\right]^{-\beta}-\left(Y^{F}(\lambda)-Y^{m}\right)\left[Y^{F}(\lambda)\right]^{-\beta}=0. \tag{A.26}$$

As Y^{P} is the lower root, $\left(\partial F_{\mathrm{A.7}}/\partial Y\right)\left(Y^{P},\beta\right)>0$. $\left(\partial Y^{P}/\partial\beta\right)$ has the opposite sign of $\left(\partial F_{\mathrm{A.7}}/\partial\beta\right)$. Evaluating,

$$\begin{aligned}\frac{\partial F_{\mathrm{A.7}}}{\partial\beta}\left(Y^{P},\beta\right)=&\left(Y^{F}(\lambda)-Y^{m}\right)\left[Y^{F}(\lambda)\right]^{-\beta}\ln\left(\frac{Y^{F}(\lambda)}{Y^{P}}\right)\\&+\left[Y^{F}(\lambda)\right]^{-\beta}\left((\beta-1)-\beta\frac{Y^{m}}{Y^{F}(\lambda)}\right)\frac{\partial Y^{F}(\lambda)}{\partial\beta}\end{aligned} \tag{A.27}$$

where Eq. (A.27) uses Eq. (A.26) to substitute for $\left[Y^{P}\right]^{-\beta}\left(Y^{P}-Y^{m}\right)$. Dividing by $\left[Y^{F}(\lambda)\right]^{-\beta}$ and substituting for the remaining $Y^{F}(\lambda)$ terms, $\left(\partial F_{\mathrm{A.7}}/\partial\beta\right)$ has the sign of

$$\left(\beta\frac{\pi^{M}}{\Delta^{F}\left(\lambda\right)}-\left(\beta-1\right)\right)\ln\left(\frac{\beta}{\beta-1}\frac{\rho-\alpha}{Y^{P}}\frac{I}{\Delta^{F}\left(\lambda\right)}\right)-\frac{\pi^{M}}{\Delta^{F}\left(\lambda\right)}+1. \tag{A.28}$$

As $Y^{P}\leq Y^{M}$, the expression above is bounded below by

$$\left(\beta\frac{\pi^{M}}{\Delta^{F}\left(\lambda\right)}-\left(\beta-1\right)\right)\ln\left(\frac{\pi^{M}}{\Delta^{F}\left(\lambda\right)}\right)-\frac{\pi^{M}}{\Delta^{F}\left(\lambda\right)}+1, \tag{A.29}$$

which is non-negative if $\beta=1$ by the logarithm inequality $\ln x\geq(x-1)/x$, $x\neq1$, and increasing in β. Therefore $\left(\partial F_{\mathrm{A.7}}/\partial\beta\right)\left(Y^{P},\beta\right)>0$ and hence $\left(\partial Y^{P}/\partial\beta\right)<0$ and

$\left(\partial Y^P/\partial\sigma\right) > 0$. $\square$

A.8. Quasiconcavity of $\hat{V}_i(y,\lambda)$ and existence of λ^V

We first establish an intermediate result:

Lemma 1. *If* $\left(\pi^D\right)''(\lambda) \leq 0$, $\left(\Delta^F\right)'(\lambda) < 0$ and $\left(\Delta^F\right)''(\lambda) \leq 0$ *then* $\hat{V}_i(y,\lambda)$ *is strictly quasiconcave in* λ.

Proof Express $\hat{V}_i(y,\lambda)$ (Eq. (3.15)) as

$$\hat{V}_i(y,\lambda) = \left(\frac{(\beta-1)\,y}{\beta(\rho-\alpha)}\right)^{\beta} I^{1-\beta}\left(\frac{\beta}{\beta-1}\left[\Delta^F(\lambda)\right]^{\beta-1}\pi^D(\lambda) - \left[\Delta^F(\lambda)\right]^{\beta}\right) \tag{A.30}$$

and normalize by $((\beta-1)\,y/\beta(\rho-\alpha))^{\beta} I^{1-\beta}$ to have only terms in λ. The first derivative is

$$\beta\left[\Delta^F(\lambda)\right]^{\beta-2}\left(\left(\Delta^F\right)'(\lambda)\,\pi^D(\lambda) + \frac{1}{\beta-1}\Delta^F(\lambda)\left(\pi^D\right)'(\lambda) - \left(\Delta^F\right)'(\lambda)\,\Delta^F(\lambda)\right) \tag{A.31}$$

and the second derivative, evaluated at a zero of the first derivative, is

$$\begin{aligned}\beta\left[\Delta^F(\lambda)\right]^{\beta-2}\Bigg(&\left(\Delta^F\right)''(\lambda)\left(\pi^D(\lambda)-\Delta^F(\lambda)\right) + \frac{\beta}{\beta-1}\left(\Delta^F\right)'(\lambda)\left(\pi^D\right)'(\lambda)\\ &+\frac{1}{\beta-1}\Delta^F(\lambda)\left(\pi^D\right)''(\lambda) - \left[\left(\Delta^F\right)'(\lambda)\right]^2\Bigg).\end{aligned} \tag{A.32}$$

As $\pi^D(\lambda)-\Delta^F(\lambda) = \lambda\left(\pi^M-\pi^D(\lambda)\right) \geq 0$, $\left(\pi^D\right)'(\lambda) > 0$ and $\left(\Delta^F\right)'(\lambda) < 0$ by assumption, $\left(\Delta^F\right)''(\lambda) \leq 0$ and $\left(\pi^D\right)''(\lambda)$ suffice to ensure the above expression is negative, establishing strict quasiconcavity. □

For Cournot competition with linear demand, and setting $(a-c)^2/b=1$ without loss of generality, we have

$$\left(\pi^D\right)'(\lambda)=\frac{1-\lambda}{(3+\lambda)^3},\ \left(\pi^D\right)''(\lambda)=\frac{2(\lambda-3)}{(3+\lambda)^4}<0 \tag{A.33}$$

and

$$\left(\Delta^F\right)'(\lambda)=\frac{4\,(1+\lambda)}{(3+\lambda)^3}-\frac{1}{4},\ \left(\Delta^F\right)''(\lambda)=-\frac{8\lambda}{(3+\lambda)^4}\leq 0 \tag{A.34}$$

so $\hat{V}_i\,(y,\lambda)$ is strictly quasiconcave in λ. Moreover as $\Delta^F\,(1)=0$, $\lim_{\lambda\to 1}\hat{V}_i\,(y,\lambda)=0$ and hence $\lambda^V\in(0,1)$.

A.9. Proof of Proposition 7

Evaluating,

$$\begin{aligned}\frac{\partial\hat{V}_i}{\partial\lambda}(y,\lambda) &= \left(\frac{y}{Y^F(\lambda)}\right)^{\beta}\left(\frac{Y^F(\lambda)}{\rho-\alpha}\left(\pi^D\right)'(\lambda)+\left((1-\beta)\frac{\pi^D(\lambda)}{\rho-\alpha}+\beta\frac{I}{Y^F(\lambda)}\right)\left(Y^F\right)'(\lambda)\right)\\ &= \frac{y^{\beta}}{(\rho-\alpha)\left[Y^F(\lambda)\right]^{\beta-1}}\left(-(\beta-1)\left(\pi^M-\pi^D(\lambda)\right)\varepsilon_{Y^F/\lambda}(\lambda)+\left(\pi^D\right)'(\lambda)\right)\end{aligned} \tag{A.35}$$

where the elasticity $\varepsilon_{Y^F/\lambda}(\lambda)=\left(Y^F\right)'(\lambda)/(Y^F(\lambda)/\lambda)$, which is positive (negative) with weak (strong) product market effects, is independent of β. Because $\left(\partial\hat{V}_i/\partial\lambda\right)(y,0)=y^{\beta}\left[Y^F(0)\right]^{-(\beta-1)}(1/(\rho-\alpha))\left(\pi^D\right)'(0)$, $\lambda^V>0$ with product market effects. If λ^V is interior, it is defined implicitly by

$$F_{\text{A.9}}\left(\lambda^{V},\beta\right):=-\left(\beta-1\right)\left(\pi^{M}-\pi^{D}\left(\lambda^{V}\right)\right)\varepsilon_{Y^{F}/\lambda}\left(\lambda^{V}\right)+\left(\pi^{D}\right)'\left(\lambda^{V}\right)=0. \tag{A.36}$$

Because $\left(\partial F_{\text{A.9}}/\partial\lambda\right)\left(\lambda^{V},\beta\right)<0$ at an interior solution, by the implicit function theorem $d\lambda^{V}/d\beta$ has the sign of $\left(\partial F_{\text{A.9}}/\partial\beta\right)\left(\lambda^{V},\beta\right)<0$ and the result follows as $\left(\partial\beta/\partial\alpha\right),\left(\partial\beta/\partial\sigma\right)<0$. $\square$

A.10. Proof of Proposition 8

Observe first that without product market effects

$$2\frac{\partial\hat{V}_{i}}{\partial\lambda}\left(y,\lambda\right)=-2\frac{\beta-1}{\rho-\alpha}\left(\frac{y}{Y^{F}(\lambda)}\right)^{\beta}\lambda\left(\pi^{M}-\pi^{D}\right)\left(Y^{F}\right)'(\lambda), \tag{A.37}$$

and that differentiation of the equilibrium condition Eq. (3.12) implies

$$\frac{dY^{P}}{dY^{F}}(\lambda)\left(\frac{Y^{F}(\lambda)}{Y^{P}(\lambda)}\right)^{\beta}=\frac{\frac{Y^{M}}{Y^{F}(\lambda)}-1}{\frac{Y^{M}}{Y^{P}(\lambda)}-1}. \tag{A.38}$$

Substituting these expressions into Eq. (4.3) and noting that without product market effects $\left(s^{D}\right)'(\lambda)=0$,

$$\frac{\partial W}{\partial\lambda}(y,\lambda)=\frac{\beta-1}{\rho-\alpha}\left(\frac{y}{Y^{F}(\lambda)}\right)^{\beta}\left(-2\lambda\left(\pi^{M}-\pi^{D}\right)+s^{M}\frac{Y^{M}-\frac{\Delta^{F}(\lambda)}{\pi^{M}}Y^{P}(\lambda)}{Y^{M}-Y^{P}(\lambda)}-s^{D}\right)\left(Y^{F}\right)'(\lambda) \tag{A.39}$$

so $(\partial W/\partial\lambda)(y,\lambda)$ has the sign of

$$\left(\pi^M-\pi^D\right)\left((1+\lambda)\frac{s^M}{\pi^M}\frac{Y^P(\lambda)}{Y^M-Y^P(\lambda)}-2\lambda\right)-(s^D-s^M). \tag{A.40}$$

Eq. (A.40) is an increasing function of Y^P, which is bounded below by $Y^P=Y^m$. Substituting and rearranging, $(\partial W/\partial\lambda)(y,0)>0$ if

$$\beta>\underline{\beta}:=1+\frac{\pi^M}{\pi^M-\pi^D}\frac{s^D-s^M}{s^M}, \tag{A.41}$$

which defines the volatility upper bound σ_0 for which $\lambda^W>0$. If $W(y,\lambda)$ increases monotonically in λ over $\left(0,\pi^D/\left(\pi^M-\pi^D\right)\right)$, there can be a continuum of maximizers. Otherwise, set $\lambda=\pi^D/\left(\pi^M-\pi^D\right)$ so $Y^P(\lambda)=Y^m$, Eq. (A.40) becomes

$$s^M\frac{Y^M}{Y^M-Y^m}-s^D-2\pi^D. \tag{A.42}$$

Then $(\partial W/\partial\lambda)\left(y,\pi^D/\left(\pi^M-\pi^D\right)\right)<0$ if

$$\beta<\overline{\beta}:=\frac{s^D+2\pi^D}{s^M}. \tag{A.43}$$

Provided $\left(s^D+2\pi^D\right)-\left(s^M+\pi^M\right)<\pi^M$, which holds for Cournot competition with constant marginal cost and concave inverse demand, $\underline{\beta}<\overline{\beta}$. For $\beta\in\left(\underline{\beta},\overline{\beta}\right)$ therefore, there exists a solution λ^W to $(\partial W/\partial\lambda)(y,\lambda)=0$ over $\left(0,\pi^D/\left(\pi^M-\pi^D\right)\right)$.

If there is a unique interior solution λ^W, we show that $\left(\partial^2 W/\partial\lambda^2\right)(y,\lambda^W) < 0$. Differentiating Eq. (A.40) with respect to λ and evaluating at λ^W gives

$$-2 + \frac{s^M}{\pi^M}\frac{Y^P(\lambda^W)}{Y^M - Y^P(\lambda^W)} + \frac{s^M}{\pi^M}\frac{Y^M}{\left(Y^M - Y^P(\lambda^W)\right)^2}\left(1+\lambda^W\right)\left(Y^P\right)'(\lambda^W) \qquad \text{(A.44)}$$

up to normalization by $\pi^M - \pi^D$ The last summand in Eq. (A.44) is negative. By Eq. (A.40),

$$\frac{s^M}{\pi^M}\frac{Y^P(\lambda)}{Y^M - Y^P(\lambda)} = 2\frac{\lambda^W}{1+\lambda^W} + \frac{1}{1+\lambda^W}\frac{s^D - s^M}{\pi^M - \pi^D} \qquad \text{(A.45)}$$

so the sum of the first two terms is

$$\frac{1}{1+\lambda^W}\left(\frac{s^D - s^M}{\pi^M - \pi^D} - 2\right) \qquad \text{(A.46)}$$

which is negative because $\left(s^D + 2\pi^D\right) - \left(s^M + \pi^M\right) < \pi^M$ □

A.11. Proof of Proposition 9

Observe that $\left(\partial\hat{V}_i/\partial\lambda\right)(y,\lambda^V) = 0$ if λ^V is interior and substitute Eq. (A.38) into Eq. (4.3) to obtain

$$\frac{\partial W}{\partial\lambda}(y,\lambda^V) = \frac{y^\beta}{(\rho-\alpha)\left[Y^F(\lambda^V)\right]^{\beta-1}}\times$$
$$\left((\beta-1)\left(-s^M\frac{Y^M - \frac{\Delta^F(\lambda^V)}{\pi^M}Y^P(\lambda^V)}{Y^M - Y^P(\lambda^V)} + s^D\left(\lambda^V\right)\right)\frac{\left(\Delta^F\right)'\left(\lambda^V\right)}{\left(\Delta^F\right)\left(\lambda^V\right)} + \left(s^D\right)'\left(\lambda^V\right)\right). \qquad \text{(A.47)}$$

The sign of $(\partial W/\partial\lambda)(y,\lambda)$ is that of the bracketed term. By Eq. (A.36), $(\beta-1)\left(\Delta^F\right)'\left(\lambda^V\right)/\Delta^F\left(\lambda^V\right)$ $=-\left(\pi^D\right)'\left(\lambda^V\right)/\lambda^V\left(\pi^M-\pi^D\left(\lambda^V\right)\right)$ so this term has the sign of

$$\left(s^M\frac{Y^M-\frac{\Delta^F\left(\lambda^V\right)}{\pi^M}Y^P(\lambda^V)}{Y^M-Y^P(\lambda^V)}-s^D\left(\lambda^V\right)\right)\frac{\left(\pi^D\right)'\left(\lambda^V\right)}{\lambda^V\left(\pi^M-\pi^D\left(\lambda^V\right)\right)}+\left(s^D\right)'\left(\lambda^V\right). \tag{A.48}$$

As the expression above is increasing in Y^P, substitute Y^m for $Y^P(\lambda^V)$ to get a lower bound.

$$\left(s^M\left(\beta-(\beta-1)\frac{\Delta^F\left(\lambda^V\right)}{\pi^M}\right)-s^D\left(\lambda^V\right)\right)\frac{\left(\pi^D\right)'\left(\lambda^V\right)}{\lambda^V\left(\pi^M-\pi^D\left(\lambda^V\right)\right)}+\left(s^D\right)'\left(\lambda^V\right). \tag{A.49}$$

After rearrangement, a sufficient condition for $(\partial W/\partial\lambda)\left(y,\lambda^V\right)>0$ is therefore

$$\beta>\frac{1}{1+\lambda^V}\frac{\frac{s^D\left(\lambda^V\right)}{s^M}-\frac{\Delta^F\left(\lambda^V\right)}{\pi^M}}{\frac{\pi^M-\pi^D\left(\lambda^V\right)}{\pi^M}}-\frac{\pi^M}{s^M}\frac{\lambda^V}{1+\lambda^V}\frac{\left(s^D\right)'\left(\lambda^V\right)}{\left(\pi^D\right)'\left(\lambda^V\right)} \tag{A.50}$$

For an arbitrary β_0 there is an $\arg\max_\lambda \hat{V}_i(y,\lambda)=\lambda_0^V$. The right-hand side of the above inequality is continuous in λ^V and therefore attains a maximum β^* over $\left[0,\lambda_0^V\right]$. Hence for any $\beta>\beta^*$ Eq. (A.50) holds, implying $(\partial W/\partial\lambda)\left(y,\lambda^V\right)>0$. $\square$