Credit risk and control rights in infrastructure projects under the Real Options approach

Carlos Armando Mejía Vega [a]
Carlos Andrés Zapata Quimbayo [b]

Abstract

This paper presents a dynamic credit risk model for debt financing in infrastructure projects. Specifically, it attempts to combine the structural models and the Monte Carlo simulation techniques by analysing the effects of extensive control rights for lenders through covenants and embedded options, like the option to renegotiate the debt agreement conditions and the option to exit, in the estimation of expected recovery rates and the expected loss along the loan life. Hence, the model proposed by Blanc-Brude and Hasan (2016) is extended to model the dynamics of debt capacity and to estimate the probability of default. Given those conditions, the option to exit and the option to renegotiate the debt conditions are evaluated. In that sense, is shown that the embedded Real Options can improve the recovery rate and the risk profile.

Keywords: *Infrastructure projects, real options, credit risk, control rights.*

1. Introduction

The measuring of credit risk (CR) has been subject to increased attention in both the theoretical and empirical literature. Furthermore, this analysis has been treated specifically under the corporate finance field. In that sense, the traditional models like structural CR models have been applied to measure the components of expected loss (EL), i.e., the probability of default (PD), the loss given default (LGD) and the exposure at default (EAD). However, project finance transactions such as infrastructure financing require more rigorous analysis. According to Gatti et al. (2007) and Gatti (2008), project finance (PF) implies the financing of the one single project, the Special Purpose Vehicle (SPV), which is funded by an off-balance sheet. Under this scheme, the SPV is created upon an ad hoc base with limited or non-resources of its shareholders and, unlike traditional corporate financing, all of the economic consequences of the project are directly attributed by the SPV. Thus, Gatti (2008) argues that PF has distinctive features compared to traditional corporate financing:

[a] Docente investigador, Observatorio de Economía y Operaciones Numéricas - ODEON, Universidad Externado de Colombia. [carlos.mejia@uexternado.edu.co]
[b] Docente investigador, Observatorio de Economía y Operaciones Numéricas - ODEON, Universidad Externado de Colombia. [carlosa.zapata@uexternado.edu.co]

i) SPV requires a level of specificity that determines its purpose;

ii) The equity is limited (its non-recourse financing) and shareholders require long terms to return the investment;

Besides, this scheme can also incorporate significant and extensive control rights for lenders (Borgonovo and Gatti, 2013, Blanc-Brude, Hasan and Ismail, 2014), where lenders are in a position to take over control of the project, as well as covenants[1] and embedded options (Gatti, 2008; Blanc-Brude and Hasan, 2017; and Blanc-Brude, Hasan and Whittaker, 2016) which limit the interests of shareholders given their high leverage. In addition, Borgonovo and Gatti (2013) suggest that into the credit agreement can be included requirements about the debt service cover ratio (DSCR) as the most relevant covenant. As a result, it is necessary to redefine the default of the project according to the cash flow available to pay the debt service. The main concern is focused on the recovery rates when the renegotiation of the debt agreement happens given that the SPV falls into default. For example, Davidson et al. (2010) and Borgonovo and Gatti (2013) show that in material breach the average aggregate recovery rate on the loan, where the lenders went through a workout (i.e. the renegotiation scenario), have been approximately on 80%. However, when the bankruptcy was declared or lenders decided to exit the average aggregate recovery rate is approximately 50%. These results are in line with the Moody's Reports of credit risk in project finance, and show the importance of the renegotiation process of the credit agreement in order to increase the recovery rates in the financing of infrastructure projects.

Given that those features, the credit analysis in infrastructure projects has been more complex. However, the models developed in traditional corporate financing to assess the credit risk such as the Merton model or the Moody's KMV model have been applied in infrastructure projects (see Freydefornt, 2001; Aragones, Blanco, and Iniesta, 2009) despite of the limitations like Gatti et al. (2007) and Blanc-Brude and Hasan (2016, 2017) suggest. For example, Blanc-Brude and Hasan (2017) argue that the Merton model and the Moody's KMV model ignore the effects of debt covenants and embedded options in infrastructure projects, and these models fail to assess the credit risk.

To overcome this, Blanc-Brude and Hasan (2016), from now on the BBH-I model, propose to estimate the probability of default in infrastructure projects from a structural model initially developed for illiquid debt, as an extension of the Moody's KMV model. They redefined the distance to default in the Moody's KMV model and estimated the probability

[1] Following to Borgonovo and Gatti (2013), covenants are defined as supplemental obligations of the borrower in addition to the basic obligation to repay the lenders the amount due on the scheduled maturity dates. Additionally, according to Blanc-Brude, Hasan and Ismail (2014) covenants are contractual clauses that impose on a borrower either obligations to do something (positive covenants) or to refrain from doing something (negative covenants). For example, debt covenants prohibiting shareholders from getting more cash through new debt or equity issuance to service existing debt. Likewise, covenants include restrictions to restructure project debt upon default or liquidate or sell the project company.

of default by modelling the project's pay-out profile stochastically, which is determined through the cash flow available for debt service (CFADS). Also, its model incorporates not only the effects of debt covenants but also the nature of the project's pay-out capacity by assuming a stochastic dynamic for the debt service coverage ratio (DSCR). Furthermore, Blanc-Brude and Hasan (2017), from now on the BBH-II model, extends the previous credit risk model in order to incorporate the effects of the covenants as well as the Black-Cox decomposition and the options embedded in the project, although they do not consider the risk-neutral framework for the valuation of the options. For instance, they estimate the value of the options to exit or the options to change (renegotiate) the credit conditions when the projects falls into default. However, the BBH-II model needs to compare the value for lenders under two scenarios: (i) when the project is under a renegotiation process given a default scenario and (ii) when it operates in normal conditions.

This paper attempts to extend both BBH-I and BBH-II models to measure the credit risk of infrastructure project's debt in an integrated framework under the Real Options approach. In that sense, we use the risk-neutral framework to value both the option to exit as well as the option to renegotiate the original conditions of the debt agreement and, therefore, we assess the effects of the technical default and the hard default when the renegotiation of debt schedule takes place or not. Likewise, it is evaluated the effects of these embedded options in the estimation of the credit risk and their components, i.e. the probability of default, the exposure and the recovery rate by using not only the structural model of credit risk (BBH-I) but also the Monte Carlo simulation (MCS) technique in order to approximate the recovery rate for lenders in the renegotiation process.

The structure of the paper is the follows: in section 2, a brief description of the credit risk in infrastructure projects. In section 3, we present the structural models for estimating credit risk and their adaptation to the field of infrastructure projects based on the BBH-I and BBH-II models. In section 4, we presented the Real Options approach to value the embedded options in infrastructure projects where an application into a toll road concession is presented. Finally, the main conclusions and related discussions are presented.

2. Credit risk analyses in infrastructure projects

Credit risk is usually measured through a model developed by the Basel Committee, known as the standard model, which consists in estimate an expected loss (EL) as follows:

$$EL = DP \ x \ LGD \ x \ EAD \quad (1)$$

where DP is the probability of default, LGD is the loss given the default and EAD is the exposure at default. Given equation (1), it is important to highlight that much of the theoretical models developed around the EL have focused on the estimation of the first component - i.e., the DP. One of the methodological approaches is known as structural models (Arora, Bohn and Zhu, 2005) with pioneer works like the ones of Merton (1974), Black and Cox (1976) and Ingersoll (1977). This approach is named the "Merton model"[2] and has been widely used to estimate the probability of default (PD), where the debt of the firm can be considered as a claim on its assets. In that sense, this approach proposes a relationship between the capital structure of the firm and its capacity to pay the debt. Under the assumption that the market value of the firm´s assets (V_A) follows a lognormal process, the model can be solved for a closed-form solution for the value of the company's debt. Specifically, under a filtered probability space $[\Omega, \mathcal{F}, (\mathcal{F}_{t\geq t}), \mathbb{P}]$, it is assumed that V_A follows a geometric Brownian motion (gBm):

$$dV_A = \mu_A\, V_A\, dt + \sigma_A\, V_A\, dW_t \quad (2)$$

where, μ_A represents the drift rate of the assets, σ_A is the volatility and W_t $(t \in [0, T])$ is a standard Wiener process. Thus, the Merton model estimates the PD (in a risk-neutral world) as the probability that V_A at time T is below the value of debt (V_D):

$$Prob\,[\,V_A \leq V_D\,] = N(-d_2) \quad (3)$$

where N (·) indicates the cumulative distribution function (c.d.f.) and d_2 is given by

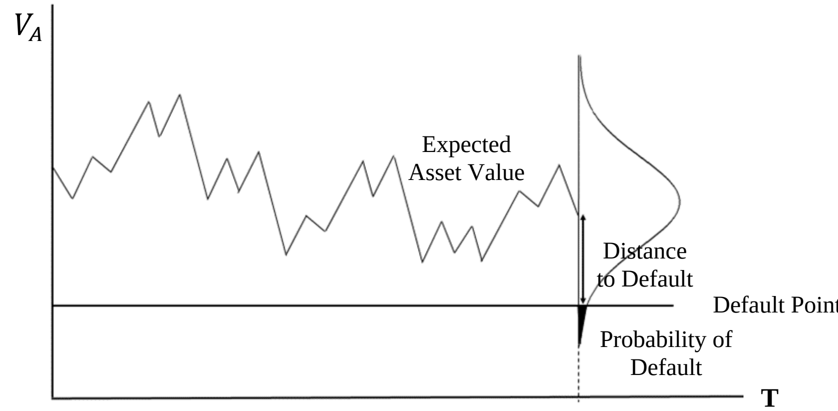
$$d_2 = \frac{ln\left(\frac{V_A}{V_D}\right) + \left(r - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}} \quad (4)$$

where r is the risk-free rate. In the same line, Vasicek (1984) developed an extension of the Merton model known as the Moody's model (from now on the KMV model), which has shown considerable success in measuring credit risk (Kealhofer, 2003). The KMV model estimates the probability of default based on the notion of distance to default (DD), by assuming that the company gets in default when the value of the assets is below the threshold, known as the default point (DP). The value of the debt determines the threshold (see figure 1). Figure 1 shows that if the value of the assets falls below the default point, then the company fails to pay the debt and gets into default. PD is represented in the shadow area of the distribution function below the default point (DP). Additionally, the KMV model estimates the DD defined as the number of standard deviations in which the value of the assets exceeds the default point:

$$DD = \frac{V_A - DP}{\sigma_A \times V_A} \quad (5)$$

[2] Starting from the seminal works of Black and Scholes (1973) and Merton (1973), the contingent claims analyses (CCA) have been adapted for the treatment of corporate problems, one of them is the measure of credit risk. This field includes the structural models for credit risk assessment initiated by Merton (1974).

Figure 1- Distance to default (DD) and the default point (DP)



Source: own elaboration based on Kealhofer (2003).

Finally, unlike the Merton model, the KMV model estimates the PD as:

$$Prob\,[\,V_A \;\leq\; V_D\,] = N(-DD) \quad (6)$$

As a result, the KMV model establishes that DD is enough to estimate the PD. However, it should be kept in mind that in infrastructure projects, the treatment of credit risk is more complex given the idiosyncratic features as we stated before. These differences were pointed out by the Basel Committee in the framework of the Basel II agreement. However, since the publication of the first version published in 2001, and despite all the developments in financial theory in credit risk, the improvements in the field of infrastructure projects have not been enough. It was not until Blanc-Brude and Hasan (2016) proposed to estimate the PD through a structural model of credit risk developed for illiquid debt, as an extension of the KMV model. This model represents an innovative proposal which allows redefining the parameters of the KMV model by considering the characteristics of infrastructure projects. It should be noted that in infrastructure projects, the cash flow is the main factor in determining the debt service capacity, therefore, it should be the main factor from the application of the credit risk model.

While the Merton and KMV models define the default based on the value of assets and debt at the maturity (T), for an infrastructure project, the default must be estimated for each period (t). The above reinforces the idea that the model must be defined regarding the cash flows where the capacity to pay the debt can be estimated by directly comparing each one of them with the service of the debt. This new idea involves to work with the debt service cover ratio:

$$DSCR_t = \; {}^{CFADS_t}/_{DS_t} \quad (7)$$

Where, DSCR is the relation between the cash available ($CFADS$) and the amount of debt to be paid, i.e., debt service (DS) at time *t*. The higher DSCR the more cash available to the project to pay its debt obligations. In that sense, Blanc-Brude and Hasan (2016) developed an explicit definition of the hard default when $DSCR = 1$ or the technical default when $DSCR = 1.x$. The $1.x$ is given by the covenants on the credit agreement. Then, the SPV can be considered in a hard default when its $DSCR < 1$ or in a technical default when $DSCR < 1.x$. Thus, the dynamics of the $DSCR$ is the only variable considered in order to estimate the DD.

Based on this, the model shows that understanding the dynamics of the $DSCR$, together with the debt repayment profile (both observable), allows implementing the structural credit risk model[3]. Again, under a filtered probability space $[\Omega, \mathcal{F}, (\mathcal{F}_{t \geq t}), \mathbb{P}]$, it is assumed that the $DSCR$ follows a geometric Brownian motion (gBm):

$$ {dDSCR_t}/{DSCR_t} = \mu \, dt + \sigma \, dW_t \quad (8) $$

Where, μ and σ are the instantaneous drift rate of the $DSCR$ and its volatility, respectively. Then, analogously to the KMV model, the distance to default (DD) at time *t* is given by:

$$ DD_t = \frac{CFADS_t - DS_t}{\sigma_{CFADS} \; CFADS_t} \quad (9) $$

Additionally, if the cash flow ($CFADS_t$) is re-expressed in terms of he $DSCR$, where $CFADS_t = DSCR_t \times DS_t$ and $\sigma_{CFADS} = \left(\frac{DS_{t-1}}{DS_t}\right) \sigma_{DSCR}$[4], then:

$$ DD_t = \frac{1}{\sigma_{DSCR}} \frac{DS_{t-1}}{DS_t} \left(1 - \frac{1}{DSCR_t}\right) \quad (10) $$

Finally, the PD under the real probability measure $\mathbb{P}$ is estimated as:

$$ P(t,T) = N(-DD_t) \quad (11) $$

where $P(t,T)$ indicates the real *PD* between time *t* and *T*. On the other side, the PD under the risk-neutral probability measure $\mathbb{Q}$ is given by:

$$ Q(t,T) = N(N^{-1}[P(t,T) + \lambda)] \quad (12) $$

where $Q(t,T)$ is the risk-neutral *PD* between time *t* and T and $\lambda = \frac{\mu - r}{\sigma} \sqrt{T}$ is the required Sharpe ratio.

[3] Moreover, according to Gatti (2008), from the decision-making viewpoint, a minimum DSCR is usually used by lenders in the loan negotiation phase and their conditions to help them decide the optimal debt-to-equity ratio of the deal.

[4] It should be noted that whether the debt repayment scheme is fixed, for example, an annuity payment, then $\sigma_{CFADS} = \sigma_{DSCR}$, given that $DS_{t-1} = DS_t$.

Although the PD can be estimated by using the model developed by Blanc-Brude and Hasan (2016) (BBH-I model), the other two components of the standard credit risk model in equation (1) such as the LGD and EAD also requires careful treatment. In general, in their effort to reduce their exposure to risk and the EL, lenders incorporate control rights throughout the covenants and embedded options in the financing agreement. Hence, the previous model should be extended in order to include the effects of the restructuring of the debt contract in a default scenario. This extension can be done under the Real Options framework.

3. Real Options, default events and the renegotiation process

3.1 An overview of the Real Options theory

The Real Options approach (ROA) arises as a useful tool for making optimal investment decisions by the adaptation of the financial option pricing models developed by Black and Scholes (1973) and Merton (1973). Additionally, many works have shown that ROA approach constitutes a better tool for assessing investment projects under uncertain market and operational conditions, which characterizes the investment projects, compared to discounted cash flows methods, such as the net present value (NPV). Some relevant and classical works in that sense are the ones of Brennan and Schwartz (1985), McDonald and Siegel (1986), Pindyck (1991), Dixit and Pindyck (1994), Trigeorgis (1996), Amram and Kulatilaka (1999), Copeland and Antikarov (2001) and others.

Furthermore, the ROA has found numerous applications in the field of project financing and specifically in the infrastructure sector. Ho and Liu (2002), Bowe and Lee (2004), Garvin and Cheah (2004), Brandão and Saraiva (2008), Iyer and Sagheer, (2011), Ashuri et al. (2012), Wibowo, Permana and Kochendörfer (2012), Pellegrino, Vajdic, and Carbonara (2013), Liu, Yu, and Cheah (2014), Zapata, Mejia and Marques (2018) among many others many others present different applications about it.

In the field of infrastructure projects, the embedded options under default risk and bankruptcy conditions have been studied by Ho and Liu (2002). Mainly, they analysed the early termination of a build-operate-transfer (BOT) project under bankruptcy risk, where the early exercise is imposed from the credit agreement for the protection of lenders. Thus, by assuming that lenders try to prevent the project value from being below the total estimated debt, they modelled the bankruptcy condition as:

$$V_t - D_t\, e^{-r_d(T-t)} < 0 \quad (13)$$

where V_t is the value of the company project, D_t is the total outstanding debt at time *t* while $D_t\, e^{-r_d(T-t)}$ is the total estimated debt at time *t* by discounting at the loan interest rate r_d, and T is defined as the maturity of the total debt. Therefore, the payoff function under bankruptcy risk is given by $max(V_t - D_t\, e^{-r_d(T-t)}; 0)$.

Blanc-Brude and Hasan (2017) extended the previous work by analysing embedded options under a default process (BBH-II model). They evaluate the effect of an exit option in the presence of defaults and they compare it with the work-out value when the renegotiation takes place. They formulated the conditions under which renegotiations can take place. In the first case, they found two significant outcomes: i) the workout scenario, i.e., the renegotiation takes place, and they obtain the value V_{WK}, and, ii) exit scenario, where the lenders sell the project company in the market by a residual value denoted by V_{EX}.

Additionally, Blanc-Brude and Hasan (2017) establish two types of renegotiation associated with the technical default (DSCR $< 1.x$) and the hard default (DSCR < 1). As such, in presence of a technical default, lenders can aim to maximise the value of the debt service according to the outstanding debt amount, while after a hard default, lenders have the control of the SPV and, although the debt renegotiation could take place, it is only considered the exit scenario to simplify the model.

Letting V_t denotes the value of the project if no change is made, i.e., the default did not occur, V_{EX} the value in the exit scenario (no renegotiation) while V_{WK} the value of the work-out scenario (when the renegotiation takes place), and X_t the exit cost, lenders aim to maximize the value of the exit value and so tend to increase the recovery rate according to the following payoff function:

$$max(V_{WK} - X_t, Cash) \quad (14)$$

We suggests some adjustments in the payoff function in order to incorporate not only the cash available when the hard default (or bankruptcy) occur but also the different amounts of cash required by the credit agreement such as the debt reserves, guarantees offered by sponsors (like owners or even the Government) as Ho and Liu (2002) and Gatti (2008) suggested.

3.2 The model

The main assumptions of the model proposed in this paper are like the ones of Blanc, Brude, and Hasan (2017), but unlike them, it will only focus on lenders. For example, whether lenders can get at least more from renegotiation or the liquidation than sponsors. In that sense, the following assumptions are formulated to apply the credit risk model where the effects of embedded options (option to exit and option to renegotiate) are included, as well as the estimation of PD, recovery rates (RR) or the LGD and EL. The assumptions are the following:

- The debt agreement includes covenants that indicate the threshold (default point) below which lenders might take control of the SPV and therefore they might renegotiate the outstanding debt in order to maximize the expected recovery rate.

- (Technical or hard) default occurs when the cash flows become insufficient to repay the debt service in each time *t*. Hence, the debt agreement includes a standard clause on minimum DSCR (1.x) to represents the technical default or a minimum DSCR (1.0) in a hard default.

- Like Borgonovo and Gatti (2013) suggest, the joint consideration of the two defaults allows to:

  i. Consider the technical default only until the renegotiation of the debt occurs.

  ii. This renegotiation is always available until DSCR reaches the lower bound when the breach of covenants becomes hard, forcing the project into bankruptcy and lenders into the exit.

For simplicity, only the two scenarios of default are considered. Additionally, by considering that the cash flow available for debt service at time *t* ($CFADS_t$) determines capacity of the debt, then the dynamic of $DSCR_t$ shows whether default may occur or not along the life of the loan. Likewise, under the assumptions given above, the next scenarios are considerd which determine the total value of the debt for lenders (V^{Loan}), the value of the debt in the restructuring scenario and the value of the embedded options in the debt contract that aim to minimize the *LGD* and *EL* (or maximize the *RR*):

1. Default does not occur since the cash flows ($CFADS_t$) become enough to repay the debt service in each of the moments of time *t*. Hence, the debt service shows a total recovery of the debt by lenders, i.e., the recovery rate of the debt reached 100%, and therefore, the restructuring debt is not necessary. In that sense, the total value for lenders (V^{Loan}) is given by the present value of debt service along the life of the loan:

$$V^{Loan} = \sum_{t=k}^{T} e^{-rt} DS_t \quad (15)$$

Where, r is the risk-free rate, *k* is the first year to repay the debt and *T* is the term to debt maturity.

2. Technical default occurs at time t_{td} and lenders and sponsors renegotiate the conditions of the debt agreement to reach the suitable credit conditions of the project. Here, lenders trigger their control rights upon the company project in to increase their expected recovery value (*ER*). However, the outstanding debt should be repaid over the subsequent periods. Following to Borgonovo and Gatti (2013), the *ER* is given by:

$$ER = \sum_{t=k}^{t_d-1} e^{-rt} DS_t + \sum_{t=t_d}^{T^{RD}} e^{-rt} DS'_t \quad (16)$$

Where, t_d is the period when the (technical) default occurs, DS_t' is the newly scheduled debt service repayment flow and T^{RD} is the new term to maturity of the loan. Here, we can associate the ER with the work out value (V_{WK}) in the model developed by Blanc-Brude and Hasan (2017). Under the debt rescheduling, lenders may incur in restructuring costs (X) to have the debt rescheduled when the renegotiation takes place[5]. In that sense, the value of the option to renegotiate is obtained from the next payoff function[6]:

$$V^{Loan} = max\left(V_{WK} - X; \check{V}\right) \quad (17)$$

where, X denotes the present value of the restructuring costs (X) and $\check{V}$ is the present value of the cash available for debt service whether lenders make no change to the debt agreement. The result of $\check{V}$ represents the scenario when lenders do not do anything, i.e., they do not take control of the SPV.

3. Hard default occurs at time t_{hd}, and the option to exit is activated given the fact that the best for lenders is to exit of the SPV by a residual value. As a result, lenders take control over the cash flow in time t_{hd}.. Therefore, they decide not to negotiate the conditions of the debt agreement. As a result, the *ER* for lenders is given by:

$$ER = \sum_{t=k}^{t_{hd}} e^{-rt} DS_t + e^{-rt}(Cash + CDR) \quad (18)$$

where $Cash$ denotes the cash available upon the SPV at time t_{hd} and CDR the present value of cash that the credit agreement requires such as debt reserves (DR) or even guarantees offered by the sponsors. Given these elements, the exit value (V_{EX}) for lenders can be determined. However, unlike the original equation proposed by Blanc-Brude and Hasan (2017), this paper suggests some adjustments in the payoff function. Therefore, the V^{Loan} is given by:

$$V^{Loan} = max\left(V_{EX}; \check{V}\right) \quad (19)$$

The exit value V_{EX} is considered upon the (hard) default scenario when the renegotiation did not occur. Also, the exit cost is deemed to be equal to zero, like Blanc-Brude and Hasan (2017) suggested. In that sense, the effect of a (technical or hard) default is estimated upon the debt agreement from the lenders' point of view and their outcomes when they may decide to take control of the SPV throughout the embedded option like the option to exit or the option to renegotiate the credit agreement.

On the other hand, unlike the traditional structural credit risk model or even the BBH-I model, it is necessary to specify the detailed characteristics from the debt repayment when the SPV falls dawn into default. Therefore, the financial model requires a complete mapping for each

[5] For simplicity, we assume that renegotiation takes place at the same time when technical default occurs.
[6] Although the proposed model is like the one of Blanc-Brude and Hasan (2017), V_{WK} incorporates the changes about the newly scheduled debt service repayment flow (DS_t') when the restructuring scenario takes place.

possible scenario. That's why it is necessary to implement the following algorithm for performing the structural credit risk model (the BBH-I and BBH-II models) joint with a based-simulation model under the MCS technique:

1. Estimate the probabilities of default by applying the BBH-I model in the initial state of the SPV. To start, we assume that the SPV satisfies the payments for debt service on time.
2. Simulate all the possible paths of the DSCR by using the MCS technique and estimate the *PD* in each one. Finally, identify whether the (technical or hard) default occurs in each iteration.
3. If the technical default is identified at time *t* (t_{td}), compute the outstanding debt and choose the new debt schedule that reaches the total debt recovery by lenders. In order to determine the optimal new debt scheduled, we applied an optimised-based method by using MCS where lenders maximize the recovery rate. Finally, estimate the total value for lenders.
4. If the hard default is identified at time *t* (t_{hd}), compute the exit value by lenders (V_{EX}).
5. If no default is encountered, compute the value of lenders.
6. Compare the value of the embedded options and choose the best outcome for lenders.
7. Estimate the *PD*, *LGD* and *EAD* and *EL* in each one.
8. Repeated the process *n* times until the total sample defined in the MCS model is reached.

4. Application

The extended model stated above is applied to the case of a hypothetical BOMT (build, operate, maintenance and transfer)[7] project which is detailed in table 1.

4.1 Main assumptions of the project and the free cash flow model

The project involves a toll road concession with a length of 80 km in Colombia and requires two-year in the construction phase and eighteen-years in the operation and maintenance phase. At the end of the period, the infrastructure will be returned to the public authority without any payment. Additionally, the operational cash flows ($CFADS$) will determine the ability to pay the company's financial obligations as debt service (i.e., syndicated loan). The assumptions of the project are summarized in Table 1.

[7] The BOMT is one of the significant non-recourse project financing schemes in practice.

Table 1 – Main assumptions of the project

Project duration	20 years
Currency	COP ($)
Annual inflation expected	4%
CAPEX (million)[i]	$272.825
O&M costs (million)	$4.400
Administration fee	10%
Average toll rate	$17.691
Average annual daily traffic (AADT)	4.080
Annual traffic growth rate	6%
Tax rate	34%
Equity	60%
MARR	12%
Debt [ii]	40%
Interest rate	9%
Loan term duration	12 years

i. Total investment represents the resources needed by the infrastructure project and includes pre-operational costs, designs, and financial costs.
ii. We assume that the debt repayment scheme is fixed (an annuity), so that $DS_{t-1} = DS_t$.

Likewise, it is assumed that the default scenario occurs if the $DSCR$ falls below 1.2 in technical default or falls below 1.0 in hard default. Additionally, if technical default occurs equity dividends are locked up until the restructuring process allow it to exit the default state.

Table 2 - Financial model outcomes and cover ratios

Year	*CFADS*	*Debt service*	*DSCR*
1	$ 40,362	$ 27,502	1.47
2	$ 44,226	$ 27,502	1.61
3	$ 48,501	$ 27,502	1.76
4	$ 53,230	$ 27,502	1.94
5	$ 58,460	$ 27,502	2.13
6	$ 64,244	$ 27,502	2.34
7	$ 70,638	$ 27,502	2.57
8	$ 77,706	$ 27,502	2.83
9	$ 85,518	$ 27,502	3.11
10	$ 94,150	$ 27,502	3.42
11	$ 103,688	$ 27,502	3.77
12	$ 114,225	$ 27,502	4.15

Based on these assumptions, initially the cash flows model is built, and by using a risk-adjusted discount rate ($WACC = 8{,}4\%$), the DCF analysis provides a NPV of $203.266

million and an internal rate of return (IRR) of 13,5%. Similarly, the project shows an appropriate capacity to pay the debt service to reach a minimum DSCR of 1.47 and an average of 2.32. Table 2 summarizes the main results of the project's financial model and cover ratios.

4.2 Risk profile: default of probability of the project

According to the equation (12), the probability of default (*PD*) for each time *t* is estimated. The first step is the estimation of the volatility; however, this estimation may represent a big concern not only in the infrastructure projects as indicated by Gatti et al. (2007) but also in the Real Options approach. In that sense, the approach proposed by Brandão, Dyer, and Hahn (2012) was applied. For that, the historical series of traffic volume were analysed and by applying the Monte Carlo simulation technique upon the logarithmic return of the project value: $k = ln\left(\hat{V}_1 / V_0\right)$ [8]. Thus, the volatility defined as the standard deviation of the returns of the present value of the *CFADS* (σ_{CFADS}) was estimated: 16%[9]. Furthermore, given that $DS_{t-1} = DS_t$, we assume that the volatility of the *CFADS* (σ_{CFADS}) is the same that the volatility of the *DSCR* (σ_{DSCR}) following to Blanc-Brude and Hasan (2017), i.e. σ_{DSCR} could represent the volatility of the project of the first year[10].

Once the estimation of the volatility has been done, the Sharpe ratio λ should be calculated by using a risk-free rate of 4.5%. So, λ is 0.1250. The next step is the estimation of the *DD* and the *PD* by using equations (10) and (12), respectively. Likewise, the *DD* was define as (technical or hard) default ($DSCR_t < 1.x$). Figure 2 shows the results.

In the first year, the project reaches the lowest *DSCR* of the loan life (1.47) therefore, the probability of default is the highest. However, as time goes on the capacity to pay the debt service improves, i.e., the relationship between *CFADS* and *DS* rise up for each year (*t*) and the probability falls. Furthermore, the *PD* falls down rapidly because of the increasing trend exhibit by the dynamic of the DSCR along the life of the loan. Therefore, the *PD* is close to zero in the later periods. Additionally, the differences presented by the default point (hard: 1.0 and technical: 1.2) determines different levels of *PD*, with a significant difference (almost 12%) in year 1 of the debt service. This difference decreases as the time goes on. These outcomes can also be analysed from the *DD* perspective, *DD* close to zero reflect a higher probability of default. Similarly, the cumulative distribution function of the default (c) shows these notable differences.

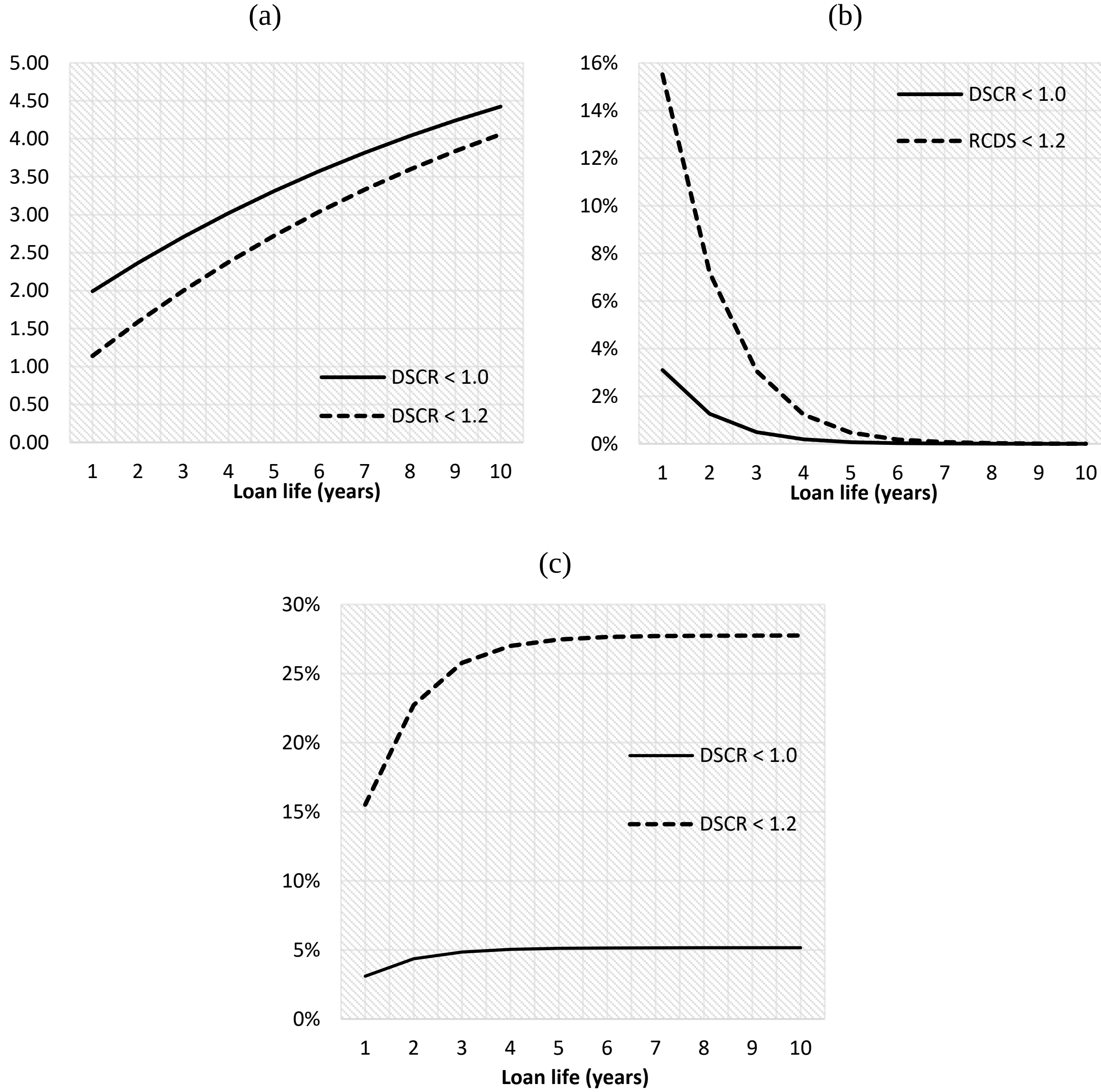
---

[8] See Brandão, Dyer, and Hahn (2012) for more details.

[9] By using Oracle Crystal Ball, the result was obtained after 100.000 iterations. The details of the estimation are omitted for simplicity.

[10] Since it is assumed that the project value follows a GBM, the volatility is constant over the project life.

Figure 2- Distance to default (a), probability of default (b), and cumulative probability of default (c)



Similarly, the estimate of *EL* takes the same dynamic throughout the loan life, given its relationship with the *PD*. The *PD* close to zero indicates that *EL* falls considerably. In other words, without considering the *LGD* and the *EAD*, the asymptotic fall of the *PD* is directly reflected in the *EL*. However, this analysis is incomplete, given that the *LGD*, although is related to probability, also have its own dynamic that must be incorporated. Like Gatti et al. (2007) suggest, it is necessary to estimate the *LGD* or, equivalently, the recovery rate (*RR*). The *RR* clearly depends on the value of the project in the event of default, which could be represented by the present value of the future cash flows. Given these conditions, it is possible to extend the BBH (I and II) models to include this joint effect on the estimation of the *EL*.

Nevertheless, this analysis should be developed under the framework of the Real Options theory. Specifically, the analysis should focus on the effect of the embedded options in the debt agreement like the option to exit and the option to renegotiate the debt conditions under the presence of default scenarios.

4.3 Real Options in practice: option to renegotiate and option to exit

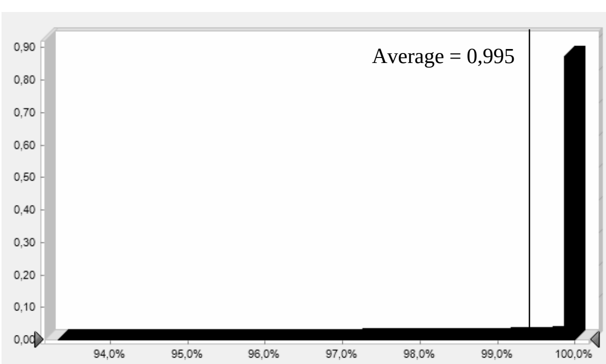
The credit risk analysis has been carried out in a framework where the *PD* is determined based on the payment debt capacity of the project. However, the previous analysis should be extended to incorporate the effect of the newly debt schedule into a default scenario.

By assuming control rights into the debt agreement and following Gatti (2008) and the BBH-II model, it is important to indicate that lenders seeks to increase the *RR* or minimize the *LGD* and *EL*. Therefore, in a (technical) default scenario, lenders will take control of the project, which will allow them to restructure the original debt agreement to extend the term of the debt, if they looking at to continue of the project, or just, they exit the agreement by taking control of the available cash. Here, they can recover not only the cash but also the reserve accounts available along the life of the loan. Additionally, in order to determine the optimal period to extend the scheduled debt, we applied an optimised-based method by using MCS technique where the recovery rate is maximised for lenders. For this easy application, we obtained an extension of 3 years (average) on the original debt schedule as the optimal period where lenders recover all of the debt.

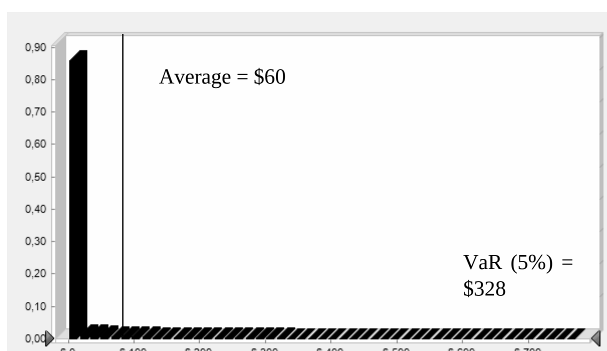
To analyse these related effects the model proposed in section 3.2 was applied[11] complemented with the MCS technique. Figure 3 shows the estimation of *RR* and *EL* in the three scenarios proposed when the default occurs.

Figure 3- Credit risk model: estimation of *RR* and *EL*

(a) Option to Renegotiate



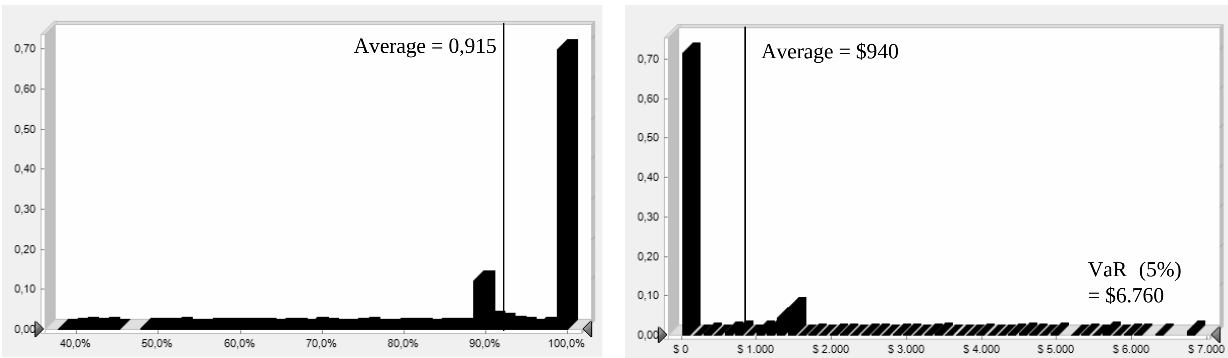
RR



EL

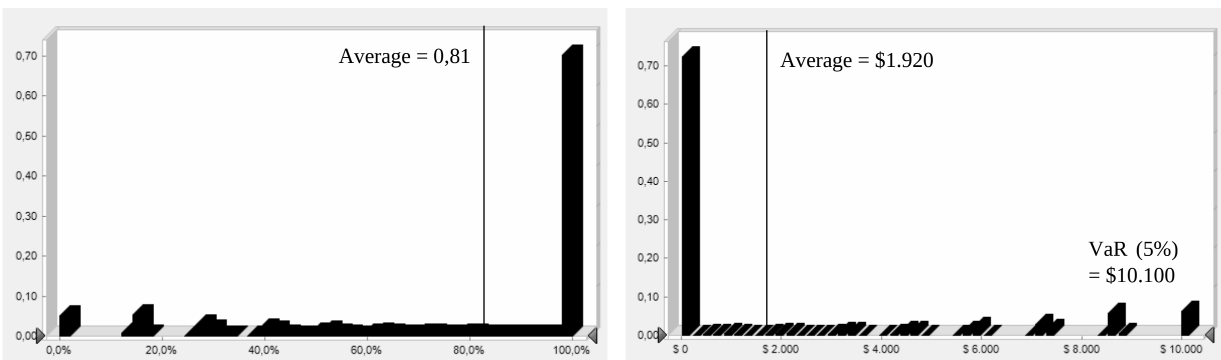
[11] Unlike the BBH-I model we do not implemented the Black and Cox (1976) model.

(b) Option to exit



RR *EL*

(c) Without options



RR *EL*

Figure 3 shows some interesting results that even through are in line with the analysis of corporate debt[12], it allow to validate previous works about credit risk in debt financing for infrastructure projects. First of all, the value for lenders in presence of both the option to exit and the option to renegotiate the debt agreement are greater than where these two options are not triggered. Hence, the *EL* for lenders is the highest ($1.920). In contrast, both options create value for lenders and also reduce the *EL* for them. In the case of the option to renegotiate, the value of the *EL* is the lowest ($60) since the dynamics of the *RR* or the *LGD*=(*1-RR*) allows them to maximize the debt value in almost all of the iterations (almost 100%) in the MCS model when lenders trigger their control rights. Thus, the control rights may represent an important mechanism to face covenant breaches in financing infrastructure projects given the high amounts of resources committed in them.

The MCS model allows the incorporation of the *LGD* or *RR* in a very simple way and its assessment in the all possible scenarios. Besides, another risk analysis can be introduced to

[12] The results are in line with Pawlina (2010) who argues that the option to renegotiate debt generally has a higher value than an analogous option to go bankrupt, which is in our case represent the option to exit.

incorporate de Value-at-Risk (VaR)[13] measure like Gatti (2008) suggested. In presence of the options, for instance, the option to renegotiate, the VaR (5% of probability) for lenders reach only $60 while the option to exit $940. However, without options, the VaR can reach $1.920, thus representing a significant expected loss, compared with the previous scenarios.

5. Conclusions

This paper attempts to present a method where the credit risk is modelled in an integrated approach by combined the dynamics of debt capacity following the models proposed by Blanc-Brude and Hasan (2016) and Blanc-Brude and Hasan (2017) where the probability of default is estimated, with the Monte Carlo simulation and the Real Options approach. In fact, the proposed model not only aim to estimate the probability of default but also the estimation of the recovery rate (or the loss given the default), and the expected loss.

In that sense, the option to exit and the option to renegotiate were estimated on the presence of covenants (debt restructuring clauses). Under the integrated approach, we show how the embedded options are affected for debt clauses and aim to maximize the recovery rate whereas minimizing the *EL* for lenders. In fact, by comparing the three scenarios analysed we found that both the option to exit and the option to renegotiate effectively increase the value for lenders.

Finally, a simulation approach may also be used to evaluate the credit risk as a complement of the structural models like the corporate model (Merton model or KMV model) or even its adaptation to analyse the features of infrastructure projects like the BBH model. Of course, although the model adaption is more complex, it is possible to incorporate the Real Options theory to overcome some limitations as the volatility estimation and the valuation in a risk-neutral work. Likewise, by modelling the uncertainties through the stochastics processes the MCS model could be easily implemented.

References

Arora, N., Bohn, J., and Zhu, F. (2005). Reduced form vs. structural models of credit risk: A case study of three models. Journal of Investment Management, 3(4), 43.

Ashuri, B., Kashani, H., Molenaar, K., Lee, S. and Lu, J. (2012). Risk-neutral pricing approach for evaluating BOT projects with government minimum revenue guarantee options. *Journal of Construction Engineering and Management*, 138(4), 545-557.

[13] According to Gatti (2008), Value at risk (VaR) is a measure of the risk of loss and it can be defined as the maximum possible loss during the time given normal market conditions.

Amram, M. and Kulatilaka, N. (1999). *Real options: Managing strategic investment in an uncertain world.* Boston: Harvard Business School Press.

Aragones, J., Blanco, C., and Iniesta, F. (2009). Modelizacion del riesgo de credito en proyectos de infraestructuras. *Innovar*, *19*(35), 65–80.

Basel Committee on Banking Supervision (2004). *Basel II: International Convergence of Capital Measurement and Capital Standards: a Revised Framework*. Bank for International Settlements, Basel.

Black, F. and Cox, J. (1976). Valuing corporate securities: Some effects of bond indenture provisions. *The Journal of Finance*, 31(2), 351–367.

Black, F. and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy*, 81(3), 637-654.

Blanc-Brude, F., Hasan, M., and Ismail, R (2014). Unlisted Infrastructure Debt Valuation & Performance Measurement. *EDHEC-Risk Institute Publications.*

Blanc-Brude, F., and Hasan, M. (2016). A Structural Model of Credit Risk for Illiquid Debt. *The Journal of Fixed Income*, *26*(1), 6–19.

Blanc-Brude, F., and Hasan, M. (2017). You can work it out! Valuation and recovery of private debt with renegotiable default threshold. *The Journal of Fixed Income*, 26(4), 113-127.

Blanc-Brude, F., Hasan, M., and Whittaker, T. (2016). *Cash Flow Dynamics of Private Infrastructure Project Debt: Empirical evidence and dynamic modelling*. EDHEC-Risk Institute Publications.

Borgonovo, E., and Gatti, S. (2013). Risk analysis with contractual default. Does covenant breach matter? *European Journal of Operational Research*, 230(2), 431–443.

Bowe, M., and Lee, D. (2004). Project evaluation in the presence of multiple embedded Real Options: evidence from the Taiwan High-Speed Rail Project. *Journal of Asian Economics*, 15(1), 71-98.

Brandão, L., Dyer, J., and Hahn, W. (2012). Volatility estimation for stochastic project value models. Eur. J. Oper. Res. 220 (3), 642–648.

Brandão, L., and Saraiva, E. (2008). The option value of government guarantees in infrastructure projects. *Construction Management and Economics*, 26(11), 1171–1180.

Brennan, M. and Schwartz, E. (1985). Evaluating Natural Resource Investments. *The Journal* *of Business*, 58(2), 135-157.

Copeland, T. and Antikarov, V. (2001). *Real Options: A practitioner´s guide*. New York: Texere Publishing Limited.

Davison, A., Keisman, D., and Kelhofer, K. (2010). *Default and Recovery Rates for Project Finance Bank Loans*, 1983–2008, Moody's Special Comment, Report no. 123903.

Dixit, A. and Pindyck, R. (1994). *Investment under uncertainty*. New Jersey: Princeton University Press.

Freydefont, M. (2001). An Approach to Credit Risk Valuation for Structured and Project Finance Transactions. *Journal of Project Finance*, 6(4), 53.

Gatti, S. (2008). *Project Finance in theory and practice: designing, structuring, and financing private and public projects*. Oxford: Academic Press.

Ingersoll J. (1977). A contingent-claims valuation of convertible securities. *Journal of Financial Economics*, 4(3), 289–321.

Ho, P. and Liu, L. (2002). An option pricing-based model for evaluating the financial viability of privatized infrastructure projects. *Construction Management and Economics*, 20(2), 143-156.

Iyer, K., and Sagheer, M. (2011). A real options based traffic risk mitigation model for build-operate-transfer highway projects in India. *Construction Management and Economics*, 29(8), 771-9.

Ingersoll J. (1977). A contingent-claims valuation of convertible securities. *Journal of Financial Economics*, 4(3), 289–321.

Kealhofer, S. (2003). Quantifying credit risk I: default prediction. *Financial Analysts Journal*, 59(1), 30-44.

Liu, J., Yu, X. and Cheah, C. Y. J. (2014). Evaluation of restrictive competition in PPP projects using real option approach. International Journal of Project Management, 32(3), 473-481.

McDonald, R. and Siegel, D. (1986). The value of waiting to invest. *Quarterly Journal Economics*, 101(4), 707-727.

Merton, R. (1973). Theory of rational option pricing. *Bell Journal of Economics and Management Science*, 4(1), 141-183.

Merton, R. (1974). On the pricing of Corporate Debt: The Risk structure of interest rates. *the Journal of Finance*, 29(2), 449-470.

Pawlina, G. (2010). Underinvestment, capital structure and strategic debt restructuring. Journal of Corporate Finance, 16(5), 679-702

Pellegrino, R., Vajdic, N., and Carbonara, N. (2013). Real option theory for risk mitigation in transport PPPs. *Built Environment Project and Asset Management*, 3(2), 199-213.

Pindyck, R. (1991). Irreversibility, Uncertainty, and Investment. *Journal of Economic Literature*, 29(3), 1110-1148.

Trigeorgis, L. (1996). *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. Cambridge: MIT Press.

Vasicek, O. (1984). *The philosophy of credit valuation: the credit valuation model*. KMV Corporation.

Wibowo, A., Permana, A., Kochendörfer, B., Kiong, R., Jacob, D., and Neunzehn, D. (2012). Modeling contingent liabilities arising from government guarantees in Indonesian BOT/PPP toll roads. *Journal of Construction Engineering and Management*, 138(12), 1403-1410.

Zapata, C., Mejia, C., and Marques, N. (2018). Minimum revenue guarantees valuation in PPP projects under a mean reverting process. *Construction Management and Economics*, 1-19, DOI: 10.1080/01446193.2018.1500024