Comparing the Forecast Accuracy and Explainability of Accounting-based Equity Valuation Models: Does Real Options Information Matter?

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ABSTRACT

We examine the forecast accuracy and explainability of landmark linear accounting-based equity valuation models (Ohlson, 1995; Feltham and Ohlson, 1995, 1996) and non-linear accounting-based valuation models with real options components (Hwang and Sohn, 2010; Ashton et al. 2003; Zhang, 2000) using historical data for the UK. Empirical results show that non-linear equity valuation models with real options characteristics provide higher forecast accuracy and stronger explainability than linear equity valuation counterparts. More specifically, we utilize the Hwang and Sohn (2010) real option adjustment in the Ohlson and Feltham framework and find the adjusted models perform better in terms of accuracy and explainability than the underlying linear models. We also find superior performance for the valuation models of Ashton et al. (2003) and Zhang (2000) and further demonstrate that this improvement in performance is due to their option characteristics. Our conclusions hold in an alternative dataset of U.S-listed firms.

JEL classification: M40, M41, G14

Keywords: Equity valuation, real options, residual income, linear models, non-linear models, forecast accuracy, explainability

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1. Introduction

Accounting-based valuation research building on what we term the 'Ohlson and Feltham framework' (OFF) continues to attract considerable interest among finance and accounting researchers and practitioners (Ohlson, 1995; Feltham and Ohlson, 1995, 1996; Dechow et al. 1999; Myers, 1999, Begley and Feltham, 2002; Callen and Segal, 2005; Pope and Wang, 2005; Choi et al, 2006; Clubb, 2013; Lyle et al. 2013; Christodoulou et al. 2016; Penman, 2016; Easton and Monahan, 2016). In addition to OFF based studies which assume that the valuation role of accounting variables can be explained in terms of linear information dynamics (LID), further studies have developed non-linear valuation approaches through the incorporation of real options information into the determination of equity value. Such models identify the importance of adaptation options (Yee, 2000; Ashton et al. 2003), abandonment options (Hwang and Sohn, 2010; Sohn, 2012) or growth options (Zhang, 2000) in determining equity value. The real options approach therefore takes into account the firms' ability to change or modify its operations in a way not captured by LID leading to the incorporation of real-options terms in the equity valuation function.

Current empirical evidence on the accuracy and explainability of linear models is not satisfactory (Courteau et al. 2001; Barth et al. 2005; Jorgensen et al. 2011). Equity values which have been calculated based on LID are known to suffer from an underestimation bias (Burgstahler and Dichev, 1997; Myers, 1999; Dechow et al. 1999; Choi et al. 2006). Empirical research regarding non-linearities in equity valuation is also still developing. Evidence on non-linear valuation concerns only the properties or systematic biases of models which arise from the omission of real option value and optionality components (Ataullah et al. 2006; Hao et al. 2011). Relatively little is known about the actual predictive ability or explainability of equity valuation models which explicitly incorporate real options information or contingent-claims characteristics. Given the unsatisfactory empirical findings associated with LID-based linear valuation models and the scarce empirical evidence on non-linear valuation models with real options, it is important that research compares and evaluates the validity of linear and non-linear equity valuation models. Furthermore, research findings showing that analysts' valuation model preferences are multi-dimensional and that multiple factors are involved in the choice of valuation models lend support to the importance of incorporating real options information (i.e. recognizing the role of abandonment and growth) in equity valuation (Demirakos et al. 2004, 2010; Iman et al. 2008, 2013).

The aim of this paper is to evaluate and compare established linear valuation models based on LID with established non-linear models with real options characteristics. More specifically, the forecast bias, accuracy and explainability of linear and non-linear valuation models are compared and tested. We focus on linear models from the OFF as developed in O95 (Ohlson, 1995), FO95 (Feltham and Ohlson 1995), and FO96 (Feltham and Ohlson, 1996) and non-linear models as developed by Hwang and Sohn (2010), Ashton et al. (2003) and Zhang (2000). The real option model of Hwang and Sohn (2010) can be applied as a real option adjustment to any specific linear model, while the models of Ashton et al. (2003) and Zhang (2000) are individual models based on alternative specific assumptions. In total, eight models are therefore tested in this paper, three linear models based on the OFF (O95, FO95, FO96), three non-linear models based on the Hwang and Sohn (2010) adaptation of the OFF models (HSO95, HSF095, HSFO96), and two non-linear models based on Ashton et al. (2003) and Zhang (2000)) respectively.

Our study contributes to the existing literature in several ways. First, to our knowledge, this paper is the first empirical research to compare the forecast bias, accuracy and explainability of linear models based on LID and non-linear models which include real options characteristics. It answers the call for more empirical work on the relation between equity value and its drivers (Burgstahler and Dichev, 1997; Ashton et al. 2003; Ataullah et al. 2009). Second, it applies the model of Hwang and Sohn (2010) as an adjustment to the OFF models and hence provides important insights on the adjustment of extant linear models with real options effects and components. Third, this is the first paper to empirically estimate and extensively compare the equity values generated by the Ashton et al. (2003) and Zhang (2000) models. Finally, and more practically, our analysis contributes to the literature by highlighting and underlining the critical valuation role of the real options flexibility associated with a firm's ability to abandon and expand its existing operating activities (see e.g., Perotti and Rossetto, 2007; de Andres et al. 2017).

2. Valuation Models

2.1 Linear Valuation Models

LID1: Unbiased accounting based on O95

The residual income based model of Ohlson (1995) was considered a landmark work in financial accounting (Lundholm, 1995, pp. 749) and one of the most important developments in capital markets research (Bernard, 1995, pp. 733). We focus on the following simplified version of the LID in O95:

$$\tilde{x}_{t+1}^{a} = \omega_1 x_t^{a} + \tilde{\varepsilon}_{t+1}, \qquad \text{(LID1)}$$

where $\tilde{x}_{t+1}^{a}$ represents future residual income at time $t + 1$, $0 \leq \omega_1 \leq 1$ represents the persistence parameter of residual income, and $\tilde{\varepsilon}_{t+1}$ is zero-mean disturbance at time $t + 1$. Assuming rational and efficient capital markets where the residual income model provides a valid representation of equity value, LID1 implies that equity value is a function of book value and residual income as follows:

$$V1_t = B_t + \beta x_t^a, \tag{V1}$$

where

$$\beta = \frac{\omega_1}{R - \omega_1}.$$

Residual income is defined as $x_t^a = x_t - (R-1)B_{t-1}$, where x_t is net (comprehensive) income at time t, B_{t-1} is book value of equity at time $t-1$, and R equals one plus the cost of capital. Ohlson (1995) is a model of 'unbiased accounting' because the LID (assuming $0 \leq \omega_1 \leq 1$) implies that residual income is expected to tend to zero in the long-run and hence that the expected market value of equity in the long-run is equal to the book value of equity.

LID2: Conservative accounting based on FO95

Feltham and Ohlson (1995) extend O95 by examining how conservatism relates to the valuation of a firm's equity in a setting where accounting variables are generated by LID. Our simplified LID based on FO95 focuses on the operating activities of the firm as follows:

$$\widetilde{ox}_{t+1}^a = \omega_{11} ox_t^a + \omega_{12} OA_t + \tilde{\varepsilon}_{1t+1}, \tag{LID2}$$

$$\widetilde{OA}_{t+1} = \omega_{22} OA_t + \tilde{\varepsilon}_{2t+1}.$$

where $\widetilde{ox}_{t+1}^a$ is residual operating income at time $t+1$, OA_{t+1} denotes net operating assets at time $t+1$, and $\tilde{\varepsilon}_{1t+1}$ and $\tilde{\varepsilon}_{2t+1}$ represent mean zero disturbance terms at time $t+1$. The model assumes that residual operating persistence is such that $0 \leq \omega_{11} \leq 1$, while $\omega_{12} \geq 0$ is a conservatism parameter and ω_{22} is the growth parameter of net operating assets such that $1 \leq \omega_{22} \leq R$. The equity value implied by LID2 is given by:

$$V2_t = B_t + \beta_1 ox_t^a + \beta_2 OA_t, \tag{V2}$$

where:

$$\beta_1 = \frac{\omega_{11}}{R - \omega_{11}},$$

$$\beta_2 = \frac{\omega_{12} R}{(R - \omega_{22})(R - \omega_{11})}.$$

FO95 is a model of 'conservative accounting' because the condition $\omega_{12} > 0$ implies that $\beta_2 > 0$. The latter condition implies that long-run expected residual operating income is positive and hence the value of firm and the value of equity value are expected to exceed the book value of operating assets and book value of equity respectively in the long-run.

LID3: Conservative accounting based on FO96

The LID in Feltham and Ohlson (1996) is described in terms of cash flow:

$$\widetilde{CR}_{t+1} = \gamma_1 CR_t + \kappa_1 CI_t + \tilde{\varepsilon}_{1t+1}, \tag{LID3}$$

$$\widetilde{CI}_{t+1} = \omega_1 CI_t + \tilde{\varepsilon}_{2t+1},$$

$$OA_{t+1} = \delta_1 OA_t + CI_{t+1}.$$

In the above LID, CR, CI, and OA respectively represent cash receipts, cash investments, and net operating assets. $\gamma_1 \in [0,1)$ is the persistence parameter of cash receipts; $\kappa_1 > 0$ is the impact parameter of cash investments on cash receipts; $\omega_1 \in [0, R)$ represents one plus the expected growth in cash investments and R equals one plus the cost of capital. Operating assets are assumed to depreciate on a reducing balance basis at the rate of $(1 - \delta)$. Employing the accounting rules, the equity value is expressed as:

$$V3_t = B_t + \beta_1 ox_t^a + \beta_2 OA_{t-1} + \beta_3 CI_t, \quad \text{(V3)}$$

where

$$\beta_1 = \Phi\gamma_1,$$

$$\beta_2 = \Phi R(\gamma_1 - \delta_1),$$

$$\beta_3 = [\Phi\kappa_1 - 1]\frac{R}{R-\omega_1},$$

$$\Phi = 1/(R - \gamma_1).$$

The FO96 analysis develops the FO95 analysis by identifying two sources of accounting conservatism. The first of these is accounting over-depreciation of operating assets which results in $\beta_2 > 0$ when $(1 - \delta_1) > (1 - \gamma_1)$. The other source of conservatism is simply due to the presence of positive net present value investments which results in $\beta_3 > 0$ because NPV per dollar of investment is given by $[\Phi\kappa_1 - 1]$. When the depreciation policy is conservative, the net operating assets are over-depreciated and OA_{t-1} has a positive coefficient in the valuation function. The second additional term, CI_t, contributes positively to the valuation when future investments have positive net present values.

2.2 Non-linear Valuation Models with Real Options

Model with Abandonment Option: Hwang and Sohn (2010)

Following Burgstahler and Dichev (1997), Hwang and Sohn (2010) develop a real option model based on the concepts of recursion value and adaptation value (hereafter, HS). Recursion value refers to the value of the equity when the firm continues to apply its business technology to its resources. Adaptation value refers to the value when resources are adapted for alternative uses.

Shareholders have the option to liquidate net assets if the recursion value is expected to be lower than the adaptation value. If the recursion value is larger than the adaptation value, shareholders can exercise their call option to take the recursion value with the exercise price of net assets value. The real option model in HS is given by (assuming the Black and Scholes (1973) apparatus):

$$V4_t = AV_t + CO_t, \tag{V4}$$

$$CO_t = V_t^{REC} * N(d_1) - AV_t * e^{-RF_t * T} * N(d_2),$$

$$d_1 = \frac{ln\left(\frac{V_t^{REC}}{AV_t}\right) + \left(RF_t + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

In V4, AV is the adaptation value (i.e., the exercise price); CO denotes the call option value with the maturity of T; V^{REC} is the recursion value of the underlying asset; and RF is the risk-free rate; σ denotes the standard deviation of the recursion value return; and $N(d)$ is the probability that a standardized, normally distributed random variable is less than or equal to d. In our empirical analysis, we assume that AV is equal to the book value of equity, we estimate the standard deviation of the recursion value return, σ, as the standard deviation of $(V_t^{REC} - V_{t-1}^{REC})/V_{t-1}^{REC}$ over the past 5 years, and we assume that the maturity of the call option, T, is 5 years.

Using put call parity, the equity value in HS can also be expressed as the recursion value plus the abandonment option value. The abandonment option represents the shareholder’s right to liquidate the net assets if the adaptation value proves superior to the recursion value. In previous research on real options and equity valuation, the estimated equity value given by O95 has been widely used as the recursion value (Burgstahler and Dichev,1997; Ashton et al. 2003; Hwang and Sohn, 2010). In this paper, the estimated equity values calculated using FO95 and FO96 are also used as measures of recursion value when testing the HS real option model. As such, the HS models therefore represent real option adjustments to the linear models and are referred to as HSO95, HSFO95 and HSFO96 hereafter.

Model with Adaptation Option: Ashton et al. (2003)

Following Ohlson (1995) and Burgstahler and Dichev (1997), Ashton et al. (2003) also suggest that the market value of equity is composed of a recursion value and real option value. The former reflects the equity value while the business continues its current operations and the latter reflects the value of flexibility to change the business operation. Ashton et al. (2003) assume that the recursion value of the firm's equity evolves in terms of a continuous time branching process, and develops a quasi-supply side generalization of the Ohlson (1995) model:

$$d\eta(t) = r\eta(t)dt + \sqrt{\eta(t)}dq(t),$$

$$\eta(t) = c_1B(t) + c_2x(t) + c_3v(t),$$

where η is the recursion value of O95 (the value calculated in $V1_t$); r is the cost of equity; $dq(t)$ is a Wiener process with a variance of ζ; B is the book value of equity; x is earnings; v is an other information variable, and c_1 to c_3 are valuation coefficients. Its expectations are that the proportionate rate of growth in recursion value will equal the cost of equity. Employing the quasi-supply-side model and standard no arbitrage conditions, Ashton et al. (2003) further develop a non-linear model for equity valuation with a close-form solution:

$$P(\eta) = \eta + \frac{P(0)}{2}\int_{-1}^{1} exp\left(\frac{-2\theta\eta}{1+z}\right)dz,$$

where $P(0) = B > 0$ is the firm's adaptation option value when the recursion value of equity equals zero and $\theta = \frac{2r}{\zeta}$ is a risk parameter which denotes the stability of the recursion value.

Ataullah et al. (2009) suggest that an orthogonal polynomial fitting procedure, in conjunction with the Riesz Representation Theorem (MacCluer, 2008, pp. 21–23), can be implemented to obtain a convergent infinite power series expansion for the Ashton et al. (2003) model.

$$V5 = B\left[h + \frac{1}{2}\int_{-1}^{1} exp\left(\frac{-2\theta Bh}{1+z}\right)dz\right] = B\left[\sum_{m=0}^{\infty} \alpha_m\, L_m\,(h)\right]. \tag{V5}$$

In the above equation, α_m is the 'Fourier-Laguerre' coefficient; L_m is the m^{th} order Laguerre polynomial and $h = \eta/B$. The functions of Laguerre coefficient and polynomial are illustrated in Table 1. Continuing the polynomial series expansion infinitely enables the determination of the Laguerre polynomial and Fourier- Laguerre coefficients of any desired order. As stated in Ataullah et al. (2009), the equity price is likely to be approximated while the order of the polynomial is expanded to twenty. The adaptation option in Ashton et al. (2003) is a mixed option. Though it theoretically focuses on abandonment, it explains the non-linear part of equity valuation associated with negative earnings and also the growth options that are available to firms (Herath et al. 2015).

Model with Abandonment Option and Growth Option: Zhang (2000)

Zhang (2000) develops an accounting-based valuation model which incorporates both abandonment and growth options. The model emphasizes the role of decision-making when faced with different investment opportunities. More specifically, a firm may choose to discontinue its operations when it is sufficiently unprofitable and to expand them when it is sufficiently profitable. The equity value in Zhang (2000) can be expressed as:

$$V_t = \frac{1}{r}x_t^E + P(q_t)as_t + C(q_t)G,$$

$$P(q_t) = \frac{1}{r(1+r)}\int_{e_l}^{q_a^*-q_t} [q_a^* - q_t - \tilde{v}_{t+1}]f(\tilde{v}_{t+1})d\tilde{v}_{t+1},$$

$$C(q_t) = \frac{1}{r(1+r)} \int_{q_g^* - q_t}^{e_u} [q_t + \tilde{v}_{t+1} - q_g^*] f(\tilde{v}_{t+1}) d\tilde{v}_{t+1}.$$

where x^E is the economic earnings; r is the cost of equity; as is the asset stock; G is potential growth investment; q is the internal rate of return on cash investments; q_a^* and q_g^* are the thresholds for exercising the put and call options; $\tilde{v}_{t+1} = \tilde{q}_{t+1} - q_t$ is the mean zero change in the internal rate of return at time $t + 1$; $P(q_t)as_t$ represents the put option value to discontinue the operations at time t and $C(q_t)G$ represents the call option value to grow at time t.

Utilizing accounting rules to establish valuation in terms of accounting variables and assuming unbiased accounting, the real option model in Zhang (2000) can be presented as follows:[1]

$$V6 = \frac{1}{r} x_t + P(ROE_t)B_t + C(ROE_t)G. \quad \text{(V6)}$$

As earnings is a product of equity book value and return on equity, the primitive accounting variables in the real option model of Zhang (2000) are the book value of equity B and return on equity ROE. While book value of equity represents the scale of investment, ROE denotes the profitability. The model of Zhang (2000) emphasizes the role of ROE in that it guides the decision making in different scenarios in time $t + 1$, which may lead to the inclusion of the put or call option value. Tables 1 summarizes the accounting-based valuation models empirically implemented in this study.

[1] The full version of Zhang (2000) includes three types of biases: the bias between book value and asset stock; the bias between accounting earnings and economic earnings as well as the bias between ROE and IRR. Following Hao et al. (2011), the reduced version of Zhang (2000) with unbiased accounting is tested in this paper.

Table 1. Summary of Accounting-Based Valuation Models

Panel A Linear Accounting-Based Valuation Models			
	O95	**FO95**	**FO96**
LID	$\frac{x_t^a}{B_{t-1}} = \omega_0 + \omega_1 \frac{x_{t-1}^a}{B_{t-1}} + \varepsilon_t.$	$\frac{ox_t^a}{B_{t-1}} = \omega_{10} + \omega_{11} \frac{ox_{t-1}^a}{B_{t-1}} + \omega_{12} \frac{OA_{t-1}}{B_{t-1}} + \varepsilon_{1t},$ $\frac{OA_t}{B_{t-1}} = \omega_{20} + \omega_{22} \frac{OA_{t-1}}{B_{t-1}} + \varepsilon_{2t}.$	$\frac{CR_t}{B_{t-1}} = \gamma_0 + \gamma_1 \frac{CR_{t-1}}{B_{t-1}} + \kappa_1 \frac{CI_{t-1}}{B_{t-1}} + \varepsilon_{1t},$ $\frac{CI_t}{B_{t-1}} = \omega_0 + \omega_1 \frac{CI_{t-1}}{B_{t-1}} + \varepsilon_{2t},$ $\frac{OA_t - CI_t}{B_{t-1}} = \delta_0 + \delta_1 \frac{OA_{t-1}}{B_{t-1}} + \varepsilon_{3t}.$
Valuation Model	$V1_t = B_t + \beta x_t^a.$	$V2_t = B_t + \beta_1 ox_t^a + \beta_2 OA_{t.}$	$V3_t = B_t + \beta_1 ox_t^a + \beta_2 OA_{t-1} + \beta_3 CI_t.$
Theoretical-Implied Coefficients	$\beta = \frac{\omega_1}{R - \omega_1}.$	$\beta_1 = \frac{\omega_{11}}{R - \omega_{11}},$ $\beta_2 = \frac{\omega_{12} R}{(R - \omega_{22})(R - \omega_{11})}.$	$\beta_1 = \Phi \gamma_1,$ $\beta_2 = \Phi R(\gamma_1 - \delta_1),$ $\beta_3 = [\Phi \kappa_1 - 1] \frac{R}{R - \omega_1},$ $\Phi = 1/(R - \gamma_1).$
Panel B Non-linear Accounting-Based Valuation Models with Real Options			
	HSO95	**HSFO95**	**HSFO96**
Valuation Model	$V4a_t = AV_t + CO_t^{O95}.$	$V4b_t = AV_t + CO_t^{FO95}.$	$V4c_t = AV_t + CO_t^{FO95}.$
Functions of Value Estimates	$CO_t^{O95} = V_t^{O95} * N(d_1) - AV_t * e^{-RF_t * T} * N(d_2),$ $d_1 = \frac{ln\left(\frac{V_t^{O95}}{BV_t}\right) + \left(RF_t + \frac{\sigma_{O95}^2}{2}\right) T}{\sigma_{O95}\sqrt{T}},$ $d_2 = d_1 - \sigma_{O95}\sqrt{T},$	$CO_t^{FO95} = V_t^{FO95} * N(d_1) - AV_t * e^{-RF_t * T} * N(d_2),$ $d_1 = \frac{ln\left(\frac{V_t^{FO95}}{BV_t}\right) + \left(RF_t + \frac{\sigma_{FO95}^2}{2}\right) T}{\sigma_{FO95}\sqrt{T}},$ $d_2 = d_1 - \sigma_{FO95}\sqrt{T}.$	$CO_t^{FO96} = V_t^{FO96} * N(d_1) - AV_t * e^{-RF_t * T} * N(d_2),$ $d_1 = \frac{ln\left(\frac{V_t^{FO96}}{BV_t}\right) + \left(RF_t + \frac{\sigma_{FO96}^2}{2}\right) T}{\sigma_{FO96}\sqrt{T}},$ $d_2 = d_1 - \sigma_{FO96}\sqrt{T}.$
	Ashton et al. (2003)		**Zhang (2000)**
Valuation Model	$V5 = B\left[h + \frac{1}{2}\int_1^1 exp\left(\frac{-2\theta Bh}{1+z}\right) dz\right] = B\left[\sum_{m=0}^{\infty} \alpha_m L_m(h)\right].$		$V6_t = \frac{1}{r} x_t + P(ROE_t) B_t + C(ROE_t) G.$
Functions of Value Estimates	$L_0(\eta) = 1, \quad L_1(\eta) = 1 - h,$ $mL_m(h) = (2m - 1 - h)L_{m-1}(h) - (m-1)L_{m-2}(h) \ \ if \ \ m \geq 2 ,$ $\alpha_0 = \theta B log\left(\frac{\theta B}{1+\theta B}\right) + 2, \quad \alpha_1 = -1 - \frac{\theta B}{1+\theta B} - \theta B \, log\left(\frac{\theta B}{1+\theta B}\right),$ $\alpha_m = \frac{\theta B(1+\theta B)^m - (m+\theta B)\theta B^m}{m(m-1)(1+\theta B)^m} \ \ if \ \ m \geq 2.$		$P(ROE_t) = \frac{1}{r(1+r)} max\{0, r - ROE_t\},$ $C(ROE_t) = \frac{1}{r(1+r)} max\{0, ROE_t - r\}.$

Notes: Table 1 summarizes the accounting-based valuation models empirically implemented in this paper. Panel A presents the linear accounting-based valuation models, while Panel B presents the non-linear accounting-based valuation models with real options.

3. Methodology

3.1 Procedures for Generating and Comparing Value Estimates

We contrast the reliability of linear and non-linear valuation models in terms of forecast bias, forecast accuracy and explainability, similar to Francis et al. (2000). Forecast bias is measured by mean and median Proportional Valuation Error (PVE). Forecast accuracy is measured by mean and median Absolute Proportional Valuation Error (APVE). We also report the central tendency of the value estimates, which is defined as the percentage of observations where the estimated value lies within 15% of the security price. Explainability is measured by the R square of the estimated values explaining the market values in pooled time series and cross-sectional regressions.

$$PVE_t = \frac{MV_t^{Est} - MV_t^{Act}}{MV_t^{Act}},$$

$$APVE_t = \frac{|MV_t^{Est} - MV_t^{Act}|}{MV_t^{Act}},$$

where MV_t^{Est} represents the estimated market value for firm j at time t and MV_t^{Act} represents the market value for firm j at time t.

For the linear models, the procedure to estimate MV_{t+1}^{Est} closely follows the one used by Dechow et al. (1999) and Choi et al. (2006). The parameters of the LID are estimated in a pooled time-series cross-sectional regression method using all historically available data from the earliest year when UK data are available on Datastream through to year t. The regressed LID parameters for each year are then used to calculate the theoretically-implied valuation multiples in each linear equity valuation model (the various β in each model). Finally, the implied valuation multiples are applied to all firms with necessary accounting data to calculate estimated market value. To reduce the influence of heteroscedasticity, following Choi et al. (2006), all accounting variables in LID are deflated by the beginning book value of equity.

For LID1, the key parameter of interest is ω_1.

$$\frac{x^a_{j,t}}{B_{j,t-1}} = \omega_{0,t} + \omega_{1,t}\frac{x^a_{j,t-1}}{B_{j,t-1}} + \varepsilon_t. \tag{1}$$

In (1), j is the firm index and t is the time index. The range of t is from the second earliest year residual income is available up to time t. $B_{j,t-1}$ is book value of equity and $x^a_{j,t-1}$ are abnormal earnings at time t-1.

For LID2, the key parameters are ω_{11}, ω_{12}, and ω_{22}.

$$\frac{ox^a_{j,t}}{B_{j,t-1}} = \omega_{10,t} + \omega_{11,t}\frac{ox^a_{j,t-1}}{B_{j,t-1}} + \omega_{12,t}\frac{OA_{j,t-1}}{B_{j,t-1}} + \varepsilon_{1t}, \tag{2}$$

$$\frac{OA_{j,t}}{B_{j,t-1}} = \omega_{20,t} + \omega_{22,t}\frac{OA_{j,t-1}}{B_{j,t-1}} + \varepsilon_{2t}. \tag{3}$$

In (2), the time index t ranges from the second earliest year residual operating income is available up to time t. $ox^a_{j,t-1}$ are residual operating income and $OA_{j,t-1}$ net operating assets at time t-1. In (3), the time index t ranges from the second earliest year net operating assets is available up to time t.

For LID3, the key parameters are γ_1, κ_1, ω_1, and δ_1.

$$\frac{CR_{j,t}}{B_{j,t-1}} = \gamma_{0,t} + \gamma_{1,t}\frac{CR_{j,t-1}}{B_{j,t-1}} + \kappa_{1,t}\frac{CI_{j,t-1}}{B_{j,t-1}} + \varepsilon_{1t}, \tag{4}$$

$$\frac{CI_{j,t}}{B_{j,t-1}} = \omega_{0,t} + \omega_{1,t}\frac{CI_{j,t-1}}{B_{j,t-1}} + \varepsilon_{2t}, \tag{5}$$

$$\frac{OA_{j,t} - CI_{j,t}}{B_{j,t-1}} = \delta_{0,t} + \delta_{1,t}\frac{OA_{j,t-1}}{B_{j,t-1}} + \varepsilon_{3t}. \quad (6)$$

In (4), the time index t ranges from the second year cash receipts is available up to time t. $CR_{j,t-1}$ is the year beginning cash receipts and $CI_{j,t-1}$ is the year beginning cash investments. In (5), the time index t ranges from the second earliest year cash investments is available up to time t. In (6), $OA_{j,t-1}$ is the year beginning net operating assets and the time index t ranges from the second earliest year net operating assets is available up to time t.

For the Hwang and Sohn (2010) adjustment, the estimated market values of equity in the three linear models (O95, FO95, FO96) are used separately as the recursion values. The Black and Scholes (1973) formula is then used to calculate each call option value, where the recursion value is regarded as the underlying asset and the book value of equity is the exercise price. The standard deviation of the recursion value return is calculated annually as the standard deviation of $(V_t^{REC} - V_{t-1}^{REC})/V_{t-1}^{REC}$ over the past 5 years, where V_t^{REC} is the recursion value of each linear model. In line with Hwang and Sohn (2010) as well as Sohn (2012), the maturity of the option is assumed to be five years and the call option value is equal to zero if the recursion value is lower than the book value of equity (option time value is ignored in this case). Finally, the estimated equity value of the Hwang and Sohn (2010) adjustment of the linear models (HSO95, HSFO95, HSFO96) is calculated as the sum of the book value of equity and the various call option value.[2]

For Ashton et al. (2003), only the estimated equity value of Ohlson (1995) η_t is used as the recursion value since the Ashton model is derived from O95. The variance of the recursion value ζ is calculated as the variance of $\eta_t - \eta_{t-1}/\eta_{t-1}$ over the past 5 years, which is consistent with the

[2] Based on put call parity, $P_t = B_t.e^{-rt} + CO_t$ should be mathematically more precise. In this paper, following Hwang and Sohn (2010), $P_t = B_t + CO_t$ is used for simplicity as the exercise price is anyway not fixed during the option maturity. The main results are not changed if the book value of equity is discounted.

variance calculated in models using the Hwang and Sohn (2010) adjustment. The parameter $\theta = 2r/\zeta$ denotes the stability of the recursion value and $h = \eta/B$ stands for the ratio of the recursion value divided by book value. These are the primitive variables in generating the Laguerre polynomials and Fourier- Laguerre coefficients. Finally, the estimated equity price of Ashton et al. (2003) is approximated while the order of polynomial is expanded to twenty. Following Ashton et al. (2003), the estimated equity value equals the book value of equity (adaptation value) when the recursion value is lower than zero.

The estimated equity value of Zhang (2000) is generated from its reduced version with unbiased accounting. According to Zhang (2000), with unbiased depreciation, the economic depreciation rate is equal to the accounting depreciation rate. In such circumstances, the book value of equity, accounting earnings and ROE are, respectively, unbiased measures of asset stock, economic earnings and IRR. Empirically, we proxy the IRR by ROE and measure asset stock by the book value of equity.[3] Growth in Zhang's model refers to prospective growth investment and hence empirically we measure growth by cash investments. To be consistent with Ashton et al. (2003) and Hwang and Sohn (2010), we assume the friction cost of adaptation equals zero.[4] After simplification, the empirical model for Zhang (2000) is as follows:

$$V_t = \frac{ROE_t B_t}{r} + P(ROE_t)B_t + C(ROE_t)G,$$

[3] We proxy q_{t+1} by ROE_t instead of ROE_{t+1} without using analysts' forecast data. Since the valuation in this paper is based on historical data, this makes the estimation consistent with other models. While using analysts' forecasts may provide more accurate results for Zhang's (2000) model, it will provide information beyond historical data.

[4] We also test the model with friction cost calculated following Berger et al. (1996). When the friction cost is not zero, the put option value is represented by $P(ROE_t) = \frac{1}{r(1+r)}\max\{0, (1-\gamma c_a)r - ROE_t\}$. We measure the economic depreciation rate γ as the accounting depreciation rate which is regressed using pooled time series and cross-sectional regressions containing all historically available data in the UK. The results (not reported) show small changes in the estimated equity value. In all cases the qualitative results of the performance of the model do not change.

$$P(ROE_t) = \frac{1}{r(1+\mathrm{r})}\max\{0, r - ROE_t\},$$

$$C(ROE_t) = \frac{1}{r(1+r)}\max\{0, ROE_t - r\}.$$

The model indicates that the equity value consists of three portions: $\frac{ROE_t B_t}{r}$ represents the portion of value if the firm continues its current operation; $P(ROE_t)B_t$ denotes the put option value and $C(ROE_t)G$ signifies the call option value. Theoretically, Zhang (2000, pp.274) denotes the first portion of value in the equation (maintaining current scale operations) as capitalized contemporaneous earnings as a result of simplifying derivation, although it is more precise to measure this value using capitalized next period earnings. To remain consistent with other models in this paper, we measure the first portion of the equation as $\frac{ROE_t B_t}{r}$ but also use capitalized contemporaneous earnings as a robustness check and obtain similar results.

3.2 Sample Selection and Data

Our sample consists of all UK non-financial companies listed on the London Stock Exchange. Dead companies are included to avoid survivorship bias. Accounting variables are collected from Worldscope and market data are retrieved from Datastream. Both databases are accessed through Thomson Reuters Datastream. The empirical analysis is conducted on a per share basis.

Our primary empirical analysis uses annual financial statement data from 1999 to 2015. Panel A in Table 2 summarizes the sample period for each valuation model, which differs because of data availability for measuring relevant variables.[5] For the estimation of LID parameters, pooled

[5] For O95 and FO95, the sample periods are both between 1990 and 2015, when the required main accounting variables are continuously available. The sample period for FO96 and Zhang (2000) is from 1995 to 2015 when the cash flow

time-series cross-sectional regressions are used with all historically available data from 1980 (the earliest year UK data are available on Datastream) through year t. Panel B summarizes the sample selection procedure for all the models. This yields an initial 46,542 observations from all UK London Stock Exchange listed non-financial firms on Worldscope and Datastream between 1980 and 2015. Excluding observations with missing earnings per share, book value per share, market value per share and negative book value of equity reduces the sample to 35,970 observations. To mitigate the effect of outliers, following Burgstahler and Dichev (1997), observations at the extreme top and bottom 1 percent of P/B and P/E ratios are trimmed. This reduces the observations to 34,589. In addition, non-missing data for various variables in each model are required in order to estimate equity value.[6] In order to compare across all models, observations with missing estimated equity value in each model are removed. The resulting final sample, thus, contains 1,624 firms and 9,768 observations from the fiscal year 1999 to 2015. For comparison, we also examined a large dataset of all US non-financial companies (2,653 firms and 15,463 observations from the fiscal year 1994 to 2017). Results are presented in the supplementary findings section.

Appendix A contains all variable definitions. All variables are stated on a per share basis, where the market value of equity is collected six months after the financial year end (6 months after time t). Earnings are measured before extraordinary items and special items as it is assumed that

variables are continuously available. For the Hwang and Sohn (2010) adjustment, the starting sample year is four years later than the original linear version because a minimum of the five most recent fiscal years including the current fiscal year is required to calculate the standard deviations of recursion value. For Ashton et al. (2003), the starting fiscal year is also four years later than O95 when the standard deviation of recursion value is available.

[6] Non-missing residual income per share is required in O95, reducing its size sample to 28,476; non-missing residual operating income per share and non-negative net operating assets per share are required in FO95, reducing its sample size to 25,871; non-missing cash investments per share, residual operating income per share and non-negative net operating assets per share are required in FO96, reducing its sample size to 21,342. For the Hwang and Sohn (2010) adjustment of the linear models, the according equity value estimated in the linear models and the according standard deviation of the recursion value are required. This reduces the sample size to 15,423 observations for HSO95, 13,386 observations for HSFO95, and 9,899 observations for HSFO96. For Ashton et al. (2003), the estimated equity value and standard deviation of the recursion value of O95 are required, which reduces its sample size to 15,423 observations. For Zhang (2000), non-missing cash investments per share is required, which reduces its observations to 23,179.

the latter are likely to be transitory items with very low persistence (Begley and Feltham, 2002). The cost of equity used in all the models is a cross-sectional constant and is calculated annually as the sum of the yield on UK 10 year treasury stock plus an average equity risk premium rate of 5%. A constant cross-sectional cost of equity is widely assumed in the literature (Ahmed et al. 2002; Choi et al. 2006), while the choice of an equity risk premium rate of 5% is based on previous UK findings of O'Hanlon and Steele (2000) and is consistent with standard texts such as Copeland, Koller, and Murrin (2000). Descriptive statistics for all variables are summarized in Table 3 for the period 1990-2015.

Table 2. Summary of Sample Selection

Panel A Summary of Sample Period for Each Model

	O95	**FO95**	**FO96**	**HSO95**	**HSFO95**	**HSFO96**	**Ashton et al. (2003)**	**Zhang (2000)**	**Comparison of All Models**
Sample Period	1990-2015	1990-2015	1995-2015	1994-2015	1994-2015	1999-2015	1994-2015	1995-2015	1999-2015
Variable Availability	Main accounting variables	Main accounting variables	Cash flow variables	Standard deviation of O95 recursion value	Standard deviation of FO95 recursion value	Standard deviation of FO96 recursion value	Standard deviation of O95 recursion value	Cash flow variables	All available variables

Panel B Summary of Sample Selection Procedure

	O95	**FO95**	**FO96**	**HSO95**	**HSFO95**	**HSFO96**	**Ashton et al. (2003)**	**Zhang (2000)**
Firm-year observations after deleting missing EPS, BVPS, MVPS and negative BV	35970	35970	35970	35970	35970	35970	35970	35970
Firm-year observations after deleting those which have the top and bottom 1 percent of PB or PE ratio	34589	34589	34589	34589	34589	34589	34589	34589
Firm-year observations after deleting missing variables in calculating the estimated equity value	28476	25871	21342	15423	13386	9899	15423	23179
Firm-year observations for comparison of the equity value of different models	9768	9768	9768	9768	9768	9768	9768	9768

Notes: Panel A presents the individual sample period for each model. The 'Variable Availability' presented in the second row in Panel A shows the variable that was only available since the start year of the sample period in each model. Panel B summarizes the sample selection procedure. The resulting final sample for comparison across all models contains 1,624 firms and 9,768 observations from fiscal year 1999 to 2015. EPS, BVPS, MVPS, BV, PB and PE respectively represent earning per share, book value per share, market value per share, book value of equity, market to book and market to earnings.

Table 3. Descriptive Statistics of Variables

Variable	Q1	Median	Q3	Mean	SD
Market Value	0.5	1.55	4.1	6.01	51.52
Book Value	0.41	0.97	2.13	3.96	38.36
Earnings	-0.00	0.08	0.24	-0.55	58.86
Cash Investments	0.01	0.06	0.22	0.3	1.73
Cash Receipts	0.02	0.14	0.37	0.45	5.59
Residual Income	-0.07	0.00	0.1	-1.06	73.2
Net Operating Assets	0.42	1.16	2.94	5.19	47.16
Operating Income	0.01	0.1	0.3	-0.45	58.39
Residual Operating Income	-0.07	0.00	0.09	-1.07	73.19
P/B Ratio	0.85	1.51	2.74	2.35	3.13
P/E Ratio	-0.19	11.38	19.48	12.04	31.87

Notes: Table 3 presents descriptive statistics of the variables, which are stated on a per share basis for the period of 1999-2015.

4. Results

4.1 LID Parameters and Implied Valuation Multiples

Table 4 presents the mean of 26 annual estimates of cost of equity, LID parameters and valuation multiples for the O95 model. ω_1 is the residual income persistence parameter from estimating Eq. (1) in pooled time-series cross-sectional regressions using all available data back to 1980. The mean residual income persistence parameter ω_1 in O95 is 0.442 with a significant Fama-MacBeth *t* value with Newey-West adjustment of 8.75. The result is close to that based on US data in Dechow et al. (1999) and Choi et al. (2006). The mean residual income valuation multiple β of 0.755 is the average of the annual β multiples based on the annual estimates of R and ω_1. Annual estimates of ω_1 and β are significantly positive for all 26 years.

Panel A in Table 5 reports the results of LID parameters and valuation multiples in FO95. With a mean of 0.473, the residual operating income consistent parameter ω_{11} is significantly positive for all years of estimates. While the average conservatism parameter on net operating assets ω_{12} of -0.019 is statistically insignificant, this parameter is significantly negative in most years. This finding is contrary to the assumption of conservatism in FO95 but is also reported by Stober (1996), Dechow et al. (1999), Myers (1999) and others.[7] Panel A also shows that ω_{22} is about 0.848, which is below the lower boundary of 1 suggested by FO95.[8] It is significantly positive from 1993 to 2015 and the significance increases as time goes on. For the 26 years of valuation multiples, β_1 is positive in all cases but β_2 is negative in 16 years. The negative sign of β_2 is due to the negative LID parameter ω_{12} and results in a lower estimated equity value for the FO95 model.

LID parameters and valuation multiples for FO96 are reported in Panel B in Table 5. The mean operating cash flow persistence parameter γ_1 of 0.584 is strongly significant as predicted. The mean κ_1 of 0.262 is significantly positive and indicates the positive impact of current capital investments on next period operating cash flow. ω_1 represents one plus growth in capital investments and a statistically significant mean of 0.725 indicates mean reversion in expected capital investment in the sample years. δ_1 represents the depreciation parameter and a statistically significant mean of 0.755 indicates an average depreciation rate of 0.245 in net operating assets. For all 21 years (i.e., 1994-2015), these parameters are also significantly positive. Turning to the valuation multiples, in all 21 sample years, the implied valuation multiple, β_1 is positive as expected. On the other hand, inconsistent with expected conservatism in equity valuation, β_2 and β_3 are negative in all years. The negative β_2 on net operating assets indicates under-depreciation rather than over-depreciation of net operating assets i.e., over-depreciation due to conservative

[7] According to Ahmed et al. (2000), the ability of ω_{12} to capture accounting conservatism is related to the sustainable profitability of the firm. Firms with negative ω_{12} generally have significantly lower return on equity, growth, persistence, size, and fixed asset intensity relative to firms with positive ω_{12}.

[8] An alternative test has been conducted using book value per share instead of net operating assets per share. The result ω_{22} is around 0.9, which is also lower than 1.

accounting practice requires that the depreciation rate $(1-\delta_1)$ (equal to 0.245 in our results) exceeds the decline rate in cash receipts $(1-\gamma_1)$ (equal to 0.416 in our results). The negative sign of β_3 is due to lack of support for the positive NPV condition in our results i.e., $\Phi\kappa_1 - 1 = \frac{\kappa_1}{R-\gamma_1} - 1$ is generally less than zero in our analysis consistent with previous U.S findings in Myers (1999) and Ahmed et al. (2000).[9] The negative β_2 and β_3 reduce the estimated equity value for the FO96 model.

Table 4. Mean of Yearly Estimates of Cost of Equity, LID Parameters and Valuation Multiples in O95

	R	$\boldsymbol{\omega_0}$	$\boldsymbol{\omega_1}$	$\boldsymbol{R^2(\omega_1)}$	$\boldsymbol{\beta}$
Estimations	1.105	-0.011	0.442***	0.331	0.755
		(-1.38)	(8.75)		
Number of positive estimates			26		26
Number of negative estimates			0		0

Notes: Table 4 shows the mean of the 26 yearly estimates of cost of equity, LID parameters and valuation multiples from 1990 to 2015 in model (V1): $V1_t = B_t + \beta x_t^a$. R is one plus the cost of equity. ω_1 is LID parameter estimated from Eq. (4.1), in pooled time-series cross-sectional regressions using all available data back to 1980. Following Choi et al. (2006), the most extreme 1% of the deflated variables are winsorized in estimating the LID parameters. *t* values (in parentheses) are based on Fama-MacBeth standard errors with Newey-West adjustments. The superscripts *** indicate significance at the 1% level.

The third and fourth rows include number of positive or negative estimates within the 26 yearly estimates. In the case of LID parameters, an estimate is designated as positive or negative only if it is significantly different from zero at the 10 percent level.

[9] With a US sample of 22 years, Myers (1999) found that residual income is positively correlated with lagged residual income, but negatively correlated with lagged book value and lagged capital expenditures. The lagged capital expenditures has a median coefficient of -0.048 in forecasting the residual income. The negative sign of capital expenditures in predicting residual operating income is also found in Ahmed et al. (2000), with a cross-sectional firm specific coefficient of -0.064. For capital investment to capture conservatism, the LID parameters have to satisfy the condition $\kappa_1 > R - \gamma_1$. As a result, we found that κ_1 is lower than $R - \gamma_1$, which indicates that average capital investments in the sample years are negative Net Present Value (NPV) investments as suggested in Myers (1999).

Table 5. Mean of Yearly Estimates of Cost of Equity, LID Parameters and Valuation Multiples in FO95 and FO96

Panel A FO95

	R	ω_{10}	ω_{11}	ω_{12}	$R^2(\omega_{12}, \omega_{12})$	ω_{20}	ω_{22}	$R^2(\omega_{22})$	β_1	β_2
Estimations	1.105	0.019	0.473***	-0.019	0.396	0.282***	0.848***	0.618	1.121	-0.197
		(1.21)	(6.89)	(-1.47)		(3.47)	(7.69)			
Number of positive estimates			26	0			23		26	0
Number of negative estimates			0	16			0		0	16

Panel B FO96

	R	γ_0	γ_1	κ_1	R^2 (γ_1, κ_1)	ω_0	ω_1	R^2 (ω_1)	δ_0	δ_1	R^2 (δ_1)	β_1	β_2	β_3
Estimations	1.096	0.033 ***	0.584 ***	0.262 ***	0.390	0.053 ***	0.725 ***	0.449	0.235 ***	0.755 ***	0.611	1.026	-0.290	-1.357
		(3.43)	(18.77)	(8.12)		(9.02)	(29.43)		(3.53)	(8.18)				
Number of positive estimates			21	21			21			21		21	0	0
Number of negative estimates			0	0			0			0		0	21	21

Notes: Panel A shows the mean of the 26 yearly estimates of the LID parameters and valuation multiples from 1990 to 2015 in model (V2): $V2_t = B_t + \beta_1 ox_t^a + \beta_2 OA_t$. R is one plus the cost of equity. ω_{11} and ω_{12} are LID parameters estimated from Eq. (4.2) and ω_{22} is estimated from Eq. (4.3).

Panel B shows the mean of the 21 yearly estimates for the items from 1995 to 2015 in model (V3): $V3_t = B_t + \beta_1 ox_t^a + \beta_2 OA_{t-1} + \beta_3 CI_t$. γ and κ are LID parameters estimated from Eq. (4.4). ω is the LID parameter estimated from Eq. (4.5) and δ is the LID parameter estimated from Eq. (4.6)

In both Panels, LID parameters are regressed in pooled time-series cross-sectional regressions using all available data back to 1980. The most extreme 1% of the deflated variables are winsorized in estimating the LID parameters. The t values (in parentheses) are based on Fama-MacBeth standard errors with Newey-West adjustments. The superscripts *** indicate significance at the 1% level. The third and fourth rows in each panel includes the number of positive or negative yearly estimates. In the case of LID parameters, an estimate is designated as positive or negative only if it is significantly different from zero at the 10 percent level.

4.2 Value Estimates with Joint Sample Dataset and the Role of Real Options Information

Table 6 reports the value estimates across all the models based on a joint sample dataset from 1999 to 2015.[10] The estimated equity value of the OFF is shown in Panel A. The median of V1 is around 0.96, compared with 0.66 for V2 and 0.36 for V3. The estimated equity value for O95 is higher than FO95 and FO96, but it is still significantly lower than MVPS (1.55) and lies close to the median estimates of BVPS (0.97). This further suggests that the median present value of residual income in O95 is close to zero.

The value estimates of the HSOFF are shown in Panel B. Within the three adjusted models, HSO95 has the highest median of value estimates of 1.17, which is lower than the median estimate of MVPS but higher than the median estimate of O95. HSO95 and HSFO96 have higher median estimates of equity value compared with FO95 and FO96, which suggests that the adjusted models with real options have higher estimated equity values than the linear models before adjustment. In terms of the standard deviation of recursion value, FO96 is the most volatile with a median of 0.74, compared with FO95 (0.32) and O95 (0.22). According to Hwang and Sohn (2010), the equity value can be represented as either the adaptation value plus the call option value, or the recursion value plus the put option value. All three models have a median call option value around 0, but a median put option value which is significantly larger than 0. This suggests that OFF highly underestimates the market equity value. Within the three models, HSFO96 has the largest median of put option value around 0.49, followed by the 0.25 median of FO95 and the 0.15 median of O95. While the recursion value is low, put option value represents a large proportion of the estimated equity value in the Hwang and Sohn (2010) adjustment models. It again explains the higher value estimates of HSOFF compared to the original OFF.

[10] To provide detailed insights for each model's performance, we also examine value estimates for each model based on full coverage of its individual sample dataset. The median value estimates for each model are close to those based on the joint sample dataset.

Panel C presents the estimated equity value, risk parameter, RB ratio (Recursion value/Book value ratio) and put option value of Ashton et al. (2003). The median estimated equity value is about 1.54, which of all the models is the closest to MVPS. A median RB ratio of around 1 is consistent with the median recursion value of O95 close to the book value of equity. The median adaptation option value is around 0.58, which is higher than the put option value in the Hwang and Sohn (2010) adjustments in Panel B. This provide insights into the superior estimation accuracy of Ashton et al. (2003), since the adaptation option contributes a large proportion of the estimated equity value.

Panel D reports the summarized value estimates of Zhang (2000). The median estimated equity value is 1.45, which is the second closest estimate to market value. The portion $\frac{X_t}{r}$ provides a median of 0.97, thereby indicating that the value generated from continuing current business operations contributes a significant proportion of the estimated equity value in Zhang (2000). Further details indicate that nearly half the observations in the sample contain a put option, and the other half observations have a call option. The result of this is that the medians of $P(ROE_t)B_t$ and $C(ROE_t)G$ are both zero. The mean of the put option value $P(ROE_t)B_t$ is 3.99, which is much larger than the mean of the call option value $(ROE_t)G$ equal to 0.21. This suggests that the put option in the half of observations with a low ROE is more valuable than the call option in the other half of observations which have a high ROE.

To summarize, value estimates of OFF indicate undervaluation bias. HSOFF provides respectively higher value estimates compared with the linear models before real option adjustments. Ashton et al. (2003) and Zhang (2000) respectively provide the two most accurate valuation estimates in comparison to MVPS among all the models. It can be inferred from Table 6 that real options do contribute significantly to the equity value. The superiority (value estimates closer to market value) of the non-linear models is due to incorporation of real options information.

Table 6. Value Estimates of Models with Joint Sample Dataset

	N	**Mean**	**S.D.**	**0.25**	**Median**	**0.75**
BVPS	9768	3.96	38.36	0.41	0.97	2.13
MVPS	9768	6.01	51.52	0.5	1.55	4.1
Panel A Ohlson and Feltham framework						
O95	$V1_t = B_t + \beta x_t^a$					
V1	9768	3.22	39.93	0.37	0.96	2.16
FO95	$V2_t = B_t + \beta_1 ox_t^a + \beta_2 OA_t$					
V2	9768	1.9	52.56	0.23	0.66	1.5
FO96	$V3_t = B_t + \beta_1 ox_t^a + \beta_2 OA_{t-1} + \beta_3 CI_t$					
V3	9768	0.43	93.32	0.05	0.36	0.96
Panel B Ohlson and Feltham framework with Hwang and Sohn (2010) adjustment						
HSO95	$V4a_t = AV_t + CO_t^{O95}$, $CO_t^{O95} = V_t^{O95} * N(d_1) - B_t * e^{-RF_t * T} * N(d_2)$					
V4a	9768	4.58	42.43	0.48	1.17	2.61
SDV^{O95}	9768	1.23	20.11	0.11	0.22	0.49
CO^{O95}	9768	0.63	6.7	0	0.05	0.41
PO^{O95}	9768	1.37	51.24	0.04	0.15	0.41
HSFO95	$V4b_t = AV_t + CO_t^{FO95}$, $CO_t^{FO95} = V_t^{FO95} * N(d_1) - B_t * e^{-RF_t * T} * N(d_2)$					
V4b	9768	4.01	38.41	0.42	1	2.2
SDV^{FO95}	9768	2.79	44.69	0.16	0.32	0.85
CO^{FO95}	9768	0.06	0.7	0	0	0
PO^{FO95}	9768	2.11	72.49	0.09	0.25	0.66
HSFO96	$V4c_t = AV_t + CO_t^{FO96}$, $CO_t^{FO96} = V_t^{FO96} * N(d_1) - B_t * e^{-RF_t * T} * N(d_2)$					
V4c	9768	3.98	38.36	0.42	0.99	2.17
SDV^{FO96}	9768	12.72	430.61	0.3	0.74	1.87
CO^{FO96}	9768	0.02	0.19	0	0	0
PO^{FO96}	9768	3.55	116.18	0.17	0.49	1.28
Panel C Ashton et al. (2003)						
Ashton et al. (2003)	$V5 = B[h + \frac{1}{2}\int_{1}^{1} exp\left(\frac{-2\theta Bh}{1+z}\right)dz] = B[\sum_{m=0}^{\infty} \alpha_m L_m(h)]$					
V5	9768	5.89	51.49	0.64	1.54	3.41
h	9768	0.89	1.24	0.93	1	1.06
θ	9768	32.27	331.43	0.73	3.55	14.43
AO	9768	2.67	53.38	0.26	0.58	1.26
Panel D Zhang (2000)						
Zhang (2000)	$V6 = \frac{1}{r}X_t + P(ROE_t)B_t + C(ROE_t)G$					
V6	9768	5.76	39.69	0.49	1.45	3.75
$\frac{X_t}{r}$	9768	1.57	106.95	-0.01	0.97	3.01
$P(ROE_t)B_t$	9768	3.99	116.38	0	0	0.6
$C(ROE_t)G$	9768	0.21	1	0	0	0.09

Notes: Table 6 shows the value estimates in each model based on a joint sample dataset from 1999 to 2015.

Panel A reports the estimated equity value of model (V1), (V2) and (V3), which are models of OFF. Panel B presents the estimated equity value, standard deviation of recursion value (SDV), call option value (CO), put option value (PO) and PVE of model (V4a), (V4b) and (V4c), which are models of HSOFF. Panel C provides the estimated equity value, put option value and PVE of model (V5), which is the model of Ashton et al. (2003). Panel D reports the estimated equity value and the specific proportion of the equity value in model (V6), which is the model of Zhang (2000).

4.3 Out of Sample Valuation Performance

4.3.1 Forecast Bias, Accuracy and Explainability

In this section, we provide out of sample forecast bias, accuracy and explainability results. This helps address our research question on the role of real options information in accounting-based equity valuation. To better illustrate the comparison of linear versus non-linear real options models, we arrange each of the following tables into four panels. Panel A includes O95 and the real option models built upon Ohlson (1995): HSO95 and Ashton et al. (2003). Panel B presents FO95 and HSFO95, which are models based on Feltham and Ohlson (1995). Panel C contains FO96 and HSFO96, which are models based on Feltham and Ohlson (1996). Panel D contains the real options model of Zhang (2000). Since Zhang's model uses capitalized earnings to capture the firm's value with stable operation, we also provide the capitalized earnings model in Panel D as a benchmark for comparison.

Table 7 reports the Proportional Valuation Error (PVE) across all models based on a joint sample dataset. For the OFF (O95, FO95 and FO96), all three linear models underestimate the market equity value. The median of PVE1, PVE2 and PVE3 are around -0.37, -0.57 and -0.75. FO96 has the greatest valuation bias among all the models, which indicates half of its estimated equity value undervalues more than 75% of the market equity value. The underestimation for FO95 and FO96 is unsurprising, given the negative valuation coefficients on net operating assets and capital investments. It can also be observed in Table 7 that the valuation bias of OFF is significantly reduced after the Hwang and Sohn (2010) adjustment. More specifically, the mean and median PVE increase significantly towards 0, with a median PVE of -0.21 for HSO95, -0.32 for HSFO95 and -0.33 for HSFO96. According to Hwang and Sohn (2010), the estimated equity value can be viewed as the recursion value plus the put option value. Thus, the reduced forecast bias is due to the put option effect. Compared with O95, Ashton et al. (2003) provides a better median PVE of

0.06. Nonetheless, the mean PVE of Ashton et al. (2003) is 0.4, which reveals right skewness and indicates a few extreme high valuations. Among all the models, Zhang (2000) provides the lowest valuation bias with a median PVE of -0.04. The relative capitalized earnings model presents a median PVE of -0.43. This further indicates that the call and put options components are crucial and that they make an important contribution to estimated equity value in the Zhang (2000) model.

In summary, HSOFF reduces the downward valuation bias of the original linear models. The real options model of Zhang (2000) has the lowest valuation bias, followed by Ashton et al. (2003). The abandonment option in Hwang and Sohn (2010) and Zhang (2000) provides an insurance policy that pays off if the firm performs below expectations. The adaption option of Ashton et al. (2003) enables the firm to convert its resources to alternative and potentially more profitable uses. The growth option in Zhang (2000) allows the firm to invest and expand the scale of its operations when it faces profitable projects. These results therefore confirm that the real options components in the non-linear models contribute significantly to equity value.

Table 7. Comparison of Proportional Valuation Errors

	Model	N	Mean	Significance Level Mean Difference=0	Median	Significance Level Median Difference=0
Panel A Models based on Ohlson (1995)						
	$V1_t = B_t + \beta x_t^a$					
PVE1	O95	9768	-0.21	0	-0.37	0
	$V4a_t = AV_t + CO_t^{O95}$, $CO_t^{O95} = V_t^{O95} * N(d_1) - B_t * e^{-RF_t*T} * N(d_2)$					
PVE4a	HSO95	9768	0.03	0	-0.21	0
	$V5 = B[h + \frac{1}{2}\int_1^1 exp\left(\frac{-2\theta Bh}{1+z}\right) dz] = B[\sum_{m=0}^{\infty} \alpha_m \, L_m\,(h)]$					
PVE5	Ashton et al. (2003)	9768	0.4	0	0.06	0
Panel B Models based on Feltham and Ohlson (1995)						
	$V2_t = B_t + \beta_1 ox_t^a + \beta_2 OA_t$					
PVE2	FO95	9768	-0.5	0	-0.57	0
	$V4b_t = AV_t + CO_t^{FO95}$, $CO_t^{FO95} = V_t^{FO95} * N(d_1) - B_t * e^{-RF_t*T} * N(d_2)$					
PVE4b	HSFO95	9768	-0.08	0	-0.32	0
Panel C Models based on Feltham and Ohlson (1996)						
	$V3_t = B_t + \beta_1 ox_t^a + \beta_2 OA_{t-1} + \beta_3 CI_t$					
PVE3	FO96	9768	-0.86	0	-0.75	0
	$V4c_t = AV_t + CO_t^{FO96}$, $CO_t^{FO96} = V_t^{FO96} * N(d_1) - B_t * e^{-RF_t*T} * N(d_2)$					
PVE4c	HSFO96	9768	-0.09	0	-0.33	0
Panel D Models based on Capitalized Earnings						
	$Vce_t = \frac{1}{r} X_t$					
	Capitalized Earnings Model	9768	-1.79	0	-0.43	0
	$V6 = \frac{1}{r} X_t + P(ROE_t)B_t + C(ROE_t)G$					
PVE6	Zhang (2000)	9768	0.28	0	-0.04	0

Notes: Table 7 shows the proportional valuation errors (PVE) of all models based on a joint sample of 9768 observations from 1999 to 2015. Panel A includes O95, HSO95 and Ashton et al. (2003). Panel B presents FO95 and HSFO95. Panel C contains FO96 and HSFO96. Panel D contains the Capitalized Earnings Model in addition to Zhang (2000).

PVE measures the forecast bias: $\mathrm{PVE_t} = (\mathrm{MV_t^{Est}} - \mathrm{MV_t^{Act}})/\mathrm{MV_t^{Act}}$, where $\mathrm{MV_t^{Est}}$ is the estimated equity value of each model and $\mathrm{MV_t^{Act}}$ is the market equity value. Significance Level Mean Difference = 0 and Significance Level Median Difference =0 represent the significance level associated with the t-statistics of (sign rank test) of whether the mean and median proportional valuation error equals zero.

Table 8 presents the Absolute Proportional Valuation Error (APVE) and valuation central tendency of each model, which respectively measure the magnitude and frequency of forecast accuracy. For the linear models, O95 has a median APVE around 0.52, substantially lower than the 0.61 median of FO95 and 0.76 median of FO96. It is also evident from Table 8 that Hwang and Sohn (2010) adjustment increases the forecast accuracy of the linear models. Specifically, HSO95 has a median APVE of 0.48, while HSFO95 and HSFO96 have a median APVE of 0.53. Comparing with HSO95, Ashton et al. (2003) does not show a better forecast accuracy and has the same median APVE (0.52) as O95. This may be due to the extreme estimated values from observations with significantly large standard deviation of recursion value. Compared with the capitalized earrings model, the Zhang (2000) model provides better forecast accuracy and, with a median APVE of 0.42, it outperforms all other models. The valuation central tendency presents similar results regarding the frequency of forecast accuracy. The model with the highest central tendency is Zhang (2000), providing a central tendency of 18.08%. Following Zhang (2000), HSO95 presents a central tendency of 14.52%. Compared with O95, Ashton et al. (2003) show more central tendency (14.06% versus 12.80%). It can also be observed that the central tendency of OFF significantly increases after the Hwang and Sohn (2010) adjustment. As a result, in terms of both the magnitude and frequency of forecast accuracy, Zhang (2000) has the best performance, followed by HSO95. This is explained by real options theory as Zhang (2000) recognizes both the role of abandonment and growth in valuation. The abandonment and growth options provide investors with flexibility to minimize bad losses and maximize future profit. Considering such flexibility in decision making and incorporating real options information in valuation contributes to the model's valuation accuracy.

Table 8. Comparison of Absolute Proportional Valuation Errors

	Model	N	Median	Significance Level Median Difference=0	Central Tendency
Panel A Models based on Ohlson (1995)					
	$V1_t = B_t + \beta x_t^a$				
APVE1	O95	9768	0.52	0	12.80%
	$V4a_t = AV_t + CO_t^{O95}$, $CO_t^{O95} = V_t^{O95} * N(d_1) - B_t * e^{-RF_t*T} * N(d_2)$				
APVE4a	HSO95	9768	0.48	0	14.52%
	$V5 = B[h + \frac{1}{2}\int_{1}^{1} exp\left(\frac{-2\theta Bh}{1+z}\right) dz] = B[\sum_{m=0}^{\infty} \alpha_m \, L_m\,(h)]$				
APVE5	Ashton et al. (2003)	9768	0.52	0	14.06%
Panel B Models based on Feltham and Ohlson (1995)					
	$V2_t = B_t + \beta_1 ox_t^a + \beta_2 OA_t$				
APVE2	FO95	9768	0.61	0	9.55%
	$V4b_t = AV_t + CO_t^{FO95}$, $CO_t^{FO95} = V_t^{FO95} * N(d_1) - B_t * e^{-RF_t*T} * N(d_2)$				
APVE4b	HSFO95	9768	0.53	0	12.80%
Panel C Models based on Feltham and Ohlson (1996)					
	$V3_t = B_t + \beta_1 ox_t^a + \beta_2 OA_{t-1} + \beta_3 CI_t$				
APVE3	FO96	9768	0.76	0	5.09%
	$V4c_t = AV_t + CO_t^{FO96}$, $CO_t^{FO96} = V_t^{FO96} * N(d_1) - B_t * e^{-RF_t*T} * N(d_2)$				
APVE4c	HSFO96	9768	0.53	0	12.73%
Panel D Models based on Capitalized Earnings					
	$Vce_t = \frac{1}{r} X_t$				
	Capitalized Earnings Model	9768	0.56	0	12.51%
	$V6 = \frac{1}{r} X_t + P(ROE_t)B_t + C(ROE_t)G$				
APVE6	Zhang (2000)	9768	0.42	0	18.08%

Notes: Table 8 shows the absolute proportional valuation errors (APVE) of all models based on a joint sample of 9768 observations from 1999 to 2015. Panel A includes O95, HSO95 and Ashton et al. (2003). Panel B presents FO95 and HSFO95. Panel C contains FO96 and HSFO96. Panel D contains the Capitalized Earnings Model in addition to Zhang (2000).

APVE measures the forecast accuracy: $\mathrm{APVE_t} = |\mathrm{MV_t^{Est}} - \mathrm{MV_t^{Act}}|/\mathrm{MV_t^{Act}}$, where $\mathrm{MV_t^{Est}}$ is the estimated equity value of each model and $\mathrm{MV_t^{Act}}$ is the market equity value. Significance Level Median Difference =0 represent the significance level associated with the t-statistics of (sign rank test) of whether the median valuation proportional error equals zero. The measure of central tendency indicates the percentage of observations with the value estimates which lie within15% of the observed security price.

Table 9 reports the results of 17 pooled time-series cross-sectional regressions of contemporaneous market equity values on estimated equity values from each model, based on a joint sample of 9,768 observations over the 17 year time period 1999-2015. The coefficients for all models are significant at the 0.01 level based on Fama-MacBeth standard errors with Newey-West adjustments. For the OFF, O95 has the highest explanatory power, while FO96 performs substantially worse than the other two models. The coefficients further indicate that estimated equity values for O95 are generally larger than for both FO95 and FO96. It is evident from the table that Hwang and Sohn (2010) adjustment improves the explainability of OFF. More specifically, the explainability of HSO95, HSFO95, and HSFO96 are 0.718, 0.689, and 0.690, compared to 0.679, 0.631, and 0.489 for O95, FO95 and FO96, respectively. The coefficients in the Hwang and Sohn (2010) adjustment models are smaller than the coefficients in the linear models, which suggests that the HSOFF provides larger estimated equity values than the original OFF. Ashton et al. (2003) outperforms O95 but provides a lower explanatory power than HSO95 with a 0.69 R square. Among all the models, Zhang (2000) has the largest average R square of 0.733. Compared with the poor explanatory power of the capitalized earnings model (with an R square of 0.32), the superior explainablity in Zhang (2000) demonstrates the importance of the growth and abandonment option value in equity valuation. Overall, non-linear real option models outperform linear models in terms of explainability. Zhang's (2000) model provides the best performance in explanatory power, followed by HSO95 and Ashton et al. (2003). The superiority of non-linear models in explainability is due to recognizing the real options effects associated with a firm's ability to abandon, adapt or expand its existing operating activities. Neglecting such effects will lead to valuation errors and inevitably reduce the model's explanatory power for market value.

Table 9. Regressions of Contemporaneous Market Equity Value on Estimated Equity Value

	N	Time Period	Average R Square	Coefficient	Standard Error	T statistics	P>t
Panel A Models based on Ohlson (1995)							
V1 O95	9768	17	0.679	1.289	0.250	5.160	0.000
V4a HSO95	9768	17	0.718	1.121	0.159	7.030	0.000
V5 Ashton et al. (2003)	9768	17	0.690	0.932	0.154	6.040	0.000
Panel B Models based on Feltham and Ohlson (1995)							
V2 FO95	9768	17	0.631	1.688	0.364	4.630	0.000
V4b HSFO95	9768	17	0.689	1.336	0.267	5.010	0.000
Panel C Models based on Feltham and Ohlson (1996)							
V3 FO96	9768	17	0.489	1.581	0.420	3.770	0.000
V4c HSFO96	9768	17	0.690	1.355	0.262	5.160	0.000
Panel D Models based on Capitalized Earnings							
Capitalized Earnings Model	9768	17	0.320	0.588	0.164	3.590	0.002
V6 Zhang (2000)	9768	17	0.733	0.948	0.116	8.200	0.000

Notes: Table 9 shows the results of pooled time-series cross-sectional regressions of contemporaneous market equity values on estimated equity values, based on a joint sample of 9768 observations and a time period of 17 years. The average R squares of the regressions, regression coefficients, Fama-MacBeth standard errors with Newey-West adjustments are provided for each model. P>t reports the significance level of the coefficients.

Panel A includes O95, HSO95 and Ashton et al. (2003). Panel B presents FO95 and HSFO95. Panel C contains FO96 and HSFO96. Panel D contains Capitalized Earnings Model in addition to Zhang (2000). The regression formula is: $\mathrm{MV}_{j,t}^{Act} = \lambda_0 + \lambda_1 \mathrm{MV}_{j,t}^{Est} + \varepsilon_t$, where $\mathrm{MV}_{j,t}^{Act}$ is the market equity value for firm j at time t, and $\mathrm{MV}_{j,t}^{Est}$ is the estimated equity value of firm j at time t for different models.

4.3.2 Explainability of Real Options Components

To provide further insights concerning the superior explainability of non-linear real options valuation models, regressions of contemporaneous market equity value on different components of value estimates are conducted. More specifically, the estimated equity value of Hwang and Sohn (2010) adjustment models are decomposed into relevant recursion value and put option value; the estimated equity value of Ashton et al. (2003) is decomposed into recursion value and adaptation value; and the estimated equity value of Zhang (2000) is decomposed into capitalized earnings value, put option value and call option value. To mitigate the influence of extreme outliers of different components, the most extreme one percent of the variables are winsorized in the regressions of contemporaneous market equity value on different components of value estimates.[11]

The regression results are illustrated in Table 10. It is evident from the table that the Hwang and Sohn (2010) adjustment models significantly increase the explanatory power of the original OFF. The Fama-MacBeth average R square of HSO95, HSFO95 and HSFO96 are 0.700, 0.670 and 0.653, compared with the 0.649, 0.623 and 0.455 R square of the original O95, FO95 and FO96. The incremental explanatory power is due to the relevant put options which are all significant at 0.01 level. In terms of Zhang (2000), the coefficients of capitalized earnings component, put option component and call option component are all close to one. The coefficients of capitalized earnings component and put option component are significant at the 0.01 level.[12] With a Fama-MacBeth average R square of 0.701, Zhang (2000) provides the highest explanatory power among all the models. The only model failing in this test is the Ashton et al. (2003) model. The theoretical adaptation option of Ashton et al. (2003) is mathematically developed through the aggregation

[11] The winsorizing process results in different regressed coefficients of O95, FO95, and FO96 (first row of Panel A, B and C) between Tables 10 and 9. These results are identical without the winsorizing process.

[12] While relaxing the Fama-Macbeth standard error without Newey-West adjustment, the coefficient of call option is also significant at the 0.10 level.

theory, which identifies the recursion value of equity as functionally proportional to its adaptation value. Thus, the adaptation option in Ashton et al. (2003) is highly correlated with the Ohlson (1995) recursion value and fails in the regression. To sum up, the above decomposition tests of the estimated equity value examine and highlight the incremental explanatory power of the option components in non-linear valuation. The findings again highlight the superior explainability of the non-linear models due to the inclusion of real options components.

Table 10. Regressions of Contemporaneous Market Value on Components of Value Estimates

Model	Observations	α_0	α_1	α_2	α_3	Average R Square
Panel A Models based on Ohlson (1995)						
			$MV_t = \alpha_0 + \alpha_1 V1_t$			
O95	9768	1.073*** (9.49)	**1.300*** ** (30.45)			0.649
			$MV_t = \alpha_0 + \alpha_1 V1_t + \alpha_2 PO_t^{O95}$			
HSO95	9768	0.913*** (6.25)	**1.012*** ** (10.94)	**1.589*** ** (4.59)		0.700
			$MV_t = \alpha_0 + \alpha_1 V1_t + \alpha_2 AO_t$			
Ashton et al. (2003)	9768	1.014*** (9.22)	**1.304*** ** (3.08)	-0.017 (-0.03)		0.690
Panel B Models based on Feltham and Ohlson (1995)						
			$MV_t = \alpha_0 + \alpha_1 V2_t$			
FO95	9768	1.206*** (17.73)	**1.800*** ** (30.33)			0.623
			$MV_t = \alpha_0 + \alpha_1 V2_t + \alpha_2 PO_t^{FO95}$			
HSFO95	9768	0.958*** (7.99)	**1.430*** ** (10.74)	**0.887*** ** (3.68)		0.670
Panel C Models based on Feltham and Ohlson (1996)						
			$MV_t = \alpha_0 + \alpha_1 V3_t$			
FO96	9768	2.168*** (13.96)	**2.071*** ** (20.20)			0.455
			$MV_t = \alpha_0 + \alpha_1 V3_t + \alpha_2 PO_t^{FO96}$			
HSFO96	9768	1.065*** (6.92)	**1.486*** ** (13.23)	**1.110*** ** (13.42)		0.653
Panel D Models based on Capitalized Earnings						
			$MV_t = \alpha_0 + \alpha_1 CE_t$			

Table 10. (Continued)

Capitalized Earnings Model	9768	2.320*** (18.02)	**0.632***** (15.48)			0.362
		$MV_t = \alpha_0 + \alpha_1 CE_t + \alpha_2 CO_t^{Zhang} + \alpha_3 PO_t^{Zhang}$				
Zhang (2000)	9768	0.553*** (5.17)	**0.800***** (19.65)	0.988 (1.47)	**1.145***** (12.98)	0.701

Notes: Table 10 shows the pooled time-series cross-sectional regressions results of contemporaneous market equity value on components of value estimates in various models based on a joint sample of 9,768 observations and a time period of 17 years. The average R squares of the regressions, regression coefficients, and Fama-MacBeth t values with Newey-West adjustments are provided for each model. The superscripts *** indicate significance at the 1% level.

4.4 Subsample Analysis on Model Performance

The results presented in the previous section reveal the dominance of non-linear real options models in forecast accuracy and explainability. The models with Hwang and Sohn (2010) adjustment, and Ashton et al. (2003) mainly emphasize the abandonment option value. As suggested by Burgstahler and Dichev (1997), Barth et al. (1998), and Herath et al. (2015), market participants tend to change to focus on the balance sheet (book value of equity) instead of income statement (earnings) when a firm gradually approaches the situation of making loss. Zhang's (2000) real option model includes both abandonment and growth options. It indicates that a firm's flexibility to grow plays a crucial role in determining the equity value when the firm is achieving high profitability. To further test whether the forecast superiority of non-linear real options models comes from their option characteristics, we divided our joint sample dataset of 9,768 observations into three subsamples. The first subsample includes $N_1 = 2475$ firm-year observation with negative earnings. In other words, losses are made in these observations. It is expected that Hwang and Sohn (2010) adjustment models (HSO95, HSFO95 and HSFO96), Ashton et al. (2003) and Zhang (2000) will reveal higher superiority in this subsample, as a result of the abandonment option. The remaining firm-year observations have positive earnings and they are divided into two subsamples involving $N_2 = 3647$ and $N_3 = 3646$ firm-year observations. The first of these subsamples N_2 comprises firm-year observations with positive but relatively low PE ratios, and the remaining subsample N_3 includes firm-year observations with positive but relatively high PE ratios. The price-to-earnings ratio is frequently used as a measure of growth potential (Siegel, 2013, Herath et al. 2015). As a result, it is assumed that Zhang (2000) will show its superior valuation performance in N_3 by capturing the growth option effect.

Table 11 reports the subsample test results on PVE, APVE and explainability. It is evident that the Hwang and Sohn (2010) adjustment models perform better than the original linear models in all three subsamples. This superiority is more apparent in the negative earnings subsample due to the put option effect. The negative median PVEs of the original OFF increases towards 0.01 after including the value of the abandonment option. Within the Hwang and Sohn (2010) adjustment models, HSO95 always provides the lowest forecast bias. It can be observed in the table that Ashton et al. (2003) does not provide a lower forecast bias in the negative earnings subsample and low positive PE subsample compared with O95. The relatively large median PVEs of Ashton et al. (2003) in such subsamples are significantly positive which indicates overestimation. Thus, the adaptation option value in Ashton et al. (2003) does increase the value estimates in these two subsamples, however, it increases to an extent which suggests over valuation bias. It can be inferred that the generally better performance in forecast bias for Ashton et al. (2003) in the whole sample tests (Section 4.3) mainly comes from its strong performance in the high positive PE subsample. The dissimilar performance of Ashton et al. (2003) in various subsamples may be due to its option duplication characteristics.[13] The option value in Ashton et al. (2003) is theoretically deduced from

[13] In Ashton et al. (2003), the equity value is represented as the recursion value plus the adaptation value. Instead of using the theory of the Black and Scholes equation, Ashton et al. (2003) employ standard no-arbitrage assumptions in conjunction with a continuous time interpretation of the Ohlson (1995) first order vector system of stochastic differential equations with dynamic programming procedures to obtain the equity value. It not only reflects the put option value when the recursion value is low, but also reflects the call option value when the firm is with high profitability. This option characteristic is revealed in the polynomial expansion procedure of Ataullah et al. (2009), which illustrates the non-linearities that arise in equity values because of the real option effects.

solving the equity stochastic differential equation. Though the option is called an adaptation option and, following Burgstahler and Dichev (1997), theoretically focuses on abandonment, the option value in Ashton et al. (2003) is a mixed option value. It simultaneously explains the non-linear part of equity valuation associated with negative earnings and also the growth options that are available to firms (Herath et al. 2015). For the high positive PE subsample, all the models tend to underestimate the market equity value and the level of underestimation is severe for the OFF models even after the Hwang and Sohn (2010) adjustment as the adjustment is focused on abandonment option effects. Thus, Zhang (2000) provides the lowest forecast bias (with a median PVE of -0.30) in this subsample by including the growth option in the valuation, followed by Ashton et al. (2003) with a median PVE of 0.32. It is unsurprising that the Zhang (2000) model displays superior performance in both the negative earnings subsample and high positive PE subsample given its dual option characteristics.

The subsample results on APVE are similar to the results on PVE. The forecast accuracy of the OFF models are significantly increased after the Hwang and Sohn (2010) adjustment in all subsamples. This advantage is most apparent in the negative earnings subsample because the Hwang and Sohn (2010) adjustment significantly reduces the median APVE towards 0.56. Ashton et al. (2003) reveals lower forecast accuracy in negative earnings subsample and low positive PE subsample because of overestimation as discussed in the subsample PVE test. It also presents the second highest forecast accuracy in the high positive PE subsample due to its mixed option characteristics. An interesting finding is that capitalized earnings model provides the best forecast accuracy in the low positive PE subsample suggesting that the role of real options information in valuation highlighted by the results for the Zhang (2000) model could be most important when firms are performing weakly or strongly. It is unsurprising to find that Zhang (2000) presents the lowest APVE in the high positive PE subsample. Compared to the low forecast accuracy of the original OFF models and even the HSOFF models in the high PE subsamples, Zhang (2000) reveals the critical role for growth options in the high growth potential companies.

Table 11 also reports the subsample test on explainability for all the models. Generally, HSOFF models provide higher explanatory power than the OFF models. The only exception is HSFO95 in the high positive PE subsample, which reveals a slightly lower R square (0.746) than FO95 (0.752). The increase in the explanatory power is most obvious in the negative earnings subsample, since the Hwang and Sohn (2010) adjustment accounts for the flexibility to abandon for firms making losses. Ashton et al. (2003) also reflects higher explainability than O95 in the negative earnings subsample and the high positive PE subsample as a result of the adaptation option. Unsurprisingly, Zhang (2000) provides the highest explanatory power in the high positive PE subsample and has relatively high explanatory power in all three subsamples.

Overall, the outcome of the subsample tests confirm our expectations on the role of real options in equity valuation. Compared with the linear OFF models, the HSOFF significantly reduces the forecast bias and increases the forecast accuracy and explainability in the negative earnings subsample, highlighting the important role of the abandonment option in loss-making firms. The Ashton et al. (2003) model also increases the value of estimates compared with O95 by including the adaptation option. Although its overestimation of equity value leads to a relatively high forecast bias and low forecast accuracy in the negative earnings subsample, it performs strongly in the high positive PE subsample. Zhang (2000) performs well in both the negative earnings subsample and high positive PE subsample by including both abandonment and growth options. The large underestimation of equity value by OFF and HSOFF models in the high positive PE subsample compared to the Zhang (2000) model highlights the critical role of growth option information in determining the equity value for high growth potential firms.

Table 11. Subsample Test on Propotional Valuation Error, Absolute Proportional Valuation Error and Explainability

Model	N_1	PVE		APVE	Explainability		N_2	PVE		APVE	Explainability		N_3	PVE		APVE	Explainability	
		Mean	*Mdn*	*Mdn*	*Coeff*	R^2		*Mean*	*Mdn*	*Mdn*	*Coeff*	R^2		*Mean*	*Mdn*	*Mdn*	*Coeff*	R^2
Panel A Models based on Ohlson (1995)																		
	Negative Earnings						*Low Positive PE*						*High Positive PE*					
O95	2475	**-0.22**	**-0.32**	**0.60**	0.457	0.626	3647	0.01	**-0.18**	**0.40**	**1.004**	0.843	3646	**-0.42**	**-0.58**	**0.61**	**2.286**	0.757
					(2.5)						(10.13)						(3.58)	
HSO95	2475	**0.26**	**0.01**	**0.56**	**0.906**	0.752	3647	**0.25**	**0.03**	**0.39**	**0.829**	0.853	3646	**-0.35**	**-0.48**	**0.53**	**2.038**	0.773
					(5.3)						(11.45)						(3.46)	
Ashton et al. (2003)	2475	**0.76**	**0.39**	**0.69**	**0.920**	0.676	3647	**0.62**	**0.30**	**0.45**	**0.641**	0.843	3646	**-0.08**	**-0.32**	**0.5**	**1.456**	0.756
					(4.34)						(10.11)						(3.58)	
Panel B Models based on Feltham and Ohlson (1995)																		
	Negative Earnings						*Low Positive PE*						*High Positive PE*					
FO95	2475	**-0.73**	**-0.63**	**0.69**	0.357	0.524	3647	**-0.25**	**-0.40**	**0.47**	**1.274**	0.815	3646	**-0.59**	**-0.70**	**0.70**	**2.913**	0.752
					(1.84)						(8.64)						(4.14)	
HSFO95	2475	**0.26**	**0.01**	**0.56**	**0.905**	0.752	3647	0.02	**-0.19**	**0.41**	**1.005**	0.834	3646	**-0.41**	**-0.58**	**0.62**	**2.270**	0.746
					(5.3)						(8.83)						(3.56)	
Panel C Models based on Feltham and Ohlson (1996)																		
	Negative Earnings						*Low Positive PE*						*High Positive PE*					
FO96	2475	**-1.51**	**-0.97**	**0.97**	-0.037	0.431	3647	**-0.51**	**-0.59**	**0.61**	**1.417**	0.735	3646	**-0.76**	**-0.81**	**0.81**	**3.144**	0.702
					(-0.18)						(9.16)						(4.66)	
HSFO96	2475	**0.26**	**0.01**	**0.56**	**0.905**	0.752	3647	0.00	**-0.20**	**0.42**	**1.034**	0.832	3646	**-0.41**	**-0.58**	**0.62**	**2.262**	0.742
					(5.3)						(9.61)						(3.54)	
Panel D Models based on Capitalized Earnings																		
	Negative Earnings						*Low Positive PE*						*High Positive PE*					
CEM	2475	**-7.01**	**-2.85**	**2.85**	-0.269	0.540	3647	**0.49**	**0.15**	**0.25**	**0.744**	0.875	3646	**-0.53**	**-0.51**	**0.51**	**2.592**	0.847
					(-2.97)						(13.53)						(4.36)	
Zhang (2000)	2475	**-0.08**	**-0.28**	**0.57**	**1.089**	0.664	3647	**1.02**	**0.44**	**0.45**	**0.579**	0.843	3646	**-0.23**	**-0.30**	**0.35**	**1.674**	0.850
					(6.75)						(13.77)						(4.42)	

Notes: Table 11 provides the results of a subsample test on PVE, APVE and explainability across all models. A joint sample dataset is divided into three subgroups: Negative Earnings, Low positive PE and High positive PE. Numbers in bold indicate significance at the 1% level: sign rank test for PVE and APVE and significance of the coefficients for explainability. The t values (in parentheses) are based on Fama-MacBeth standard errors with Newey-West adjustments.

5. Robustness Checks and Additional Results

5.1 Cost of Equity

Issues regarding cost of equity measurement are an unavoidable problem in empirical accounting research (Callen and Segel, 2005). In this paper, following Choi et al. (2006) and Ahmed et al. (2002), a cross-sectional cost of equity which equals annual average yield of British Government security 10-year nominal par yield plus 5% risk premium is used. Some other research also uses a constant cost of equity such as 12% (Dechow et al. 1999; Begley and Fetlham, 2002; Ashton and Wang, 2013). As a result, we carry out sensitivity tests on the cost of equity based on a constant cost of equity of 8%, 10% and 12%. The main results do not change with different constant costs of equity.

5.2 Different Time to Maturity in the Hwang and Sohn (2010) adjustment

Following Hwang and Sohn (2010) and Sohn (2012), a time to maturity of 5 years is used in the Hwang and Sohn (2010) adjustment of the Ohlson and Feltham framework in this paper. To examine whether the time to maturity affects the main results of the models, a time to maturity of 3 years and 8 years is also considered in alternative tests. The test results show that time to maturity does affect the Hwang and Sohn (2010) adjustment value. As expected, the call option value increases with the time to maturity (also the put option value due to put call parity). Out of the three models, HSFO95 and HSFO96 are the least sensitive to time to maturity because they have a large number of observations with recursion value lower than book value of equity (these estimates have zero call option value regardless of time to maturity according to the Hwang and Sohn (2010) model). Even for model HSO95, the increase in call option value to time to maturity is not significant. An increase in time to maturity from 5 years to 8 years increases the median of call option from 0.05 to 0.06 (and the relative put option with same strike price from 0.15 to 0.19). Thus, the performance of HSO95 is also improved, but not significantly (the median of PVE from -0.21 to -0.19, the median of APVE is not changed, median of average R square from 0.718 to 0.725). A further examination using an extreme time to maturity of 50 years (10 times the original) shows that the option value of HSO95 is increased and model performance is improved but that this improvement is not statistically significantly (the median call option value from 0.05 to 0.12, the median put option value from 0.12 to 0.33, the median of PVE from -0.21 to -0.05, the median of APVE from 0.48 to 0.46 and the median of average R square from 0.718 to 0.743). Though the time to maturity of option in Hwang and Sohn (2010)'s adjustment increases the option value and thus increases the performance of the models in this paper, it is unreasonable to use an extreme time to maturity (such as 50 years) for all the firms. What is more, changing the time to maturity of the Hwang and Sohn (2010) adjustment does not qualitatively change the main results. Specifically, Zhang (2000) still outperforms other models in forecast bias, accuracy and explainability, followed by HSO95 and Ashton et al. (2003).

5.3 Alternative Deflators

In this paper, all of the accounting variables in LID are deflated by the beginning book value per share. According to Akbar and Stark (2003a and 2003b) and Shen and Stark (2011), book value is the best deflator for mitigating the bias caused from scale effect in the UK data and is widely adopted in UK empirical market-based accounting research (Dedman et al. 2009; Dedman et al. 2010; Rees and Valentincic, 2013). Nevertheless, we also consider opening market value and opening net operating assets as alternative deflators together with unscaled variables in sensitivity tests. According to Akbar and Stark (2003a and 2003b), opening market value is the second strongest deflator after book value, while Liu and Ohslon (2000) suggest that opening operating assets as a conservatism variable is a relevant potential deflator. Unscaled regressions are widely used in the US literature. Although alternative deflators cause coefficients to vary in the LID, the qualitative results for bias, accuracy and explainabilty remain unchanged for all the models. Zhang (2000) continues to outperform other models in forecast bias, accuracy and explainability, followed by HSO95 and Ashton et al. (2003). The

HSOFF performs better than the original OFF throughout the robustness test for deflators.

5.4 Alternative Book Value Growth/Depreciation Rate

We also conduct robustness tests of book value of equity growth rate and net operating assets depreciation rate in FO95 and FO96. Instead of using regression method, we utilize the method used by Choi et al. (2006) which involves all previous available book value data. Following Choi et al. (2006), at each valuation date t, an alternative year-specific book value growth parameter G_t, corresponding to ω_{22} in original FO95, is estimated using all available book value of equity data up to t, as follows:

$$G_t = \frac{\sum_{s=k}^{s=t} \sum_{j=1}^{j=N_s} b_{j,s}}{\sum_{s=k}^{s=t} \sum_{j=1}^{j=N_s} b_{j,s-1}},$$

where j is a firm index; s is a time index; N_s is the number of firms for which data are available for year s, and k is the second year for which book value data are available. Meanwhile, at each valuation date t, an alternative year-specific net operating assets deprecation rate Dep_t, corresponding to δ_1 in original FO96, is estimated using all available net operating assets and cash investments data up to t, as follows:

$$NOALCI_{j,t} = OA_{j,t} - CI_{j,t},$$

$$Dep_t = \frac{\sum_{s=k}^{s=t} \sum_{j=1}^{j=N_s} NOALCI_{j,s}}{\sum_{s=k}^{s=t} \sum_{j=1}^{j=N_s} NOALCI_{j,s-1}},$$

where j is a firm index; s is a time index; N_s is the number of firms for which data are available for year s, and k is the second year for which book value data are available. When the Hwang and Sohn (2010) adjustments of FO95 and FO96 are now based on this alternative book value growth/depreciation rate, the main findings of the paper remain unchanged.

5.5 OFF with Intercepts in LID

The empirical literature also provides different treatments for the valuation effects of intercepts in linear information dynamics. Following Dechow et al. (1999), we estimate the intercepts in the LID but exclude their valuation impact on equity valuation. This approach allows our empirical models to link more closely with the theoretical dynamic (Dechow et al. 1999; Gregory et al. 2005). Meanwhile, another stream of research identifies the valuation impact of intercepts (Myers, 1999; Choi et al. 2006) and indicates that valuation should recognize non-zero LID intercepts when other information is ignored. For robustness, we therefore also consider the valuation impacts of non-zero intercepts in our OFF valuations. Specifically, when the Hwang and Sohn (2010) adjustment of OFF and the Ashton et al. (2003) model are tested for these alternative OFF valuations, the main findings do not change.

5.6 Additional Findings on US Data

In order to provide additional findings beyond our UK dataset, we also implemented our analysis for a large sample of U.S. firm-year observations obtained from the CRSP/Compustat Merged database for the period 1994-2017 (excluding utilities and financial firms with SIC codes 4900-4999 and 6000-6411, as well as firms with SIC codes 9000 or above). After conducting the similar sample selection process in the main text, our final data set consists of 15463 firm-year observations of NYSE, AMEX and Nasdaq firms.

The table in Appendix C reports results for forecast bias, forecast accuracy and explainability of the linear and non-linear accounting-based valuation models based on U.S. data. The findings are similar to our UK results. According to median of PVE and APVE, OFF continues to underestimate the market equity value with the weak performance of FO95 and FO96 again resulting from negative valuation multiples for book value and investment variables based on estimated LID parameters. Similar to the UK, HSOFF models perform better than the original OFF models in terms of forecast bias, forecast accuracy and explainability, and the Ashton et al. (2003) model provides higher forecast accuracy and explainability than O95 through the inclusion of the adaptation option. Once again, the Zhang (2000) model has the best results for forecast accuracy and explainability due to the incorporation of both put and call options.

In summary, these further findings again indicate that non-linear models with real options information provide a better estimation of equity value than linear models. It is evident from both UK and U.S. data that real options make a significant contribution to the overall market value of equity.

6. Conclusion

This paper contrasts the forecast bias, accuracy and explainability of linear accounting-based valuation models and non-linear accounting-based valuation models with real options. The linear models considered are based on the landmark studies of Ohlson (1995) and Feltham and Ohlson (1995,1996) (which we refer to as the OFF), while the non-linear real option models include the Hwang and Sohn (2010) adjustment of the OFF, Ashton et al. (2003) and Zhang (2000). Following Francis et al. (2000), forecast bias is measured by mean and median Proportional Valuation Error (PVE) and forecast accuracy is measured by mean and median Absolute Proportional Valuation Error (APVE). Explainability is measured by the R square of the estimated values as explanatory variables for market value of equity in pooled time series and cross-sectional regressions. Following Dechow et al. (1999) and Choi et al. (2006), the equity value of linear models is estimated using historical data utilizing cross-sectional valuation multiples calculated from estimated LID coefficients. The equity value of the Hwang and Sohn (2010) adjustment is calculated based on various recursion values from the linear valuation models and the estimated standard deviation of the recursion value. We obtain the equity value for the Ashton et al. (2003) model through the orthogonal polynomial fitting procedure suggested by Ataullah et al. (2009) and estimate the equity value of the Zhang (2000) model directly using ROE, earnings and book value of equity data.

To summarize our findings, it is confirmed that the negative sign of valuation multiples calculated from LID coefficients in the theoretical models of Feltham and Ohlson (1995) and Feltham and Ohlson (1996) contribute to significant value underestimation for these two models. For the OFF, Ohlson (1995) therefore has the lowest forecast bias as well as the highest forecast accuracy and explainability. Meanwhile, the Hwang and Sohn (2010) adjustment of the OFF models perform better than the original linear models in terms of forecast bias, forecast accuracy and explainability. Subsample tests reveal that the superior performance mostly comes from observations with negative earnings. While the recursion value reveals the proportion of equity value if the firm performs under its existing investment opportunity set, the put option value in Hwang and Sohn (2010)'s adjustment reflects the firm's ability to abandon its current operations and this contributes to the market equity value especially when the firm is in financial stress. The non-linear model of Ashton et al. (2003) generally provides higher forecast accuracy and explainability than the linear Ohlson (1995) model through inclusion of an adaptation option. Though this model tends to overestimate the equity value in negative earnings and low positive PE subsamples, it performs relatively well for the high PE subsample due to its mixed and duplicate option characteristics. Finally, among all the models, Zhang (2000) has superior performance in relation to forecast bias, forecast accuracy and explainability. By including both put and call options, it reflects the flexibility to capture abandonment value when a firm is making a loss and growth value when a firm has high expansion potential. Overall, the non-linear models with real options generally provide better estimation of equity value than their linear counterparts. It is therefore evident that real options make a significant contribution to the overall market equity value and are important in equity valuation.

This paper contributes to capital market research in several ways. Our empirical research provides large-sample evidence in UK which compares the performance of both linear models with linear information dynamics and non-linear models with real options. In total, eight models are tested. The non-linear models include the most representative real option models with close-form solutions in the accounting-based valuation area. Through applying the model of Hwang and Sohn (2010) as an adjustment to the OFF models, our paper provides insights for future research concerned with adjustment of linear accounting valuation models using real option information. Again, to the best of our knowledge, this is the first paper to empirically estimate and compare the equity values in Ashton et al. (2003) and Zhang (2000) and answers the call for more empirical work which focuses on the actual impact of real options in accounting-based equity valuation.

Recently the collective effort of researchers has given rise to a literature focusing on the role of real options in the equity valuation (Chen et al. 2015; Livdan and Nezlobin, 2017; Rao et al. 2018). From a practical perspective, as a specialist valuation technique, the real options approach has been increasingly adopted to complement equity valuation, especially in hedge funds (Pinto et al. 2015). It has long been found that enhanced long-term value and performance are likely to follow when real options are identified and exploited or exercised appropriately through increased awareness and financial flexibility (Ioulianou et al. 2017; Aabo et al., 2016). As a result, we confirm that real options information does matter in equity valuation, and it contributes to enhancing the forecast accuracy and explainability of non-linear valuation models. Future research could pay attention to refining the theoretical modelling in this area. Meanwhile, econometric and empirical implementation procedures could be developed to provide inspiration for future improvement and validation of equity valuation incorporating real options information.

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Appendix A. Variable Definitions

Variables	Label	Data Items and Definition
Market Value	MV_t	Market Value = Market Value Capital **(MV)** / Common Shares Used to Calculate Earnings Per Share **(WC05191)** For accounting periods ending before the 20th January 2007, UK firms had up to six months after the financial year-end to publish accounting data. This was reduced to four months for accounting periods ending after that date following the implementation of the Transparency Directive 2004/109/EC. To maintain consistency, we collect the market value of equity (MV) six months after the financial year-end (6 months after time t). MVPS = Market Value / Common Shares Used to Calculate Earnings Per Share at time t.
Book Value	B_t	Book Value = Common Equity (**WC03501)** / Common Shares Used to Calculate Earnings Per Share (**WC05191)**
Earnings	x_t	Earnings = Net Income Available to Common **(WC01751) /** Common Shares Used to Calculate Earnings Per Share (**WC05191)**
Cash Investments	CI_t	Cash Investments = Capital Expenditures Additions to Fixed Assets **(WC04601)** / Common Shares Used to Calculate Earnings Per Share **(WC05191)**
Cash Receipts	CR_t	Cash Receipts = Net Cash Flow Operating Activities **(WC04860) /** Common Shares Used to Calculate Earnings Per Share (**WC05191)**
Residual Income	x_t^a	$x_t^a = x_t - r_t * B_{t-1}$
Net Operating Assets	OA_t	Net Operating assets = [Common Equity (**WC03501)** + Net financial Obligation + Minority Interest Balance Sheet **(WC03426)**] / Common Shares Used to Calculate Earnings Per Share (**WC05191)** Net Financial Obligation = Short Term Debt and the Current Portion of Long Term Debt **(WC03051)** + Long Term Debt **(WC03251)** + Preferred Stock **(WC03451)** – Cash and Short Term Investment **(WC02001)**
Operating Income	ox_t	Operating Income = [Net Financial Expense + Net Income Available to Common **(WC01751)** + Minority Interest Income Statement **(WC01501)**] / Common Shares Used to Calculate Earnings Per Share (**WC05191)** Net Financial Expense = Interest Expense On Debt **(WC01251)** * [Income Tax **(WC01451)** / Pretax Income **(WC01401)**] + Preferred Dividends **(WC05401)**
Residual Operating Income	ox_t^a	$ox_t^a = ox_t - (r_t * OA_{t-1})$
Cost of Equity	r_t	Cross Sectional: An annual Average of British Government Securities Ten Year Nominal Par Yield (10 Year Par Yield) plus average risk premium rate of 5%

Notes: Appendix A presents descriptive statistics of the variables, which are stated on a per share basis in the period of 1999-2015.

Appendix B. Sensitivity Test of Time Period of Option in Hwang and Sohn (2010) Adjustment

Model	N	Value Estimates					PVE		APVE	Explainability	
		Mean	*S.D.*	*0.25*	*Mdn*	*0.75*	*Mean*	*Mdn*	*Mdn*	*Coeff*	R^2
MVPS	9768	6.01	51.52	0.5	1.55	4.1					
Panel A Ohlson and Feltham framework											
V4a	9768	3.22	39.93	0.37	0.96	2.16	**-0.21**	**-0.37**	**0.52**	1.289	**0.679**
V4b	9768	1.9	52.56	0.23	0.66	1.5	**-0.5**	**-0.57**	**0.61**	1.688	**0.631**
V4c	9768	0.43	93.32	0.05	0.36	0.96	**-0.86**	**-0.75**	**0.76**	1.581	**0.489**
Panel B Ohlson and Feltham framework with Hwang and Sohn (2010) adjustment (T5)											
V4a(T5)	9768	4.58	42.43	0.48	1.17	2.61	**0.03**	**-0.21**	**0.48**	1.121	**0.718**
V4b(T5)	9768	4.01	38.41	0.42	1	2.2	**-0.08**	**-0.32**	**0.53**	1.336	**0.689**
V4c(T5)	9768	3.98	38.36	0.42	0.99	2.17	**-0.09**	**-0.33**	**0.53**	1.355	**0.690**
CO4a(T5)	9768	0.63	6.7	0	0.05	0.41					
CO4b(T5)	9768	0.06	0.7	0	0	0					
CO4c(T5)	9768	0.02	0.19	0	0	0					
PO4a(T5)	9768	1.37	51.24	0.04	0.15	0.41					
PO4b(T5)	9768	2.11	72.49	0.09	0.25	0.66					
PO4c(T5)	9768	3.55	116.18	0.17	0.49	1.28					
Panel C Ohlson and Feltham framework with Hwang and Sohn (2010) adjustment (T3)											
V4a(T3)	9768	4.44	41.28	0.47	1.12	2.51	0	**-0.24**	**0.49**	1.164	**0.712**
V4b(T3)	9768	4.00	38.40	0.42	1	2.19	**-0.08**	**-0.32**	**0.53**	1.341	**0.689**
V4c(T3)	9768	3.97	38.36	0.42	0.99	2.17	**-0.09**	**-0.33**	**0.53**	1.355	**0.690**
CO4a(T3)	9768	0.48	4.97	0	0.04	0.32					
CO4b(T3)	9768	0.04	0.53	0	0	0					
CO4c(T3)	9768	0.02	0.17	0	0	0					
PO4a(T3)	9768	1.22	51.06	0.04	0.12	0.31					
PO4b(T3)	9768	2.10	72.49	0.09	0.25	0.65					
PO4c(T3)	9768	3.55	116.18	0.17	0.49	1.27					
Panel C Ohlson and Feltham framework with Hwang and Sohn (2010) adjustment (T8)											
V4a(T8)	9768	4.76	43.92	0.49	1.22	2.73	**0.06**	**-0.19**	**0.48**	1.073	**0.725**
V4b(T8)	9768	4.02	38.42	0.43	1.01	2.20	**-0.08**	**-0.32**	**0.53**	1.330	**0.688**
V4c(T8)	9768	3.98	38.36	0.42	0.99	2.17	**-0.09**	**-0.33**	**0.53**	1.355	**0.690**
CO4a(T8)	9768	0.80	8.86	0	0.06	0.53					
CO4b(T8)	9768	0.07	0.90	0	0	0					
CO4c(T8)	9768	0.02	0.21	0	0	0					
PO4a(T8)	9768	1.54	51.54	0.05	0.19	0.54					
PO4b(T8)	9768	2.13	72.49	0.10	0.26	0.67					
PO4c(T8)	9768	3.55	116.18	0.18	0.50	1.28					

Notes: T3 indicates option with time length of 3 years. T8 indicates option with time length of 8 years. The original models are with time length of 5 years. CO and PO respectively represent the call and put option in Hwang and Sohn (2010) adjustment. Numbers in bold indicate significance at the 1% level: sign rank test for PVE and APVE and significance of the coefficients for explainability. The t values (in parentheses) are based on Fama-MacBeth standard errors with Newey-West adjustments.

Appendix C: Additional U.S. Findings

Model	N	Time Period	PVE		APVE		Explainability	
			Mean	**Mdn**	**Mdn**	**Central Tendency**	**Coeff**	R^2
Panel A Models based on Ohlson (1995)								
$V1_t = B_t + \beta x_t^a$								
O95	15463	24	**-0.42**	**-0.51**	**0.54**	10%	**1.744**	0.489
							(26.20)	
$V4a_t = AV_t + CO_t^{O95}$, $CO_t^{O95} = V_t^{O95} * N(d_1) - B_t * e^{-RF_t * T} * N(d_2)$								
HSO95	15463	24	**-0.29**	**-0.41**	**0.48**	13%	**1.521**	0.509
							(26.52)	
$V5 = B[h + \frac{1}{2}\int_1^1 exp\left(\frac{-2\theta Bh}{1+z}\right)dz] = B[\sum_{m=0}^{\infty} \alpha_m L_m(h)]$								
Ashton et al. (2003)	15463	24	**-0.05**	**-0.22**	**0.43**	17%	**1.115**	0.489
							(26.56)	
Panel B Models based on Feltham and Ohlson (1995)								
$V2_t = B_t + \beta_1 ox_t^a + \beta_2 OA_t$								
FO95	15463	24	**-0.85**	**-0.83**	**0.83**	2%	**1.589**	0.195
							(18.42)	
$V4b_t = AV_t + CO_t^{FO95}$, $CO_t^{FO95} = V_t^{FO95} * N(d_1) - B_t * e^{-RF_t * T} * N(d_2)$								
HSFO95	15463	24	**-0.37**	**-0.49**	**0.54**	11%	**1.684**	0.447
							(23.54)	
Panel C Models based on Feltham and Ohlson (1996)								
$V3_t = B_t + \beta_1 ox_t^a + \beta_2 OA_{t-1} + \beta_3 CI_t$								
FO96	15463	24	**-1.12**	**-0.93**	**0.93**	1%	**0.455**	0.041
							(5.43)	
$V4c_t = AV_t + CO_t^{FO96}$, $CO_t^{FO96} = V_t^{FO96} * N(d_1) - B_t * e^{-RF_t * T} * N(d_2)$								
HSFO96	15463	24	**-0.36**	**-0.49**	**0.54**	11%	**1.652**	0.448
							(22.15)	
Panel D Models based on Capitalized Earning								
$Vce_t = \frac{1}{r} X_t$								
CEM	15463	24	**-0.91**	**-0.49**	**0.56**	11%	**0.735**	0.462
							(23.04)	
$V6 = \frac{1}{r} X_t + P(ROE_t)B_t + C(ROE_t)G$								
Zhang (2000)	15463	24	**-0.11**	**-0.30**	**0.41**	17%	**0.790**	0.517
							(24.80)	

Notes: This table provides the results of the supplementary findings on PVE, APVE and explainability across all models based on US data set. Numbers in bold indicate significance at the 1% level: sign rank test for PVE and APVE and significance of the coefficients for explainability. The t values (in parentheses) are based on Fama-MacBeth standard errors with Newey-West adjustments.