A Real Options Model of Debt Ratio in Leveraged Buyouts

Makoto Goto*
Hokkaido University

Ryuta Takashima
Tokyo University of Science

Motoh Tsujimura
Doshisha University

February 1, 2019

Abstract

We investigate the optimal timing for acquiring a target firm and the optimal capital structure of the new firm in a leveraged buyout (LBO) transaction using a real options model. Furthermore, we examine the case in which the target firm changes its capital structure by issuing bonds by itself. We compare the latter firm's capital structure to the capital structure in the LBO case and show that the leverage ratio of the LBO firm is higher.

Keywords: Leveraged buyouts; Debt ratio; Real options

1 Introduction

Leveraged buyouts (LBOs) play an important role in the efficient allocation of resources in the economy by improving the performance of the acquired firms. In an LBO transaction, a firm is acquired by an investor who typically borrows against the target firm's future cash flows to finance the acquisition. An LBO recapitalizes the acquired firms and redistributes returns and risks among the providers of the capital. See, for example, Arzac (2008) and the references therein for more details.

Another aspect of LBOs is the investor's uncertainty regarding the target's future cash flows. Due to flexibility in the decision-making of the investment, it is possible for the investor to wait before acquiring the target firm. That is, the investor has a real option in the timing of the acquisition of the target firm (see Dixit and Pindyck, 1994). Therefore, it is important to explain the mechanism of LBOs, such as the capital structures and the options to acquire the target firms. In this paper, we investigate these mechanisms using a real options model.

We focus on an LBO transaction and explore the optimal capital structure by determining the size of the debt financing. Furthermore, we examine the case in which the target firm changes its capital structure through issuing bonds by itself. We compare its capital structure to that of the LBO case and show that the leverage ratio of the LBO is higher than that of the target firm issuing bonds itself, which is due to the fact that an investor issues high-yield bonds with a higher risk of default in an LBO transaction.

*Corresponding author. Graduate School of Economics and Business Administration, Hokkaido University, Kita 9, Nishi 7, Kita-ku, Sapporo 060-0809, Japan. E-mail: goto@econ.hokudai.ac.jp

2 The LBO model

In this section, we develop a model of an LBO transaction by investigating the option value of acquiring the target firm. To this end, we examine the values of the target firm and the new firm.

Suppose that an investor establishes a special purpose vehicle (SPV) using the investor's own equity I and non-recourse debt D_N to buy a 100% of the stock in an all-equity target firm. The debt is financed by issuing bonds whose coupon payment is c_L. After acquiring the target firm, the SPV and the target firm merge into a new firm as a subsidiary firm of the investor. We assume that the investor, the creditors, and the shareholders of the target firm are risk neutral.

Let X_t be the target firm's EBIT , given by

$$\mathrm{d}X_t = \alpha X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t,$$

where α is the instantaneous expected growth rate of X_t, σ (> 0) is the associated volatility, and W_t is a standard Brownian motion.

2.1 Target firm's value

The target firm is assumed to maintain its activity until the EBIT falls to the level X_T. The value of the target firm, V_T, is given by

$$V_T(x) = \sup_{t_T} \mathbb{E}\left[\int_t^{t_T} \mathrm{e}^{-r(s-t)}(1-\tau)X_s \mathrm{d}s + \mathrm{e}^{-r(t_T-t)}K \middle| X_t = x\right],$$

where t_T is the liquidation time, r is the discount rate, τ is the effective corporate tax rate, and K is the liquidation value.

Following the standard technique of the real options approach, the value of the target firm is

$$V_T(x) = \begin{cases} A_T x^{\beta_2} + \dfrac{(1-\tau)x}{r-\alpha}, & \text{for } x > X_T, \\ K, & \text{for } x \leq X_T, \end{cases}$$

$$A_T = \left(K - \frac{(1-\tau)X_T}{r-\alpha}\right) X_T^{-\beta_2},$$

$$X_T = \frac{\beta_2}{\beta_2 - 1}\frac{r-\alpha}{1-\tau}K,$$

where β_2 is the negative root of the following characteristic equation:

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0. \tag{1}$$

2.2 New firm's value

The value of the new subsidiary firm, V_N, consists of the equity E_N and debt D_N values:

$$V_N(x) = E_N(x) + D_N(x). \tag{2}$$

Recall that the investor is a shareholder of the new firm. The investor has the right to liquidate the firm if the EBIT falls to the level X_N. For analytical simplicity, we assume that the creditors rather than the investor receive the liquidation value. Therefore, the equity value of the new firm, $E_N(x)$, is formulated as

$$E_N(x) = \sup_{t_N} \mathbb{E}\left[\int_t^{t_N} \mathrm{e}^{-r(s-t)}(1-\tau)(X_s - c_L)\mathrm{d}s \middle| X_t = x\right],$$

where t_N is the liquidation time of the new firm. Therefore, $E_N(x)$ is

$$E_N(x) = \begin{cases} A_E x^{\beta_2} + \dfrac{(1-\tau)x}{r-\alpha} - \dfrac{(1-\tau)c_L}{r}, & \text{for } x > X_N, \\ 0, & \text{for } x \leq X_N, \end{cases}$$

$$A_E = \left(\frac{(1-\tau)c_L}{r} - \frac{(1-\tau)X_N}{r-\alpha}\right) X_N^{-\beta_2},$$

$$X_N = \frac{\beta_2}{\beta_2 - 1}\frac{r-\alpha}{r} c_L.$$

Further, the investor indirectly decides the debt value of the new firm, $D_N(x)$, via determining the liquidation time t_N. Therefore, the value of debt is given by

$$D_N(x) = \mathbb{E}\left[\int_t^{t_N} \mathrm{e}^{-r(s-t)} c_L \mathrm{d}s + \mathrm{e}^{-r(t_N - t)}(1-\theta)K \middle| X_t = x\right],$$

where $\theta \in (0,1)$ is the default cost parameter. Therefore, $D_N(x)$ is given by

$$D_N(x) = \begin{cases} A_D x^{\beta_2} + \dfrac{c_L}{r}, & \text{for } x > X_N, \\ (1-\theta)K, & \text{for } x \leq X_N, \end{cases}$$

$$A_D = \left((1-\theta)K - \frac{c_L}{r}\right) X_N^{-\beta_2}.$$

Finally, the total value of the new firm, V_N, is given by

$$V_N(x) = \begin{cases} (A_E + A_D)x^{\beta_2} + \dfrac{(1-\tau)x}{r-\alpha} + \dfrac{\tau c_L}{r}, & \text{for } x > X_N, \\ (1-\theta)K, & \text{for } x \leq X_N. \end{cases} \tag{3}$$

The first term of the right-hand side in the first line of Eq. (3) represents the option value to bankrupt the new firm. The second term represents the present value of the net operating profit after tax and the third term represents the tax benefit from debt.

2.3 Option value of acquiring

We now proceed to characterizing the timing and the option value of acquiring the target firm. The investor acquires the target firm if its revenue rises to the level X_L. Then, the investor's problem is to choose the investment time, t_L, in order to maximize the expected net present value of acquiring the target firm:

$$F_L(x) = \sup_{t_L} \mathbb{E}\left[\mathrm{e}^{-rt_L}(E_N(X_{t_L}) - I)\right].$$

Notice that there is the following capital constraint at investment time t_L:

$$V_T(X_{t_L}) = I + D_N(X_{t_L}), \tag{4}$$

which determines the size of debt financing c_L. From Eqs. (2) and (4), the investor's problem can also be formulated as

$$F_L(x) = \sup_{t_L} \mathbb{E}\left[\mathrm{e}^{-rt_L}(V_N(X_{t_L}) - V_T(X_{t_L}))\right]. \tag{5}$$

Solving Eq. (5) gives

$$F_L(x) = \begin{cases} (A_E + A_D - A_T)x^{\beta_2} + \dfrac{\tau c_L}{r}, & \text{for } x \geq X_L, \\ A_L x^{\beta_1}, & \text{for } x < X_L, \end{cases} \tag{6}$$

$$A_L = \left((A_E + A_D - A_T)X_L^{\beta_2} + \frac{\tau c_L}{r}\right) X_L^{-\beta_1},$$

$$X_L = \left(\frac{\beta_1}{\beta_2 - \beta_1} \frac{1}{A_E + A_D - A_T} \frac{\tau c_L}{r}\right)^{1/\beta_2},$$

where β_1 is the positive root of Eq. (1). The right-hand side in the second line of Eq. (6) represents the option value of acquiring the target firm.

3 Debt financing by target firm

We investigate the case in which there is no LBO transaction against the target firm in this section. The target firm raises funds through debt and changes its capital structure by itself. This implies the optimal debt finance of the target firm (ODF).

Suppose that the target firm finances the amount D_O by issuing bonds when the EBIT rises to the level X_O. Let c_O be the coupon payment of debt D_O. Note that $D_O \neq D_N$ and $c_O \neq c_L$. Then the target firm's problem is to choose the timing of the issuance of the bond in order to maximize the financial leverage effect, defined as the increase in the value of firm from the debt financing. Let V_O be the value of the target firm when it is a leveraged firm, given by

$$V_O(x) = E_O(x) + D_O(x),$$

where E_O is the value of the equity.

Then, the target firm's problem is formulated as follows:

$$F_O(x) = \sup_{t_O} \mathbb{E}\left[\mathrm{e}^{-rt_O}(V_O(X_{t_O}) - V_T(X_{t_O}))\right], \tag{7}$$

where t_O is the time of debt financing. Comparing Eqs. (5) and (7), it is clear that $F_O = F_L|_{c_L=c_O}$. The target firm chooses the coupon payment c_O to maximize the financial leverage effect at optimal issuing time:

$$c_O = \arg\max_c F_O(X_O; c),$$

where $X_O = X_L|_{c_L=c_O}$.

4 Numerical examples

In this section, we numerically examine coupon payments, leverage ratio (LR), interest rate of debt (IRD), and tolerance for default (TFD) and compare these values between the two cases, LBO and ODF. Further, we investigate the impacts of changes in uncertainty, σ, on these values.

Table 1: Comparative statics with respect to σ.

LBO	σ	D_N	V_N	X_N	X_L	c_L	LR	IRD	TFD
	0.15	38.92	54.93	1.636	3.483	2.618	0.709	0.067	1.847
	0.20	26.34	40.21	0.996	2.558	1.855	0.655	0.070	1.562
	0.25	18.76	31.29	0.621	1.970	1.343	0.600	0.072	1.349
ODF	σ	D_O	V_O	X_N	X_O	c_O	LR	IRD	TFD
	0.15	9.98	14.50	0.361	0.849	0.578	0.688	0.058	0.488
	0.20	11.05	17.61	0.363	1.054	0.677	0.628	0.061	0.691
	0.25	12.47	21.90	0.375	1.339	0.812	0.570	0.065	0.963

LR and IRD are respectively defined by D_N/V_N (D_O/V_O) and c_L/D_N (c_O/D_O) in the LBO (ODF) case. TFD is defined as the difference between the time of debt issuance, t_L, and the default time, t_N. Because these times are defined by thresholds, the TFDs for the two cases are $X_L(c_L) - X_N(c_L)$ and $X_O(c_O) - X_N(c_O)$, respectively. Recall that we assumed that the investor raises funds and acquires the target firm simultaneously.

We use the following parameter values for the base case: $\sigma = 0.2$, $\alpha = 0$, $r = 0.05$, $\tau = 0.3$, $I = 5$, $K = 10$, and $\theta = 0.5$. Table 1 displays the impacts of changing the magnitude of uncertainty.

First, even for changes of the uncertainty, the LR, IRD, and TFD of LBO are higher than those of ODF. This result for the values of LR and IRD is a property of LBO transactions, as previously mentioned. The result for TFD is contrary to our expectation based on the debt financing . The value of debt in the LBO case is higher than in the ODF case.

Second, according to Arzac (2008), in an ideal LBO transaction, a target firm has cash-generating capacity, and its cash flow is foreseeable. Our model demonstrates these properties. The lower the uncertainty of cash flow is, the higher the value of debt D_N and the new firm's value V_N are. Combining these results, a decrease in uncertainty of cash flow raises the LR. The acquiring threshold X_L implies the target firm's cash-generating capacity. Therefore, a decrease in the uncertainty of cash flow raises the acquiring threshold. Consequently, a decrease in the uncertainty of cash flow also raises the default threshold X_N. These results of changing the uncertainty imply that a decrease in uncertainty raises TFD. Furthermore, a decrease in uncertainty raises coupon payment. The combined changes in the values of the debt and of the coupon payment imply that a decrease in uncertainty reduces the IRD.

5 Conclusion

In this paper, we have investigated the investor's option to acquire the target firm in an LBO. To this end, we solved the investor's problem using a real options approach. Then, we found the firm values and thresholds. Finally, we conducted a comparative static analysis for uncertainty, which showed the representative property of LBO transactions .

For future work, we will consider levered target firms and the separation of owner and manager as in Lambrecht and Myers (2007, 2008).

References

Arzac, E.R., 2008. Valuation for Mergers, Buyouts, and Restructuring, second ed. John Wiley & Sons.

Dixit, A., Pindyck, R.S., 1994. Invetment under Uncertainty, Princeton University Press, New Jersey.

Lambrecht, B.M., Myers, S.C., 2007. A Theory of Takeovers and Disinvestment, Journal of Finance, 62, 809–845.

Lambrecht, B.M., Myers, S.C., 2008. Debt and managerial rents in a real-options model of the firm, Journal of Financial Economics, 89, 209–231.