Application of Real Options to Valuation and Decision-making in the Petroleum E&P Industry

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Abstract

This study establishes a risk-neutral binomial lattice method to apply real options theory to valuation and decision-making in the petroleum exploration and production (E&P) industry under uncertain oil prices.

The research is applied to the switching time from primary to water flooding oil recovery. First, West Texas Intermediate (WTI) oil price evolution in the past 25 years, from January 2, 1986 to May 28, 2010, is studied and modeled with geometric Brownian motion (GBM) and one-factor mean reversion price models. Second, production profile for primary and water flooding oil recovery for a synthetic onshore oil reservoir is generated using UTCHEM simulator. Third, the binomial lattice real options evaluation (ROE) method is established to value the project with flexibility in switching time from primary to water flooding oil recovery.

Seven results and conclusions are reached: 1) for GBM price model, the assumptions of constant drift rate and volatility do not hold for WTI oil prices; 2) one-factor mean reversion model is better to fit WTI oil prices than GBM model; 3) the evolution of WTI oil prices in the past 25 years was according to three price regimes and since 2003, the world economy has increased its tolerance to higher oil prices and to higher price fluctuation from its long run price; 4) the established ROE method can be used to identify the best time to switch from primary to water flooding oil recovery; 5) with one-factor mean reversion oil price model and the most updated cost data, the ROE method finds that water flooding switching time is earlier than that from traditional net present value optimizing method; 6) the ROE results reveal that most of time water flooding should start when oil prices are high; and 7) water flooding switching time is sensitive to oil price models and to the investment and operating costs.

The established ROE framework enhances the valuation and decision-making for petroleum E&P industry including when to switch from one enhanced oil recovery method to another and when to switch from conventional to unconventional hydrocarbon production.

Introduction

The petroleum industry is characterized by large investment and asset base. In addition to high level of technical complexity in exploration and production, it is very vulnerable to the fluctuation in oil and gas prices. The value of a petroleum company relies on its oil and gas reserves and the evaluation of producing an oil and gas reserve depends on many certain and uncertain factors.

Taking only oil production into consideration, after an oil field is discovered and developed, there are three major stages in the production of the oil field, *i.e.*, primary, secondary, and tertiary production. How should a petroleum company arrange the oil production according to specific reservoir conditions and the changing oil prices so that the total value which the company receives from producing the natural resource can be maximized? Traditionally, net present value (NPV) is used to value the project with the deterministic oil prices. However, this method cannot capture the uncertainty of oil prices and value of flexibility in decision making, such as the flexibility to decide when to switch from one oil production option to another, contingent on uncertain factors such as oil prices.

The real options approach, built on the theory for pricing financial options, addresses this challenge more successfully than conventional techniques such as the NPV method. The real options method takes uncertainty in oil prices, oil production rate change, and managers' flexibility in decision making into consideration and discovers the best time for switching from one production option to another. Thus the maximum value of producing the oil reservoir is obtained.

With a synthetic petroleum company and a synthetic oil exploration and production (E&*P*) project, this research establishes a binomial lattice real options evaluation method for the petroleum E&*P* industry to perform project evaluations under uncertainty. With the unique industry needs oriented, this research builds binomial lattices for two stochastic oil price models, i.e., the geometric Brownian motion (GBM) and one-factor mean reversion models, generates oil production profile with different water flooding switching times through reservoir simulations, and creates binomial lattice cash flow models. The research integrates the established real options evaluation framework with the binomial lattices of stochastic oil price models, oil production profile, and binomial lattice cash flow to achieve the evaluation functions. This real options evaluation method is then applied to identify the best switching time at changing oil prices with the flexibility of switching from primary to water flooding oil recovery, and to capture the maximum value of the project for producing the synthetic oil reservoir. The results from the real options evaluation method are compared with those from the traditional NPV evaluation method.

The research and results for this study are presented through five sections. 1). The historical West Texas Intermediate (WTI) oil prices are analyzed, the parameters for the geometric Brownian motion (GBM) and one-factor mean reversion oil price models are calibrated using the historical WTI oil prices, and simulations and comparisons of the GBM and one-factor mean reversion price models for the historical WTI oil prices are conducted. 2). For the purpose of real options evaluations, specific reservoir simulations are designed and conducted, and the oil production profile for primary and water flooding oil recovery is generated. 3). The theory on risk- neutral world and risk-neutral probability is presented; the oil price stochastic processes are reconstructed into the risk-neutral binomial lattices; and the binomial real options evaluation method is established for determining the switching time from primary to water flooding oil recovery and for evaluating the petroleum E&*P* project under oil price uncertainty and with the flexibility in water flooding switching time. 4). The conclusions from this study are reached. 5) The recommendations for future work are proposed.

Modeling West Texas Intermediate (WTI) Oil Prices

The oil prices for this research are the West Texas Intermediate (WTI) crude oil free on board (FOB) historical spot prices from January 2, 1986 to April 7, 2010 (U. S. Energy Information Administration, 2010). Figure 1 shows the evolution of WTI daily spot prices during this period of time. Both the geometric Brownian motion (GBM) and ond-factor mean reversion price models are applied to analyze and model the WTI oil prices.

The Geometric Brownian Motion (GBM) Price Model and Parameter Calibrating

The GBM price model can be expressed with the following equation:

$$dX = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW, \qquad (1)$$

where $X = LnP$ is oil price return, P is the stock price at time t, W is the Wiener process with $dW = N(0,1)\sqrt{dt}$.

Historical WTI daily, weekly, and monthly price data from January 2, 1986 to April 7, 2010 are used to calibrate and analyze the drift rate μ and volatility σ for the GBM oil price model. Tables 1 and 2 are the resutls in three periods: whole price data from January 2, 1986 to April 7, 2010; low oil price data from January 2, 1986 to December 30, 1999; and high oil price data from January 4, 2000 to April 7, 2010. Table 1 shows that for all the four kinds of price data, the period from January 4, 2000 to April 7, 2010 has the highest annualized drift rate; the period from January 2, 1986 to December 30, 1999 has the lowest annualized drift rate. Within the same period, yearly price data results in the highest annualized drift rate; the weekly and monthly data have very close and lowest annualized drift rate. Table 2 shows that, unlike the annualized drift rate, for all of the four kinds of price return, the volatility in the three time periods is very close to each other.

Tables 1 and 2 show that both drift rate μ and volatility σ in the GBM model are not constant for the WTI historical oil prices.

One-factor Mean Reversion Price Model and Parameter Calibrating

In this paper, one-factor mean reversion price model is expressed as:

$$dX = \eta(\bar{x} - X)dt + \sigma dW, \qquad (2)$$

where $X = LnP$, $\bar{x} = Ln\bar{p} - \frac{\sigma^2}{2\eta}$, and

P is the price at time t,

$\bar{p}$ is the long run price,

η is the mean reversion rate,

W is the Wiener process and $dW = N(0,1)\sqrt{dt}$,

σ is the volatility of the random process σdW.

The chosen historical WTI oil price data for parameter calibrating are from January 2, 1986 to May 28, 2010, including daily, weekly, monthly, and yearly prices. Figures 2, 3, 4 and Table 3 show the results of long run price, annulized mean reversion rate, annulized mean reversion volatility for WTI daily, weekly, and monthly oil price data in different price regimes. The results of the mean reversion parameter estimations give insight on how different the world economy is in responding to the changes of oil prices in different periods of time, and propose the possibilities on how future oil prices may be evolving according to the mean reversion price theory.

First, with the estimated mean reversion parameters, the mean reversion oil price model reveals that, the evolution of historical WTI oil prices from January 2, 1986 to May 28, 2010 is classified into three price regimes: 1) B4-2K regime (1986 to 1999) with a long run price of $19.50/bbl; 2) 2K-2.3K price regime (2000-2003) with a long run price of $29.24/bbl; and 3) AF-2.3K regime (2004-2010) with a long rune price of about $77.00/bbl.

Second, the long run price and mean reversion rate reveal that in the AF-2.3K price regime, the world economy has increased its tolerance to the higher oil prices and to the higher price fluctuation from its long run price comparing with that in the B4-2K and 2K-2.3K price regimes.

Third, the relatively stable mean reversion volatility, which is from 0.4129 per year to 0.4347 per year among three price regimes with daily price data, over the period from 1986 to 2010, reveals that the difference in oil price changes from 1986 to 2010 are mainly caused by the deterministic force, that is, mean reversion rate and long run price, not by the random effect- mean reversion volatility.

Forth, limitations of applying mean reversion theory to historical oil prices include: 1) The results of the calibrated mean reversion parameters from historical oil prices are very data dependant. Meaningful parameter estimations for the model demand that the historical oil price data used are within the same price regime, cover enough length of time, evolve in complete up and down cycles, and are with right type of data. 2) Extending parameters to future oil prices involves the risk that the future oil prices may not be in the same price regime as the one the mean reversion parameters are calibrated from. By the end of 1999, meaningful mean reversion parameters could be estimated with oil prices from 1986 to 1999. However, the calibrated long run price and mean reversion rate cannot be applied to the price evolution after 1999 because of the switch of the oil price regime since the year of 2000. 3) Very low values of t-Statistic may occur for the regression intercept and slope in parameter calibrating, which may cause the unreliability of the calibrated results.

It takes time for a long run price pattern to be established. The long run price pattern is developed after there are several complete price cycles around the same long run price from below and above; the price data that are not evolving in cycles may give wrong mean reversion parameters. Prices evolving in one direction, or without complete cycles, should be avoided to use to calibrate the mean reversion parameters, even after the price regime has already be identified and the data are all inside the same price regime; full range historical WTI oil price data from 1986 to 2010, which cover several price regimes, are not recommended to use for calibrating the mean reversion parameters.

There may be three possibilities for the future oil price movements in and after 2010: 1) Future prices may still be in the AF-2.3K price regime and keep the fluctuation momentum with a long run price of about $77.00/bbl, a daily price mean reversion rate of 1.2242 per year, and a daily price mean reversion volatility of 42.51% per year. Future prices may be repeating the cycles of moving up and down across the same long run price of about $77.00/bbl. 2) Since the AF-2.3K price regime may be an ongoing regime, variation in long run price and other mean reversion parameters may be observed when more price data are included. 3) Future prices may be evolving to another price regime and another long run price pattern may be developed several years later with a different set of mean reversion parameters from those calbrated with the oil price data from 2004 to 2010.

Simulations and Comparisons of the Geometric Brownian Motion (GBM) and One-factor Mean Reversion Price Models for the WTI Oil Prices

Monte Carlo simulations are designed and performed in order to understand how the West Texas Intermediate (WTI) oil prices evolve according to the two price models, that is, the geometric Brownian motion (GBM) and the one-factor mean reversion price models. With the Monte Carlo simulation results, future WTI oil price distributions are obtained. The simulation results from the two price models are compared with each other and with the actual price movement of the WTI oil prices. Through the designed Monte Carlo simulations and their results, which price model better fits the evolution of historical WTI oil prices is observed and analyzed.

Monte Carlo Simulations for the Geometric Brownian Motion Price Model for the WTI Oil Prices

From the continuous form of the GBM price model as shown in the following equation:

$$dLnP = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW,$$

the price change in the time period of $[\mathrm{t},\ \mathrm{t}+\Delta\mathrm{t}]$ with the discrete form of the GBM price model is written as:

$$\Delta LnP = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma N(0,1)\sqrt{\Delta t}\,.$$

That is

$$P_{t+\Delta t} = P_t Exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma N(0,1)\sqrt{\Delta t}\right]. \qquad (3)$$

Equation 3 is used to run the simulations for the GBM price model.

Monte Carlo Simulations for the One-factor Mean Reversion Oil Price Model

With the one-factor mean reversion price model as shown in the following equation:

$$\frac{dP}{P} = \eta(Ln\bar{p} - LnP)dt + \sigma dW.$$

The future prices can be generated with the Monte Carlo simulations. For any given time interval $[\mathrm{t},\ \mathrm{t}+\Delta\mathrm{t}]$, the prices at the end of the time interval can be derived according to the following equation:

$$P_{t+\Delta t} = Exp\left[e^{-\eta\Delta t}(LnP_t) + (1-e^{-\eta\Delta t})\left(Ln\bar{p} - \frac{\sigma^2}{2\eta}\right) + N(0,1)\sigma\sqrt{\frac{(1-e^{-2\eta\Delta t})}{2\eta}}\right]. \qquad (4)$$

Comparisons between the Geometric Brownian Motion and One-factor Mean Reversion Oil Price Models

The Monte Carlo simulations are designed as follows: First, the price at time $\mathrm{t} = 0$ is set as the actual weekly price in the first week of year 2000, that is, $\mathrm{P}_0 = \$24.95/\mathrm{bbl}$. Second, the Monte Carlo simulations are run from the second week of year 2000 to the week of May 28, 2010. Third, the simulation results are compared with the actual prices during the same period of time. This way, assume that it was at the beginning of the first week of year 2000, equipped with the knowledge of the actual oil prices in the future and the price model parameters calibrated from these prices, simulations are run to forecast the future oil price movement. The difference between the actual prices and the simulation results are then observed and compared. The results from the two price models are also compared and analyzed.

The paramters for the two price models are calibrated from the actual WTI weekly oil prices from January 4, 2000 to April 7, 2010. The paramters for the GBM model are: $\mu = 0.17463$, $\sigma = 0.3393$, and $\Delta t = 0.01923$. The parameters for the one-factor mean reversion price model are: $\eta = 0.2822$, $\sigma = 0.3402$, $\Delta t = 0.01923$ year, $\bar{p} = \$79.74/\text{bbl}$.

Figures 5 and 6 demonstrate that the maximum simulation prices from the GBM price model are much higher than those from the one-factor mean reversion price model.

In Figure 7, the simulation results are shown in two charts with the same scale. From the two charts, it is clearly observed that the price range simulated from the GBM price model becomes wider with the increase of the simulation time, indicating that the GBM process is a non-stationary stochastic process. On the other hand, the price range simulated from the one-factor mean reversion model becomes very stable after a certain simulation time. In addition, many of the simulated prices from the GBM price model are much higher than the actual prices.

Figures 8 shows that the simulated mean prices with the GBM price model, which are very close to the expected forecast future price distribution when the iteration number of simulations is large enough, are at the upside of the actual prices; and the simulated mean prices from the mean reversion price model are along the center of the actual prices.

From Figure 9, it is observed that: 1) The 95th percentiles from the GBM price model are much higher than the actual prices can reach. 2) The ranges of the 5th and 95th percentiles from the mean reversion price model are very close to the fluctuation ranges of actual prices. 3) The simulated price ranges from the GBM price model are much wider than those from the mean reversion price model.

Figures 10 and 11 show that the variance of the simulated prices from the GBM price model is much higher than that from the mean reversion price model. Volatility is one of the most important factors in option pricing. When the variance from the simulation prices is higher than that from the actual prices, so is the annualized standard deviation, *i.e.*, volatility, then the options will be overpriced.

In summary, the above simulation results demonstrate that the mean reversion price model fits the historical WTI oil prices better than the GBM price model.

Reservoir Construction, Reservoir Simulations, and Oil Production Profile

A three-dimensional synthetic oil reservoir is constructed as follows: the reservoir is at a vertical depth of 4,370 feet with a 60 feet net pay; and the sizes in both X and Y directions of the reservoir are 2,640 feet. For reservoir simulations, the grid size is 64.4 feet in both X and Y directions and 20 feet in Z direction. So the total simulation number of grid blocks is 5,043 ($41 \times 41 \times 3$).

The reservoir is a carbonate mixed-wet reservoir. The reservoir formation compressibility is 0.000004 psi^{-1}. The initial reservoir pressure is 4,000 psi. The formation temperature is around 130 °F (or 54.5 °C).

The properties of the oil in the reservoir is very close to the West Texas Intermediate (WTI), in both API gravity and sulfur content; thus, the oil produced in this reservoir can be sold at the same prices as WTI when transportation costs from well-head to spot market and inventory cost are neglected. The API gravity of the oil in the reservoir is 39.6. The specific gravity of the oil is 0.827 g/cm^3, or 0.3585 psi/ft, at 60 °F; and 0.805 g/cm^3, or 0.3490 psi/ft, at 130 °F. The viscosity of the oil is 9.7cp at 60 °F and 2.8174 cp at 130 °F. Formation oil and water compressibility are 0.00002 psi^{-1} and 0.000001 psi^{-1} respectively.

In order to take the underground reservoir uncertainty into consideration, three categories of reservoir heterogeneity, *i.e.*, initial water saturation, permeability, and porosity, are taken into consideration.

The initial oil saturation is normally distributed with a mean of 69.75% and a standard deviation of 9.54%. 5,043 initial oil saturation data are generated according to this distribution and assigned to each grid block of the reservoir.

The Matrix Decomposition Method (MDM) (Yang, 1990) is used to generate a permeability heterogeneity field for the synthetic reservoir. The generated permeability heterogeneity field has a mean of 174.7 mD, a Dykstra-Parsons coefficient of 0.70, and a standard deviation of 320.9 mD. The permeability heterogeneity in the Y direction is the same as that in the X direction. The permeability heterogeneity in the Z direction is 10% of that in the X direction.

The reservoir porosity heterogeneity field is generated according to the relationship between permeability and porosity for a typical carbonate reservoir as shown in the following equation (Ghomian, 2010):

$k = 7.38 \times 10^6 \times \emptyset^{6.72}$,

where k is reservoir permeability in mD, and $\emptyset$ is reservoir porosity in fraction. The resulting porosity field has a mean of 18.66 and a standard deviation of 3.39%.

In order to conduct the real options analysis and capture the value of the flexibility in water flooding switching time, 29 oil production cases are designed to run the reservoir simulations, as shown in Table 4. The difference among cases is the starting time of water flooding after primary oil production with an increment of 91 or 92 days. The reservoir simulations are conducted through the University of Texas Chemical Flooding Simulator (UTCHEM) (Center for Petroleum and Geosystems Engineering, 2000) on the above constructed synthetic onshore oil reservoir.

Oil is produced with a 5-spot well configuration. The oil production well is controlled by bottom hole pressure (BHP) at 500 psi while the water injection wells are controlled by the water injection rate at 4,000 bbl/day for each well. The simulation results, including the production profiles for the 29 cases, cumulative oil production and water cut for the selected eight cases are shown in Figures 12-14.

Binomal Real Options Evaluation

In this study, the value of the flexibility in water flooding switching time is through the comparison of the values of the project with and without the flexibility in water flooding switching time. Oil prices are separated from oil production. In other words, there is no need to model the cash flow or project value process. Oil prices are first modeled with GBM and mean reversion models; then the binomial lattice method is applied to the GBM and mean reversion price models; and then for each binomial lattice of oil prices, with the oil production profile, a cash flow lattice is generated. The value lattice of the project is then calculated step by step with the cash flow lattice. The following sections present the details about the process of calculating the values of the flexibility in water flooding switching time.

Binomial Lattice for a Stochastic Process

A stochastic price process can be reconstructed with a binomial lattice developed by Cox, Ross, and Rubinstein (1979) for the GBM model, and by Nelson and Ramaswamy (1990) for the mean reversion model. At any time interval $[t,\ t+\Delta t]$, a stochastic process of a random variable X with X_t at the beginning of the time interval t, will either move up to X_t^+ with a probability of q_t, or move down to X_t^- with a probability of $(1-q_t)$, at the end of the time interval $t+\Delta t$. In a risk-neutral world, X_t^+, X_t^-, and q_t can be calculated from the parameters in the stochastic process. q_t is then called risk-neutral probability or martingale equivalent probability.

With a general stochastic process described as

$$dX = \alpha(X,t)dt + \sigma(X,t)dW\ ,$$

where W is the Wiener process, X_t^+, X_t^-, and q_t can be calculated as

$$X_t^+ \equiv X_t + \sigma(X,t)\sqrt{\Delta t},$$
$$X_t^- \equiv X_t - \sigma(X,t)\sqrt{\Delta t}$$
$$q_t \equiv \frac{1}{2}\left(1 + \frac{\alpha(X,t)}{\sigma(X,t)}\sqrt{\Delta t}\right)$$

When the above risk-neutral probabilities are applied to the up and down movements of a random variable, the expectations of future values of the random variable can be discounted using the risk-free discount rate.

Binomial Lattice for the GBM Oil Price Model

For the stochastic price P evolving according to the GBM process, that is

$$X = \mathrm{Ln}P,\ \ dX = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW,$$

where μ and σ are assumed constant in the process, then

$$X_t^+ = X_t + \sigma\sqrt{\Delta t}, \tag{5}$$
$$X_t^- = X_t - \sigma\sqrt{\Delta t} \tag{6}$$
$$q_t \equiv \frac{1}{2}\left[1 + \frac{(\mu - \frac{\sigma^2}{2})}{\sigma}\sqrt{\Delta t}\right] \tag{7}$$

$$P_t^+ = e^{\sigma\sqrt{\Delta t}}P_t = P_t u\ ,$$
$$P_t^- = e^{-\sigma\sqrt{\Delta t}}P_t = P_t d$$

where

$$u = e^{\sigma\sqrt{\Delta t}}\ ,$$
$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}.$$

According to Eq. 7, along the price evolution following the GBM stochastic process, the risk-neutral probabilities for price up and down movements are the same since μ and σ are assumed constant.

Binomial Lattice for the One-factor Mean Reversion Oil Price Model

For a stochastic price P evolving according to the one-factor mean reversion process, that is

$$X = \mathrm{Ln}P,\quad dX = \eta(\bar{x} - X)dt + \sigma dW,$$

where $\bar{x} = \mathrm{Ln}\bar{p} - \frac{\sigma^2}{2\eta}$, η , $\bar{p}$, and σ are assumed constant in the process. Then according to Nelson and Ramaswamy (1990),

$$X_t^+ = X_t + \sigma\sqrt{\Delta t}, \tag{8}$$
$$X_t^- = X_t - \sigma\sqrt{\Delta t}, \tag{9}$$

$$q_t \equiv \begin{cases} \frac{1}{2}\left[1+\frac{\eta(\bar{x}-X_t)}{\sigma}\sqrt{\Delta t}\right] & \text{if } 0 \le \frac{1}{2}\left[1+\frac{\eta(\bar{x}-X_t)}{\sigma}\sqrt{\Delta t}\right] \le 1 \\ 0 & \text{if } \frac{1}{2}\left[1+\frac{\eta(\bar{x}-X_t)}{\sigma}\sqrt{\Delta t}\right] \le 0 \\ 1 & \text{if } \frac{1}{2}\left[1+\frac{\eta(\bar{x}-X_t)}{\sigma}\sqrt{\Delta t} \ge 1\right] \end{cases}, \tag{10}$$

or

$$q_t = \max\left\{0, \min\left[1, \frac{1}{2}\left(1+\frac{\eta(\bar{x}-X_t)}{\sigma}\sqrt{\Delta t}\right)\right]\right\}. \tag{11}$$

$$P_t^{+} = e^{\sigma\sqrt{\Delta t}}P_t = P_t u,$$

$$P_t^{-} = e^{-\sigma\sqrt{\Delta t}}P_t = P_t\, d,$$

$$u = e^{\sigma\sqrt{\Delta t}},$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}.$$

Since X_t changes along the price evolution process, the risk-neutral probabilities for price up and down movements change accordingly for the one-factor mean reversion price model.

Risk-neutral World and Risk-neutral Probability

In financial mathematics, there are two parallel worlds: the physical world and the risk-neutral world. In the physical world, investors are risk averse; investors demand high return when bearing high risk. The performance of an asset return related to risk can be measured by the Sharpe ratio.

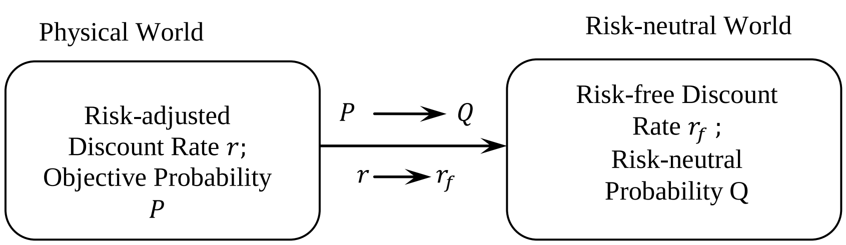
$$\text{Sharpe Ratio} = \frac{\text{Excess Return of Asset or Portfolio}}{\text{Standard Deviation of Asset or Portfolio Return}} = \frac{\bar{r}-r_f}{\sqrt{\text{var}(r)}}$$

The higher the Sharpe ratio, the higher the excess return per unit of risk. Generally, investors choose a project, among many investment opportunities, with the highest Sharpe ratio. In the risk-neutral world, investors are risk indifferent and demand only a risk-free rate of return.

In the physical world, the evaluation of an investment with uncertainty involves three parts: 1) future outcomes; 2) probability measure (denoted as P) for future events; and 3) discounting the expected future outcomes, which is calculated from 1) and 2), at the risk-adjusted discount rate. In this case, the probability measure P is objective. The measure of risk is incorporated in the risk-adjustment discount rate such as using the capital asset pricing model (CAPM).

In the risk-neutral world, risk is incorporated into the probabilities. The objective probabilities are adjusted so that the expected value under the adjusted probabilities can be discounted using the risk-free rate. The adjusted probabilities are called risk-neutral probabilities, or equivalent martingale probability measure Q.

These two worlds, i.e., the physical world and the risk-neutral world, can be connected when there are no arbitrage opportunities in a complete market.



Under the no arbitrage assumption, there is one unique set of risk-adjusted probabilities, *i.e.*, risk-neutral probabilities, to give the same value when future expected outcomes under risk-neutral probabilities are discounted with risk-free rate as the value discounted using risk-adjusted discount rate with the objective probabilities for future events (Duffie, 1992). Converting one probability distribution to another is called probability measure change. The following section discusses the rigor mathematic theory on the probability measure change.

Girsanov's Theorem

Girsanov's theorem provides the mathematical approach to transform one probability measure into another. When applying Girsanov's theorem to finance, it tells how to change a real probability distribution to a risk-neutral probability distribution, or called equivalent martingale probability distribution, so that a risk-free rate can be applied to value assets and derivatives.

For a stochastic process such as a price process

$dP_t = \alpha_t P_t dt + \sigma_t P_t dW_t$, or

$$\frac{dP_t}{P_t} = \alpha_t dt + \sigma_t dW_t \qquad (12)$$

W_t is a Brownian motion under the probability measure P, such that

$E^P(W_t) = W_0 = 0,$

$Var^P(W_t) = t.$

And the density function for W_t is

$$f^P(W_t) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2}\left(\frac{W_t}{t}\right)^2}.$$

In Eq. 12, add the right side of the equation with

$(\alpha_t - r)dt - (\alpha_t - r)dt.$

Then

$$\frac{dP_t}{P_t} = rdt + \sigma_t d\left(W_t + \frac{\alpha_t - r}{\sigma_t} t\right).$$

Let

$$W_t^Q = W_t + \frac{\alpha_t - r}{\sigma_t} t, \qquad (13)$$

and

$$\gamma = -\frac{\alpha_t - r}{\sigma_t},$$

then

$W_t^Q = W_t - \gamma t,$

and then

$$\frac{dP_t}{P_t} = rdt + \sigma_t dW_t^Q. \qquad (14)$$

Compared with W_t , W_t^Q has a mean of $-\gamma t$ and variance of t under the original probability measure of P. That is

$E^P(W_t^Q) = E^P(W_t - \gamma t) = E^P(W_t) - \gamma t = -\gamma t$,

and

$$f^P(W_t^Q) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2}\left(\frac{W_t^Q - \gamma t}{t}\right)^2}.$$

Therefore, W_t is a Brownian motion under probability measure P. But W_t^Q is not a Brownian motion under probability measure P, since other than at time zero, the expectation of W_t^Q is not zero. A new probability measure Q is desired so that under which

$E^Q(W_t^Q) = W_0^Q = 0$,

$Var^Q(W_t^Q) = t$,

$$f^Q(W_t^Q) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2}\left(\frac{W_t^Q}{t}\right)^2}.$$

W_t^Q is a Brownian motion under probability measure Q. W_t is not a Brownian motion under probability measure Q, since other than at time zero, the expectation of W_t under probability measure Q is not zero, but changes with time. That is

$E^Q(W_t) = E^Q(W_t^Q + \gamma t) = E^Q(W_t^Q) + \gamma t = \gamma t$.

With the transformation of Eq. 13, the process of ${dP_t}/{P_t}$, which originally has a drift rate of α_t under probability measure of P, has a drift rate of r under probability measure Q once the probability measure Q is determined so that W_t^Q is a Brownian motion. With the transform of probability measure from P to Q, the drift rate of the process changes to a risk-free rate automatically. Girsanov's theorem states that there exists a probability measure Q under which W_t^Q is a Brownian motion. And when there is no arbitrage opportunity, the probability measure Q is unique.

For many years, the petroleum industry could not accept that a risk-free rate was used to discount the values from cash flows when real options were included in the decision making. The argument was based on the Sharpe ratio. From the above analysis of probability transformation, it is clear that risk is not neglected or unrecognized when converting the probability measure from one into another. For example, oil prices in the future can still be as high as \$100/bbl or as low as \$40/bbl. But when a different set of probabilities is assigned to future oil price states, the present value of future cash flows based on oil prices will be different. Future price states will not be altered by the change in probability measure. The change in probability measure according to Eq. 13 changes the likelihood of the occurrence of each price state so that the discounted expectations of future values will be the same with a different discount rate.

Attention needs to be paid to that a probability measure needs to match the discount rate. In the binomial lattice method of reconstructing the stochastic process, the risk-neutral probability is calculated with Equations 7 and 10. Then the risk-free rate needs to be used to discount the future events which are governed by the risk-neutral probability distribution.

Real Options Evaluation Method for the Water Flooding Switching Time Flexibility

For the production of the synthetic reservoir studied in this research, assume that there are only two production stages: primary and secondary (water flooding) oil recovery. The total production time is 25 years when the water cut in production flow reaches 97 percent according to the reservoir simulation results. The project is evaluated at the third quarter of year 2010.

Water flooding can start at time zero; and at year seven, no matter what state the oil price is at, water flooding switching starts if it does not start earlier. Water flooding switching is an irreversible process. That is, once water flooding starts, it cannot switch back to primary oil production. It is assumed that the managerial decision on water flooding switching is revised every three months and it is long enough to build all the water flooding facilities in three months. If water flooding does not start at time zero, it can start by the end of three months, or six months, etc., until reaching the end of year seven. Therefore, there are 28 decision-making opportunities. Figure 15 illustrates the decision-making process of water flooding switching with eight switching opportunities. Both the geometric Brownian motion (GBM) and one-factor mean reversion price models are used to value the water flooding switching for the purpose of comparison.

The 25 years' oil production time is divided into 100 yearly quarters with three months in one yearly quarter. From time zero to the end of project life, the time series is defined as T_0, T_1, T_2, …, T_{99}, T_{100}. The investment costs are made in T_0; all the exploration and development activities are conducted in T_0; and all the primary production facilities are built in T_0. The water flooding well and facilities are completed one time lag ahead of the starting time of water flooding. For example, if water flooding starts in T_3 , the water flooding wells are drilled and completed, and the related facilities are build in T_2 .

The amount of oil sold at the oil price of $P(T_i)$, which is the quarterly price for the oil produced from the synthetic oil reservoir, is the total amount of oil produced from the reservoir from the beginning of time T_i , which is the end of time T_{i-1} , to the end of time T_i . Therefore, the gross revenue in T_i is equal to the price $P(T_i)$ multiplied by the amount of oil produced from the beginning of time T_i to the end of time T_i . Based on the assumption that the oil produced in the synthetic oil reservoir can be sold at the same prices as those of the West Texas Intermediate (WTI), the historical WTI oil prices and the parameters calibrated from the historical WTI oil prices are used to forecast the future oil price $P(T_i)$ produced from the synthetic oil reservoir.

To forecast the quarterly future price distributions for the oil produced in the synthetic oil reservoir, the annualized price model parameters from the WTI monthly oil prices are used to closely represent the model parameters for the quarterly oil prices.

Establishment of the Lattices for Probabilities, Prices, and Cash Flows

The establishment of the lattices for probabilities, prices, and cash flows is a forwarding process from T_0 to T_{100}. In Excel, probability, price and cash flow lattices are triangles with 101 columns from row 1 to row 101.

According to Equations 5 through 7, the probability and price lattices can be calculated for the GBM price model. In the model, P_0 is the WTI oil price at the third quarter of 2010, which is \$76.05/bbl; Δt is 0.25 years; μ (annualized) equals 0.16247; and σ (annualized) equals 0.3217. Both μ and σ are parameters calibrated from the historical WTI monthly prices from January 2000 to April 7, 2010.

According to Equations 8 through 10 or 11, the lattices of oil prices, the logarithm of oil prices, which is $Ln(P)$, and probabilities, are calculated for the one-factor mean reversion price model. Figures 16, 17, and 18 are illustrations for the partial binomial lattices, at a three-month time step (Δt is 0.25 years), of oil prices, the logarithm of oil prices, and probability for the one-factor mean reversion oil price model, starting from the third quarter of 2010. The parameters for the one-factor mean reversion price model are calibrated from the historical WTI monthly prices from January 2004 to May 28, 2010. That is, $\eta = 0.9235$, $\sigma = 0.3512$, and $\bar{p} = \$78/\text{bbl}$. Both η and σ are annualized.

Based on the price and probability lattices, the cash flow lattices, for both the GBM and one-factor mean reversion oil price models, are established. Cash flows are calculated according to two cost cases: a low cost case and a high cost case. All the cost data are based on an onshore oil well of about 4,000 foot depth.

For the low cost case, operating costs are \$1.00/bbl oil produced for primary recovery and \$3.00/bbl oil produced for water flooding with a yearly escalation of 1%. The exploration and development cost data for the low cost case are based on academic data with adjustments: the sum of exploration drilling and development costs is \$751,545 for one production well and the cost for four water injection wells and facilities is \$3,141,180. For the high cost cases, the operating costs are adjusted as follows: \$30.00/bbl oil produced for primary production and \$50.00/bbl oil produced for water flooding with a yearly escalation rate of 3%. The exploration and development costs for high cost case are the most current data. That is, \$1,951,000 for one exploration well and \$1,617,000 for one development well. The cost for four water injection wells and facilities is \$14,272,000 for the high cost case. 4.97% annual escalation is applied to the water injection well and facility cost for both the low cost and high cost cases. For both the low cost and high cost cases, when the production rate is high, the variable cost is the controlling cost factor. Thus, the fixed cost is neglected. When the production rate is low, the fixed cost becomes the controlling cost factor and then the variable cost is neglected. Therefore, there are four minimum operating costs for primary and water flooding oil production: $2,000/month for primary recovery and $6,000/month for water flooding for the low cost case; $6,000/month for primary production and $18,000/month for water flooding for high cost case. 1% and 3% annual escalation rates are applied for the above minimum operating costs for the low cost and high cost cases respectively.

Establishment of Project Value Lattice

The determination of project value at each time T_i at each price state is a backward process starting from T_{100}. For the convenience of describing value calculation and determination process, the cash flows at different price states for different switching times are first defined.

Let $CF^j_{i,k}$ denote cash flow at T_i and at price state of k for water flooding switching at T_j ($j = 0, 1, 2, \ldots, 27, 28$; $i = j+1, j+2, \ldots, 99, 100$; $k = 0, 1, 2, \ldots, i, i+1$ for any i). For example, if $j = 3$, there will be cash flows for the oil production when water flooding starts at T_3, which are

$CF^3_{4,k}(k = 0, 1, \ldots, 4)$,
$CF^3_{5,k}(k = 0, 1, \ldots, 5)$,
…
$CF^3_{99,k}(k = 0, 1, \ldots, 99)$,
$CF^3_{100,k}(k = 0, 1, \ldots, 100)$.

And let $CF^{PP}_{i,k}$ denote cash flows for primary oil production only ($i = 1, 2, 3, \ldots, 27, 28$). Since there is no primary oil production after year seven, $CF^{PP}_{i,k}$ ends at T_{28}. Then

1) $$V^j_{i,k} = CF^j_{i,k} + \left[q_{i,k}V^j_{i+1,k+1}(u) + \left(1 - q_{i,k}\right)V^j_{i+1.k}(d)\right]e^{-r\Delta t}, \qquad (15)$$

for any $i > j$ and $j = 0, 1, 2, \ldots, 27, 28$, where

$q_{i,k}$: the probability of price up move from T_i to T_{i+1} at price state k ;

$\left(1 - q_{i,k}\right)$: the probability of a downward price move from T_i to T_{i+1} at price state k ;

$V^j_{i,k}$: value of the project at T_i at price state k for the oil production starting water flooding switching at T_j ;

$V^j_{i+1,k+1}(u)$: value of the project at T_{i+1} starting water flooding at T_j when oil price moves up from price state k at T_i ;

$V^j_{i+1,k}(d)$: value of the project at T_{i+1} starting water flooding at T_j when oil price moves down from price state k at T_i ;

r : risk-free discount rate;

Δt : a quarter of year, or 0.25 year.

Since $q_{i,k}$ is the risk-neutral probability, values of the project at different times can be related to each other with a risk-free discount rate.

2) $$V^j_{j,k} = CF^{PP}_{j,k} - SWC(j) + \left[q_{j,k} \times V^j_{j+1,k+1}(u) + \left(1 - q_{j,k}\right) V^j_{j+1,k}(d)\right] e^{-r\Delta t} \qquad (16)$$

$(j = 0, 1, 2, \ldots, 27, 28)$,

where $V^j_{j,k}$ is the value of the project at T_j at price state k for oil production starting water flooding at T_j ; $SWC(j)$ is the switching cost at time T_j . $V^j_{j,k}$ captures the value of water flooding which starts at T_j at price state k. This value is compared with the project value $V^{PP}_{j,k}$, for which water flooding does not start at T_j at price state k, to determine whether or not water flooding should start at T_j at price state k. Following two equations are designed to perform the comparison:

3) $$V^{PP}_{i,k} = CF^{PP}_{i,k} + \left[q_{i,k} \times V_{i+1,k+1}(u) + (1 - q_{i,k})V_{i+1,k}(d)\right]e^{-r\Delta t} \qquad (17)$$

4) $$V_{i,k} = \mathrm{Max}\left(V^{j=i}_{i,k}, V^{PP}_{i,k}\right) \qquad (18)$$

Four steps need to follow using Equations 15 through 18 to calculate the maximum value of the project when water flooding option can be exercised at every price state at each of the possible switching times:

1). Start at T_{100}, for each water flooding switching option, i.e., water flooding starts at T_0, T_1, T_2, … , T_{27}, T_{28}, calculate the project values for each price state all the way to one lag behind the water flooding switching time according to Eq. 15;

2). Calculate the project values at water flooding switching time according to Eq. 16 for each switching option;

3). The value lattice of water flooding switching at T_{28} is used as the maximum value-determining lattice. In this lattice, for each price state k, starting from T_{27}, calculate the value of $V^{PP}_{27,k}$ according to Eq. 17 which is the project value when water flooding happens at T_{28} but does not happen at T_{27}. $V^{PP}_{27,k}$ is compared with $V^{27}_{27,k}$, which is the value at T_{27} for the oil production starting water flooding at T_{27}. The maximum value is assigned to T_{27} according to Eq. 18 which is $V_{27,k}$;

4). The process continues until T_0 is reached and the maximum value of the project is calculated and the best switching time is determined at each price state.

Figure 19 illustrates the process of determining the maximum value at T_j for price state k. Figure 20 is the illustration of determining the maximum values at different price states for different water flooding switching times.

For each water flooding switching option, there is a 101×101 triangular matrix for cash flow and value calculations. Since there are 29 water flooding switching options, there are 29 production curves, 29 of the 101×101 cash flow and value lattices. Comparing and determining the maximum value need to be conducted among the 29 water flooding switching options based on the values of different price states. In order to meet the large computation need, a computer program is developed to facilitate the real options evaluation process.

Development of the Computer Program for Binomial Lattice Real Options Evaluation

In this study, a binomial lattice real options evaluation computer program is developed. The program is designed to fulfill seven functions: 1) generate quarterly oil production rates from the reservoir simulation results for the 29 water flooding switching options; 2) generate binomial lattices of risk-neutral probabilities and oil prices, for both the geometric Brownian motion (GBM) and one-factor mean reversion price models; 3) make a series of specially designed cash flow calculators to calculate cash flows for oil prices at different times and different price states for the 29 water flooding switching options; 4) generate cash flow binomial lattices for the 29 water flooding switching options; 5) generate binomial value lattices for the 29 water flooding switching cases; 6) conduct real options evaluations; 7) conduct base case analysis under deterministic oil prices for both the GBM and one-factor mean reversion price models. Figure 21 shows the data flow for the real options evaluation computer program. Figure 22 shows one example of the generated cash flow lattices for the one-factor mean reversion oil price model.

With the cash flow lattices, the real options evaluation computer program automatically establishes the value lattices for the 29 water flooding switching options and then obtains the real options evaluation results according to Equations 15 through 18 following the four steps described in the above section. The real options evaluation results are then exported in an Excel spread sheet which shows the project value at each price state for each of the 100 yearly quarters and highlights the time and price state when water flooding switching occurs.

For the base case analysis, the real options evaluation computer program automatically creates 29 Excel spread sheets of cash flows with the traditional net present value (NPV) evaluation results, each sheet representing one switching option; generates a chart of the NPVs for the 29 water flooding switching options; and obtains the highest project value under the deterministic oil prices and the corresponding switching time. The results from base cases are then compared with the real options evaluation results regarding switching time and project values.

Results of Real Options Evaluations and Analysis

In order to establish a theoretically sound, mathematically solid, specific industry needs oriented, and feasible and reliable method to conduct real options evaluations for the petroleum E&*P* industry, not only is one complete set of standard real options case used to test the method with the desirable results, but other cases are designed and studied in order to give insight on how this real options evaluation method should be applied to real world needs, to which aspects attention should be paid when using this method, and what conclusions are reached from using this method.

The cases that have been studied include: 1) Base Cases without Options; 2) High Cost Mean Reversion case; 3) High Cost GBM case; 4) High Cost Mean Reversion with Stop Options case; 5) Low Cost Mean Reversion case; 6) Low Cost GBM case; and 7) Oil Production Rate Cut case. The High Cost Mean Reversion case is the standard real options evaluation case in this study.

In the Base Cases without Options, the traditional NPV method is used to evaluate different water flooding switching opportunities. For the one-factor mean reversion price process, the constant long run oil price $\bar{p}$ of $78.00/bbl, according to the WTI historical monthly oil prices from January 5, 2004 to May 28, 2010, is used as the deterministic price. For the GBM price model, the expected future price $E(P_{T_i}) = P_0 \mathrm{Exp}(\mu T_i)$ is calculated as the deterministic price and applied to each time T_i, where $P_0 = \$76.05/\mathrm{bbl}$, the WTI oil price at the third quarter of 2010; $\mu = 0.16247$ (annualized); $\sigma = 0.3217$ (annualized). Both μ and σ are calibrated from the historical WTI monthly oil prices from January 4, 2000 to April 7, 2010. The switching time with the highest NPV is the best switching time, which is also called the NPV optimization result, for the Base Cases without Options.

Since negative net income occurs when oil prices are low and/or oil production rate is low as well, an approximate stop option is included for the analysis. When net income is negative, it indicates that cost is higher than the benefit from operation, and that the operation should not be continued once the costs of stop and re-start the operation are taken into consideration. For simplicity, in this study, to include a stop option, the corresponding cash flow is set to zero once negative net income occurs. This way, since the costs to stop and re-start the operation are not included, the stop option just approximately represents the real cases.

As observed from the oil production curves, the oil production rates are very high at the beginning of primary and water flooding oil production: about 1,600 bbl/day for primary oil production and 2,600 bbl/day for water flooding oil production. A synthetic production profile is generated by cutting all of the oil production rates by a factor of (1/4.5), which results in 22% of the original oil production rates, to create the Oil Production Rate Cut case.

Table 5 contains the summary of the real options evaluation results, including water flooding switching times and the values of the project according to different water flooding switching times. Emphasis should be put on the corresponding changes in water flooding switching times and project values, not on the absolute project values, when changes are made for a specific factor for the evaluation purpose.

Figure 23 shows the real options evaluation results for the High Cost Mean Reversion case, the standard real options evaluation case. For the High Cost Mean Reversion case, real options evaluation shows that water flooding does not start at T_0 or T_1. Water flooding starts at T_2 when oil price is high at T_2; but it does not starts at T_2 when oil price is low at T_2. The project value is $2.7E+7. The real options evaluation results for this case are differenct from the base case NPV optimization results. The traditional NPV optimization method shows that for the base case with high cost and mean reversion price model, the best water flooding switching time is T_5 with a project value of $1.1E+7, as shown in Table 5 and Figure 24. The inclusion of stop options does increase the project value for the real options evaluation, but it does not change the best water flooding switching time.

Table 5 also shows that the real options evaluation results are very sensitive to oil price models. When GBM oil price model is used, no matter whether in high cost cases or low cost cases, no matter whether production rates are cut or not, the best water flooding switching time moves to T_{28} , which is the latest water flooding switching time, in all designed cases. With GBM oil price model, the expected oil prices increase dramatically with the increase of time. About 15 years from the third quarter of 2010, the expected oil price will reach about $900/bbl, which is very unlikely to happen. The oil prices become the dominant factor that impacts cash flows. The later the water flooding switching happens, the later the high oil production rates occur because of the water flooding, and the higher the project values. The real options evaluation method does respond to this price model and moves the best water flooding switching time to T_{28} and it agrees with the base case NPV optimization result on water flooding switching time.

When the one-factor mean reversion oil price model is used, the real options evaluation results are sensitive to the combining effects of initial investment for primary oil production, switching costs for water flooding, and the operating costs. In the Low Cost Mean Reversion and the Low Cost Mean Reversion with Stop Option cases, the best time of water flooding switching moves to T_0. In these cases, since the costs are so low, production rates and the time value of money become the controlling factors on project values. The more oil that is produced at an earlier time, the higher the project values. The real options evaluation results agree with those from the traditional NPV optimization method on water flooding switching time, as shown in Table 5. Figure 25 shows the results of Low Cost Mean Reversion case. The values in the cells that are highlighted are the project values for which water flooding switching starts at the times as the cells can indicate. As it is shown, water flooding switching starts at T_0 in this case.

As shown in Table 5, for all the cases designed and studied, with the inclusion of the approximate stop options, the best switching time of water flooding does not change, compared with the corresponding cases without stop options, from the real options evaluation results. The results from the real options evaluation agree with those from the traditional NPV optimization method for those cases.

For the studied High Cost cases, the oil rate cut does not change the water flooding switching time from the real options evaluation results for both the GBM and one-factor mean reversion oil price models. However, with the traditional NPV optimization method, when one-factor mean reversion oil price model is applied, the oil rate cut moves the water switching time from T_5 to T_{28}, along with a negative resulting project NPV.

In summary, the established real options evaluation method can be used to identify the best time to switch from primary oil recovery to water flooding oil recovery, and thus, gives the maximum project value according to the best water flooding switching time. With the mean reversion oil price model, the real options evaluation finds that the best water flooding switching time is earlier than the traditional NPV optimizing method. The method also reveals that the best water flooding switching time can be much different when different oil price models are applied and different cost data are used. The method challenges the knowledge on oil price models, the sound judgment in selecting the appropriate oil price model, and the reasonable investment and operating costs in the project evaluation.

Conclusions

1. The results of parameter calibrations for the GBM price model for the historical WTI oil prices reveal that the assumptions of constant drift rate and constant volatility for the GBM price model do not hold for the historical WTI daily, weekly, and monthly oil price data.
2. The simulation results for the GBM and one-factor mean reversion price models show that the one-factor mean reversion model is a better model to fit the historical WTI oil prices than the GBM model.
3. Applying the one-factor mean reversion price model to the historical WTI oil prices reveals that the evolution of the historical WTI oil prices from January 2, 1986 to May 28, 2010 can be classified into three price regimes:

1) B4-2K regime (1986 to 1999) with a long run price of $19.50/bbl; 2) 2K-2.3K price regime (2000-2003) with a long run price of $29.24/bbl; and 3) AF-2.3K price regime (2004-2010) with a long run price of about $77.00/bbl. Extending parameters to the future oil prices involves the risk that the future oil prices may not be in the same price regime as the one from which the mean reversion parameters are calibrated.

4. The calibrated long run prices and mean reversion rates for the historical WTI oil prices reveal that in the AF-2.3K price regime, the world economy has increased its tolerance to higher oil prices and to the higher price fluctuation from its long run price compared with that in the B4-2K and 2K-2.3K price regimes.
5. The established binomial lattice real options evaluation method can be successfully used to identify the best time to switch from primary oil recovery to water flooding when stochastic oil price models are included.
6. The real options evaluation method reveals that the best switching time for water flooding can be much different when different oil price models are applied and different cost data are used.
7. With the one-factor mean reversion oil price model and the most updated cost data, the real options evaluation method finds that the best water flooding switching time is earlier than the traditional net present value (NPV) optimizing method.
8. The real options evaluation results reveal that most of time water flooding should start when oil prices are high, and should not start when oil prices are low.

Table 1: Annualized Drift Rates of the WTI Oil Prices for the GBM Price Model Using Historical Price Data from January 2, 1986 to April 7, 2010

	January 2, 1986 - April 7, 2010	January 2, 1986 - December 30, 1999	January 4, 2000 - April 7, 2010
Daily	0.13740	0.08539	0.20921
Weekly	0.10190	0.05262	0.17463
Monthly	0.10032	0.05465	0.16247
Yearly	0.17781	0.11859	0.20891

Table 2: Volatility of the WTI Oil Prices for the GBM Price Model Using Historical Oil Price Data from January 2, 1986 to April 7, 2010

	January 2, 1986 - April 7, 2010	January 2, 1986 - December 30, 1999	January 4, 2000 - April 7, 2010
Daily	0.4186	0.4119	0.4277
Weekly	0.3281	0.3196	0.3393
Monthly	0.3098	0.3012	0.3217
Yearly	0.4823	0.4457	0.5094

Table 3: Summary of the Parameter Estimations for the Mean Reversion Price Model and Average Prices for WTI Historical Oil Prices in the Three Price Regimes

Mean Reversion Parameters		η Annualized	σ Annualized	Long Run Price $\bar{p}$ $/bbl	Average Price	*t*-Stat for $\hat{a}$	*t*-Stat for $\hat{b}$
BF-2K Price Regime (1986-1999)	Daily Price	2.2813	0.4129	19.49	19.09	3.99	440.15
	Weekly Price	1.3876	0.3220	19.45	19.09	3.10	114.66
	Monthly Price	1.2828	0.3106	19.55	19.09	2.88	25.53
	Yearly Price	2.4211	0.3214	19.67	19.09	3.39	0.33
2K-2.3K Price Regime (2000-2003)	Daily Price	4.3989	0.4347	29.11	28.41	2.93	166.07
	Weekly Price	2.6455	0.3392	29.25	28.41	2.26	43.09
	Monthly Price	2.1402	0.2991	29.35	28.41	1.99	10.12
	Yearly Price	NA	NA	NA	NA	NA	NA
AF-2.3K Price Regime (2004-2010)	Daily Price	1.2242	0.4251	76.36	67.23	2.37	476.16
	Weekly Price	0.9136	0.3441	77.51	67.23	2.23	122.21
	Monthly Price	0.9235	0.3512	78.00	67.23	2.23	26.98
	Yearly Price	0.7483	0.3281	76.37	67.23	2.17	1.86

Table 4: Water Flooding Switching Schedule for the 29 Oil Production Cases

Case Number	UTCHEM Run Case Name	Switching Time, Year	Switching Time, Days	Time Length between Two Switching Time, Days
1	G440T0	0.00	0	
2	G44002	0.25	91	91
3	G44005	0.50	182	91
4	G44007	0.75	274	92
5	G440T1	1.00	365	91
6	G44012	1.25	456	91
7	G44015	1.50	547	91
8	G44017	1.75	639	92
9	G440T2	2.00	730	91
10	G44022	2.25	821	91
11	G44025	2.50	912	91
12	G44027	2.75	1004	92
13	G440T3	3.00	1095	91
14	G44032	3.25	1186	91
15	G44035	3.50	1277	91
16	G44037	3.75	1369	92
17	G440T4	4.00	1460	91
18	G44042	4.25	1551	91
19	G44045	4.50	1642	91
20	G44047	4.75	1734	92
21	G440T5	5.00	1825	91
22	G44052	5.25	1916	91
23	G44055	5.50	2007	91
24	G44057	5.75	2099	92
25	G440T6	6.00	2190	91
26	G44062	6.25	2281	91
27	G44065	6.50	2372	91
28	G44067	6.75	2464	92
29	G440T7	7.00	2555	91

Table 5: Real Options Evaluation Results

	Real Options				Base Case NPV Optimization			
	GBM		MR		GBM		MR	
	Time to Start Switch	Project Value	Time to Start Switch	Project Value	Time to Start Switch	Project Value	Time to Start Switch	Project Value
High Cost	T_{28}	$5.9E+8	T_2	$2.7E+7	T_{28}	$2.4E+8	T_5	$1.1E+7
Low Cost	T_{28}	$7.1E+8	T_0	$1.7E+8	T_{28}	$3.1E+8	T_0	$1.4E+8
High Cost, Net Income ≥ 0	T_{28}	$6.0E+8	T_2	$3.5E+7	T_{28}	$2.4E+8	T_5	$1.1E+7
Low Cost, Net Income ≥ 0	T_{28}	$7.1E+8	T_0	$1.7E+8	T_{28}	$3.1E+8	T_0	$1.4E+8
High Cost, Oil Rate Cut	T_{28}	$1.4E+8	T_2	$4.8E+6	T_{28}	4.37E+8	T_{28}	$-8.7E+6

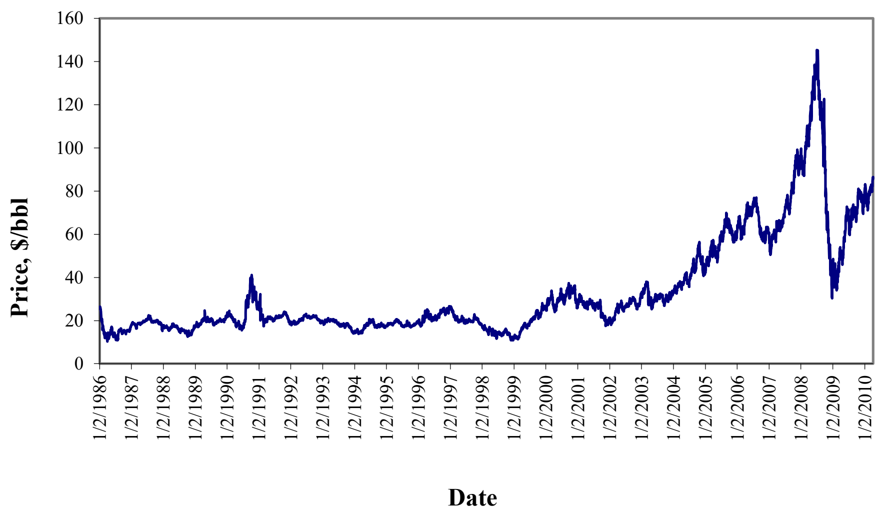


Figure 1: Evolution of WTI Daily Spot Prices (FOB) from January 2, 1986 to April 7, 2010
(Source: U. S. Energy Information Administration, 2010a)

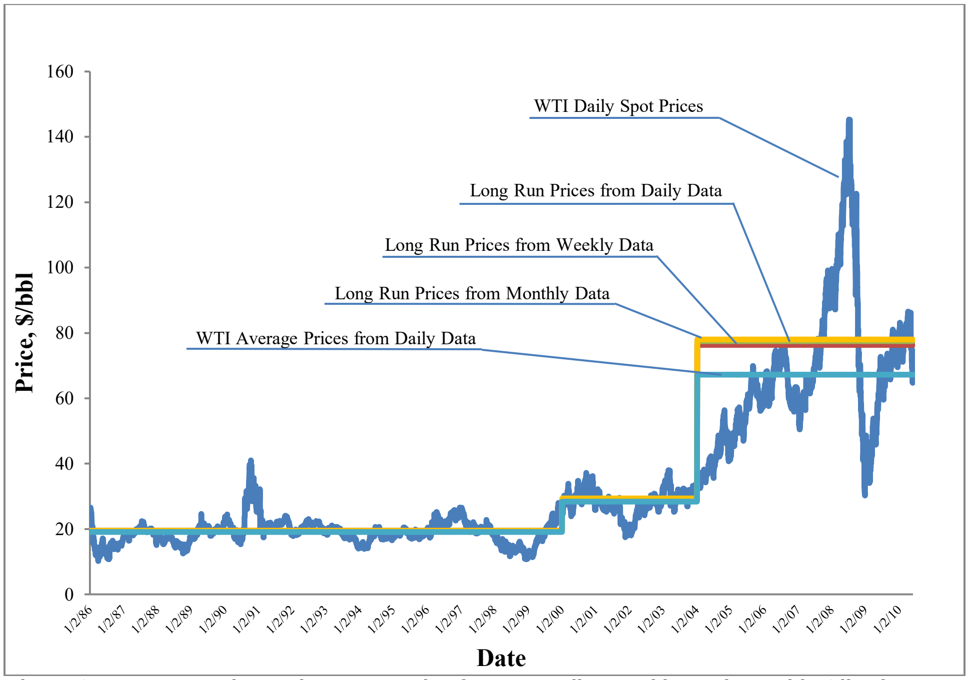


Figure 2: Long Run Price and Average Price for WTI Daily, Weekly, and Monthly Oil Price Data in Different Price Regimes from 1986 to 2010

Figure 3: Annualized Mean Reversion Rate for WTI Daily, Weekly, and Monthly Oil Price Data in Different Price Regimes from 1986 to 2010

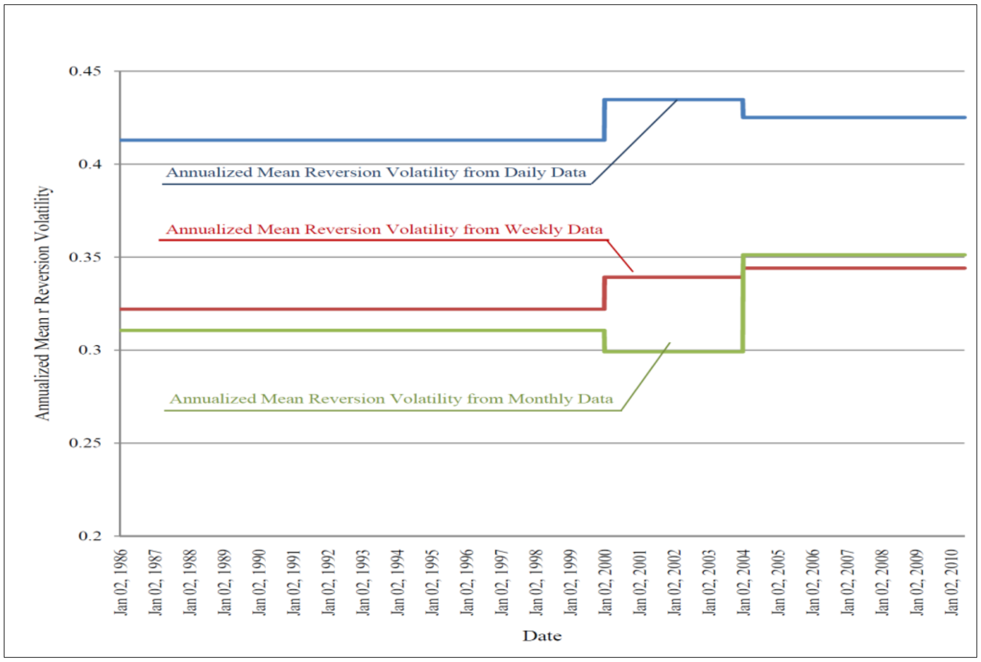


Figure 4: Annualized Mean Reversion Volatility for WTI Daily, Weekly, and Monthly Oil Price Data in Different Price Regimes from 1986 to 2010

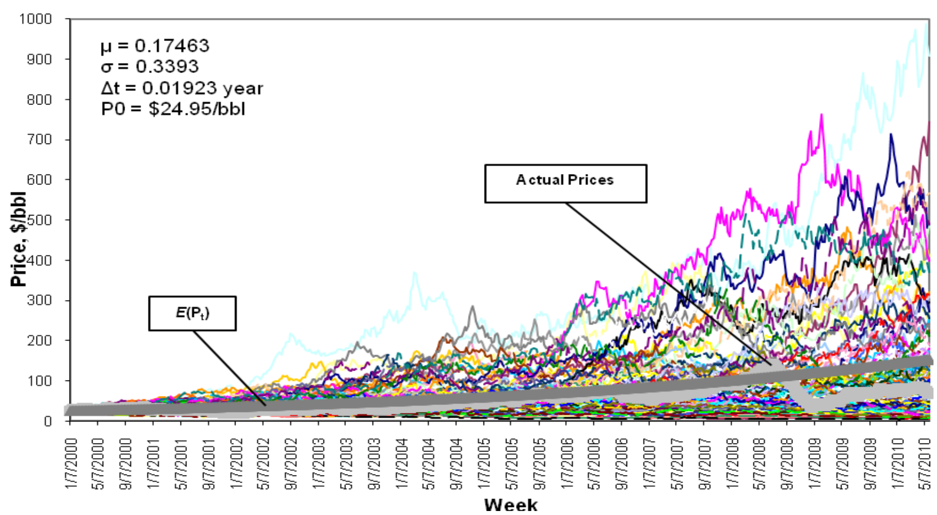


Figure 5: **Simulation Results of GBM Price Model for the WTI Weekly Oil Prices from January 4, 2000 to May 28, 2010 (100 Iterations)**

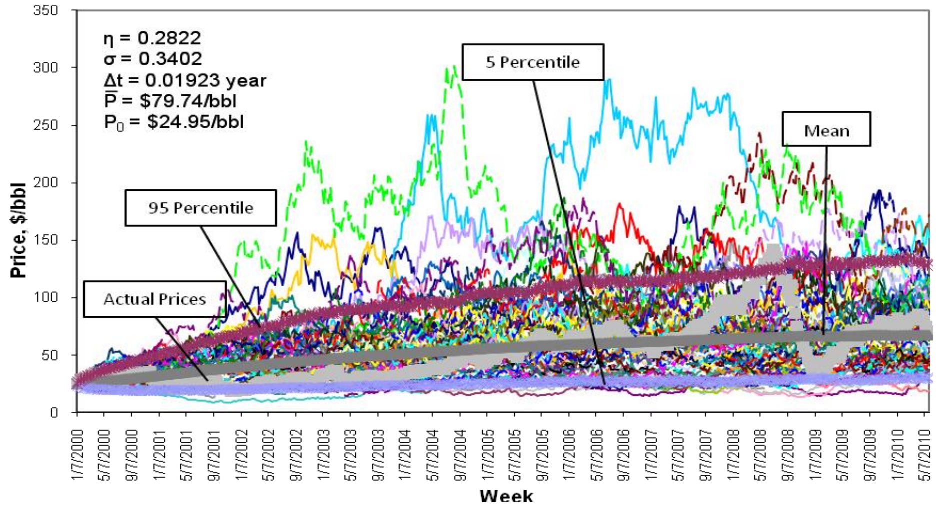


Figure 6: Simulation Results of One-factor Mean Reversion Price Model Compared with the Actual Price Evolution for the WTI Weekly Oil Prices from January 4, 2000 to May 28, 2010 (100 Iterations)

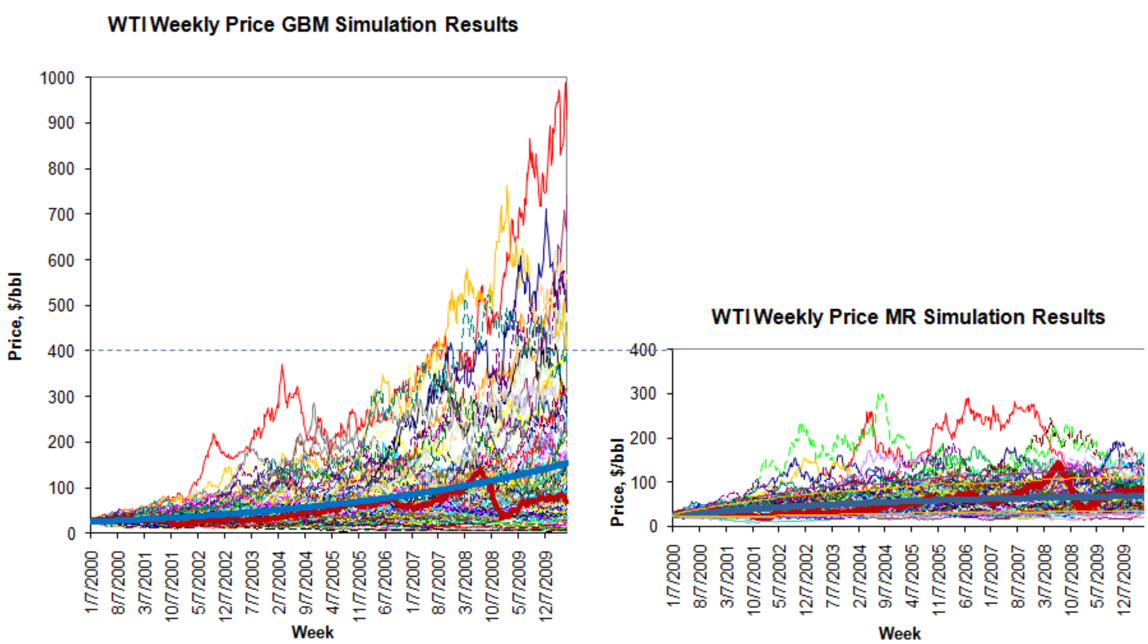


Figure 7: Comparison of the Simulation Results of the GBM and One-factor Mean Reversion Price Models for the WTI Weekly Oil Prices from January 4, 2000 to May 28, 2010

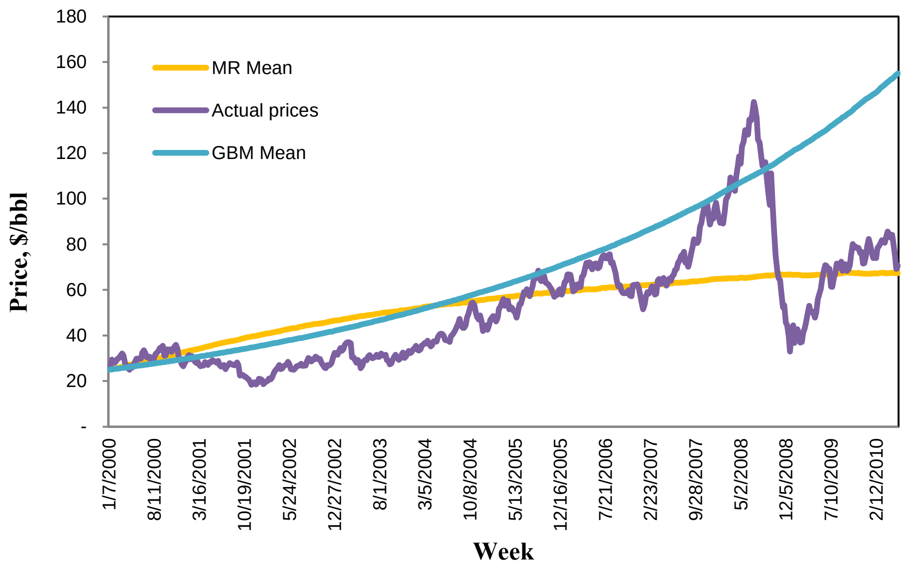


Figure 8: Comparison of the Simulation Mean of One-factor Mean Reversion and GBM Price Models for the WTI Weekly Prices from January 4, 2000 to May 28, 2010

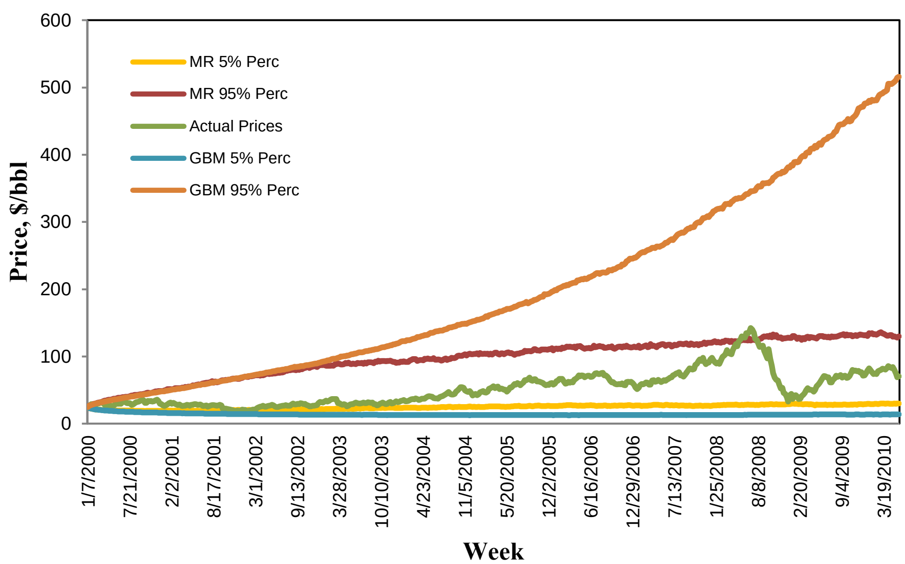


Figure 9: Comparison of the Simulation 5th and 95th Percentiles of the GBM and One-factor Mean Reversion Price Models for the WTI Weekly Prices from January 4, 2000 to May 28, 2010

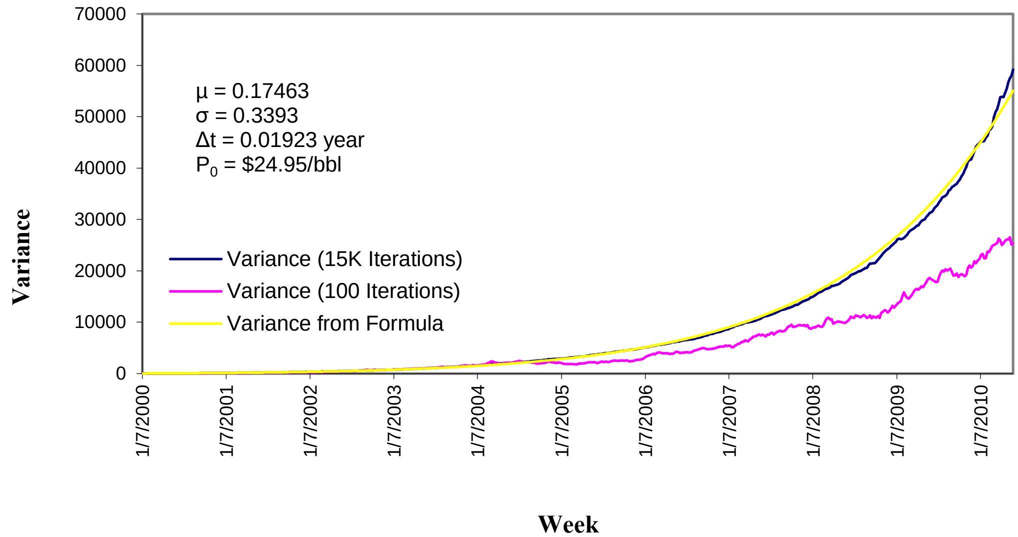


Figure 10: Comparison between the Variance of the Simulated Prices with Different Numbers of Iterations and the Calculated Variance with GBM Price Model for the WTI Weekly Oil Prices from January 4, 2000 to May 28, 2010

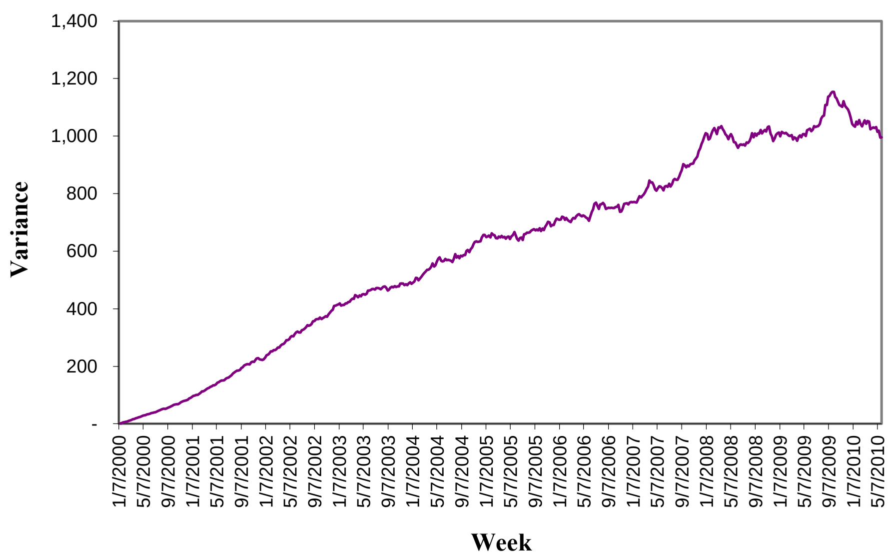


Figure 11: Simulation Variance of One-factor Mean Reversion Price Model for the WTI Weekly Oil Prices from January 4, 2000 to May 28, 2010

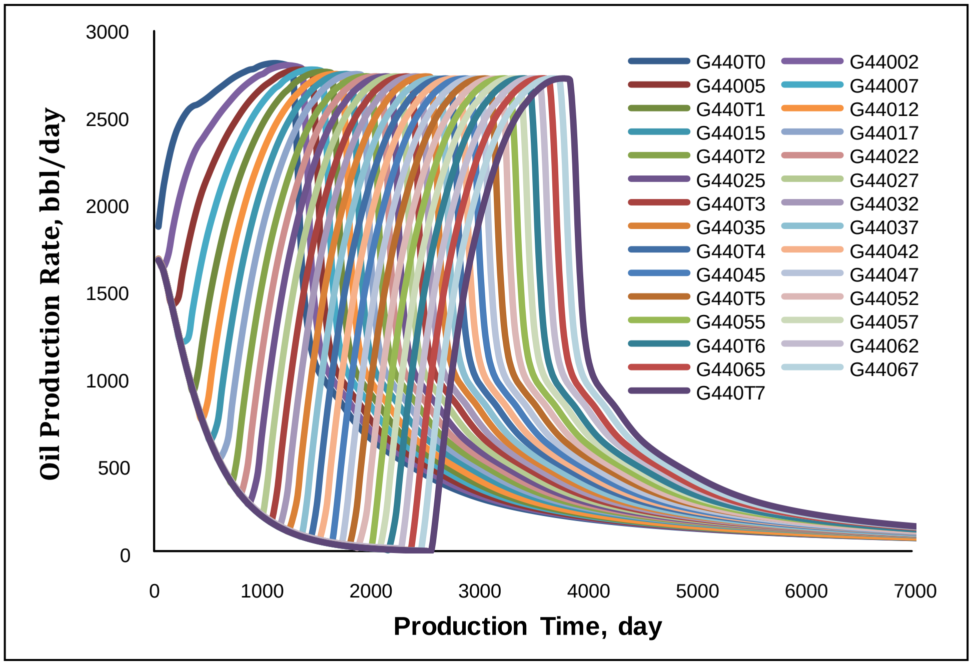


Figure 12: Oil Production Rate Change with Oil Production Time for 29 Cases at Different Water Flooding Switching Times

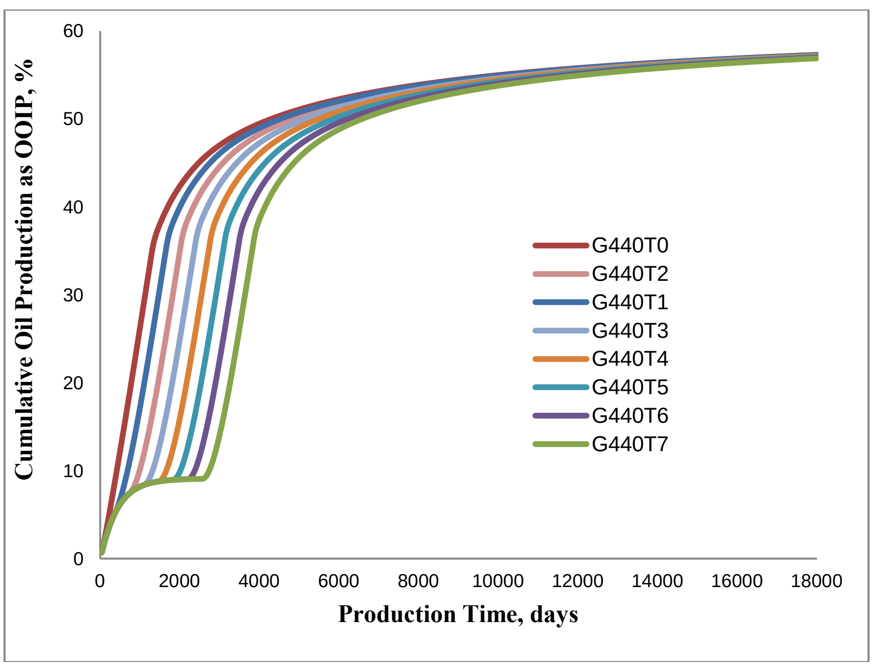


Figure 13: Cumulative Oil Production (as % OOIP) with Oil Production Time for Eight Cases of Different Water Flooding Switching Times

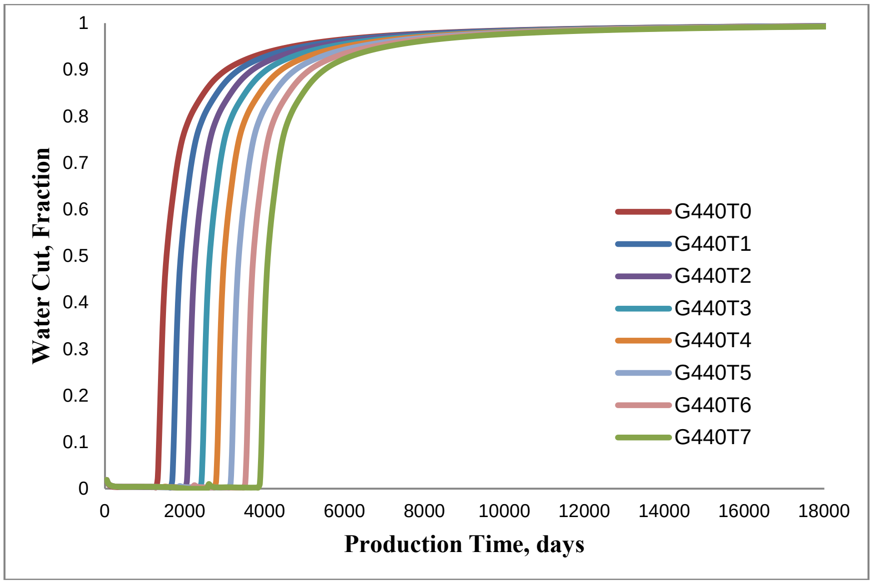


Figure 14: Water Cut Change with Oil Production Time for Eight Cases of Different Water Flooding Switching Times

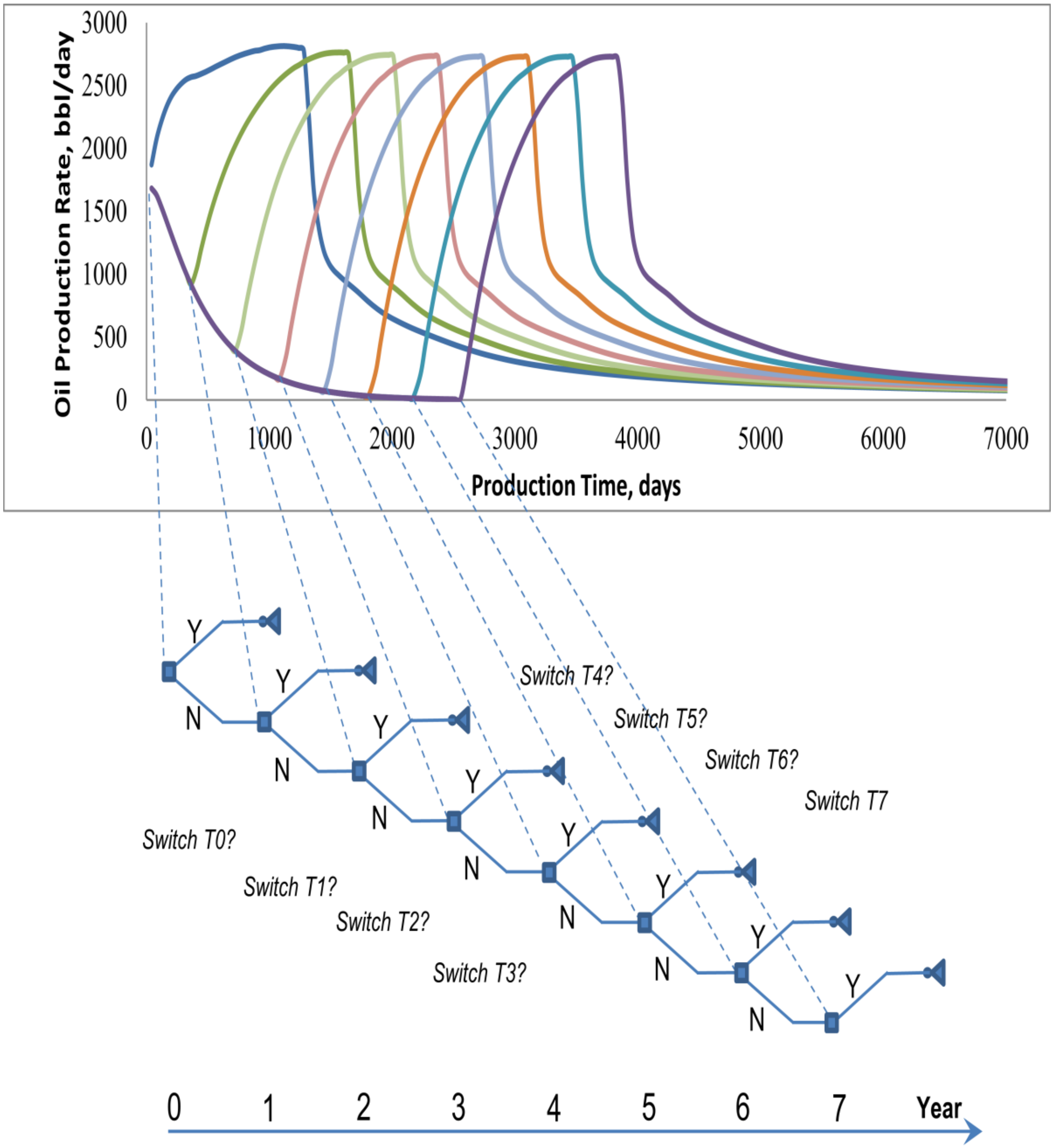


Figure 15: Decision Making Process of Water Flooding Switching

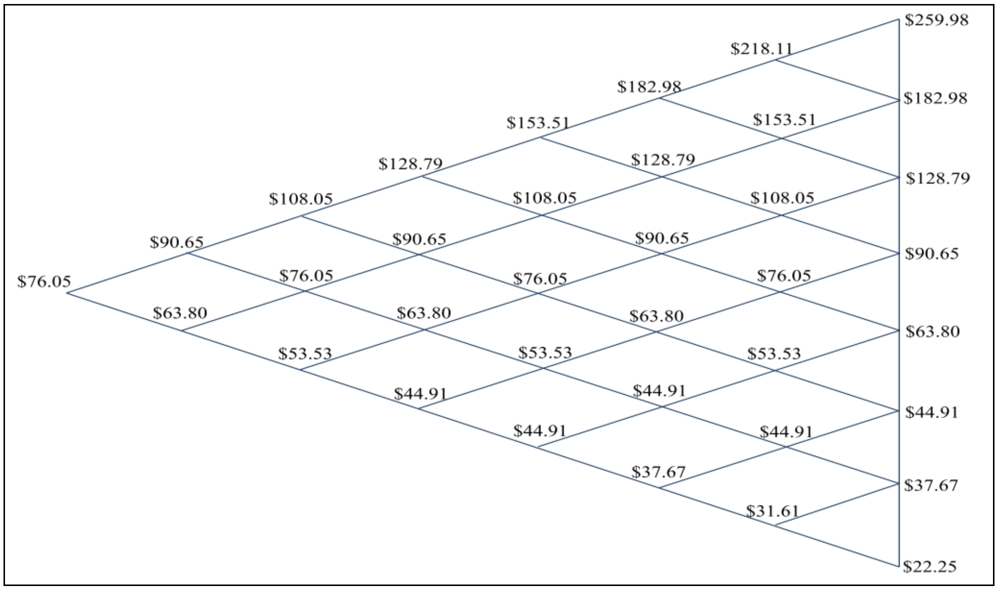


Figure 16: Illustration of Partial Binomial Lattice on Oil Prices (P) according to the One-factor Mean Reversion Price Model at the Three-month Time Step Starting from the Third Quarter of Year 2010 for the Oil Produced in the Synthetic Oil Reservoir

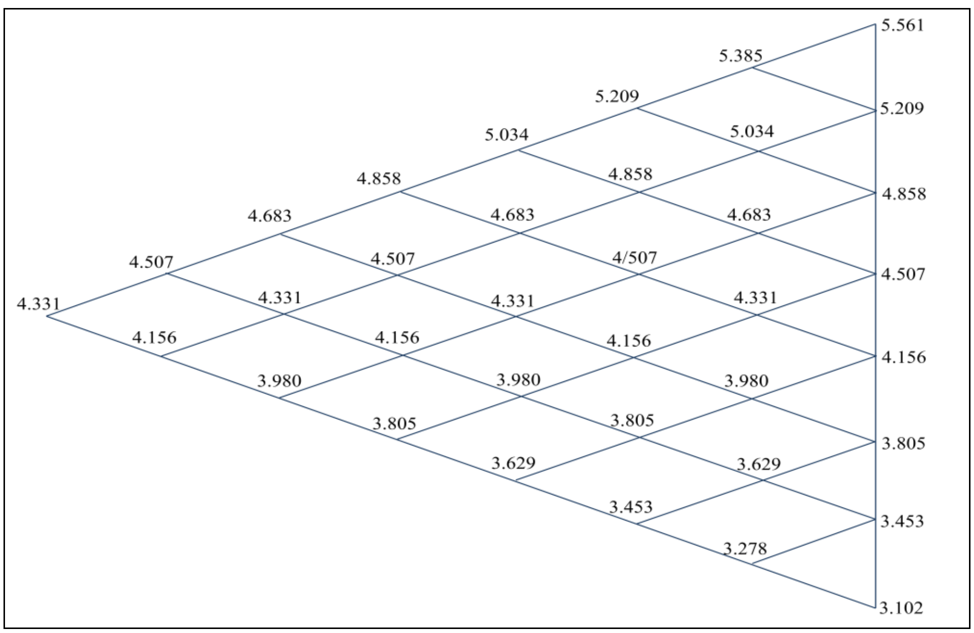


Figure 17: Illustration of Partial Binomial Lattice on the Logarithm of Oil Prices (LnP) according to the One-factor Mean Reversion Price Model at the Three-month Time Step Starting from the Third Quarter of Year 2010 for the Oil Produced in the Synthetic Oil Reservoir

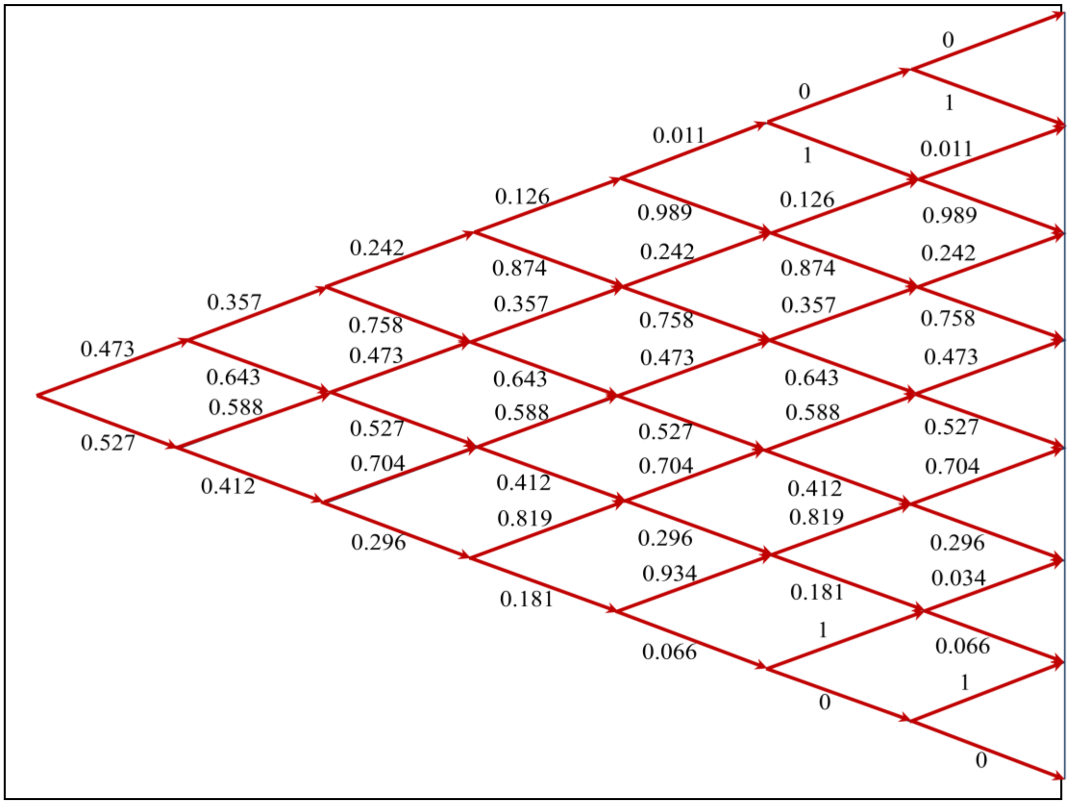
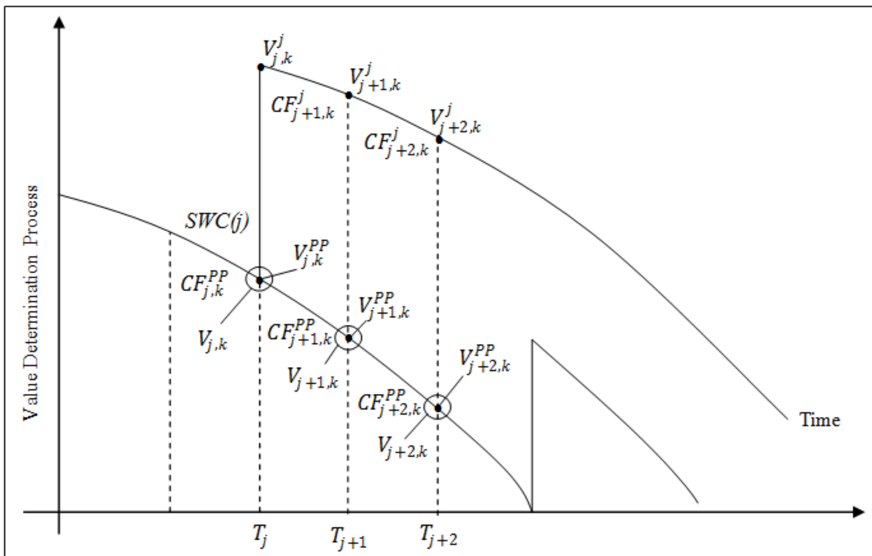


Figure 18: Illustration of Partial Binomial Probability Lattice According to the One-factor Mean Reversion Price Model at the Three-month Time Step Starting from the Third Quarter of Year 2010 for the Oil Produced in the Synthetic Oil Reservoir



$$V^j_{j,k} = CF^{PP}_{j,k} - SWC(j) + \ [V^j_{j+1,k+1}(u)q_{j,k} + V^j_{j+1,k}(d)\left(1 - q_{j,k}\right)]e^{-r\Delta t}$$

$$V^j_{j+1,k} = CF^j_{j+1,k} + \ [V^j_{j+2,k+1}(u)q_{j+1,k} + V^j_{j+2,k}(d)\left(1 - q_{j+1,k}\right)]e^{-r\Delta t}$$

$$V_{j,k} = Max(V^{PP}_{j,k}, V^j_{j,k})$$

Figure 19: Illustration of the Process to Determine the Maximum Value at T_j for Price State k

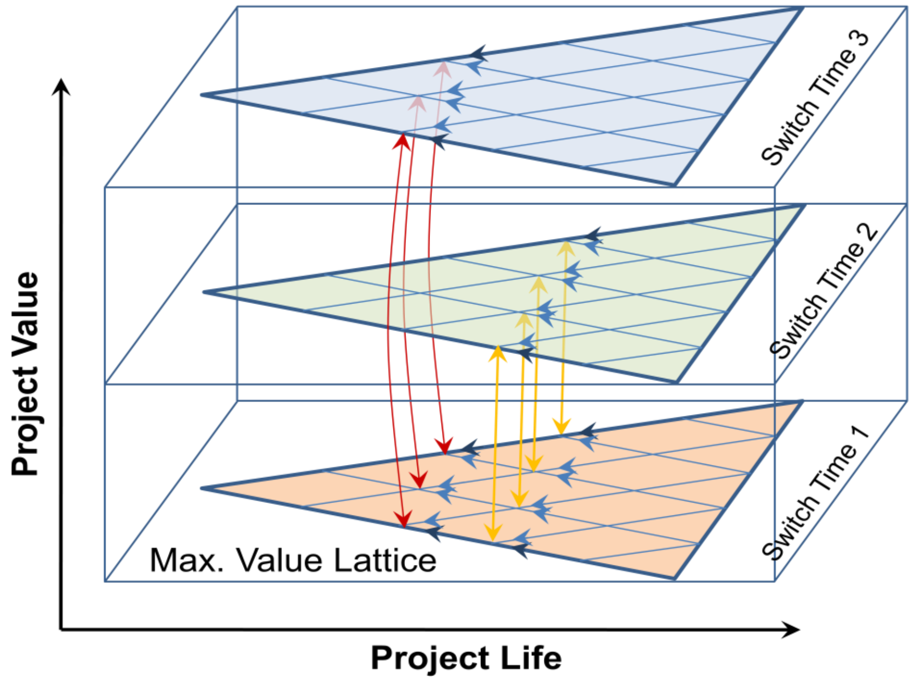


Figure 20: Illustration of Determining the Maximum Values at Different Price States for Different Water Flooding Switching Times

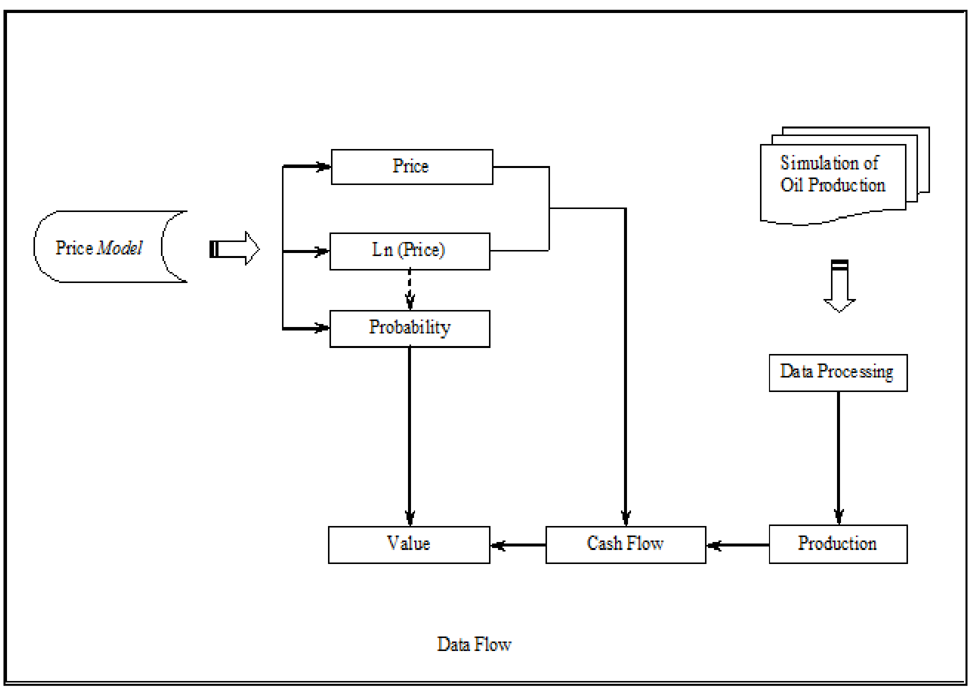
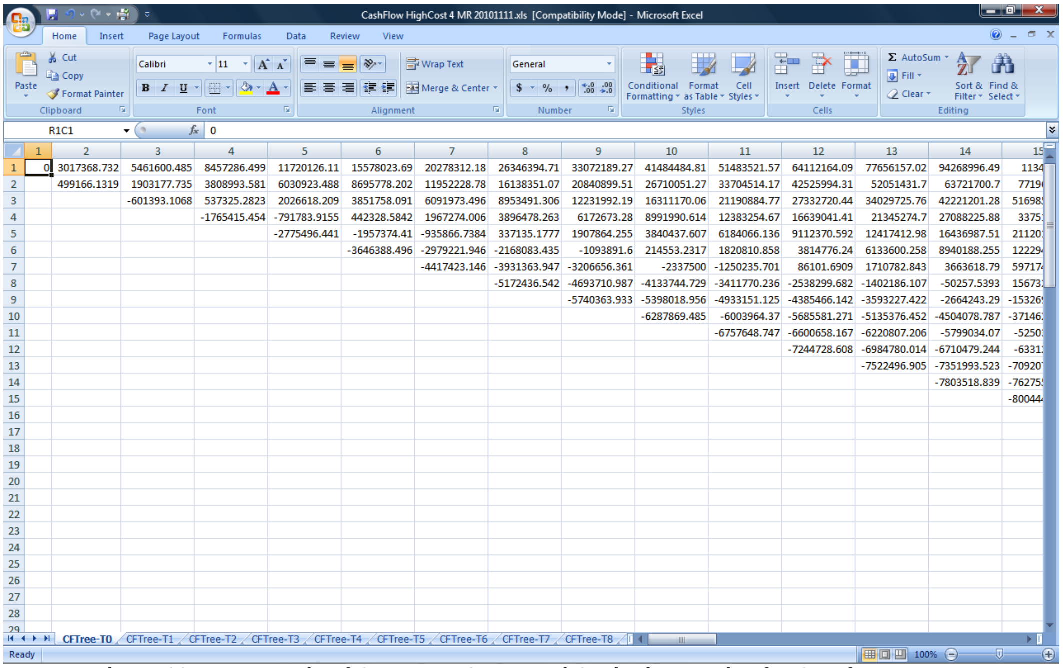
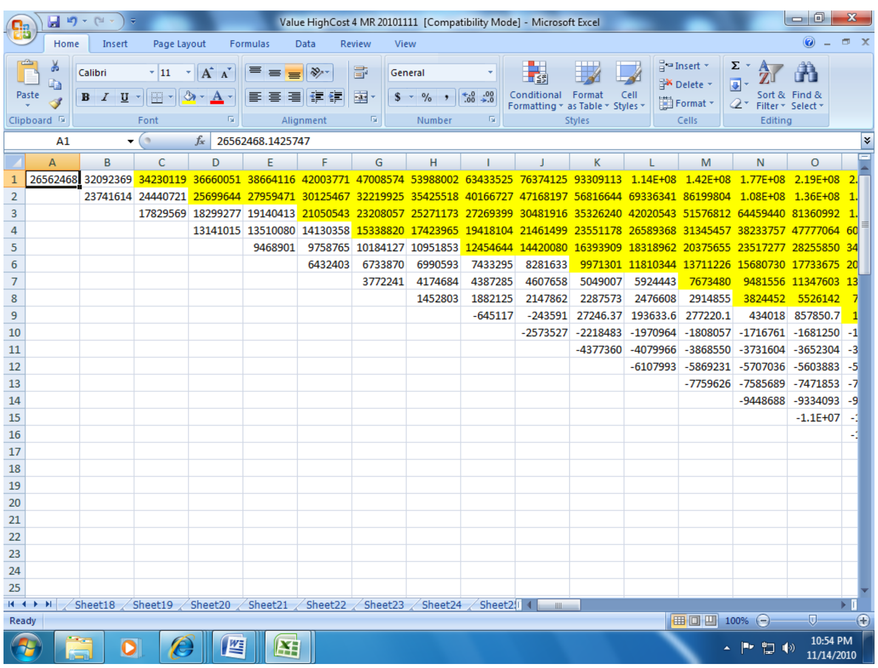


Figure 21: Data Flow for Real Options Evaluation Program



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	3017368.732	5461600.485	8457286.499	11720126.11	15578023.69	20278312.18	26346394.71	33072189.27	41484484.81	51483521.57	64112164.09	77656157.02	94268996.49	1134
2		499166.1319	1903177.735	3808993.581	6030923.488	8695778.202	11952228.78	16138351.07	20840899.51	26710051.27	33704514.17	42525994.31	52051431.7	63721700.7	7719
3			-601393.1068	537325.2823	2026618.209	3851758.091	6091973.496	8953491.306	12231992.19	16311170.06	21190884.77	27332720.44	34029725.76	42221201.28	51698
4				-1765415.454	-791783.9155	442328.5842	1967274.006	3896478.263	6172673.28	8991990.614	12383254.67	16639041.41	21345274.7	27088225.88	3375
5					-2775496.441	-1957374.41	-935866.7384	337135.1777	1907864.255	3840437.607	6184066.136	9112370.592	12417412.98	16436987.51	21120
6						-3646388.496	-2979221.946	-2168083.435	-1093891.6	214553.2317	1820810.858	3814776.24	6133600.258	8940188.255	12229
7							-4417423.146	-3931363.947	-3206656.361	-2337500	-1250235.701	86101.6909	1710782.843	3663618.79	59717
8								-5172436.542	-4693710.987	-4133744.729	-3411770.236	-2538299.682	-1402186.107	-50257.5393	15673
9									-5740363.933	-5398018.956	-4933151.125	-4385466.142	-3593227.422	-2664243.29	-15326
10										-6287869.485	-6003964.37	-5685581.271	-5135376.452	-4504078.787	-37146
11											-6757648.747	-6600658.167	-6220807.206	-5799034.07	-5250
12												-7244728.608	-6984780.014	-6710479.244	-6331
13													-7522496.905	-7351993.523	-70920
14														-7803518.839	-76275
15															-80044

Figure 22: An Example of Computer Generated Cash Flow Lattice for One-factor Mean Reversion Price Model for the Oil Produced in the Synthetic Oil Reservoir



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	26562468	32092369	34230119	36660051	38664116	42003771	47008574	53988002	63433525	76374125	93309113	1.14E+08	1.42E+08	1.77E+08	2.19E+08
2		23741614	24440721	25699644	27959471	30125467	32219925	35425518	40166727	47168197	56816644	69336341	86199804	1.08E+08	1.36E+08
3			17829569	18299277	19140413	21050543	23208057	25271173	27269399	30481916	35326240	42020543	51576812	64459440	81360992
4				13141015	13510080	14130358	15338820	17423965	19418104	21461499	23551178	26589368	31345457	38233757	47777064
5					9468901	9758765	10184127	10951853	12454644	14420080	16393909	18318962	20375655	23517277	28255850
6						6432403	6733870	6990593	7433295	8281633	9971301	11810344	13711226	15680730	17733675
7							3772241	4174684	4387285	4607658	5049007	5924443	7673480	9481556	11347603
8								1452803	1882125	2147862	2287573	2476608	2914855	3824452	5526142
9									-645117	-243591	27246.37	193633.6	277220.1	434018	857850.7
10										-2573527	-2218483	-1970964	-1808057	-1716761	-1681250
11											-4377360	-4079966	-3868550	-3731604	-3652304
12												-6107993	-5869231	-5707036	-5603883
13													-7759626	-7585689	-7471853
14														-9448688	-9334093
15															-1.1E+07

Figure 23: Real Options Evaluation Results for the High Cost Mean Reversion Case

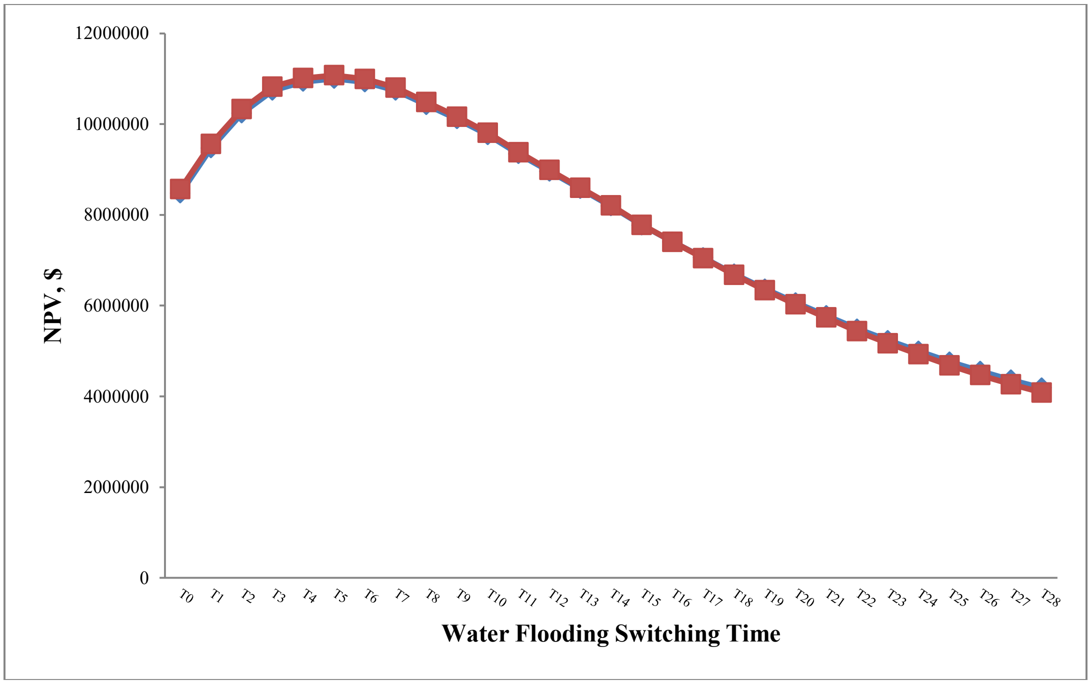
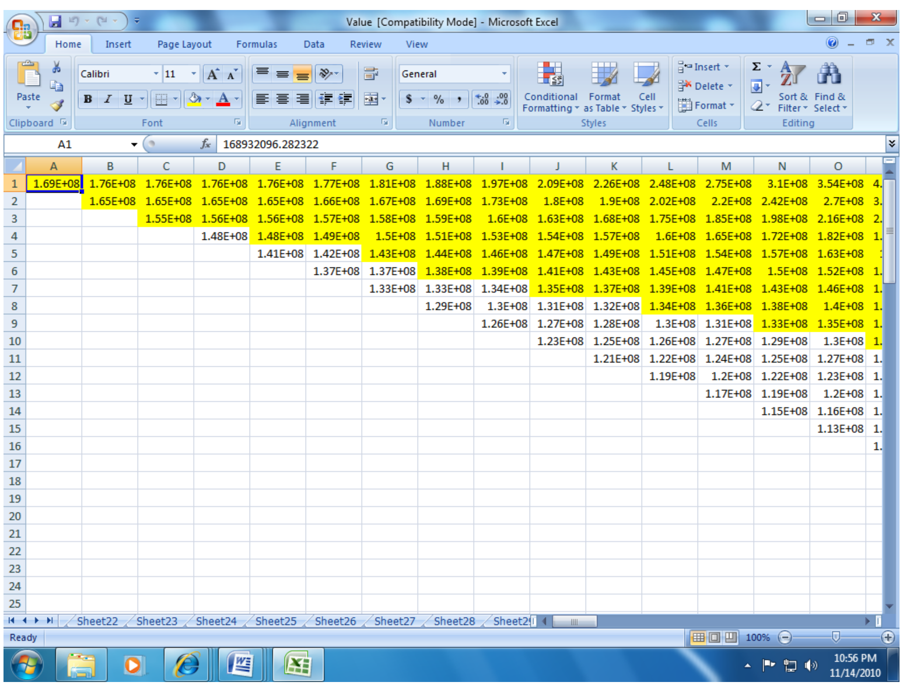


Figure 24: NPV of the Base Case with High Cost and Mean Reversion Price Model at Different Water Flooding Switching Times



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
1	1.69E+08	1.76E+08	1.76E+08	1.76E+08	1.76E+08	1.77E+08	1.81E+08	1.88E+08	1.97E+08	2.09E+08	2.26E+08	2.48E+08	2.75E+08	3.1E+08	3.54E+08	4.
2		1.65E+08	1.65E+08	1.65E+08	1.65E+08	1.66E+08	1.67E+08	1.69E+08	1.73E+08	1.8E+08	1.9E+08	2.02E+08	2.2E+08	2.42E+08	2.7E+08	3.
3			1.55E+08	1.56E+08	1.56E+08	1.57E+08	1.58E+08	1.59E+08	1.6E+08	1.63E+08	1.68E+08	1.75E+08	1.85E+08	1.98E+08	2.16E+08	2.
4				1.48E+08	1.48E+08	1.49E+08	1.5E+08	1.51E+08	1.53E+08	1.54E+08	1.57E+08	1.6E+08	1.65E+08	1.72E+08	1.82E+08	1.
5					1.41E+08	1.42E+08	1.43E+08	1.44E+08	1.46E+08	1.47E+08	1.49E+08	1.51E+08	1.54E+08	1.57E+08	1.63E+08	
6						1.37E+08	1.37E+08	1.38E+08	1.39E+08	1.41E+08	1.43E+08	1.45E+08	1.47E+08	1.5E+08	1.52E+08	1.
7							1.33E+08	1.33E+08	1.34E+08	1.35E+08	1.37E+08	1.39E+08	1.41E+08	1.43E+08	1.46E+08	1.
8								1.29E+08	1.3E+08	1.31E+08	1.32E+08	1.34E+08	1.36E+08	1.38E+08	1.4E+08	1.
9									1.26E+08	1.27E+08	1.28E+08	1.3E+08	1.31E+08	1.33E+08	1.35E+08	1.
10										1.23E+08	1.25E+08	1.26E+08	1.27E+08	1.29E+08	1.3E+08	1.
11											1.21E+08	1.22E+08	1.24E+08	1.25E+08	1.27E+08	1.
12												1.19E+08	1.2E+08	1.22E+08	1.23E+08	1.
13													1.17E+08	1.19E+08	1.2E+08	1.
14														1.15E+08	1.16E+08	1.
15															1.13E+08	1.
16																1.

Figure 25: Real Options Evaluation Results for the Low Cost Mean Reversion Case

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