Real Option Valuation Using Simulation and Exercise Boundary Fitting – Extended Abstract *

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1 Introduction

Real option analysis (ROA) is recognized as a superior method to quantify the value of real-world investment opportunities where managerial flexibility can influence their worth, as compared to standard discounted cash-flow methods typically used in industry. ROA stems from the work of Black and Scholes (1973) on financial option valuation. Myers (1977) recognized that both financial options and project decisions are exercised after uncertainties are resolved. Early techniques therefore applied the Black-Scholes equation directly to value put and call options on tangible assets (see, for example, Brennan and Schwartz (1985)). Since then, ROA has gained significant attention in academic and business publications, as well as textbooks (Copeland and Tufano (2004), Trigeorgis (1996)). However, realistic models that try to account for a number of risk factors can be mathematically complex, and in situations where many future outcomes are possible, many layers of analysis may be required. The focus of this research is the development of a real options valuation methodology geared towards practical use. A key innovation of the methodology to be presented is the idea of fitting optimal decision making boundaries to optimize the expected value, based on Monte Carlo simulated stochastic processes that represent important uncertain factors. We show how the methodology can be used to value a simple Bermudan put option and discuss convergence and accuracy issues. Then, we apply the methodology to a real options optimal build / abandon problem for a single stochastic factor.

2 Relevant Literature

The academic literature is very rich in the field of mining valuation and we begin by making the case that real option valuation is the best approach for the task at hand. Next, we provide a summary of real option methods applied in mining valuation, followed by simulation based American option valuation.

Mining projects are laced with uncertainty and many discounted cash-flow (DCF) methods have been proposed in the literature (Bastante, Taboada, Alejano, and Alonso (2008), Dimitrakopoulos

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(2011), Everett (2013), Ugwuegbu (2013)). However, the ability for managers to react to uncertainties at a future time adds value to projects, and since this value is not captured by standard DCF methods, erroneous decision making may result (Trigeorgis (1996)). An excellent empirical review of ex-post investment decisions made in copper mining showed that fewer than half of investment timing decisions were made at the right time and 36 of the 51 projects analyzed should have chosen an extraction capacity of 40% larger or smaller (Auger and Guzman (2010)). The authors were unaware of any mining firm basing all or part of their decision making on the systematic use of ROA and emphasize that the "failure to use ROA to assess investments runs against a basic assumption of neoclassical theory: under uncertainty, firms ought to maximize their expected profits". They make the case that irrational decision making exists within the industry due to a lack of real option tools available for better analysis. A number of surveys across industries have found that the use of ROA is in the range of 10-15% of companies, and the main reason for lack of adoption is model complexity (Hartmann and Hassan (2006), Block (2007), Truong, Partington, and Peat (2008), Bennouna, Meredith, and Marchant (2010), Dimitrakopoulos and Abdel Sabour (2007)). As mentioned, this work is focused on developing a practical Monte Carlo simulation-based real options methodology as Monte Carlo simulation can be easily understood by managers and allows for the modelling of multiple stochastic factors (Longstaff and Schwartz (2001)).

Several guidelines/codes have been developed to standardize mining valuation (CIMVAL (2003), VALMIN (2015)). The main mining valuation approaches are income (i.e. cash-flows), market or cost based and the focus of this paper is on income-based real option valuation, which resemble American type financial options. Earlier real option works focused on modelling price uncertainty only (Brennan and Schwartz (1985), Dixit and Pindyck (1994), Schwartz (1997)) however the complexity in mining is significant and there are numerous risk factors. Simpler models based on lattice and finite difference methods (FDM) are difficult to implement in a multi-factor setting (Longstaff and Schwartz (2001)) and, also, it is extremely difficult to account for time dependant costs with multiple decision making points (Dimitrakopoulos and Abdel Sabour (2007)). However, the simpler models continue to merit attention (Haque, Topal, and Lilford (2014), Haque, Topal, and Lilford (2016)). Dimitrakopoulos and Abdel Sabour (2007) utilize a multi-factor least squares Monte Carlo (LSMC) approach to account for price, foreign exchange and ore body uncertainty under multiple pre-defined operating scenarios (states). However, the model only allows for operation and irreversible abandonment - aspects such as optimal build time, expansion and mothballing are not considered. Similarly, Mogi and Chen (2007) use ROA and the method developed by Barraquand and Martineau (2007) to account for multiple stochastic factors in a four-stage gas field project. Abdel Saboura and Poulin (2010) develop a multi-factor LSMC model for a single mine expansion. A review of 92 academic works found that most real options research is focused on dealing with very specific situations where usually no more than two real options are considered (Savolainen (2016)). While the LSMC allows for a more realistic analysis, methods presented to date are applicable only for the case where changes from one state to another does not change the fundamental stochastic factors with time. For example, modular expansion would be difficult to implement in such a model if the cost to expand was a function of time and impacts extracted ore quality due to the changing rate of extraction – these issues were considered in Davison, Lawryshyn, and Zhang (2015) and Kobari, Jaimungal, and Lawryshyn (2014). Also, modeling of multiple layers is still complex and will not lead to a methodology that managers can readily utilize.

A somewhat recent review of the valuation of American options was provided by Barone-Adesi (2005) where the LSMC of Longstaff and Schwartz (2001) was highlighted as the most innovative, but other similar Monte Carlo based approaches have been proposed (Barraquand and Martineau (2007)) and the literature is abundant on the utilization of simulation and dynamic programming to value American options. While there are many articles providing numerical or analytical approximations to an American exercise boundary (e.g. Barone-Adesi and Whaley (1987), Ju (1998), Tung (2016), Del Moral, Remillard, and Rubenthaler (2012)), we only found the work of Del Moral, Remillard, and Rubenthaler (2012) where a forward Monte Carlo valuation method was proposed, however the exercise boundary was estimated using the analytical method of Barone-Adesi and Whaley (1987), which negates the ability to develop a general model. One reason why our proposed approach may not have been presented is that most works are focused on improving efficiency and accuracy of the pricing models. In the real options context, where many assumptions are required to estimate the cash-flows, accuracy is not as important – what is important is ease of implementation and comprehension by decision makers.

3 Theory

We begin the theoretical discussion with a motivating example. Consider the case of a greenfield site, where the life of the mine lease is 2 years, construction will take half a year, S_t, the ore price follows geometric Brownian motion (GBM) and the per unit costs are K to construct, C_{ab} to abandon and C_{op} is the operating cost rate. For a given set of parameters, the scenarios are depicted in Figure 1 in a binomial tree. The S_t process of the first panel is used to determine the operating cash-flow, calculated as $CF_t = S_t - C_{op}$. For this case, we assume that abandonment can occur at year 2 only, with cost C_{ab}. The real option can be valued in a recursive manner and the different scenarios are presented in Figure 2. Since it takes half a year for construction, the latest we would construct the mine is at year 1. In this case, only the cash-flows associated with the last period are of value and these are discounted twice to year 1 (relevant probabilities and discounting factor were used) to determine the expected value. At year 1, there are 3 possible values for S_t and thus three possible valuations for the cash-flows. Clearly, we would only invest if the total expected value of the cash-flows minus the investment cost, K, is greater than 0. As shown, only one of the three scenarios has a positive value, the others are set to 0. We continue to discount these expected values to reach a valuation of \$1.0 at year 0. Similar valuations are done for the case of building at years 0.5 and 0. Based on the analysis, we see that it is best to wait one period (half year) before constructing and the overall project value is determined to be \$2.9. Note that even for this very simple problem, a separate binomial tree was required at each decision making time point. If we allowed for early abandonment, many more trees would be required. If we added a second stochastic factor, we would have another spatial dimension. Clearly, to value a complex real option the model's complexity increases substantially. This complexity leads us to the overall objective of developing a practical simulation based real options methodology that can model realistic decision-making scenarios encountered in industry. Our specific emphasis in this work will be to explore theoretical / numerical aspects associated with the simulation methodology as they pertain to 1) a Bermudan put option, 2) a Bermudan-like option with variable strike price K and 3) a build / abandon real option example.

Price Process (St)					**Cash-Flow per Period**				
0	**0.5**	**1**	**1.5**	**2**	**0**	**0.5**	**1**	**1.5**	**2**
				13.3					15.3
			12.4					11.8	
		11.5		11.5			7.6		6.6
	10.7		10.7			3.7		3.7	
10.0		10.0		10.0	0.0		0.0		-1.0
	9.3		9.3			-3.4		-3.4	
		8.7		8.7			-6.6		-7.6
			8.1					-9.6	
				7.5					-13.3

Figure 1: Price process and resulting cash-flow.

Build at Year 1					**Build at Year 0.5**					**Build at Year 0**				
0	**0.5**	**1**	**1.5**	**2**	**0**	**0.5**	**1**	**1.5**	**2**	**0**	**0.5**	**1**	**1.5**	**2**
				15.3					15.3					15.3
			11.6					23.4					23.4	
		3.1		6.6			16.4		6.6			24.0		6.6
	1.8		3.4			5.0		7.1			14.5		7.1	
1.0		~~0.0~~		-1.0	**2.9**		1.3		-1.0	0.6		1.3		-1.0
	0.0		-3.7			~~0.0~~		-7.1			-6.8		-7.1	
		~~0.0~~		-7.6			-11.9		-7.6			-18.5		-7.6
			-9.8					-19.4					-19.4	
				-13.3					-13.3					-13.3

Figure 2: Real option valuation based on different build options.

3.1 Bermudan Put Option

For the Bermudan put option, we consider a GBM stock price process, S_t, as

$$dS_t = rS_t dt + \sigma S_t d\widehat{W}_t, \tag{1}$$

where r is the risk-free rate, σ is the volatility and $\widehat{W}_t$ is a Wiener process in the risk-neutral measure. We assume the payoff of the option to be $\max(K - S_t, 0)$ and can be exercised at times $t = \tau$ and $t = T$ where $\tau < T$. The value of the put option can be written as

$$V_0 = e^{-r\tau} \int_0^\infty \max\left(K - x, P_{BS_{put}}(x, \tau, T, r, \sigma, K)\right) f_{S_\tau}(x|S_0) dx, \tag{2}$$

where $P_{BS_{put}}(x, \tau, T, r, \sigma, K)$ is the Black-Scholes formula for the value of a European put option with current stock price x, maturity $T - \tau$, risk-free rate r, volatility σ and strike K, and $f_{S_\tau}(x|S_0)$ is the density for S_τ given S_0. As can be seen in equation (2), the optimal exercise occurs when

$$K - \theta^* = P_{BS_{put}}(\theta^*, \tau, T, r, \sigma, K), \tag{3}$$

where θ^* is used to denote the exercise price at $t = \tau$. Equation (3) can be solved using numerical methods and thus the option value simplifies to

$$V_0 = e^{-r\tau} \left(\int_0^{\theta^*} (K - x) f_{S_\tau}(x|S_0) dx + \int_{\theta^*}^\infty P_{BS_{put}}(x, \tau, T, r, \sigma, K) f_{S_\tau}(x|S_0) dx \right), \tag{4}$$

which can be solved using standard numerical methods.

To explore numerical issues regarding the proposed boundary fitting methodology in the context of the Bermudan put option, we simulate N risk-neutral paths for S_t. For a given exercise price θ the value of the option for the i-th path is given by

$$V_0^{(i)}(\theta) = \mathbb{1}_{S_\tau^{(i)} \le \theta} \left(K - S_\tau^{(i)}\right) e^{-r\tau} + \mathbb{1}_{S_\tau^{(i)} > \theta} \max\left(K - S_T^{(i)}, 0\right) e^{-rT}. \tag{5}$$

where $S_t^{(i)}$ represnets the value of S_t of the i-th simulated path. The optimal exercise price can then be estimated as

$$\theta^* = \arg\max_\theta \frac{1}{N} \sum_{i=1}^N V_0^{(i)}(\theta), \tag{6}$$

and the option value estimate becomes

$$V_0^{sim} = \frac{1}{N} \sum_{i=1}^N V_0^{(i)}(\theta^*). \tag{7}$$

Note that $\lim_{N\to\infty} V_0^{sim} = V_0$, as required.

3.2 Bermudan Option with Variable Strike

Next, we consider a Bermudan-like option with a variable strike K. This scenario represents a simplification of the idea of the optimal plant build size of a real option project valuation. We utilize the same stock price process as above (equation (1)). In this scenario, the option holder has the opportunity to exercise the option at τ ($\tau < T$) at a cost of

$$C_K = \mathbb{1}_{K>0}(aK + b), \tag{8}$$

where $a > 0$ and $b > 0$ are some constants, to receive a payoff of $\min(S_T, K)$ at time T.

The value of the option at $t = \tau$ if exercised is

$$V_\tau^+ = e^{-r(T-\tau)} \int_0^\infty \min(x, K) f_{S_T}(x|S_\tau) dx \tag{9}$$

$$= S_\tau \Phi(A) + e^{-r(T-\tau)} K \left(1 - \Phi(B)\right) \tag{10}$$

where $\Phi(\cdot)$ is the standard normal distribution and

$$A \equiv \frac{\ln \frac{K}{S_\tau} - \left(r + \frac{\sigma^2}{2}\right)(T-\tau)}{\sigma\sqrt{T-\tau}}, \; B \equiv \frac{\ln \frac{K}{S_\tau} - \left(r - \frac{\sigma^2}{2}\right)(T-\tau)}{\sigma\sqrt{T-\tau}}. \tag{11}$$

To find the optimal K we set $\frac{\partial\left(V_\tau^+ - C_K\right)}{\partial K} = 0$ and solve for K

$$K_{opt}(S_\tau) = S_\tau e^{\left(r - \frac{\sigma^2}{2} - \sqrt{2}\sigma \operatorname{erf}^{-1}\left(2ae^{r(T-\tau)} - 1\right)\right)(T-\tau)}. \tag{12}$$

Furthermore, if we assume a maximum capacity of K_{max} then we can define

$$K^*(S_\tau) \equiv \min(K_{opt}(S_\tau), K_{max}) \tag{13}$$

and substituting $K = K^*(S_\tau)$ in equation (10),

$$V_\tau^{+*}(S_\tau) \equiv S_\tau \Phi(A) + e^{-r(T-\tau)} K^*(S_\tau) \left(1 - \Phi(B)\right) \tag{14}$$

The option value at $t = 0$ is thus

$$V_0 = e^{-r\tau} \mathbb{E}\left[\max\left(V_\tau^{+*}(S_\tau) - C_{K^*}(S_\tau), 0\right)\right] \tag{15}$$

$$= e^{-r\tau} \int_0^\infty \max\left(V_\tau^{+*}(x) - C_{K^*}(x), 0\right) f_{S_\tau}(x|S_0) dx. \tag{16}$$

To utilize simulation to estimate the option value, we assume a parametric function for K^* as a function of S_τ for the form $g(S_\tau|\vec{\theta})$, where $\vec{\theta} = [\theta_1, \theta_2, ..., \theta_n]'$ is a vector of constants. For a given $\vec{\theta}$, the option value of the i-th path is given by

$$V_0^{(i)}(\vec{\theta}) = e^{-rT} \min\left(S_T^{(i)}, g(S_\tau^{(i)}|\vec{\theta})\right) - e^{-r\tau} C_K\left(g(S_\tau^{(i)}|\vec{\theta})\right) \tag{17}$$

and the optimal parameters can be determined by

$$\vec{\theta}^* = \arg\max_{\vec{\theta}} \frac{1}{N} \sum_{i=1}^N V_0^{(i)}(\vec{\theta}) \tag{18}$$

from which the option value can be estimated as

$$V_0^{sim} = \frac{1}{N} \sum_{i=1}^N V_0^{(i)}(\vec{\theta}^*). \tag{19}$$

3.3 Build / Abandon Real Option Example

In this subsection we develop our boundary fitting methodology for a build / abandon real option example. As above, we simulate N risk-neutral paths for S_t. We assume parametric functions $f_B(S_t, t; \vec{\theta}_B)$ for the construction (build) boundary and $f_A(S_t, t; \vec{\theta}_A)$ for the abandon boundary. Defining $\lambda_t^{(i)} = \{0, 1, 2, 3\}$ as the state variable of the i-th simulation such that $\lambda_0^{(i)} = 0$, where 0 denotes the state where no construction has taken place, 1 denotes state where the plant is under consturction, 2 denotes the state where the plant is in operation and 3 donotes the state where the plant has been abandoned. We define the first passage of time when $S_t^{(i)}$ hits the build boundary,

$$\tau_B^{(i)} \equiv \min\{t > 0\,;\, S_t^{(i)} \geq f_B(S_t^{(i)}, t; \vec{\theta}_B)\}. \tag{20}$$

Similarly, the first passage of time when $S_t^{(i)}$ hits the abandon boundary after construction has begun can be defined as

$$\tau_A^{(i)} \equiv \min\left\{t > 0\,;\, S_t^{(i)} \geq f_A(S_t^{(i)}, t; \vec{\theta}_A),\, \lambda_t^{(i)} \in \{1, 2\}\right\}. \tag{21}$$

Clearly, the state variable is set as follows,

$$\lambda_t^{(i)} = \begin{cases} 0, & \text{for } t < \tau_B^{(i)} \text{ or } \tau_B^{(i)} \in \emptyset, \\ 1 & \text{for } \tau_B^{(i)} \leq t < \tau_B^{(i)} + \tau_c, \\ 2 & \text{for } \left\{\tau_B^{(i)} + \tau_c \leq t < \tau_A^{(i)}\right\} \text{ or } \left\{\tau_B^{(i)} + \tau_c \leq t \text{ and } \tau_A^{(i)} \in \emptyset\right\}, \\ 3 & \text{for } t \geq \tau_A^{(i)}, \end{cases} \tag{22}$$

where τ_c is a constant representing the time required for construction.

The real option value of the i-th path can be written as

$$V_0^{(i)}(\vec{\theta}) = -\mathbb{1}_{\lambda_T^{(i)} \geq 1}\left(Ke^{-r\tau_B^{(i)}} + C_{ab}\, e^{-r\left(\mathbb{1}_{\lambda_T^{(i)}=3}\tau_A^{(i)} + \mathbb{1}_{\lambda_T^{(i)} \in \{1,2\}}T\right)}\right) + \int_{\tau_B^{(i)}+\tau_c}^{T} \mathbb{1}_{\lambda_t^{(i)}=2} e^{-rs}\gamma\left(S_s^{(i)} - C_{op}\right) ds \tag{23}$$

where, C_{ab} is the cost to abandon or close the plant, C_{op} is the per unit operating cost and γ is the rate of extraction of the mineral. Defining $\vec{\theta} \equiv [\vec{\theta}_B, \vec{\theta}_A]'$, the optimal parameters defining the build and abandon exercise boundaries can be determined as

$$\vec{\theta}^* = \arg\max_{\vec{\theta}} \frac{1}{N}\sum_{i=1}^{N} V_0^{(i)}(\vec{\theta}) \tag{24}$$

from which the option value can be estimated as

$$V_0^{sim} = \frac{1}{N}\sum_{i=1}^{N} V_0^{(i)}(\vec{\theta}^*). \tag{25}$$

4 Results

In the following subsections we present some results of the simulation experiments that were performed for 1) the Bermudan put option, 2) the Bermudan-like option with variable strike price K and 3) the build / abandon real option example.

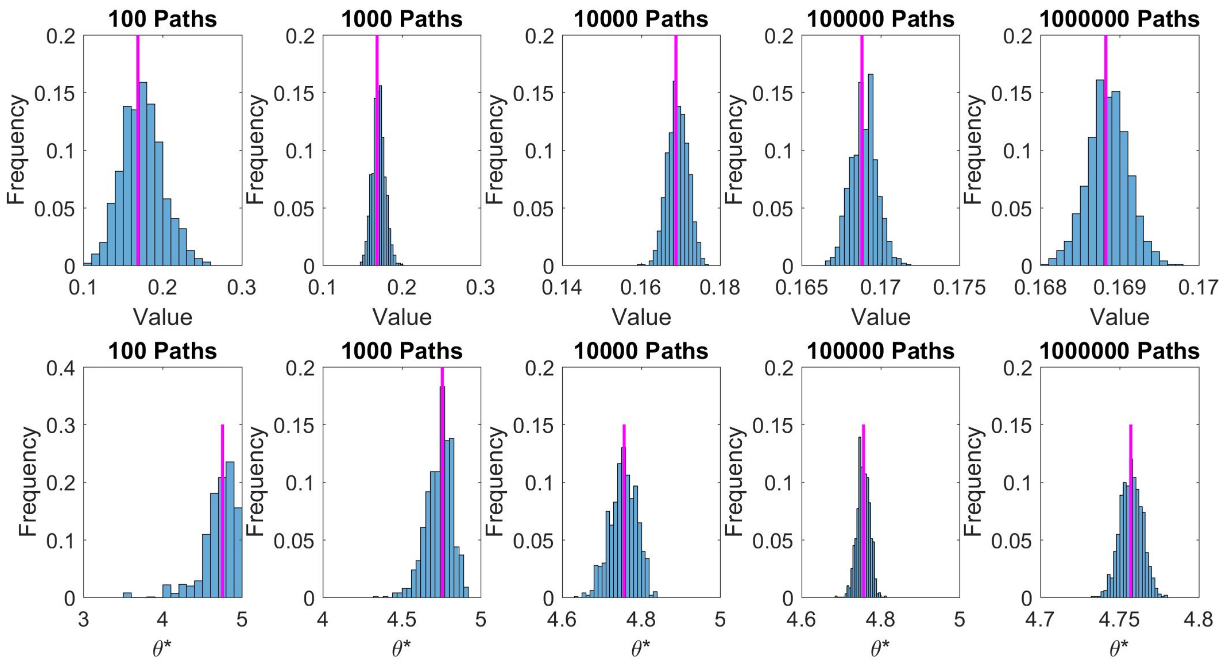

Figure 3: Histograms of V_0^{sim} for the Bermudan put option (note that each case was simulated 200 times).

4.1 Bermudan Put Option

For the Bermudan option, we assume the following parameters:

- $S_0 = 5$
- $K = 5$
- $\tau = 1$
- $T = 2$
- $r = 3\%$
- $\sigma = 10\%$.

For these parameters the pseudo-analytical results, using equations (4) and (3) respectively, are:

- $V_0 = 0.1688$
- $\theta^* = 4.7571$.

Histograms of V_0^{sim} of equation (7) resulting from the simulations are presented in Figure 3, where the number of simulation paths was varied from $N = 10^2$ to $N = 10^6$. In each case, 200 simulations were performed. In Figure 4 we present the standard deviation of the 200 simulations as a function of N. As expected, the variance in the results reduces as N is increased and the converged values for V_0 and θ^* approach those of the pseudo-analytical solution, as expected.

4.2 Bermudan Option with Variable Strike

In this subsection we present the results of the Bermudan-like option where the strike K is variable. We use the same parameter values as in Subsection 4.1 with $a = 0.5$ and $b = 1.0$ of equation (8).

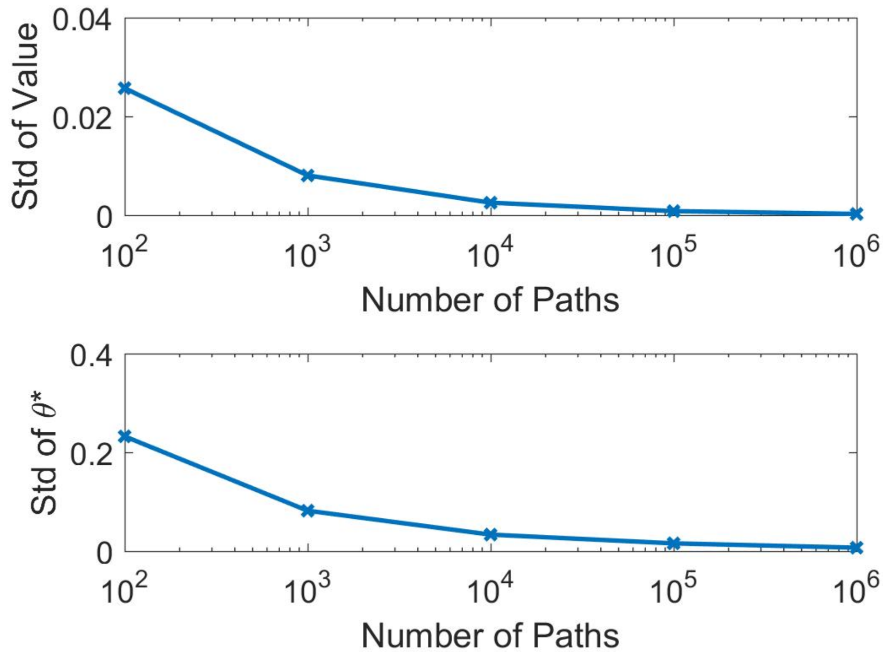

Figure 4: Simulation convergence for the Bermudan put option.

For the parametric function representing K^* we use a second order polynomial,

$$g(x|\vec{\theta}) = \theta_1 x^2 + \theta_2 x + \theta_3. \tag{26}$$

In Figure 5 we plot $V_\tau^+ - C_K$ as a function of S_τ and K using equations (10) and (8), respectively. Setting $K_{max} = 10$, the resulting histograms of V_0^{sim} of equation (19) are plotted in Figure 6 and those of $\vec{\theta}^*$ of equation (18) in Figure 7. Note the bi-modal distribution for θ_3^* is likely due to the fact that we are using a second order polynomial where a line will likely suffice. In Figure 8 we present the standard deviation of the option value using 200 simulations as a function of N. As expected, the variance in the results reduces as N is increased and the converged values approach those of the analytical solution. A plot of the simulated and actual K^* as a function of S_τ in Figure 9 shows that as N is increased, the simulated results converge to the actual analytical ones.

4.3 Build / Abandon Real Option Example

For the build / abaondon real option example, we assume the following parameters:

- $S_0 = 100$
- $K = 5$
- $C_{op} = 100$
- $C_{ab} = 1$
- $T = 2$ years

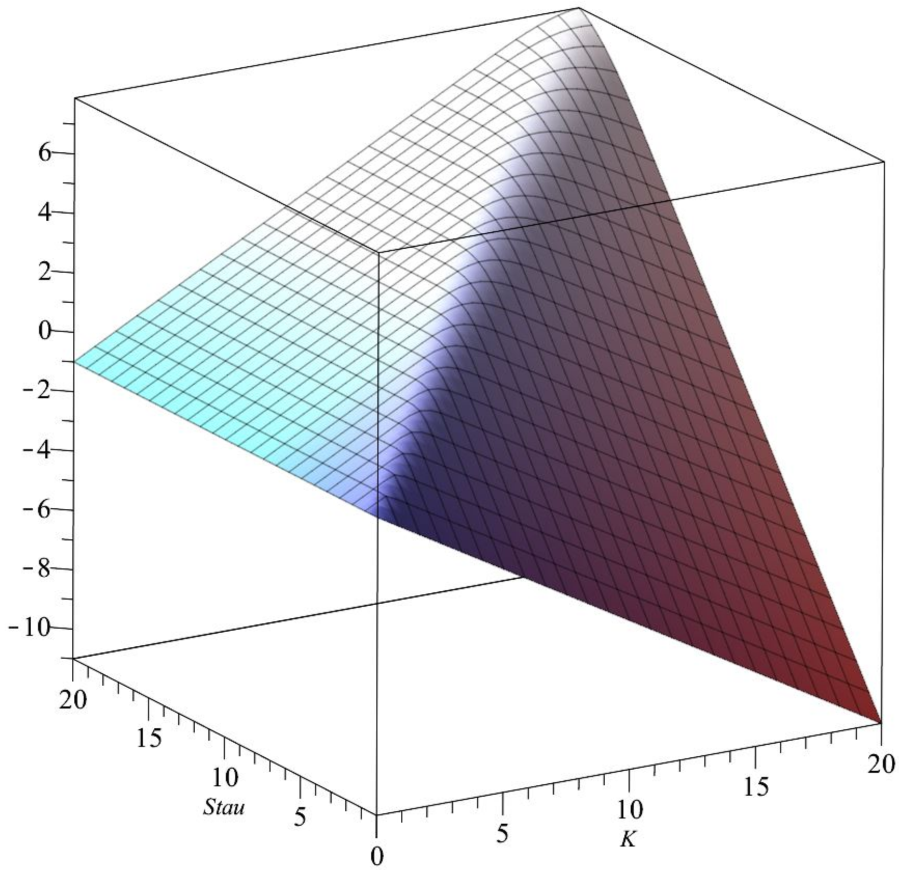

Figure 5: $V_{\tau+} - C_K$ as a function of S_τ and K.

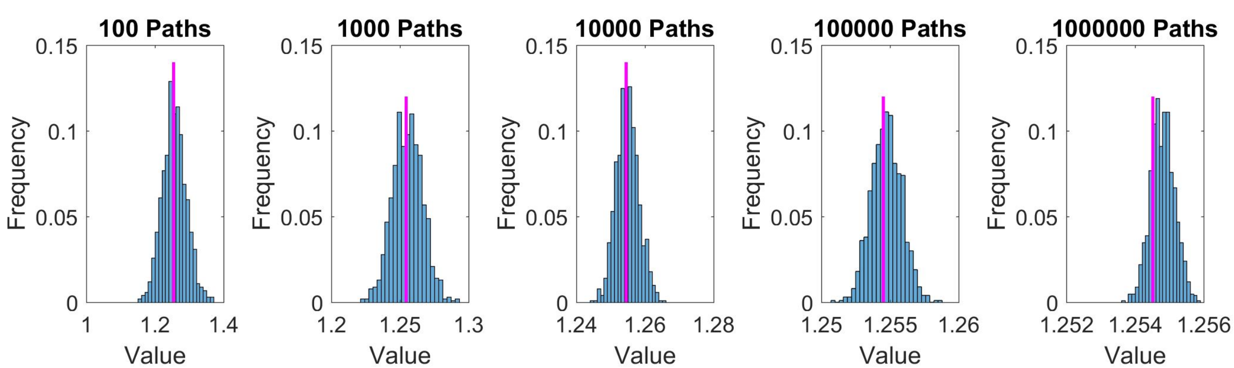

Figure 6: Histograms of V_0^{sim} for the variable strike Bermudan-like option (note that each case was simulated 200 times).

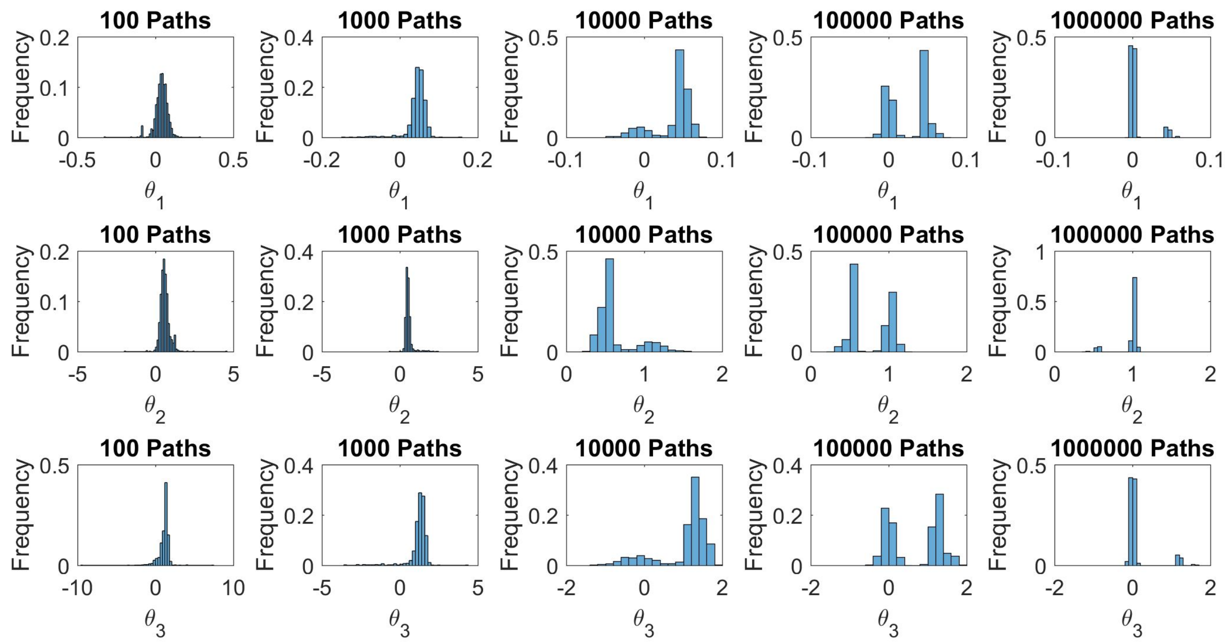

Figure 7: Histograms of $\vec{\theta}^*$.

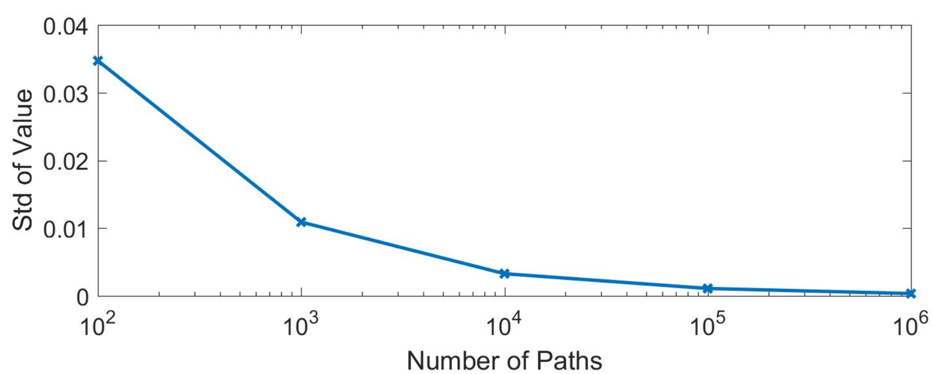

Figure 8: Simulation convergence for the variable strike Bermudan-like option.

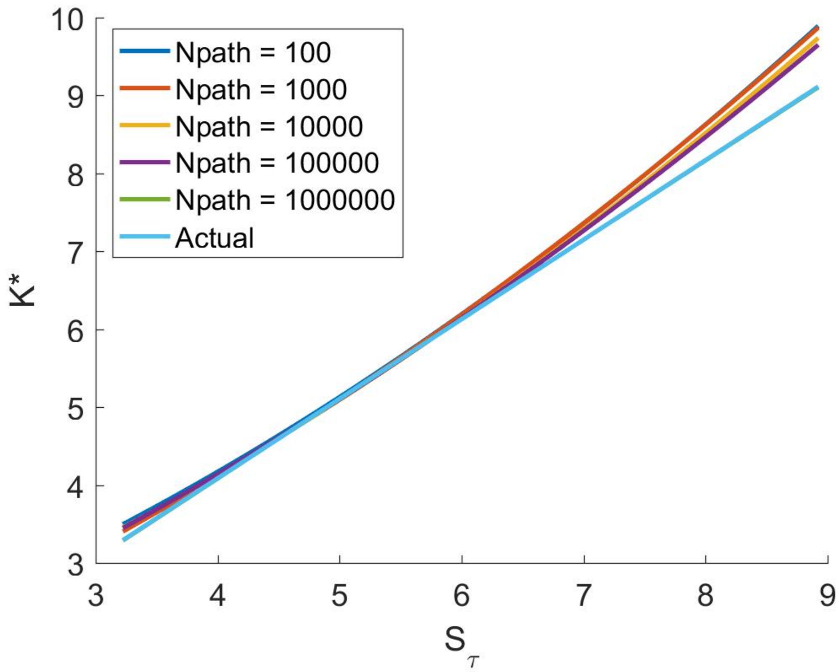

Figure 9: Simulated and actual K^* as a function of S_τ for the variable strike Bermudan-like option (note that the green line for the simulated case of $N = 10^6$ lies directly under the light blue (actual) line).

- $\tau_c = 0.5$ years
- $\gamma = 1.0$
- $r = 3\%$
- $\sigma = 10\%$.

In Figure 10 we plot the histograms for V_0^{sim} of equation (25) for varying N, using, as before 200 simulations. A few select build / abandon boundaries for varying N are plotted in Figure 11. Again, we see convergence is achieved.

5 Conclusions

The focus of this research was to present of a real options valuation methodology geared towards practical use. A key innovation of the methodology is the idea of fitting optimal decision making boundaries to optimize the expected value, based on Monte Carlo simulated stochastic processes that represent important uncertain factors. We showed how the methodology can be used to value a simple Bermudan put option. Then, we presented a Bermudan-like option where the strike was variable. This type of option is a simplification of the situation where managers have the option to build an optimal sized plant. For both the Bermudan and the Bermudan-like variable strike options convergence to the analytical values was achieved as the number of simulation paths were increased. Finally, we presented a simple build / abandon real option. As mentioned, to value a complex real option with multiple stochastic factors leads to model complexity that may make

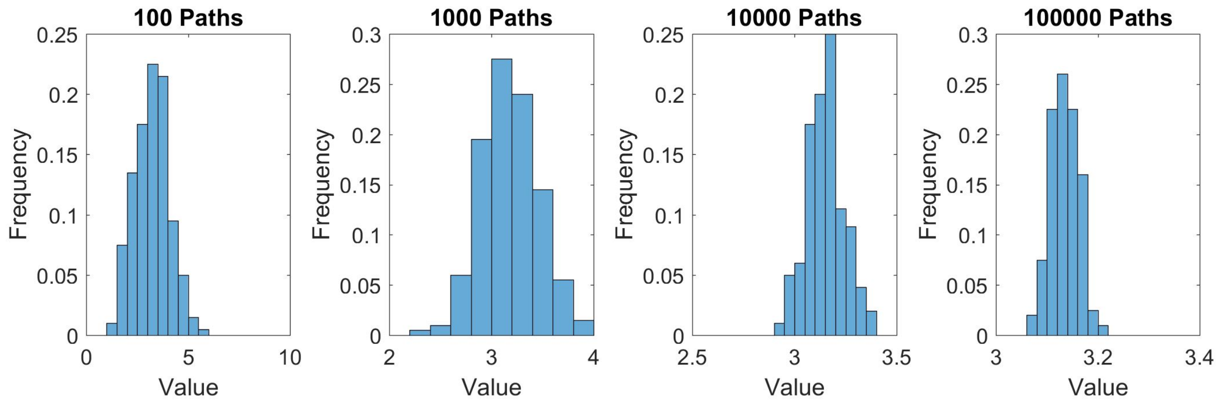

Figure 10: Histograms of V_0^{sim} for the build / abaondon real option (note that each case was simulated 200 times).

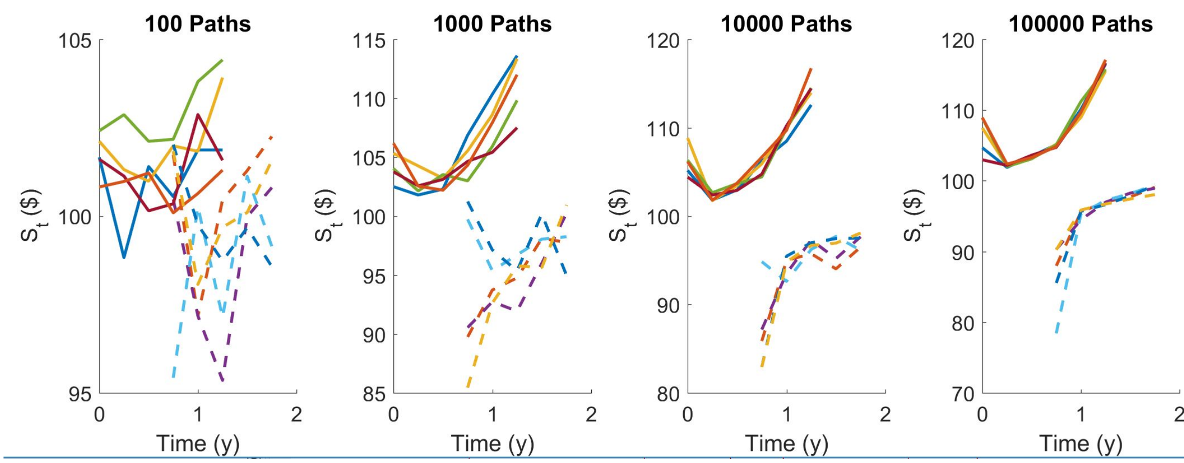

Figure 11: Example build / abaondon real option boundaries.

the analysis intractable. Our theoretical and numerical presentation of exercising boundary fitting shows how the complexity can be overcome through the use of Monte Carlo simulation. We feel that the methodology presented here is much more tractable in an industry setting for it is simple enough for managers to understand, yet can account for important real world factors that make the real options model suitable for valuation.

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