Investment Decisions with Finite-Lived Collars

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Abstract

Most collar arrangements provided by governments to encourage early investment in infrastructure, renewable energy facilities, or other projects with social objectives are finite, not perpetual. We provide an analytical solution for finite American collars, subtracting from the value of a perpetual collar the discounted forward start collar. These perpetual and finite collars are composed of perpetual floors and ceilings, and floor and ceiling annuities, and pairs of put and call American options. What is the difference between perpetual and finite collars? Lots, including different vega signs, and substantially different values for different current price levels. A critical consideration in negotiating the floors/ceilings/duration of finite collars is the current price level and expected volatility over the life of the contract.

Keywords: Perpetual and Finite Collar Options, Price Floors and Ceilings, Investment Opportunities

JEL Classification Codes: C72, D81, G31, H25, Q48

1 Introduction

The analysis of collars adopts a real option formulation because the guarantee on the downside and bonus compensation for the government on the upside are expressible as real options. We use an American perpetuity model and forward start European model, and show both the present values at any stage of the floor or ceiling, and the separate values for each of the four options.

There are several examples of finite collars. Couture and Gagnon (2010) describe a Spanish 2007 "variable premium" that involves a floor and cap, where the highest premium (over the market electricity price) is paid when the electricity price is low, and zero when the price exceeds a ceiling (so the facility owner receives all of the higher price). González (2008) provides some detail on these premium collars, which were (€cents/kWh) 25.4-34.4 for solar, 7.3-8.5 for small on-shore wind, and 15.4-16.6 for energy crops in 2007. de Miera et al. (2008) illustrates how this system that government guarantees for infrastructure projects should involve a European collar. Fernandes et al. (2015) suggest a collar-type insurance for wind power in Brazil, where the generator has promised to supply power even during times where there is little wind.

There are some analytical studies for perpetual floors/ceilings. Takashima et al. (2010) design a private-public partnership (PPP) deal involving government debt participation that incorporates a floor on the future maximum loss level where the investor has the right to sell back the project whenever adverse conditions emerge. Barbosa et al. (2017) develop a model for a feed-in tariffs contract with a minimum price guarantee (price-floor regime) with regulatory uncertainty. Armada et al. (2012) make an analytical comparison of various subsidy policies including minimum revenue guarantees. Adkins and Paxson (2017) provide analytical solutions for perpetual collars, floors and ceilings, plus partial floors and ceilings, and show the sensitivity of these collars to changes in most of the parameter values.

Our contribution consists of analytical models for a post-investment (ACTIVE) finite collar, based on the forward start model in Shackleton and Wojakowski (2007), also used in Pereira and Rodrigues (2014), which are extended to cover investment opportunities in projects with collars.

This paper is organized in the following way. In the next section, we outline the basic real option investment model, for later comparisons. In section 3, we show the analytical solution for perpetual collars, and derive an analytical solution for finite collars. In section 4, we show the analytical solution for floor only and ceiling only finite arrangements. In section 5, further insights are gained from performing a numerical sensitivity analysis, and discussing potential applications and interpretations. Section 6 is a conclusion, and suggests several extensions.

2 The plain investment opportunity

Let us start by presenting the well known solution for a plain perpetual investment opportunity (for details see, for instance, Dixit and Pindyck (1994)).

Consider a monopolistic firm with the option to invest in a project whose value depends on a single source of uncertainty that, in our case, corresponds to the unitary output price P, exogenously defined, which is assumed to follow a geometric Brownian motion process:

$$dP = \alpha P dt + \sigma P dz \tag{1}$$

where α and σ denote the risk-neutral drift and the volatility, respectively, and dz is an increment of the standard Wiener process. Additionally, $\alpha = r - \delta$, where r stands for the risk-free rate and δ is a return shortfall. Assume the project requires an investment cost K, allowing the firm to produce a fixed output quantity Q. For the sake of simplicity, operating costs and taxes are not considered. After investing the value of the active project is simply:

$$V(P) = \frac{PQ}{\delta}. \tag{2}$$

Following the standard arguments, the value of a monopolistic opportunity to invest in this project, $F(P)$, given by:

$$F(P) = \begin{cases} \left(V(\hat{P}) - K\right)\left(\dfrac{P}{\hat{P}}\right)^{\beta_1} & \text{for } P < \hat{P} \\ V(P) - K & \text{for } P \geqslant \hat{P} \end{cases} \tag{3}$$

where $\hat{P}$ corresponds to the investment trigger:

$$\hat{P} = \frac{\beta_1}{\beta_1 - 1}\frac{\delta}{Q}K \tag{4}$$

and β_1 is the positive root the characteristic quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0$, i.e.,

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{5}$$

3 The investment opportunity with a collar

Consider a concessionaire (e.g., the government) offering the firm a contract with a collar, establishing some restriction on the output price P. In particular, consider the price floats freely subject to a price floor (corresponding to low price P_L) and a price cap (corresponding to high price P_H), where $P_H \geqslant P_L$. Whenever P lies between P_L and P_H the firm receives the market price P; however, if the price happens to be smaller that P_L or greater than P_H, the firm receives P_L or P_H instead, respectively. For the particular case where $P_L = P_H$, the collar reduces to a fixed price payment. The instantaneous revenue received by the company can be expressed as $R(P, P_L, P_H, Q) = \min\{\max\{P_L, P\}, P_H\}Q$.

On the one hand, concessionaire (the government) subsidizes prices below P_L, compensating the firm in the case of low market prices, which receives $(P_L - P)Q$ with the floor protection. On the other hand, the firm transfers excess profits to the government by paying $(P_H - P)Q$, in the case of $P > P_H$. Being arbitrarily defined, price floors and caps can be used to reduce firm's downside risk and to limit firm's profitability, i.e., the government guarantees a floor in the case of adverse circumstances, but, at the same time, captures the abnormal high returns occurring under sufficiently favorable circumstances.

The inclusion of a collar produce mixed effects on the investment opportunity. On the one hand, the price floor reduces the down-side risk impacting positively on the value of the project. On the other hand, the price cap puts a limit on the potential profits, reducing the project value. Naturally, the terms of the collar will also impact the timing of the investment.

Furthermore, regarding investment incentives policy, i.e., policy for hastening investment, it is be possible for the concessionaire to offer a collar, combining price caps and floors, such that the firm finds optimal to prompt the investment undertaking the project for the current level of the state variable P.

Let us start by presenting the solution for an investment opportunity with a perpetual collar (Adkins and Paxson 2016), and then we derive the model to assess a project with a finite-lived collar, both for the active and idle stages.

3.1 Investments with perpetual collars

The solutions for an investment opportunity with a perpetual collar can be found in Adkins and Paxson (2016). Let $V_{pC}(P)$ represent the value of an active project whose output price P is bounded by a price floor P_L and a price cap P_H, as previously explained. The solution for $V_{pC}(P)$ must satisfy the following non-homogeneous differential equation:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V_{pC}(P)}{\partial P^2} + \alpha P \frac{\partial V_{pC}(P)}{\partial P} - rV_{pC}(P) + R(P) = 0 \tag{6}$$

where $R(P) \equiv R(P, P_L, P_H, Q) = \min\{\max\{P_L, P\}, P_H\}Q$. Notice that $R(P)$ equals $P_L Q$ if $P < P_L$, PQ if P stays between P_L and P_H, and $P_H Q$ if $P > P_H$. The solution for the homogeneous part of equation (6) has the form $A_a P^{\beta_1} + A_b P^{\beta_2}$, where β_1 is as in equation (5) and $\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$. The solutions for the non-homogeneous part (the particular solutions) depend where P stands in relation to P_L and P_H. Accordingly, the particular solution for $P < P_L$ is $\frac{P_L Q}{r}$, for $P \in [P_L, P_H)$ is $\frac{PQ}{\delta}$, and for $P > P_H$ becomes $\frac{P_H Q}{r}$. Considering that $V_{pC}(0) = 0$, then $A_b = 0$ for $P < P_L$. Additionally, given that $V_{pC}(P)$ has an upside limit of $\frac{P_H Q}{r}$ whenever $P \geqslant P_H$, then A_a must be set equal to 0 in this region. Putting together the solutions for all the regions we get:

$$V_{pC}(P) = \begin{cases} \frac{P_L Q}{r} + A_{11} P^{\beta_1} & \text{for } P < P_L \\ \frac{PQ}{\delta} + A_{21} P^{\beta_1} + A_{22} P^{\beta_2} & \text{for } P_L \geqslant P < P_H \\ \frac{P_H Q}{r} + A_{32} P^{\beta_2} & \text{for } P \geqslant P_H \end{cases} \tag{7}$$

As usually, the constants $A_{11}, A_{21}, A_{22}, A_{32}$ must be found by ensuring that $V_{pC}(P)$ is continuous and differentiable along P. The solutions for the constants are as follows:

$$A_{11} = \left[\frac{P_H Q}{P_H^{\beta_1}} - \frac{P_L Q}{P_L^{\beta_1}}\right] \frac{\beta_2(r-\delta)-r}{(\beta_1-\beta_2)r\delta} \tag{8}$$

$$A_{21} = \frac{P_H Q}{P_H^{\beta_1}} \frac{\beta_2(r-\delta)-r}{(\beta_1-\beta_2)r\delta} \tag{9}$$

$$A_{22} = \frac{-P_L Q}{P_L^{\beta_2}} \frac{\beta_1(r-\delta)-r}{(\beta_1-\beta_2)r\delta} \tag{10}$$

$$A_{32} = \left[\frac{P_H Q}{P_H^{\beta_2}} - \frac{P_L Q}{P_L^{\beta_2}}\right] \frac{\beta_1(r-\delta)-r}{(\beta_1-\beta_2)r\delta} \tag{11}$$

where β_2 corresponds to the negative root of the characteristic quadratic equation presented above, i.e.,

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{12}$$

Moving back to the idle stage, the value of the option to invest in a project with a perpetual collar, $F_{pC}(P)$ must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 F_{pC}(P)}{\partial P^2} + \alpha P \frac{\partial F_{pC}(P)}{\partial P} - r F_{pC}(P) = 0 \tag{13}$$

The general solution has the form $F_{pC}(P) = B_a P^{\beta_1} + B_b P^{\beta_2}$. Considering that $F_{pC}(0) = 0$ then we set $B_b = 0$. The arbitrary constant B_a is found using the value matching condition $F_{pC}(P^*) = B_a P^{*\beta_1} = V_{pC}(P^*) - K$, i.e., $B_a = (V_{pC}(P^*) - K)\left(\frac{1}{P^*}\right)^{\beta_1}$. Accordingly, the complete solution for $F_{pC}(P)$ comes:

$$F_{pC}(P) = \begin{cases} (V_{pC}(P^*) - K)\left(\frac{P}{P^*}\right)^{\beta_1} & \text{for } P < P^* \\ V_{pC}(P) - K & \text{for } P \geq P^* \end{cases} \tag{14}$$

The investment trigger, P^*, is obtained by solving numerically the following equation:

$$\beta_1(V_{pC}(P^*) - K) - V'_{pC}(P^*)P^* = 0 \tag{15}$$

which corresponds to the well known smooth-pasting condition.[1]

Technically, the solutions for $F_{pC}(P)$ and for the trigger are obtained using the so-called value-matching (VM) and smooth-pasting (SP)conditions, as presented in the Appendix.

Notice that the trigger P^* can be found either below or above P_H (but above P_L), which means the VM and the SP can be placed in all the domain $P \in [P_L, \infty)$. As we are going to see in our numerical example, there are market conditions (e.g., high volatility) or project-specific

[1]The smooth-pasting condition ensures that $\beta_1 B_a P^{*\beta_1 - 1} = V'_{pC}(P^*)$. Multiplying both sides of the equation by P^*, and substituting according to the value matching solution, we get (15).

conditions (e.g., high investment cost) for which the investment trigger is above the price cap P_H. The justification is straightforward. Consider, for instance, a project with high volatility. The investor may find optimal to wait and only invest for a sufficiently large $P(> P_H)$, even knowing that he only receive P_H, for accommodating the larger probability of significantly lower values of P in the future.

3.2 Investments with finite-lived collars

Let us now derive the model for assessing investment opportunities with finite-lived collars. We assume the government offers a similar contract that guarantees the firm a minimum price P_L but limits its gains to a maximum P_H. However, we now assume the contract has a finite duration of $T < \infty$ years. This means that, during the finite period $(t^*,\ t^* + T)$, where t^* is the investment timing, the collar is in place. After $t^* + T$ the contract ends and the firm's profits start to depend entirely on the stochastic behavior of P.

Let us begin by the solution for the active project. Immediately after being undertaken, the value of a project protected by a collar that lasts for T years is equivalent to a portfolio that includes: (i) a long position in a perpetual collar, (ii) a short position in a forward-start perpetual collar (that start after T years) and (iii) a long position in the expected profits that will start after T years. Combining (i) and (ii) replicates the finite-collar, whereas (iii) captures the value in operating the project without restrictions in P perpetually after the end of the collar.[2]

Accordingly, the value of an active project with a finite-lived collar is given by:

$$V_{fC}(P) = V_{pC}(P) - S(P) + \frac{PQ}{\delta} e^{-(r-\alpha)T} \tag{16}$$

The first term, $V_{pC}(P)$, is as presented in equation (7). The second term, $S(P)$, represents the forward-start perpetual collar (a collar that starts in the future moment T), which is value given by:[3]

$$\begin{aligned} S(P) = {} & \frac{P_L Q}{r} e^{-rT} N(-d_1(P, P_L)) + A_{11} P^{\beta_1} N(-d_{\beta_1}(P, P_L)) \\ & + \frac{PQ}{\delta} e^{-(r-\alpha)T} \left(N(d_0(P, P_L)) - N(d_0(P, P_H))\right) \\ & + A_{21} P^{\beta_1} \left(N(d_{\beta_1}(P, P_L)) - N(d_{\beta_1}(P, P_H))\right) \\ & + A_{22} P^{\beta_2} \left(N(d_{\beta_2}(P, P_L)) - N(d_{\beta_2}(P, P_H))\right) \\ & + \frac{P_H Q}{r} e^{-rT} N(d_1(P, P_H)) + A_{32} P^{\beta_2} N(d_{\beta_2}(P, P_H)) \end{aligned} \tag{17}$$

[2]Our generic model assumes a perpetual concession with a finite collar. Naturally, the model also applies for finite concessions with the same duration of the finite collar. In this case component (iii) should be ignored.

[3]See Shackleton and Wojakowski (2007) and Pereira and Rodrigues (2014) for details on the valuation of forward-start options.

where $N(.)$ is the standard normal cumulative distribution, and

$$d_\beta(P,x) = \frac{\ln\left(\dfrac{P}{x}\right) + \left(\alpha + (\beta - 0.5)\sigma^2\right)T}{\sigma\sqrt{T}}, \quad \beta \in \{0, 1, \beta_1, \beta_2\},\ x \in \{P_L, P_H\} \tag{18}$$

Naturally, the negative sign represents the short position on the forward-start perpetual collar. Finally, the last term represents the present value of the expected profits that will start after T.

Using the standard arguments (see the Appendix for details) we find that, in the stage prior investment, the value of the option to invest in the project granted with a finite-lived collar (F_{fC}) is:

$$F_{fC}(P) = \begin{cases} (V_{fC}(P^{**}) - K)\left(\dfrac{P}{P^{**}}\right)^{\beta_1} & \text{for } P < P^{**} \\ V_{fC}(P) - K & \text{for } P \geq P^{**} \end{cases} \tag{19}$$

where the optimal trigger to invest, P^{**}, is the numerical solution of the following equation:

$$\beta_1(V_{fC}(P) - K) - V'_{fC}(P)P = 0 \tag{20}$$

For the finite collar, no restrictions are required for the VM and SP, meaning that the transition between the idle and the active stages can occur for any P ($<>$ P_H).

4 Numerical Example

Some important features of the model are analyzed with a numerical example. Consider an investment option for which the following parameters apply:

Parameter	Description	Value
P	Current price of the output	\$2
P_L	Price floor	\$2
P_H	Price cap	\$6
σ	Volatility	0.25
r	Risk-free rate	0.04
δ	Return shortfall	0.04
Q	Output quantity	1
K	Investment cost	\$75
T	Duration of the collar (years)	10

Table 1: The base case parameters.

In Figure 1 we analyze the effects of the main parameters on the value of the active project for two different values of the state variable (both for the base case $P = \$2$, as well as for $P = \$4$). 1(a) and 1(b) show different sensitivities in respect to volatility. For $P = \$2$ the value of the perpetual collar reveals to be non-monotonic. For a low volatility, the moneyness of the *long* put option protection (floor) dominates. As the volatility increases, the probability for entering in the in-the-money region of the *short* call option position (cap) increases, decreasing the value of the perpetual collar. For the finite collar, theses effects are not so evident, as they are not permanent. The predominance of the *short* call option position effect is clear in 1(b), where $P = \$4$ is closer to $P_H = \$6$. Additionally, in 1(c), 1(d), 1(e), and 1(f) we see the value of both the perpetual and the finite collar increases as P_H and P_L increase. Interestingly, for low (high) price floors, the finite (perpetual) collar reveals to be more valuable. The level of P_L for the separation region increases as P increases, which means the current level of P is critical in negotiating the floor level and the finite collar duration. Additionally, the finite collar reveals not too sensitive to increases in P_H, so there would be little advantage after an initial arrangement of renegotiating the ceiling if the collar duration is short. Finally, in 1(g) and 1(h) we see that the value of the active finite collar may increases or decreases as duration T increases, depending on whether P is closer to P_L or P_H. Referring to the option concept of Θ (the sensitivity of the option value with respect to remaining time to expiration) we see that, when P is closer to P_H, the short position in the call option dominates, producing an overall value increase in the active finite collar as T decreases, i.e., $\Theta < 0$ (see Figure 1(h)). In practical terms, due to the short position, the firm benefits as the call option approaches the maturity date. On the other hand, whenever the long position in the put option becomes dominant (if P is closer to P_L) it produces an overall value decrease as T decays, revealing the traditional $\Theta > 0$ (see Figure 1(g)). Table 2 shows in detail all these effects, presenting the value decomposition for different durations and moneyness.

	$T = 10$		$T = 30$		$T = \infty$	
	$P = \$2$	$P = \$4$	$P = \$2$	$P = \$4$	$P = \$2$	$P = \$4$
Plain active project	\$50.00	\$100.00	\$50.00	\$100.00	\$50.00	\$100.00
Short Call	-\$0.21	-\$2.71	-\$2.73	-\$13.41	-\$8.99	-\$29.98
Long Put	\$3.28	\$0.66	\$10.76	\$4.69	\$20.21	\$12.13
Active with Collar	**\$53.07**	**\$97.95**	**\$58.03**	**\$91.28**	**\$61.22**	**\$82.15**

Table 2: Decomposition of the value of an active project with a collar, both for finite durations ($T = 10$ and $T = 30$) and for the perpetual case ($T = \infty$). The results are for $P = \$2$ and $P = \$4$.

[Figure 1 about here]

Figure 2 shows the sensitivities of the main parameters on the investment triggers, both for finite and perpetual collars as well as for the plain (without collar) investment opportunity. Figure 2(a) depicts the well known effect of uncertainty on investment timing (a higher volatility implies a higher trigger). In particular, it shows that the trigger for a finite-lived collar lies between those of a perpetual collar and a plain investment. Additionally, also shows that the triggers, depending on the levels of uncertainty, can be placed below or above the price cap P_H. As we already said, there is economic reasoning for the latter situation. When the uncertainty is significant, the investor may find optimal to wait and only invest for a P sufficiently large, knowing that he only receive P_H if the price trigger surpasses the cap, for accommodating the larger probability of significantly lower values of P in the future, resulting from the high volatility.

Figure 2(b) reveals the impact of the price cap P_H on the trigger and its effectiveness in promoting investment. Three regions appear. For $P_H \leqslant 3.212$, the plain project (without collars) is preferable for hastening the investment, as its trigger remains below the other triggers. If $3.212 < P_H \leqslant 4.561$, the finite collars reveals to be the most effective, whereas for $P_H > 4.561$ the perpetual collar is the one that more contributes in promoting investment.

The impact of the price floor is analyzed in Figure 2(c). Curiously, for very low P_L the investment takes place for output prices above P_H. Also interesting is the fact that for high P_L ($P_L > 4.981$ for our base case parameters), the investment trigger for the finite collar stays below the price floor. As before, the figure reveals which contract is more effective for hastening the investment for different levels of P_L.

Figure 2(d) reports the obvious positive relation between the investment cost and trigger. However, depending on the level of investment one type of collar contract (either finite or perpetual) can be preferred in hastening investment. For our parameters, if $K < 85.393$ a perpetual collar should be set, otherwise a finite collar reveals more effective. This figure also reveals that for relatively expensive projects, the investment takes place for output prices above the cap.

Finally, 2(e) shows that the trigger of a finite-lived collar lies between those of a plain project and perpetual collar. Also, for low durations of the collar the trigger can be optimally placed above the price cap P_H.

[Figure 2 about here]

In Figure 3 we study the effect of the various parameters on the value of the idle project. In particular, 3(a) shows that the value of a finite collar increases as the uncertainty increases. However, the effect of uncertainty is ambiguous when a perpetual collar is concerned. The justification is similar to the one presented for the active project value (see Figure 2(a)). The other relations are as expected. The value of both a perpetual and finite collar increases with P_H and P_L, and decreases with the investment cost (Figures 3(b), 3(c), 3(d)). Additionally, 3(e) shows that the value of the idle project lies between the value of a plain project ($T = 0$) and a perpetual collar ($F_{fC} \to F_{pC}$ as $T \to \infty$). Figure 3 also reveals that value of the finite collar dominates that of the perpetual collar.

[Figure 3 about here]

5 Partial contract: only floors or caps

The model also allows to shows the value of the investment option when only a floor or a cap is in place.

[Figure 4 about here]

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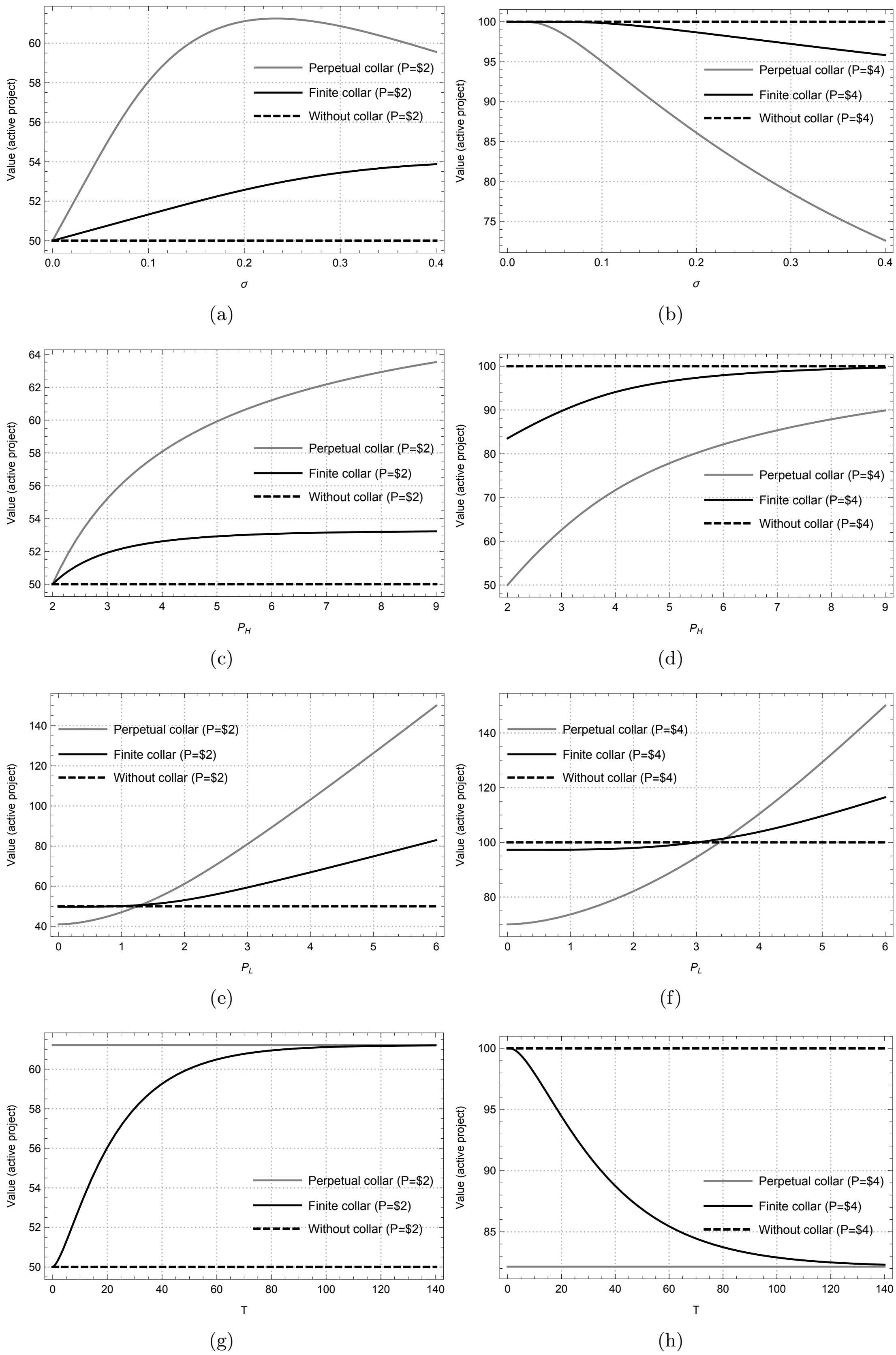
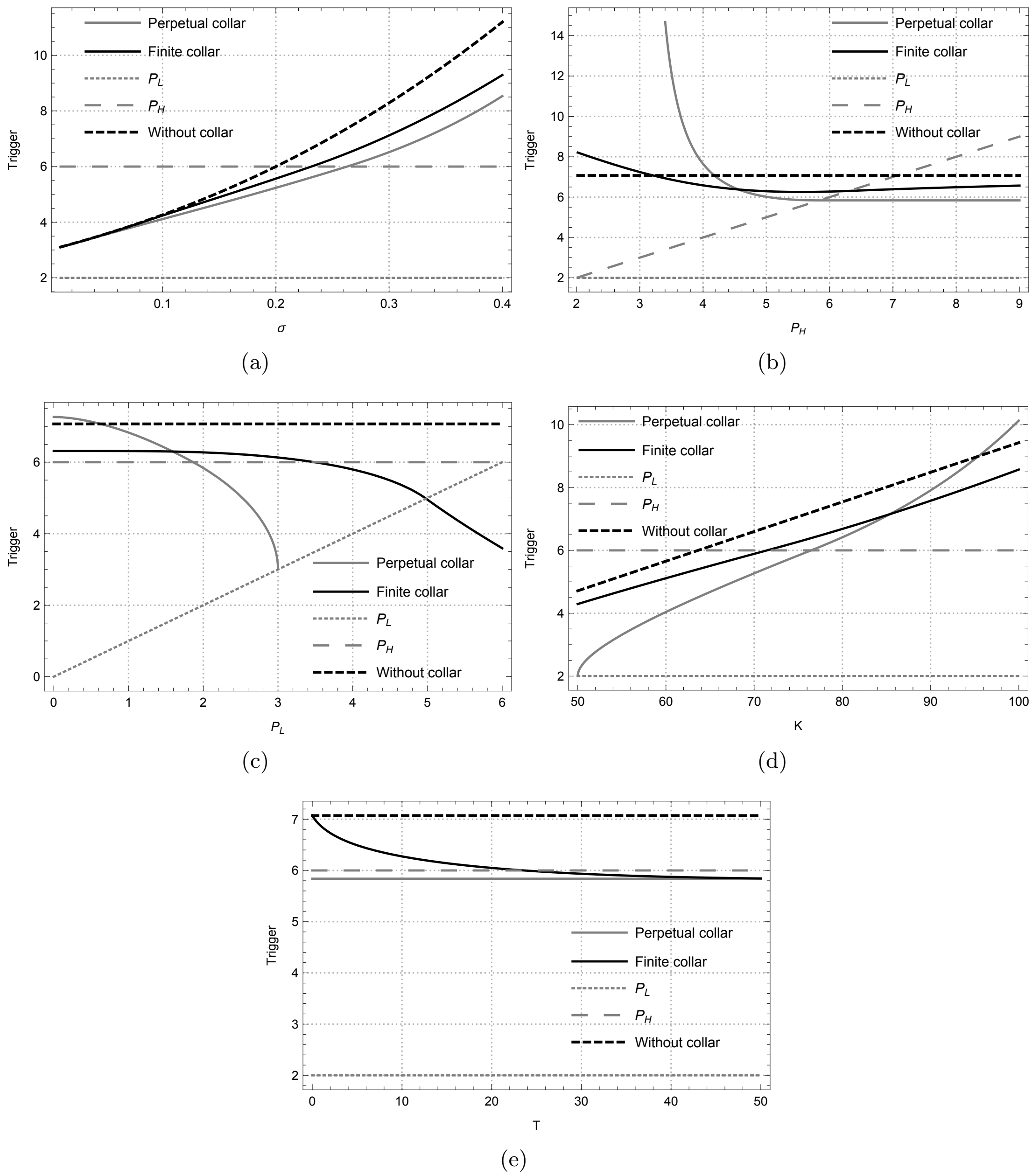
Figures



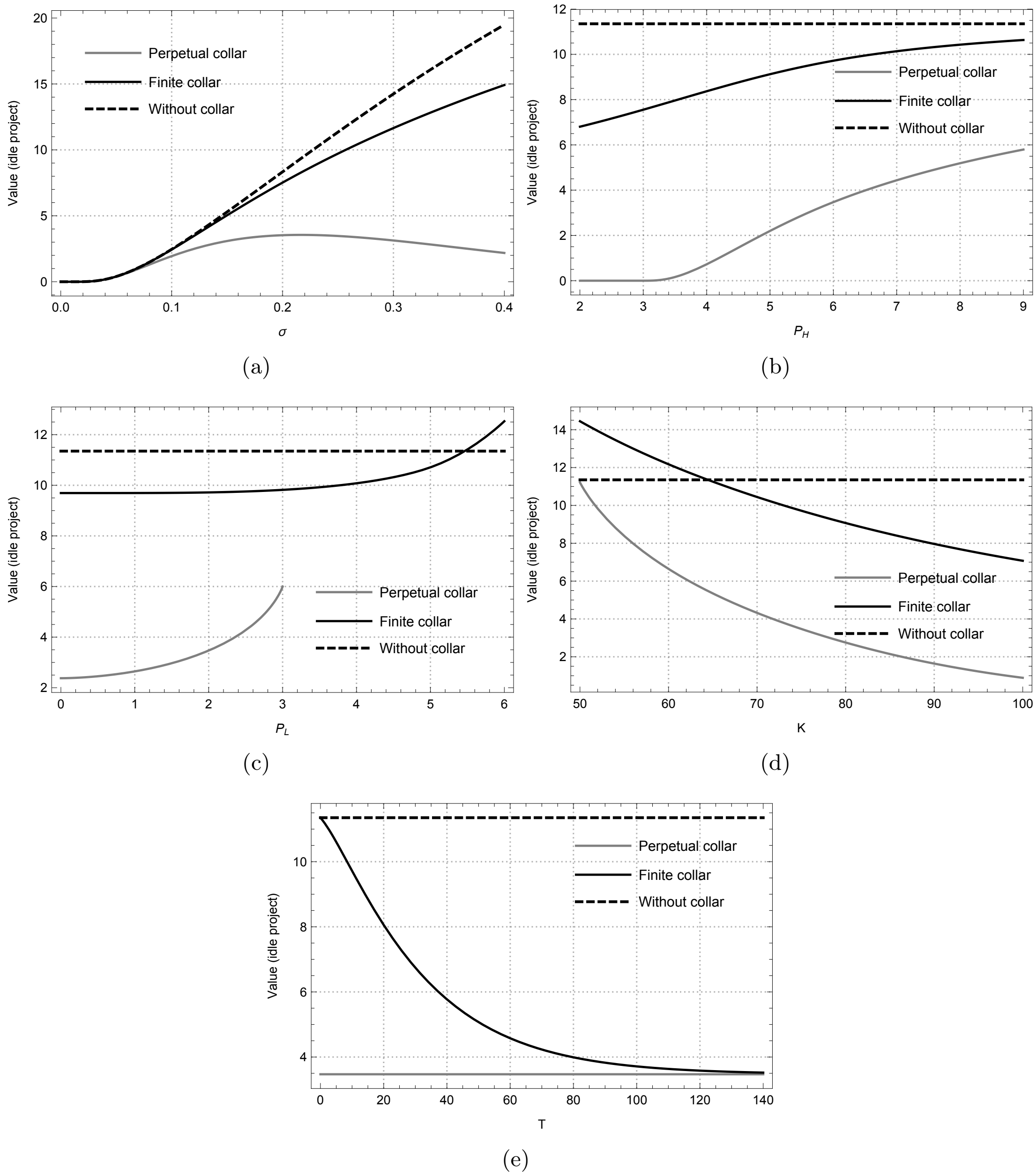
$P = \$2$ and $\$4; P_L = \$2; P_H = \$6; \sigma = 0.25; r = 0.04; \delta = 0.04; Q = 1; K = \$75; T = 10.$

Figure 1: The sensitivity analysis of the effect main parameters on the project active value: 1(a) and 1(b) for the impact volatility, 1(c) and 1(d) for the impact of the price cap, 1(e) and 1(f) for price floor, and finally 1(g) and 1(h) for the duration of the collar.



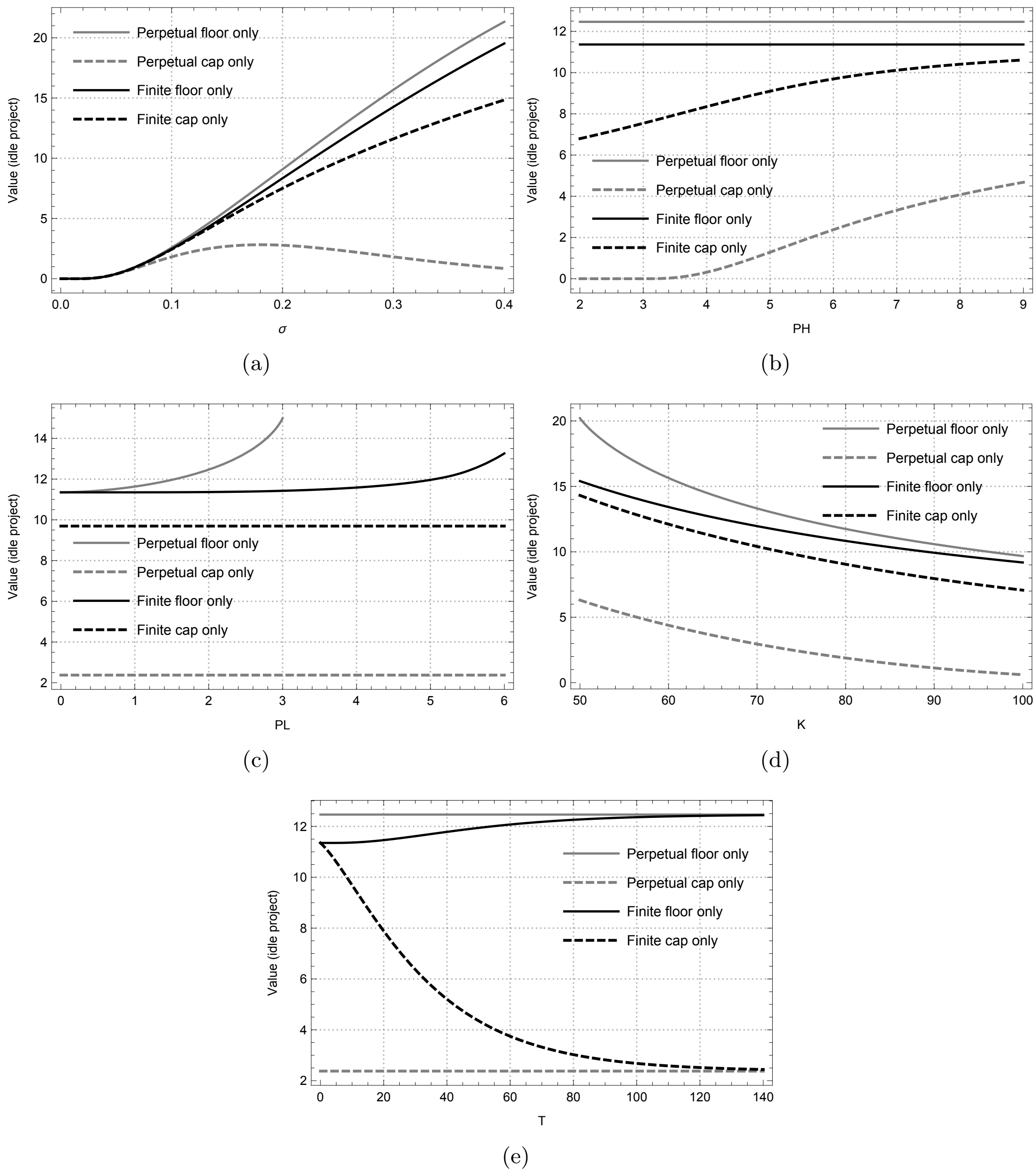
$P = \$2; P_L = \$2; P_H = \$6; \sigma = 0.25; r = 0.04; \delta = 0.04; Q = 1; K = \$75; T = 10.$

Figure 2: The sensitivity analysis of the effect main parameters on the investment trigger: 2(a) for the impact volatility, 2(b) for the impact of the price cap, 2(c) for price floor, 2(d) for the impact of investment cost, and 2(e) for the duration of the collar.



$P = \$2; P_L = \$2; P_H = \$6; \sigma = 0.25; r = 0.04; \delta = 0.04; Q = 1; K = \$75; T = 10.$

Figure 3: The sensitivity analysis of the effect main parameters on the project value: 3(a) for the impact volatility, 3(b) for the impact of the price cap, 3(c) for price floor, 3(d) for the impact of investment cost, and 3(e) for the duration of the collar.



$P = \$2; P_L = \$2; P_H = \$6; \sigma = 0.25; r = 0.04; \delta = 0.04; Q = 1; K = \$75; T = 10.$

Figure 4: The sensitivity analysis of the effect main parameters on the project active value: 4(a) for the impact volatility, 4(b) for the impact of the price cap, 4(c) for price floor, 4(d) for the investment cost and 4(e) for the duration of the collar.

A The value of a forward start collar

Shackleton and Wojakowski (2007) value separately caps and floors. Following similar arguments the value of a forward start collar is given by:

$$S(P) = e^{-rT}\mathbf{E}_0^Q\left[V_{pC}(P_T)\right] \tag{21}$$

where P_T if the price P at time T.

The value of perpetual collar starting at level P_T is given by Equation (7):

$$V_{pC}(P_T) = \begin{cases} \dfrac{P_L Q}{r} + A_{11}P^{\beta_1} & \text{for } P_T < P_L \\ \dfrac{PQ}{\delta} + A_{21}P^{\beta_1} + A_{22}P^{\beta_2} & \text{for } P_L \leqslant P_T < P_H \\ \dfrac{P_H Q}{r} + A_{32}P^{\beta_2} & \text{for } P_T \geqslant P_H \end{cases} \tag{22}$$

or in a compact notation:

$$\begin{aligned} V_{pC}(P_T) = \left(\frac{P_L Q}{r} + A_{11}{P_T}^{\beta_1}\right)\mathbf{1}_{P_T<P_L} + \left(\frac{P_T Q}{\delta} + A_{21}{P_T}^{\beta_1} + A_{22}{P_T}^{\beta_2}\right)\mathbf{1}_{P_L\leqslant P_T<P_H} \\ + \left(\frac{P_H Q}{r} + A_{32}{P_T}^{\beta_2}\right)\mathbf{1}_{P_T\geqslant P_H} \end{aligned} \tag{23}$$

where the indicator $\mathbf{1}_{condition}$ equals 1 if the *condition* is met or 0 otherwise.

From the Appendix A of Shackleton and Wojakowski (2007):

$$e^{-rT}\mathbf{E}_0^Q\left[P_T^\beta\mathbf{1}_{P_T<P_L}\right] = e^{q(\beta)T}P^\beta N(-d_\beta(P,P_L)) \tag{24}$$

$$\begin{aligned} e^{-rT}\mathbf{E}_0^Q\left[P_T^\beta\mathbf{1}_{P_L\leqslant P_T<P_H}\right] &= e^{-rT}\mathbf{E}_0^Q\left[P_T^\beta\mathbf{1}_{P_T\geqslant P_L}\right] - e^{-rT}\mathbf{E}_0^Q\left[P_T^\beta\mathbf{1}_{P_T\geqslant P_H}\right] \\ &= e^{q(\beta)T}P^\beta\left(N(d_\beta(P,P_L)) - N(d_\beta(P,P_H))\right) \end{aligned} \tag{25}$$

$$e^{-rT}\mathbf{E}_0^Q\left[P_T^\beta\mathbf{1}_{P_T\geqslant P_H}\right] = e^{q(\beta)T}P^\beta N(d_\beta(P,P_H)) \tag{26}$$

where

$$d_\beta(P,x) = \frac{\ln\left(\dfrac{P}{x}\right) + \left(\alpha + (\beta-0.5)\sigma^2\right)T}{\sigma\sqrt{T}}, \quad \beta\in\{0,1,\beta_1,\beta_2\},\ x\in\{P_L,P_H\} \tag{27}$$

$$q(0) = -r \tag{28}$$

$$q(1) = -(r-\alpha) \tag{29}$$

$$q(\beta_1) = 0 \tag{30}$$

$$q(\beta_2) = 0 \tag{31}$$

Rearranging, we obtain:

$$\begin{aligned}
S(P) = {} & \frac{P_L Q}{r} e^{-rT} N(-d_1(P, P_L)) + A_{11} P^{\beta_1} N(-d_{\beta_1}(P, P_L)) \\
& + \frac{PQ}{\delta} e^{-(r-\alpha)T} \left(N(d_0(P, P_L)) - N(d_0(P, P_H)) \right) \\
& + A_{21} P^{\beta_1} \left(N(d_{\beta_1}(P, P_L)) - N(d_{\beta_1}(P, P_H)) \right) \\
& + A_{22} P^{\beta_2} \left(N(d_{\beta_2}(P, P_L)) - N(d_{\beta_2}(P, P_H)) \right) \\
& + \frac{P_H Q}{r} e^{-rT} N(d_1(P, P_H)) + A_{32} P^{\beta_2} N(d_{\beta_2}(P, P_H))
\end{aligned} \tag{32}$$