M&A Dynamic Games, Threat Values and the Market for Corporate Control^{*}

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Abstract

This paper builds on recent advances in the domain of option games under uncertainty and looks closer at determinants that drive friendly mergers. Each firm calculates its payoff resulting from either a friendly merger or hostile takeover that then serves as a credible threat when jointly negotiating the terms of a merger. In contrast to similar papers, we show that the firms still have an incentive to delay the merger. Moreover, the results indicate that threat values are important for the asymmetric firm case, i.e. when firms have different bargaining power. The weaker firm can improve its position in the merger as uncertainty increases, i.e. its share in the new entity increases. The same holds true if synergies increase.

Keywords: M&A; Real Options; Cooperative and Non-cooperative Bargaining JEL codes: C73; D43; D81; D92; G31.

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1 Introduction

Research on Merger & Acquisition (M&A) strategies, their performance and important determinants have constantly received considerable attention in the finance literature. To date, there is great consensus regarding some empirical features of M&A activity. First, M&As occur in waves where periods of low takeover activity are followed by periods of high takeover activity and second, merger activity within a wave is considerably clustered by industries (see e.g. Andrade et al. (2001); Rhodes-Kropf and Viswanathan (2004); Martynova and Renneboog (2008); Betton et al. (2008), among others). Alike, M&A activity happens in response to major shocks and is procyclical, i.e. the number of deals is higher in economics booms than in recessions. Yet, however, there is much about the M&A process we do not fully understand. One prominent issue is the choice between hostile takeovers and friendly mergers. Both types usually coexist within a merger wave but their importance varies from one merger wave to the other. Exemplary, while hostile takeovers triggered the M&A wave in 1980s their worldwide number dropped significantly during the subsequent M&A wave in the 1990s. While this trend has continued worldwide, it does not necessarily imply that hostile takeovers are becoming less important. Rather, examples like the EURO 190 billion takeover of the German firm Mannesmann by the British mobile operator Vodafone in early 2000 or the \$162 billion takeover of US-media giant TimeWarner by America Online (AOL) the same year shows that despite their low numbers hostile takeovers account for the majority stake when comparing M&A activity by deal volume. In addition, hostile takeovers serve as credible threats in friendly merger negotiations and often induce managers to accept friendly mergers later on (Browne and Rosengren (1987)). Exemplary, the German steelmaker Krupp-Hoesch announcement in 1997 that it would seek to buy its far larger German rival Thyssen either friendly or unfriendly lead to a *David* versus *Goliath* battle in which Thyssen finally agreed to merge

with the smaller contender. Finally, Continental Europe and Japan are recent examples where the number of hostile takeovers is rising against the overall worldwide trend thereby stoking fears of losing competitiveness to foreign bidders.

While previous literature in the domain of theoretical corporate finance has predominantly looked at the effect of uncertainty, synergies, means of payment, and debt level among others has on M&A outcome, less attention has been on the firms' negotiation tactics and outside options and their impact on M&A outcome. In particular, how does the threat of a hostile bid affect merger bargaining and timing? Alike, does such a threat make friendly merger more likely? Under what circumstances are hostile takeovers superior over friendly mergers? And finally, how does uncertainty impact the negotiating tactics? This paper contributes to the M&A literature on dynamic bargaining under uncertainty and tries to provide answers to the aforementioned questions.

The paper unfolds as follows. Section 2 provides a brief overview of recent literature while Section 3 presents the derivation of the model. Section 4 analyses the optimal strategy choices and presents numerical results based on comparative-static analysis. Finally, Section 5 concludes.

2 Literature review

Even though the issue whether firms should choose to merge friendly or accept/launch hostile bids has gained less attention as opposed to other issues related to M&A waves it is not completely ignored in academia. In particular, finance research has revealed some empirical features that affect the choice. Exemplary, Betton et al. (2009) and Browne and Rosengren (1987), among others find that targets of tender offers tend to be larger than other publicity traded targets acquired in mergers. Given expensive pre-takeover advertising cost, costlier ex-post integration and the fact that targets might opt for takeover defenses Schnitzer (1996) and McSweeney (2012) argue that transaction cost are generally higher for hostile takeovers than for mergers. Other findings reveal that hostile takeovers become very likely when target firms perform poorly (Hasbrouck, 1985; Palepu, 1986; Morck et al., 1989; Mitchell and Lehn, 1990) thereby confirming the view that hostile

takeovers act as a means to discipline management. Alike, Jensen and Ruback (1983) find that average synergies achieved in an M&A are greater for hostile takeovers than for mergers and that targets do better ex-post under tender offers. Finally, the findings of Betton et al. (2008) indicate that greater concentration in an industry make it less likely that bidders are to select mergers over tender offers. Moreover, economic theory has argued that disciplinary takeovers are mainly hostile takeovers while synergetic takeovers are mainly friendly Morck et al. (1988). While achieving synergies due to increases in market power, economies of scale and scope, among others is at the core of every merger attempt, disciplinary takeovers indicate the bidder's preferences to replace the target's management because of their incapacity of maximizing shareholder wealth. This alternative view on M&A has emanated from the market for corporate control theory where according to Jensen and Ruback (1983) managers compete for the rights to control and manage corporate resources. Obviously, when such firms operate sub-optimally they signal upside potential in profitability and hostile takeovers are an efficient means to capitalize on these efficiency gains. While in the extreme this lead to a complete replacement of the target's management even the threat can act as a disciplining instrument.

Surprisingly, only a few theoretical models in finance exist that look at these issues from a classical microeconomic perspective. Exemplary, Berkovitch and Khanna (1991); Betton et al. (2009); Aktas et al. (2010), and Calcagno and Falconieri (2014), among others present models of merger negotiations in which the outside option is a tender offer. The central assumption within this domain of literature is that merger negotiations take place privately while hostile takeover bids are announced publicly and signal synergy gains to outsider that might become potential additional bidders. The findings reveal that under such a threat a unique level of synergies exist in equilibrium that motivate bidders to refrain from attempting to take over the target by means of a hostile takeovers should replacing the management generate lower synergies. Alike, the findings also indicate that if a bidder has a toehold in the target firm it is very likely that she wins the auction but at the same time increase the propensity for the target management to reject hostile bids from such bidders. All of these papers, however, have in common that uncertainty is neglected.

On the other hand, the literature on investment under uncertainty has acknowledged that bargaining and negotiation are pivotal pieces of M&As and has analyzed how uncertainty affects dynamic decision making of the party's involved. In particular, several papers have used the real options approach to advance the analysis of contract design under uncertainty (Lambrecht, 2004; Morellec and Zhdanov, 2005; Alvarez and Stenbacka, 2006; Lambrecht and Myers, 2007; Thijssen, 2008; Lukas and Welling, 2012). The results have provided answers with respect to how hostile takeover negotiation and merger negotiation, respectively, have an impact on takeover timing and sharing of the surplus under uncertainty. Exemplary, Morellec and Zhdanov (2005) and Lambrecht (2004) model a friendly merger of two firms where the timing and terms of takeovers are endogenous and result from value-maximizing decisions. The findings reveal, that M&As are usually timed in periods of economic expansion and that competition among heterogeneous firms speeds up the acquisition process. Alike, Lambrecht and Myers (2007) show that M&As are not solely triggered by positive economic shocks but can also be efficient when industries decline. Recent papers have furthermore stressed the importance of takeover type, i.e. friendly or hostile, on timing and wealth distribution. The results indicate that hostile takeovers occur inefficiently late when compared with the friendly merger as being the first-best. However, the bidder can claim a majority stake in the new entity due to its first-mover advantage and thus improve his bargaining position (See e.g. Lambrecht, 2004; Lukas and Welling, 2012). Yet, however, the choice between hostile takeovers and friendly merger and the impact credible threats as in the case of ThyssenKrupp have been neglected in this literature domain so far.

The goal of our paper is to bridge these two strands of literature, i.e. the real option view on M&A with the literature on market for corporate control. Hence, we build on the literature that hostile takeovers are an efficient means to replace target management and take advantage of upside potential. In particular, while we allow the takeover to generate synergies in general we add to this the possibility for the bidder to profit from additional synergy gains whenever the synergies due to replacing target's management are higher than conjoint control due to a friendly merger. In contrast to the beforementioned literature, however, we deviate from the assumption that hostile takeovers occur in the shadow of an auction. Neglecting competitive bidding, however, seems reasonable as empirical data indicates that only a few hostile takeovers suffer from bidding contests and the majority of hostile takeovers are single-bid contests (Betton et al., 2008). The bargaining process develops as follows: The raider has two options. It may either make a hostile bid for the target knowing that if the bid is not accepted subsequent friendly merger negotiations are still possible. On the other hand, the raider can enter into negotiating a friendly merger in the first round knowing that if negotiations fail subsequent hostile takeovers are still possible. Whether the parties become a target or a bidder in this subsequent hostile takeover game is derived endogenously.

The closest paper to ours is Thijssen (2008). Here, both the bidder and the target can make a bid for the other firm at any time. The two firms maximize expected profits and face different, but correlated, risk. A friendly merger occurs when both simultaneous bid for the other firm. In contrast, no simultaneous bids indicate a hostile takeover. There are two important results: First, if the roles the player take up, i.e. bidder or seller, are determined endogenously then the value to delay the M&A disappears due to the threat of preemption. This is in contrast to situations where the roles are assigned exogenously. Here, an incentive for the parties arise to delay the merger. Second, merger can be observed in both declining and expanding industries.

Our paper, however, differs in several ways. First, we endogenously derive an optimal synergy level which indicates when hostile takeovers are more profitable than friendly mergers for the raider. Consequently, both forms of M&A activity can occur in equilibrium. Moreover, we do not consider null threat values in friendly mergers. Rather, both firms can threaten the other party to either act as a bidder or target in a subsequent hostile takeover game. Our results indicate that friendly mergers are always first-best when there are little or no extra synergies achievable when replacing target's management in a hostile takeover. In such cases, however, the threat of a possible subsequent hostile takeover deters the sharing rule. In particular, the larger the weaker partner in a friendly merger

the more likely hostile takeovers become should friendly negotiation fail. This is due to the fact that by doing so, the weaker partner can escape the curse of being weak and threaten to accept a hostile bid. As a consequence, the stronger partner will concede a higher equity share in the new entity to the weaker one in order to circumvent a suboptimal late hostile takeover. Greater uncertainty, however, mitigates the effect of threat value and in the extreme, i.e. for very high levels of uncertainty, subsequent hostile takeovers will never occur in response to a failed merger. On the contrary, should the raider be able to achieve considerably high synergies when replacing the target's management then hostile takeovers can become first-best. However, these additional gains have to surpass a critical level in order to eliminate the risk that the hostile bid will be rejected and the final outcome becomes a friendly merger. Our findings reveal that this threshold is sensitive to size, cost differential and uncertainty. In particular, the larger (smaller) the raider (the poor performing target) the more likely hostile takeovers become should the industry exhibit poor performing targets. Interestingly, at the same time the risk of rejected hostile bids, increases, too. In addition, the more volatile an industry the more likely friendly merger become, i.e. the higher the gains from replacing target's management to justify hostile takeovers.

3 The Model

Consider two firms active in the market labeled as i and j. In the merging process two roles B and T can be assumed by the firms, where B stand for bidder and T for target. For the sake of simplicity, we will assume that each firm is endowed with a capital stock $K_{i,j}$ and subject to an industry wide shock modeled by means of a geometric Brownian motion, i.e.:

$$dx(t) = \alpha x(t)dt + \sigma x(t)dW \tag{1}$$

where $\alpha \in \mathbb{R}$ denotes the instantaneous drift, $\sigma \in \mathbb{R}^+$ denotes the instantaneous variance and dW denotes the standard Wiener increment. Under risk-neutrality we set $\alpha = r - \delta$, where r is the risk-free rate and δ is a return shortfall. Additionally, we will assume that:

$$V_{i,j}(t) = K_{i,j}x(t) \tag{2}$$

were $V_{i,j}(t)$ approximates the firms' individual stand-alone values.

Upon merging, the value of the new entity M is give by:

$$V_M(t) = (\omega + K_B + K_T)x(t) \tag{3}$$

where $\omega > 0$ denote synergies arising from the merger.

Our merger game unfolds as follows. At the outset, and before start negotiating the terms of the merger, each of the firms (i and j) assess in what position it would end up if the negotiations fail. This *pre-assessment* stage seems to be relevant in situations where the interest in the M&A process is not restricted to friendly mergers, and when firms can move to a hostile takeover if cooperation breaks. In other words, before starting the negotiation, each firm considers the alternative (hostile) option to merge, evaluating its own position in this scenario.

In technical terms, the alternative for the hostile takeover is an outside option, which will emerge in the case of disagreement. If credible, the alternative position of each firm acts as a threat value. These are known as disagreement points (the values each plays expect to obtain in the case of a break down of the negotiations) which should be considered for finding the terms of the friendly merger. In fact, the firms play a threat game, by choosing the role (hostile bidder or target) that maximizes their own position in the cooperative negotiation. The optimal roles to be played by each firm will depend, as we will see, on the their relative bargaining power. By considering, realistically, the existence of a hostile outside option for both parties, we depart from the existing dynamic M&A literature, where a null threat value is assumed (Thijssen, 2008).

Then, the players move to the second stage which refers to setting the cooperative game, accounting for the existence of the disagreement points. Given each firm's bargaining power the game is solved by means of the Nash bargaining solution.

3.1 Hostile takeover

In order to model the acquisition process, we will rely on a non-cooperative bargaining solution following Lukas and Welling (2012), where the bidder firm offers a premium $\psi > 0$ and the target times the acquisition. Let $\epsilon_B Y$ and $\epsilon_T Y = (1 - \epsilon_B) Y$ denote the transaction costs assigned to each party where $\epsilon_B \in (0, 1)$ indicates the fraction of the irreversible transaction costs (Y) assigned to the bidder.

Consequently, the target firm T receives a premium $\psi K_T x(t)$ in exchange for its asset worth $K_T x(t)$ and has to bear transaction cost of size $(1 - \epsilon_B)Y$. Following standard real option reasoning, for any given premium level, ψ , T's timing decision to sell the company solves the following optimization problem:

$$f(x) = \max_{\tau} \left[\mathbf{E} \left[\left((\psi - 1) K_T x(t) - (1 - \epsilon_B) Y \right) e^{-r\tau} \right] \right], \tag{4}$$

$$= \max_{x_{h}^{*}(\psi)} \left[\left((\psi - 1) K_{T} x_{h}^{*}(\psi) - (1 - \epsilon_{B}) Y \right) \left(\frac{x(t)}{x_{h}^{*}(\psi)} \right)^{\beta_{1}} \right]$$
(5)

where $\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$ is the positive root of the standard fundamental quadratic equation (see Dixit and Pindyck, 1994). On the other side, the bidder anticipates the reaction function of the target and grants an optimal premium such that it maximizes its objective function, i.e.:

$$\max_{\psi} \left[\left(\left(\left(\omega + K_B + K_T \right) - K_B - \psi K_T \right) x_h^*(\psi) - \epsilon_B Y \right) \left(\frac{x(t)}{x_h^*(\psi)} \right)^{\beta_1} \right] \tag{6}$$

Solving both objective functions recursively leads to the following result:

Proposition 1. The hostile takeover of firm j by firm i takes place, if firm j receives an optimal premium ψ^* and waits until x(t) hits the optimal trigger value $x(t) = x_h^*$ where ψ^* and x_h^* are given by:

$$\psi^* = 1 + \frac{(\beta_1 - 1)(1 - \epsilon_B)}{\beta_1 - \epsilon_B} \frac{\omega}{K_T}$$
(7)

$$x_h^*(\omega, Y, \epsilon_B) \equiv x_h^*(\psi^*) = \frac{\beta_1}{(\beta_1 - 1)^2} \frac{(\beta_1 - \epsilon_B)Y}{\omega}$$
(8)

Option values:

$$B(\omega, Y, \epsilon_B) x^{\beta_1} = \frac{(\beta_1 - \epsilon_B) Y}{(\beta_1 - 1)^2} \left(\frac{x}{x_h^*(\omega, Y, \epsilon_B)} \right)^{\beta_1}$$
(9)

$$T(\omega, Y, \epsilon_B) x^{\beta_1} = \frac{(1 - \epsilon_B) Y}{\beta_1 - 1} \left(\frac{x}{x_h^*(\omega, Y, \epsilon_B)} \right)^{\beta_1}$$
(10)

Proof. See Appendix.

A higher uncertainty (higher σ , lower β_1) induces the bidder to offer a lower premium and to wait for a higher level of the state variable x. If the merger produces more synergies, it will occur sooner with a higher premium (Corollary 1).

Corollary 1. The sensitivities of the optimal solution are as follows: $\partial \psi^* / \partial \sigma < 0$, $\partial x_h^* / \partial \sigma > 0$, $\partial \psi^* / \partial \omega > 0$, $\partial x_h^* / \partial \omega < 0$, $\partial \psi^* / \partial Y = 0$, $\partial x_h^* / \partial Y > 0$, $\partial \psi^* / \partial \epsilon_B < 0$, $\partial x_h^* / \partial \epsilon_B < 0$.

3.2 Friendly merger

Let us start by considering a friendly merger between firm 1 and 2. In particular, let us assume that after the merger, each firm holds an equity stake γ_i in the new firm. Each firm will give up his stand-alone value $V_i = K_i x(t)$ and receives upon paying the transaction cost $\epsilon_i Y$ a stake in the new venture thereby profiting from the synergies ω that arise out of the merger. Hence, firm *i*'s net gain becomes:

$$(\gamma_i(\omega + K_1 + K_2) - K_i)x(t) - \epsilon_i Y \qquad i \in \{1, 2\}$$
(11)

where $\omega + K_1 + K_2$ denotes the size of the merged firm.

Assuming that both firms possess a certain amount of bargaining power, η_1 for firm 1 and $\eta_2 = 1 - \eta_1$ for firm 2, then the optimal share each firm has in the new venture solves

the following optimization problem:

$$\max_{\gamma_{i}} \left[\left(\gamma_{i}(\omega + K_{1} + K_{2}) - K_{i})x(t) - \epsilon_{i}Y - A_{i}x^{\beta_{1}} \right)^{\eta_{i}} \\ \left(((1 - \gamma_{i})(\omega + K_{1} + K_{2}) - K_{j})x(t) - (1 - \epsilon_{i})Y - A_{j}x^{\beta_{1}} \right)^{1 - \eta_{i}} \right] \qquad i, j \in \{1, 2\}, i \neq j$$

$$(12)$$

The terms $A_i x^{\beta_1}$ and $A_j x^{\beta_1}$ represent, in generic terms, the disagreement points of *i* and *j* respectively. Notice that the constants A_i and A_j will reflect, as we will see, the concrete position each firm will assume by playing the threat game.

Solving the cooperative bargaining game by means of the Nash-Bargaining solution leads to the following proposition:

Proposition 2. Both firms will agree to merge, if x(t) hits the optimal timing threshold x_f^* from below:

$$x_f^*(\omega, Y) = \frac{\beta_1}{\beta_1 - 1} \frac{Y}{\omega}$$
(13)

Firm i's optimal stake $\gamma_i^*(x_f^*(\omega, Y))$ in the merger amounts to:

$$\gamma_i^* = \frac{K_i}{\omega + K_1 + K_2} + \left(\frac{(\beta_1 - 1)\epsilon_i + \eta_i}{\beta_1} + \theta_i\right) \frac{\omega}{\omega + K_1 + K_2} \tag{14}$$

where

$$\theta_{i} = \frac{\beta_{1} - 1}{\beta_{1}} \times \frac{(1 - \eta_{i})A_{i} - \eta_{i}A_{j}}{Y} x_{f}^{*}(\omega, Y)^{\beta_{1}}$$
(15)

Proof. See Appendix.

Now that we have derived the optimal policy for the two firms to merge, we can deduce firm i's ex-ante option value for the friendly merger, i.e.:

$$F_{i}(x,\omega,Y,\epsilon_{i}) = \begin{cases} \left(\left(\gamma_{i}^{*}(\omega+K_{1}+K_{2})-K_{i}\right)x_{f}^{*}(\omega,Y)-\epsilon_{i}Y\right) \left(\frac{x(t)}{x_{f}^{*}(\omega,Y)}\right)^{\beta_{1}} & x(t) < x_{f}^{*}(\omega,Y) \\ \left(\gamma_{i}^{*}(\omega+K_{1}+K_{2})-K_{i}\right)x(t)-\epsilon_{i}Y & x(t) \ge x_{f}^{*}(\omega,Y) \end{cases}$$

$$(16)$$

where $\gamma_i^* \equiv \gamma_i^*(x_f^*(\omega, Y)).$

The threat values do not impact the timing, but only the sharing rule (γ_i^*) and, therefore, the option value F_i . Choosing the highest option value is the same as choosing the highest share in the merged firm.

Since we are focusing on a cooperative game the optimal investment trigger equals the central planner's optimal investment threshold. The central planner's objective function equals:

$$G(x,\omega,Y) = \max_{\tau} \left[\mathbf{E} \left[\left(\omega x(t) - Y \right) e^{-r\tau} \right] \right]$$
$$= \max_{x_{f}^{*}(\omega,Y)} \left[\left(\omega x_{f}^{*}(\omega,Y) - Y \right) \left(\frac{x(t)}{x_{f}^{*}(\omega,Y)} \right)^{\beta_{1}} \right]$$
(17)

Proposition 3. The trigger for merging of the individual firms is the same as that of a central planner maximizing the overall payoff $\omega x(t) - Y$.

Proof. See Appendix.

The two firms bargain the share of a constant merger surplus or, equivalently, the share of the overall option to merge.

3.3 The threat game

Let us now move to the initial stage where the firms make a pre-assessment of their individual positions in the case of a negotiation failure. The firms play a threat game, by choosing the role (hostile bidder or target) that maximizes their own position in the cooperative negotiation, which naturally influences the outcome of the game.

Additionally, let our model be sufficiently flexible to allow that synergies in an hostile takeover could be different according to acquiring firm, and that the costs could be different depending on the type of deal. In particular, let as assume that, in the case of an hostile takeover, a firm (acting as a bidder) has the chance to fully replace the (poor performer) management team of the target by its our management team, extracting in this case a higher synergy, ω_h . In the case of a friendly merger, this fully replacement may not be possible, and so a lower synergy is produce, ω_f . Accordingly, we allow for $\omega_h > \omega_f$, where ω_h and ω_f are the synergies in the case of an hostile takeover of a poor performing target or a friendly merger, respectively. Notice that a poor performing firm can also make an hostile bid, but it will be unable to extract the same synergies as the good performing firm. For the sake of simplicity, let us assume that the synergies are same as the friendly synergies (ω_f). In addition, we also consider asymmetric merging costs. Here, we allow for higher costs in the case of an hostile takeover than in that of a friendly merger, i.e. $Y_h > Y_f$. This could be due to higher consultancy and legal costs or severance pays to compensate managers for their replacement.

Let us present the possible combinations of the roles (bidder and target) each firm can threat to assume.

Case 1: both bidders (null threat)

Consider the case where both firm threaten to assume the role of a hostile bidder if the negotiations breakdown. Naturally, the threat reveals not credible in this context and so a null disagreement point is assumed, as no serious outside option emerges in this context. Accordingly, under a null threat value we set $A_i = A_j = 0$. Substituting in equation 14, we obtain the following solutions for the optimal sharing rule and for option value in the continuation region:

$$\gamma_i^{BB} = \frac{K_i}{\omega_f + K_1 + K_2} + \frac{(\beta_1 - 1)\epsilon_i + \eta_i}{\beta_1} \times \frac{\omega_f}{\omega_f + K_1 + K_2}$$
(18)

$$F_i^{BB}(x) = \eta_i \frac{Y_f}{\beta_1 - 1} \left(\frac{x(t)}{x_f^*(\omega_f, Y_f)} \right)^{\beta_1}, \qquad x(t) < x_f^*(\omega_f, Y_f)$$
(19)

Case 2: both targets

If both plays threaten to assume the target position, both passively waiting for the hostile bidding from the other party, the disagreement points are as follows:

$$A_i = T(\omega_f, Y_h, 1 - \epsilon_i)$$
$$A_j = T(\omega_h, Y_h, \epsilon_i)$$

Firm j is the poor performing firm that when bidding in an hostile takeover of firm i generates a smaller synergy (ω_f) than when it a target a target of the biding firm $i (\omega_h)$. The following solutions arise for the sharing rule:

$$\gamma_i^{TT} = \frac{K_i}{\omega_f + K_1 + K_2} + \left(\frac{(\beta_1 - 1)\epsilon_i + \eta_i}{\beta_1} + \theta_i^{TT}\right) \frac{\omega_f}{\omega_f + K_1 + K_2} \tag{20}$$

where

$$\theta_i^{TT} = \frac{Y_h}{Y_f} \left[\frac{(1-\eta_i)\epsilon_i}{\beta_1} \left(\frac{\beta_1 - 1}{\beta_1 - (1-\epsilon_i)} \frac{Y_f}{Y_h} \right)^{\beta_1} - \frac{\eta_i(1-\epsilon_i)}{\beta_1} \left(\frac{\beta_1 - 1}{\beta_1 - \epsilon_i} \frac{Y_f}{Y_h} \frac{\omega_h}{\omega_f} \right)^{\beta_1} \right]$$
(21)

and for the option to merge:

$$F_i^{TT}(x) = \left[\frac{Y_h}{Y_f} \left((1 - \eta_i)\epsilon_i \left(\frac{\beta_1 - 1}{\beta_1 - (1 - \epsilon_i)} \frac{Y_f}{Y_h} \frac{\omega_h}{\omega_f} \right)^{\beta_1} - \eta_i (1 - \epsilon_i) \left(\frac{\beta_1 - 1}{\beta_1 - \epsilon_i} \frac{Y_f}{Y_h} \right)^{\beta_1} \right) + \eta_i \right] \\ \times \frac{Y_f}{\beta_1 - 1} \left(\frac{x(t)}{x_f^*(\omega_f, Y_f)} \right)^{\beta_1}, \quad x(t) < x_f^*(\omega_f, Y_f)$$
(22)

Case 3: i the bidder, and j the poor performing target

Consider that the outside options for firm i is to be a hostile bidder and for the poor performing firm j is to assume a target position. For this case, the threat values take the form:

$$A_i = B(\omega_h, Y_h, \epsilon_i)$$
$$A_j = T(\omega_h, Y_h, \epsilon_i)$$

and the sharing rule become:

$$\gamma_i^{BT} = \frac{K_i}{\omega_f + K_1 + K_2} + \left(\frac{(\beta_1 - 1)\epsilon_i + \eta_i}{\beta_1} + \theta_i^{BT}\right) \frac{\omega_f}{\omega_f + K_1 + K_2} \tag{23}$$

where

$$\theta_i^{BT} = \frac{Y_h}{Y_f} \left[\frac{(1-\eta_i)(\beta_1 - \epsilon_i) - \eta_i(\beta_1 - 1)(1-\epsilon_i)}{\beta_1(\beta_1 - 1)} \left(\frac{\beta_1 - 1}{\beta_1 - \epsilon_i} \frac{Y_f}{Y_h} \frac{\omega_h}{\omega_f} \right)^{\beta_1} \right]$$
(24)

and the options value is as follows:

$$F_i^{BT}(x) = \left[\frac{(1-\eta_i)(\beta_1-\epsilon_i) - \eta_i(\beta_1-1)(1-\epsilon_i)}{\beta_1-1} \frac{Y_h}{Y_f} \left(\frac{\beta_1-1}{\beta_1-\epsilon_i} \frac{Y_f}{Y_h} \frac{\omega_h}{\omega_f}\right)^{\beta_1} + \eta_i\right]$$
(25)
$$\times \frac{Y_f}{\beta_1-1} \left(\frac{x(t)}{x_f^*(\omega_f, Y_f)}\right)^{\beta_1}, \quad x(t) < x_f^*(\omega_f, Y_f)$$

Case 4: i the target, and i the poor performing bidder

Consider that the outside options the poor performing firm j is to be a hostile bidder and for firm j is to assume a target position. For this case, the threat values take the form:

$$A_i = T(\omega_f, Y_h, 1 - \epsilon_i)$$
$$A_j = B(\omega_f, Y_h, 1 - \epsilon_i)$$

and the sharing rule become:

$$\gamma_i^{TB} = \frac{K_i}{\omega_f + K_1 + K_2} + \left(\frac{(\beta_1 - 1)\epsilon_i + \eta_i}{\beta_1} + \theta_i^{TB}\right) \frac{\omega_f}{\omega_f + K_1 + K_2} \tag{26}$$

where

$$\theta_i^{TB} = \frac{Y_h}{Y_f} \left[\frac{(1-\eta_i)(\beta_1-1)\epsilon_i - \eta_i(\beta_1-(1-\epsilon_i))}{\beta_1(\beta_1-1)} \left(\frac{\beta_1-1}{\beta_1-(1-\epsilon_i)} \frac{Y_f}{Y_h} \frac{\omega_h}{\omega_f} \right)^{\beta_1} \right]$$
(27)

and the options value is as follows:

$$F_i^{TB}(x) = \left[\frac{(1-\eta_i)(\beta_1-1)\epsilon_i - \eta_i(\beta_1-(1-\epsilon_i))}{\beta_1-1}\frac{Y_h}{Y_f}\left(\frac{\beta_1-1}{\beta_1-(1-\epsilon_i)}\frac{Y_f}{Y_h}\frac{\omega_h}{\omega_f}\right)^{\beta_1} + \eta_i\right]$$
(28)

$$\times \frac{Y_f}{\beta_1-1}\left(\frac{x(t)}{x_f^*(\omega_f, Y_f)}\right)^{\beta_1}, \quad x(t) < x_f^*(\omega_f, Y_f)$$

Figure 1 shows the possible combinations for the roles of firms 1 and 2 (the poor performer), with respective equity stake.

Since $1 - \gamma_1^{BT} (= \gamma_2^{BT}) \ge 1 - \gamma_1^{TT} (= \gamma_2^{TT})$ and $\gamma_1^{BT} \ge \gamma_1^{TT}$ firm *i* always prefers to threat to be the hostile bidder when *j* opts for the hostile target threat. Therefore the

		Firm 2	
		bidder	target
Firm 1	bidder	$\left(\gamma_1^{BB}, 1-\gamma_1^{BB}\right)$	$\left(\gamma_1^{BT}, 1 - \gamma_1^{BT}\right)$
	target	$\left(\gamma_1^{TB}, 1 - \gamma_1^{TB}\right)$	$\left(\gamma_1^{TT}, 1 - \gamma_1^{TT}\right)$

Figure 1: The threat game

two firms will never use as threat values the hostile target values simultaneously.

When firm 2 opts for the bidder threat, Firm 1 prefers the bidder threat when $\gamma_1^{BB} \ge \gamma_1^{TB}$, which occurs when

$$\eta_1 \ge \eta_1^L = \frac{(\beta_1 - 1)\epsilon_1}{\beta_1 - 1 + \beta_1\epsilon_1} \tag{29}$$

If firm 2 has not enough bargaining power, it will opt for the target threat when $1 - \gamma_1^{BT} \ge 1 - \gamma_1^{BB}$, which occurs when

$$\eta_1 \ge \eta_1^H = \frac{\beta_1 - \epsilon_1}{\beta_1 - 1 + \beta_1(1 - \epsilon_1)}$$
(30)

Proposition 4. Nash equilibrium in pure strategies is the following:

- (i) Firm 1 chooses target and firm 2 chooses bidder if the bargaining power of firm 1 is small $(0 \leq \eta_1 < \eta_1^L);$
- (ii) Firm 1 choose bidder and firm 2 chooses target if the bargaining power of firm 2 is small $(\eta_1^H < \eta_1 \leq 1);$
- (iii) For intermediate bargaining powers the two firms want to threaten being hostile bidders, and end up with zero threat value (non-credible threat) $(\eta_1^L \leq \eta_1 < \eta_1^H)$.

3.4 When do hostile bids take place?

Let us consider the situation where one firm is in a position of obtaining a higher synergy if acquiring the other firm by means of takeover. Typically, as we said before, this could be the case where one firm has a management team that has a relatively better performance than the other (which will be substituted after the takeover). Suppose that one firm can extract ω_h from the hostile takeover, but only ω_f if a friendly merger takes place. This firm will tend to move sooner towards the other one by means of an hostile takeover, if the trigger of the hostile bid reveals to be smaller than of a friendly merger, after considering the higher transactions costs, $x_h^*(\omega_h, Y_h) < x_f^*(\omega_f, Y_f)$.

The condition for an hostile takeover to occur before a friendly merger can be simply derived from Equations (8) and (13):

$$\omega_h > \omega_b = \frac{\beta_1 - \epsilon_i}{\beta_1 - 1} \omega_f \left(> \omega_f \right) \tag{31}$$

When x hits $x_h^*(\omega_h, Y_h)$, the bidder firm offers the optimal premium $\psi^*(\omega_h)$ (see Equation (7)) to the other firm which in turn will decide on its own best interest. In fact, two decisions can be taken by the target firm: either accepts the offer or rejects it. By accepting the bid, the target benefits from the premium paid by the bidder as a result of a higher synergy $\omega_h > \omega_f$. If the hostile bid is refused, the target, and the bidder, have the chance to merge at later stage (i.e., at $x_f^*(\omega_f, Y_f)$).

By deciding based on value, the target firm will only accept the bid if the intrinsic value of the hostile takeover reveals to be larger than the value of a friendly merger, i.e. if $T(\omega_h, Y_h, 1 - \epsilon_j) > F_j^{BB}$.

The condition for the takeover to be accepted by the target is as follows:

$$\omega_h > \omega_a = \frac{\beta_1 - \epsilon_i}{\beta_1 - 1} \left(\frac{1 - \eta_i}{1 - \epsilon_i}\right)^{1/\beta_1} \left(\frac{Y_h}{Y_f}\right)^{(\beta_1 - 1)/\beta_1} \omega_f \tag{32}$$

Notice that if ω_h is not sufficiently large to produce enough premium to be accepted by the target (i.e., higher than ω_a), the only possibility for the bidder firm is to wait and agree on a friendly merger based poor synergy ω_f , which will happen in a later moment. For modeling purposes, we assume that there are no threat values in the case of this subsequent friendly merger, as the hostile takeover threshold has already been passed.



Figure 2: Sensitivity of the threat values to β_1 and ϵ_i

4 Comparative statics

In this section we present a comparative statics of the main drivers of merger and acquisitions timing and terms, starting with friendly mergers and showing next when hostile takeovers become optimal.

4.1 Friendly mergers

From Equations (29) and (30), the choice of the optimal strategy for the threat game is only determined by the level of uncertainty and the merger costs incurred by each firm.

Corollary 2. A higher the uncertainty increases the bargaining power wedge over which firms prefer to chose the bidder threat value $(\partial \eta_i^L/\partial \beta_1 > 0 \text{ and } \partial \eta_i^H/\partial \beta_1 < 0)$. The choice of the target (bidder) threat is more likely for firms with a low (high) bargaining power the lower the uncertainty is.

Corollary 3. The choice of the target (bidder) threat is more (less) likely for firms with a low (high) bargaining power when the fraction of the merger costs incurred increases $(\partial \eta_i^L / \partial \epsilon_i > 0 \text{ and } \partial \eta_i^H / \partial \epsilon_i > 0).$

Figure 2 illustrates these effects. Notice that size can impact the strategy choice if the fraction of the merger costs paid by each firm is not independent of size.



Figure 3: Sensitivity of the merger terms to β_1

Corollary 4. A higher uncertainty deters mergers $(\partial x_f^*/\partial \sigma > 0)$ and induces a higher (lower) share for the firm with a bargaining power higher (lower) than its fraction in the merger costs $(\partial \gamma_i/\partial \sigma > 0, if \eta_i > \epsilon_i)$. If the bargaining power is proportional to the merger costs, uncertainty has no effect on the merger terms.

Figure 3 illustrates the effect of uncertainty on the merger terms, stated in Corollary 4. The Figure plots three cases: a base-case where $n_1 = \epsilon_i$ and the limiting cases of a full ($\eta_i = 1$) and a null ($\eta_i = 0$) bargaining power. The Figure shows that the effect of uncertainty on the merger terms is more pronounced for high levels of uncertainty (low β_1).

The bargaining power of each firm in the merger negotiation determines the merger terms. Our model suggests that a higher bargaining power allows a firm to capture a higher share of the merged firm, but also increases the negotiation power of those firms that can credible commit to a bidder role in a subsequent hostile takeover.

Corollary 5. The bargaining power of firms has no effect on the timing of mergers $\left(\partial x_f^*/\partial \eta_i = 0\right)$ and has an ambiguous effect on the merger terms $\left(\partial \gamma_i/\partial \eta_i \leq 0\right)$. This effect is augmented both by the level of synergies and uncertainty.

Figure 4 shows the effect of the bargaining power on the merger terms for different



Figure 4: Sensitivity of the merger terms to η_i

levels of uncertainty (Figure 4(a)) and synergies (Figure 4(b)). Figure 4(a) illustrates the effect of uncertainty also shown in Figure 3, but additionally suggests that the effect of the bargaining power on the merger terms is smaller and can be negative when one of the firms can credibly threat with an hostile takeover (both for low and high values of η_i). The target firm benefits from the threat of accepting an hostile takeover, which increases its stake the friendly merger. The bidder agrees to concede some of its bargaining power to escape from a sub-optimally late hostile takeover. The lower the uncertainty, the less pronounced is the effect of the bargaining power on the merger terms up to limit of not having any effect ($\beta_1 \rightarrow \infty$). Another interesting result suggested by this Figure is that for high levels of uncertainty ($\beta_1 \rightarrow 1$) the effect of the bargaining power is maximum and the threat of hostile takeovers is not credible. Figure 4(a) the effect of the bargaining power is smaller for a low or a high bargaining power.

Corollary 6. Higher merger synergies hasten mergers $(\partial x_f^*/\partial \omega_f < 0)$. When the bargaining power of a firm is higher (lower) than its proportional size $(\eta_i > (<)K_i/(K_i + K_j))$ higher merger synergies allow the firm to capture a higher (lower) share of the merger surplus $(\partial \gamma_i/\partial \omega_f > (<)0)$. If the bargaining power is proportional to the firm sizes, synergies have no effect on the merger terms.

Corollary 7. The synergies of the hostile takeover have no effect on the timing of friendly



Figure 5: Sensitivity of the merger terms to ω

mergers $(\partial x_f^*/\partial \omega_h = 0)$. When the bargaining power of the best performing firm is higher (lower) than the critical value that allows it to use the hostile takeover threat $(\eta_i > (<)\eta_i^H)$, higher takeover synergies reduce (have no effect) the share of the merger surplus it captures $(\partial \gamma_i/\partial \omega_h < (=)0)$. If the bargaining power is proportional to the firm sizes, synergies have no effect on the merger terms.

The effect of synergies stated in corollaries 6 and 7 are shown in Figure 5.

Corollary 8. Holding the bargaining power constant, bigger firms capture higher shares in mergers $(\partial \gamma_i / \partial K_i > 0)$.

Figure 6 illustrates the effect of firm size on the merger terms.

Corollary 9. Holding the bargaining power constant, firms incurring in higher relative costs capture higher shares in mergers $(\partial \gamma_i / \partial \epsilon_i > 0)$.

Figure 7 illustrates the effect of the merger costs on the merger terms. The effect is augmented both by uncertainty (Figure 7(a)) and synergies (Figure 7(b)).

4.2 Hostile takeovers vs friendly mergers

A bidder will announce an hostile takeover when condition (31) holds. The target will reject the bid if it is better off in a friendly merger (condition (32) does not hold).



Figure 7: Sensitivity of the merger terms to ϵ_i

 $K_i = 1, K_j = 1, Y_f = Y_h = 0.1, \epsilon_i = K_i/(K_i + K_j) = 0.5, \eta_i = 0.4, \beta_1 = 2.$ TA = Hostile takeover accepted; TR = Hostile takeover rejected (followed by a friendly merger); M = Friendly merger.

Figure 8: Optimal M&A strategy

The hostile synergies determine the likelihood of occurring an hostile takeover versus a friendly merger, and also the likelihood of the target response. Figure 8 illustrates what are the level of the takeover synergies that prompt an hostile bid and how the target responds. In the dark gray area the hostile takeover is accepted because the target benefits from a premium induced by higher bidder takeover synergies ($\omega_h > \omega_b > \omega_f$), even considering possibly higher takeover costs, when compared to the less synergistic friendly merger, albeit possibly less expensive. In the light gray area, the target rejects the offer because it prefers to negotiate a merger considering its bargaining power. In the white area the takeover synergies are not sufficient to make the hostile takeover optimal prior to a friendly merger.

From conditions (31) and (32), the following corollary holds:

Corollary 10. Hostile bids require higher synergies when uncertainty is higher $(\partial \omega_b / \partial \beta_1 <$

 $K_i = 1, K_j = 1, \omega = 0.1, Y_f = Y_h = 0.1, \epsilon_i = K_i/(K_i + K_j) = 0.5, \eta_i = 0.4.$ TA = Hostile takeover accepted; TR = Hostile takeover rejected (followed by a friendly merger); M = Friendly merger.

Figure 9: Optimal M&A strategy: sensitivity to uncertainty

 $0, \partial \omega_a / \partial \beta_1 < 0).$

Figure 9 illustrates this corollary. Because the level of synergies required is higher, firms operating in more volatile industries or periods will tend to enter less in hostile takeovers (preferring friendly mergers) and their hostile bids will be less successful.

Corollary 11. Hostile bids require higher synergies when the fraction of the merger costs incurred by the bidder is lower $(\partial \omega_b / \partial \epsilon_i < 0)$, but they will be less successful as the costs incurred increase $(\partial \omega_a / \partial \epsilon_i > 0)$.

If the fraction of the costs incurred by each firm is related to firm sizes, this corollary implies that bigger firms tend to make more often hostile offers, but the probability of success decreases with the bidder size. Figure 10 illustrates this result.

Corollary 12. A higher bargaining power (in a friendly merger) increases the probability

 $K_i = 1, K_j = 1, \omega = 0.1, Y_f = Y_h = 0.1, \epsilon_i = K_i/(K_i + K_j), \eta_i = 0.4.$ TA = Hostile takeover accepted; TR = Hostile takeover rejected (followed by a friendly merger); M = Friendly merger.

 $K_i = 1, K_j = 1, \omega = 0.1, Y_f = Y_h = 0.1, \epsilon_i = K_i/(K_i + K_j) = 0.5, \beta_1 = 2.$ TA = Hostile takeover accepted; TR = Hostile takeover rejected (followed by a friendly merger); M = Friendly merger.

Figure 11: Optimal M&A strategy: sensitivity to the bargaining power

of success of an hostile takeover $(\partial \omega_a / \partial \eta_i < 0)$, but does not impact the likelihood of an hostile bid $(\partial \omega_b / \partial \eta_i = 0)$.

Figure 11 illustrates this result.

5 Conclusions

Given the increasing prominence of M&A deals in today's global economy, their increasing valuation levels, and their strategic importance for firms? competitiveness it is surprising how little about their trends and in particular their dynamics has been explored in depth. The paper at hand builds on recent advances in the domain of option games under uncertainty and looks closer at determinants that drive the choice between friendly mergers and hostile takeovers. In particular, the model considers two firms that independently commit themselves to grow by means of M&A. Given that the outcome is uncertain, each firm calculates its payoff resulting from either a friendly merger or hostile takeover thereby taking into account that a friendly merger can be followed by a hostile takeover and vice a versa. Consequently, the paper advances recent literature by explicitly considering both takeover strategies simultaneously and accounts for the associated threat values during negotiation. Our results indicate that friendly mergers are always first-best when there are little or no extra synergies achievable when replacing target's management in a hostile takeover. However, the threat of a possible subsequent hostile takeover deters the sharing rule significantly and weaker firms can improve its position the larger they are. Alike, the importance of threat values is higher the lower the industry's uncertainty becomes. Similar to e.g. Berkovitch and Khanna (1991), we determine an optimal synergy level that motivate raiders to refrain from attempting to merge friendly should replacing the management generate higher synergies. Consequently, in the presence of poor performing targets, hostile takeovers are more likely to be observed when the raider is bigger and the M&A activity is clustered in less volatile industries.

Of course, our paper is not without limitations. Exemplary, we do not consider the risk, that a merger is not approved by governmental authorities. In addition, the hostile bid does not induce competitive bids by other firms nor do we explicitly model the shareholdermanager conflict that could arise in such a setting. Hence, these aspects represent fruitful avenues that might motivate further research in this domain.

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